Title: Representing quantum states with spiking neural networks

Speakers: Stefanie Czischek

Collection: New Frontiers in Machine Learning and Quantum

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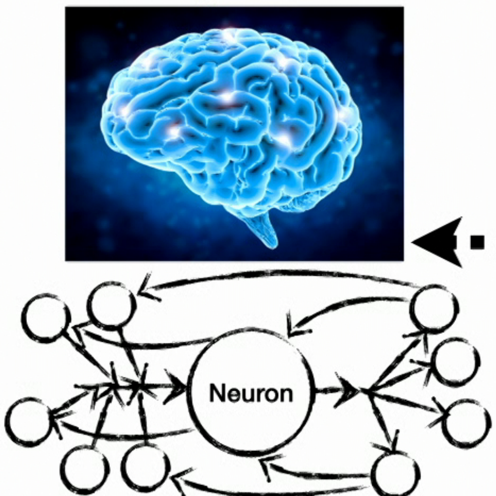
Representing quantum states with spiking neural networks

Stefanie Czischek

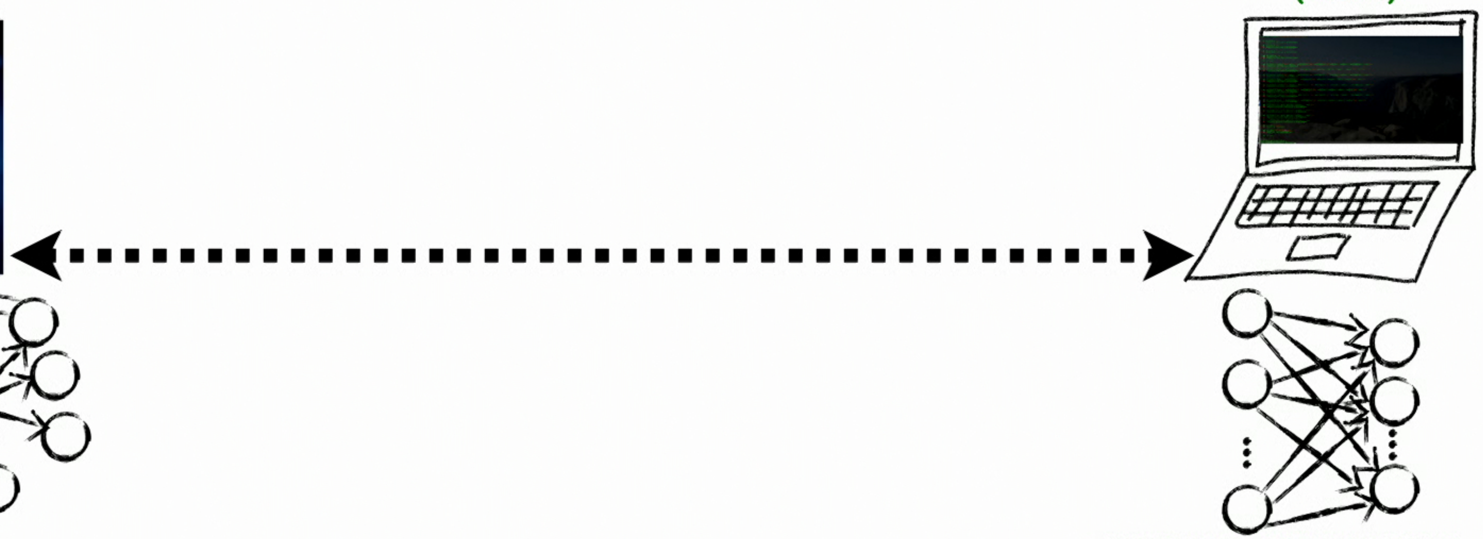
November 23, 2022

Spiking neural networks

Biological Brains



Artificial Neural Networks (ANNs)



Fast, energy efficient, small

Efficient computation

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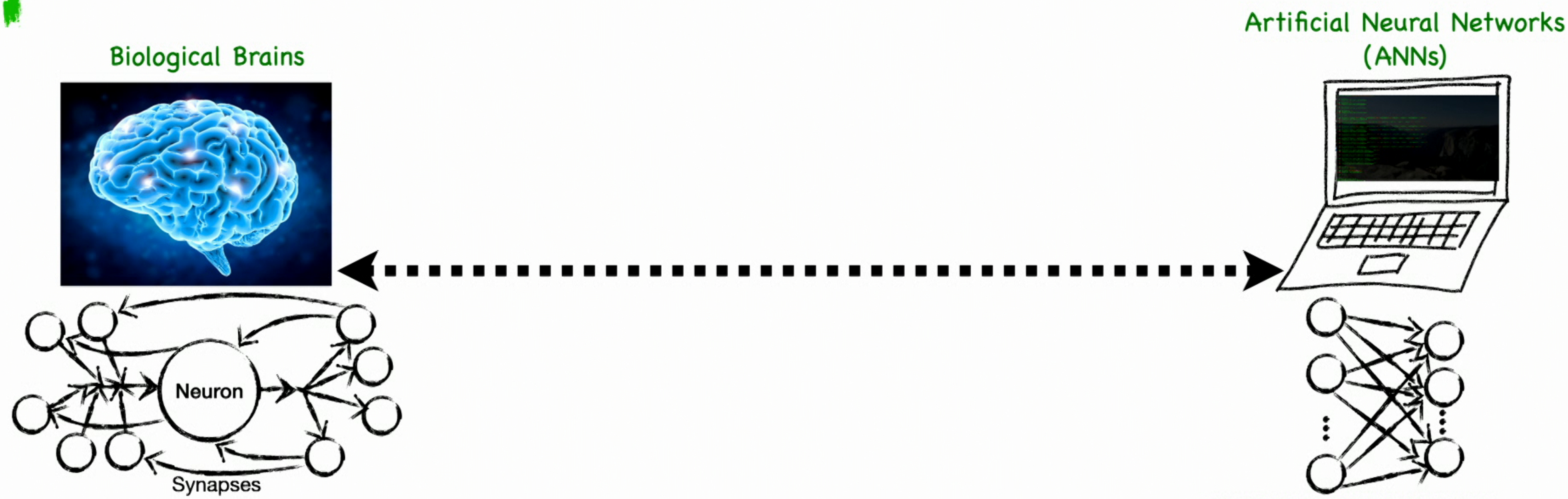
Spiking neural networks



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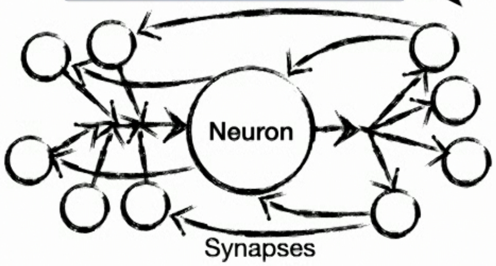

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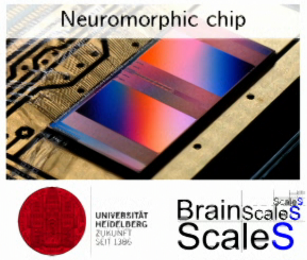
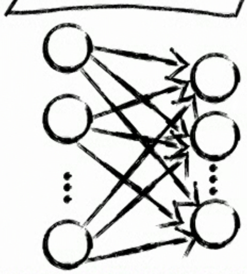


Spiking Neural Networks

- Efficient approximation of biology
- Signal-driven information propagation

Artificial Neural Networks (ANNs)



Efficient computation

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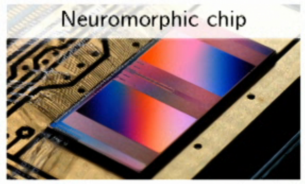
Spiking neural networks

Biological Brains



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BrainScaleS

Spiking Neural Networks

- Efficient approximation of biology
- Signal-driven information propagation

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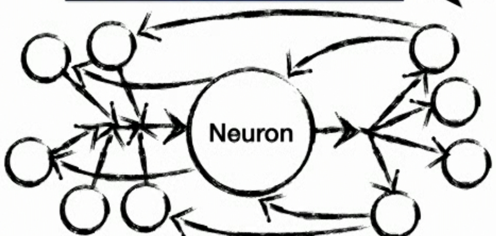

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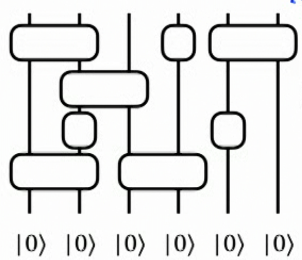
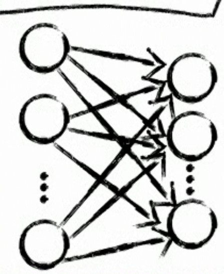


[J. Carrasquilla, Adv. Phys. X **5** (2020)]
[A. Dawid et al., arXiv:2204.04198 (2022)]

ANNs for quantum technologies



Efficient computation

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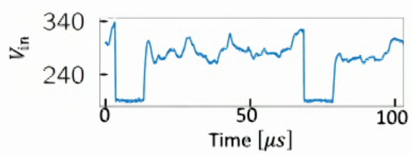
Biological neurons

Artificial neuron:

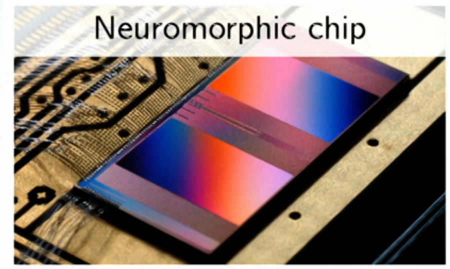
$$h_i^k = f\left[\sum_j h_j^{k-1} W_{i,j}^{k-1,k}\right]$$

discrete evaluation

Biological neuron:



continuous in time



Neuromorphic chip

- Common approximation:
  Leaky Integrate-and-Fire (LIF) neurons
  - Can be modelled in electronic circuits

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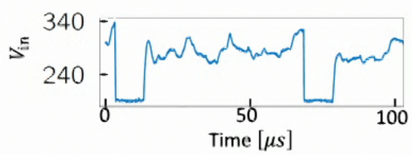

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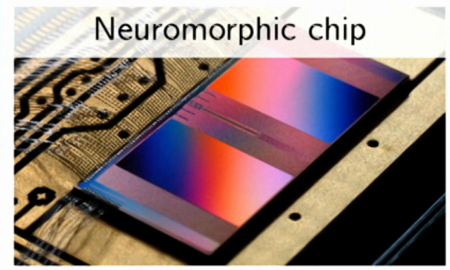

- Common approximation:
  Leaky Integrate-and-Fire (LIF) neurons
  - Can be modelled in electronic circuits

Spiking neurmorphic hardware emulates network of LIF neurons

Spiking neuromorphic devices:

- BrainScaleS (Heidelberg University)
- SpiNNaker (University of Manchester)
- Loihi (Intel)

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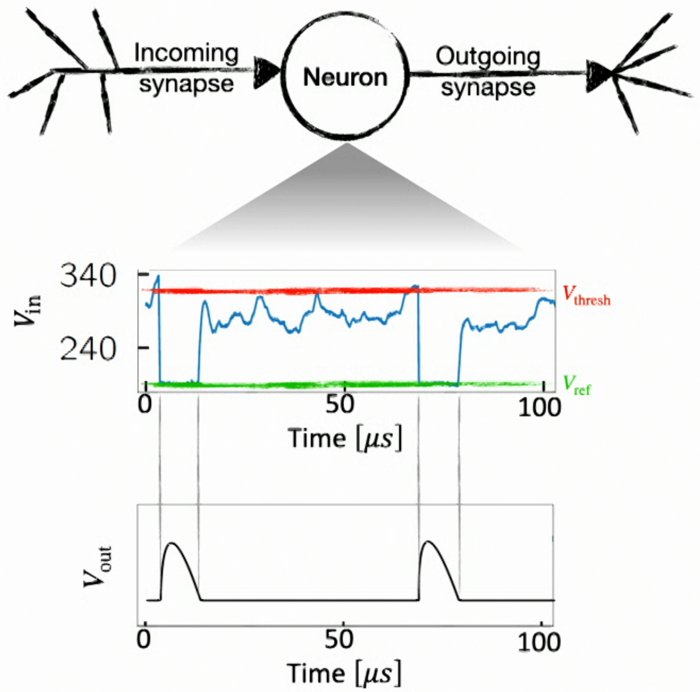
LIF neurons



$$V_{in}(i) \propto \sum_j V_{out}(j)\, W_{i,j}$$

- Membrane potential V_{in} evolves in time
- Spike: V_{in} crosses V_{thresh}
  - V_{in} fixed to V_{ref}
  - Leaky signal sent on V_{out}

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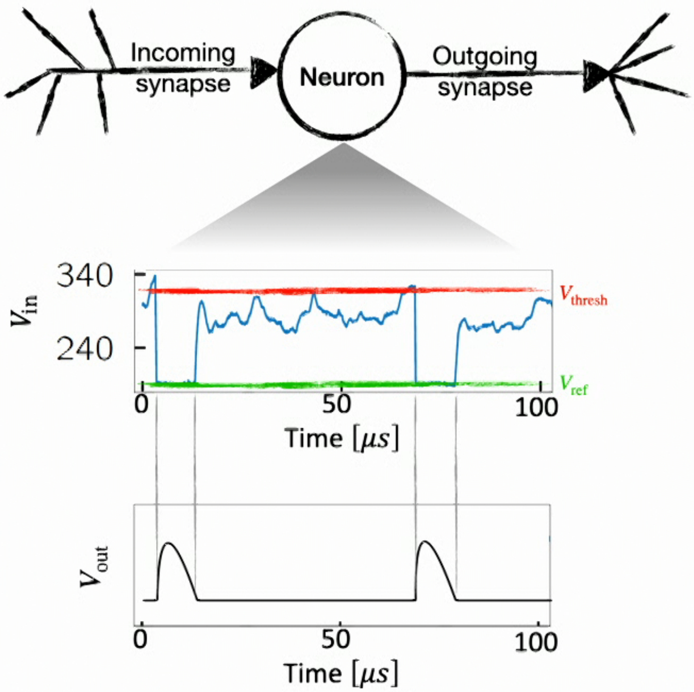
LIF neurons



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- Membrane potential V_{in} evolves in time
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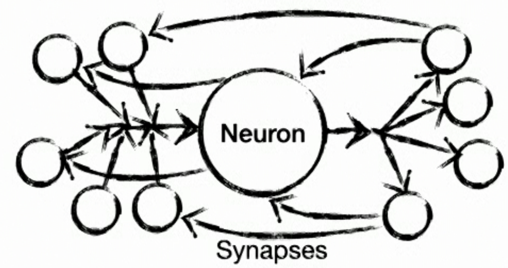
  - Leaky signal sent on V_{out}
- Neuron connections are weighted similar to ANNs: trainable



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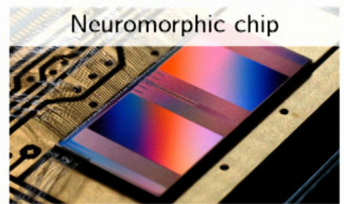
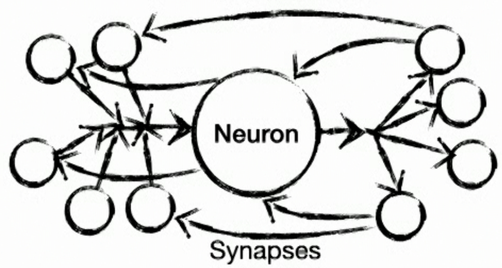
Spiking neurons for sampling

- Applications:
  - classification
  - time-dependent data, recurrent nature,…
  - data generation

BrainScaleS-2





[SC, et al., SciPost Physics **12** (2022)]

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Spiking neurons for sampling

- Applications:
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- Binary interpretation: refractory or not
- Read neuron states
  - Sample from probability distribution
- Train connecting weights

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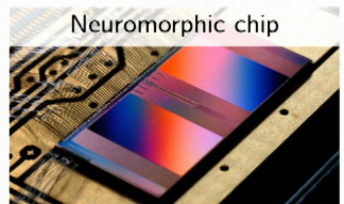

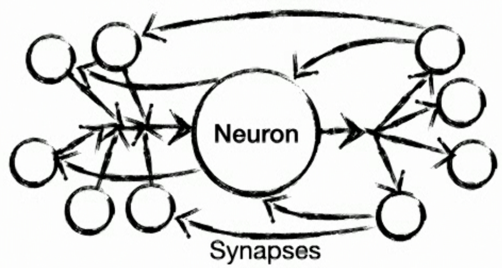

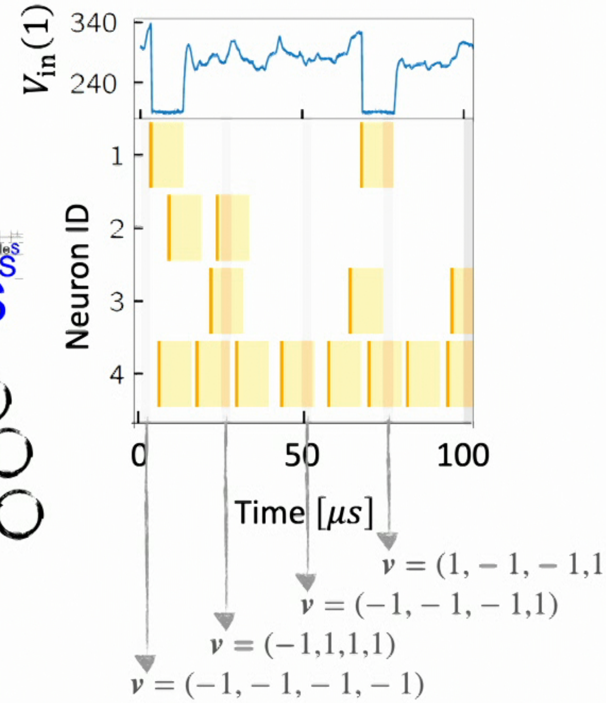

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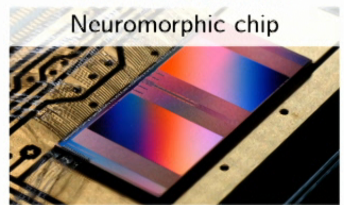

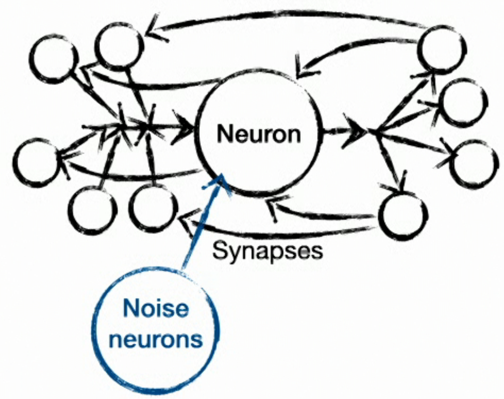

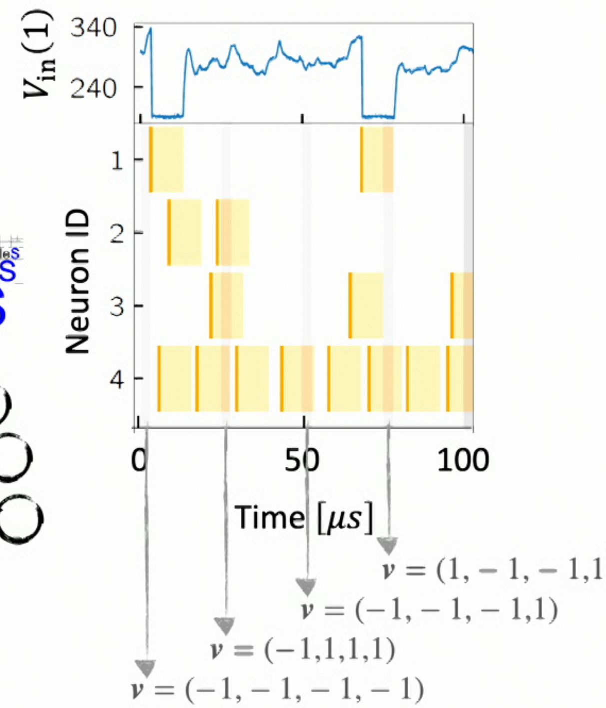

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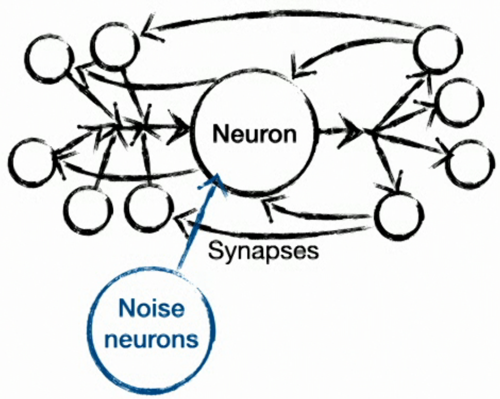

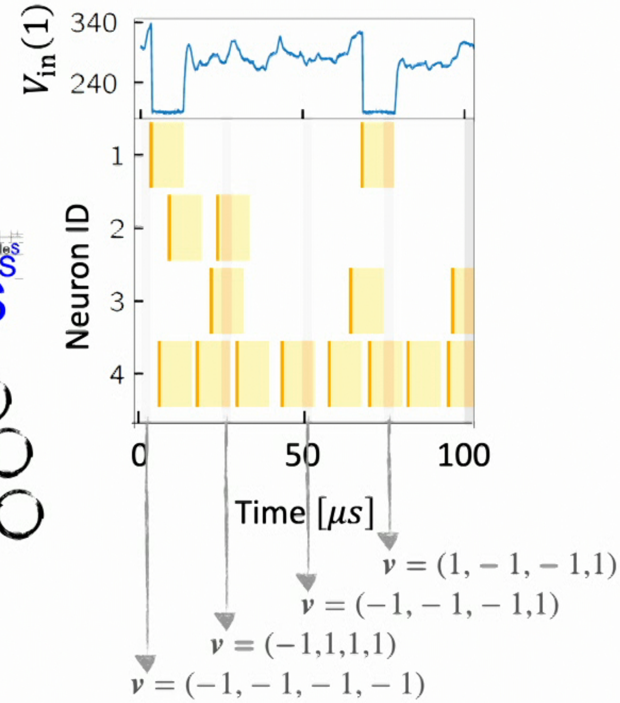

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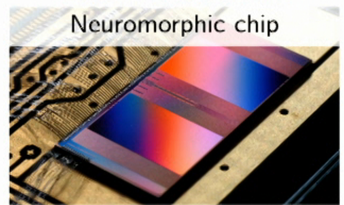

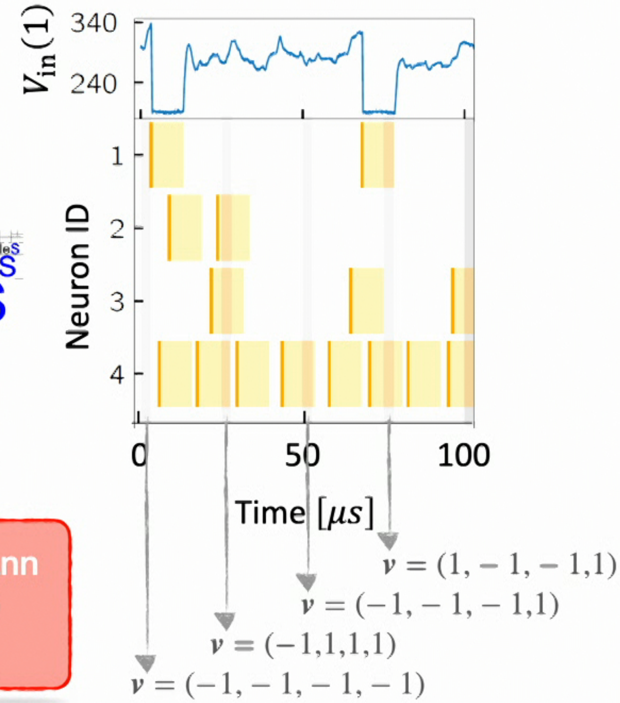

Approximate restricted Boltzmann machine: ANN model used to represent quantum states

[M. Petrovici et al., BMC Neurosci. **16** (2015)]

[SC, et al., SciPost Physics **12** (2022)]

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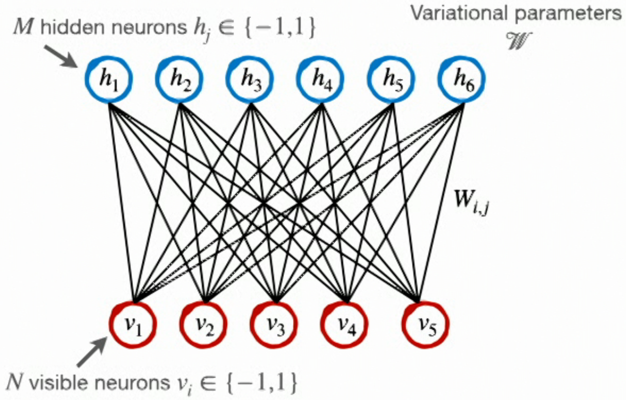
Neural network quantum states

Restricted Boltzmann Machine 🦑



$$E\left[\boldsymbol{v},\boldsymbol{h};\mathcal{W}\right] = -\sum_{i,j} v_i W_{i,j} h_j$$

$$P_{\mathcal{W}}\left[\boldsymbol{v}\right] = \frac{1}{Z\left(\mathcal{W}\right)} \sum_{\{\boldsymbol{h}\}} e^{-E\left[\boldsymbol{v},\boldsymbol{h};\mathcal{W}\right]}$$

$$Z\left(\mathcal{W}\right) = \sum_{\{\boldsymbol{v}\},\{\boldsymbol{h}\}} e^{-E\left[\boldsymbol{v},\boldsymbol{h};\mathcal{W}\right]}$$

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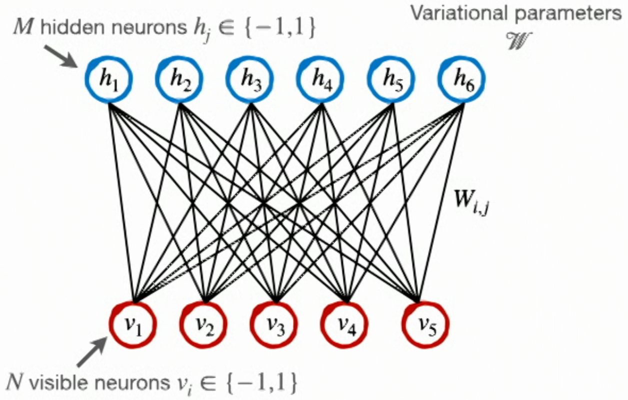
Neural network quantum states

Restricted Boltzmann Machine



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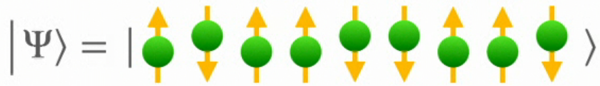
Quantum State



$$|\Psi\rangle = \sum_{\{\boldsymbol{\varphi}\}} \Psi(\boldsymbol{\varphi}) |\boldsymbol{\varphi}\rangle \in \mathbb{C}^N \qquad \Psi(\boldsymbol{\varphi}) \in \mathbb{C}$$

$$\left\langle \mathcal{O}_{\text{diag}} \right\rangle = \sum_{\{\boldsymbol{\varphi}\}} \mathcal{O}_{\text{diag}}\left(\boldsymbol{\varphi}\right) \Psi(\boldsymbol{\varphi})\Psi(\boldsymbol{\varphi})^*$$

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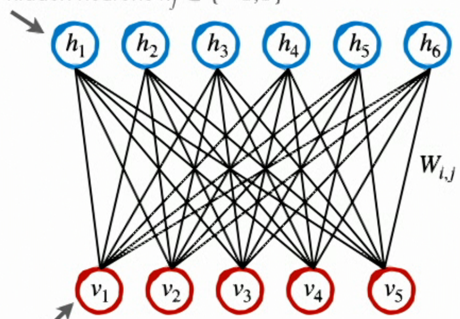
Neural network quantum states

Restricted Boltzmann Machine

M hidden neurons $h_j \in \{-1,1\}$

Variational parameters $\mathscr{W}$



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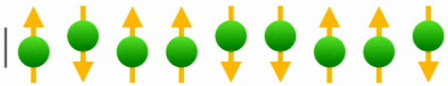
Quantum State

$|\Psi\rangle = |$  $\rangle$

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Train the restricted Boltzmann machine such that $P_{\mathscr{W}}\left[\boldsymbol{v}\right] \approx \Psi(\boldsymbol{v})\Psi(\boldsymbol{v})^*$

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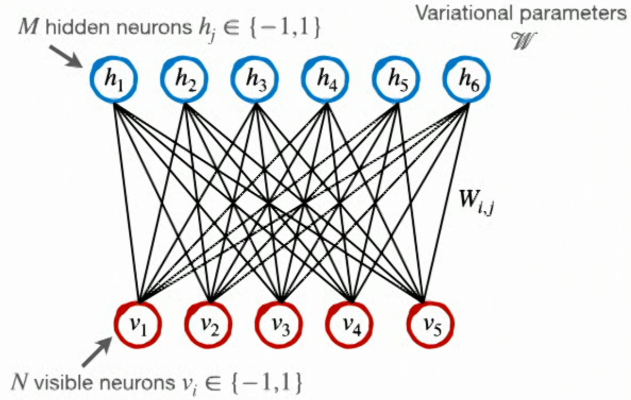
Neural network quantum states

Restricted Boltzmann Machine



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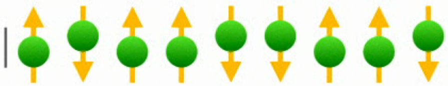
$|\Psi\rangle = |$  $\rangle$

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Train the restricted Boltzmann machine such that $P_{\mathscr{W}}\left[\boldsymbol{v}\right] \approx \Psi(\boldsymbol{v})\Psi(\boldsymbol{v})^*$

Works only if
$\mathrm{Im}\left[\Psi(\boldsymbol{v})\right] = 0 \ \forall \boldsymbol{v}$

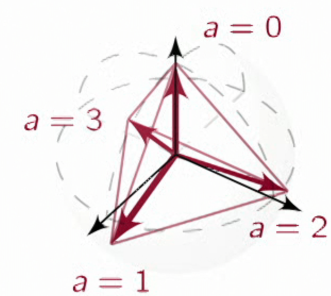
Positive operator-valued measure (POVM) representation

$P(\boldsymbol{a}) \in \mathbb{R}$

$P(\boldsymbol{a}) \geq 0 \qquad \sum_{\{\boldsymbol{a}\}} P(\boldsymbol{a}) = 1$

$\rho = \sum_{\{\boldsymbol{a}\}} P(\boldsymbol{a})\, \mathcal{Q}(\boldsymbol{a})$

Mapping operators



[J. Carrasquilla, et al., Nat. Machine Intel. (2019)]

Uniquely represent density matrix in terms of probability distribution

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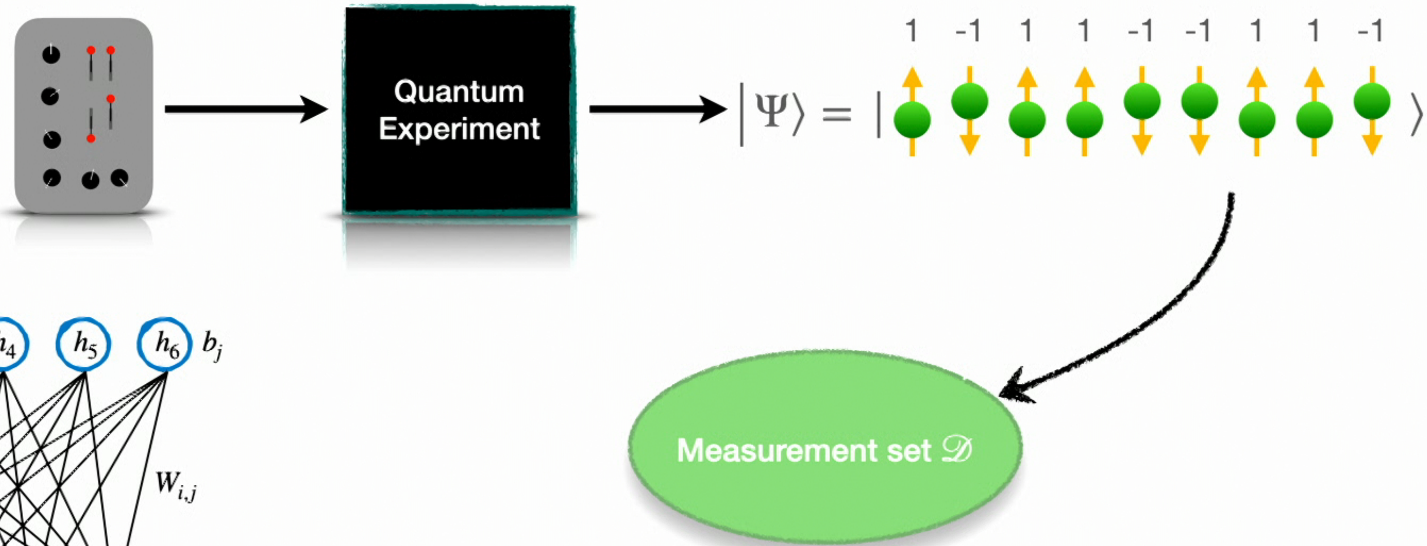
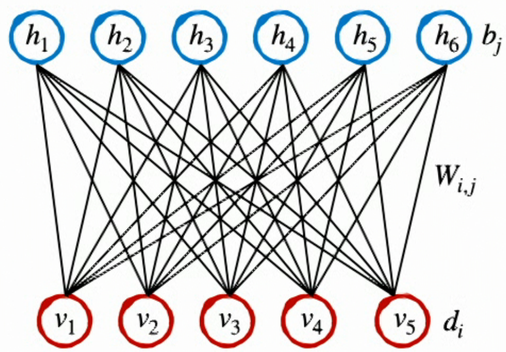
Network training





$p_{\mathscr{D}}(\boldsymbol{\varphi})$

Approximate

$$P_{\mathscr{W}}[\boldsymbol{v}] = \frac{1}{Z(\mathscr{W})} \sum_{\{\boldsymbol{h}\}} e^{-E[\boldsymbol{v},\boldsymbol{h};\mathscr{W}]}$$

- Optimize $p_{\mathscr{W}}[\boldsymbol{v}] \approx p_{\mathscr{D}}[\boldsymbol{\varphi}]$
- Minimize Kullback-Leibler divergence

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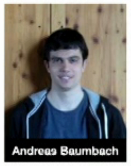
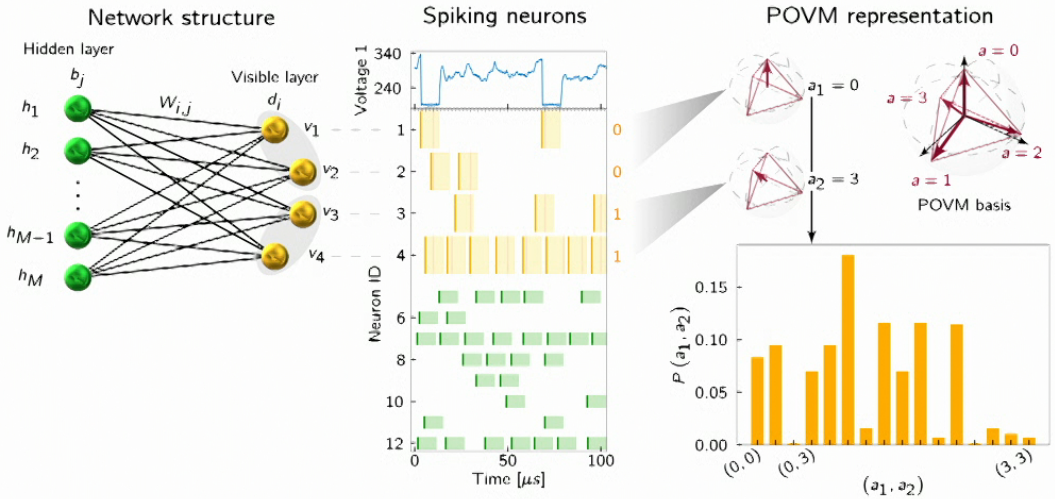
Spiking quantum states





[SC, et al., SciPost Physics **12** (2022)]

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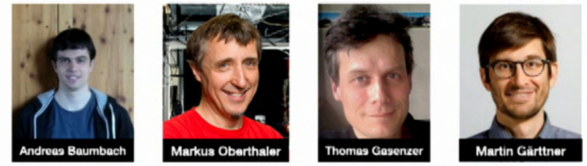

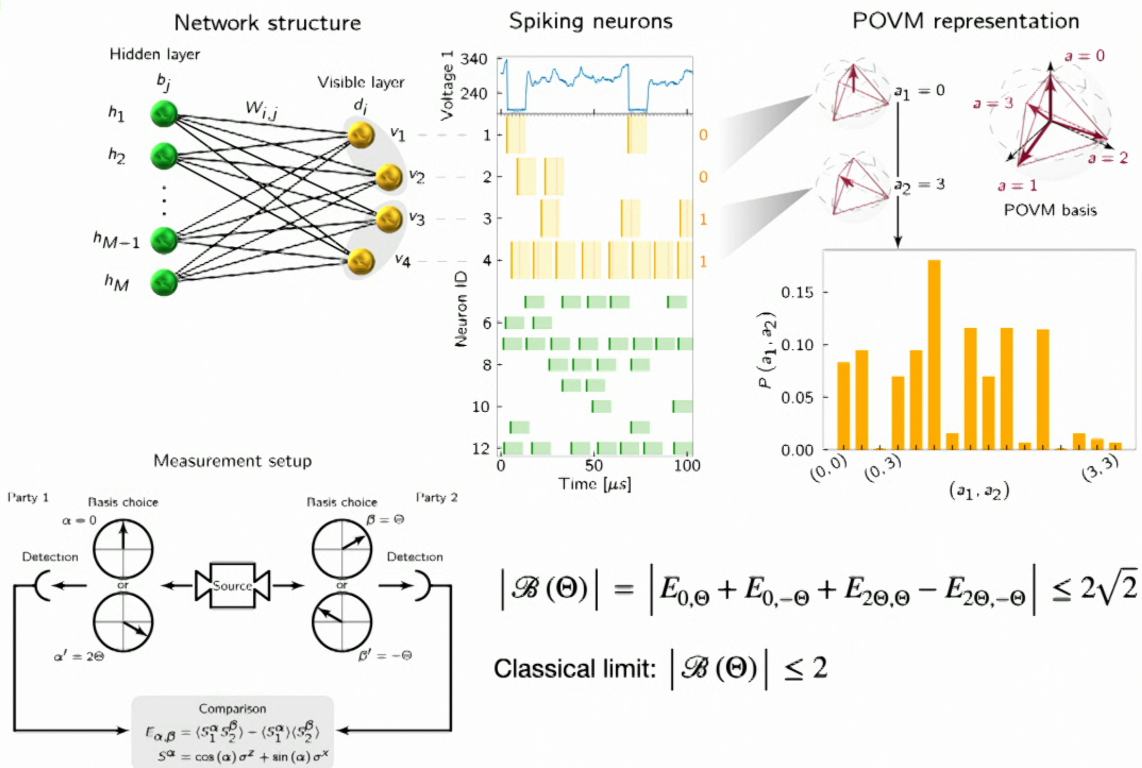

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\right]$$

$$\left|\mathscr{B}(\Theta)\right| = \left|E_{0,\Theta} + E_{0,-\Theta} + E_{2\Theta,\Theta} - E_{2\Theta,-\Theta}\right| \leq 2\sqrt{2}$$

Classical limit: $\left|\mathscr{B}(\Theta)\right| \leq 2$

[SC, et al., SciPost Physics **12** (2022)]

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Spiking quantum states

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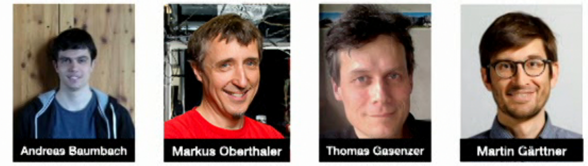
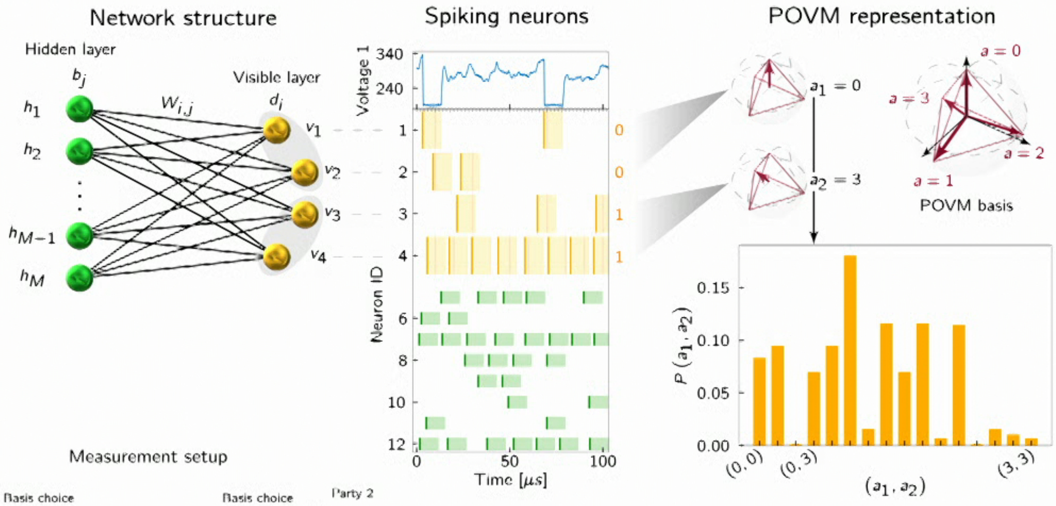
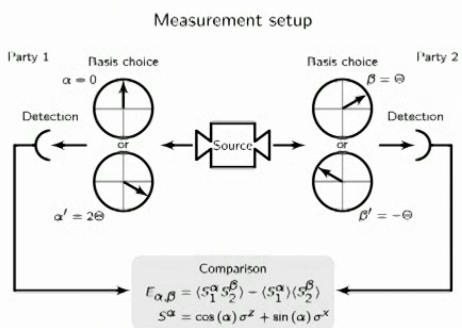


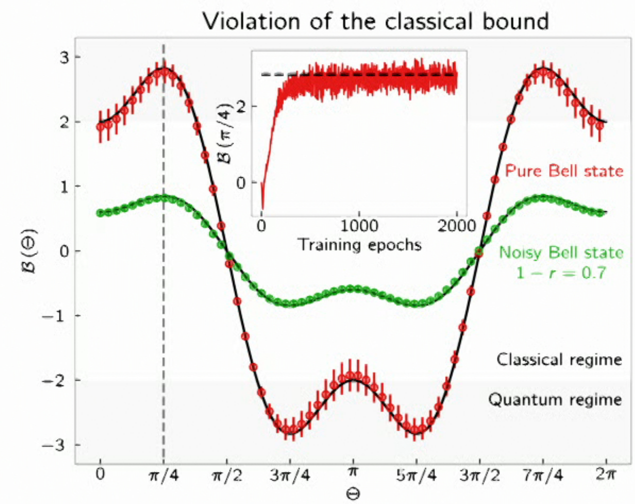




$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\downarrow\downarrow\right\rangle + \left|\uparrow\uparrow\right\rangle\right]$$

$$\left|\mathscr{B}(\Theta)\right| = \left|E_{0,\Theta} + E_{0,-\Theta} + E_{2\Theta,\Theta} - E_{2\Theta,-\Theta}\right| \leq 2\sqrt{2}$$

Classical limit: $\left|\mathscr{B}(\Theta)\right| \leq 2$



$M = 20$ hidden neurons

[SC, et al., SciPost Physics **12** (2022)]

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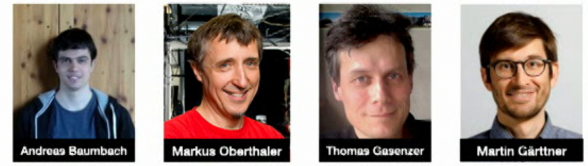
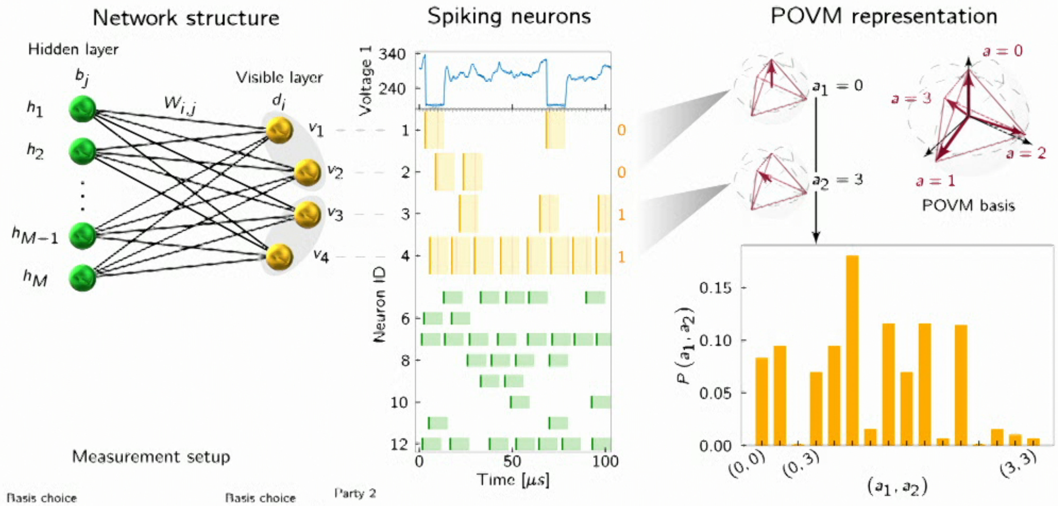
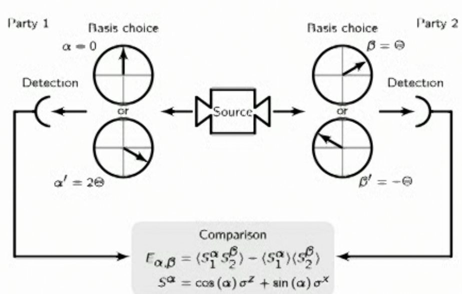
Spiking quantum states







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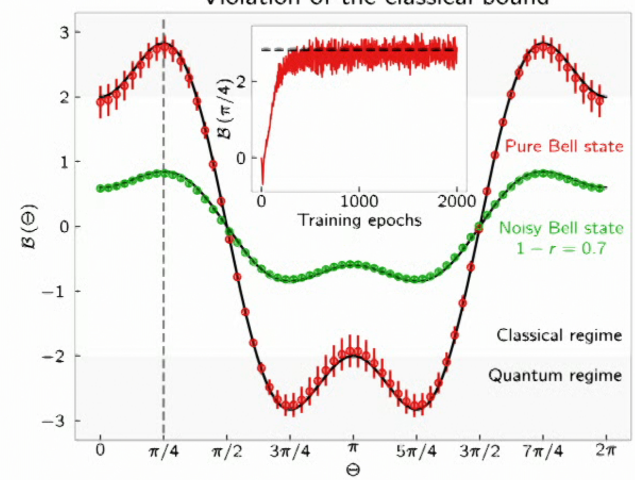

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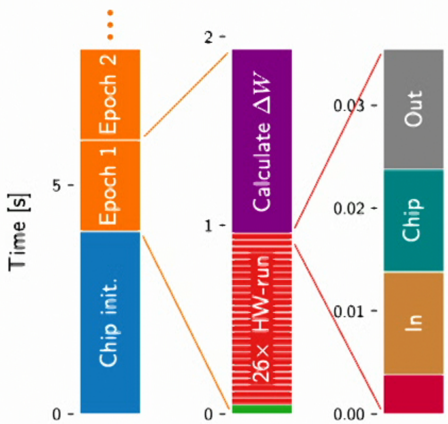
Why spiking hardware?

Runtime



- One training epoch: ~2 seconds
- Generating 125,000 samples: ~1 second
- Pure sampling process (5,000 samples): ~0.01 second

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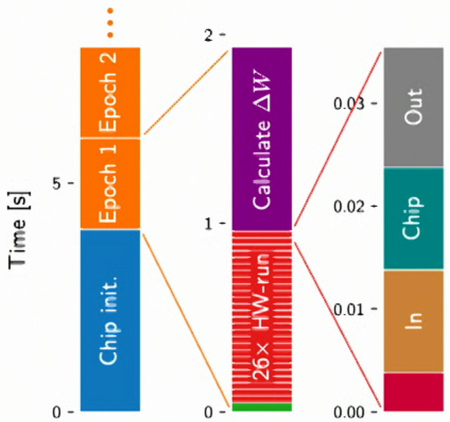

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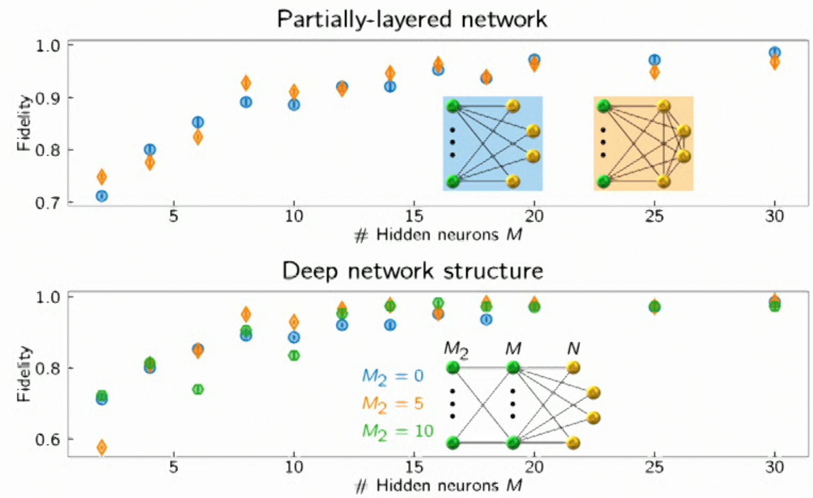
Scaling



- More hidden neurons and complex network architectures do not increase the computational cost

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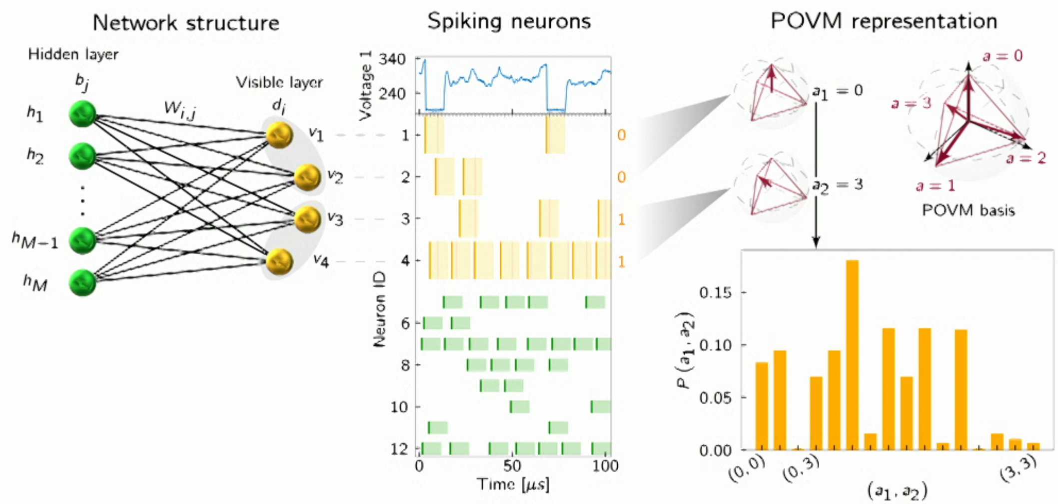
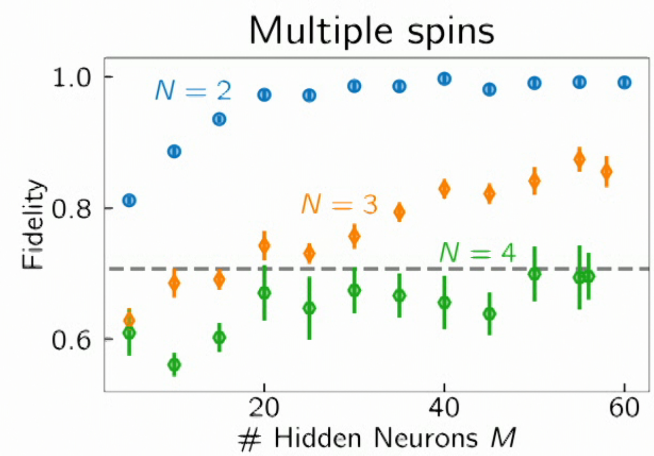
Spiking quantum states



$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\downarrow \cdots \downarrow\rangle + |\uparrow \cdots \uparrow\rangle\right]$$

$$\mathcal{F}(\rho_T, \rho_R) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho_T}\rho_R\sqrt{\rho_T}}\right]$$



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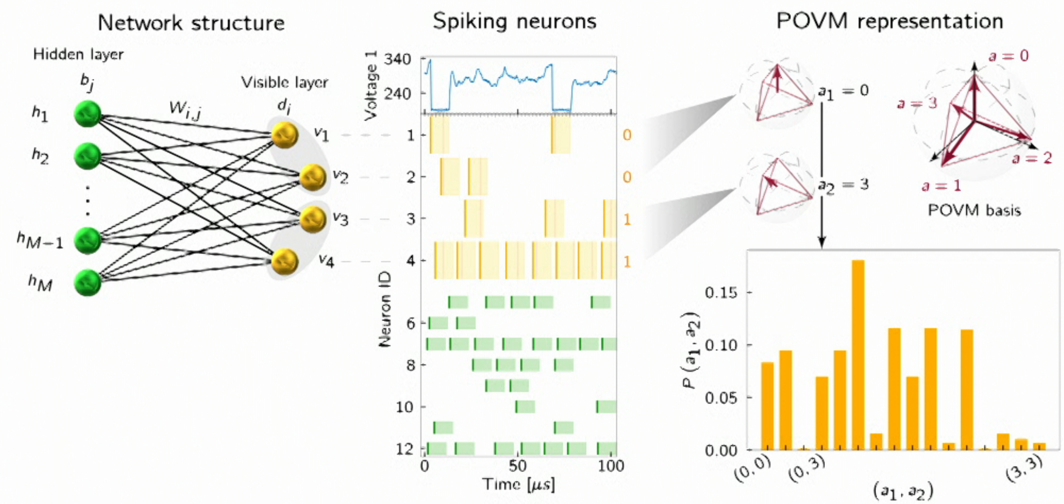
Spiking quantum states



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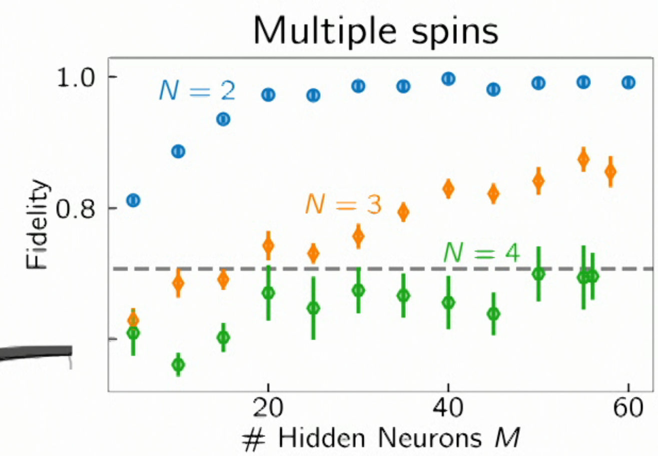
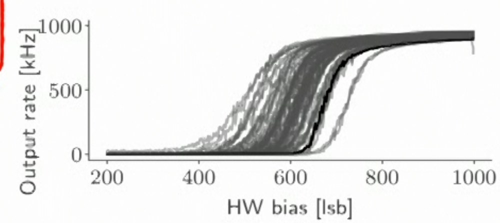


Representable system sizes are strongly limited by the amount of neurons on the chip



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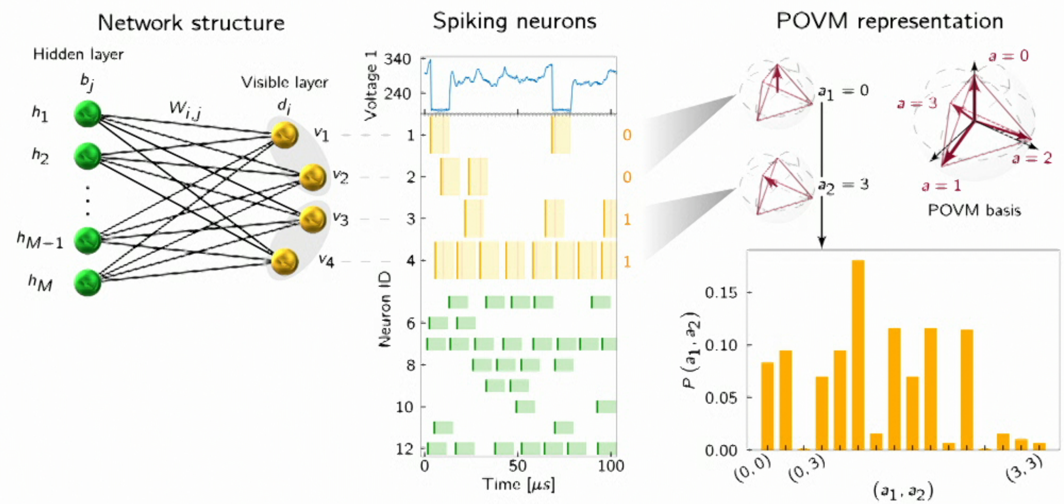
Spiking quantum states



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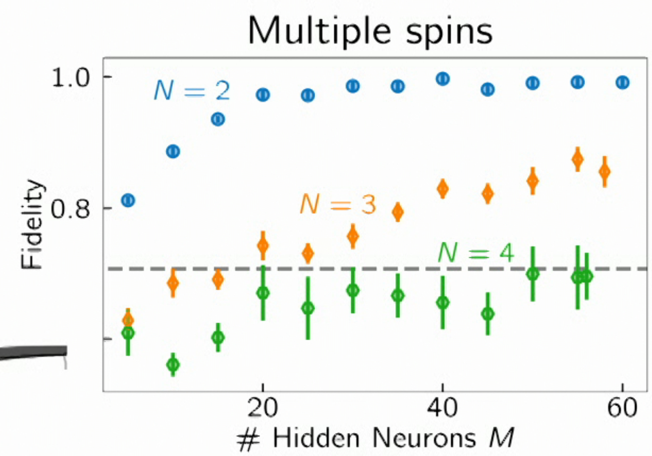
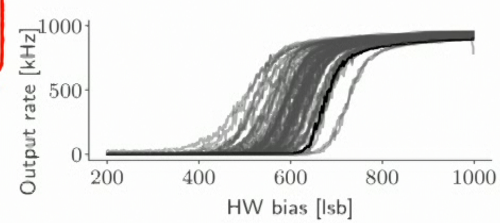


Representable system sizes are strongly limited by the amount of neurons on the chip

Can we find more suitable algorithms?



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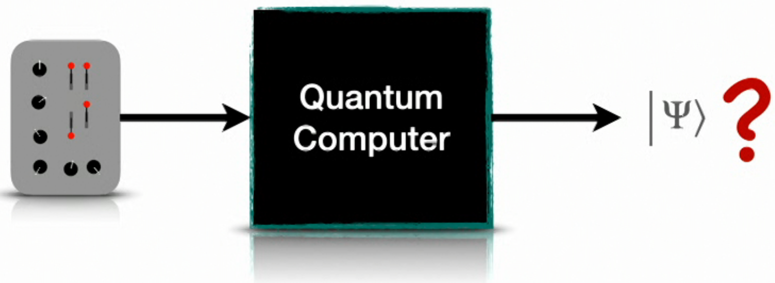
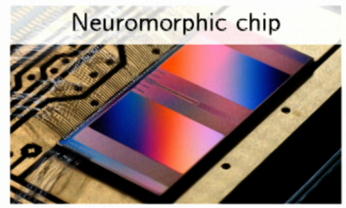
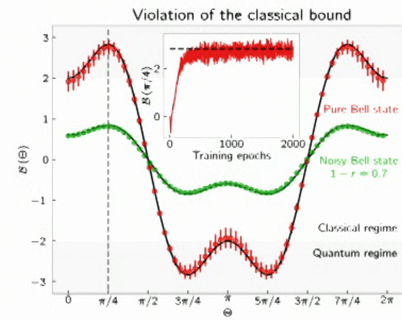
Summary

Reconstruct quantum states from measurement data

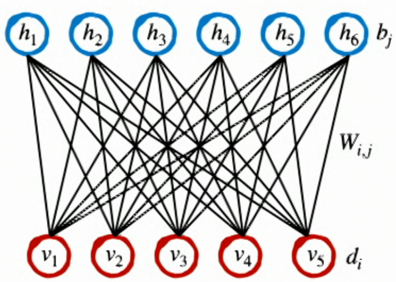


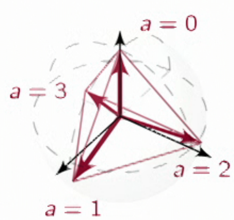
Spiking neuromorphic hardware: fast and energy-efficient





Use restricted Boltzmann machine and POVMs





- Calibrate quantum computer close to qubit system
- Generate a lot of measurement data

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