Title: Intrinsically gapless symmetry-protected topology

Speakers: Andrew Potter

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Abstract: While sharply-quantized topological features are conventionally associated with gapped phases of matter, there are a growing number of examples of gapless systems with topologically protected edge states. A particularly striking set of examples are "intrinsically gapless" symmetry-protected topological states (igSPTs), which host topological surface states that could not arise in a gapped system with the same symmetries. Examples include familiar non-interacting Weyl semimetals with Fermi arc surface states, as well as more exotic examples like deconfined quantum critical points with topological edge states. In this talk, I will discuss recent progress in formally understanding the bulk-boundary correspondence of strongly-interacting igSPTs using tools from group cohomology. In these examples, the gapless-ness of the bulk and presence of topological surface states can be understood in a unified way due to the presence of an emergent anomaly. Our formalism allows construction of lattice-models with such emergent anomalies whose topological properties can be deduced exactly.

Intrinsically gapless SPTs

PI Quantum Matter Workshop
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Andrew C. Potter (UBC)

Rui Wen, ACP arXiv:2208.09001

Stewart Blusson Quantum Matter Institute
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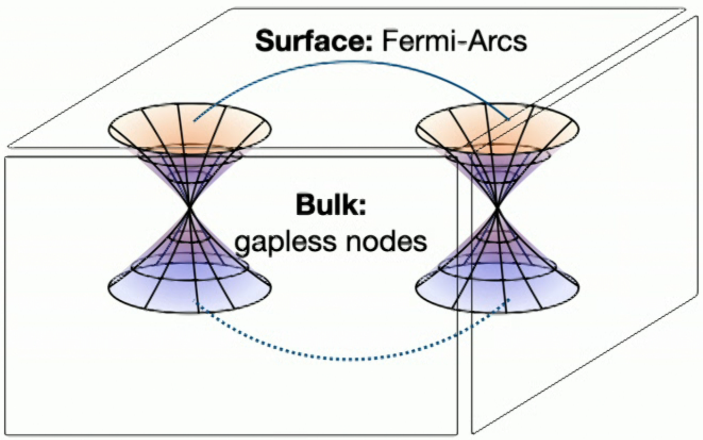
Motivating example: Topological semimetal [Last talk!]

Weyl Semimetal



- Intrinsically gapless: cannot have Fermi-arcs w/ gapped bulk
- Symmetry protected: Translation + Charge-U(1)
- Emergent anomaly: Valley-U(1) has chiral anomaly $\partial_\mu j^v_\mu \sim \frac{e^2}{4\pi} E \cdot B$
- Bulk Boundary correspondence: surface arcs tied to emergent chiral anomaly

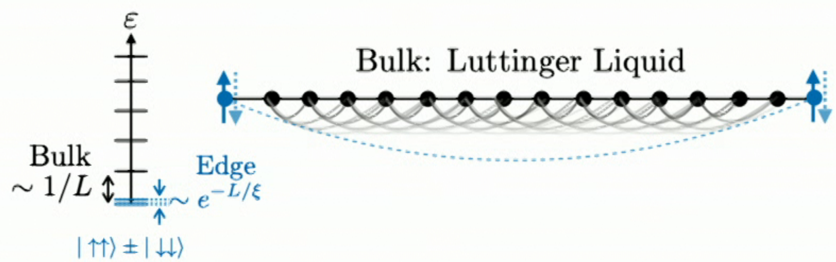
Quantized!

Q: Can we have intrinsically interacting + gapless topological states?

[E.g. Gapless spin/boson SPTs in Mott insulators?]

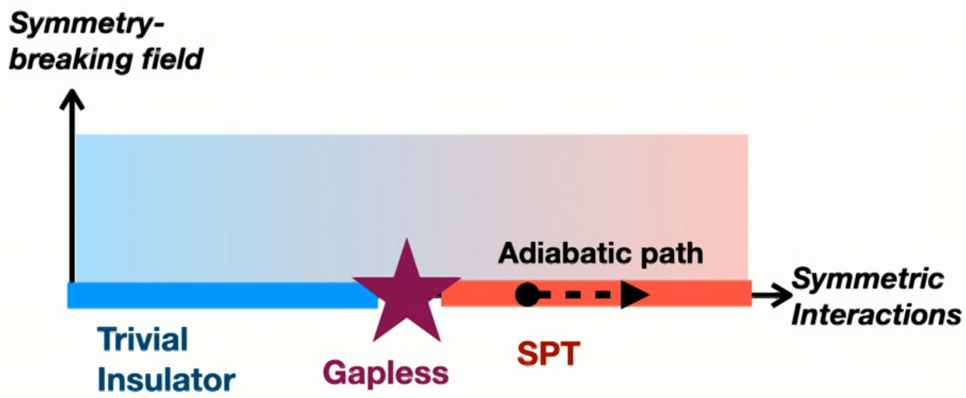
Today's talk

- Gapped Bosonic SPTs and their anomalous surfaces
  [Quick recap]
- Intrinsically gapless SPTs
  - Intrinsic = Edge cannot be cancelled by gapped system (with same symmetries)
  - Emergent Anomalies
  - Bulk-Boundary correspondence
  - Open questions

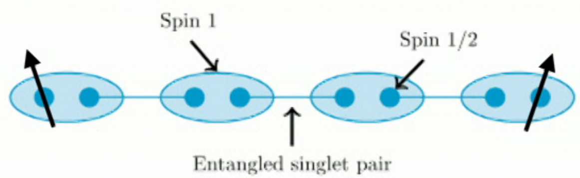


<u>Reminder: Gapped SPTs</u>

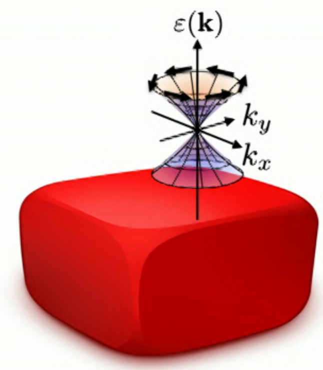
Definition:



Examples:

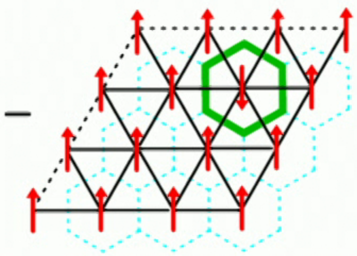
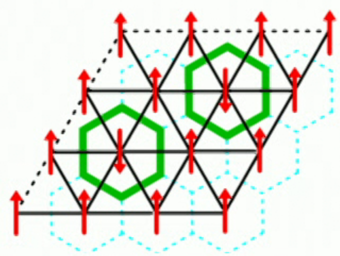


Haldane/AKLT spin-chain

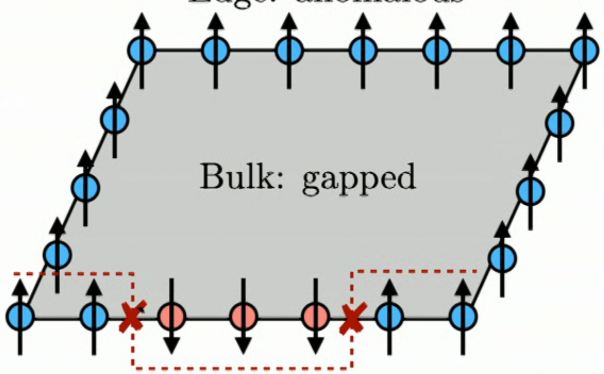


Electronic topological insulators and superconductors

Anatomy of a gapped boson SPT (Levin-Gu model)

$|\Psi_1\rangle = -$  $+$  $+ \ldots$

$$G = \mathbb{Z}_2 = \{1, s\} \quad U_s = \prod_i X_i$$



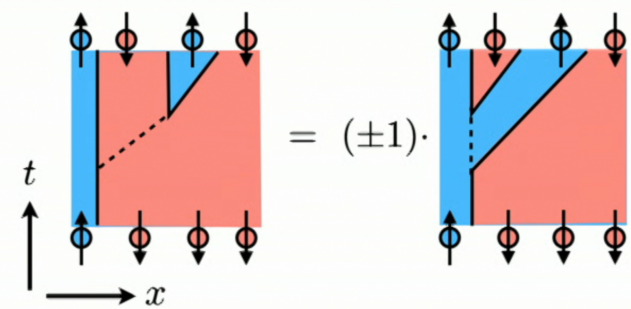
Anomalous symmetry action:

$$U_{\text{edge}} = \text{DW-phases} * \prod_{i \in \text{edge}} \tilde{X}_i$$

DW-phases $\in H^{d+1}(G, U(1))$

Chen, Gu, Wen

Anomaly = Obstacle to Symmetric+SRE:

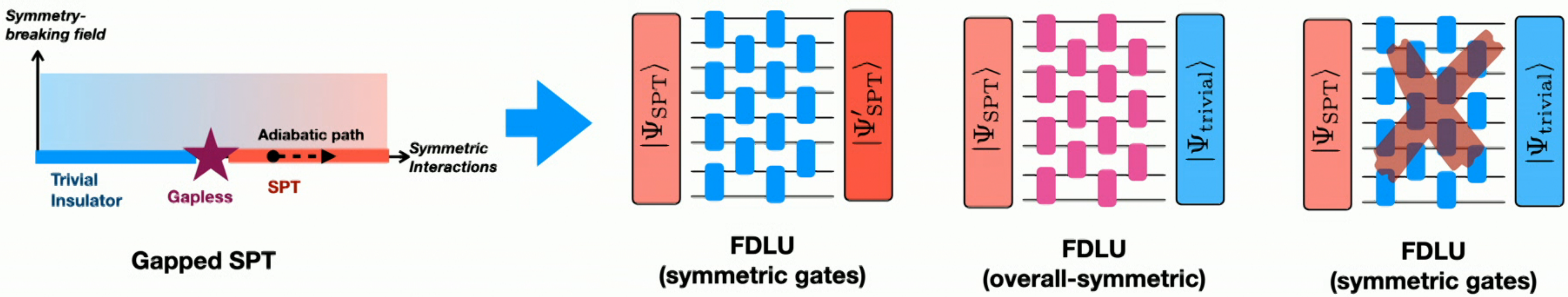


Else, Nayak; Kawagoe, Levin

Possible Edge Fates:

- Gapless + Symmetric (Luttinger liquid)
- Gapped + SSB
- Gapped + Symmetric + LRE (d>2)

Discussion: Defining Gapless SPTs (via Circuit Equivalence)



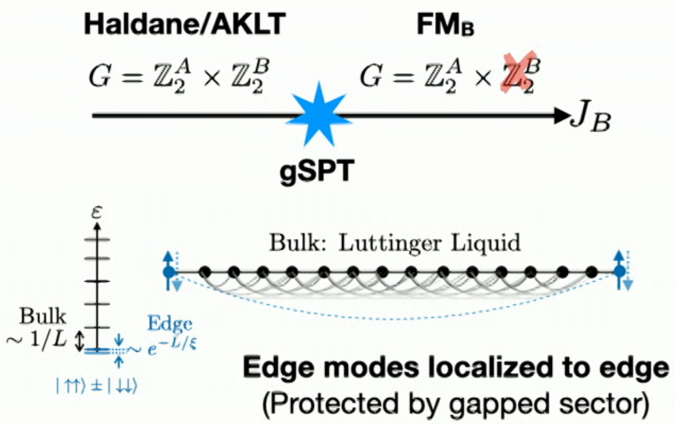
Def: Two G-symmetric ***gapless*** systems are in inequivalent SPT phases if:

1) they *can* be related by a overall-symmetric FDLU
2) they *cannot* be related by a symmetrically-generated FDLU

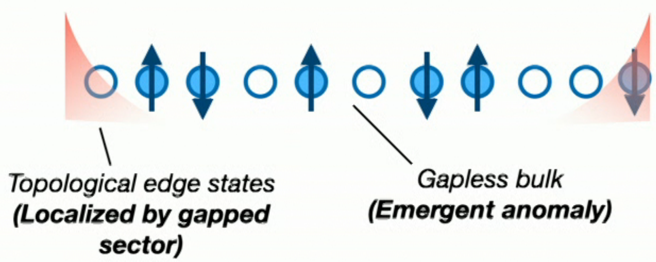
Rmk: *no assumption made about stability, these could be gapless phases or (multi) critical points*

Gapless SPTs: Zoology

Purely Gapless → / Intrinsically gapless? ↓	No	Yes
No Can kill edge states by stacking w/ gapped SPT	see figure below (Scaffidi, Parker, Vasseur)	
Yes No Gapped SPT has this edge	see figure below (Thorngren, Verresen, Vishwanath)	???



Scaffidi, Parker, Vasseur



Thorngren, Verresen, Vishwanath

Gapless SPTs: Zoology

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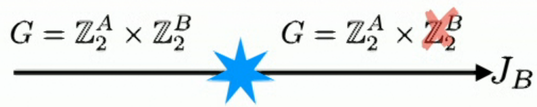
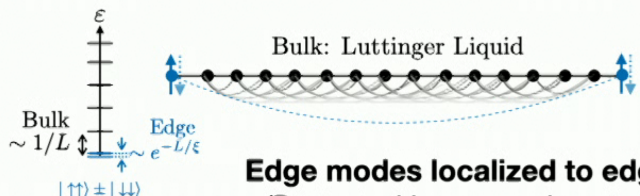
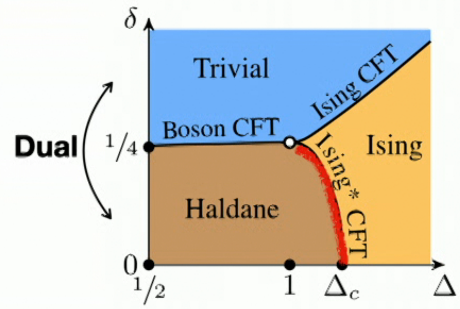
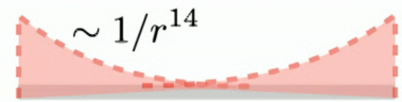
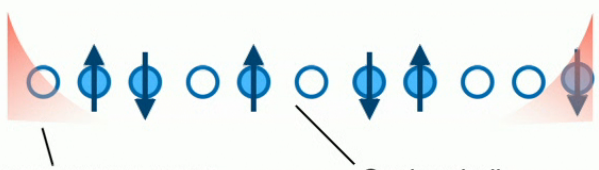
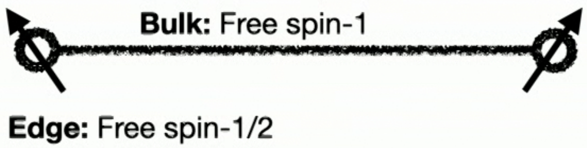
No Can kill edge states by stacking w/ gapped SPT	 **Edge modes localized to edge** (Protected by gapped sector) *Scaffidi, Parker, Vasseur*	 **Edge modes algebraically confined** *(Their magnetization decays >> faster than bulk magnetization correlations)* *Verresen, Thorngren, Jones, Pollmann*
Yes No Gapped SPT has this edge	 *Topological edge states* ***(Localized by gapped sector)*** *Gapless bulk* ***(Emergent anomaly)*** *Thorngren, Verresen, Vishwanath*	???

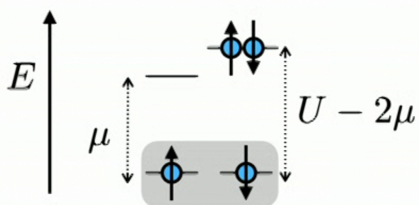
Emergent anomalies (Proto-example in 0d)

Haldane/AKLT Chain



Edge anomaly compensated by higher-d bulk

Anomalous edge w/out the bulk



Single Hubbard site @ 1/2-filling:

$0 < \mu < U/2$

"Anomaly" lifted by larger (extended) symmetry

UV symmetry: $G_{\mathrm{UV}} = SU(2)$

$$R_z(\pi)R_z(\pi+\theta) = (-1)^{N_f} R_z(\theta)$$

$(-1)^{N_f} \in \mathcal{N}_{\mathrm{gapped}}$

Formalism: (Central) Group extensions

$$\mathbb{Z}_2 \xrightarrow{\text{normal subgroup}} SU(2) \xrightarrow{\text{quotient}} SO(3)$$

Notation: $\mathcal{N} \to \Gamma \to G$

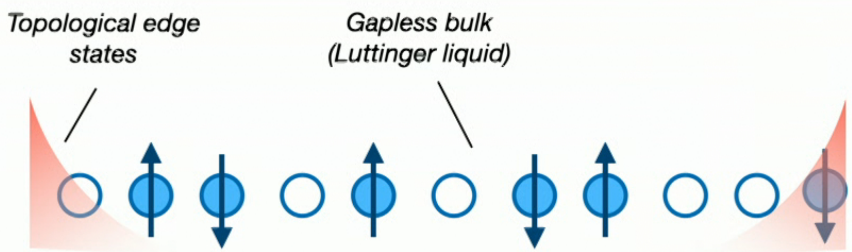
Extension   UV   IR

"Twisted" multiplication rules: $\gamma = (n, g) \in \Gamma$

$$\gamma \cdot \gamma' = (n \cdot n' \cdot e_2(g, g'), g \cdot g')$$

$e_2 \in H^2(G, N) : G \times G \to \mathcal{N}$

1d Intrinsically Gapless SPT (igSPT) in an Ising-Hubbard Chain

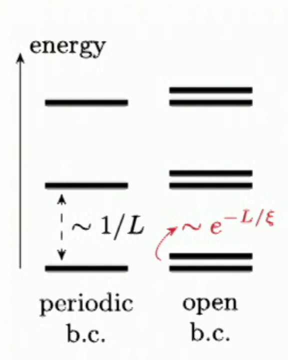


$$H = H_{\mathrm{Hubbard}}(t, \mu, U) + H_{\mathrm{Ising}}(J, h)$$

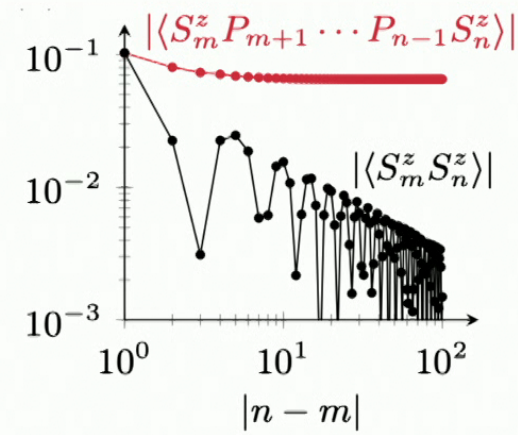
$$H_{\mathrm{Ising}} = \sum_i \left(J S_i^z S_{i+1}^z - h S_i^x \right)$$

igSPT phase: U>>t, partial filling, J>>h

Spectrum



Long Range String Order



Extreme limit: h=0

Bulk: Frozen AFM hidden in Luttinger Liquid of holes

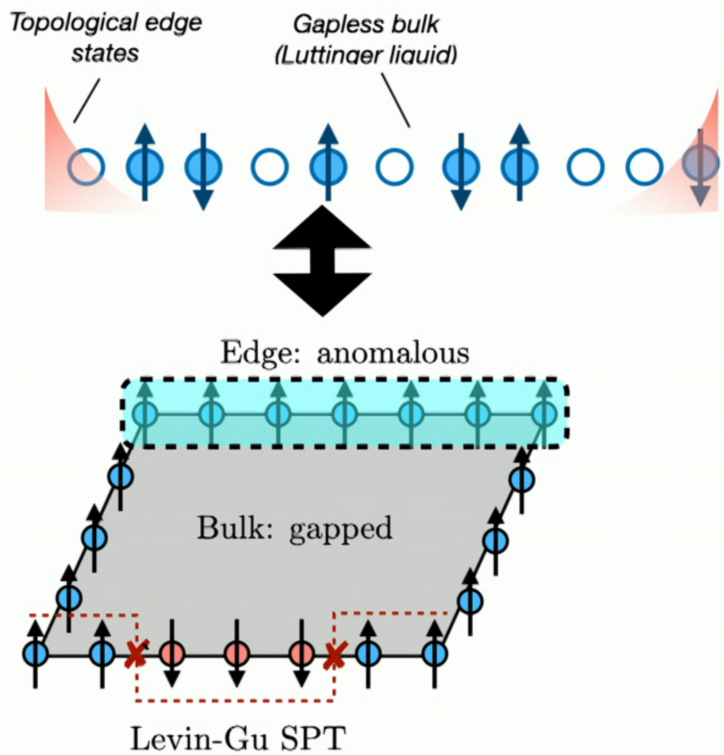
Periodic BCs: Unique ground-state

Open BCs:

- 1st spin up or down (local to edge)
- Symmetry-breaking field pins this DOF

Thorngren, Verresen, Vishwanath

Ising-Hubbard Chain: Emergent Anomaly



UV Symmetry group: $\Gamma = \mathbb{Z}_4 = \left\{1, R_x = e^{i\pi S^x}, R_x^2 = (-1)^{N_F}, R_x^3\right\}$

IR Symmetry group: $G = \mathbb{Z}_2$

$(R_x^2)\,|_{\mathrm{IR}} \sim 1$ *(Odd fermion-parity excitations frozen out)*

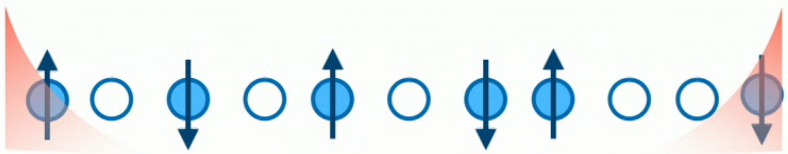
Q1: (How) are the symmetry extension, emergent bulk anomaly and edge states related?

Q2: Can we use this paradigm to construct general igSPTs?

Thorngren, Verresen, Vishwanath

String order <=> Lower-d SPT pumping symmetry

1d: Ising-Hubbard Chain



UV Symmetry: $\Gamma = \mathbb{Z}_4 = \{1, R_x, R_x^2 = (-1)^{N_f}, R_x^3\}$

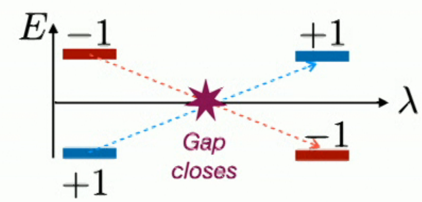
Gapped sector: $\mathcal{N} = \{1, (-1)^{N_f}\}$

IR symmetry: $G = \mathbb{Z}_2 = \{1, [R_x]\}$

String order: $S \approx \sigma_1^z (-1)^{N_f} \sigma_L^z$

N-transformation
(acts trivially on bulk
since N-DOF are gapped)

0d SPT pump
Changes Rx quantum
number of end by x(-1)

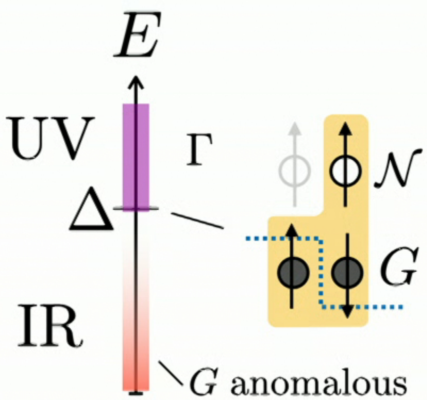


Thorngren, Verresen, Vishwanath

General Construction

Thm. [Tachikawa]: Given: G-anomaly in d-dimensions characterized by w_{d+2} in $H^{d+2}(G,U(1))$
∃ symmetry extension: N→Γ→G s.t. Γ is anomaly free

Our construction: Design Γ-rotor model w/ exactly solvable Hdelta s.t. emergent G anomaly in IR



Bulk-Boundary correspondence

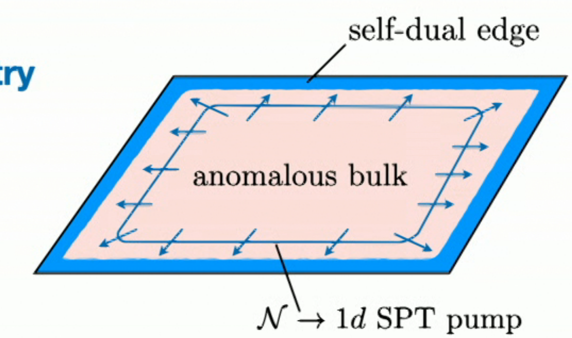
$$\omega_{d+2} = b_d \cup e_2$$

$e_2 \in H^d(G,\mathcal{N})$ **Specifies the group extension**

SPT Pumping Symmetry

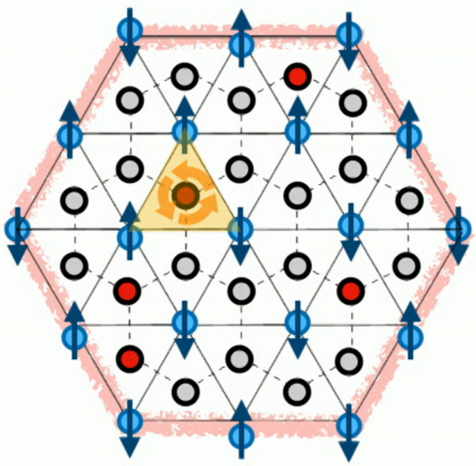
$b_d \in H^d(G,\hat{\mathcal{N}})$

Map from n∈N to (d-1)-dimensional G-SPT



R.Wen, ACP arXiv:2208.09001

2d Time-reversal symmetric igSPT



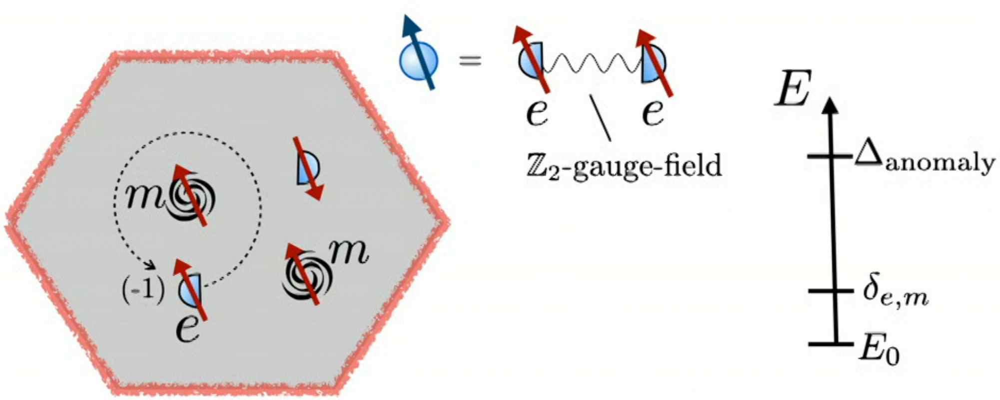
Sites: non-Kramers qubits (G)
Plaquettes: Kramers bosons (N)
IR: Bosons bound to spin vortices

Bulk:
- 3d boson SPT surface anomaly (eTmT)
- Gapless+symmetric option: SO(5) WZW_1 model (DQCP)

Senthil, Vishwanath
Bi, Slagle, Xu

Edge:
- Haldane SPT pumping symmetry
- Pure 1d: Self-dual point ($SU(2)_1$ CFT)
- Edge/bulk decouple? *Ma, Zou, Wang*

Quotient Symmetry Enriched Topological order (QSET)



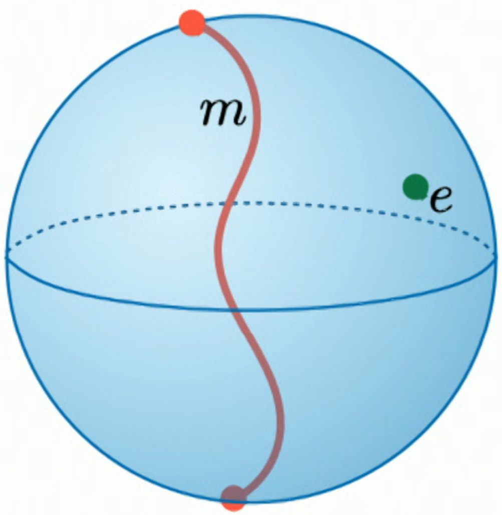
Exactly solvable model: fractionalization + gauging
Bulk: Toric code with Anomalous TRS (eTmT)
Edge: Self-dual critical point between Haldane SPT/PM
- Sharply defined when Kramers bosons are gapped
- Anomaly can be lifted in the UV (requires edge phase transition)

R.Wen, ACP arXiv:2208.09001

Higher-d Examples:

R.Wen, ACP arXiv:2208.09001

3d: Fully gapped QSET: anomalous bulk and (even more anomalous) surface topological order



Bulk SET:

- Z2 gauge theory (toric code)
- Point Charge (e): Kramers boson
- Flux line (m): Carries 1d igSPT (gapless)

Outlook

- Classification:
    - Other mechanisms for emergent anomalies besides extended symmetry?
    - igSPTs with beyond group-cohomology anomalies? Fermionic anomalies?
- More physical models of higher-d igSPTs? (Experimental realizations?)
- How do gapless bulk and boundary interplay?
- How do we understand the edge states of “purely gapless”