Title: Adaptive Quantum State Tomography with Active Learning

Speakers: Hannah Lange

Collection: New Frontiers in Machine Learning and Quantum

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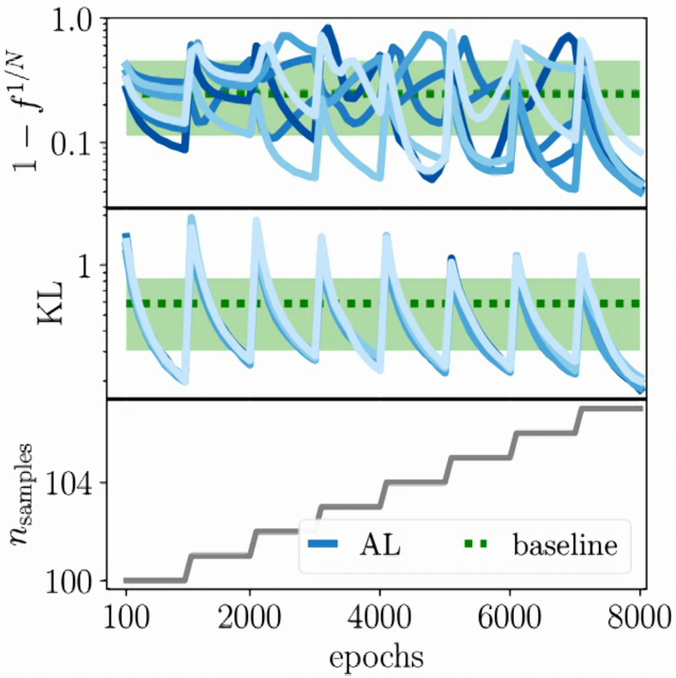
Adaptive Quantum State Tomography with Active Learning



Hannah Lange,

Matjaz Kebric, Maximilian Buser,

Ulrich Schollwöck,

Fabian Grusdt, Annabelle Bohrdt

LMU Munich / MPQ Munich

arXiv:2203.15719

QUANTUM STATE TOMOGRAPHY

quantum wave function

[https://www.ibm.com/blogs/digitale-perspektive/2021/06/quantum-opening-in-ehningen]

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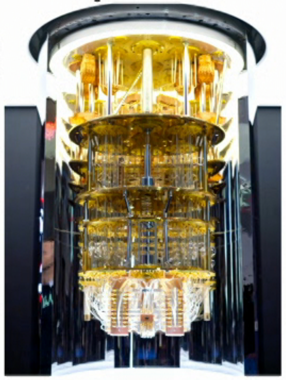

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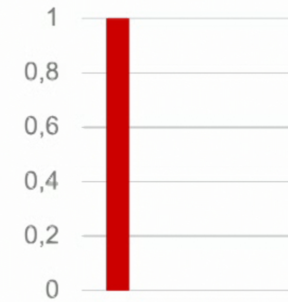
→ **Measurement** →



Quantum State Tomography

= reconstruction of quantum states from measurements

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QUANTUM STATE TOMOGRAPHY

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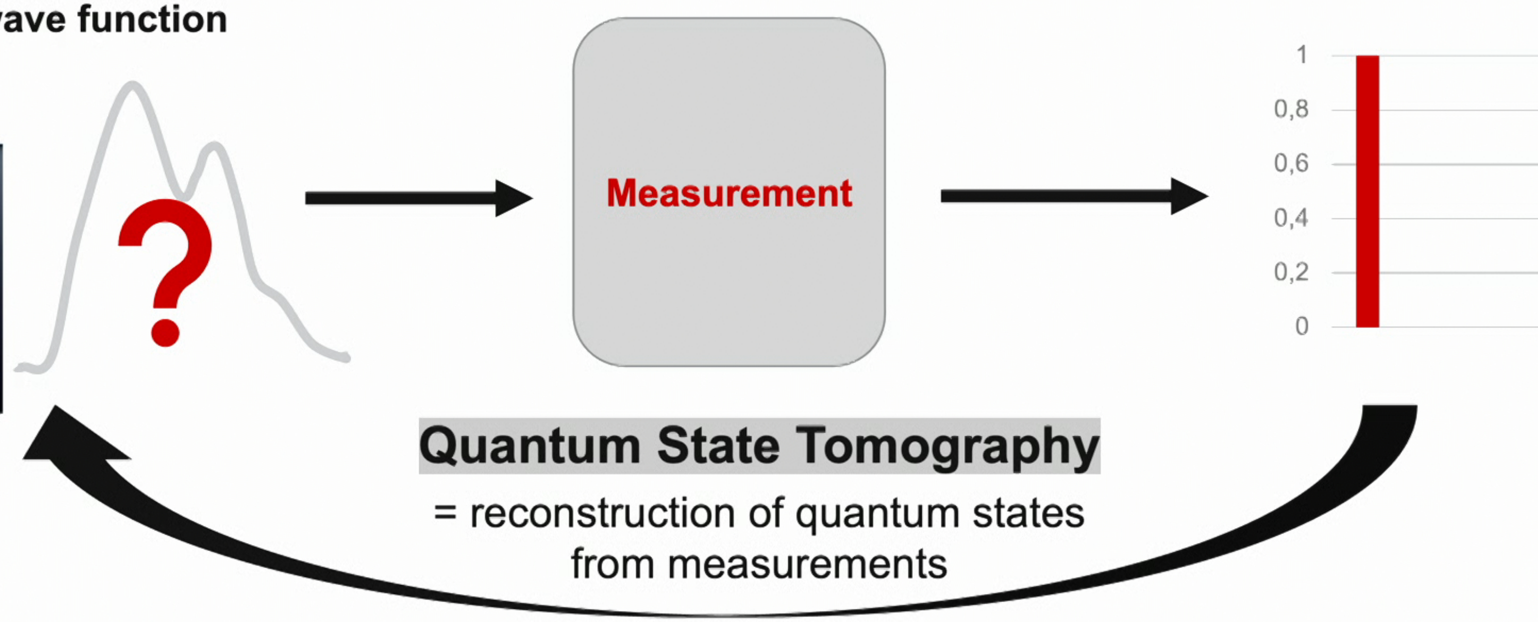
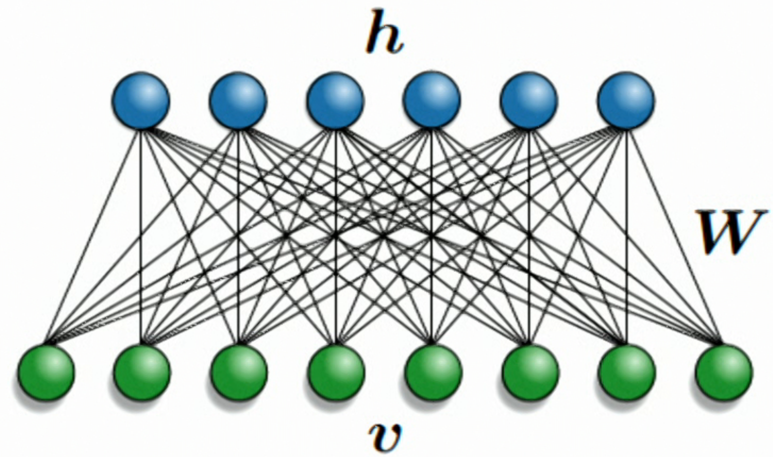


→ use neural networks!

G. Torlai, G. Mazzola, J. Carrasquilla, M. Troyer, R. Melko, and G. Carleo (2018)
J. Carrasquilla, G. Torlai, R. G. Melko, and L. Aolita (2019)
G. Torlai and R. G. Melko (2021)
J. Carrasquilla and G. Torlai (2021)
S. Czischek et al. (2022), …

RESTRICTED BOLTZMANN MACHINES

Neural-network based QST using Restricted Boltzmann Machines (RBMs):



[P.I.Q.I.Lab. RBMs - A Short Tutorial. (2019)]

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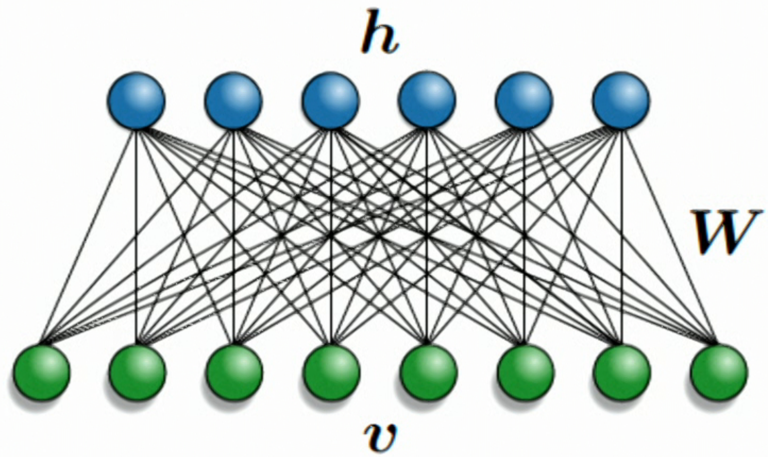
Neural-network based QST using Restricted Boltzmann Machines (RBMs):

- Similar to the Ising model in statistical physics, we can assign a **probability distribution** $p_{\{\boldsymbol{W},\boldsymbol{b},\boldsymbol{c}\}}(\boldsymbol{v}) = \sum_{\boldsymbol{h}} \frac{1}{Z}\, e^{-E_{\{\boldsymbol{W},\boldsymbol{b},\boldsymbol{c}\}}(\boldsymbol{v},\boldsymbol{h})}$ to this system



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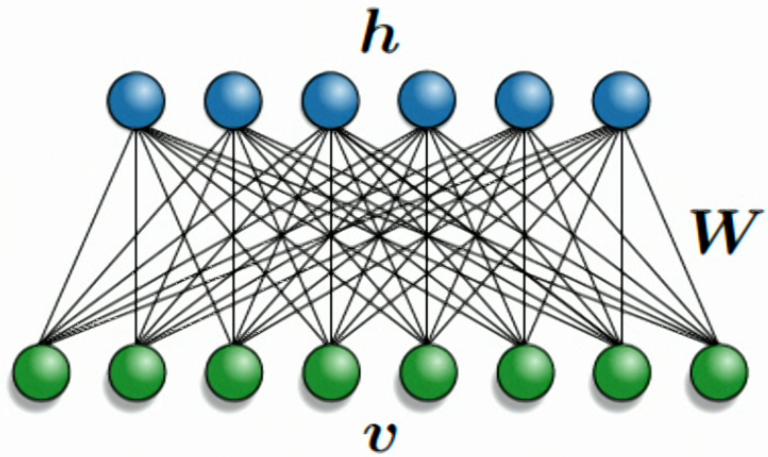

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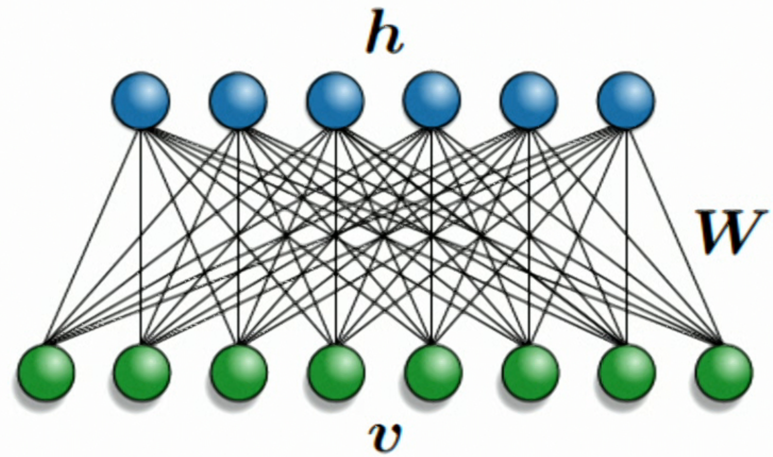

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- QST: two RBMs (following G. Torlai et al. (2018)):
  a. for learning the **amplitude** of the state
  b. for learning the **phase structure** of the state
     → (local) rotations to different measurement configurations are needed!

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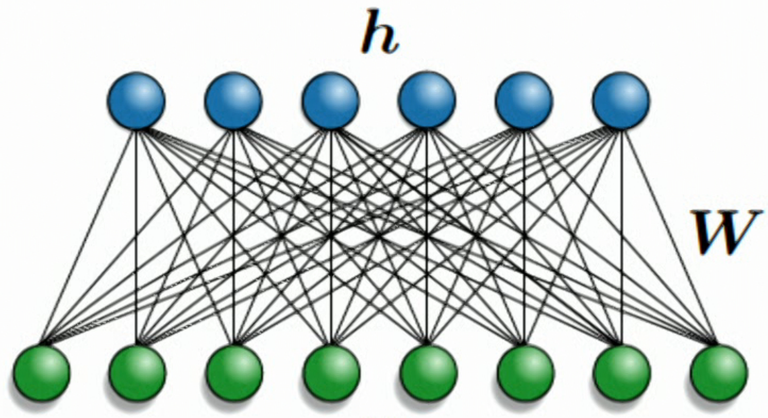

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$$\psi_{\{W,b,c\},\{W',b',c'\}}(\boldsymbol{x}) = \sqrt{\frac{p_{\{W,b,c\}}(\boldsymbol{x})}{Z_{\{W,b,c\}}}}\, e^{i\theta_{\{W',b',c'\}}(\boldsymbol{x})/2}$$

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NN BASED QUANTUM STATE TOMOGRAPHY

Neural-network based QST using RBMs:

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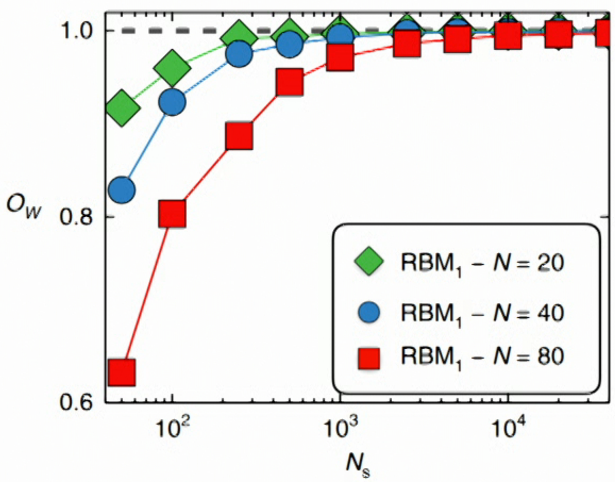

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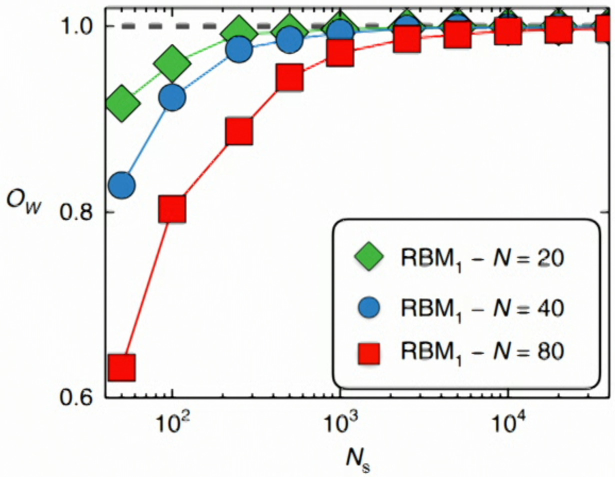

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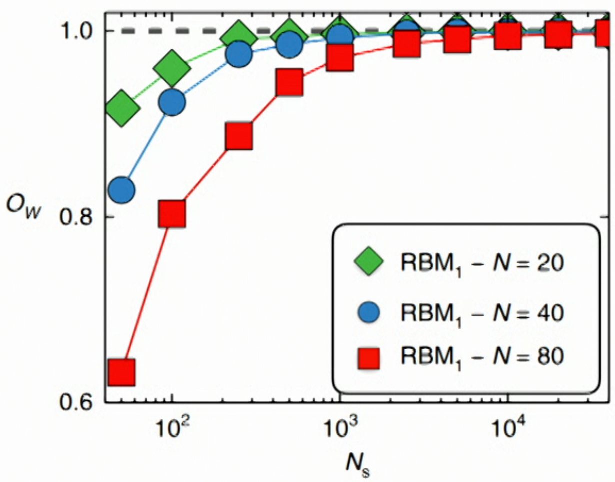
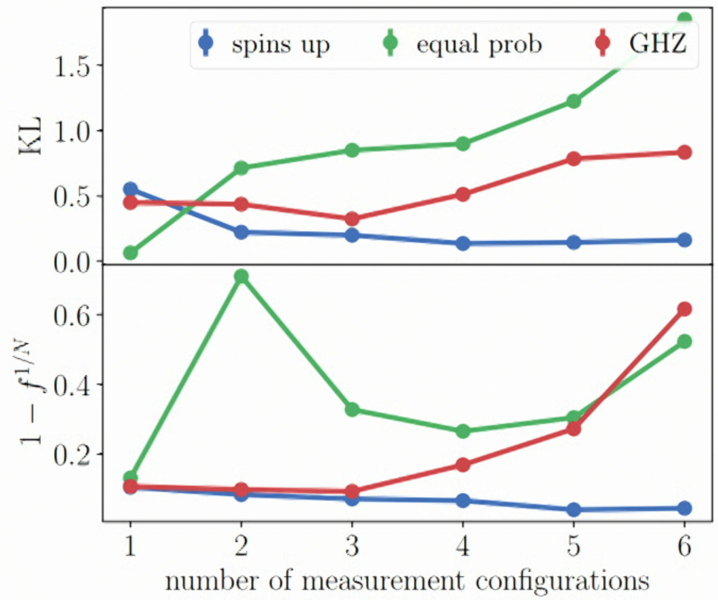


[G. Torlai et al. (2018)]

- **BUT:** when I want to do this…



QST WITH ACTIVE LEARNING

passive machine learning,
e.g. RBMs

active learning

e.g. B. Settles (2009)

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ACTIVE LEARNING SCHEME

$$\psi_{\lambda,\mu}(\boldsymbol{x}) = \sqrt{\frac{p_{\lambda}(\boldsymbol{x})}{Z_{\{\boldsymbol{W},\boldsymbol{b},\boldsymbol{c}\}}}}\, e^{i\theta_{\mu}(\boldsymbol{x})/2}$$

rotations

reference basis

Lange, Kebric, Buser, Schollwöck, Grusdt and Bohrdt, arXiv:*2203.15719*

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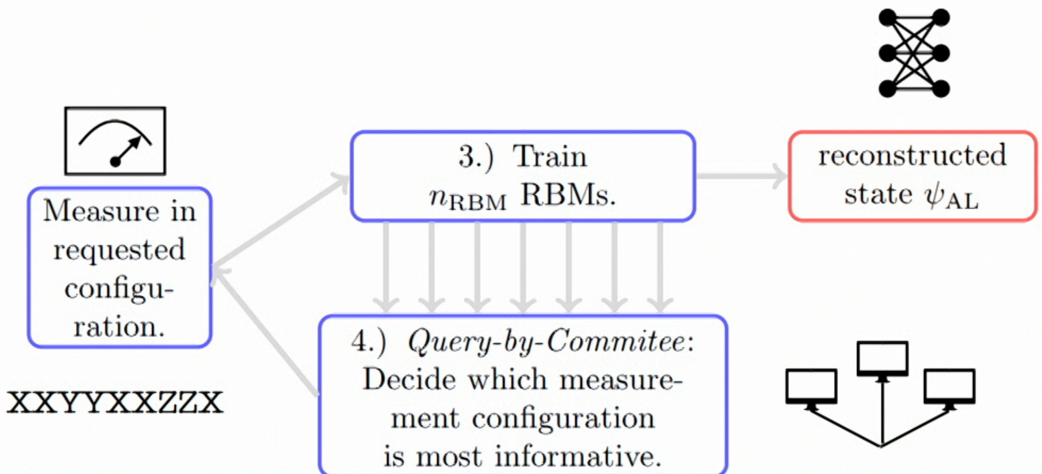

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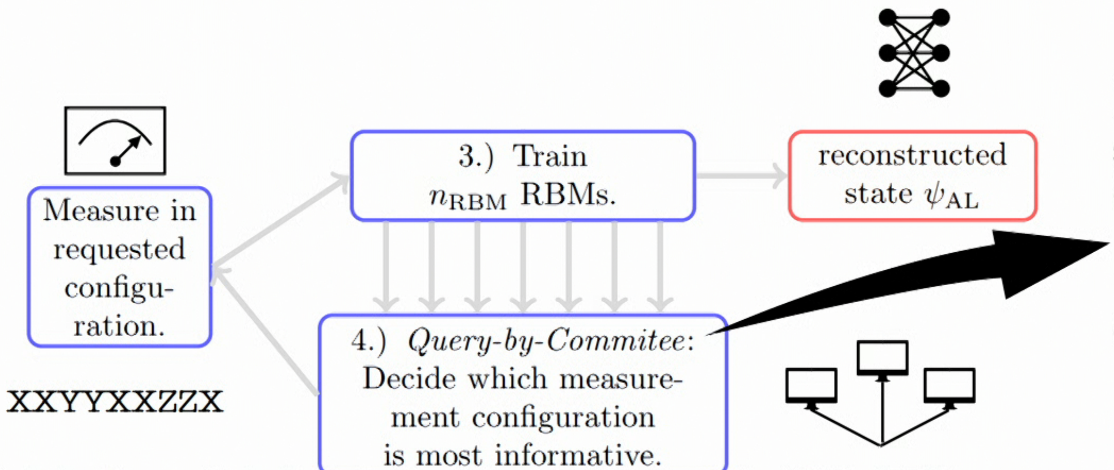

Look at phases and amplitudes of all RBMs separately:

- if **var(amplitudes) > var(phases)**: request samples from the reference configuration
- **else**: rotate $\psi_{\lambda,\mu}(\boldsymbol{x})$ for all RBMs to different measurement configurations and select the one where RBMs disagree most

Lange, Kebric, Buser, Schollwöck, Grusdt and Bohrdt, arXiv:*2203.15719*

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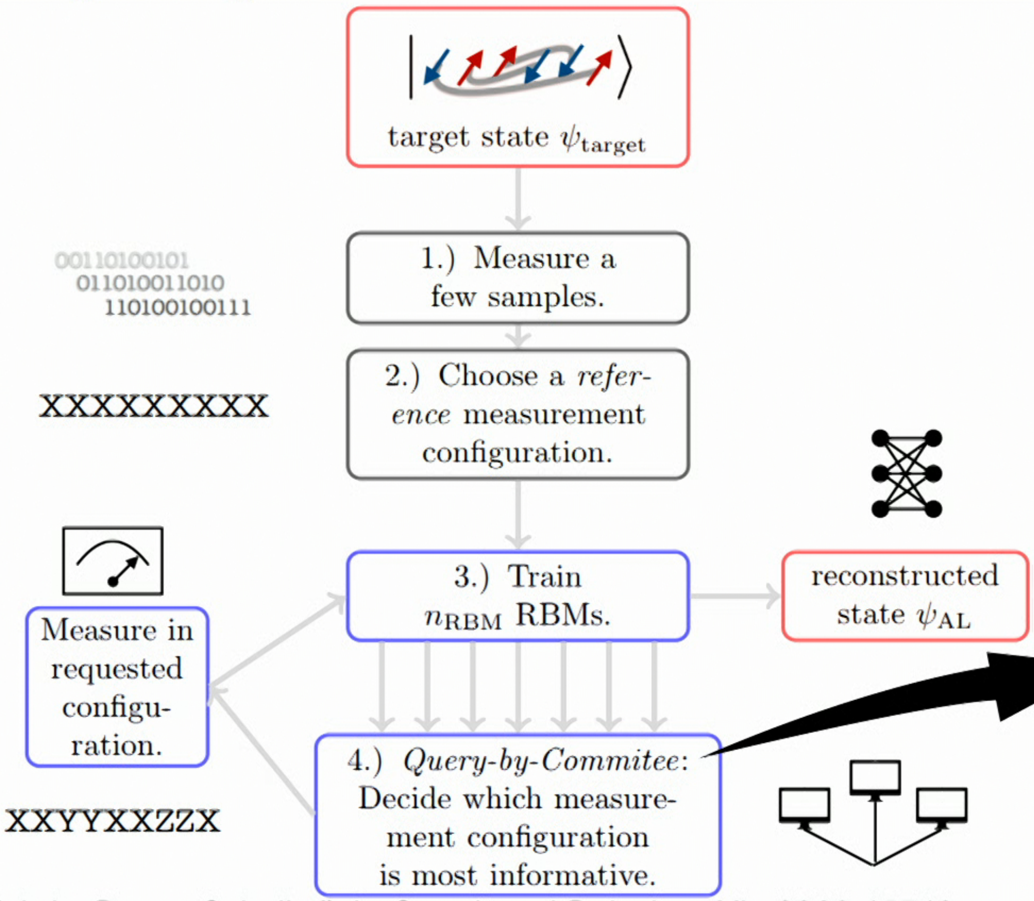

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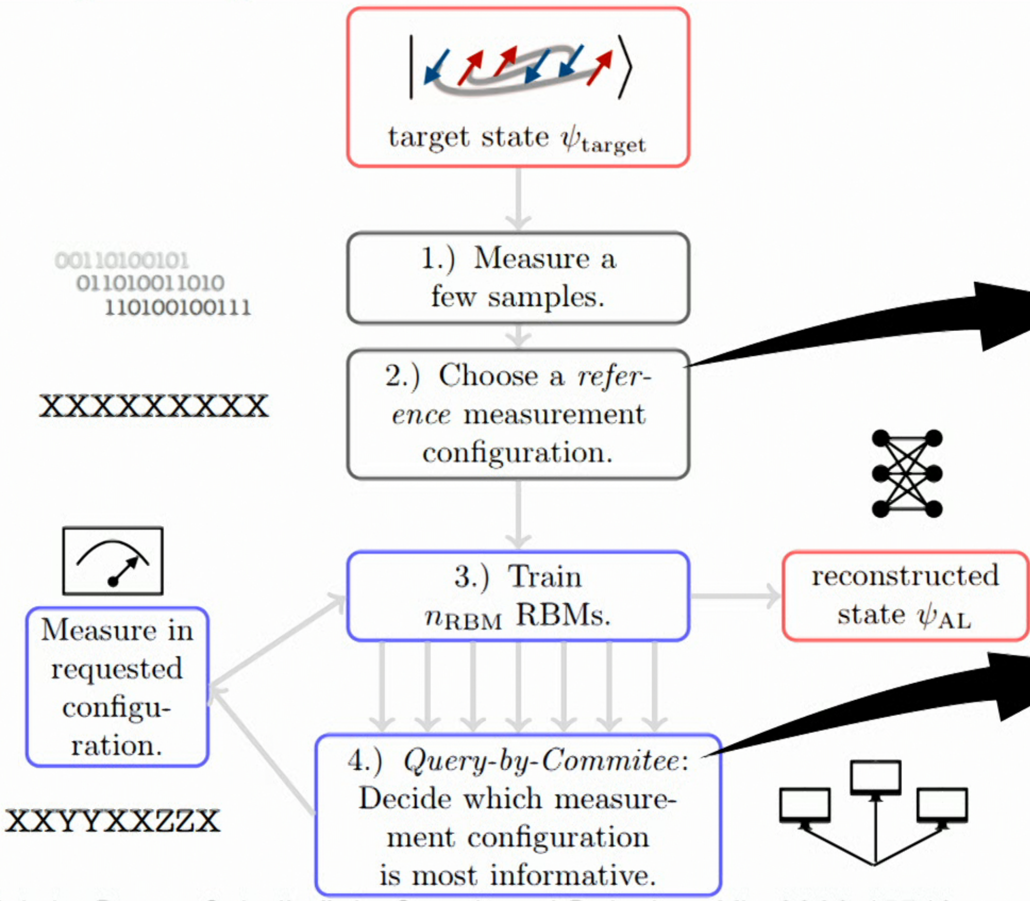

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reference basis — rotations

take measurements from the *XX…X*, *YY…Y* and *ZZ…Z* configurations and choose the one where RBMs **agree most** on the reconstructed state

Look at phases and amplitudes of all RBMs separately:

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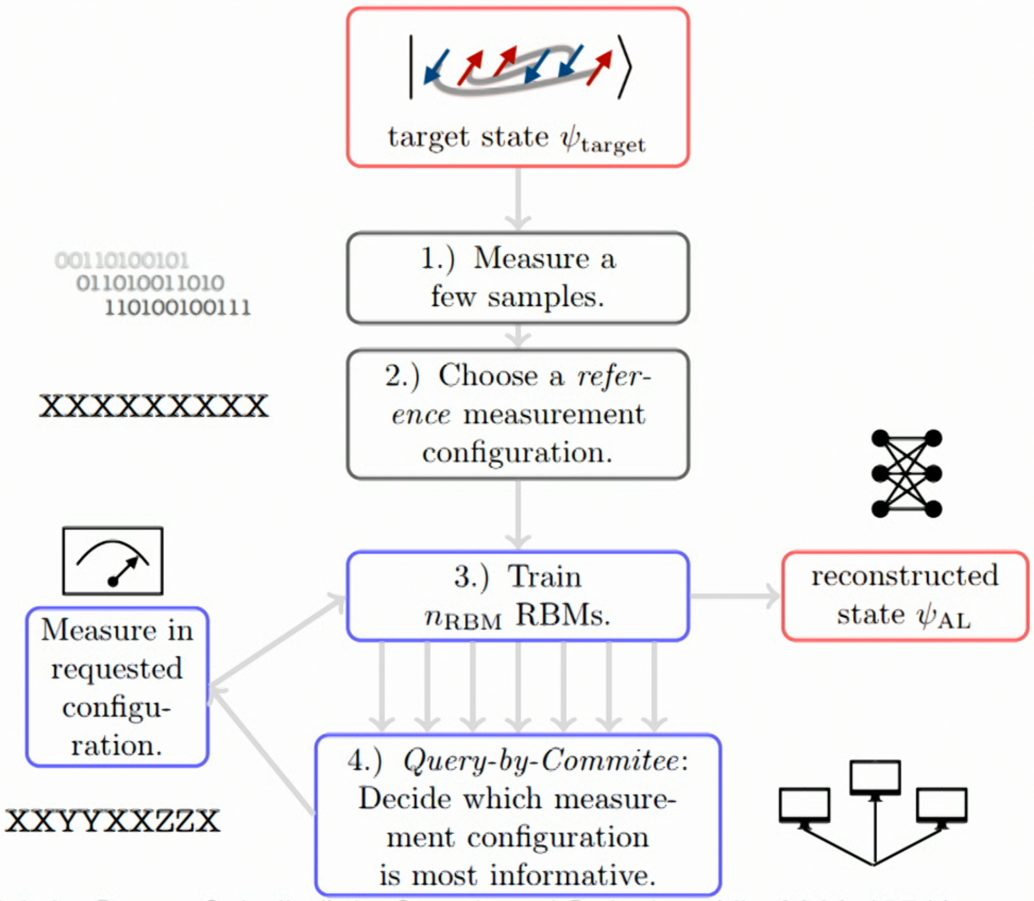
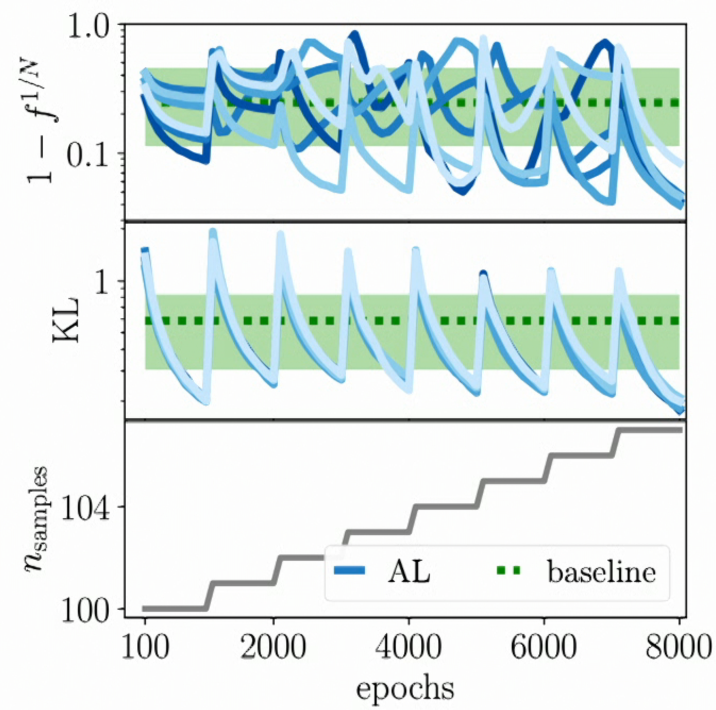


GHZ state with phase structure (5 qubits):



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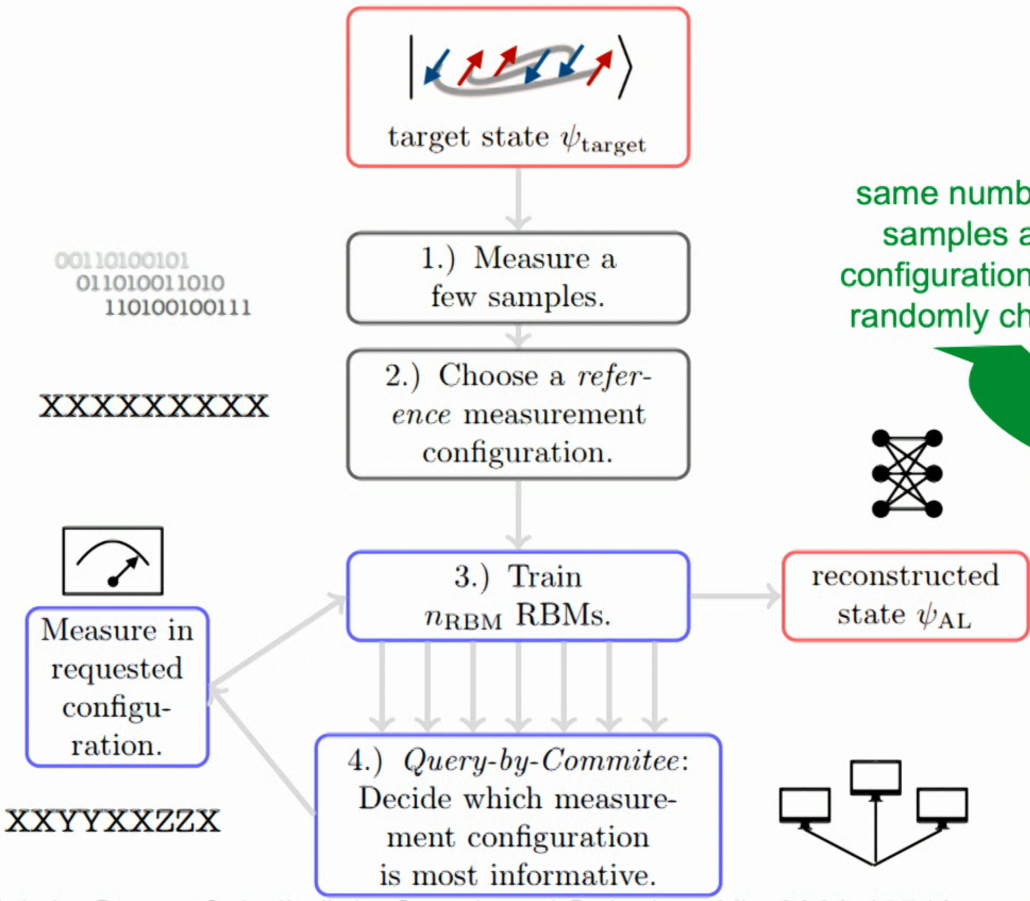

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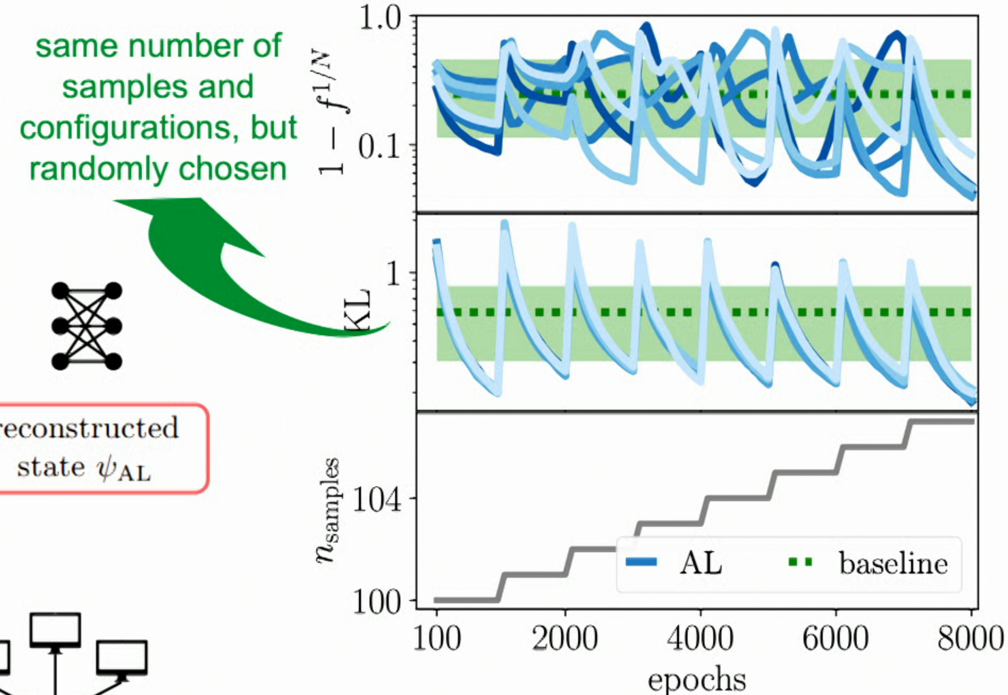

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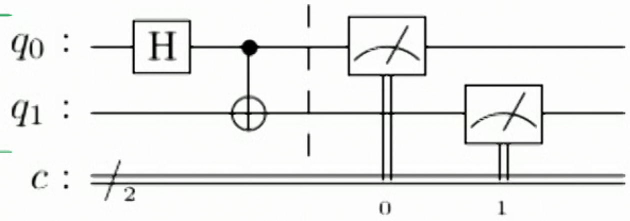
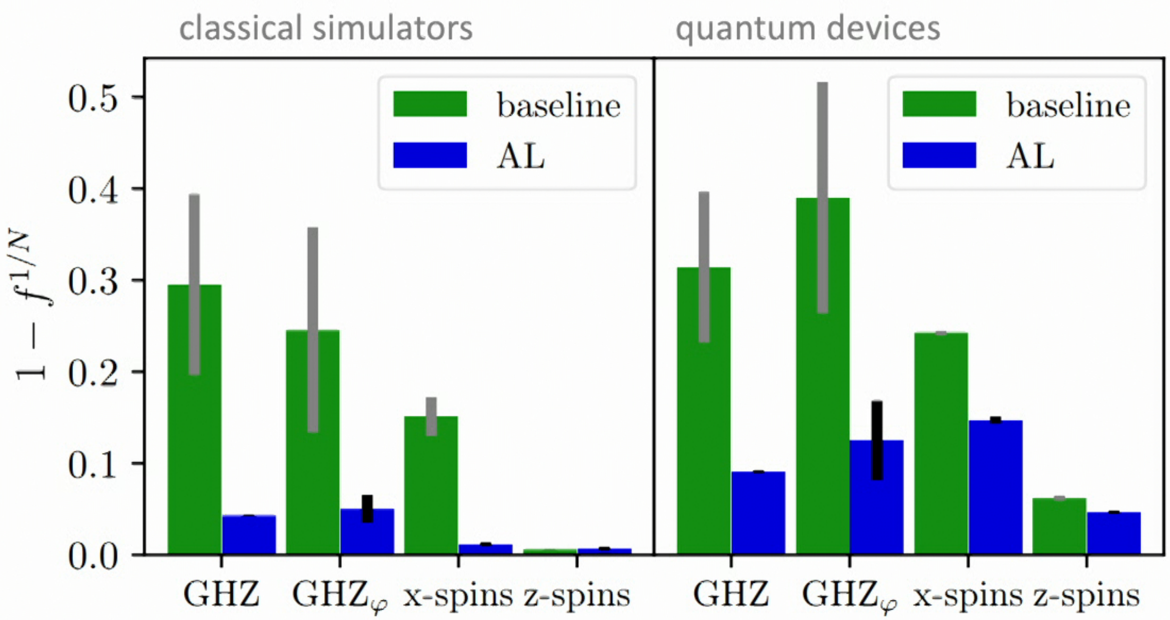
RESULTS



IBM Quantum Experience:

- Greenberger–Horne–Zeilinger (GHZ) state: $|\mathrm{GHZ}\rangle = 1/\sqrt{2}\,(|0\ldots0\rangle + |1\ldots1\rangle)$
- GHZ with phase $|\mathrm{GHZ}_\varphi\rangle = 1/\sqrt{2}\,(|0\ldots0\rangle + i\,|1\ldots1\rangle)$
- polarized states with all spins in z or x direction



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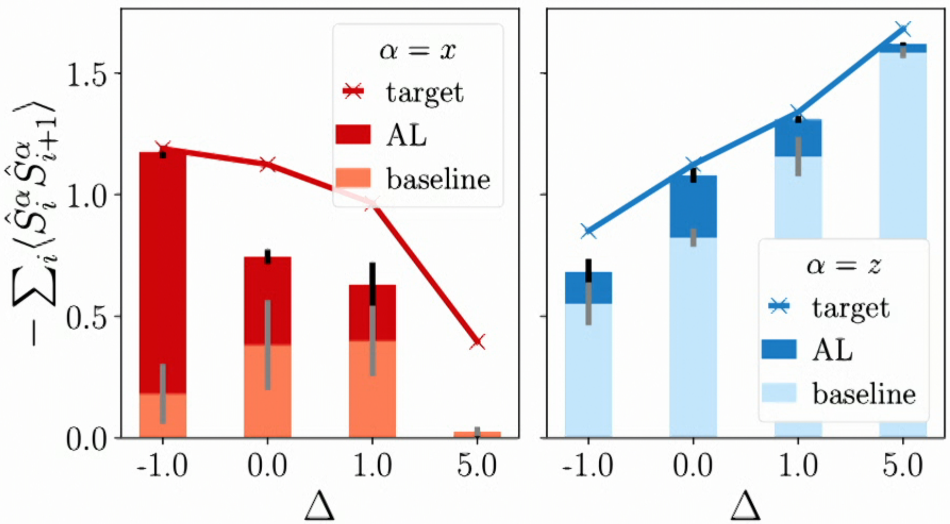
RESULTS

DMRG Simulations: 1) XXZ Model:

$$H_{XXZ} = \sum_i J\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right) + J(1+\Delta) S_i^z S_{i+1}^z$$



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RESULTS

Matjaz Kebric

DMRG Simulations: 2) $\mathbb{Z}_2$ Lattice Gauge Model [Borla et al., Phys.Rev.Lett. (2018)]

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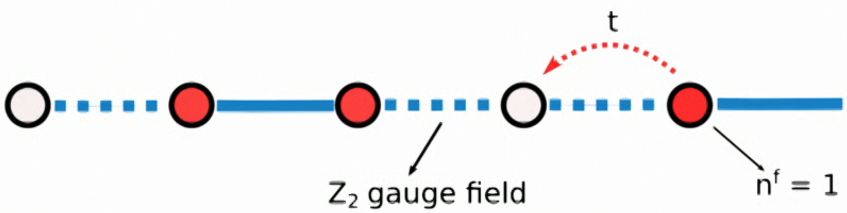

$$H_{KCS} = -t\sum_i (c_i^+ \tau_{i,i+1}^z c_{i+1} + H.c.) - h\sum_i \tau_{i,i+1}^x$$

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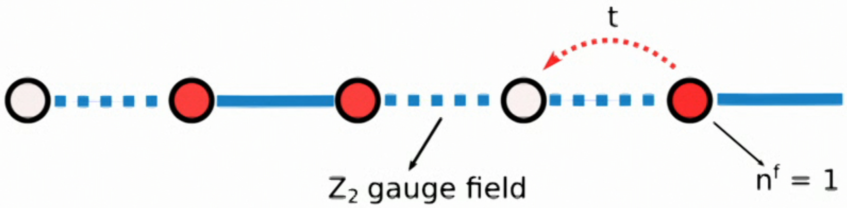

$$H_{KCS} = -t \sum_i (c_i^\dagger \tau^z_{i,i+1} c_{i+1} + H.c.) - h \sum_i \tau^x_{i,i+1}$$

+ physical constraint:

Gauss' Law:

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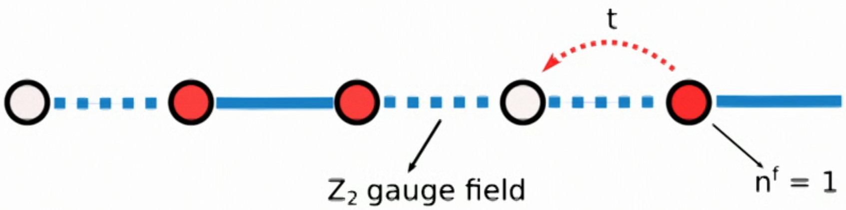

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Gauss' Law: 

$$\tau^x_{i-1,i}\, \tau^x_{i,i+1} = \begin{cases} -1, & n_i^f = 1 \\ 1, & n_i^f = 0 \end{cases} = (1 - 2n_i^f)$$

$$\Leftrightarrow \quad n_i^f = \frac{1}{2}(1 - \tau^x_{i-1,i}\, \tau^x_{i,i+1})$$

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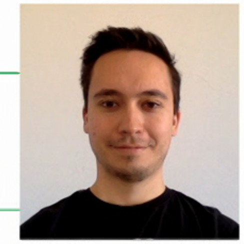
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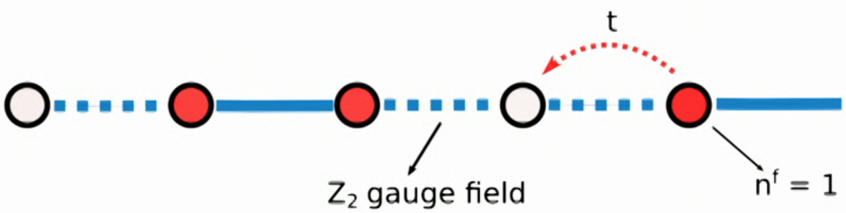

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+ physical constraint: conservation of particles $+\mu\sum_i n_i^f$

→ can be mapped to spin-$\frac{1}{2}$ model:

$$H_{KCS} = t\sum_i (4S^x_{i-1}S^x_{i+1} - 1)S^z_i - 2h\sum_i S^x_i + \mu\sum_i S^x_{i+1}S^x_i$$

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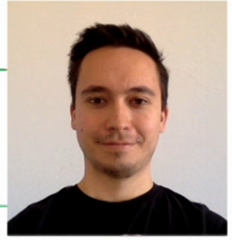

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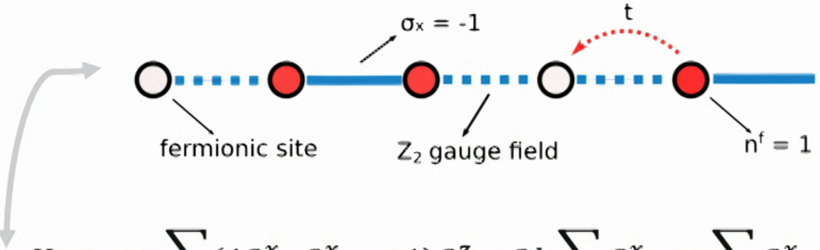

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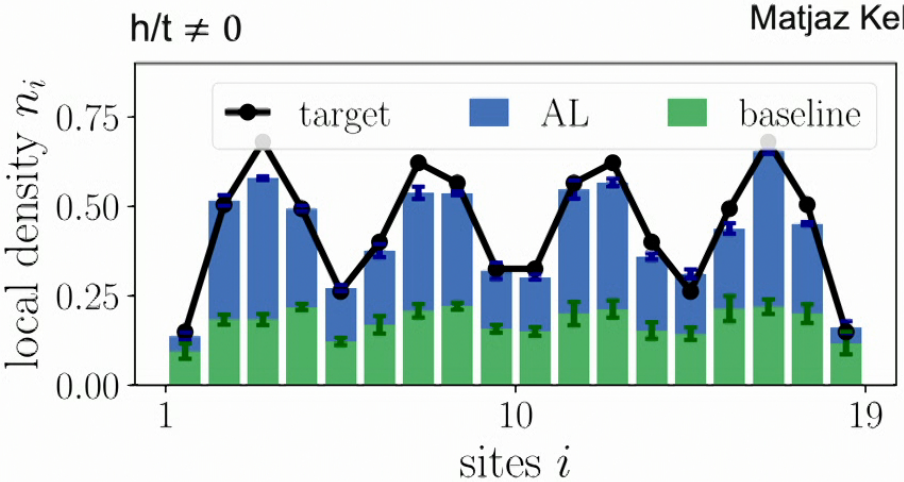

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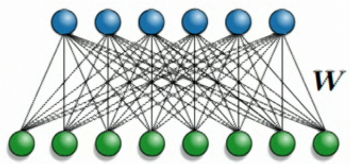
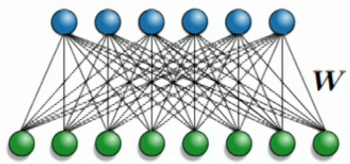
CONCLUSION



- RBMs as building blocks of our adaptive scheme

CONCLUSION



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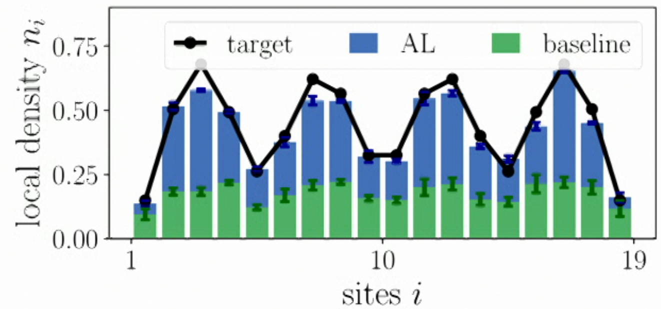

- The reconstruction results for
  - states generated on IBM devices
  - XXZ and KCS Model states

  were improved by requesting the most informative samples after each learning cycle (active learning = smartly choosing the measurement configurations) compared to a RBM reconstruction with the same number of samples and measurement configurations.

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FUTURE DIRECTIONS?

- representations with more efficient sampling? [e.g. Hibat-Allah et al. (2020), Czischek et al. (2022),…]

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FUTURE DIRECTIONS?

- representations with more efficient sampling? [e.g. Hibat-Allah et al. (2020), Czischek et al. (2022),…]
- Extensions to finite temperatures, fermionic systems…
  - ➢ mixed states [e.g. Torlai (2018)]
  - ➢ antisymmetric wavefunctions [e.g. Humeniuk et al. (2021), Moreno et al. (2022), Inui et al. (2021), …]

THANK YOU!

Matjaz Kebric

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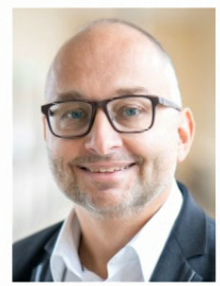
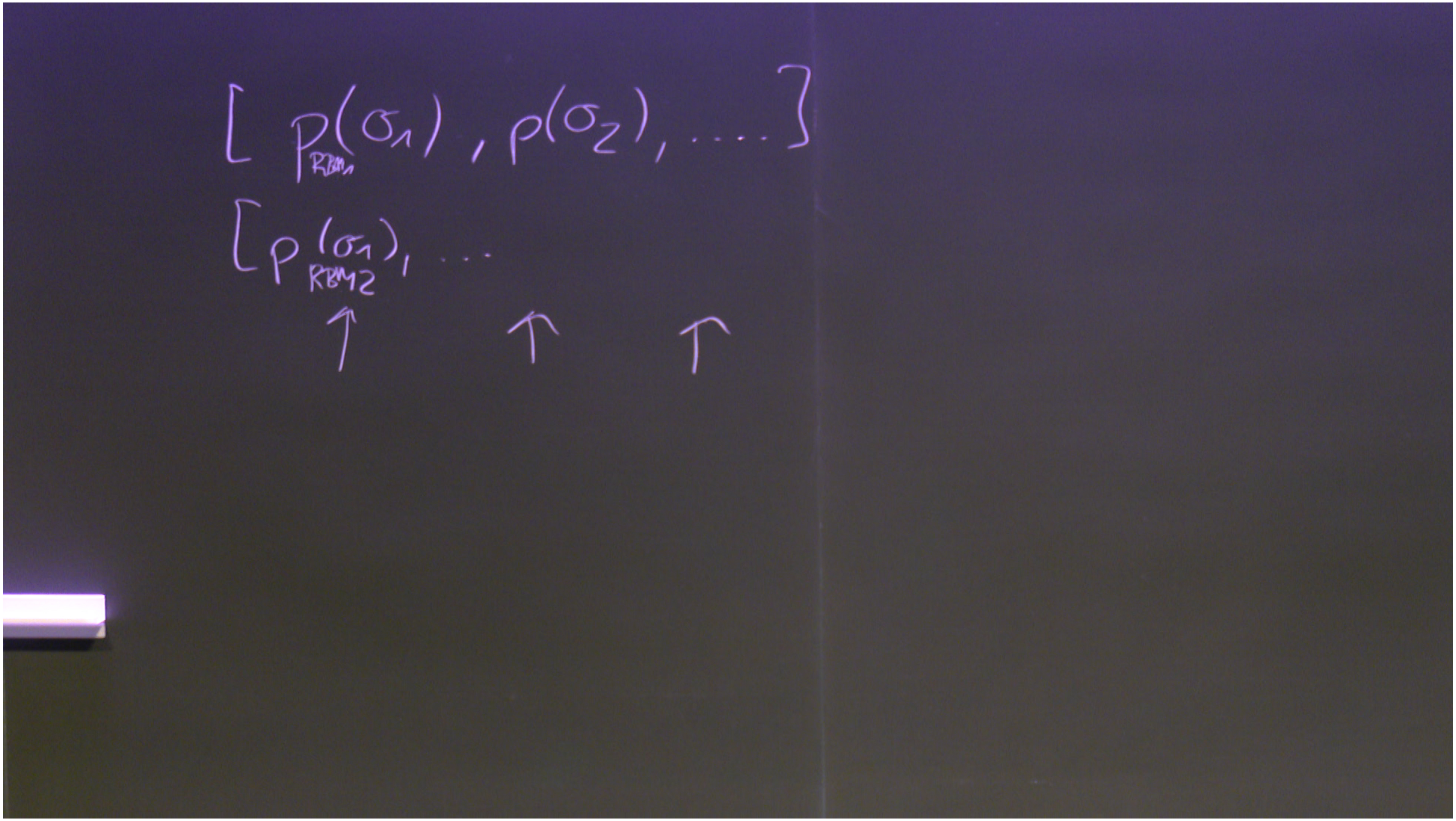


Uli Schollwöck


[p_RBM1(σ1) , p(σ2), ...]
[p_RBM2(σ1), ...