Title: Counting the microstates of the cosmic horizon

Speakers: Vasudev Shyam

Series: Quantum Gravity

Date: November 10, 2022 - 2:30 PM

URL: https://pirsa.org/22110078

Abstract: I will describe a holographic model for the three dimensional de Sitter static patch where the boundary theory is the so called $T\bar{T}+\Lambda_2$ deformation of the conformal field theory dual to AdS_3 quantum gravity. This identification allows us to obtain the cosmic horizon entropy from a microstate count, and the microstates themselves are a dressed version of those that account for the entropy of certain black holes in AdS space. I will also show how the effect of this dressing at the cosmic horizon is to replace the spacetime dependence of the fields of the undeformed holographic CFT with dependence on the indices of large matrices.

Zoom link: https://pitp.zoom.us/j/95396921570?pwd=NGFoOGlGY1ZDU2pnNFRwWit3b2w0Zz09

Counting the micro-states of the cosmic horizon

Vasu Shyam

The Branco Weiss Fellowship
Society in Science

Based on:

arXiv > hep-th > arXiv:2106.10227

$T\bar{T} + \Lambda_2$ Deformed CFT on the Stretched dS$_3$ Horizon

Vasudev Shyam

arXiv > hep-th > arXiv:2110.14670

de Sitter Microstates from $T\bar{T} + \Lambda_2$ and the Hawking-Page Transition

Evan Coleman, Edward A. Mazenc, Vasudev Shyam, Eva Silverstein, Ronak M Soni, Gonzalo Torroba, Sungyeon Yang

arXiv > hep-th > arXiv:2209.11753

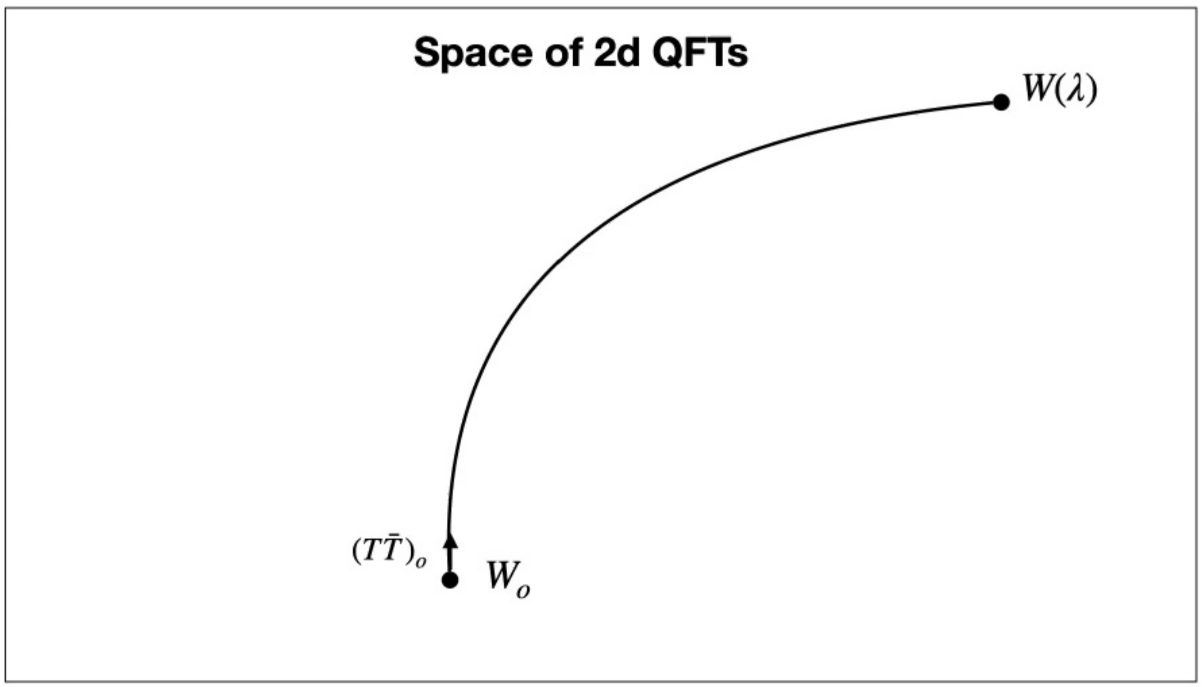
$T\bar{T}$ deformed scattering happens within matrices

Vasudev Shyam, Yigit Yargic

Deforming QFTs in $d = 2$ by $T\bar{T}$

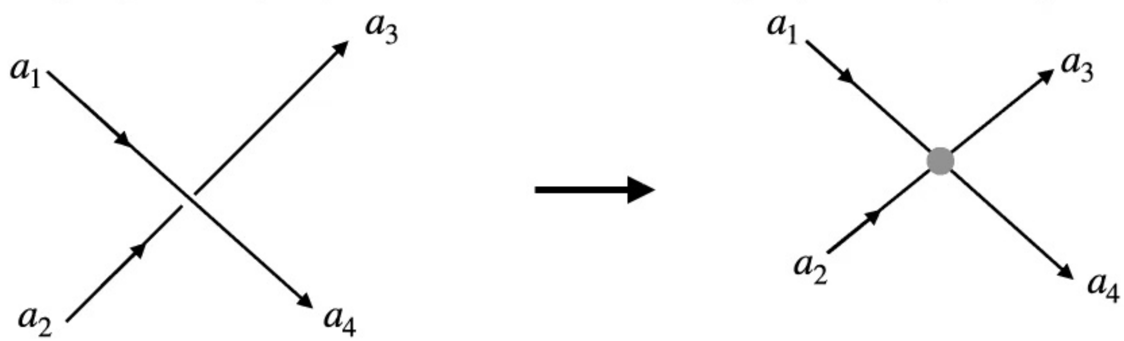
Smirnov & Zamolodchikov, 16'
Cavaglia et. al. 16'

$$\partial_\lambda W(\lambda) = \int_x \langle T\bar{T} \rangle \equiv \int d^2x \det f \left\langle \left(T^{\mu\nu}T_{\mu\nu} - (T^\alpha_\alpha)^2 \right) \right\rangle, \;\; W(\lambda = 0) = W_o$$

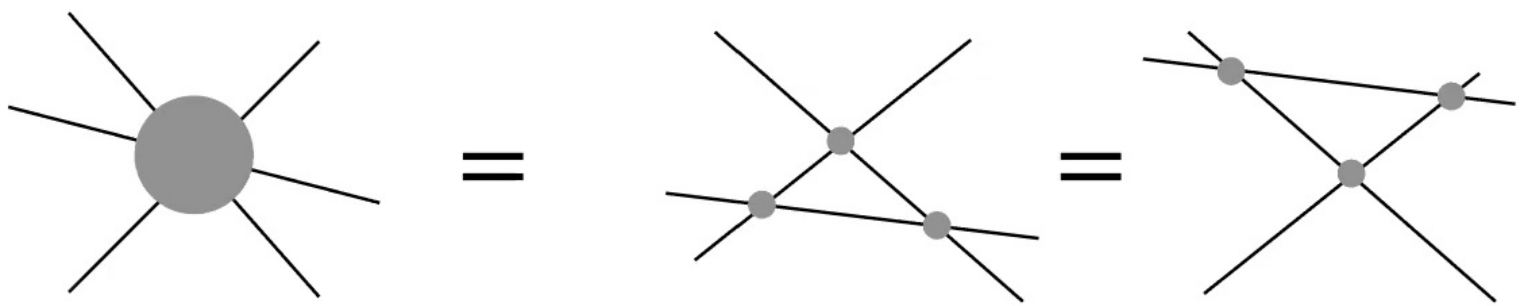


What $T\bar{T}$ does

$$S(\{p_{a_1}, p_{a_2}\} \to \{p_{a_3}, p_{a_4}\}) = 1 \qquad S(\{p_{a_1}, p_{a_2}\} \to \{p_{a_1}, -p_{a_2}\}) = \exp(\lambda(p_{a_1} \wedge p_{a_2}))$$



Integrability:



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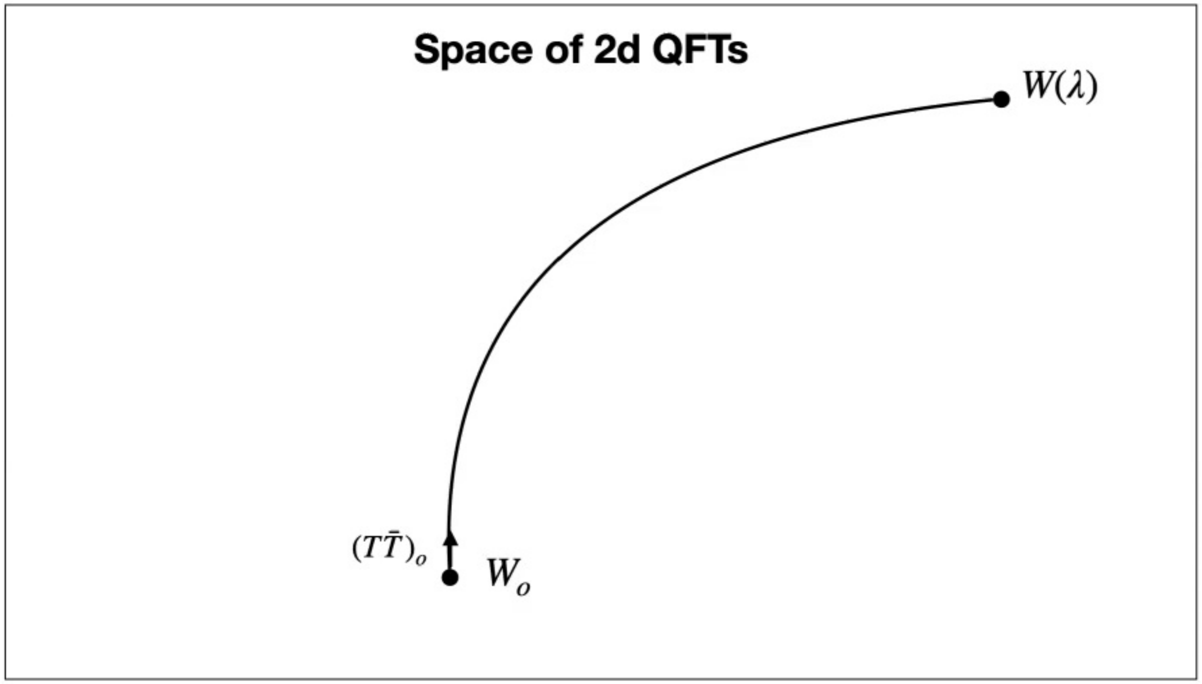
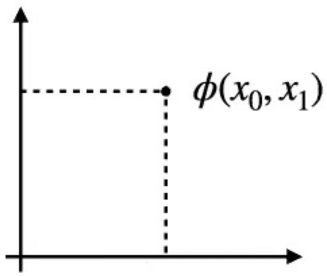


$T\bar{T}$ turns fields into matrices

Yigit Yagrıç, V.S.



$$\longrightarrow \quad \begin{pmatrix} \phi_{11} & \cdots & \phi_{1N} \\ \vdots & \ddots & \vdots \\ & & \phi_{NN} \end{pmatrix}$$

$$\phi_{ij} = \phi_{\lambda^{-1}\left(\frac{i-j}{2}\right), \frac{i+j}{2}} \equiv \phi\left(p_0 = \frac{1}{\lambda}\left(\frac{i-j}{2}\right), x_1 = \frac{i+j}{2}\right) \qquad \phi(p_0, p_1) = \int dx_1\, e^{ip_1x_1}\phi(p_0, x_1)$$

$$\phi \cdot \psi \to \phi \star \psi$$

$$\phi(p_0^a, p_1^b) \star \phi(p_0^b, p_1^b) = e^{i\lambda p^a \wedge p^b}\phi(p_0^a, p_1^b) \cdot \phi(p_0^b, p_1^b)$$

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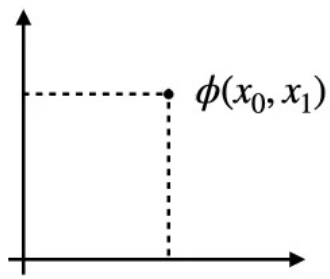
Yigit Yagrıç, V.S.



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Dressing the number 1

$$S_n(p_{a_1}, \ldots, p_{a_n}) = \prod_i (p^2 + m^2)_i \langle \Omega | \mathcal{T}\left(\varphi(p_{a_1}) \cdots \varphi(p_{a_n}) \right) | \Omega \rangle \Big|_{p_i^2 + m_i^2 = 0,\ \forall i.} = 1$$

$$\Big\downarrow T\bar{T}$$

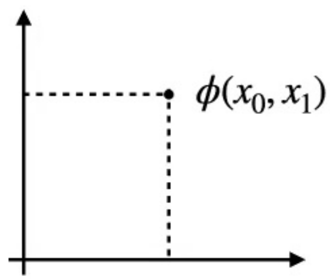
$$\prod_i (p^2 + m^2)_i \langle \Omega | \mathcal{T}\left(\varphi(p_{a_1}) \star \cdots \star \varphi(p_{a_n}) \right) | \Omega \rangle \Big|_{p_i^2 + m_i^2 = 0,\ \forall i} =$$

$$= \exp\left(-i\lambda \sum_{i<j=1}^{l} \left(p_{a_i} \wedge p_{a_j} \right) \right) \times \exp\left(i\lambda \sum_{k<r=l}^{n} \left(p_{a_k} \wedge p_{a_r} \right) \right) \times 1$$

[Dubovsky, Gorbenko, Mirbabayi 18']

$T\bar{T}$ turns fields into matrices

Yigit Yagrıç, V.S.



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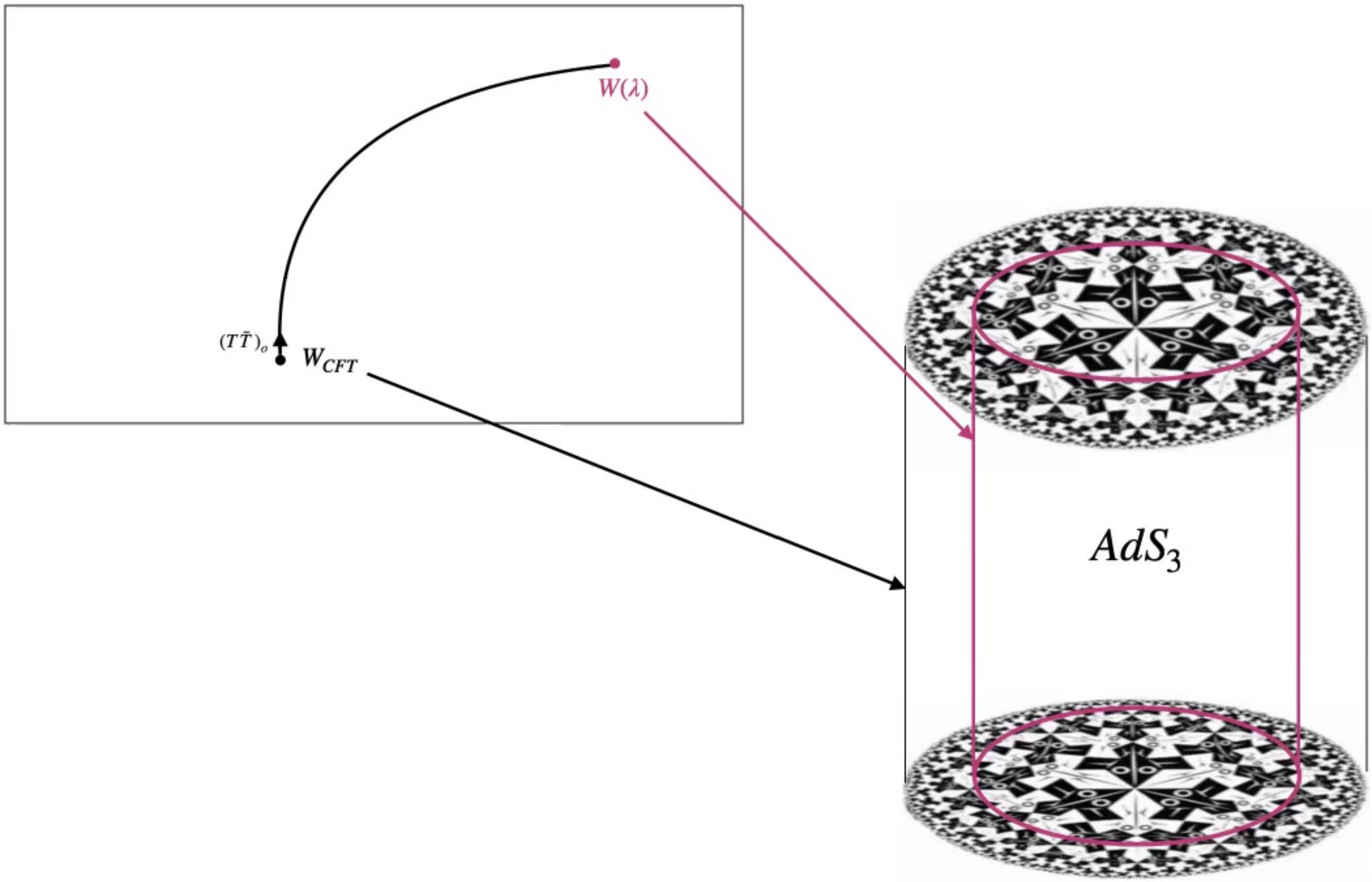
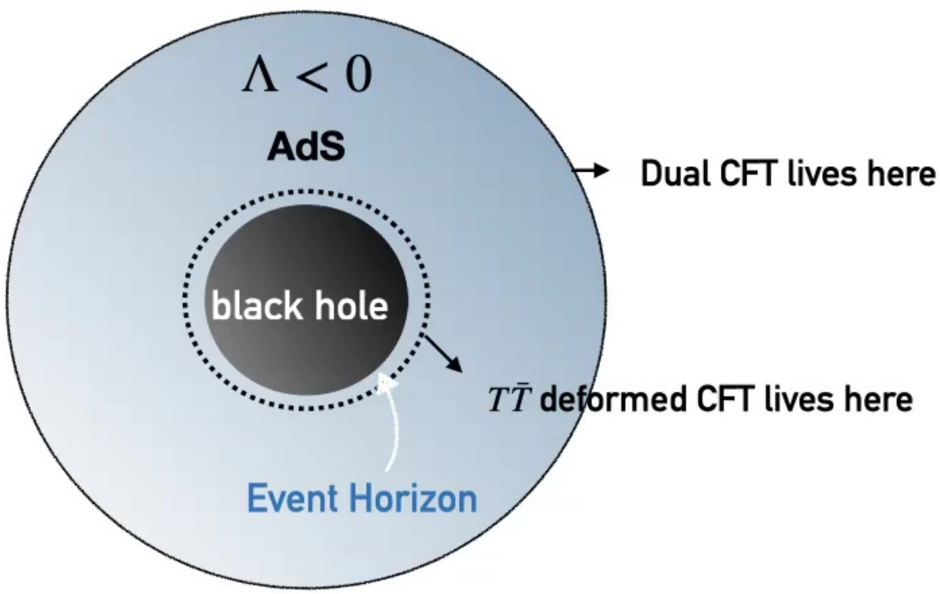
$T\bar{T}$ holography: moving "into" the bulk

McGough, Mezei, Verlinde 16'

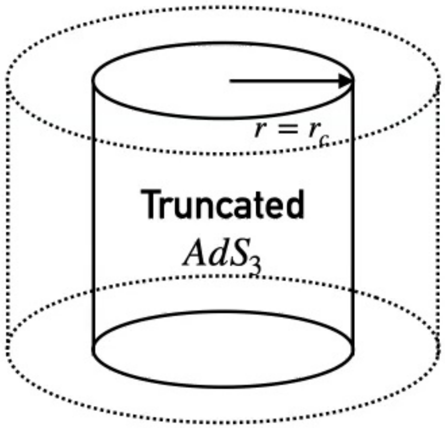


Quasi local energy of cutoff AdS_3



Quasi Local energy:

$$E = \frac{r_c}{4G}\left[1 - \sqrt{1 - \frac{8GM}{r_c^2} + \frac{16G^2J^2}{r_c^4}}\right]$$



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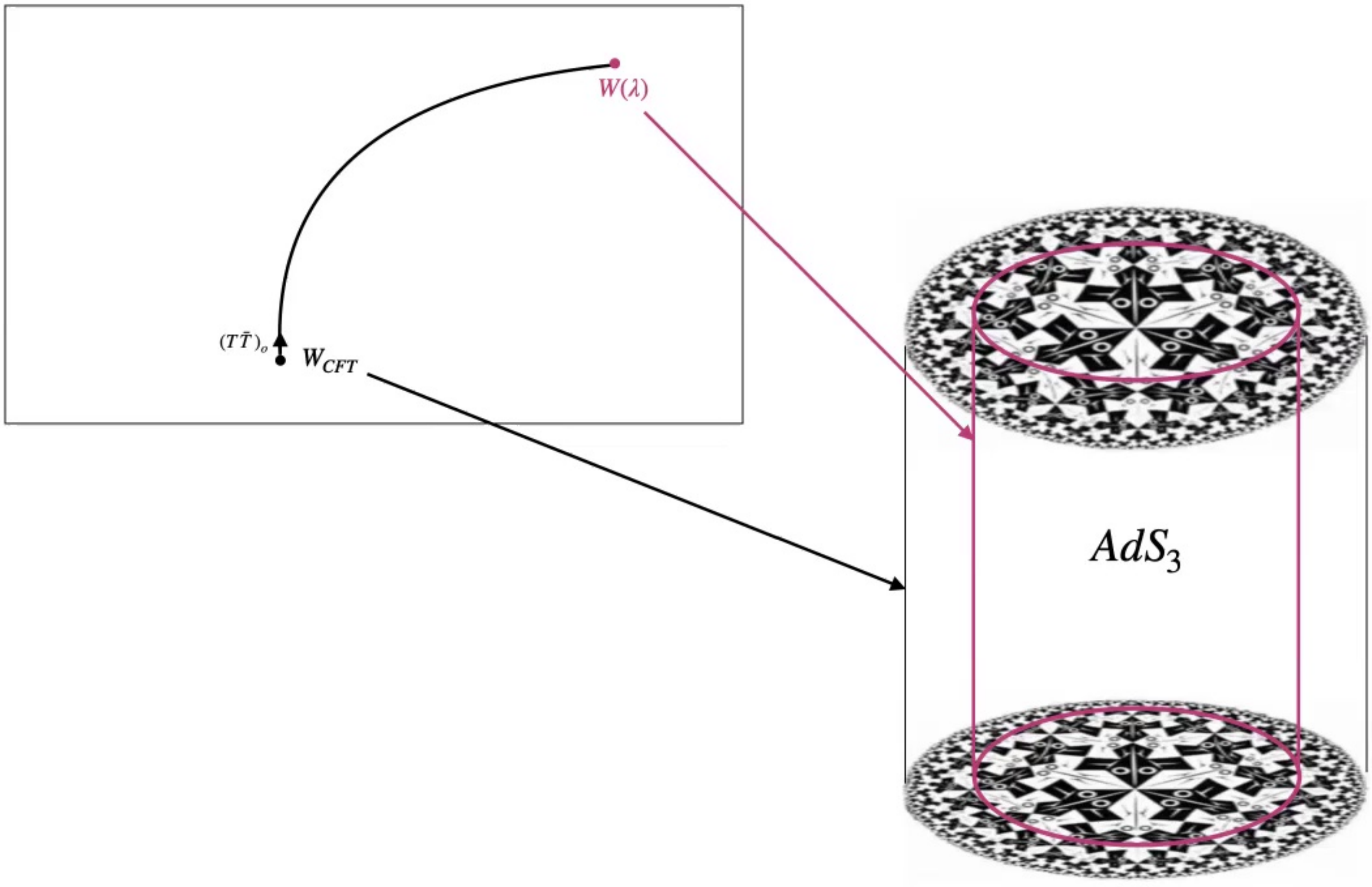
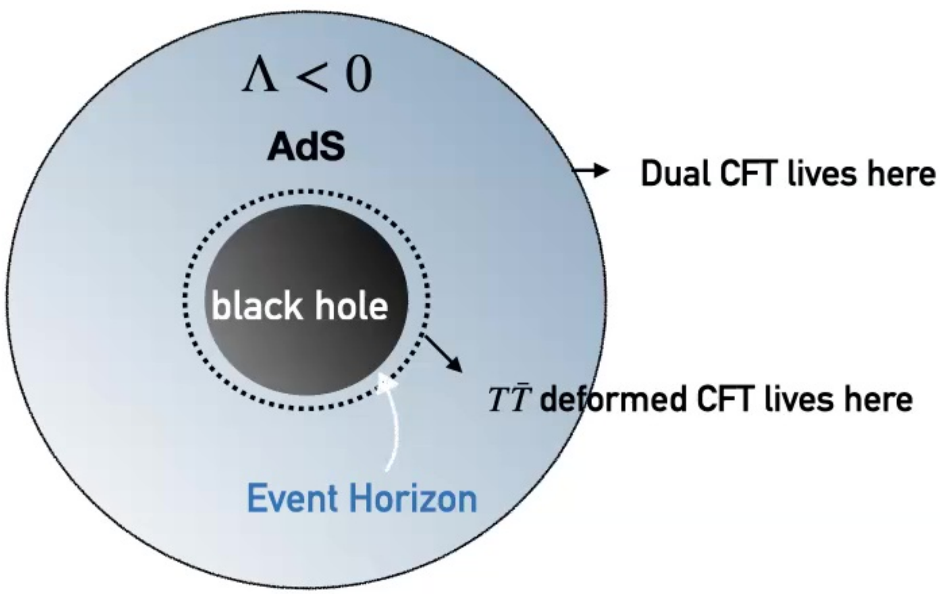


Quasi local energy of cutoff AdS_3



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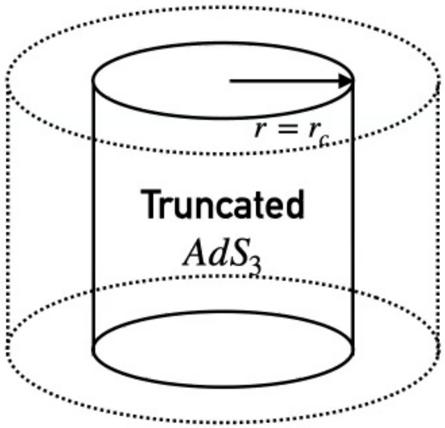
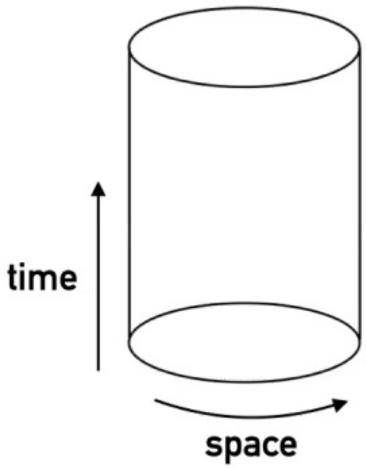


Energy levels flow under $T\bar{T}$



$$\partial_\mu E_n(\mu, L) = L\langle n \,|\, T\bar{T} \,|\, n\rangle = -\frac{1}{4}\left(E_n \partial_L E_n + \frac{P_n^2}{L^2}\right)$$

$$E_n(\mu, L) = \frac{2L}{\mu}\left(1 - \sqrt{1 \pm 2\pi\mu\left(\frac{\Delta_n + \bar{\Delta}_n - \frac{c}{12}}{L^2}\right) + \pi^2\mu^2\left(\frac{(\Delta_n - \bar{\Delta}_n)^2}{L^4}\right)}\right)$$

$$E_n^o = \frac{1}{L}\left(\Delta_n + \bar{\Delta}_n - \frac{c}{12}\right),\; j_n = \frac{1}{L}(\Delta_n - \bar{\Delta}_n)$$

Matching

[McGough, Mezei, Verlinde 16']

$$E_n(\mu, L) = \frac{2L}{\mu}\left(1 - \sqrt{1 \pm 2\pi\mu\left(\frac{\Delta_n + \bar{\Delta}_n - \frac{c}{12}}{L^2}\right) + \pi^2\mu^2\left(\frac{(\Delta_n - \bar{\Delta}_n)^2}{L^4}\right)}\right)$$

$$\Big\downarrow \quad \Delta_n + \bar{\Delta}_n - \frac{c}{12} = M,\ \Delta_n - \bar{\Delta}_n = J \qquad L = \frac{r_c}{4G}$$

$$E = \frac{r_c}{4G}\left[1 - \sqrt{1 - \frac{8GM}{r_c^2} + \frac{16G^2J^2}{r_c^4}}\right]$$

Honorary mention: Laurent's formula

Freidel 08'

$$\Psi[f] = \int \mathscr{D}e\, e^{-\frac{2}{\lambda}\int d^2x \det(e-f)} Z_{CFT}[e]$$

$$f^a_\mu \frac{\delta \Psi[f]}{\delta f^a_\mu} = \underbrace{\lambda \epsilon^{ab}\epsilon_{\mu\nu}\frac{\delta^2\Psi[f]}{\delta f^a_\mu \delta f^b_\nu}}_{\equiv \langle T\bar{T}\rangle \Psi[f]} + R[f]\det f\,\Psi[f]$$

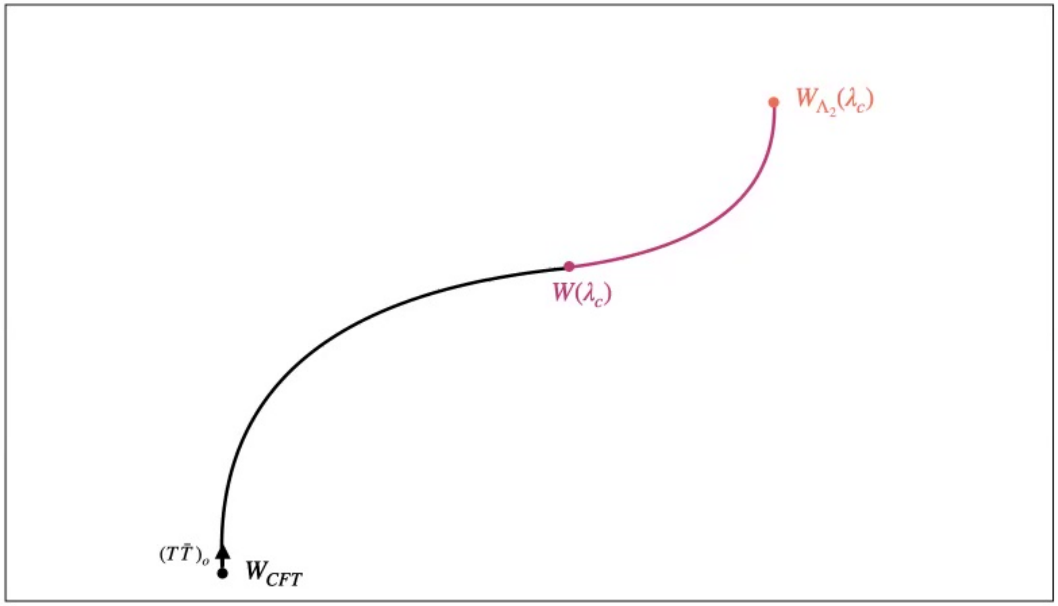
Deformation $\longleftarrow$

$$e^a_\mu \frac{\delta}{\delta e^a_\mu} Z_{CFT}[e] = \frac{c}{24\pi} \det e R(e) Z_{CFT}[e]$$

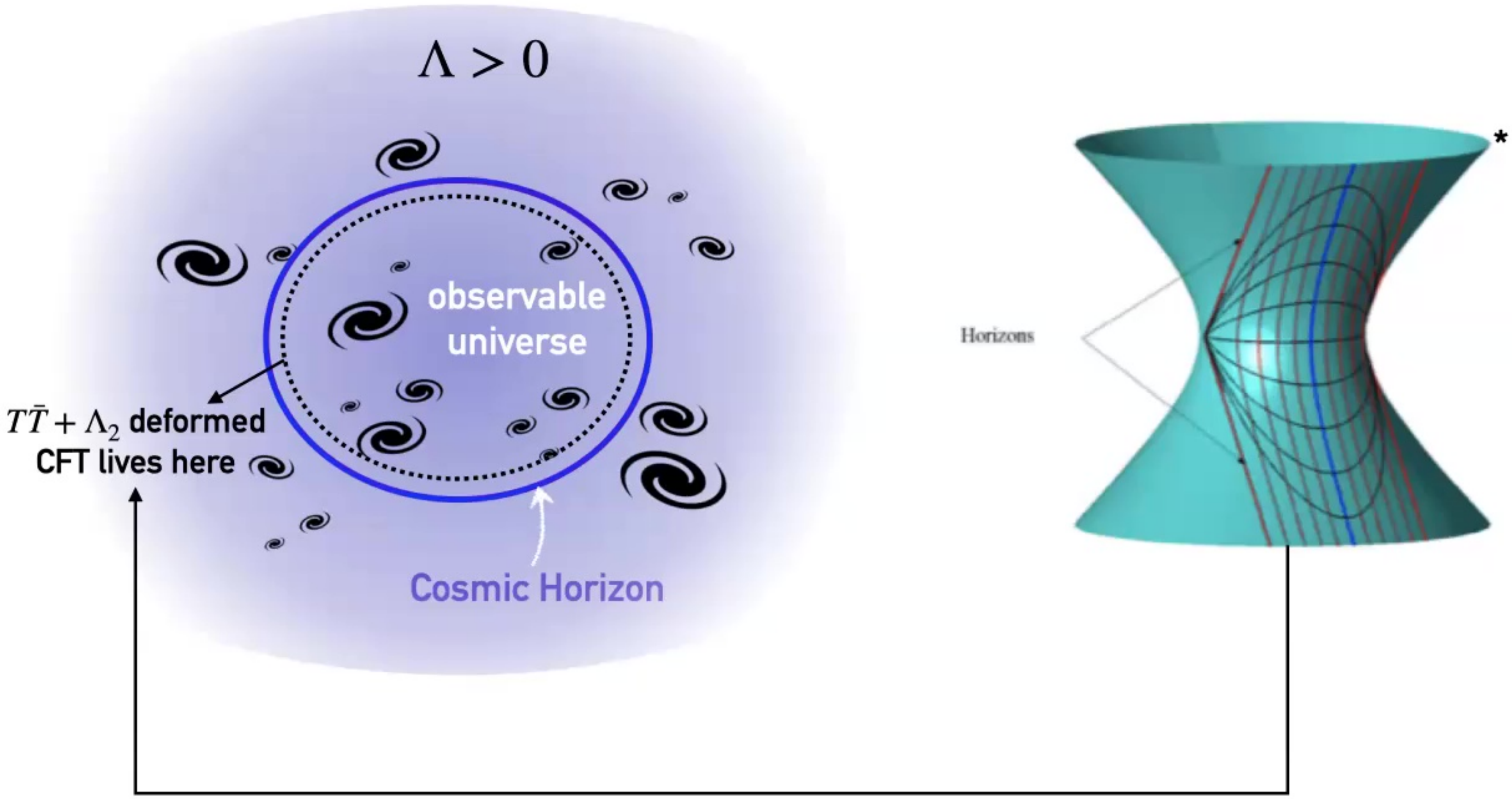
The $T\bar{T} + \Lambda_2$ deformation

Gorbenko, Silverstein, Torroba 18'

$$\partial_\lambda W_{\Lambda_2}(\lambda) = \int d^2x \det f \left\langle \left(T^{\mu\nu}T_{\mu\nu} - (T^\alpha_\alpha)^2 \right) \right\rangle - \frac{1-\eta}{4\pi\lambda^2} \int d^2x \det f, \quad W_{\Lambda_2}(\lambda = \lambda_c) = W(\lambda_c)$$

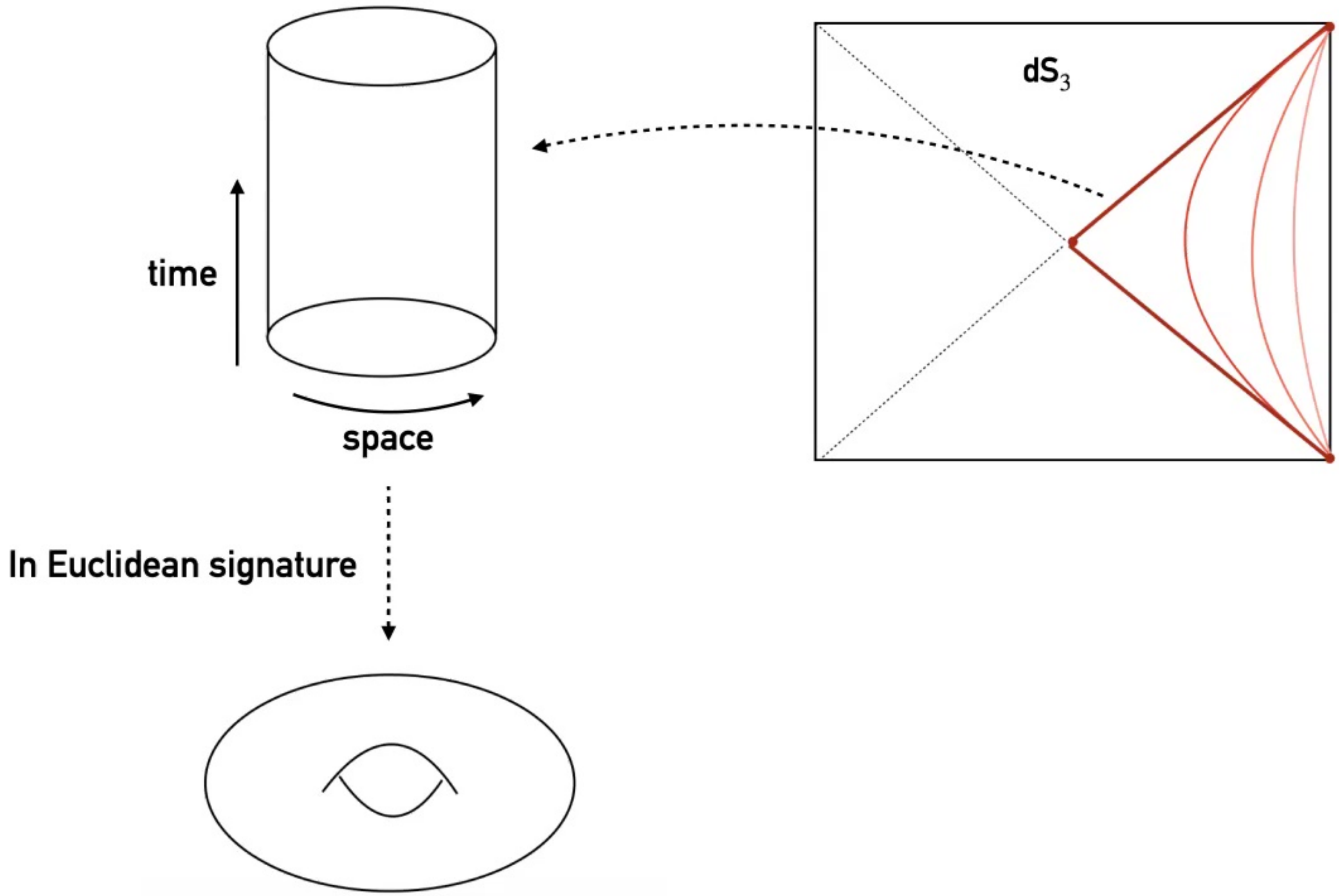


Cutoff Holography in dS_3

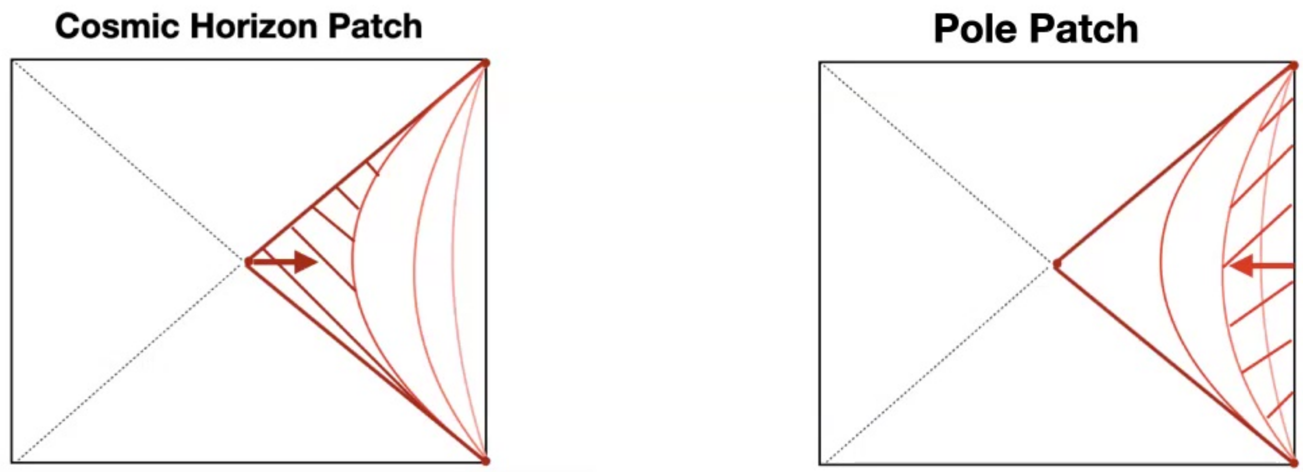


*from U.Moschella,
Prog.Math.Phys.
47 (2006) 120-133,
10.1007/3-7643-7436-5_4

Boundary | Bulk



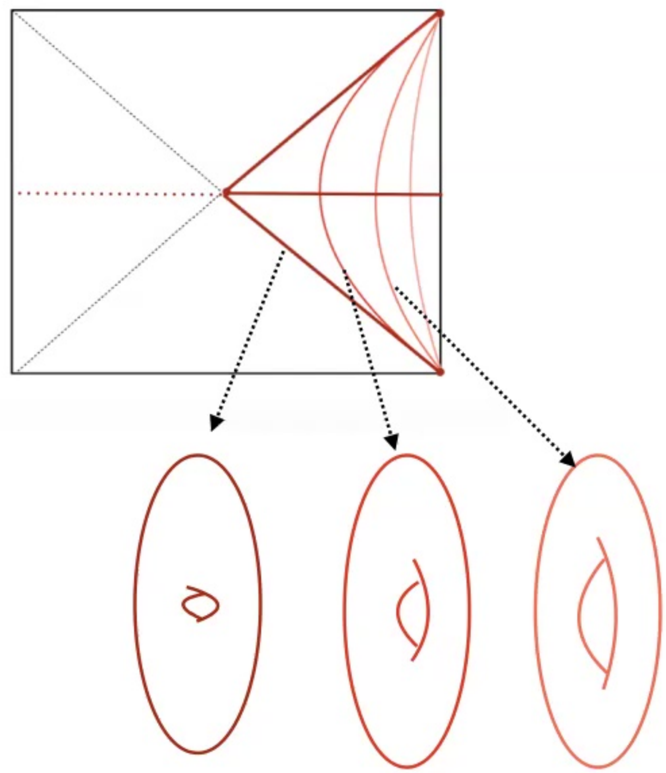
$\eta = -1$: Patches of dS



$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right)$ $\longleftarrow$ related by $\pm\sqrt{\ }$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$

Euclidean dS$_3$ Horizon

$$ds^2 = \left(1 - \frac{R^2}{\ell^2}\right) dt_E^2 + \frac{1}{\left(1 - \frac{R^2}{\ell^2}\right)} dR^2 + R^2 d\phi^2$$

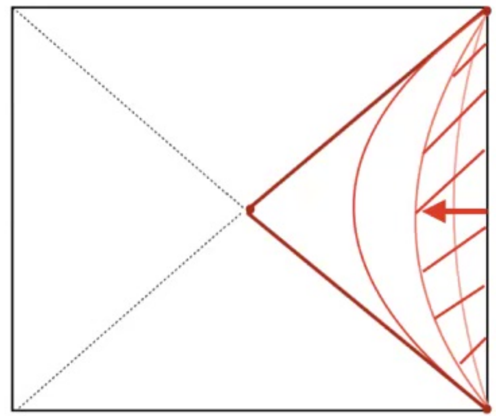


$\therefore$ **Approaching the de Sitter horizon**
$<=>$
Taking the $\beta \to 0$ limit in the dual theory

[Varadarajan 02']

[Gorbenko, Silverstein, Torroba 18']

Pole Patch

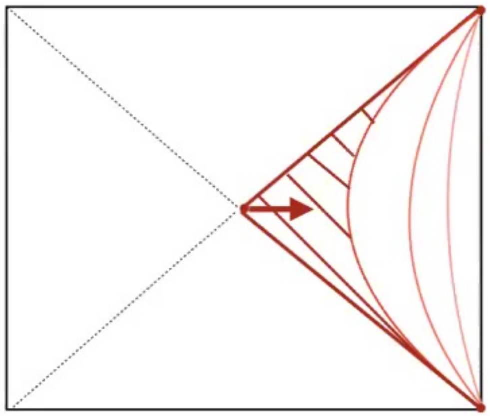


$$\mathscr{E} = \frac{L^2}{\pi\mu}\left(1 - \sqrt{-1 - \frac{4\pi^2\mu}{L^2}\left(\Delta - \frac{c}{12}\right) + 4\pi^4 J^2 \frac{\mu^2}{L^4}}\right)$$

Initial condition fixed by:

$$\mathscr{E}_{\eta=-1}(y \gg 1) = \mathscr{E}_{\eta=1}(y \gg 1)$$

Cosmic Horizon Patch



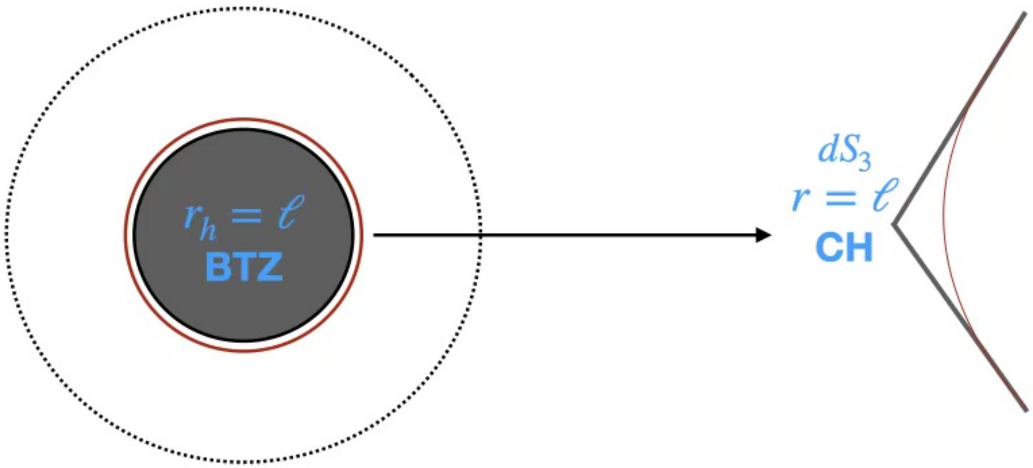
$$\mathscr{E} = \frac{1}{\pi y}\left(1 + \sqrt{-1 + \frac{2y}{y_0} - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right) \qquad y_0 = \frac{1}{4\pi^2(\Delta_* - c/12)}, \Delta_* = \frac{c}{6} + \delta$$

Initial condition fixed by:

$$\mathscr{E}_{\eta=-1}(y = y_0) = \mathscr{E}_{\eta=1}(y = y_0)$$

$$ds_3^2 = -\left(\frac{r^2 - r_h^2}{l^2}\right) d\tau^2 + \left(\frac{l^2}{r^2 - r_h^2}\right) dr^2 + r^2 d\phi^2, \qquad ds_3^2 = -\left(\frac{\ell^2 - r^2}{\ell^2}\right) d\tau^2 + \left(\frac{\ell^2}{\ell^2 - r^2}\right) dr^2 + r^2 d\phi^2$$

$$\eta = 1, \Delta = \frac{c}{6}$$



$$\eta = -1, \Delta = 0$$

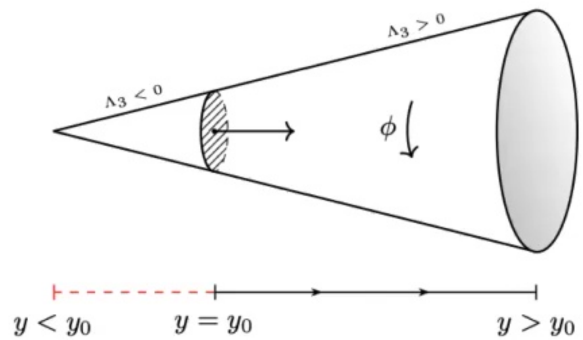
$$r \equiv r_h + \delta r = \ell + \delta r, \quad \delta r = \frac{w^2}{2\ell} \ll r_h = \ell \qquad r \equiv \ell - \delta r, \quad \delta r = \frac{w^2}{2\ell} \ll \ell$$

Near Horizon: $ds_3^2 = -\frac{w^2}{\ell^2} d\tau^2 + dw^2 + \left(\ell^2 + \eta w^2\right) d\phi^2$

$$\mathscr{E} = \frac{L^2}{\pi\mu}\left(1 + \sqrt{-1 - \frac{4\pi^2\mu}{L^2}\left(-\frac{c}{12}\right)}\right) = \frac{1}{\pi y}\left(1 + \sqrt{-1 + \frac{y}{y_0}}\right)$$

Real Excited States

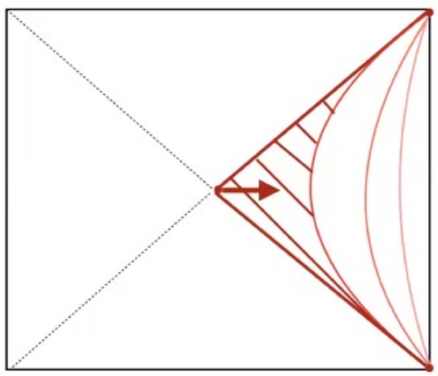
$\Delta < c/6$



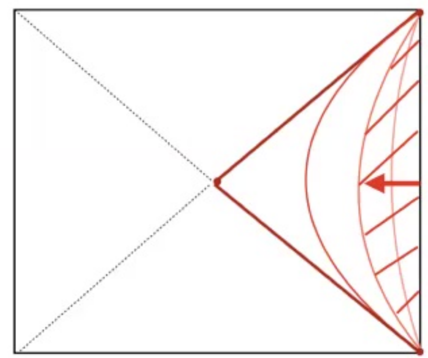
$$\mathscr{E}_{\eta=-1} = \frac{1}{\pi y}\left(1 + \sqrt{-1 + 4\pi^2 y\left(\Delta_* - \frac{c}{12} + (\Delta_* - \Delta)\right)}\right) \qquad \Delta_* = \frac{c}{6} + \delta$$

Hawking-Page like transition

$$\log Z \simeq \max\left\{-\beta E_{vac}(L), -LE_{vac}(\beta)\right\} = -\frac{\beta L}{\pi\lambda}\min\left\{1-\frac{\beta}{L}, 1-\frac{L}{\beta}\right\}$$



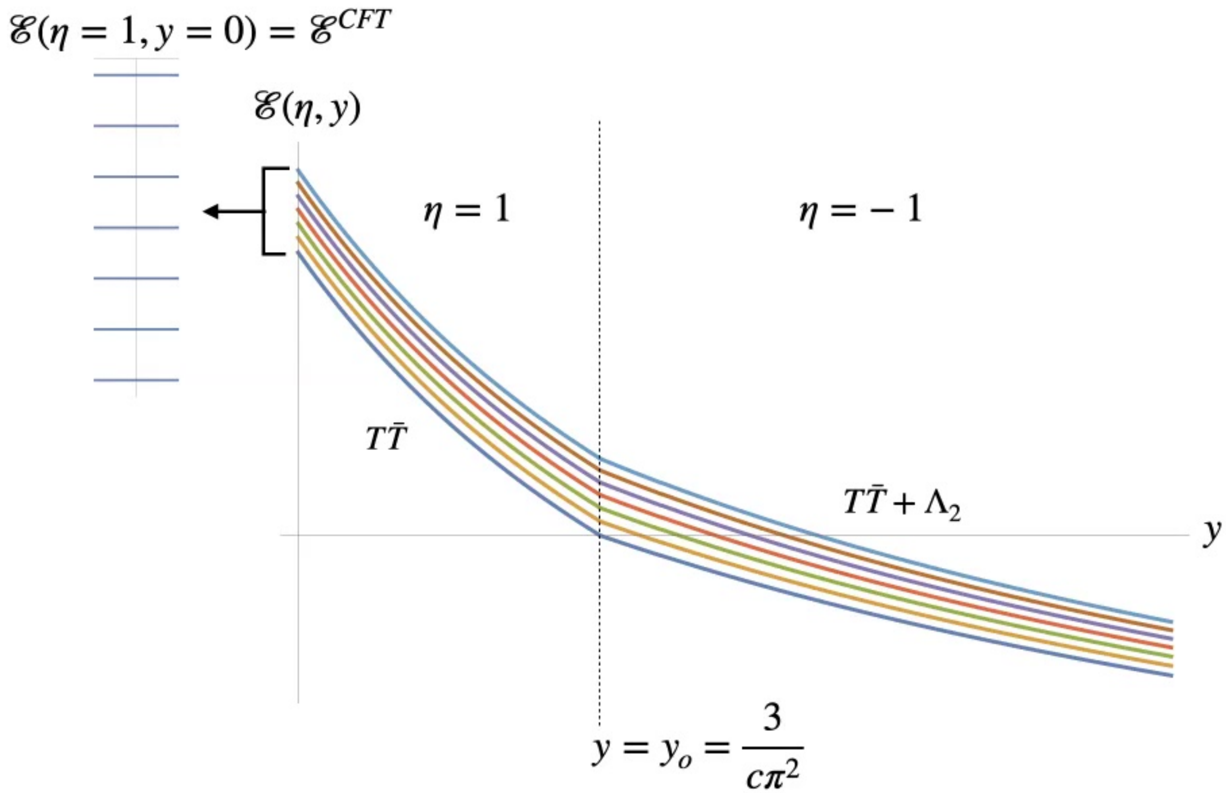
Crossover at $\beta \sim L$



When $\beta < L$ **:**

$$\log Z\Big|_{\beta<L} \simeq -LE_{vac}(\beta) = S_{Cardy}(\Delta = c/6) - \beta E_{\Delta=c/6}(L)$$

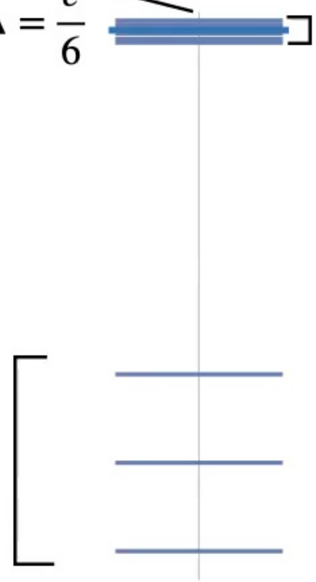
Integrable flow - No level crossing!



Truncated Hilbert Space

Cutoff @ $\Delta = \frac{c}{6} + \delta$

$\Delta = \frac{c}{6}$



$O(1)$ $\quad \rho(\Delta) \propto \exp\left(\sqrt{\frac{c\pi}{3}\left(\Delta - \frac{c}{12}\right)}\right)$

sparse light spectrum (sub-leading, model dependent)

$\Delta = 0$

$$\rho(\Delta) \propto \sum_n \delta(\Delta - \Delta_{n_i})$$

Flowing the Density of States

$$\log Z(\beta) = \min_{\mathscr{E}} \left(\log \rho(\mathscr{E}) - \beta \mathscr{E}\right)$$

Deformation: $\mathscr{E} \to f_y(\mathscr{E})$

$$\log Z(\beta, y) = \min_{\mathscr{E}} \left(\log \underline{\rho(f_y^{-1}(\mathscr{E}))} - \beta f_y(\mathscr{E})\right)$$

Undeformed d.o.s. written in terms of deformed energy levels

Final Result

$$S_{\eta=-1}(\mathscr{E}, y) = \sqrt{\frac{2c\pi}{3}\left(\frac{y-y_o}{\pi y} + \mathscr{E} - \frac{\pi y \mathscr{E}}{2}\right)} - 3\log\left(\sqrt{\frac{2c\pi}{3}\left(\frac{y-y_o}{\pi y} + \mathscr{E} - \frac{\pi y \mathscr{E}}{2}\right)}\right) + \log c + \cdots$$

Near horizon limit: $y = y_o = \frac{3}{c\pi^2}$, $\mathscr{E} = \frac{1}{\pi y}$, **where:**

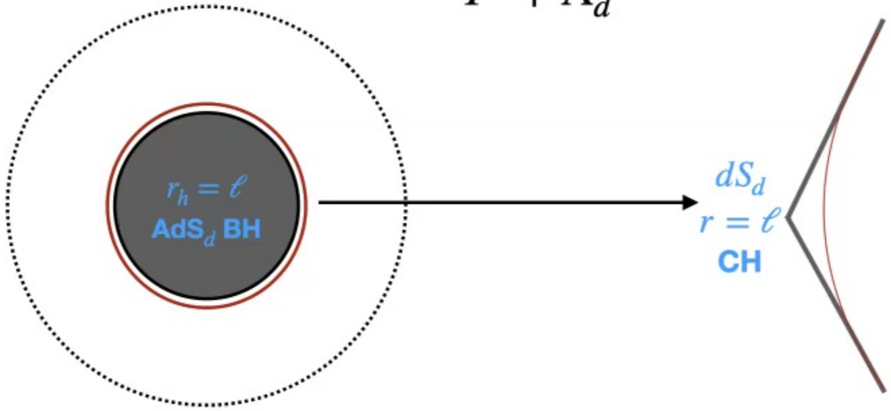
$$S = \frac{c\pi}{3} - 3\log\left(\frac{c\pi}{3}\right) + \cdots.$$

In bulk terms:

$$\boxed{S_{hor} = \frac{\pi\ell}{2G} - 3\log\left(\frac{\pi\ell}{2G}\right) + \cdots}$$

Higher dimensions

$T^2 + \Lambda_d$



Thank you for your attention!