Title: Causality and Ideal Measurements of Smeared Fields in Quantum Field Theory

Speakers: Ian Jubb

Series: Quantum Foundations

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Abstract: The usual quantum mechanical description of measurements, unitary kicks, and other local operations has the potential to produce pathological causality violations in the relativistic setting of quantum field theory (QFT). While there are some operations that do not violate causality, those that do cannot be physically realisable. For local observables in QFT it is an open question whether the projection postulate, or more specifically the associated ideal measurement operation, is consistent with causality, and hence whether it is physically realisable in principle.

In this talk I will recap a criteria that distinguishes causal and acausal operations in real scalar QFT. I will then focus on operations constructed from smeared field operators - the basic local observables of the theory. For this simple class of operations we can write down a more practical causality criteria. With this we find that, under certain assumptions - such as there being a continuum spacetime - ideal measurements of smeared fields are acausal, despite prior heuristic arguments to the contrary. For a discrete spacetime (e.g. a causal set), however, one can evade this result in a 'natural' way, and thus uphold causality while retaining the projection postulate.

Zoom link: https://pitp.zoom.us/j/94464896161?pwd=UkhPQnJONmlxYy9pQXJINThpY3l4QT09

Causality and Ideal Measurements in Quantum Field Theory

Ian Jubb
Perimeter Institute - 16/11/2022

Borsten, Kells, IJ, *arxiv:1912.06141* , IJ, *arxiv:2106.09027*

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Introduction

- **Textbook measurements in QFT in terms of scattering amplitudes, which are approximations. If we ask to describe multiple measurements in finite spacetime regions we get causality issues...**
- **Two aspects of measurements to note:**
  - i How to compute probabilities of outcomes, expectation values, variances, etc.
  - ii How to encode the effect of a measurement on the statistics of future measurements: ***update map***. Only needed if we have multiple measurements. *Example*: ideal measurement map from projection postulate.
- ***What is the space of causally consistent update maps in QFT?*** **Such maps are physically realisable in principle.**
- **Other examples of maps include those coming from tracing out probes of main field of interest, e.g. UDW, or probe quantum fields.**
  - Probes measured once (traced out), so no need for update map for probe statistics.

Introduction

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  i. How to compute probabilities of outcomes, expectation values, variances, etc.
  ii. How to encode the effect of a measurement on the statistics of future measurements: ***update map***. Only needed if we have multiple measurements. *Example*: ideal measurement map from projection postulate.
- ***What is the space of causally consistent update maps in QFT?*** **Such maps are physically realisable in principle.** ***Is there a principle analogous to the projection postulate in QFT?***
- **Other examples of maps include those coming from tracing out probes of main field of interest, e.g. UDW, or probe quantum fields.**
  - Probes measured once (traced out), so no need for update map for probe statistics.
  - Multiple measurements of main field achieved with multiple probes, one measurement per probe.
  - Probes are effective systems.
  - Still need update maps at a more fundamental level.

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Plan

- Recap
  - QFT
  - Local Operations
  - Sorkin's Scenario
  - Locality and Causality
- Ideal Measurements
  - Background
  - Kraus operations and Causality
  - Ideal Measurements of Smeared Fields
- Summary

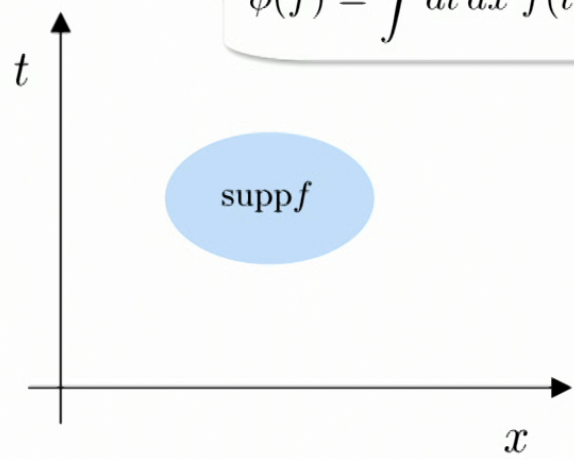
Quantum Field Theory

Recap

Quantum Field Theory

- Real scalar field theory in some globally hyperbolic spacetime
- Generate algebra $\mathfrak{A}$ with identity and smeared field operators, $\phi(f)$, for test functions f

$$\phi(f) = \int dt\, dx\; f(t,x)\,\phi(t,x)$$



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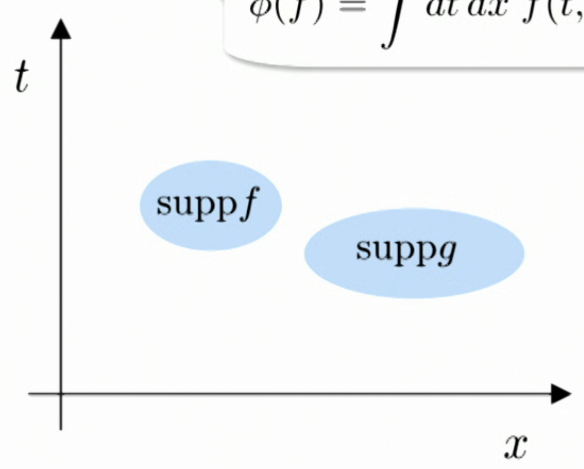
- e.g. $\phi(f)\phi(g) + 2i\phi(h)^3 - 4I$, $e^{i\phi(f)}$
- Commutation relations:

$$[\phi(f), \phi(g)] = i\Delta(f,g)$$



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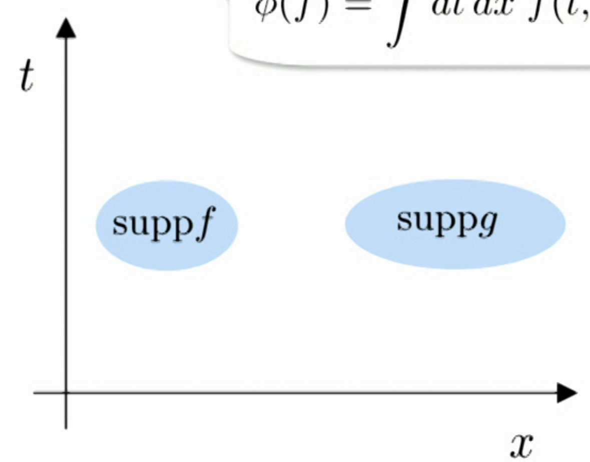
Spacelike commutativity (Einstein Causality)



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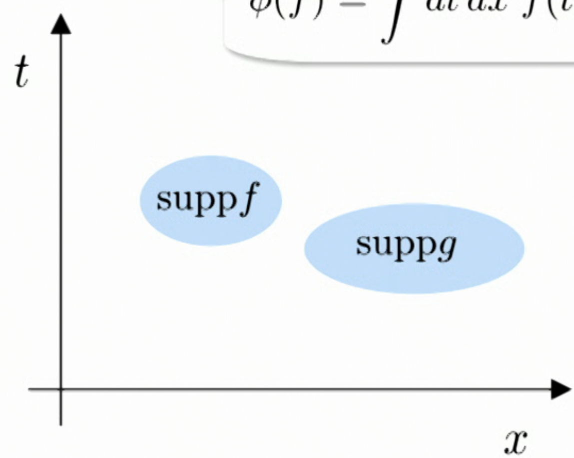
- 2-point function

$$W(f,g) = \langle\Omega|\phi(f)\phi(g)|\Omega\rangle$$



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Local Operations

Borsten et al, *arxiv:1912.06141*
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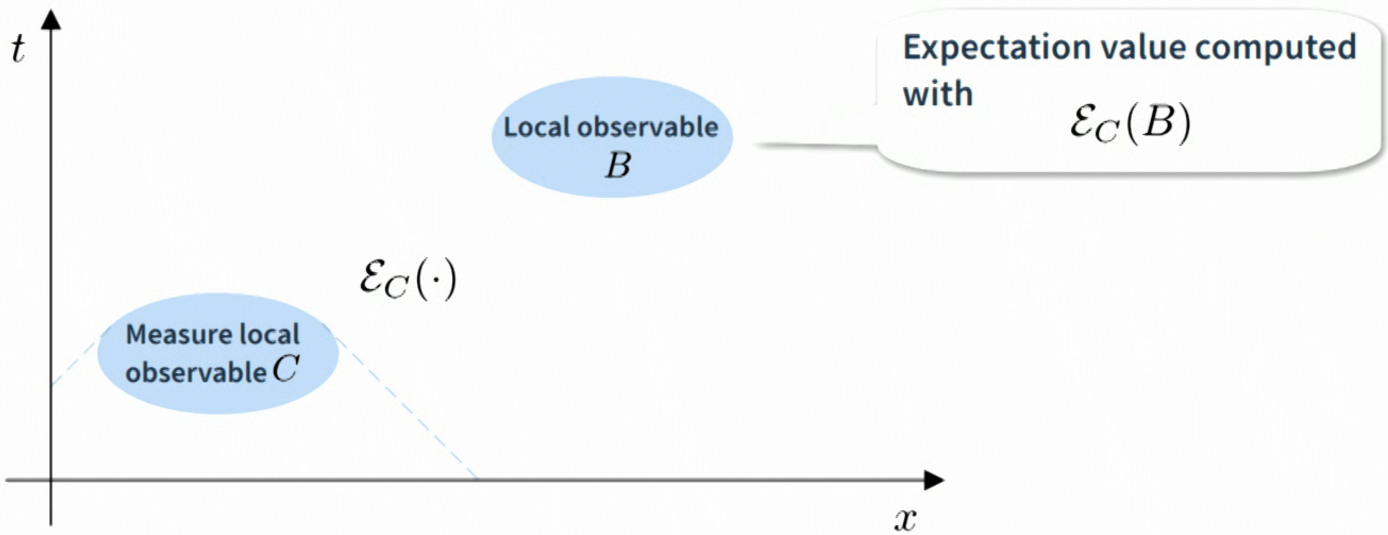
- Operation described by completely positive, unit preserving map $\mathcal{E}_C : \mathfrak{A} \to \mathfrak{A} \quad , \quad \mathcal{E}_C(1) = 1$



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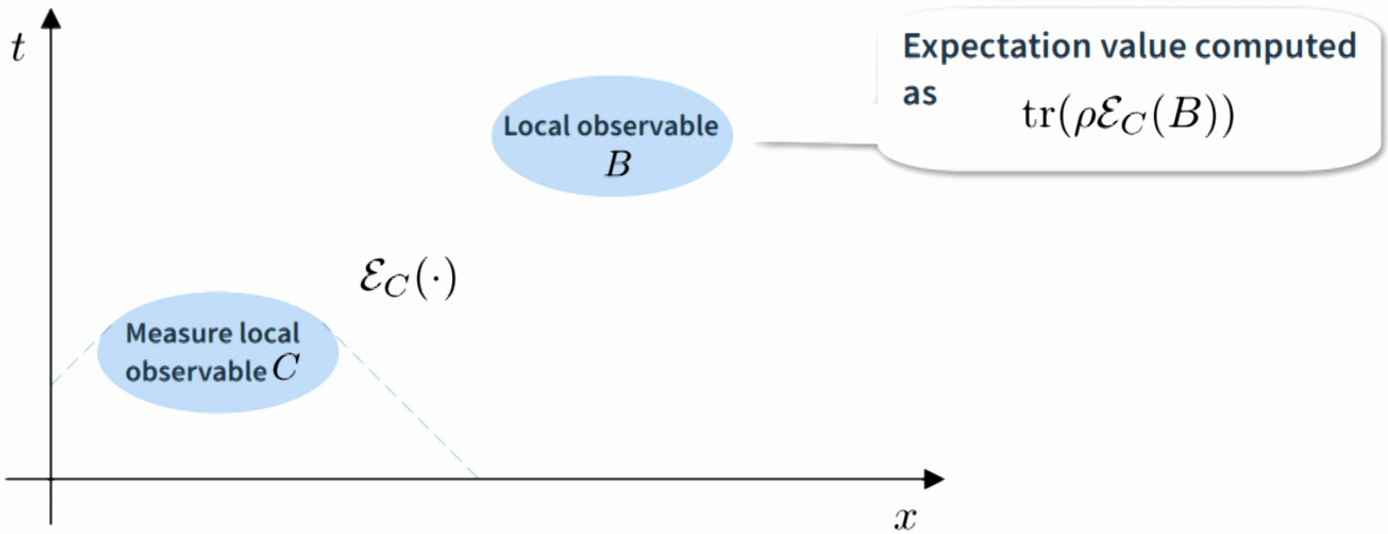

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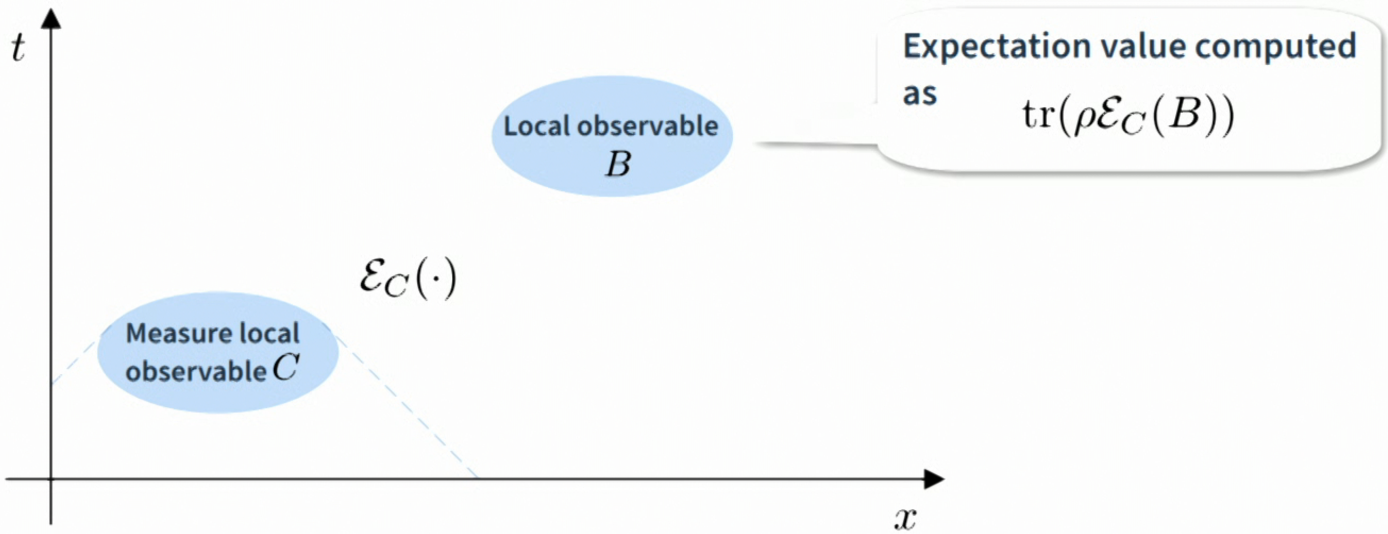
- **Operation described by completely positive, unit preserving map** $\mathcal{E}_C : \mathfrak{A} \to \mathfrak{A} \quad , \quad \mathcal{E}_C(1) = 1$
- **e.g. Ideal/projective measurement:** $C = \sum_n \lambda_n P_n \quad , \quad B \mapsto \mathcal{E}_C(B) = \sum_n P_n B P_n$



Local Operations

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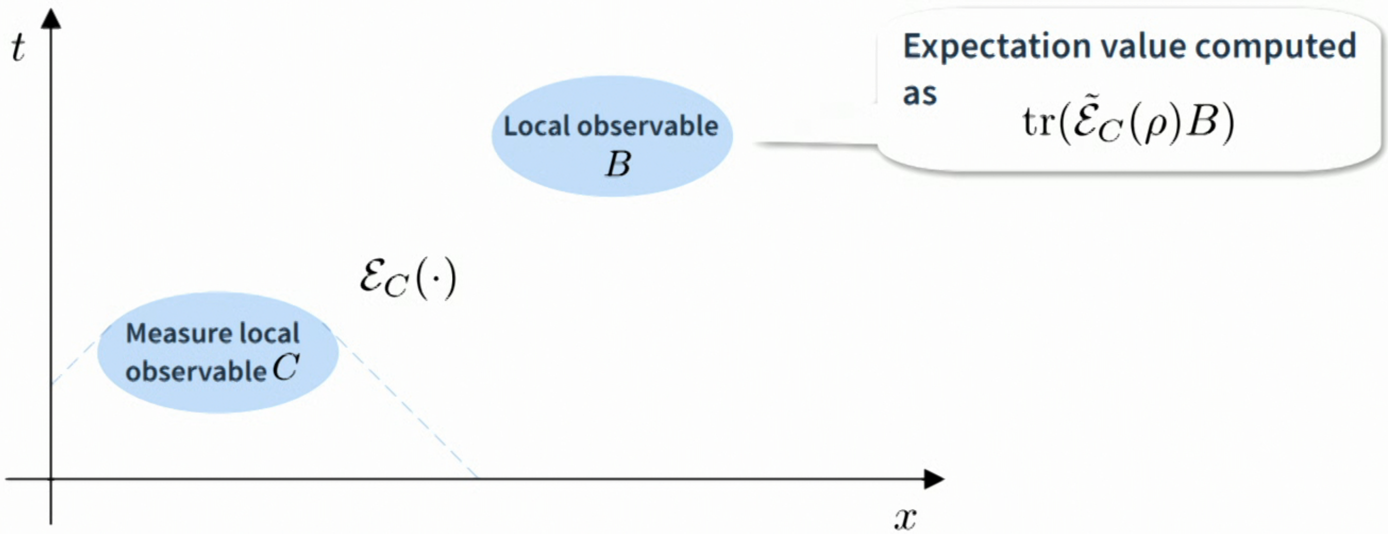
- Operation described by completely positive, trace preserving map $\tilde{\mathcal{E}}_C \quad , \quad \mathrm{tr}(\tilde{\mathcal{E}}_C(\rho)) = 1$
- e.g. Ideal/projective measurement: $C = \sum_n \lambda_n P_n \quad , \quad \rho \mapsto \tilde{\mathcal{E}}_C(\rho) = \sum_n P_n \rho P_n$

Dual picture



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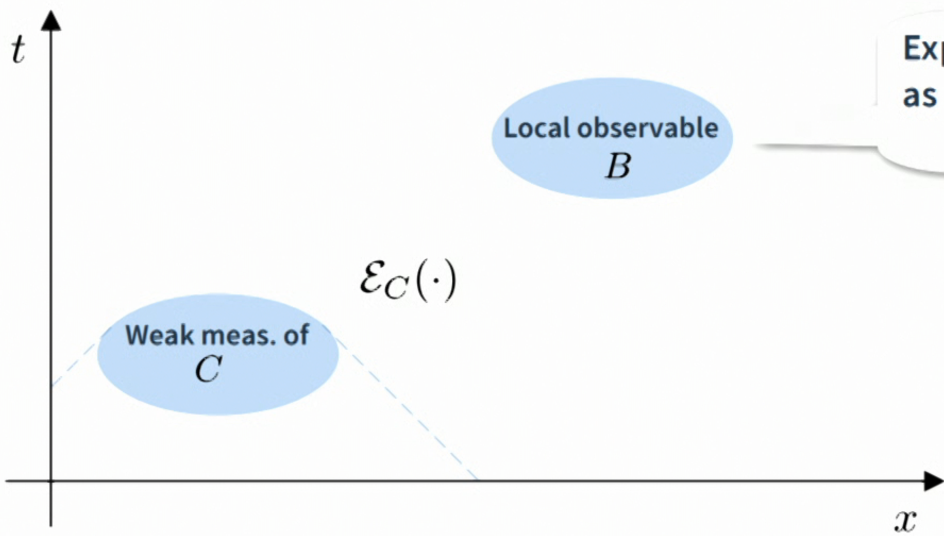
- e.g. weak measurement: $B \mapsto \mathcal{E}_C(B) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} d\alpha \, e^{-\frac{(C-\alpha)^2}{4\sigma^2}} B e^{-\frac{(C-\alpha)^2}{4\sigma^2}}$



Expectation value computed as $\mathrm{tr}(\rho \mathcal{E}_C(B))$

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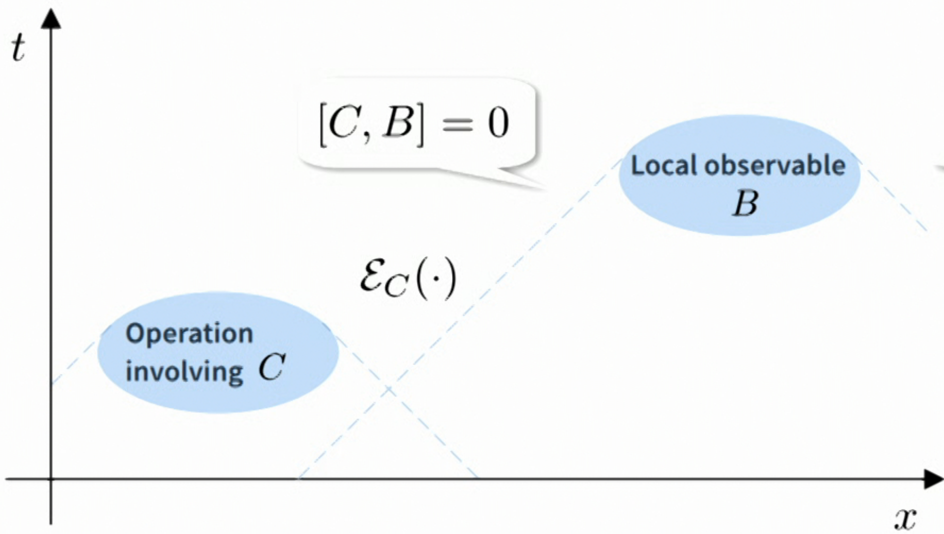

Commutativity of spacelike observables ensures the ***locality*** of the map, i.e.

$$\mathcal{E}_C(B) = B$$

Physically, expectation values are unchanged

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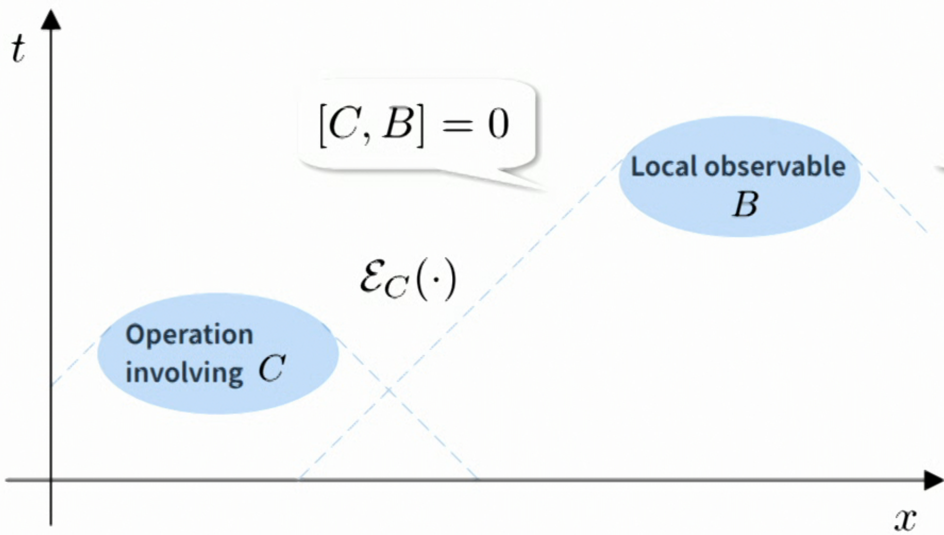

Commutativity of spacelike observables ensures the ***locality*** of the map, i.e.

$$\mathrm{tr}(\rho \mathcal{E}_C(B)) = \mathrm{tr}(\rho B)$$

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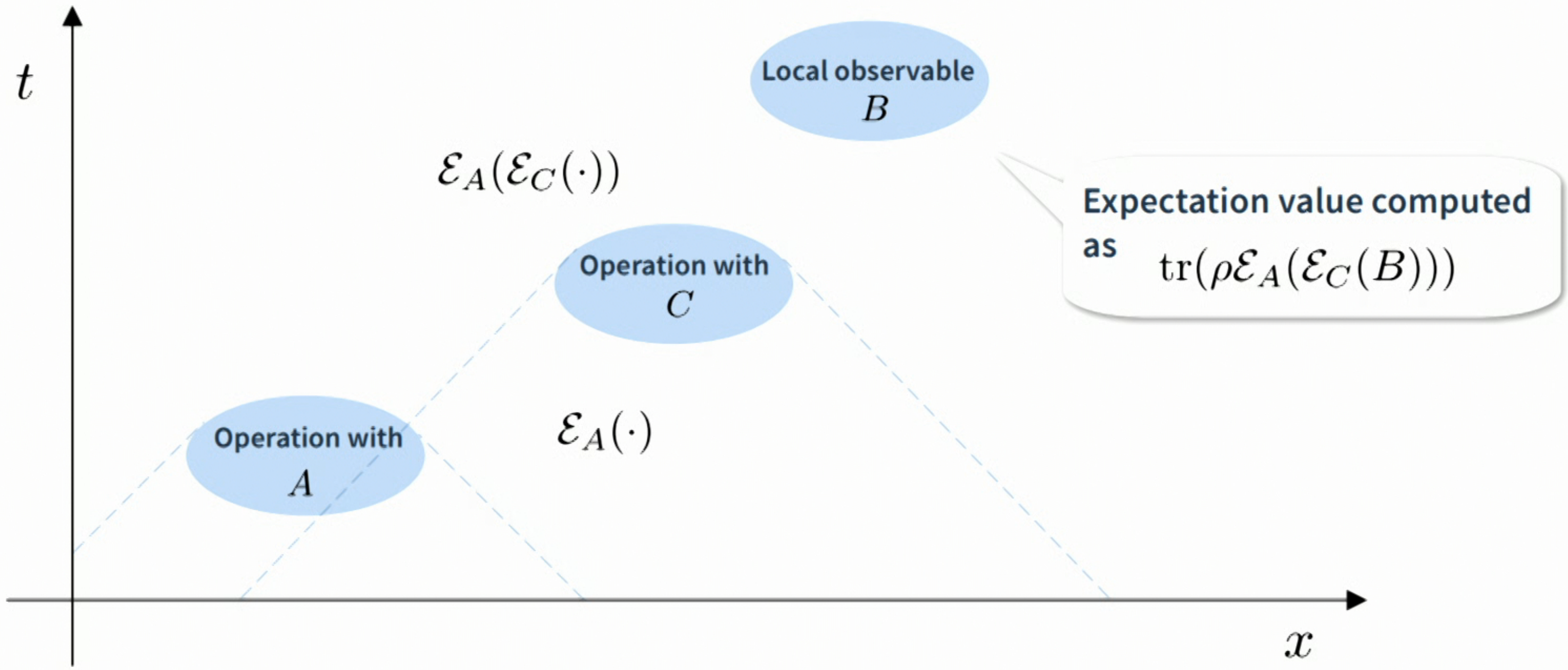
Multiple Local Operations

Hellwig, Kraus, *Phys. Rev. D 1*, 566

- Multiple local operations:



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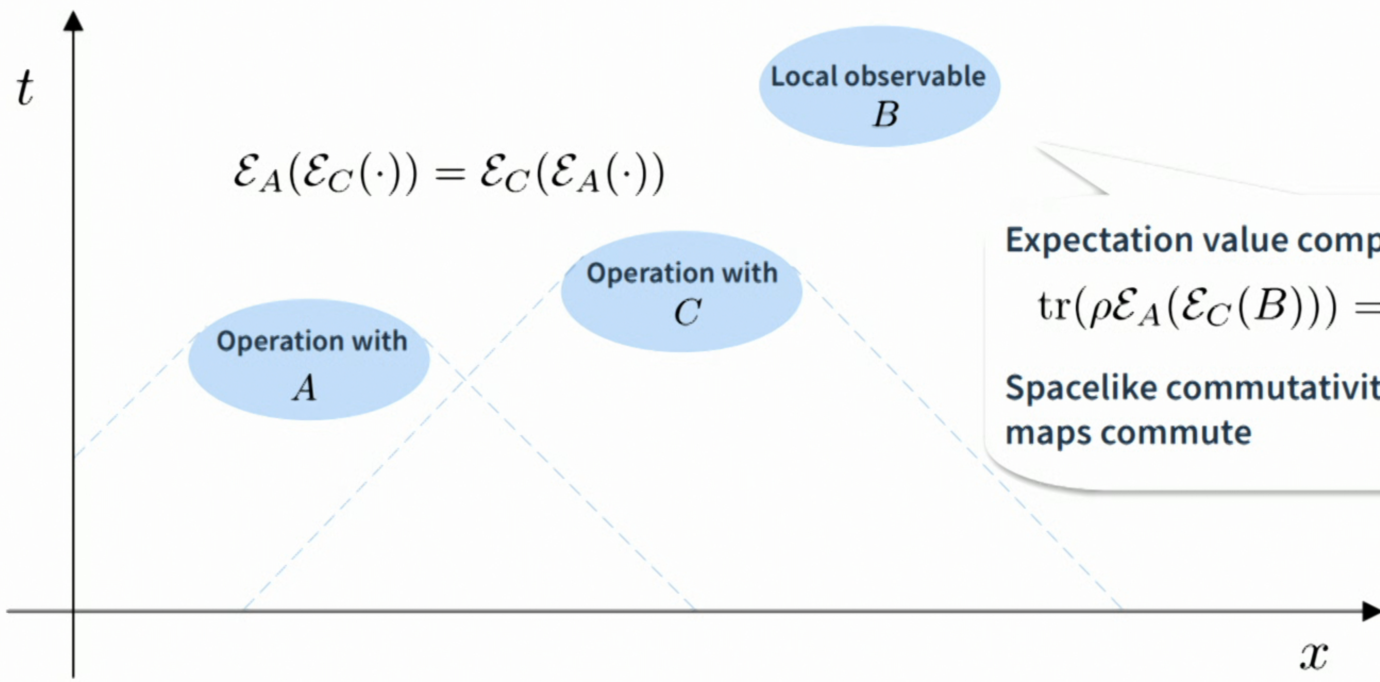

$$\mathcal{E}_A(\mathcal{E}_C(\cdot)) = \mathcal{E}_C(\mathcal{E}_A(\cdot))$$

Expectation value computed as

$$\mathrm{tr}(\rho\mathcal{E}_A(\mathcal{E}_C(B))) = \mathrm{tr}(\rho\mathcal{E}_C(\mathcal{E}_A(B)))$$

Spacelike commutativity ensures update maps commute

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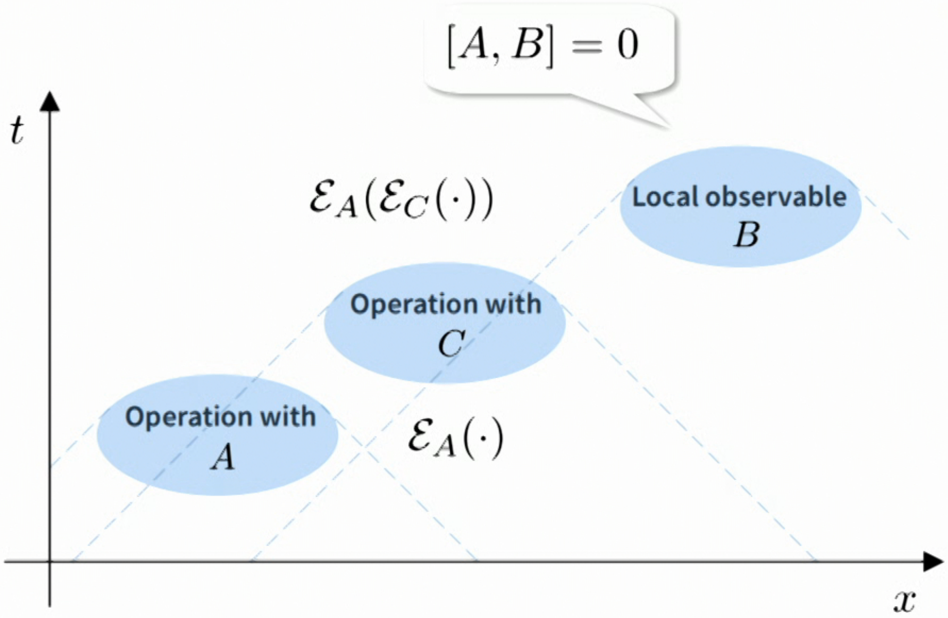
Sorkin's Scenario

Recap

Sorkin's Scenario

Sorkin *arxiv:9302018*

- Consider 3 agents, Aoife, Caoimhe, and Beolagh, acting in their respective regions:



- Composition rule says this expectation value given by

$$\mathrm{tr}(\rho\,\mathcal{E}_A(\mathcal{E}_C(B)))$$

- Physically, exp. val.'s shouldn't depend on spacelike operations, so should have

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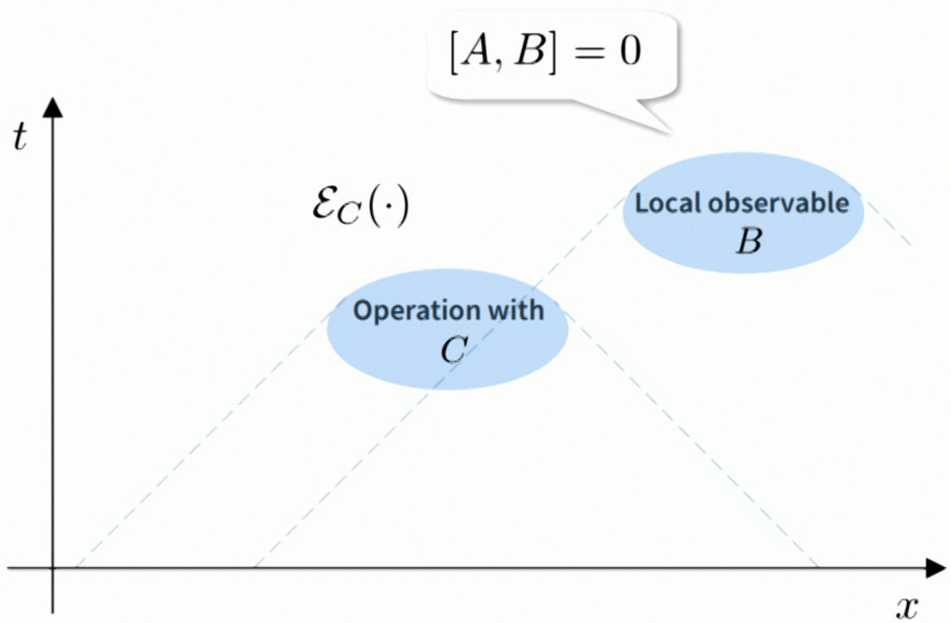

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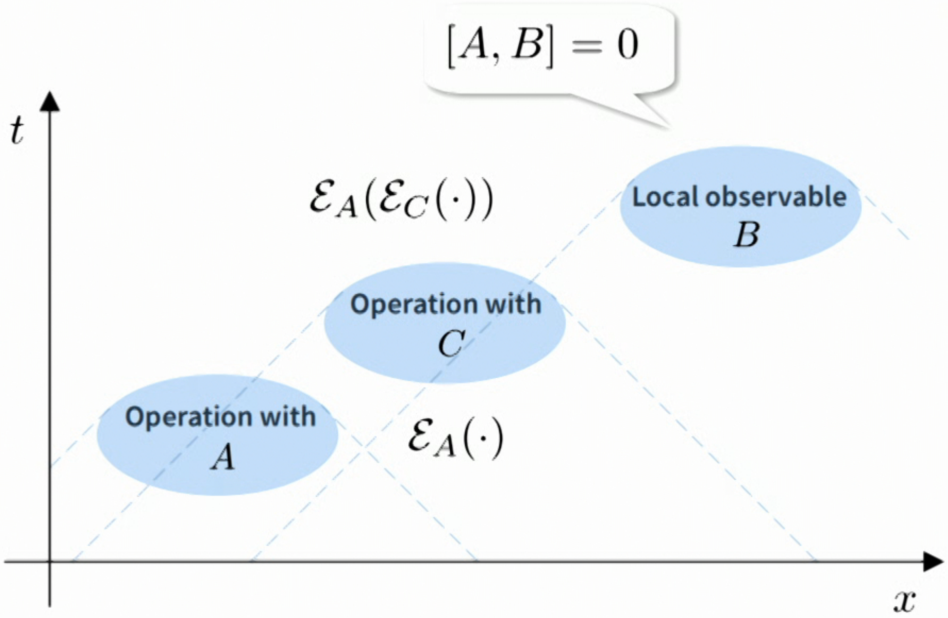

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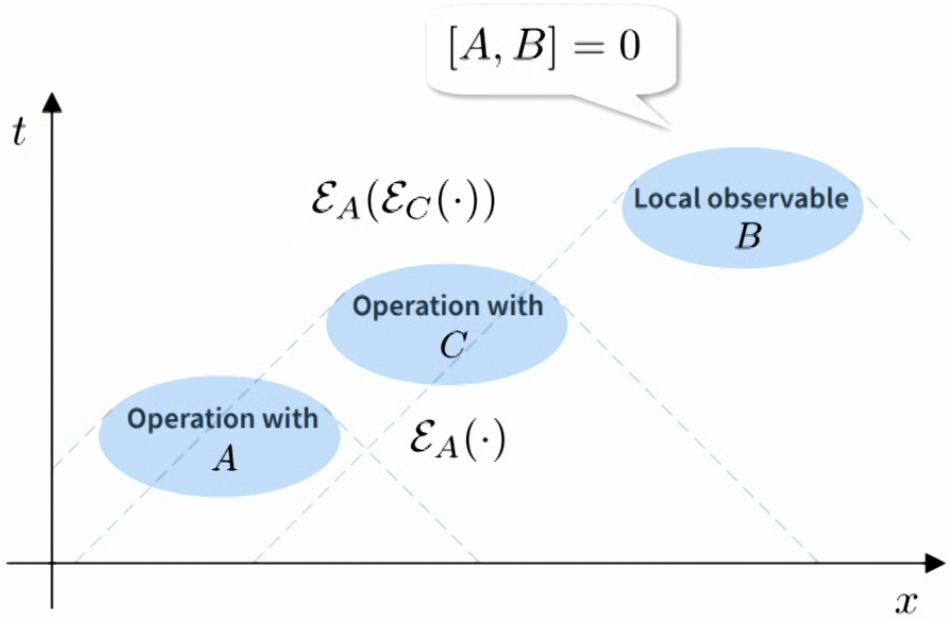

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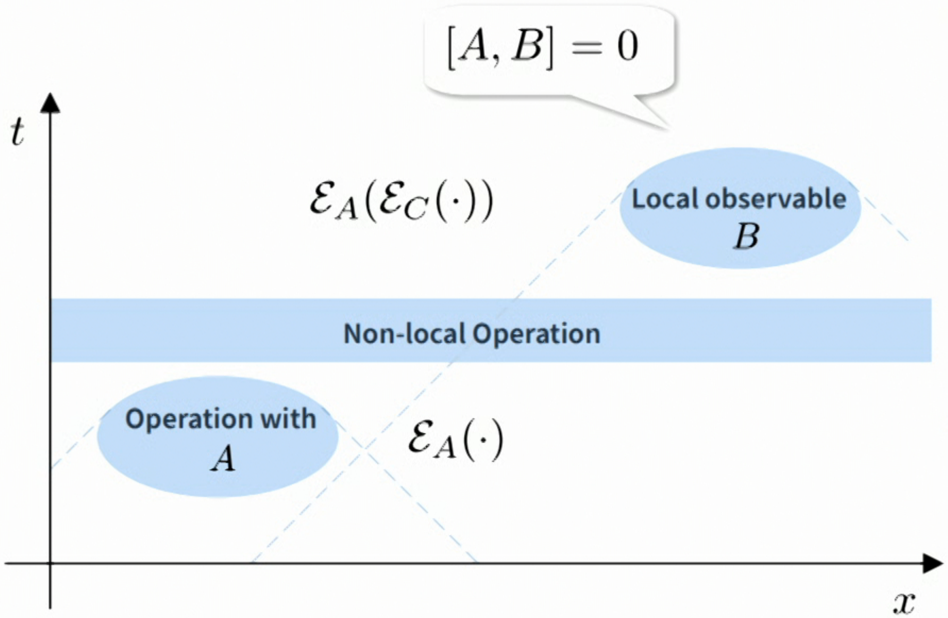

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- If Caoimhe's map describes a physical process, then it must satisfy a further ***causality*** condition:

$$\mathcal{E}_A(\mathcal{E}_C(B)) = \mathcal{E}_C(B)$$

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Locality and Causality

Recap

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IJ, *arxiv:2106.09027*

- Operation described by completely positive, unit preserving map $\mathcal{E}_C : \mathfrak{A} \to \mathfrak{A}$, $\mathcal{E}_C(1) = 1$
- **Locality**: $\mathcal{E}_C(B) = B$ for all B spacelike to C
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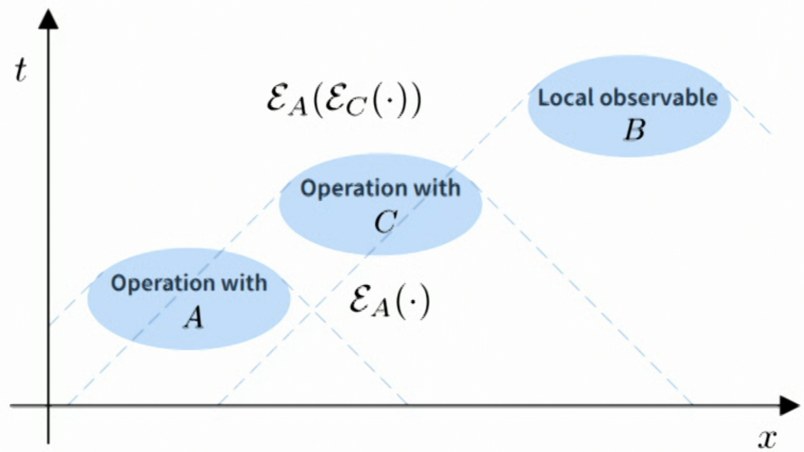

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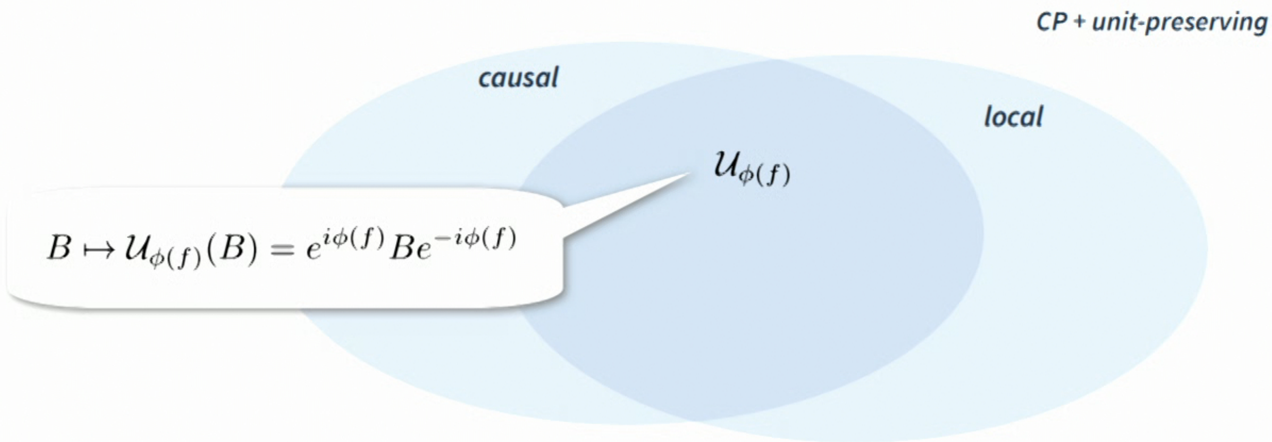

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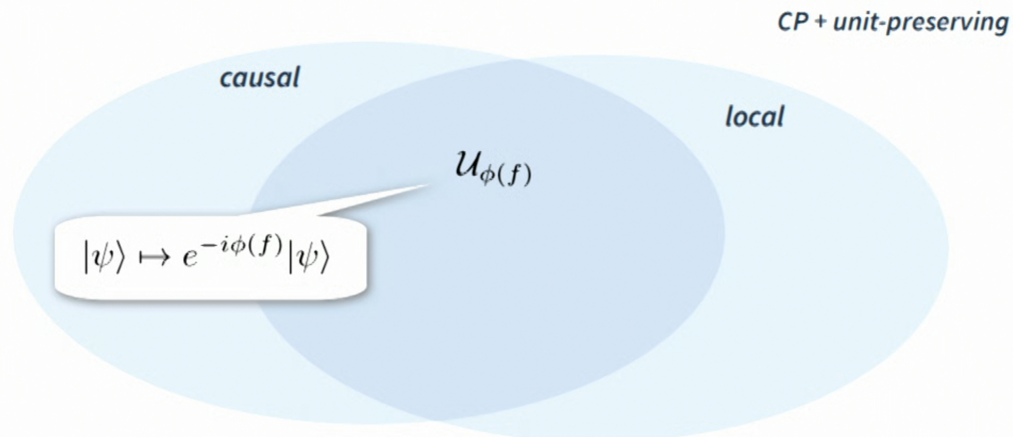

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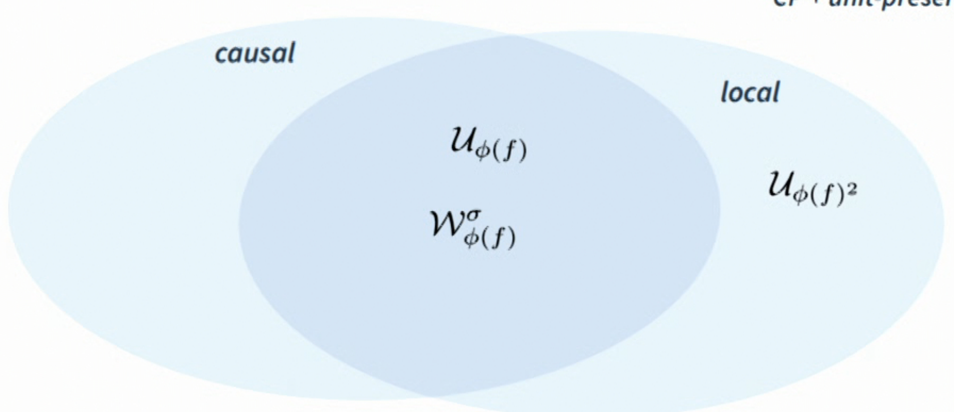

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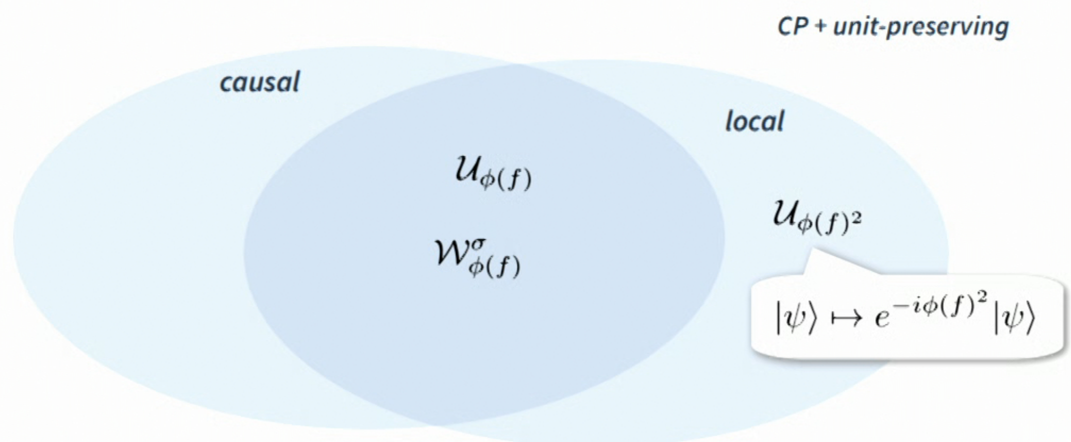

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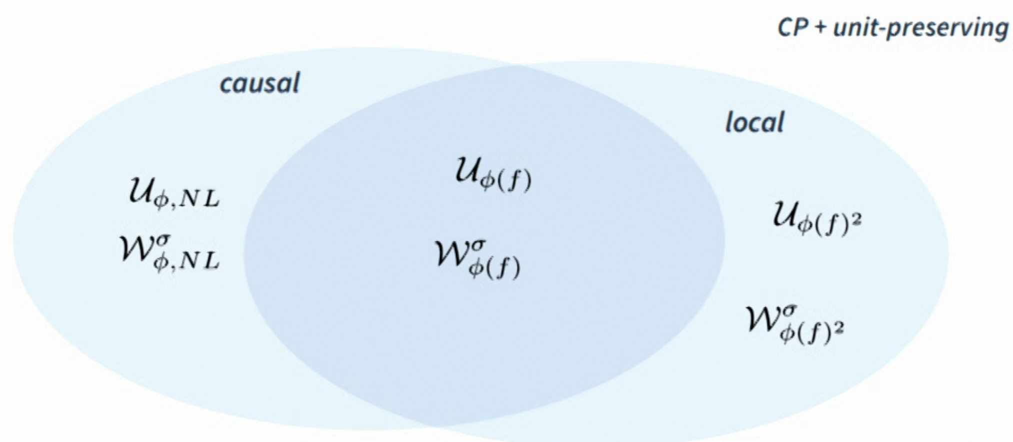

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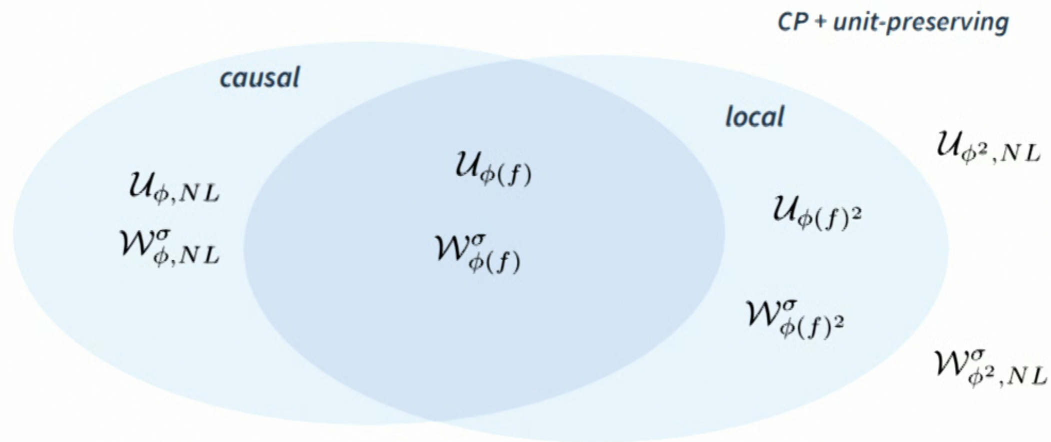

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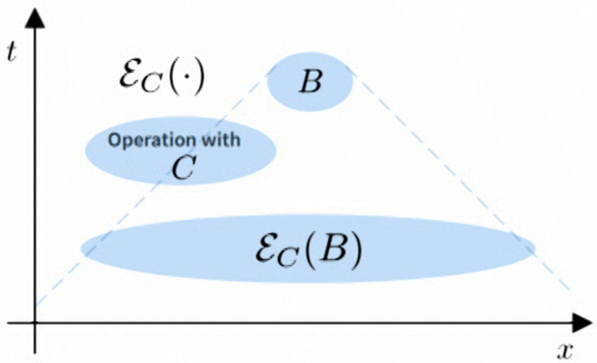
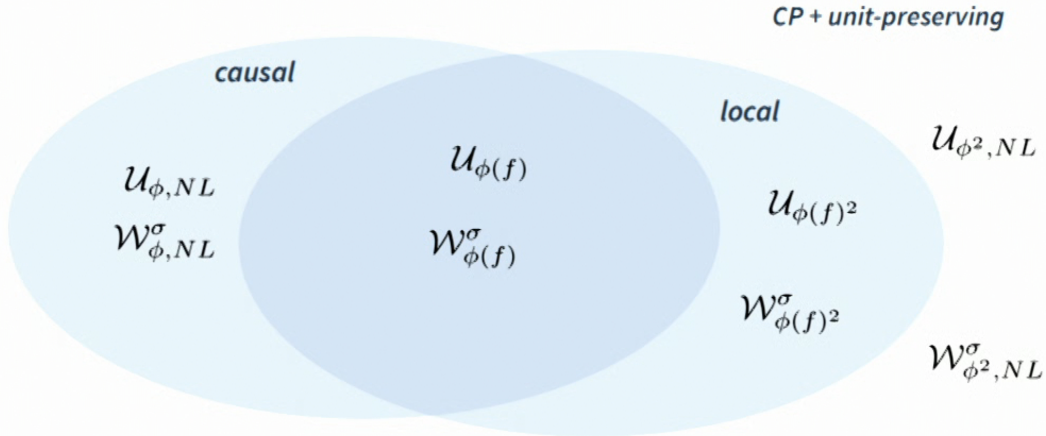
- **Past support non-increasing (PSNI)**

  Update map keeps operator in past-lightcone:





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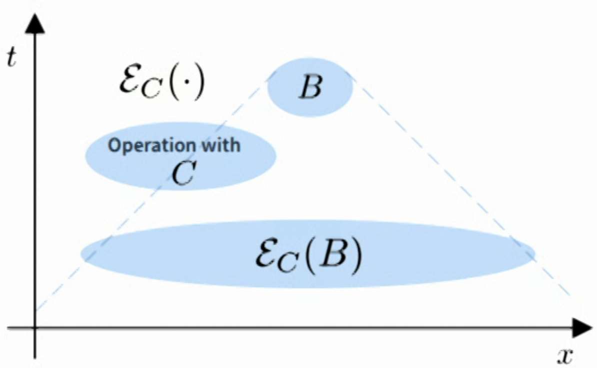

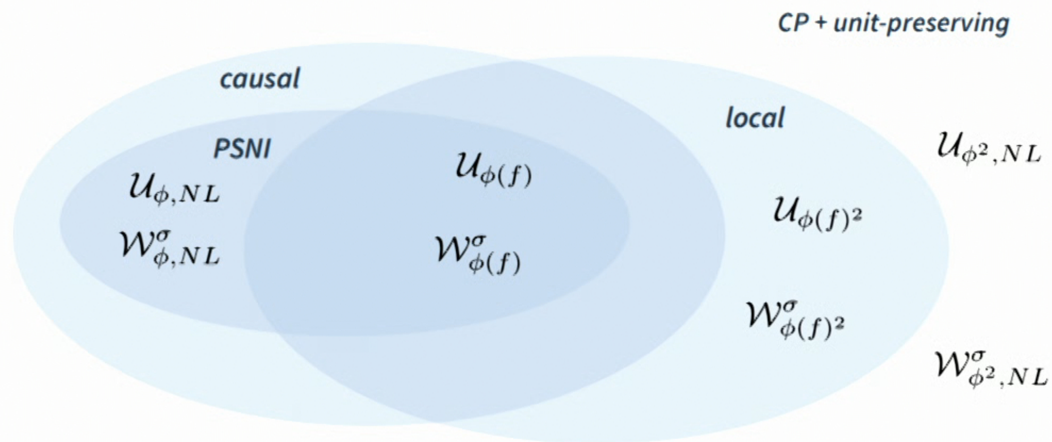

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Locality and Causality

IJ, *arxiv:2106.09027*

- Operation described by completely positive, unit preserving map $\mathcal{E}_C : \mathfrak{A} \to \mathfrak{A} \quad , \quad \mathcal{E}_C(1) = 1$
- **Locality**: $\mathcal{E}_C(B) = B$ for all B spacelike to C

Locality of $\mathcal{E}_A$ *assumed, i.e.* $\mathcal{E}_A(B) = B$

- **Causality**: $\mathcal{E}_A(\mathcal{E}_C(B)) = \mathcal{E}_C(B)$ for all A *not* in the future of C, all B *not* in the past of C, and A spacelike to B
- **Support non-increasing (SNI)**
  Update map doesn't change operator region
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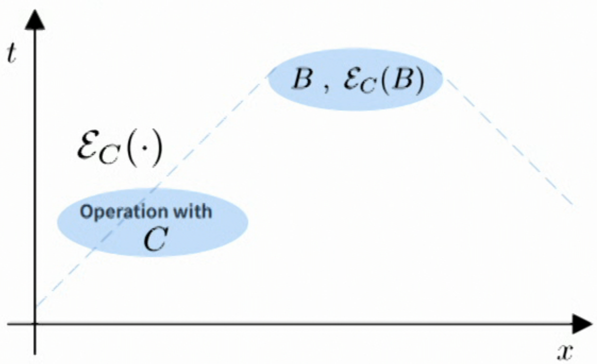

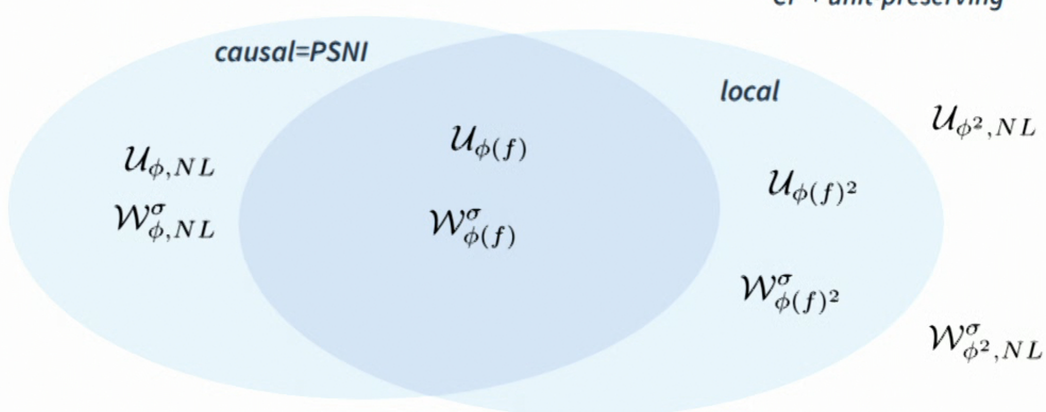

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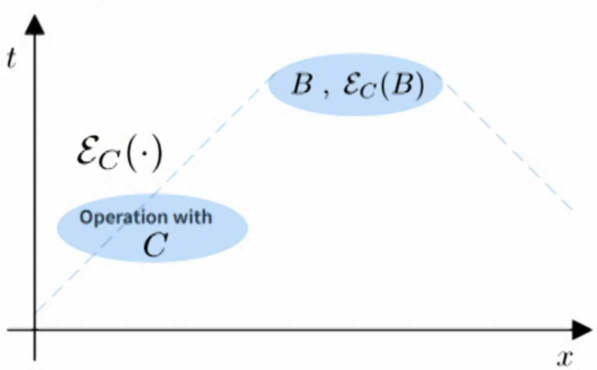

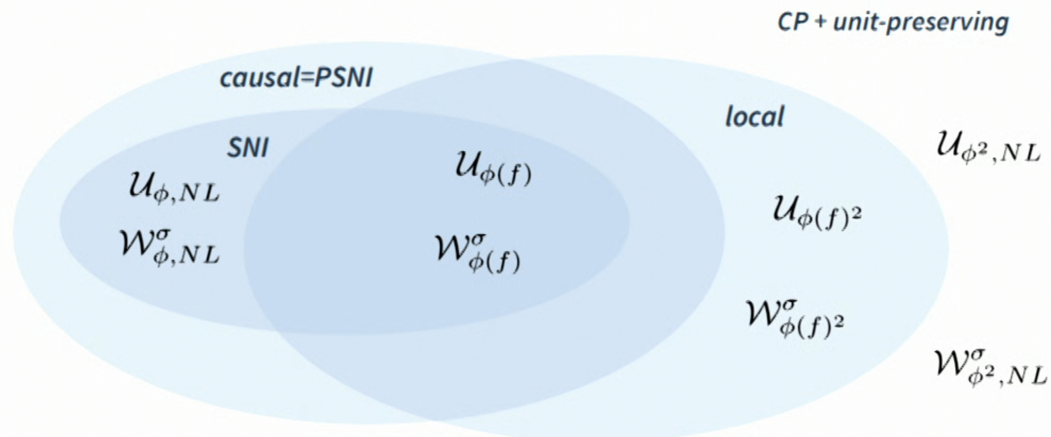

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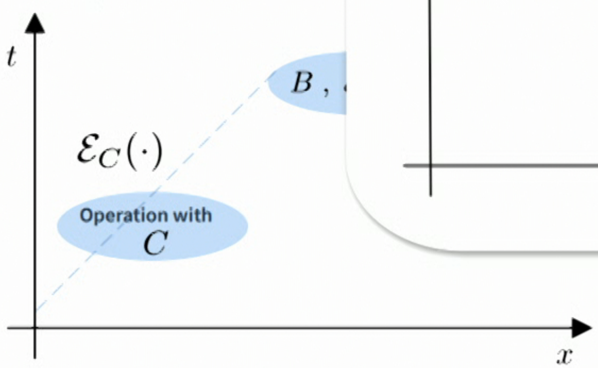

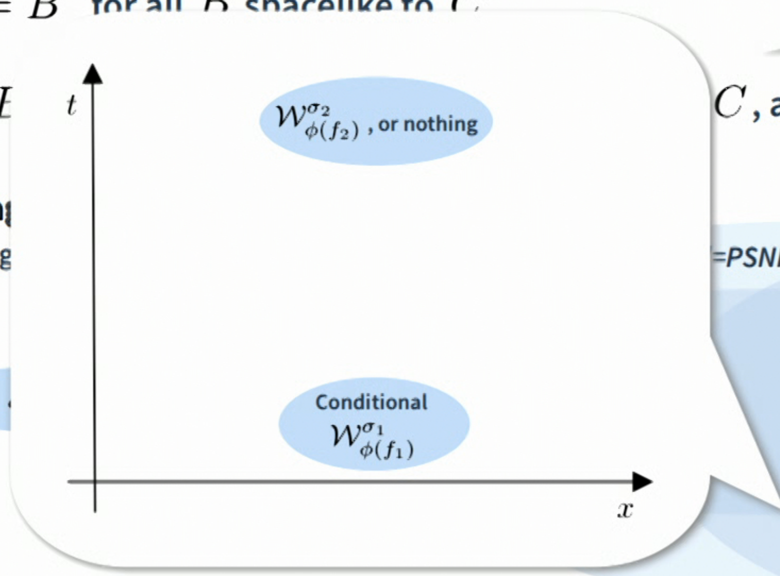

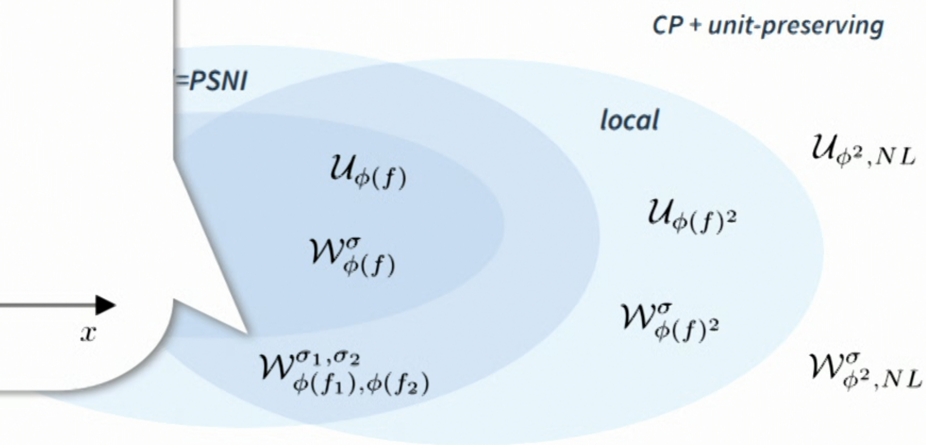

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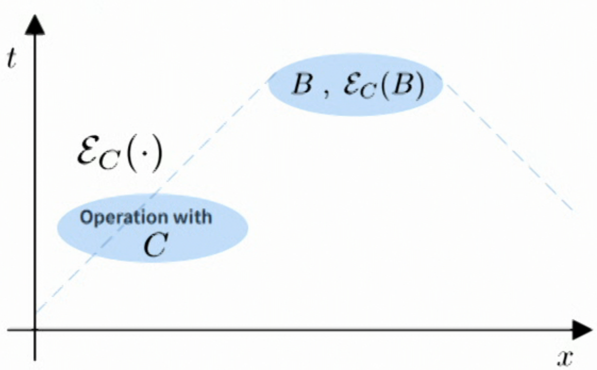

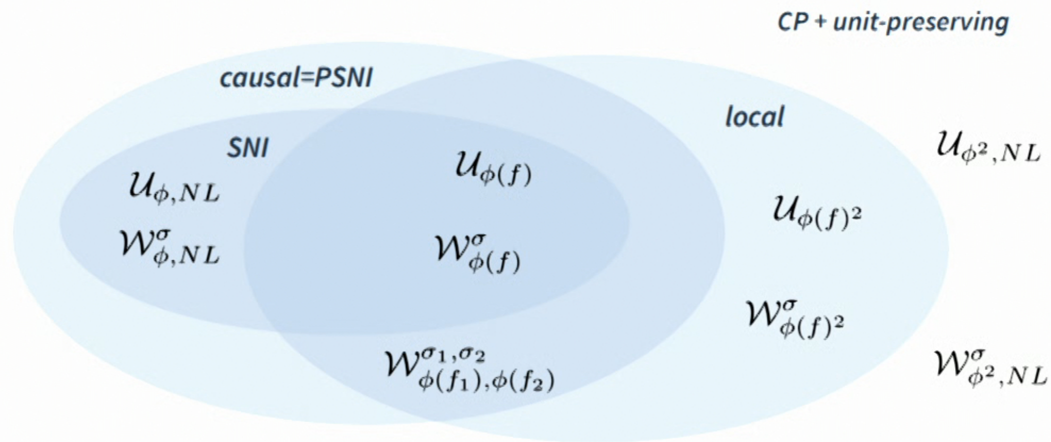

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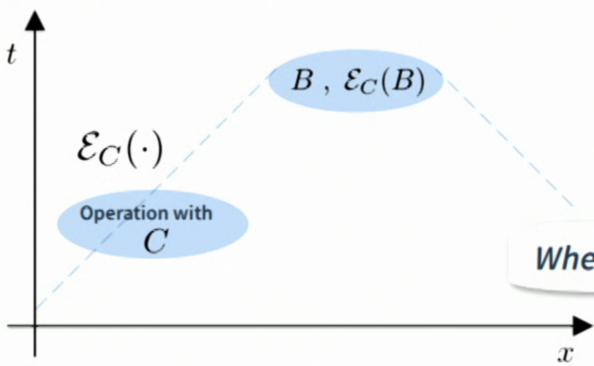

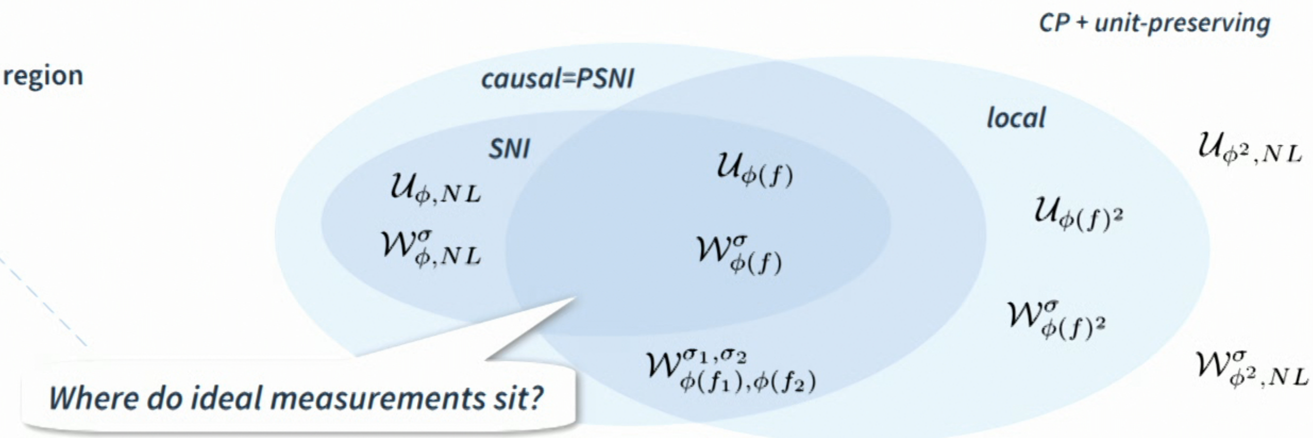

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Background

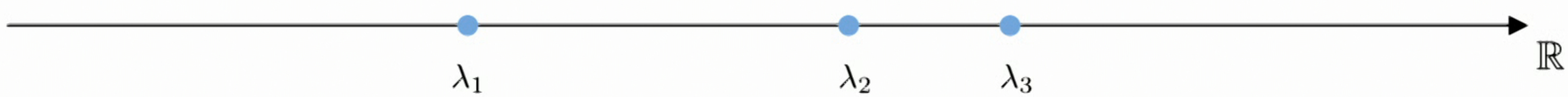
Ideal Measurements

Projectors

- Self adjoint A on $\mathbb{C}^3$: $A = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3$
- In eigenbasis:

$$A = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

- Spectrum:



Projectors

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$$F(A) = \begin{pmatrix} F(\lambda_1) & 0 & 0 \\ 0 & F(\lambda_2) & 0 \\ 0 & 0 & F(\lambda_3) \end{pmatrix}$$

For a function $F : \mathbb{R} \to \mathbb{C}$

- Spectrum:

Projectors

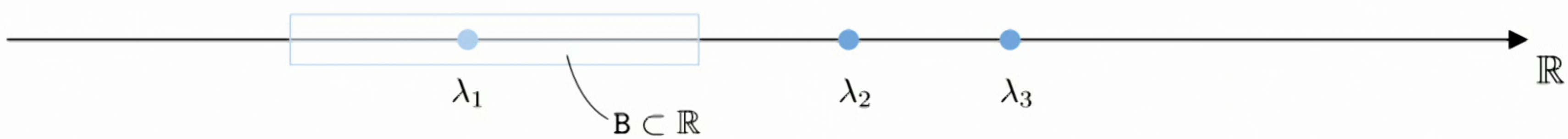
- **Self adjoint** A **on** $\mathbb{C}^3$**:** $A = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3$
- **In eigenbasis:**

$$1_{\mathrm{B}}(A) = \begin{pmatrix} 1_{\mathrm{B}}(\lambda_1) & 0 & 0 \\ 0 & 1_{\mathrm{B}}(\lambda_2) & 0 \\ 0 & 0 & 1_{\mathrm{B}}(\lambda_3) \end{pmatrix}$$

Indicator function

$$1_{\mathrm{B}}(\lambda) = \begin{cases} 1\,, & \lambda \in \mathrm{B} \\ 0\,, & \lambda \notin \mathrm{B} \end{cases}$$

- **Spectrum:**



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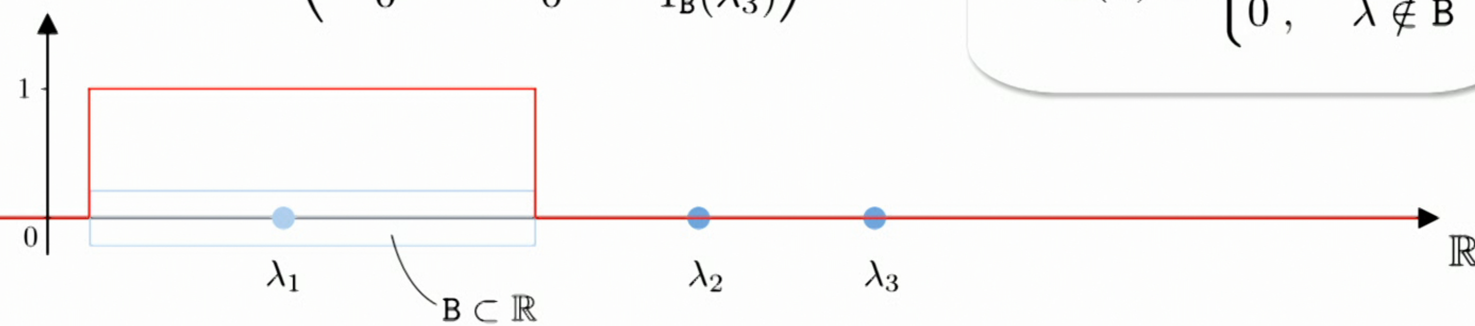

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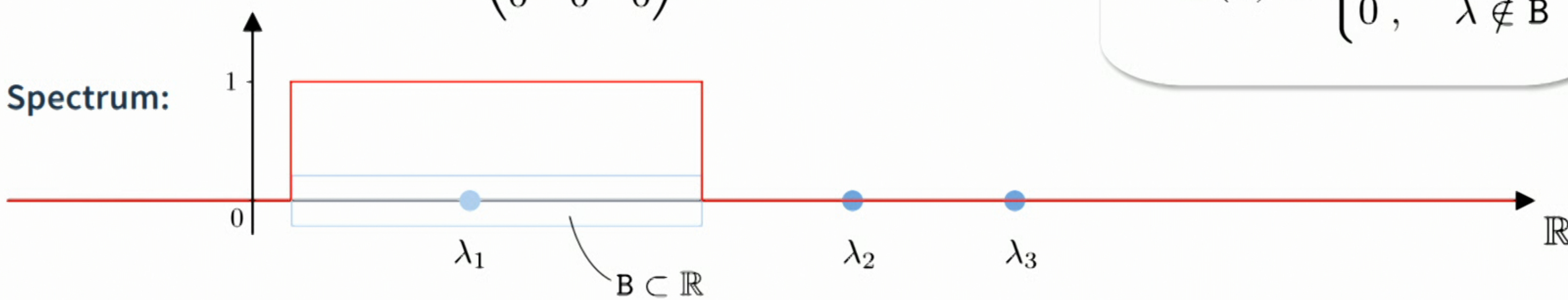

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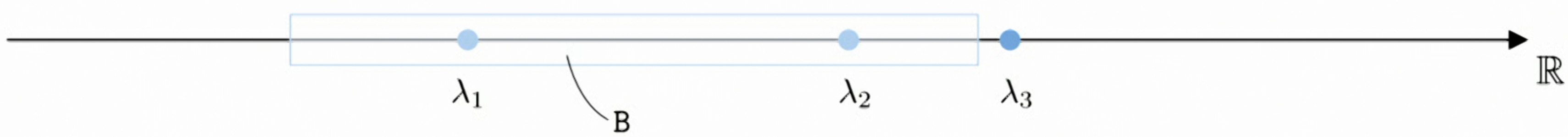

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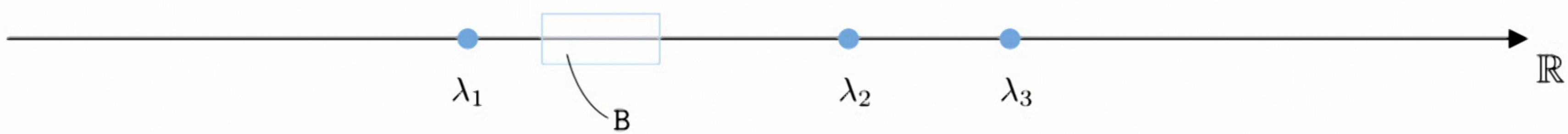

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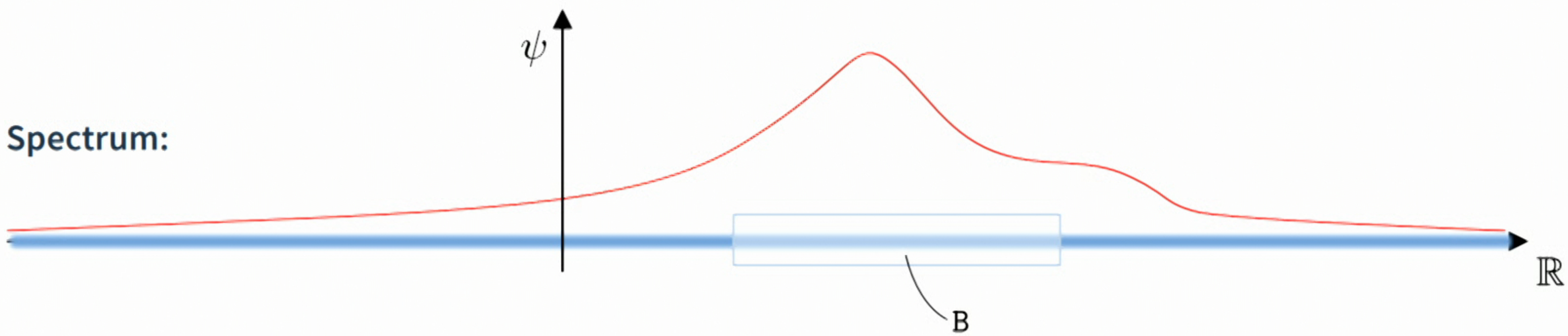
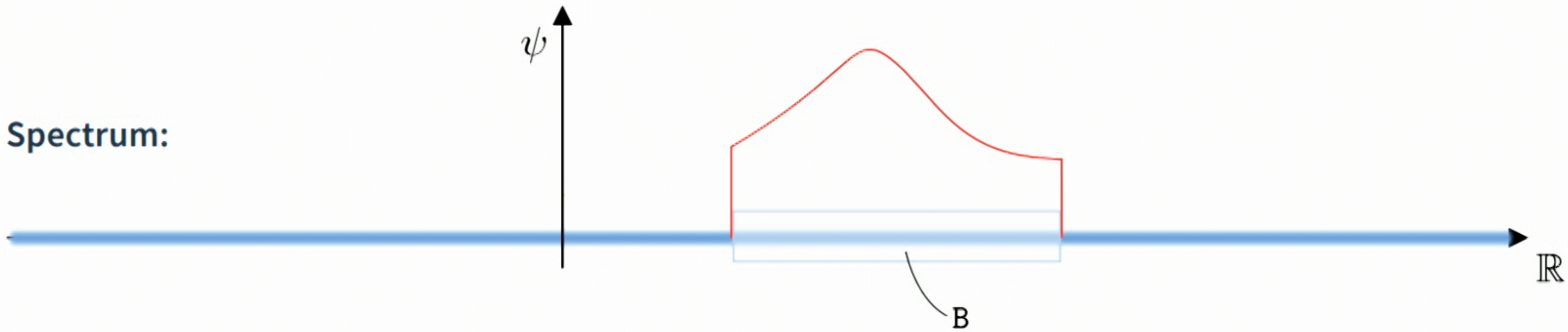
Projectors

- Self adjoint $\hat{x}$ on $L^2(\mathbb{R})$ (unbounded)
- Can still take indicator function:
- Spectrum:



Projectors

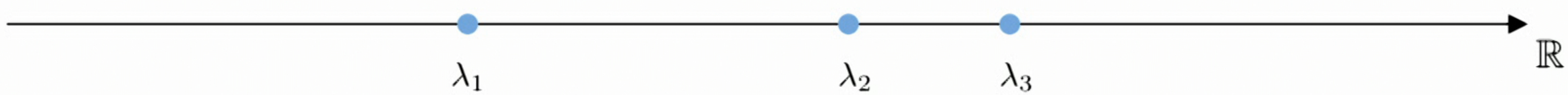
- Self adjoint $\hat{x}$ on $L^2(\mathbb{R})$ (unbounded)
- Can still take indicator function: $(1_{\mathrm{B}}(\hat{x})\psi)(x) = 1_{\mathrm{B}}(x)\psi(x)$
- Spectrum:



For a general self adjoint A, given (Borel) set $\mathrm{B} \subseteq \mathbb{R}$, $1_{\mathrm{B}}(A)$ is a projector on the Hilbert space

Ideal Measurements

- **Ideal measurement of** $A = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3$
- **Update map:** $$\mathcal{E}_A(X) = \sum_{n=1}^{3} P_n X P_n$$
- **Spectrum:**



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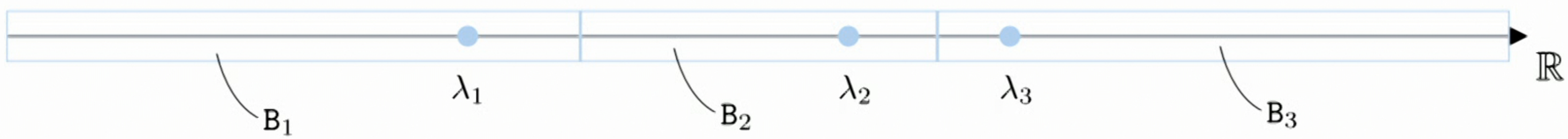

Ideal Measurements

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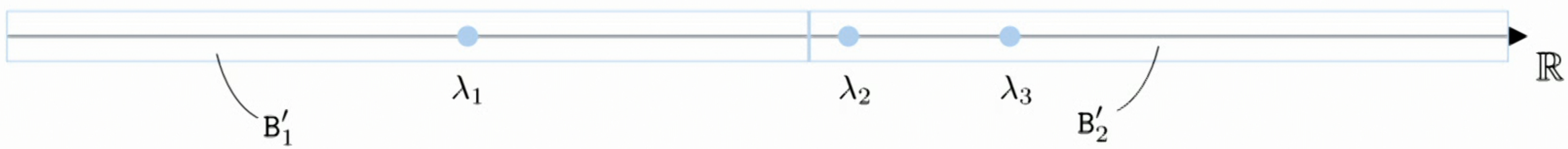
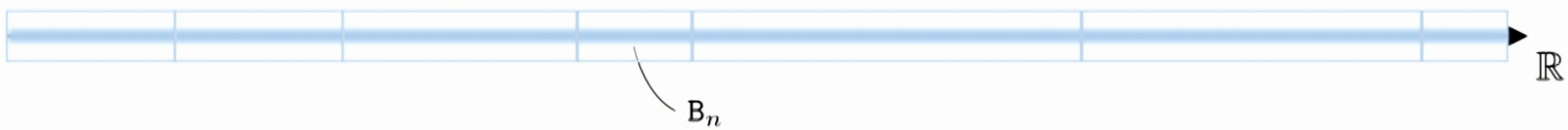
$$\begin{aligned} \mathcal{E}_A(X) &= 1_{\mathrm{B}'_1}(A) X 1_{\mathrm{B}'_1}(A) + 1_{\mathrm{B}'_2}(A) X 1_{\mathrm{B}'_2}(A) \\ &= P_1 X P_1 + (P_2 + P_3) X (P_2 + P_3) \end{aligned}$$

- Spectrum:



Ideal Measurements

- Ideal measurement of $\hat{x}$
- Update map: $$\mathcal{E}_{\hat{x}}(X) = \sum_{n \in I} 1_{\mathrm{B}_n}(\hat{x}) X 1_{\mathrm{B}_n}(\hat{x})$$
- Spectrum:



Resolution $\mathrm{R} = \{\mathrm{B}_n\}_{n \in I}$

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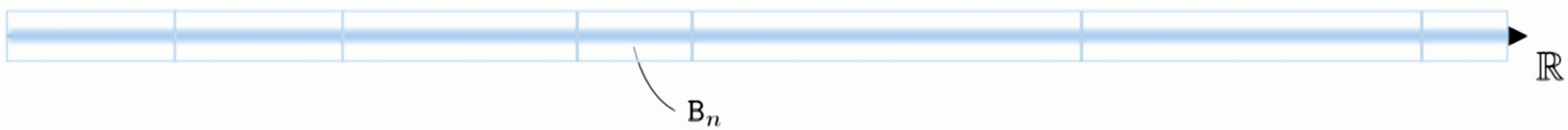

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Ideal Measurements

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Resolution $\mathrm{R} = \{\mathrm{B}_n\}_{n \in I}$. For a general self adjoint A and some resolution R ,

ideal measurement with
that resolution gives update map: $$\mathcal{E}_{A,\mathrm{R}}(X) = \sum_{n \in I} 1_{\mathrm{B}_n}(A) X 1_{\mathrm{B}_n}(A)$$

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Kraus Operations and Causality

Ideal Measurements

Kraus operations

$$\mathcal{E}_{A,\mathtt{R}}(X) = \sum_{n \in I} 1_{\mathtt{B}_n}(A) X 1_{\mathtt{B}_n}(A)$$

$$\mathcal{U}_A(X) = e^{iA} X e^{-iA}$$

$$\mathcal{W}_A^{\sigma}(X) = \int_{\mathbb{R}} d\alpha \, \frac{e^{-\frac{(A-\alpha)^2}{4\sigma^2}}}{(2\pi\sigma^2)^{1/4}} X \frac{e^{-\frac{(A-\alpha)^2}{4\sigma^2}}}{(2\pi\sigma^2)^{1/4}}$$

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Kraus operation with A . Specify labelling set Γ

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Kraus operation with A . Specify labelling set Γ , non-neg. measure ν , func's $\kappa(\cdot,\gamma) : \mathbb{R} \to \mathbb{C}$ for $\gamma \in \Gamma$

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Kraus operation with A . Specify labelling set Γ , non-neg. measure ν , func's $\kappa(\cdot, \gamma) : \mathbb{R} \to \mathbb{C}$ for $\gamma \in \Gamma$

$$\mathcal{E}_{A,\kappa}(X) = \int_\Gamma d\nu(\gamma) \, \kappa(A, \gamma) X \kappa(A, \gamma)^\dagger$$

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Causality condition for Kraus operations with a smeared field

Let $$\tilde{\kappa}(\lambda, t) = \int_\Gamma d\nu(\gamma)\, \kappa(\lambda, \gamma)\kappa(\lambda - t, \gamma)^* = \langle \kappa(\lambda - t, \cdot), \kappa(\lambda, \cdot)\rangle_{L^2(\Gamma;\nu)}$$

***Condition*:**

$\tilde{\kappa}(\cdot, t)$ is a constant func. for all t → For any smeared field $\phi(f)$ — $\mathcal{E}_{\phi(f),\kappa}$ is ***SNI***. No signal for any state

$\tilde{\kappa}(\cdot, t)$ is not a constant func. for some t → For any smeared field $\phi(f)$ — If ***ABS***, then $\mathcal{E}_{\phi(f),\kappa}$ signals in the ground state $|\Omega\rangle$

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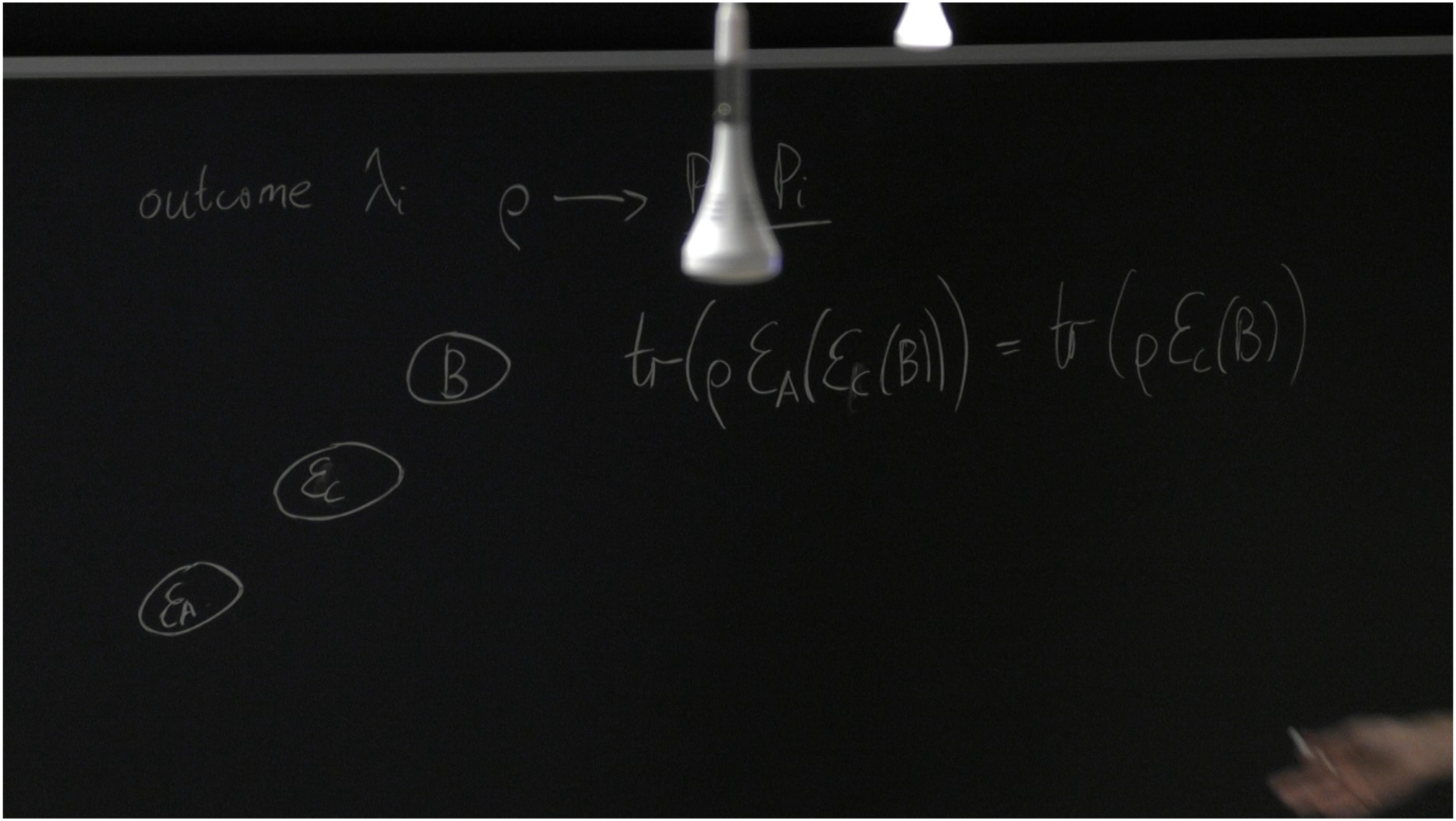
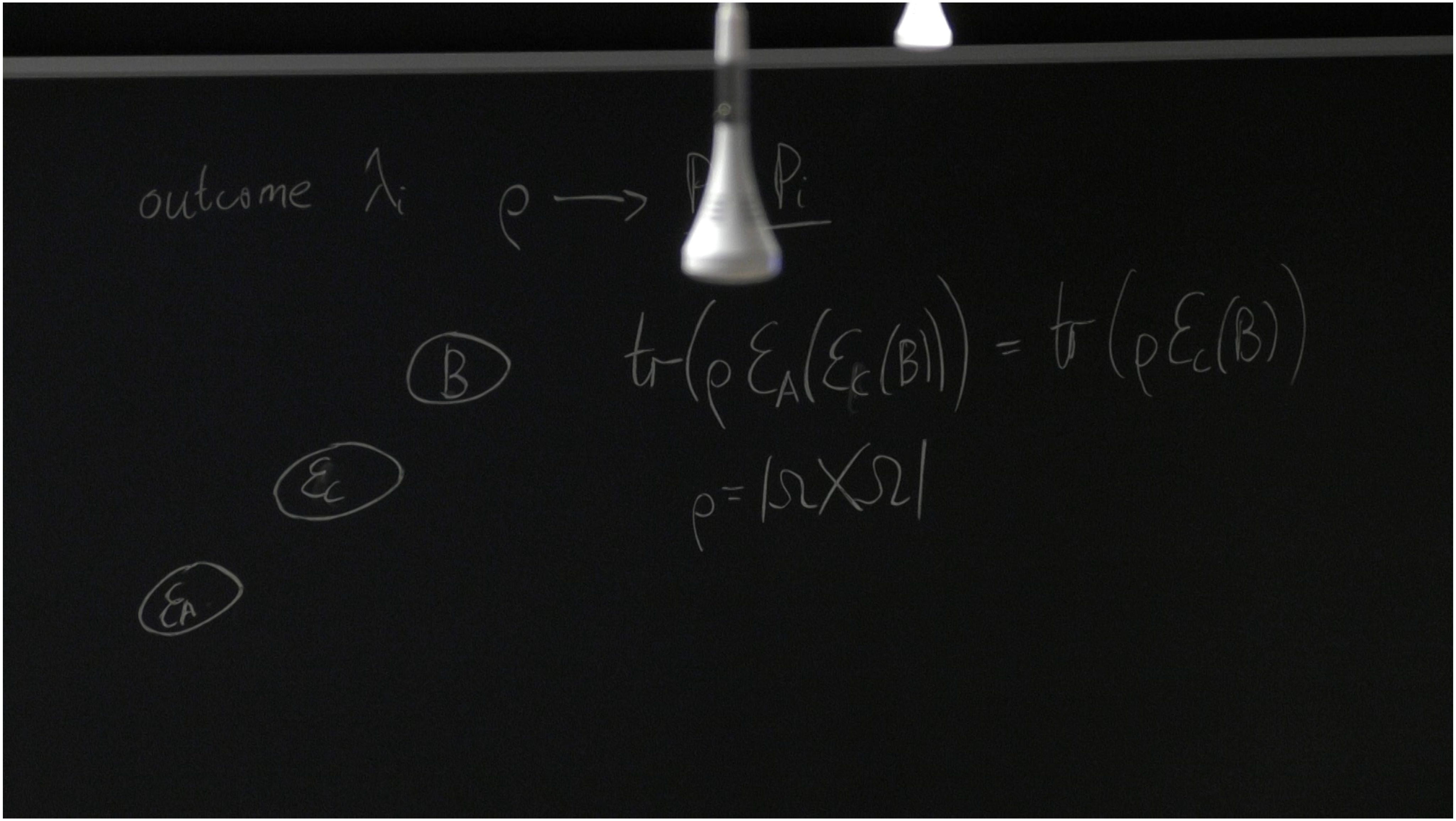

outcome λ_i $\rho \longrightarrow$ ρ_i
$\mathrm{tr}(\rho\, \mathcal{E}_A(\mathcal{E}_C(B))) = \mathrm{tr}(\rho\, \mathcal{E}_C(B))$
B
$\mathcal{E}_C$
$\mathcal{E}_A$



outcome λ_i
$\rho \longrightarrow \rho_i$
B
$\mathcal{E}_C$
$\mathcal{E}_A$
$\mathrm{tr}(\rho\, \mathcal{E}_A(\mathcal{E}_C(B))) = \mathrm{tr}(\rho\, \mathcal{E}_C(B))$
$\rho = |\Omega\rangle\langle\Omega|$

Causality condition for Kraus operations with a smeared field

Let $$\tilde{\kappa}(\lambda, t) = \int_\Gamma d\nu(\gamma)\, \kappa(\lambda, \gamma)\kappa(\lambda - t, \gamma)^* = \langle \kappa(\lambda - t, \cdot), \kappa(\lambda, \cdot)\rangle_{L^2(\Gamma;\nu)}$$

Condition :

$\tilde{\kappa}(\cdot, t)$ is a constant func. for all t → For any smeared field $\phi(f)$: $\mathcal{E}_{\phi(f),\kappa}$ is ***SNI***. No signal for any state

$\tilde{\kappa}(\cdot, t)$ is not a constant func. for some t → For any smeared field $\phi(f)$: If ***ABS***, then $\mathcal{E}_{\phi(f),\kappa}$ signals in the ground state $|\Omega\rangle$

Example: $\mathcal{U}_{\phi(f)}(X) = e^{i\phi(f)} X e^{-i\phi(f)}$

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Example: $\mathcal{U}_{\phi(f)}(X) = e^{i\phi(f)} X e^{-i\phi(f)}$ ➡ $\Gamma = \{\gamma\}$, $\kappa(\lambda, \gamma) = e^{i\lambda}$ ➡ $\tilde{\kappa}(\lambda, t) = e^{i\lambda} e^{-i(\lambda - t)} = e^{it}$ ➡ *No signal*

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Ideal Measurements of Smeared Fields

Ideal Measurements

The case of ideal measurements

For some ***resolution*** $\mathtt{R} = \{\mathtt{B}_n\}_{n\in I}$, $\kappa(\lambda, n) = 1_{\mathtt{B}_n}(\lambda)$, $\tilde{\kappa}(\lambda, t) = \int_\Gamma d\nu(\gamma)\kappa(\lambda,\gamma)\kappa(\lambda - t,\gamma)^*$

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$\mathtt{R}(t) = \cup_{n \in I} (\mathtt{B}_n \cap (\mathtt{B}_n + t))$

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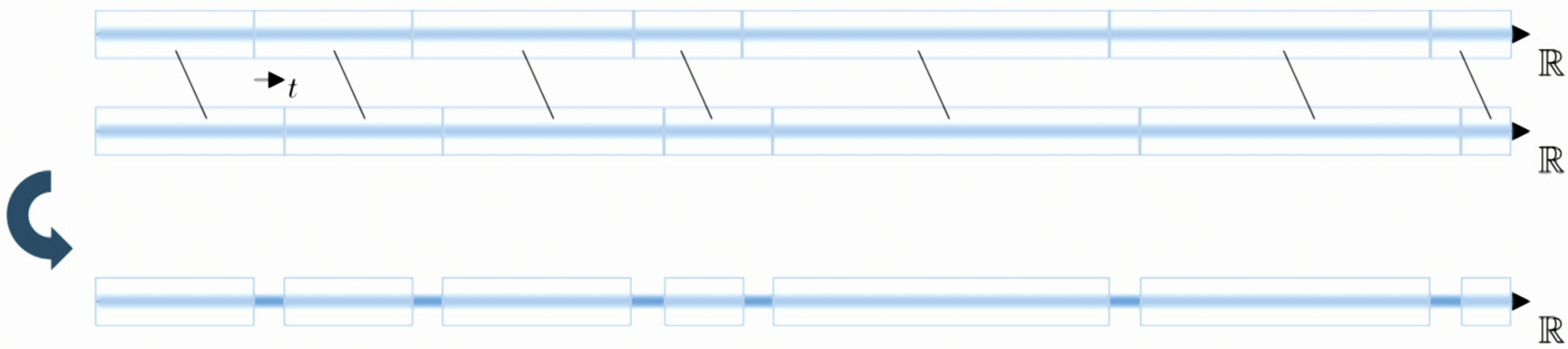

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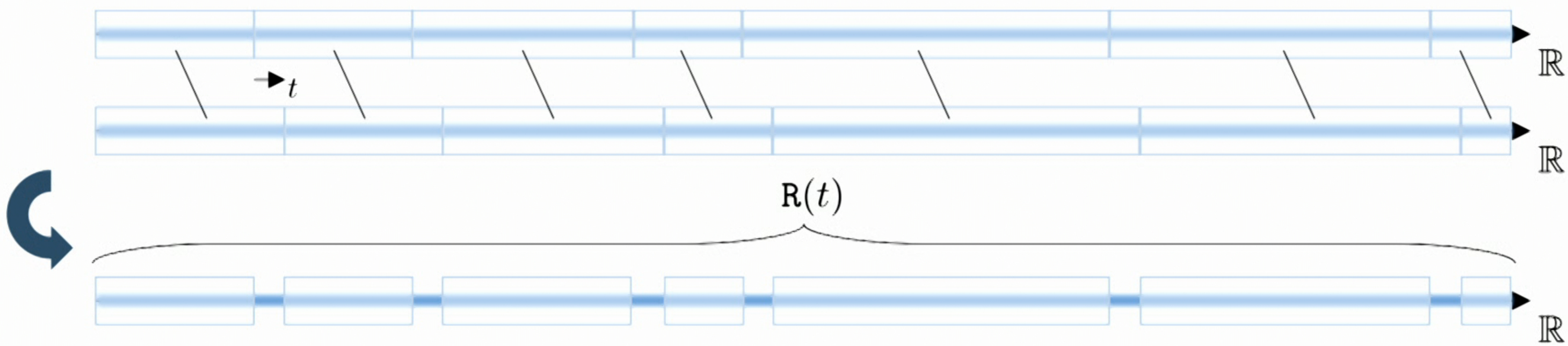

$\tilde{\kappa}(\lambda, t) = 1_{\mathrm{R}(t)}(\lambda)$ **not constant**

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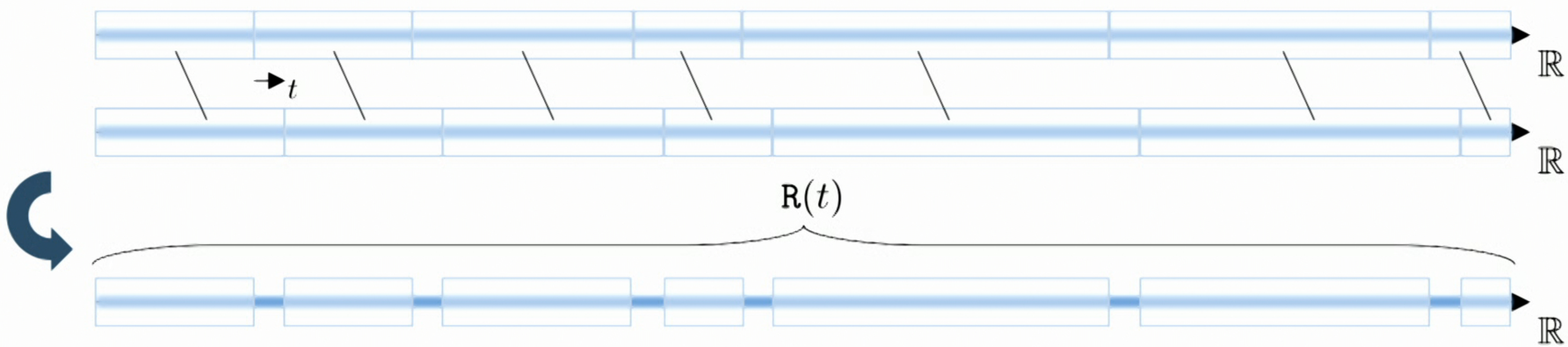

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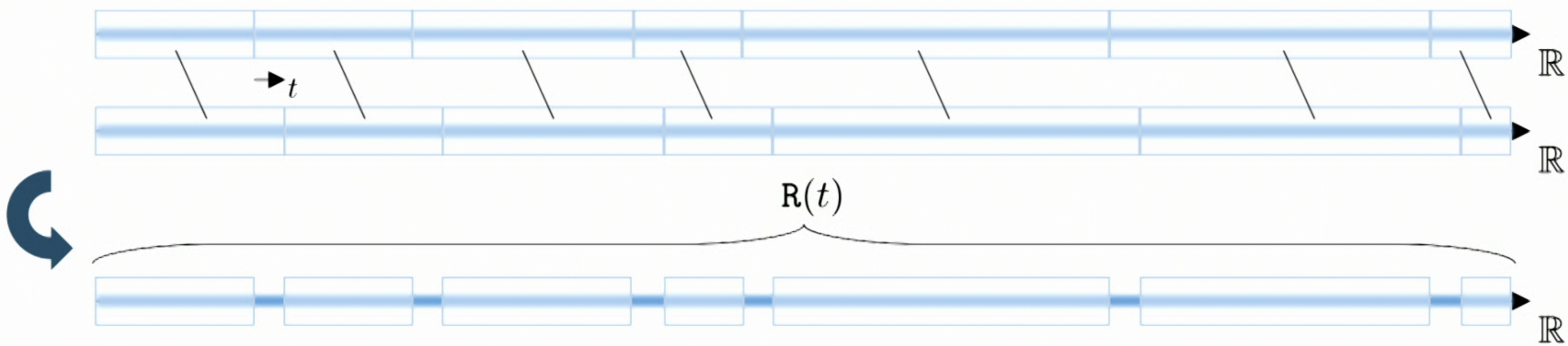

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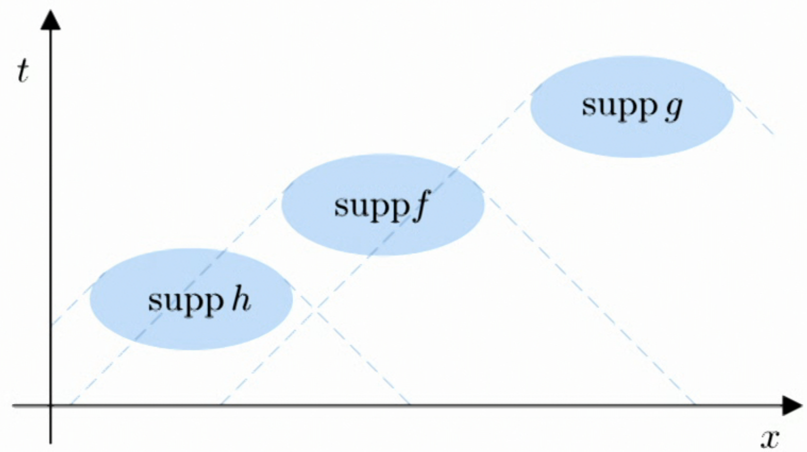
ABS assumptions

A. Aoife can kick with a smeared field: $\mathcal{U}_{\phi(h)}$ is allowed

B. Beolagh can measure a smeared field and determine the exp. val. $\langle e^{i\phi(g)} \rangle$

S. Sorkin scenario exists:



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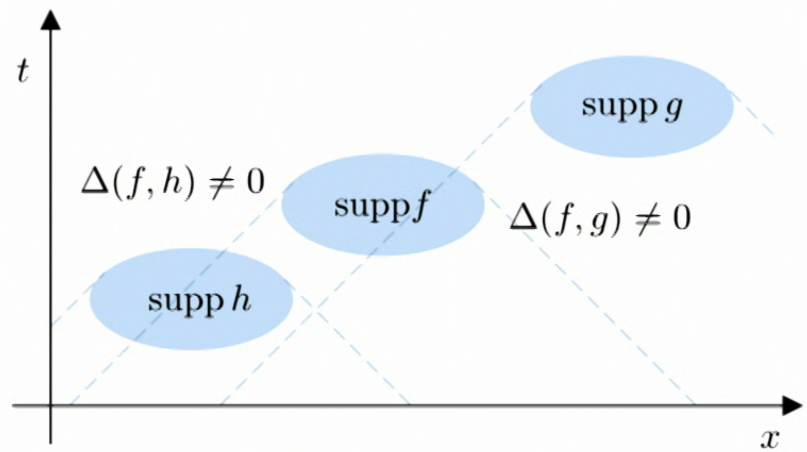
Does not exist if measurement region is ***transitive***. Examples:

- Cylinder spacetime



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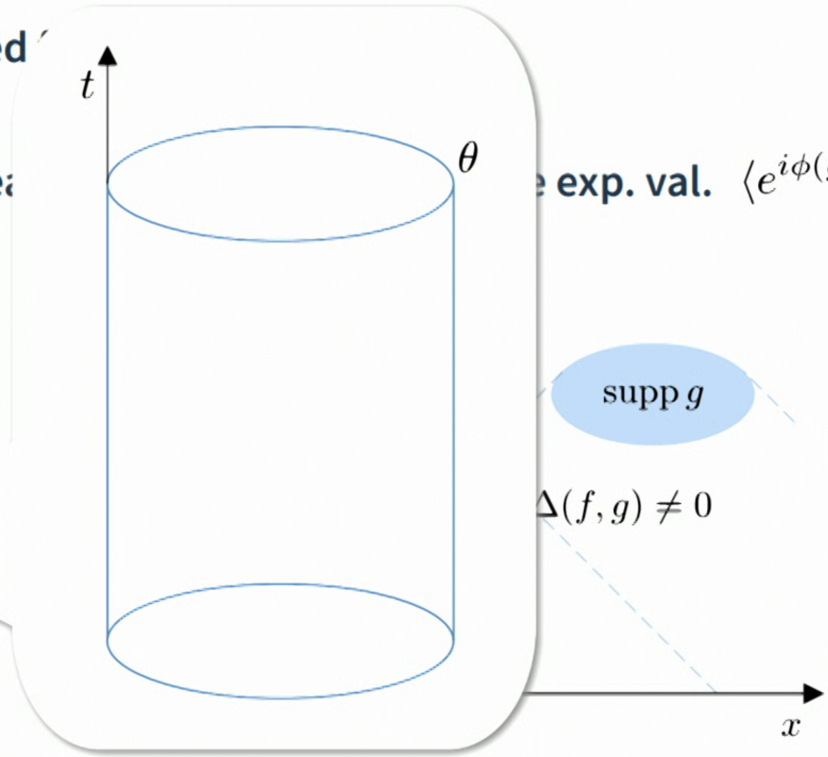

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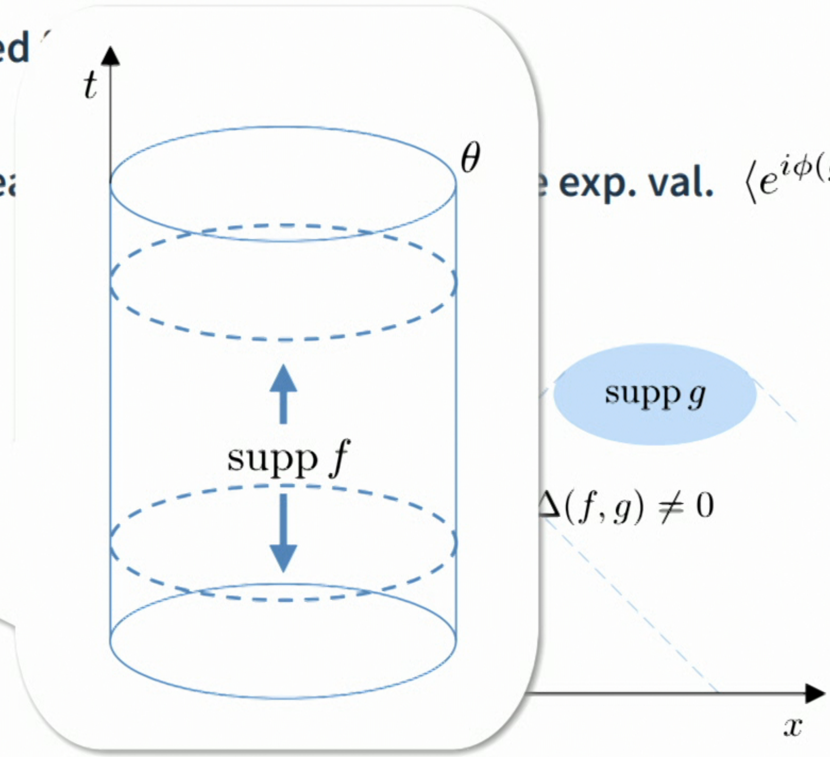

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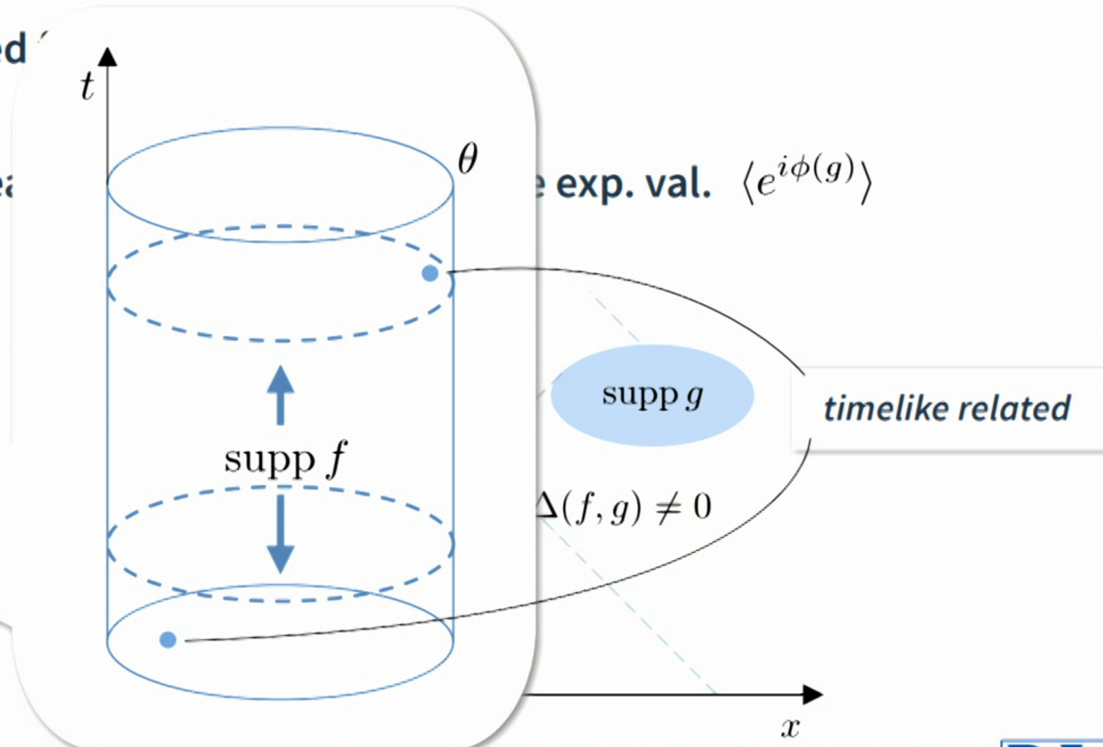

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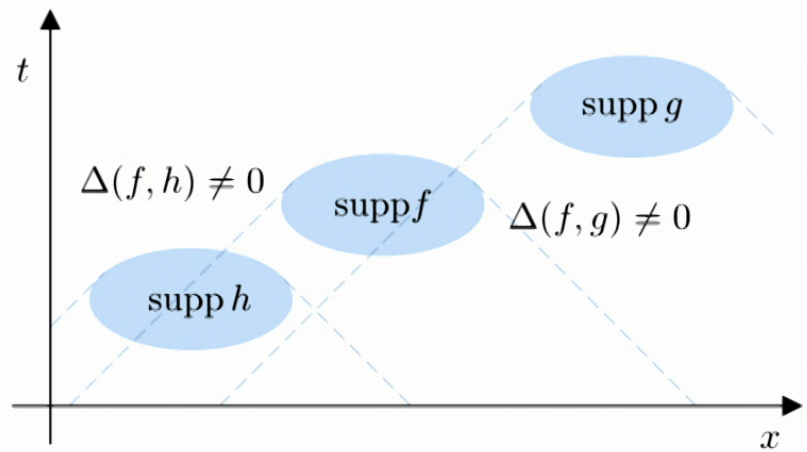

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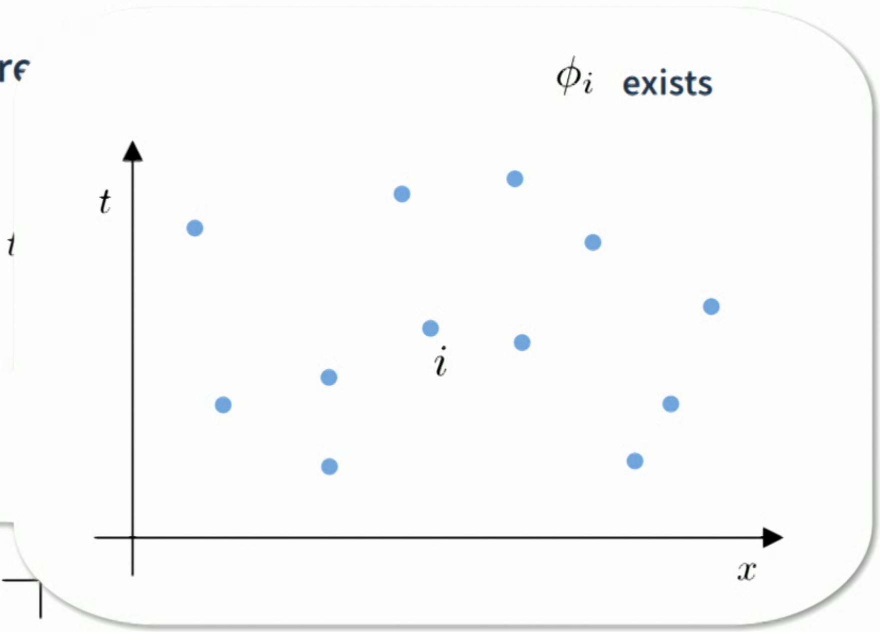

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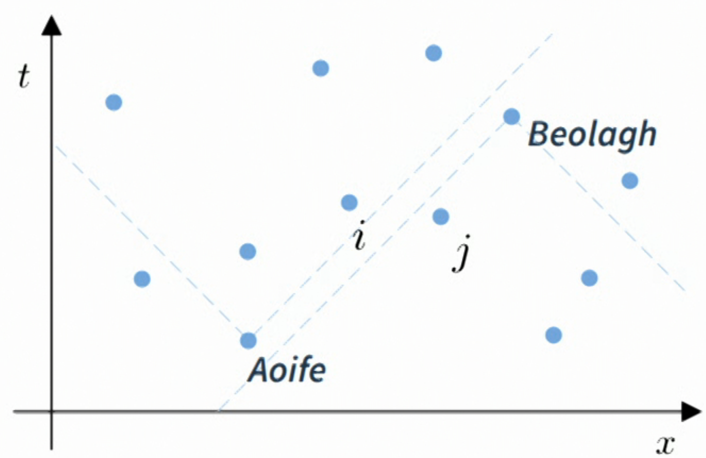
Smeared field: $f_i\phi_i + f_j\phi_j$



Key points

- Signals appear in causal set case if support is not transititve, e.g. pair of spacelike points
- In continuum, there may be operators with ideal measurements that do not signal. We have only tested smeared fields – the generators of the algebra.
- No projection postulate in QFT does not mean no projection postulate in non-relativistic systems. In those systems, the ideal measurement map may arise from some other (causally consistent) map on the underlying quantum field.

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- No projection postulate in QFT does not mean no projection postulate in non-relativistic systems. In those systems, the ideal measurement map may arise from some other (causally consistent) map on the underlying quantum field.
- In both continuum and discrete spacetimes, there are other ways to describe measurements of smeared fields that are *always* causal, e.g. $\mathcal{W}^{\sigma}_{\phi(f)}$.

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Summary

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- Operations are described by update maps/quantum channels. In QFT, maps must be ***local*** and ***causal***/***PSNI*** if they are to be physically realisable.
- For operations constructed from smeared fields, we have a simple causality condition in terms of the Kraus functions $\kappa(\lambda, \gamma)$.
- The ideal measurement map, arising from the projection postulate, fails this condition.
- Thus, if we have ***ABS*** for a smeared field, an ideal measurement of it is prohibited by causality.
- The ***S***orkin scenario in ***ABS*** does not exist in certain situations, e.g. single points in a discrete spacetime.

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Thank you!

Any questions?