Title: Quantum adiabatic speedup on a class of combinatorial optimization problems

Speakers: Madelyn Cain

Collection: New Frontiers in Machine Learning and Quantum

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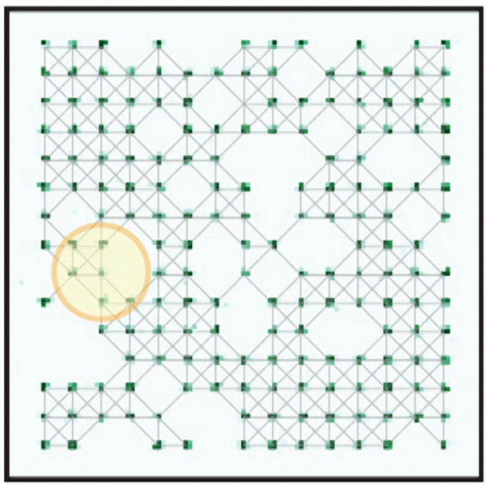
Abstract: "One of the central challenges in quantum information science is to design quantum algorithms that outperform their classical counterparts in combinatorial optimization. In this talk, I will describe a modification of the quantum adiabatic algorithm (QAA) [1] that achieves a Grover-type speedup in solving a wide class of combinatorial optimization problem instances. The speedup is obtained over classical Markov chain algorithms including simulated annealing, parallel tempering, and quantum Monte Carlo. I will then introduce a framework to predict the relative performance of the standard QAA and classical Markov chain algorithms, and show problem instances with quantum speedup and slowdown. Finally, I will apply this framework to interpret results from a recent Rydberg atom array experiment [2], which suggest a superlinear speedup in solving the Maximum Independent Set problem on unit-disk graphs.

[1] Farhi et al. (2001) Science 292, 5516
[2] Ebadi et al. (2022) Science 376, 6598"

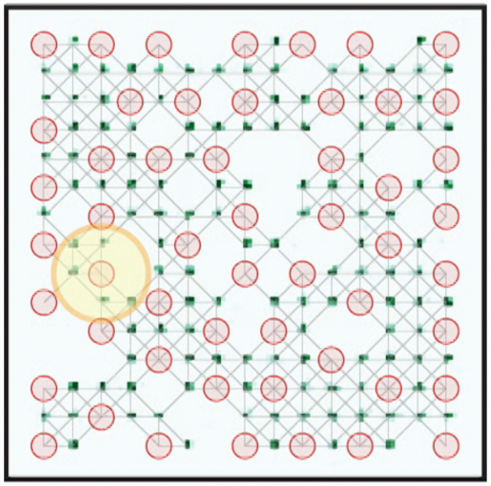
Quantum adiabatic speedup on a class of combinatorial optimization problems



Madelyn Cain

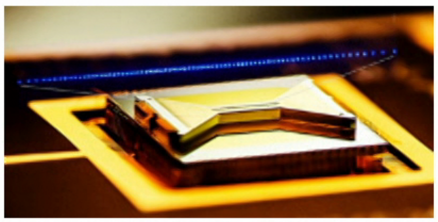
New Frontiers in ML and Quantum

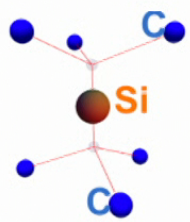
November 22, 2022

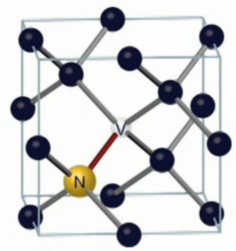


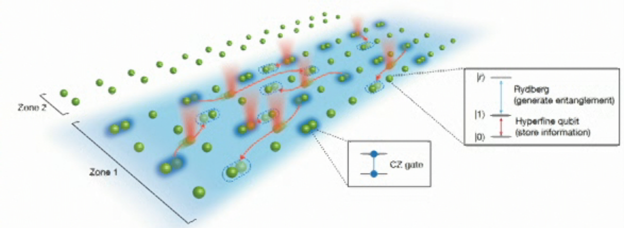
Demonstrating near-term quantum speed-up

Control quantum systems with ≤ 1000 qubits, and no error correction.

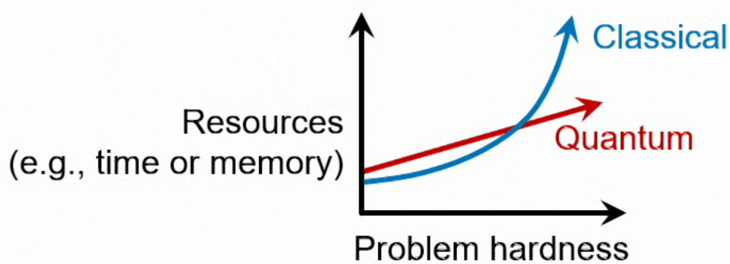








Can we demonstrate quantum speed-up on useful problems in the near-term?



- Shor's algorithm
- Grover's database search
- Solving linear systems
- Quantum dynamics
- Combinatorial optimization

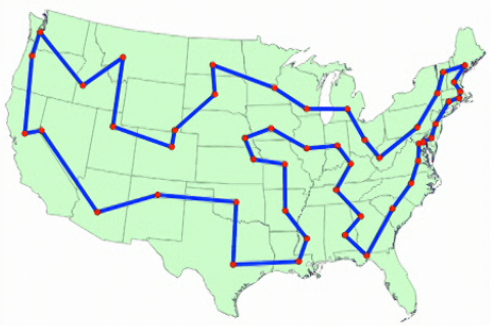
etc…

Combinatorial optimization problems

Broad set of problems with the goal: optimize a cost function, subject to constraints.

- NP-hard, intractable to solve in worst-case
- Solving one NP-hard problem is general enough to solve all NP-hard problems

Traveling salesman problem



Minimize: total distance traveled
Constraint: path must include all cities

This talk: Maximum Independent Set

Given a graph (nodes connected by edges),

Maximize: size of set of nodes
Independent set constraint: no nodes in set are connected by an edge

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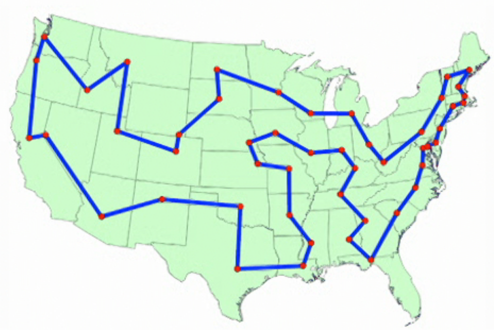

Minimize: total distance traveled
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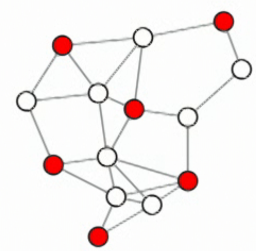
This talk: Maximum Independent Set

Maximum independent set (MIS) in red



Given a graph (nodes connected by edges),

Maximize: size of set of nodes
Independent set constraint: no nodes in set are connected by an edge

Hardware-efficient encoding of Maximum Independent Set

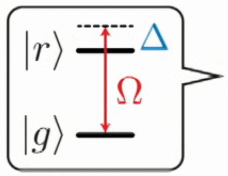
Rydberg atom array platform

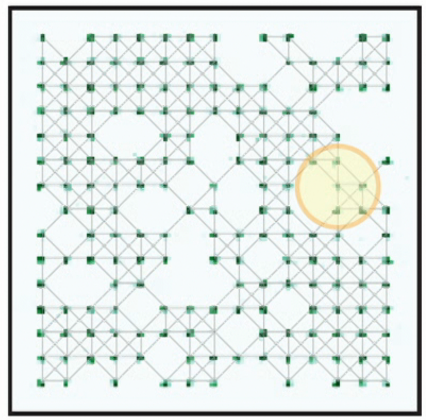
Several hundred Rb atoms (qubits) arranged deterministically

Each qubit represents a node in the graph

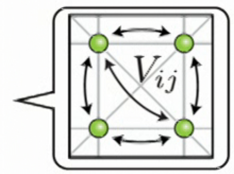
In independent set

Out of independent set



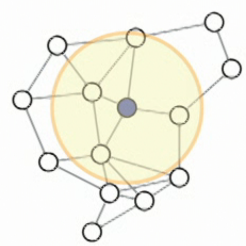


$V_{ij} = V_0/|\mathbf{x_i} - \mathbf{x_j}|^6$



Rydberg blockade: simultaneous Rydberg excitation forbidden in blockade radius

Rydberg interaction naturally encodes the independent set constraint on unit disk graphs
(edge between nodes within a unit radius)



Blockade radius

The Quantum Adiabatic Algorithm

System Hamiltonian

Quantum driver

$$H_q = \Omega \sum_{u \in V} |g_u\rangle \langle r_u| + \text{h.c.}$$

Cost Hamiltonian

$$H_{\text{cost}} = -\delta \sum_{u \in V} n_u + \sum_{u,v} V_{uv} n_u n_v$$

Favors many nodes in set

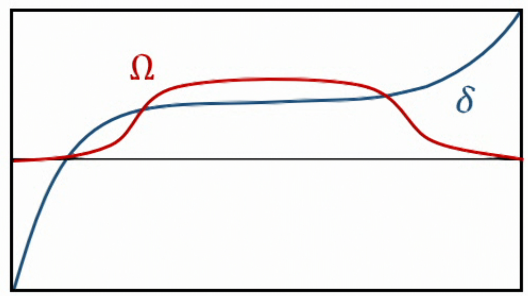
Penalizes independent set violations

Notice that ground state of H_{cost} is the MIS!

Adiabatically prepare the ground state of H_{cost}

1. Initialize in easy to prepare many-body ground state
2. Slowly change Hamiltonian
3. Final, hard to prepare, ground state encodes MIS

$\delta < 0$
$\Omega = 0$



$\delta > 0$
$\Omega = 0$

Closed-loop optimization to optimize δ over time

Time

Initialize atoms in atomic ground state $|g\rangle^N$

Maximize atoms in $|r\rangle$ subject to blockade (corresponds to MIS of graph)

Outline

1. Experimental signatures of a speedup over simulated annealing on certain problem instances

2. Mechanism for observed experimental performance, and a new quantum algorithm with guaranteed speedup

Hard instances for SA

Expected runtime to randomly select MIS $\sim \frac{D_{|\mathrm{MIS}|-1}}{D_{|\mathrm{MIS}|}}$

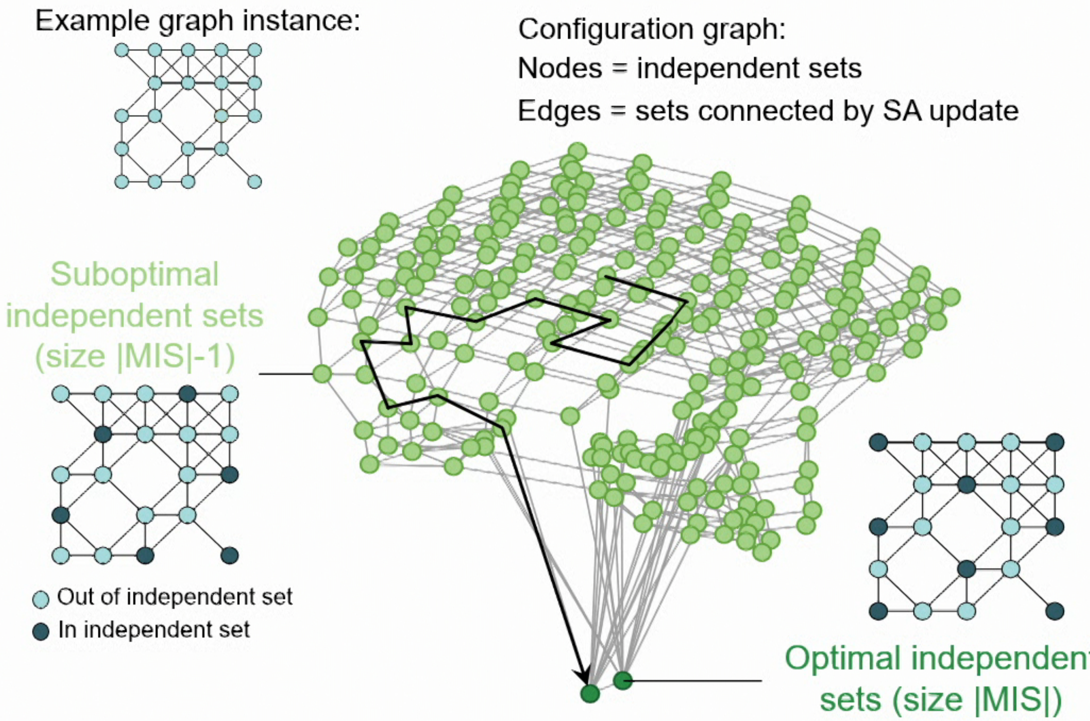
$D_k =$ Number (degeneracy) of independent sets of size *k*

Runtime of any SA algorithm (no. proposed updates)

$$\gtrsim \max_k \frac{D_{k-1}}{nD_k}$$

Among states with the same energy, SA dynamics are a random walk!

Hard MIS unit-disk graphs for SA: exponentially many suboptimal solutions with same energy



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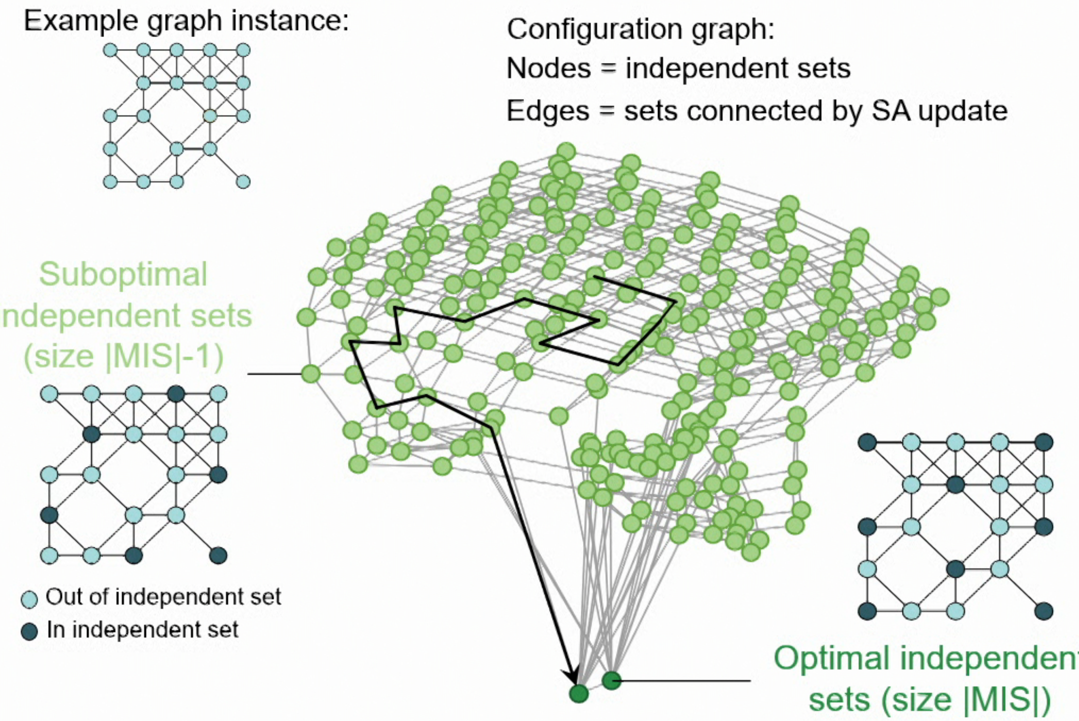

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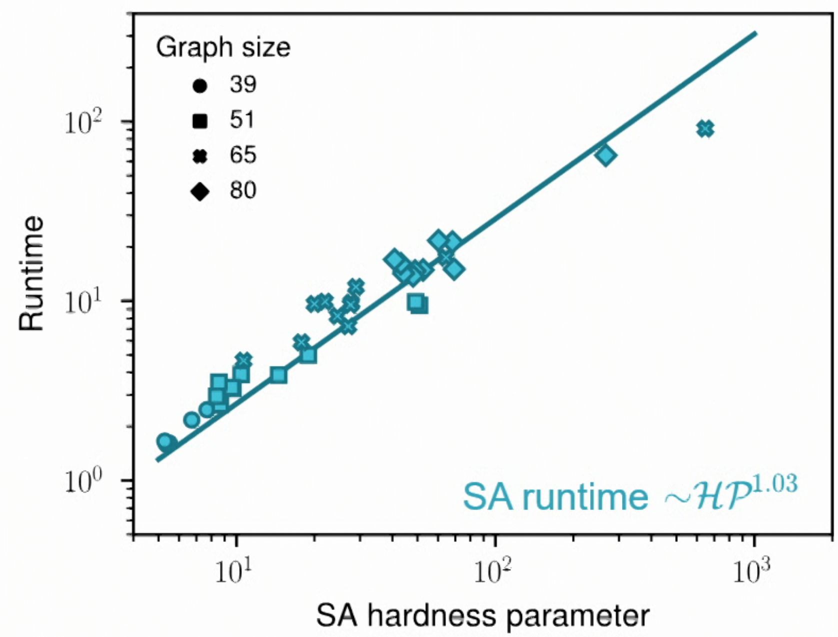
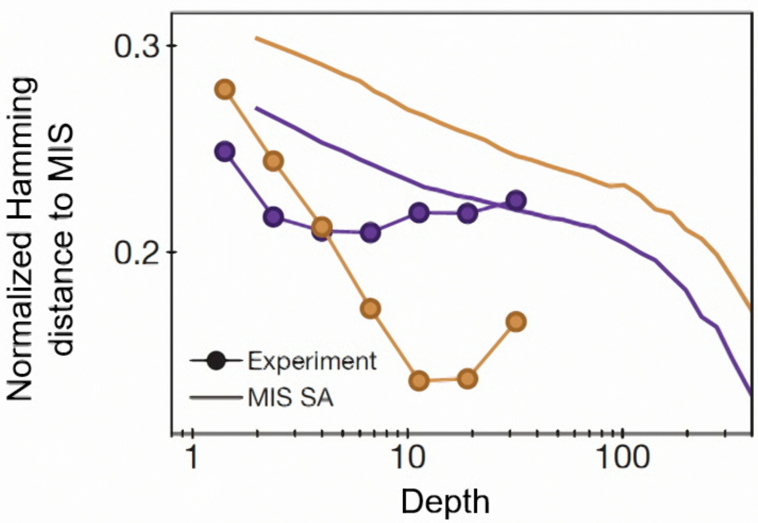
Generate top 2% hardest instances maximizing SA hardness parameter



Experimentally test for quantum speedup

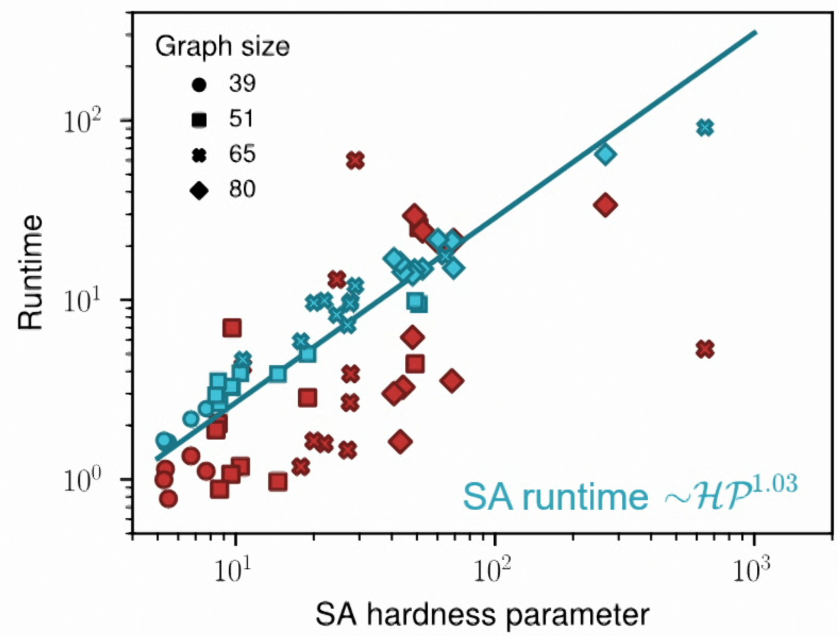
Quantum performance varies on instances with similar SA hardness

- Easy graph for quantum
- Hard graph for quantum



Quantum depth = evolution time / time to flip spin

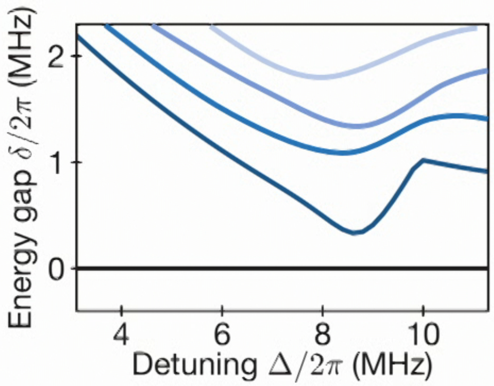
Generate top 2% hardest instances maximizing SA hardness parameter



Quantum hardness not determined by SA hardness parameter!

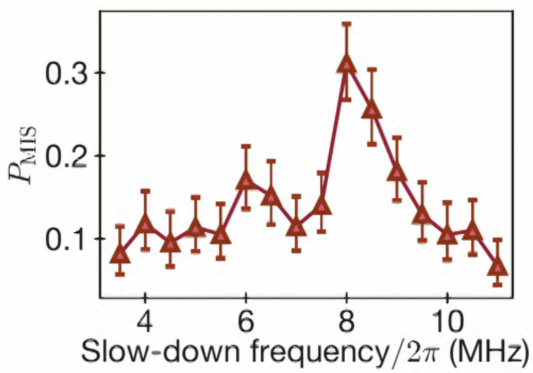
Quantum performance controlled by minimum energy gap

Adiabaticity is limited by the minimum energy gap

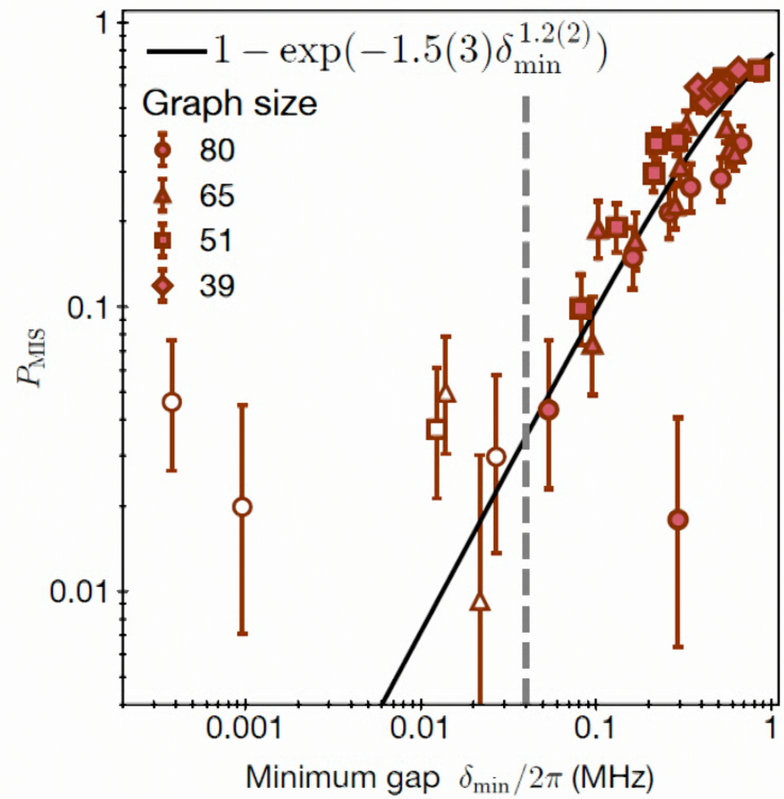


Experiment sensitive to many-body adiabatic gap

MIS probability increases when detuning sweep slowed at gap



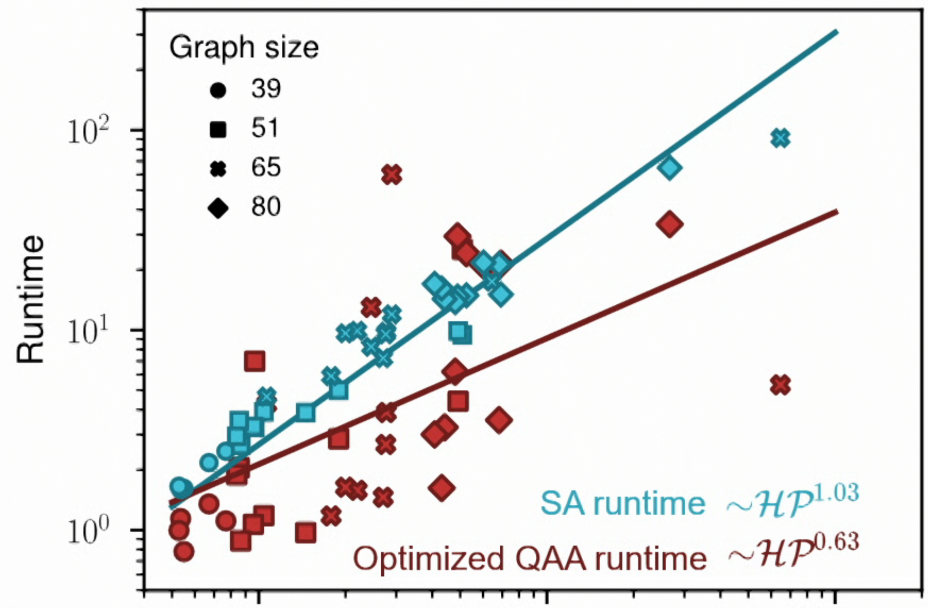
Hardness controlled by adiabatic gap
Dependence on gap near-optimal (linear)

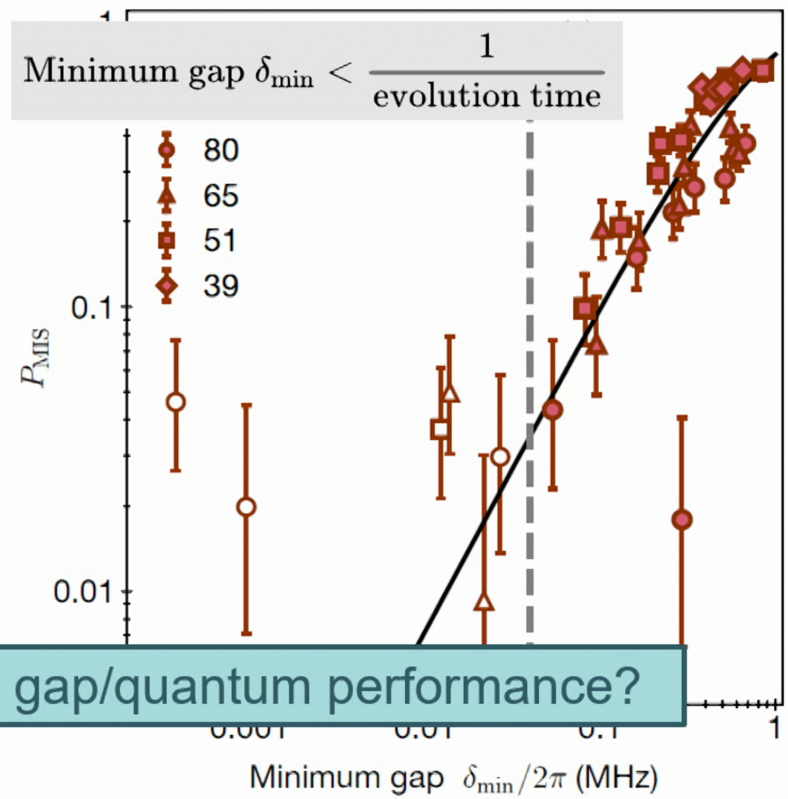


Quantum performance controlled by minimum energy gap

Emergence of superlinear speedup in deep circuit regime
S. Ebadi, A. Keesling*, MC* et al, Science 376, 6598

Focus on instances we can optimize (runtime large enough to stay adiabatic)

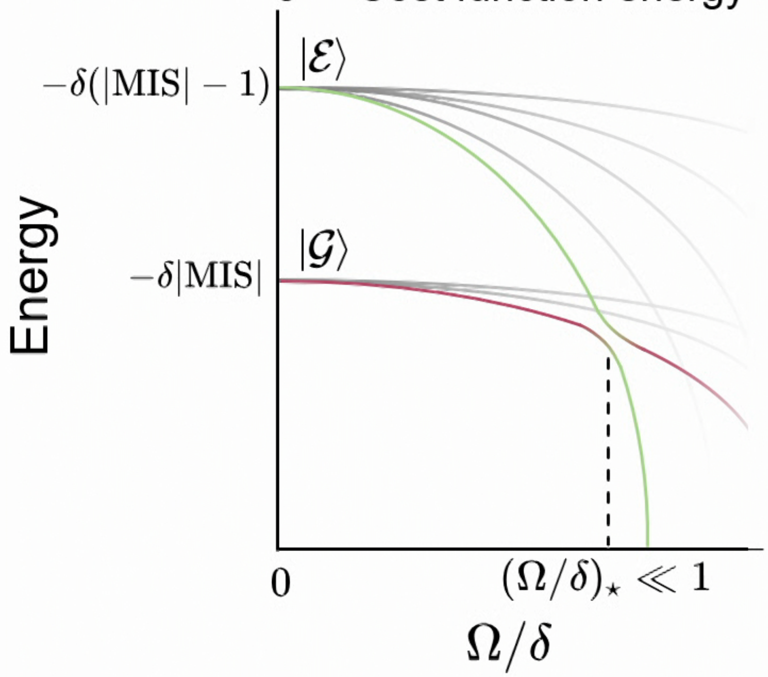




What controls the minimum energy gap/quantum performance?

Understanding the minimum gap

$\Omega =$ Spin-flip energy
$\delta =$ Cost function energy



$$|\mathcal{G}\rangle = \sum_{i \text{ size } |\mathrm{MIS}|} \mathcal{G}_i |i\rangle$$

$$|\mathcal{E}\rangle = \sum_{i \text{ size } |\mathrm{MIS}|\text{-}1} \mathcal{E}_i |i\rangle$$

Gap decided by how strongly Hamiltonian connects $|\mathcal{G}\rangle, |\mathcal{E}\rangle$ via spin-flips

Understand gap from states $|\mathcal{G}\rangle, |\mathcal{E}\rangle$
Can estimate with perturbation theory for $(\Omega/\delta)_\star \ll 1$

Understanding the minimum gap

$|\mathcal{G}\rangle, |\mathcal{E}\rangle$ ground states of spin-exchange Hamiltonian by perturbation theory

$$|\mathcal{G}\rangle = \sum_{i \text{ size } |\text{MIS}|} \mathcal{G}_i |i\rangle$$

$$|\mathcal{E}\rangle = \sum_{i \text{ size } |\text{MIS}|\text{-}1} \mathcal{E}_i |i\rangle$$

Gap decided by how strongly Hamiltonian connects $|\mathcal{G}\rangle, |\mathcal{E}\rangle$ via spin-flips

Gap behavior in two limits:

1. When $|\mathcal{G}\rangle, |\mathcal{E}\rangle$ are **localized** on certain independent sets,

   gap $\sim (\Omega/\delta)_\star^{\text{hamming distance}}$

2. When $|\mathcal{G}\rangle, |\mathcal{E}\rangle$ are **delocalized** (uniform superpositions),

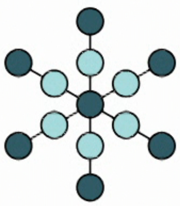
   gap $\sim \sqrt{\dfrac{D_{|\text{MIS}|}}{D_{|\text{MIS}|-1}}}$

   Optimized quantum runtime $\sim \sqrt{\dfrac{D_{|\text{MIS}|-1}}{D_{|\text{MIS}|}}}$ SA runtime $\sim \dfrac{D_{|\text{MIS}|-1}}{D_{|\text{MIS}|}}$

   Grover-type quadratic speedup over SA!

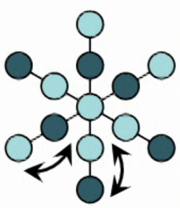
Instance-by-instance performance variation

Star graph has b branches of length ℓ



$|\mathcal{G}\rangle$ = unique MIS

$b = 6, \ell = 2$

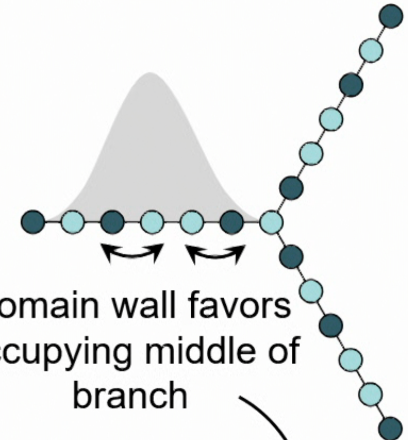


$|\mathcal{E}\rangle$ = superposition of $\sim \left(\frac{\ell}{2} + 1\right)^b$ |MIS|-1 states

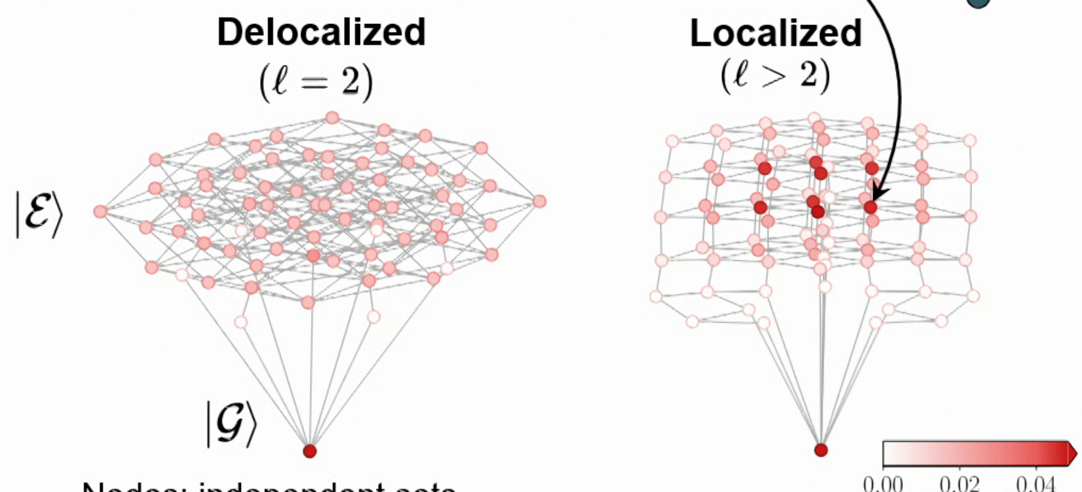
Domain walls

(correspond to the movement of a domain wall on each branch)

Estimate $|\mathcal{E}\rangle$ perturbatively: minimize driver Hamiltonian at second order in Ω/δ (spin-exchange hopping)



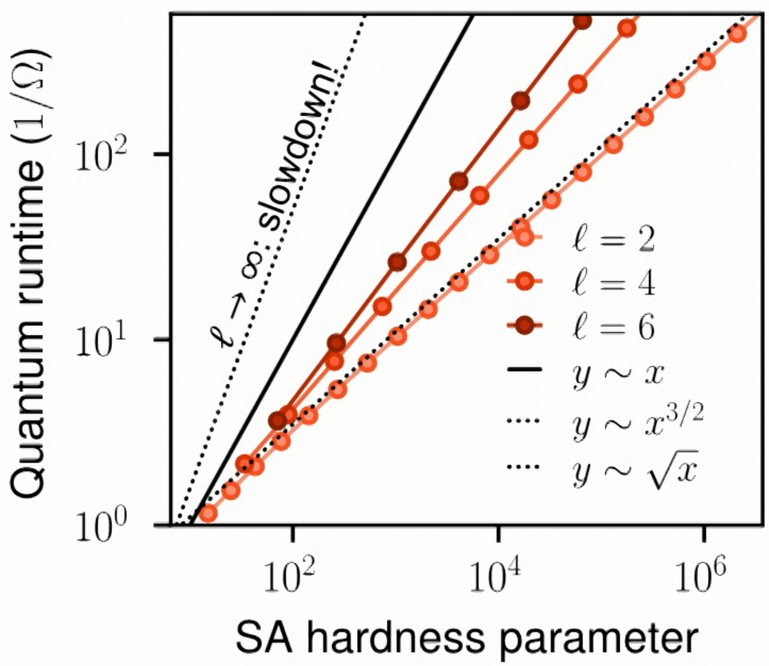
Domain wall favors occupying middle of branch



Nodes: independent sets
Edges: states connected by spin exchange / flip

Instance-by-instance performance variation

Localization at large Hamming distance $\approx b\ell/2$ causes quantum slowdown!



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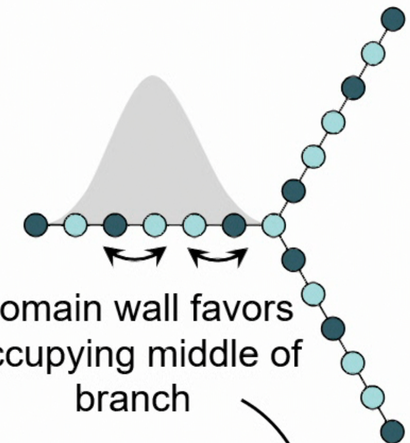
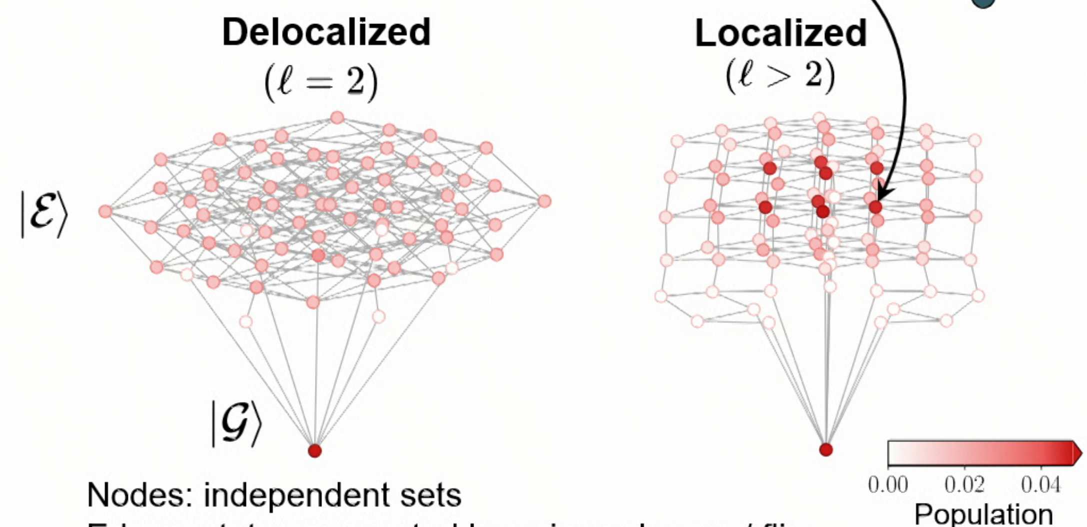


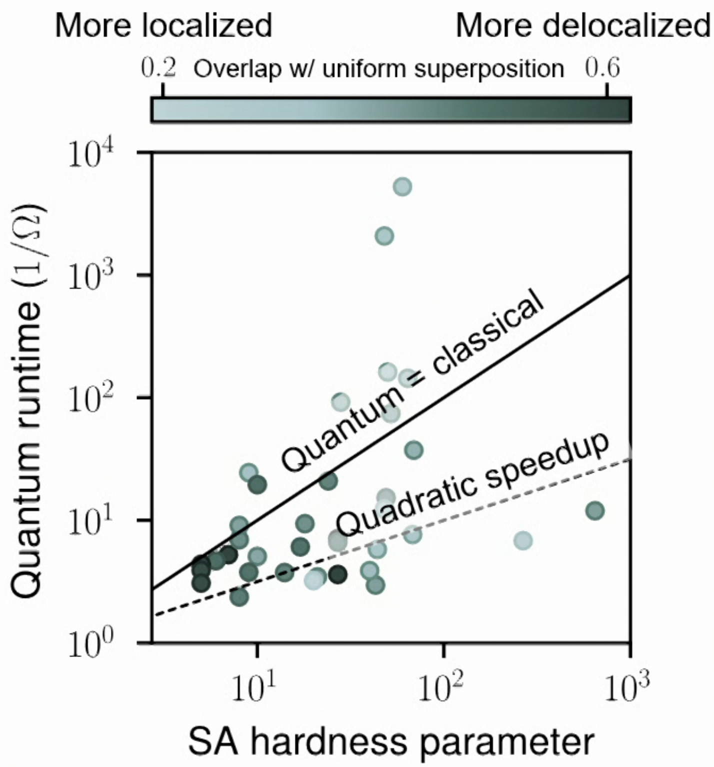


Nodes: independent sets

Edges: states connected by spin exchange / flip

Understanding experimental observations

Study unit-disk graph instances with large SA hardness parameter



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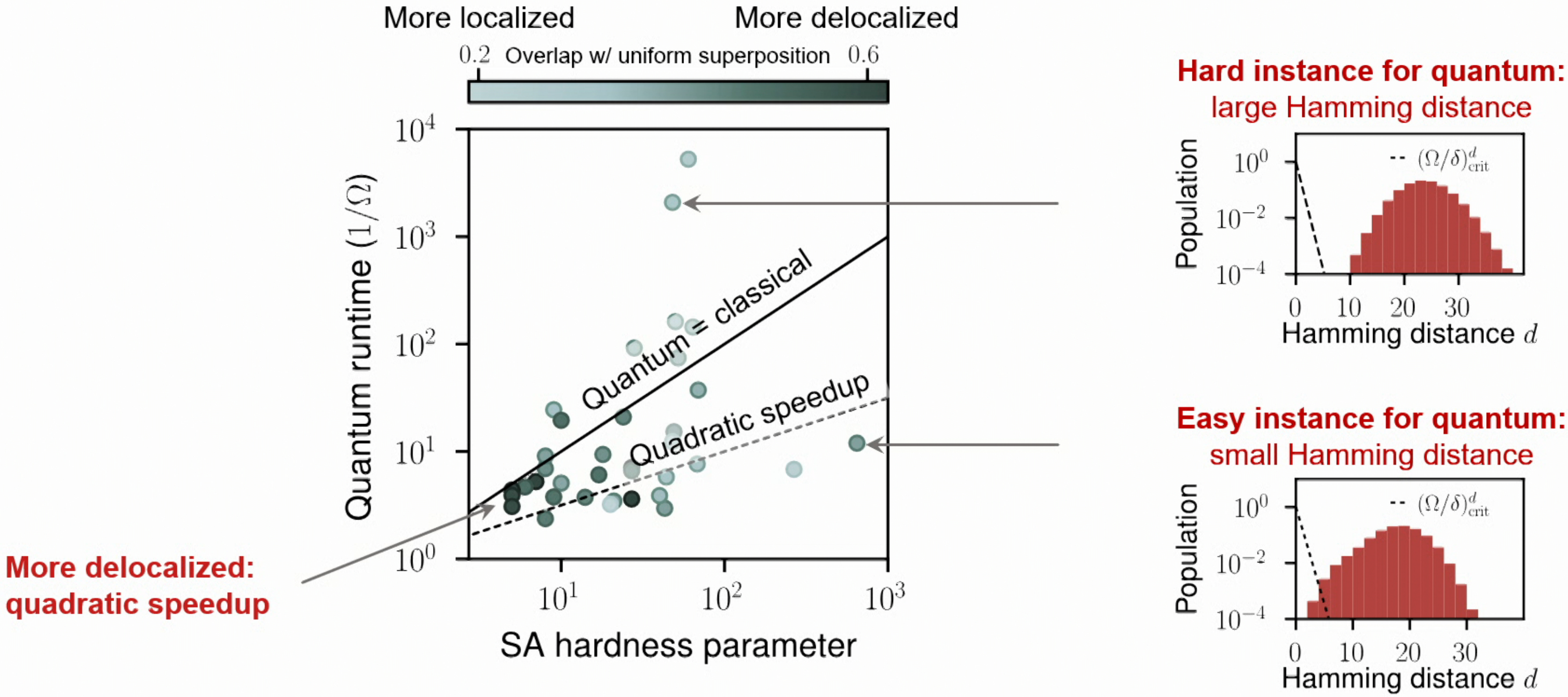

We’ve seen that quantum performance is highly dependent on the problem instance.

Is there a way to generically guarantee a quadratic speedup?

Engineering a quadratic speedup

We want to force $|\mathcal{G}\rangle, |\mathcal{E}\rangle$ to be uniform superpositions.

Add a strong "delocalizing Hamiltonian" λH_ℓ whose ground states are uniform superpositions

$$|\bar{b}\rangle = \frac{1}{\sqrt{D_b}} \sum_{i \text{ size } b} |i\rangle \qquad b \in \{1, \cdots, |\mathrm{MIS}|\}$$

Inspiration from single-particle quantum mechanics: *kinetic energy* $\sim -\nabla^2$ promotes delocalization → Delocalizing Hamiltonian: A *Laplacian* in spin configuration space

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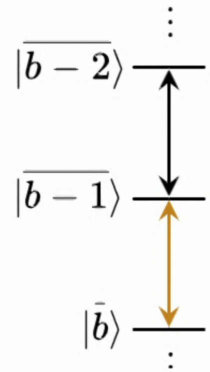
λ large: dynamics restricted to H_ℓ ground states



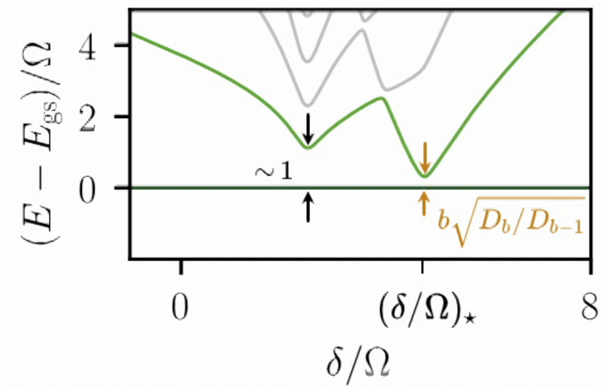
Coupling $= \langle \bar{b}|H|\overline{b-1}\rangle \sim \Omega\sqrt{\frac{D_b}{D_{b-1}}}$

is coherently enhanced

Inspiration from single-particle quantum mechanics: *kinetic energy* $\sim -\nabla^2$ promotes delocalization → Delocalizing Hamiltonian: A *Laplacian* in spin configuration space

$$H_\ell = -H_{\text{spin-exchange}} + \sum_{(u,v)\in E} n_u(1-n_v) \prod_{(y,v)\in E, y\neq u} (1-n_y)$$

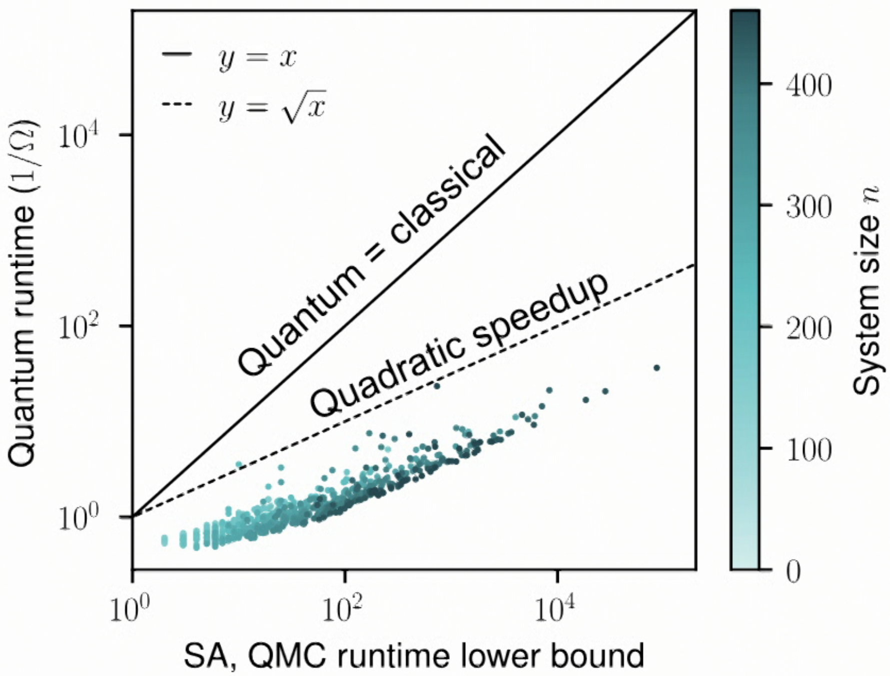
Local terms, no sign problem



Analytic argument that gap goes as smallest coupling – quadratic speedup!

Engineering a quadratic speedup

Quadratic speedup obtained on hard unit-disk graph instances



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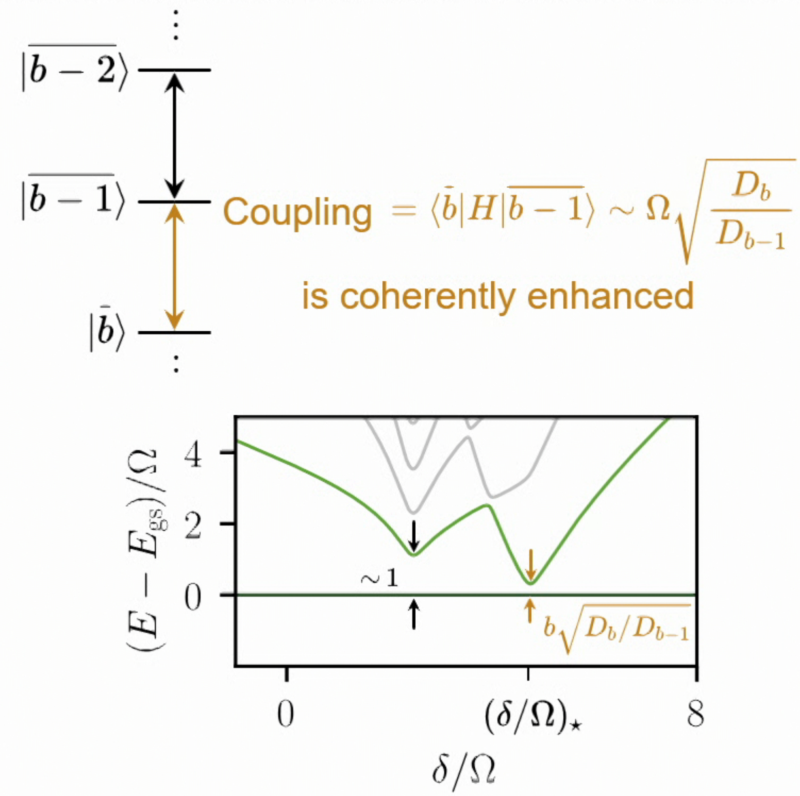


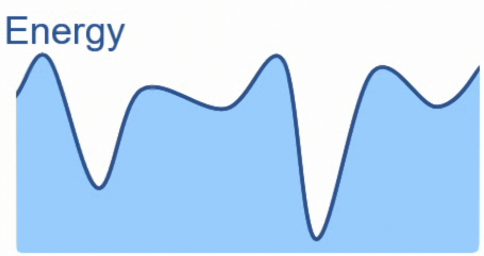
Analytic argument that gap goes as smallest coupling – quadratic speedup!

Summary

- Modification of the QAA provides quadratic speedup over SA
- Framework to understand instance-by-instance performance of the standard QAA
  - Key factor: localization/delocalization of eigenstates at level crossing

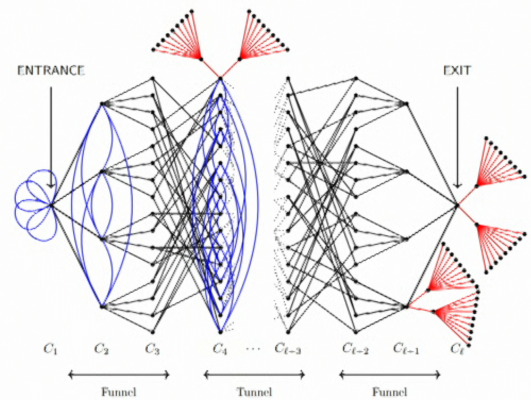
Outlook

Rugged energy landscapes

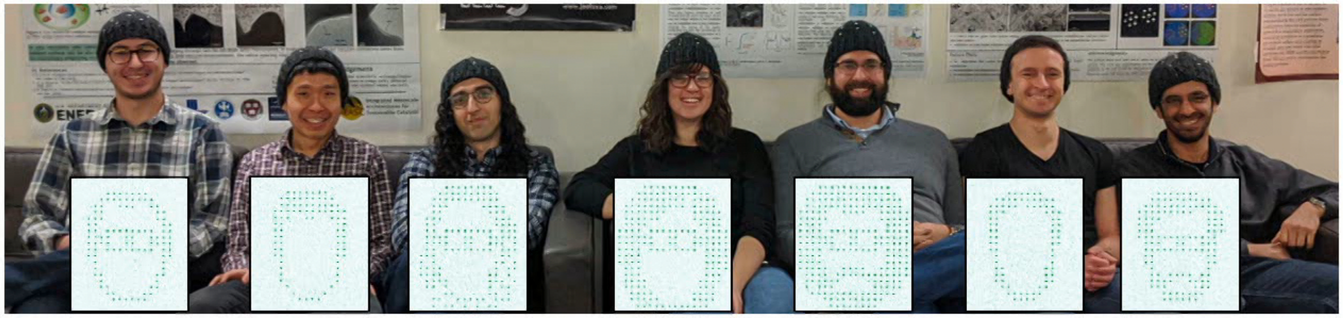


Provably hard to find solution for *all* local classical algorithms
(Overlap Gap Property, Gamarnik 2021)

Superpolynomial speedup with unphysical (oracle) Hamiltonians
[Hastings 2020, Gilyen + Vazirani 2020]



Acknowledgements



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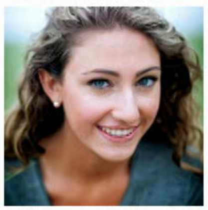
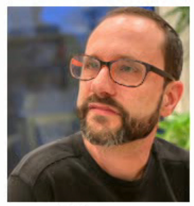


Beatrice Nash

Leo Zhou



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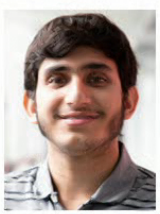

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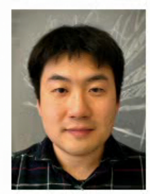

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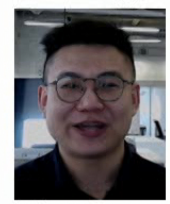
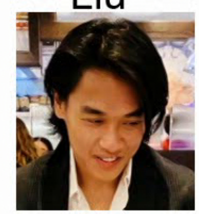


Roger Luo



Mao Lin