Title: On the system loophole of generalized noncontextuality

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Series: Quantum Foundations

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Abstract: Generalized noncontextuality is a well-studied notion of classicality that is applicable to a single system, as opposed to Bell locality. It relies on representing operationally indistinguishable procedures identically in an ontological model. However, operational indistinguishability depends on the set of operations that one may use to distinguish two procedures: we refer to this set as the reference of indistinguishability. Thus, whether or not a given experiment is noncontextual depends on the choice of reference. The choices of references appearing in the literature are seldom discussed, but typically relate to a notion of system underlying the experiment. This shift in perspective then begs the question: how should one define the extent of the system underlying an experiment? Our paper primarily aims at exposing this question rather than providing a definitive answer to it. We start by formulating a notion of relative noncontextuality for prepare-and-measure scenarios, which is simply noncontextuality with respect to an explicit reference of indistinguishability. We investigate how verdicts of relative noncontextuality depend on this choice of reference, and in the process introduce the concept of the noncontextuality graph of a prepare-and-measure scenario. We then discuss several proposals that one may appeal to in order to fix the reference to a specific choice, and relate these proposals to different conceptions of what a system really is.

arXiv link: https://arxiv.org/abs/2209.04469

Zoom link: https://pitp.zoom.us/j/97393198973?pwd=dWhCOUJQLytxeXVIVmEvOHRnRHc1QT09

ETH zürich

On the system loophole of generalized noncontextuality

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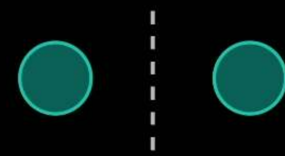
arXiv:2209.04469

Systems in nonclassicality

Bell nonlocality:



Noncontextuality:

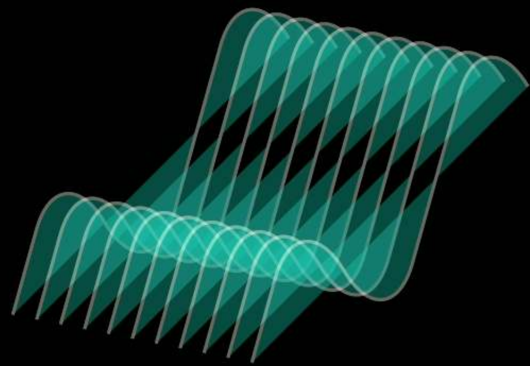
Systems in noncontextuality

P is equivalent to P' if

$$p(k|P,M) = p(k|P',M) \text{ for all } M. \qquad (1)$$

[Spekkens 2005]

Universe-subsystem ontologies



Fuzzy universe

Effective subsystems

Composite universe

Fundamental subsystems

Typical NCOMs

NCOMs here?

Roadmap:

I. Relative noncontextuality $\simeq$ noncontextuality without systems

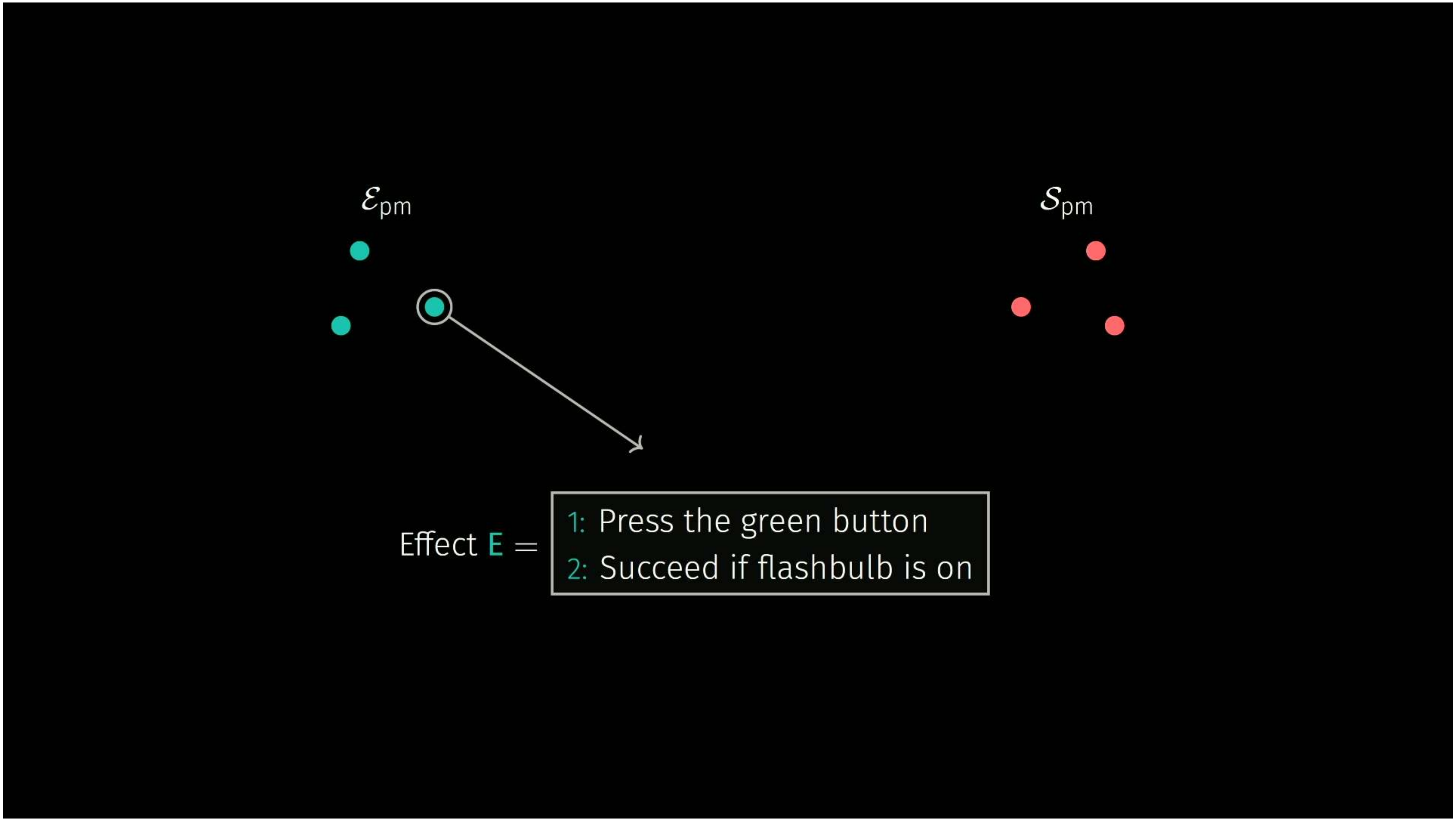
I. Relative noncontextuality

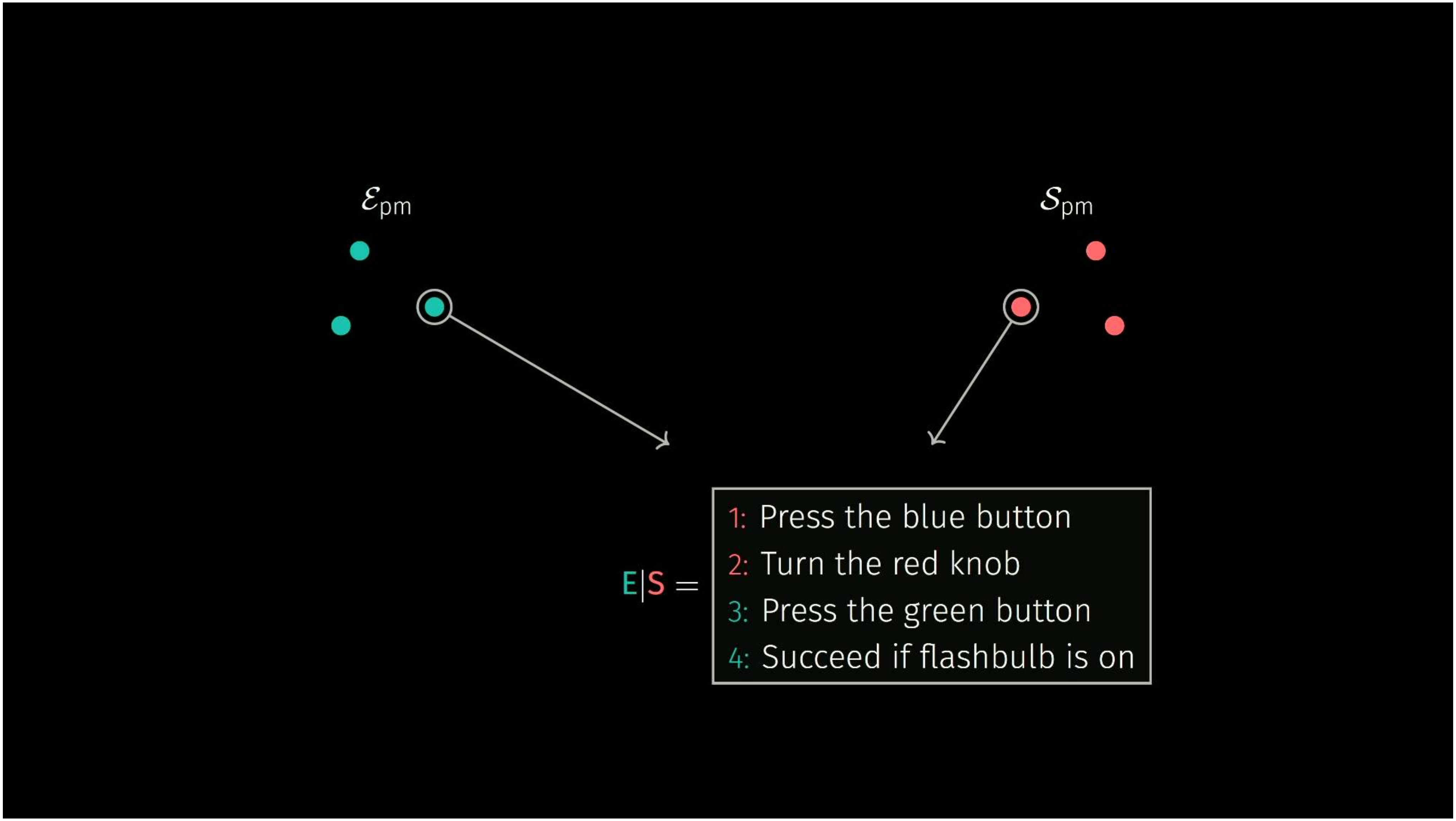
Roadmap:

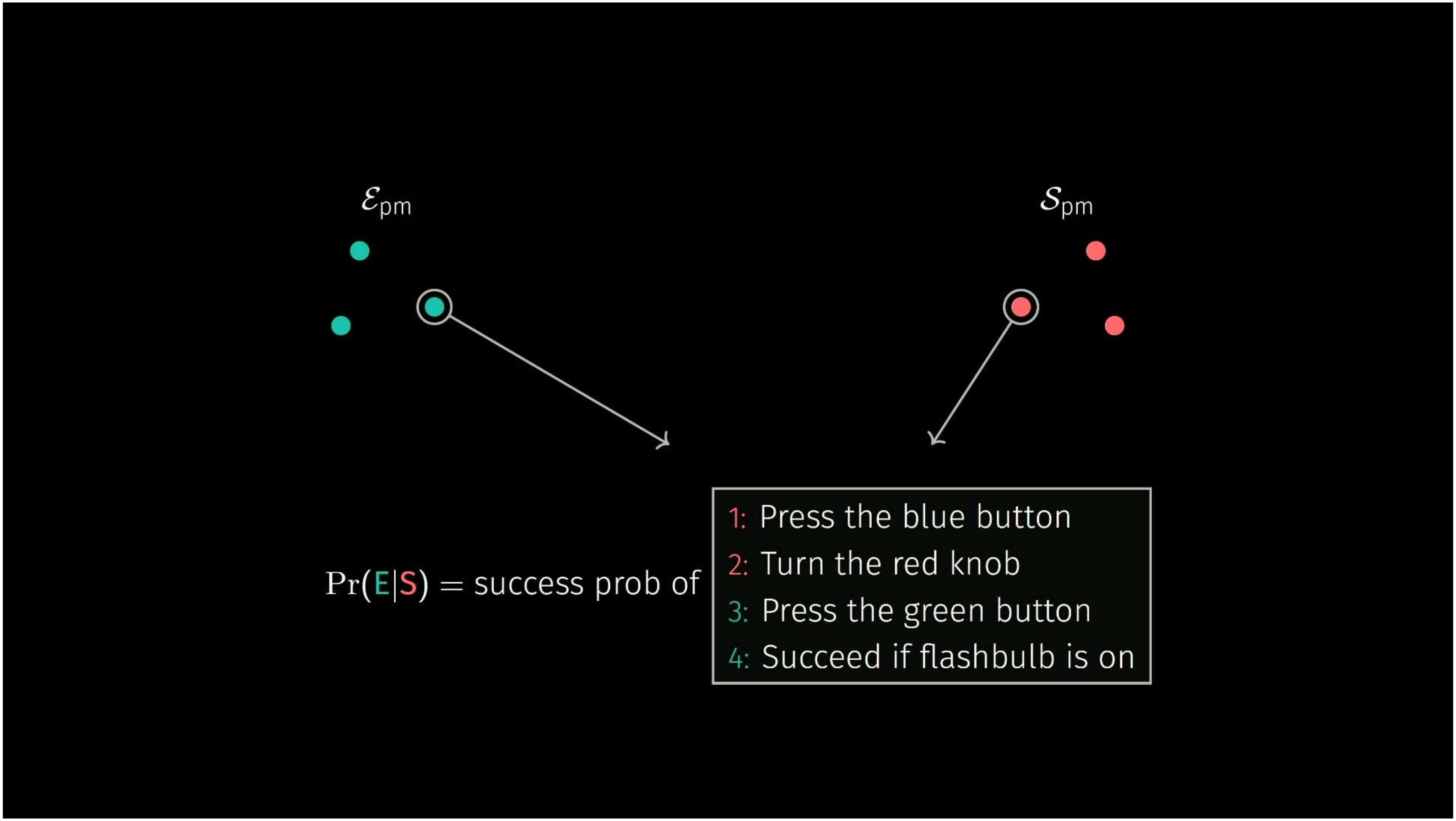
I. Relative noncontextuality $\simeq$ noncontextuality without systems

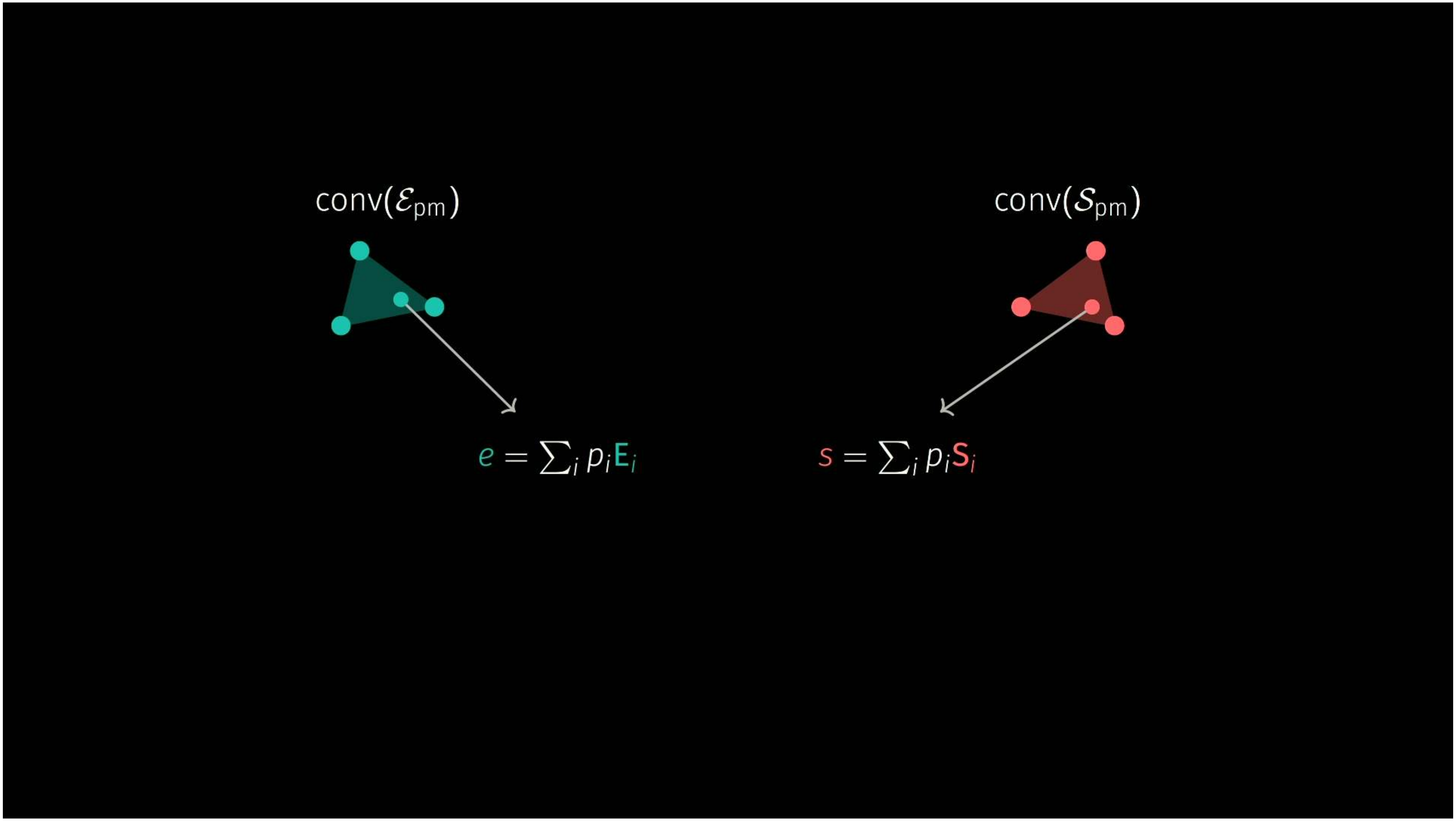
$\downarrow$

II. Reintroduce different notions of systems

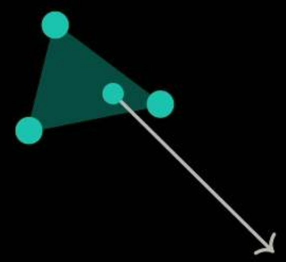

$\mathcal{E}_{\text{pm}}$
$\mathcal{S}_{\text{pm}}$
Effect E =
1: Press the green button
2: Succeed if flashbulb is on


$\mathcal{E}_{pm}$
$\mathcal{S}_{pm}$
E|S =
1: Press the blue button
2: Turn the red knob
3: Press the green button
4: Succeed if flashbulb is on

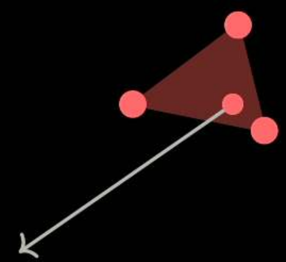

$\mathcal{E}_{\text{pm}}$
$\mathcal{S}_{\text{pm}}$
$\Pr(\mathsf{E}|\mathsf{S})$ = success prob of
1: Press the blue button
2: Turn the red knob
3: Press the green button
4: Succeed if flashbulb is on


conv($\mathcal{E}_{\text{pm}}$)
conv($\mathcal{S}_{\text{pm}}$)
$e = \sum_i p_i \mathsf{E}_i$
$s = \sum_i p_i \mathsf{S}_i$

$\mathrm{conv}(\mathcal{E}_{\mathrm{pm}})$



$\mathrm{conv}(\mathcal{S}_{\mathrm{pm}})$

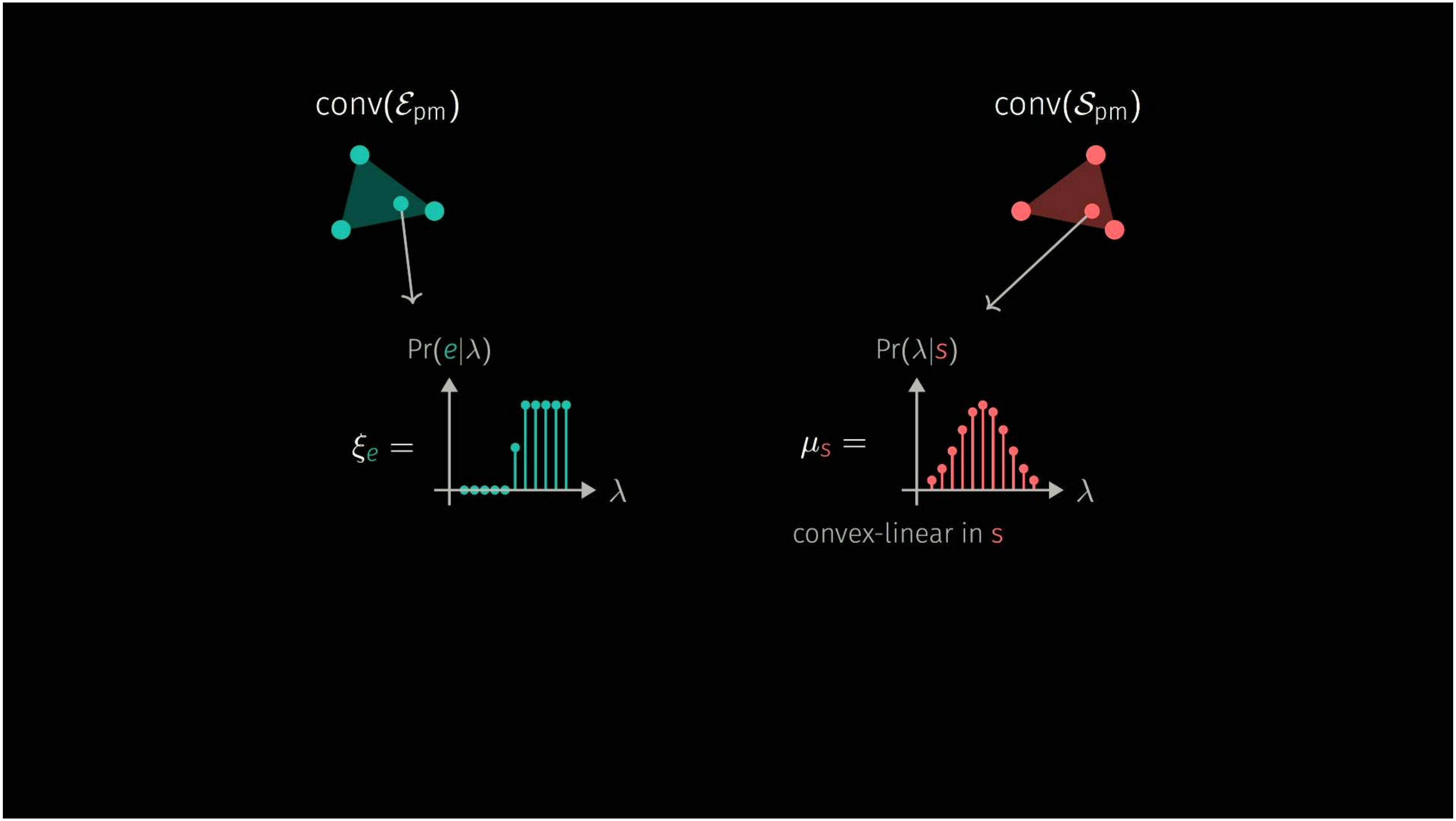


$e = \sum_i p_i \mathsf{E}_i$

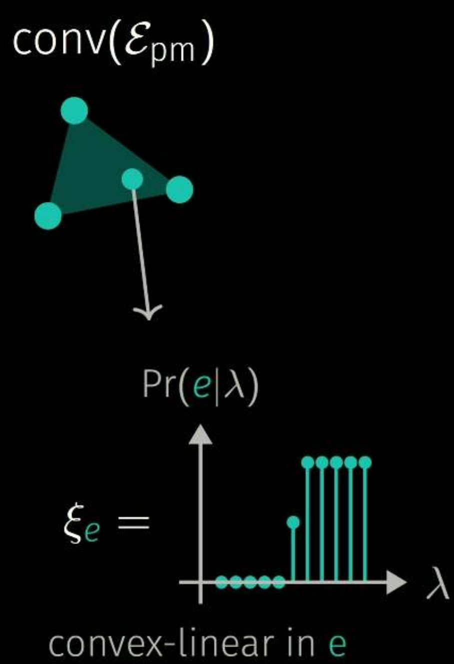
$s = \sum_i p_i \mathsf{S}_i$

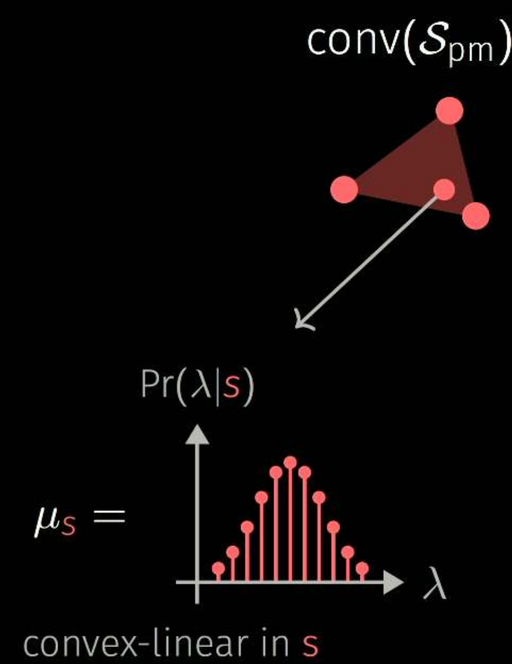
Data: $\{\Pr(e|s)\}_{s,e}$

with $\Pr(\cdot|\cdot)$ convex-linear in both arguments



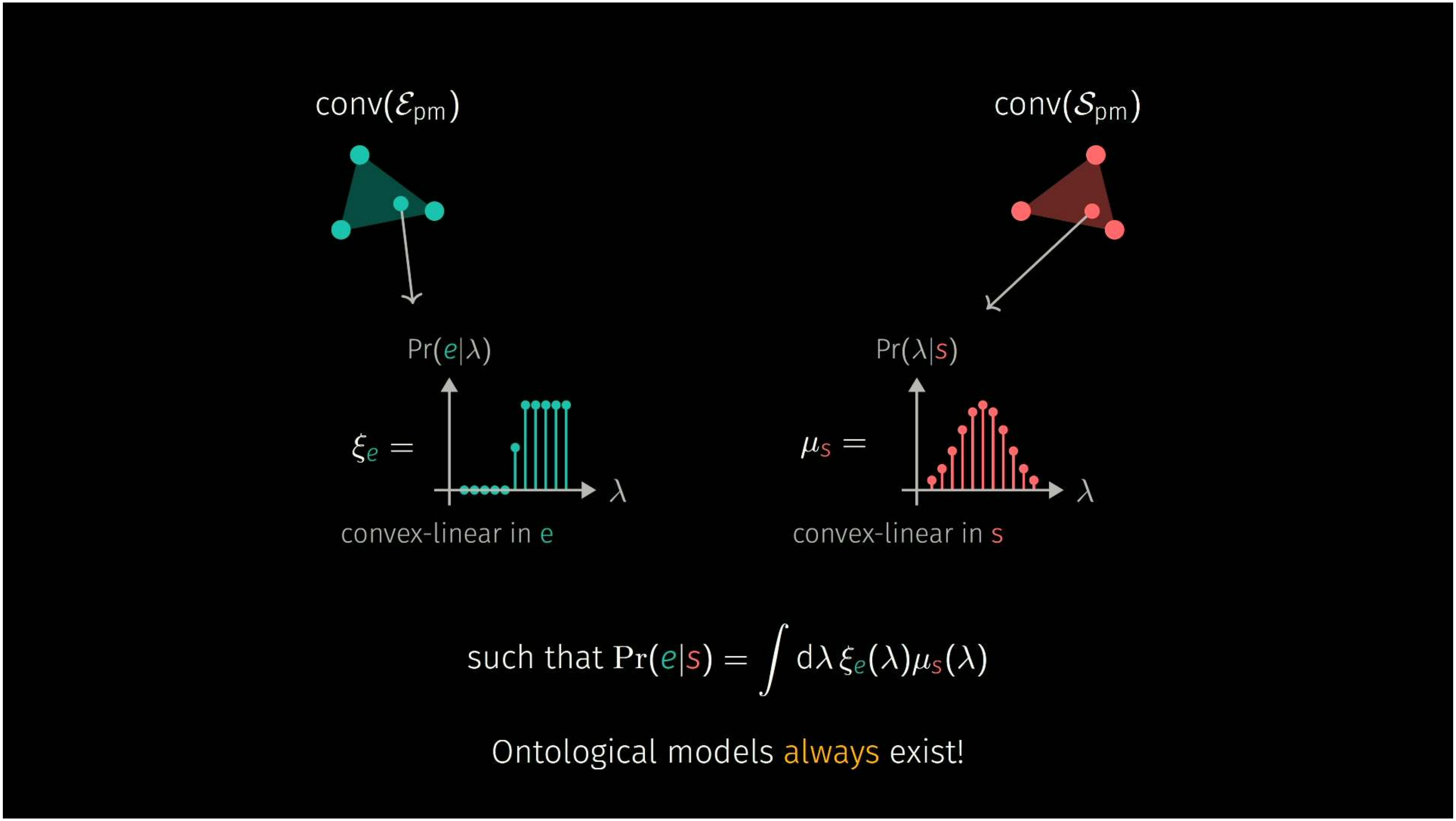
$\mathrm{conv}(\mathcal{E}_{\mathrm{pm}})$
$\mathrm{conv}(\mathcal{S}_{\mathrm{pm}})$
$\Pr(e|\lambda)$
$\Pr(\lambda|s)$
$\xi_e =$
$\mu_s =$
λ
λ
convex-linear in s



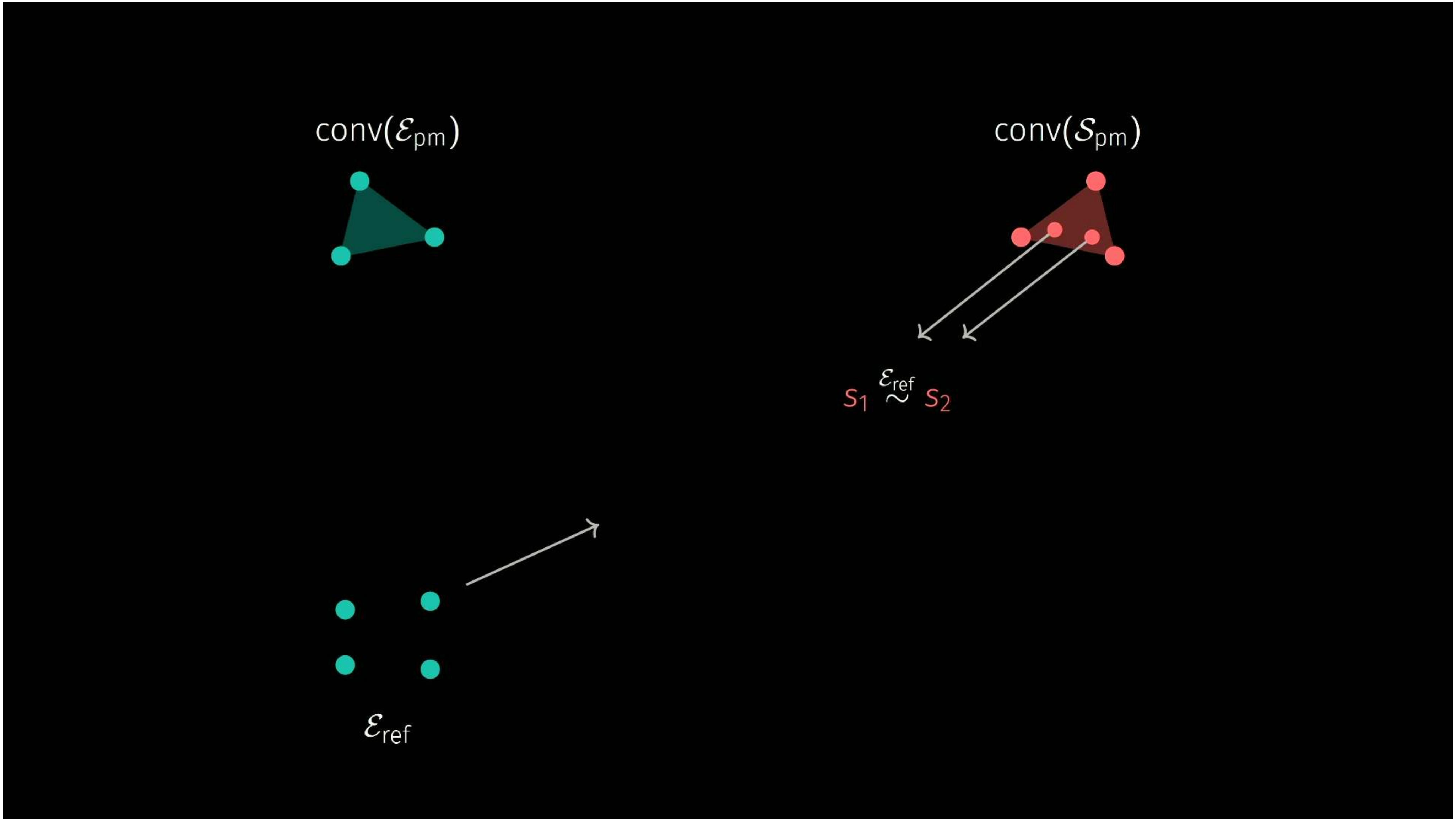


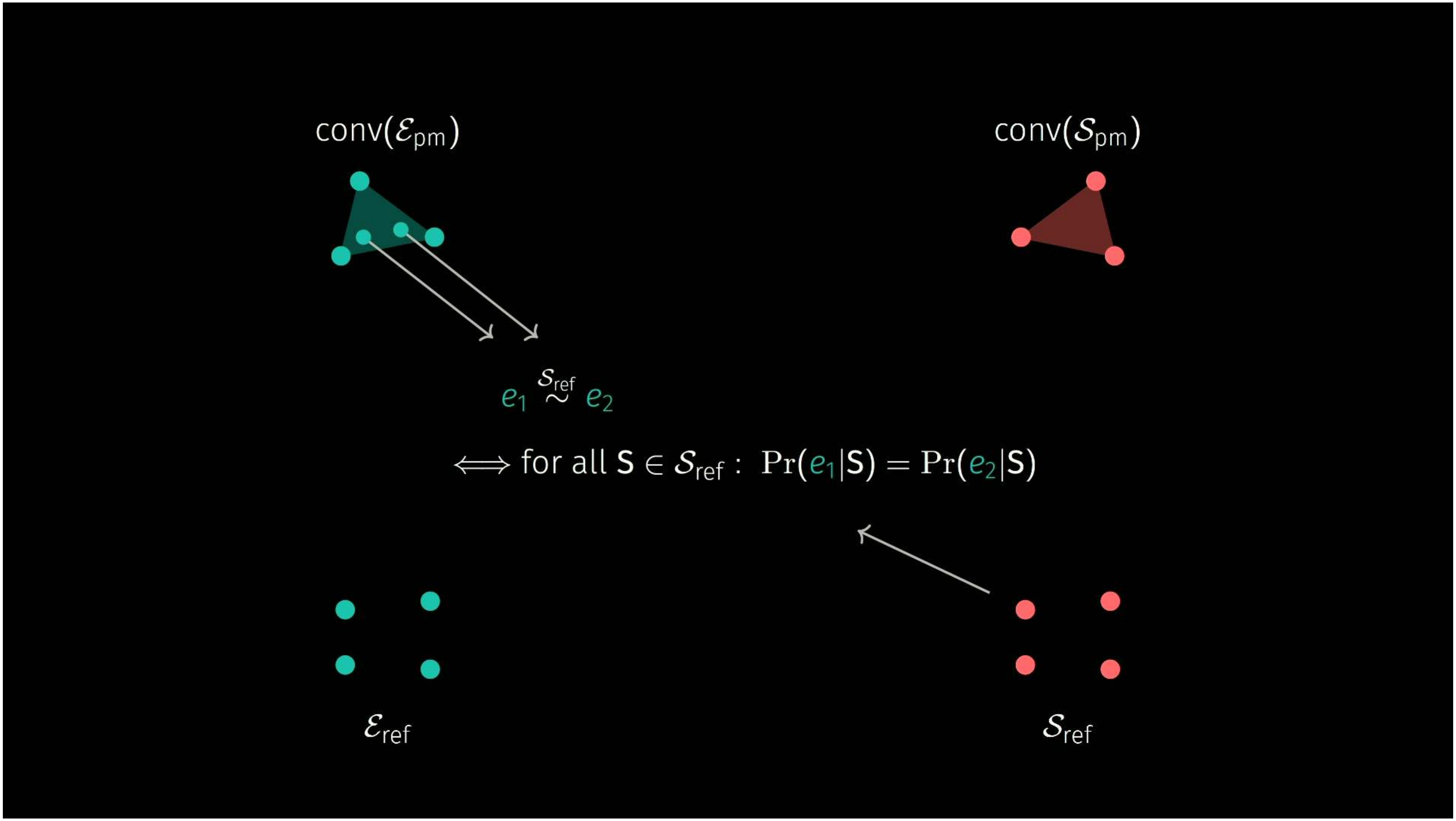
such that $\mathrm{Pr}(e|s) = \int \mathrm{d}\lambda\, \xi_e(\lambda)\mu_s(\lambda)$

Reference of indistinguishability



conv($\mathcal{E}_{\mathrm{pm}}$)
conv($\mathcal{S}_{\mathrm{pm}}$)
Pr($e|\lambda$)
$\xi_e =$
λ
convex-linear in e
Pr($\lambda|s$)
$\mu_s =$
λ
convex-linear in s
such that $\Pr(e|s) = \int \mathrm{d}\lambda\, \xi_e(\lambda)\mu_s(\lambda)$
Ontological models always exist!


$\mathrm{conv}(\mathcal{E}_{\mathrm{pm}})$
$\mathrm{conv}(\mathcal{S}_{\mathrm{pm}})$
$s_1 \overset{\mathcal{E}_{\mathrm{ref}}}{\sim} s_2$
$\mathcal{E}_{\mathrm{ref}}$


conv($\mathcal{E}_{\text{pm}}$)
conv($\mathcal{S}_{\text{pm}}$)
$e_1 \overset{\mathcal{S}_{\text{ref}}}{\sim} e_2$
$\iff$ for all $\mathsf{S} \in \mathcal{S}_{\text{ref}}$: $\Pr(e_1|\mathsf{S}) = \Pr(e_2|\mathsf{S})$
$\mathcal{E}_{\text{ref}}$
$\mathcal{S}_{\text{ref}}$

Noncontextual OM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $(\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$

$$\iff \text{OM for } (\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}}) \text{ such that } \begin{cases} s_1 \overset{\mathcal{E}_{\text{ref}}}{\sim} s_2 \implies \mu_{s_1} = \mu_{s_2} \\ e_1 \overset{\mathcal{S}_{\text{ref}}}{\sim} e_2 \implies \xi_{e_1} = \xi_{e_2} \end{cases}$$

Perfect distinguishability

Suppose that $\{\mathsf{E_S}\}_{\mathsf{S}\in\mathcal{S}_{\mathrm{pm}}} \subseteq \mathcal{E}_{\mathrm{ref}}$

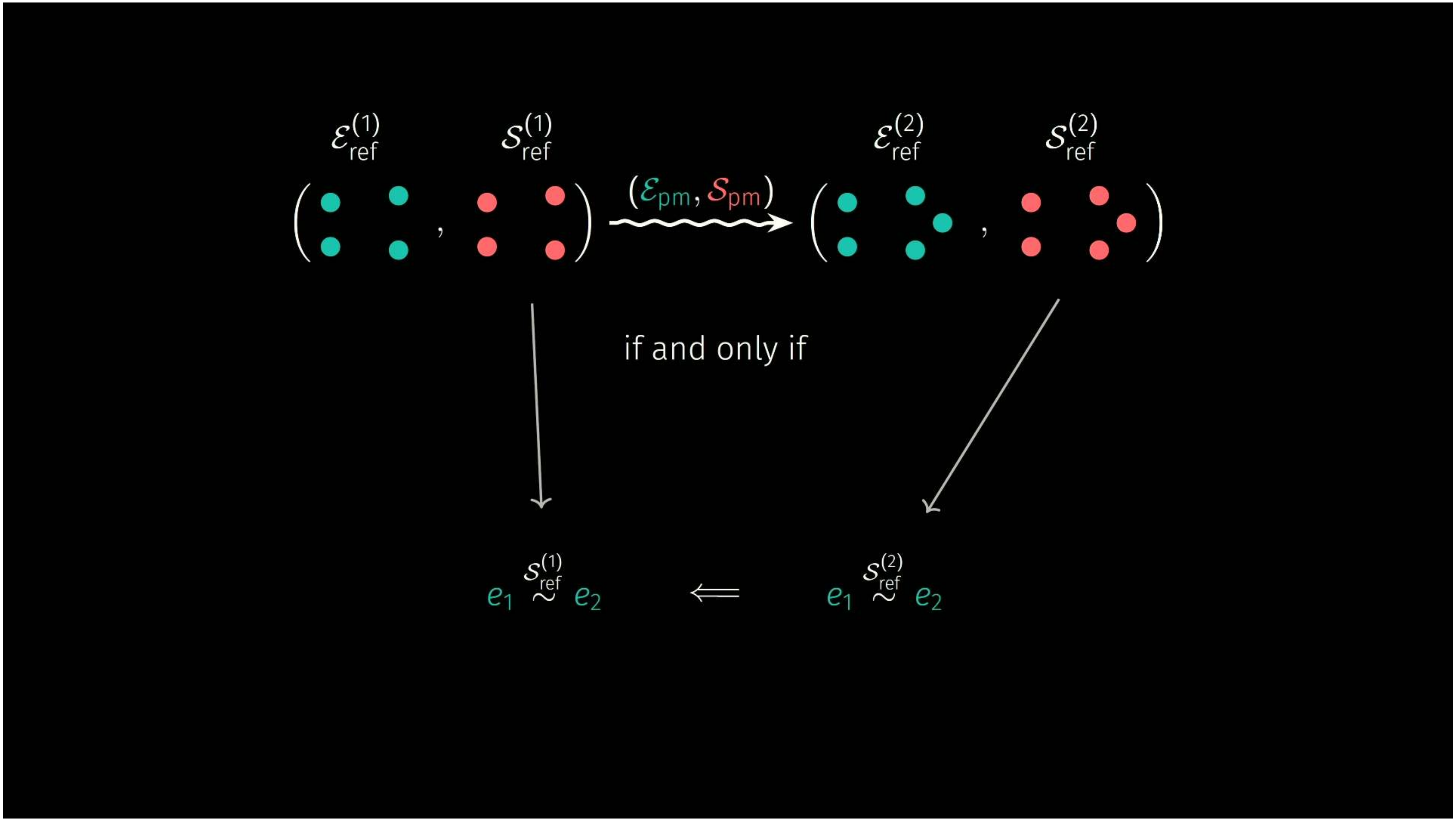
with $\Pr(\mathsf{E_S}|\mathsf{S}') = \begin{cases} 1 \text{ if } \mathsf{S} = \mathsf{S}' \\ 0 \text{ else} \end{cases}$

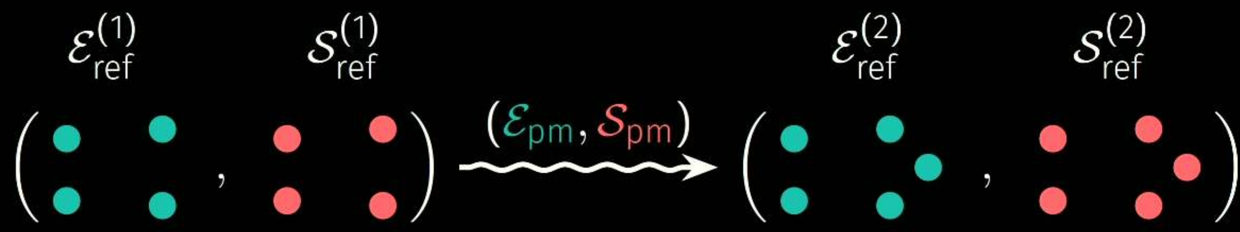
Suppose that $\{E_S\}_{S \in \mathcal{S}_{pm}} \subseteq \mathcal{E}_{ref}$

with $\Pr(E_S | S') = \begin{cases} 1 \text{ if } S = S' \\ 0 \text{ else} \end{cases}$

then there exists a NCOM for $(\mathcal{E}_{pm}, \mathcal{S}_{pm})$ relative to $(\mathcal{E}_{ref}, \mathcal{S}_{ref})$

Should restrict the reference $(\mathcal{E}_{ref}, \mathcal{S}_{ref})$!


$\mathcal{E}_{\text{ref}}^{(1)}$
$\mathcal{S}_{\text{ref}}^{(1)}$
$\mathcal{E}_{\text{ref}}^{(2)}$
$\mathcal{S}_{\text{ref}}^{(2)}$
$(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$
if and only if
$e_1 \overset{\mathcal{S}_{\text{ref}}^{(1)}}{\sim} e_2 \Longleftarrow e_1 \overset{\mathcal{S}_{\text{ref}}^{(2)}}{\sim} e_2$



if and only if

$$s_1 \overset{\mathcal{E}_{\text{ref}}^{(1)}}{\sim} s_2 \quad \Longleftarrow \quad s_1 \overset{\mathcal{E}_{\text{ref}}^{(2)}}{\sim} s_2$$

$$e_1 \overset{\mathcal{S}_{\text{ref}}^{(1)}}{\sim} e_2 \quad \Longleftarrow \quad e_1 \overset{\mathcal{S}_{\text{ref}}^{(2)}}{\sim} e_2$$

Preorder: reflexive, transitive

If $\left(\mathcal{E}_{\text{ref}}^{(1)}, \mathcal{S}_{\text{ref}}^{(1)}\right) \xrightarrow{(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})} \left(\mathcal{E}_{\text{ref}}^{(2)}, \mathcal{S}_{\text{ref}}^{(2)}\right)$:

$\exists$ NCOM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $\left(\mathcal{E}_{\text{ref}}^{(1)}, \mathcal{S}_{\text{ref}}^{(1)}\right)$ $\implies$ $\exists$ NCOM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $\left(\mathcal{E}_{\text{ref}}^{(2)}, \mathcal{S}_{\text{ref}}^{(2)}\right)$

If there exists a NCOM for $(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$ relative to $(\mathcal{E}_{\mathrm{ref}}, \mathcal{S}_{\mathrm{ref}})$

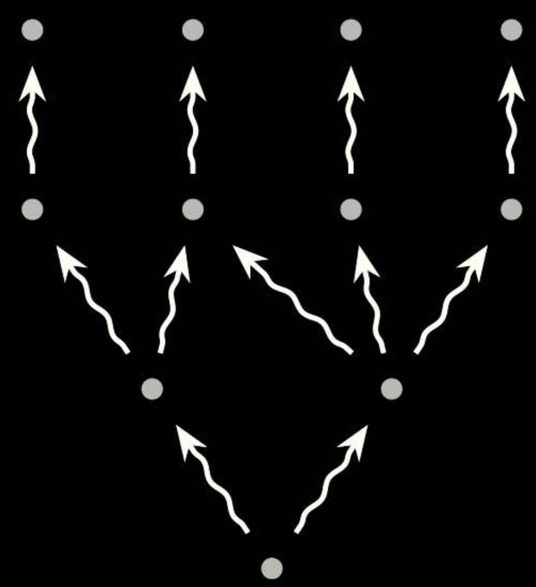
If there exists a NCOM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $(\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$

then $$(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}}) \overset{(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})}{\rightsquigarrow} (\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$$

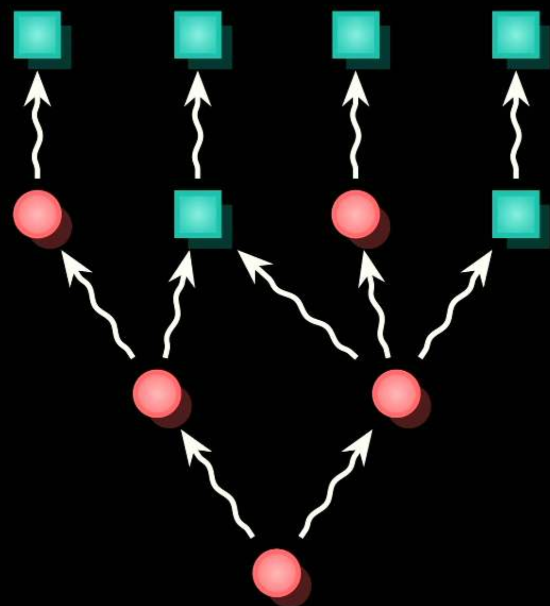
i.e., by definition, $(\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$ is a faithful reference

Relative noncontextuality graph

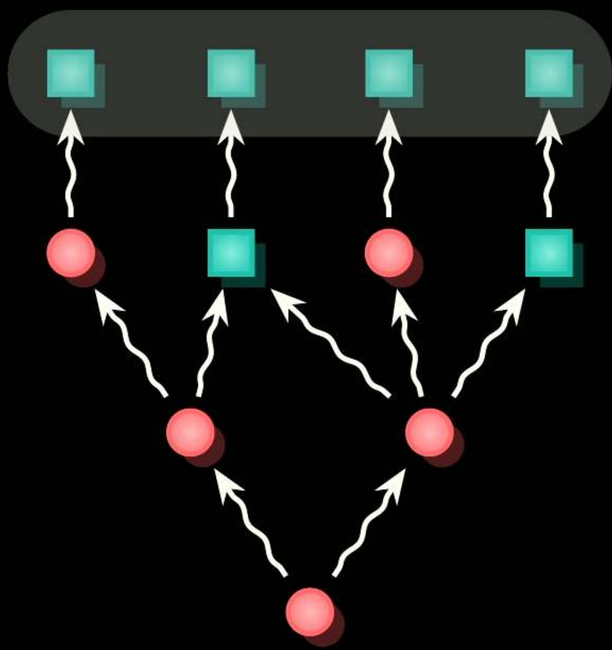
Fix $(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$ and vary the reference:



Fix $(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$ and vary the reference:

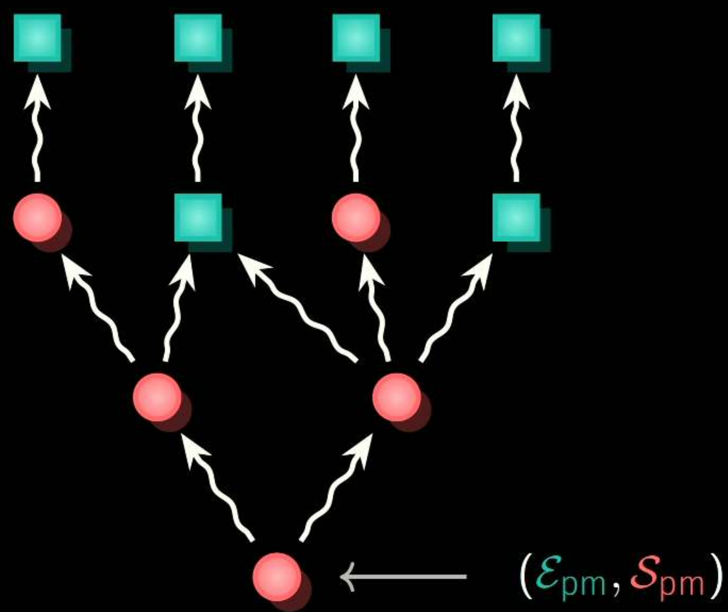


Fix $(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$ and vary the reference:



expect classicality

Fix $(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$ and vary the reference:

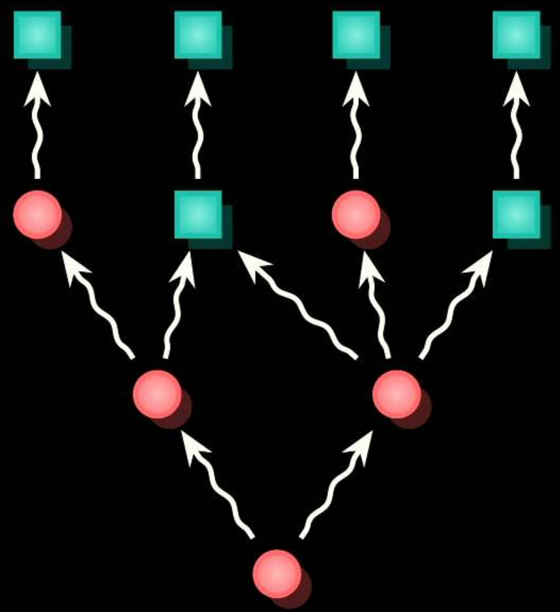


If there exists a NCOM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $(\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$

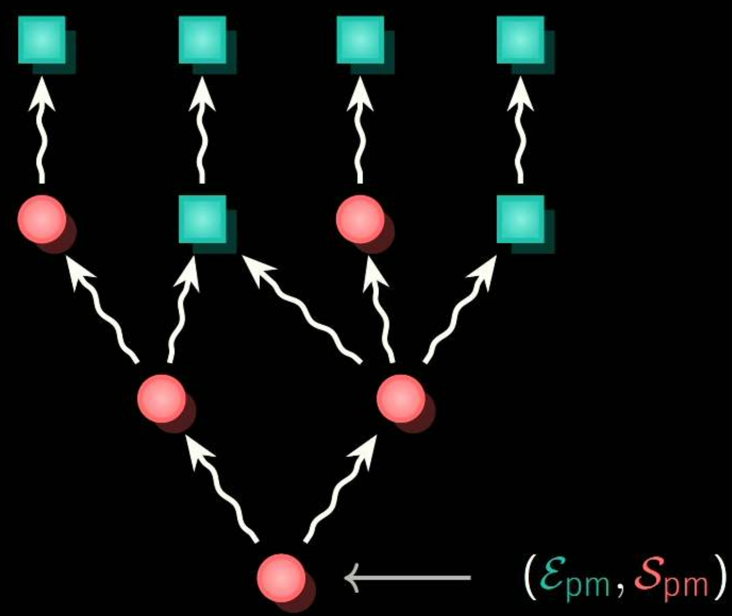
$$\text{then} \quad (\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}}) \xrightarrow{(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})} (\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$$

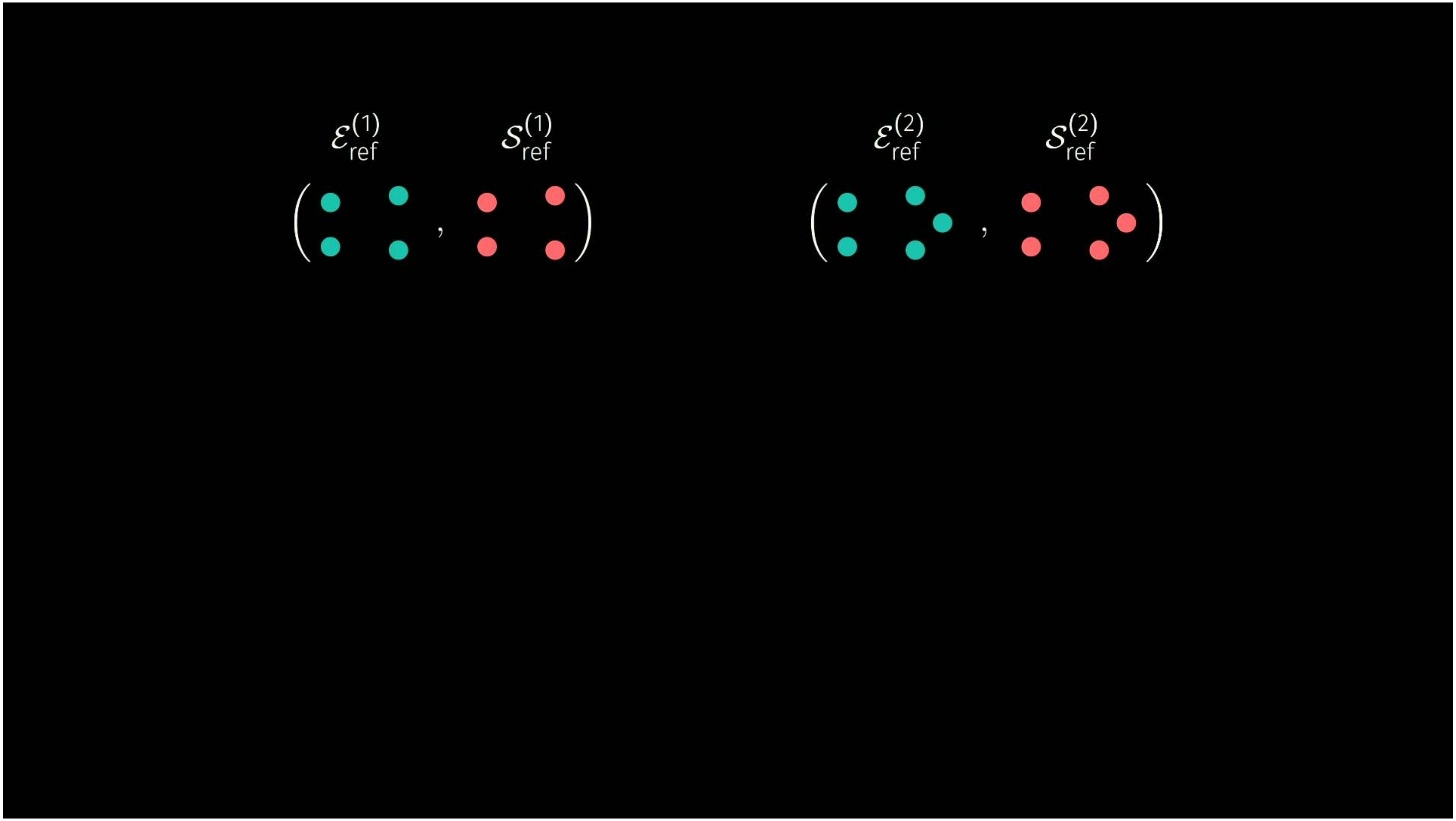
i.e., by definition, $(\mathcal{E}_{\text{ref}}, \mathcal{S}_{\text{ref}})$ is a faithful reference

Fix $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ and vary the reference:



Fix $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ and vary the reference:




$\mathcal{E}_{\text{ref}}^{(1)}$
$\mathcal{S}_{\text{ref}}^{(1)}$
$\mathcal{E}_{\text{ref}}^{(2)}$
$\mathcal{S}_{\text{ref}}^{(2)}$

$(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ is operationally noncontextual [VG,MPW 22]

if and only if

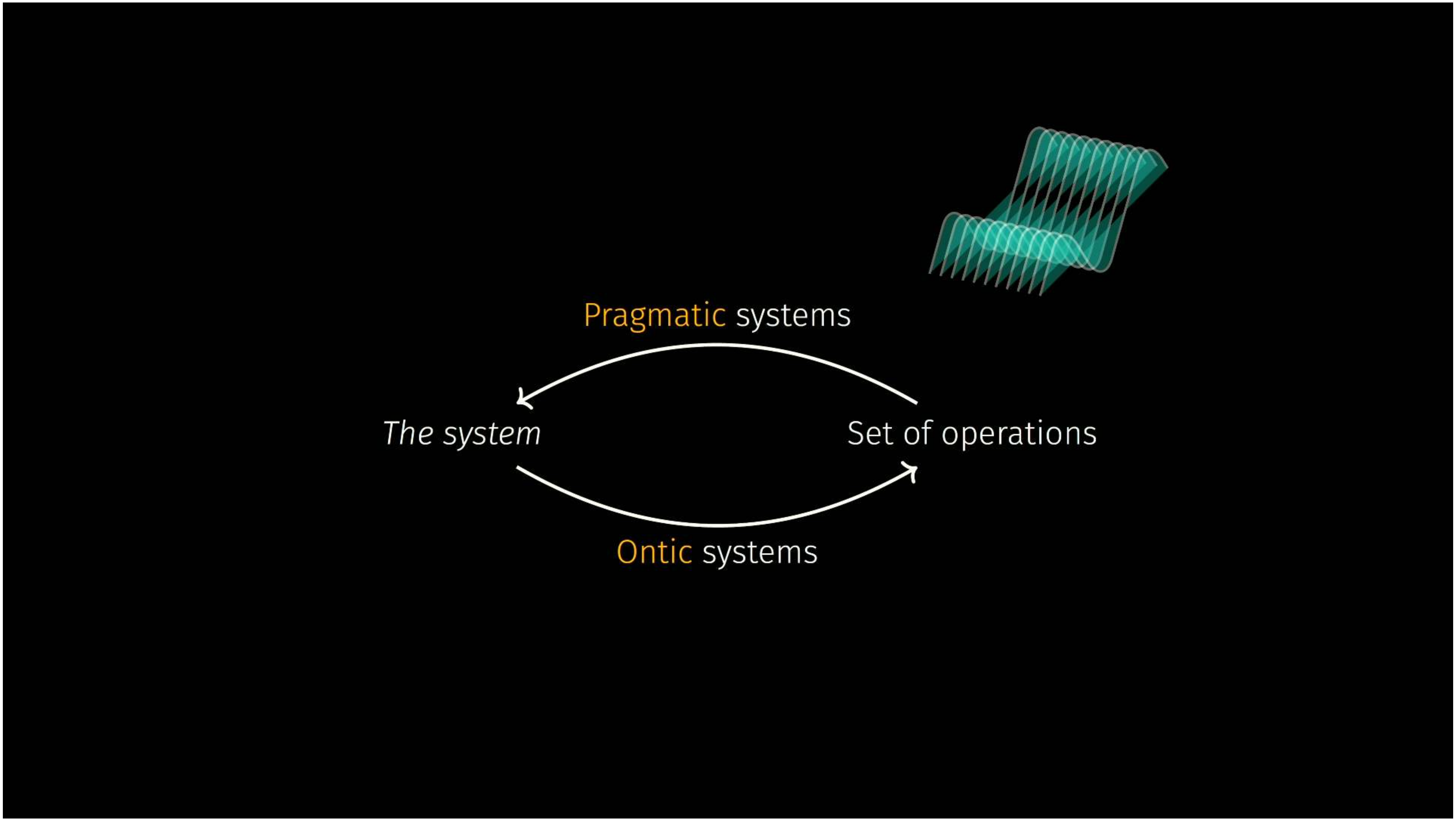
$\exists$ a noncontextual OM for $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$ relative to $(\mathcal{E}_{\text{pm}}, \mathcal{S}_{\text{pm}})$

Pros: simple

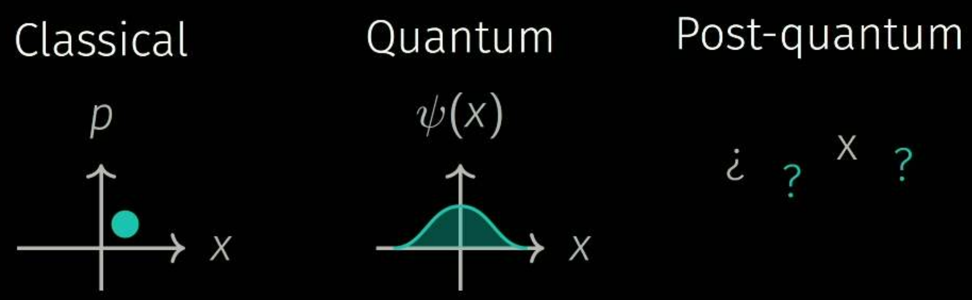
Notions of systems

The system

Set of operations

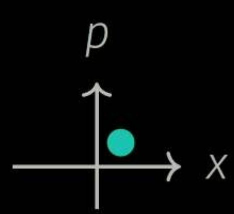

Pragmatic systems
The system
Set of operations
Ontic systems

The case of a particle

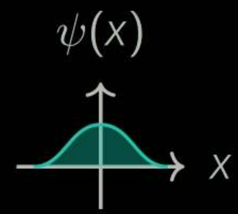


As pragmatic systems: all valid (related) choices

Classical



Quantum



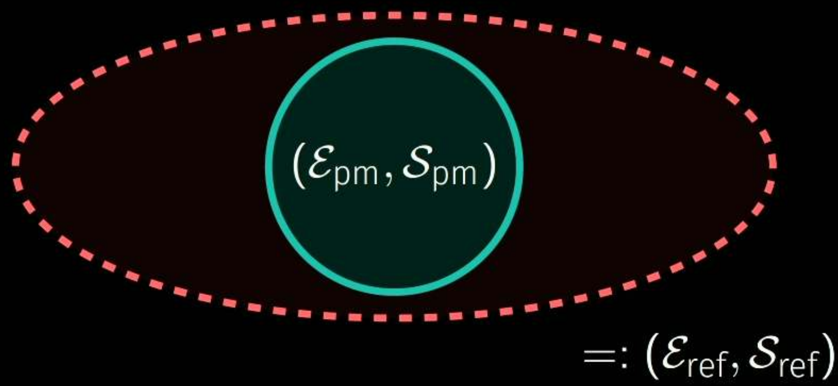
Post-quantum

¿ ? x ?

As pragmatic systems: all valid (related) choices

As ontic systems: quantum is valid (unless post-quantum exists)

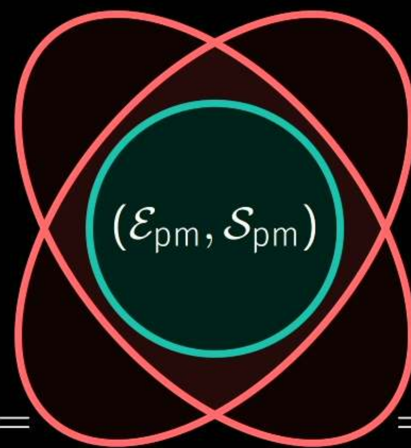
Smallest supporting ontic system



Open: justify ontic system partition?

Two choices of pragmatic systems

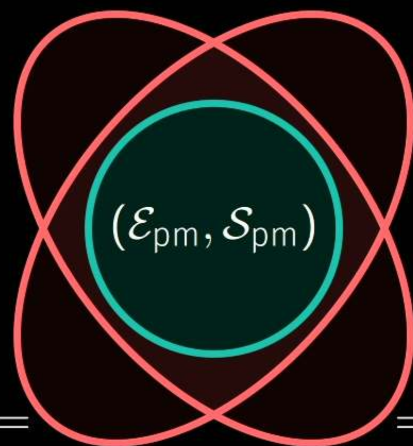
$(\mathcal{E}^{(1)}_{\text{ref}}, \mathcal{S}^{(1)}_{\text{ref}}) :=$



$=: (\mathcal{E}^{(2)}_{\text{ref}}, \mathcal{S}^{(2)}_{\text{ref}})$

Two choices of pragmatic systems

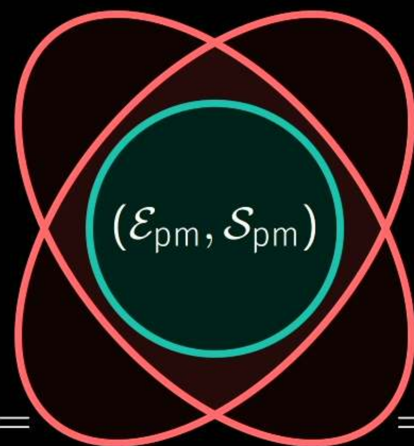
$(\mathcal{E}_{\text{ref}}^{(1)}, \mathcal{S}_{\text{ref}}^{(1)}) :=$



$=: (\mathcal{E}_{\text{ref}}^{(2)}, \mathcal{S}_{\text{ref}}^{(2)})$

Open: restrict to well-justified pragmatic systems.
Are verdicts of noncontextuality consistent?

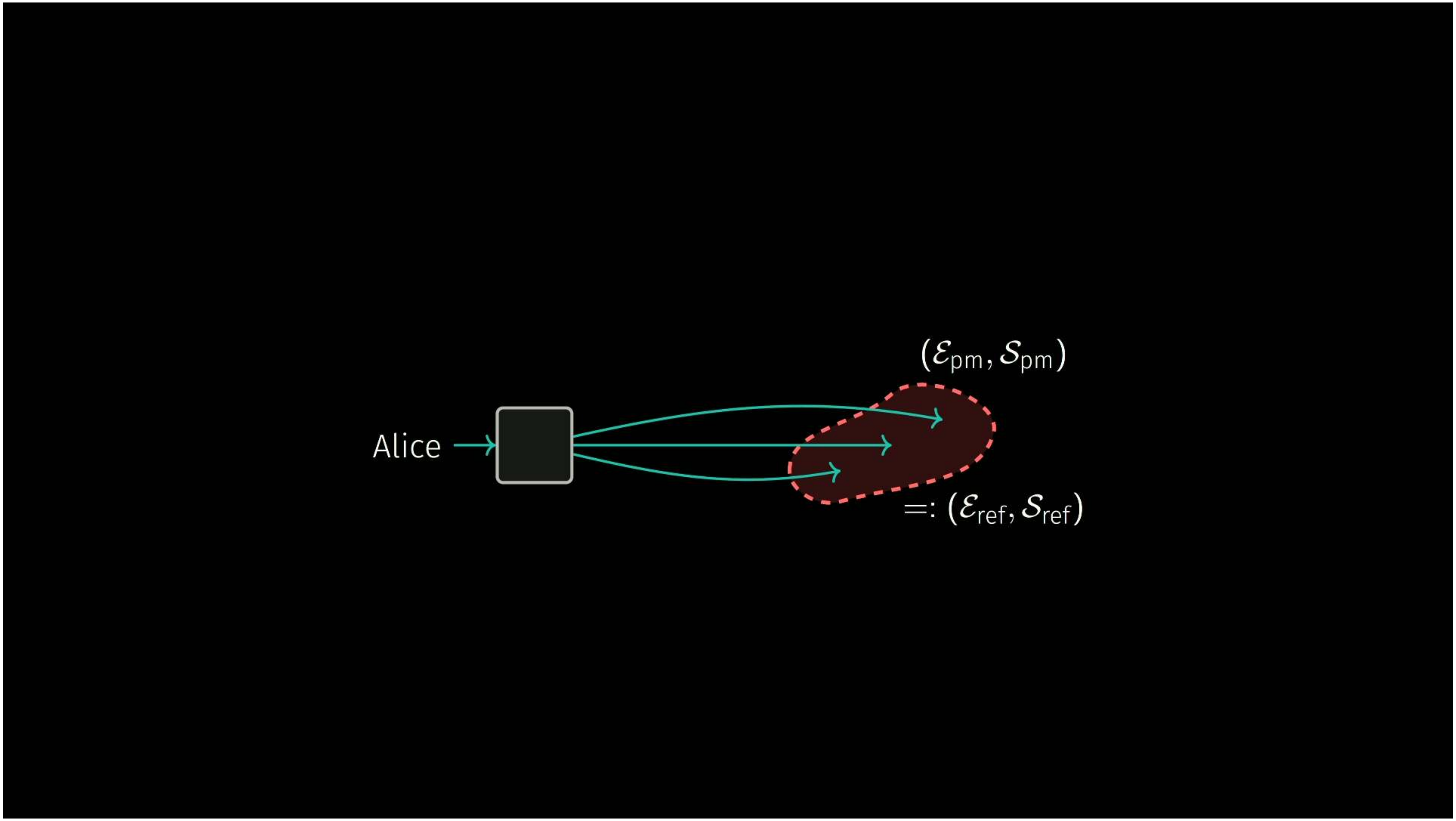
Two choices of pragmatic systems



$(\mathcal{E}^{(1)}_{\text{ref}}, \mathcal{S}^{(1)}_{\text{ref}}) :=$ $=: (\mathcal{E}^{(2)}_{\text{ref}}, \mathcal{S}^{(2)}_{\text{ref}})$

If inconsistent verdicts:
let noncontextuality influence our choices of systems?

Local noncontextuality


$(\mathcal{E}_{\mathrm{pm}}, \mathcal{S}_{\mathrm{pm}})$
Alice
$=: (\mathcal{E}_{\mathrm{ref}}, \mathcal{S}_{\mathrm{ref}})$

Outlook

What do prepare-transform-measure scenarios add?

Clear methodology of individuation?

Thank you