Title: Emergent anomalies and generalized Luttinger theorems in metals and semimetals

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Collection: Quantum Matter Workshop

Date: November 16, 2022 - 9:30 AM

URL: https://pirsa.org/22110074

Abstract: Luttinger's theorem connects a basic microscopic property of a given metallic crystalline material, the number of electrons per unit cell, to the volume, enclosed by its Fermi surface, which defines its low-energy observable properties. Such statements are valuable since, in general, deducing a low-energy description from microscopics, which may perhaps be regarded as the main problem of condensed matter theory, is far from easy. In this talk I will present a unified framework, which allows one to discuss Luttinger theorems for ordinary metals, as well as closely analogous exact statements for topological (semi)metals, whose low-energy description contains either discrete point or continuous line nodes. This framework is based on the 't Hooft anomaly of the emergent charge conservation symmetry at each point on the Fermi surface, a concept recently proposed by Else, Thorngren and Senthil [Phys. Rev. X {\bf 11}, 021005 (2021)]. We find that the Fermi surface codimension p plays a crucial role for the emergent anomaly. For odd p, such as ordinary metals ($p=1$) and magnetic Weyl semimetals ($p=3$), the emergent symmetry has a generalized chiral anomaly. For even p, such as graphene and nodal line semimetals (both with $p=2$), the emergent symmetry has a generalized parity anomaly. When restricted to microscopic symmetries, such as $U(1)$ and lattice symmetries, the emergent anomalies imply (generalized) Luttinger theorems, relating Fermi surface volume to various topological responses. The corresponding topological responses are the charge density for $p=1$, Hall conductivity for $p=3$, and polarization for $p=2$. As a by-product of our results, we clarify exactly what is anomalous about the surface states of nodal line semimetals.

Emergent anomalies and generalized Luttinger theorems

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CATS
Center for the Advancement of Topological Semimetals

Quantum Matter Workshop, Perimeter Institute, November 16, 2022

Thanks

Lei Gioia Yang (Waterloo/Perimeter)

Chong Wang (Perimeter)

Alex Hickey (Waterloo)

Xuzhe Ying (Waterloo)

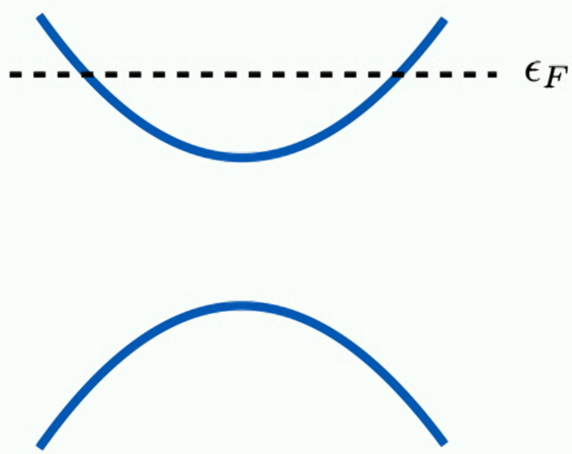
Outline

- Introduction: metals and topological semimetals.
- Topological response of metals and semimetals and generalized mixed anomalies of emergent symmetries.
- Interaction effects: 3D FQH liquid by gapping magnetic Weyl semimetal.

PRB 104, 235113 (2021) PRR 3, 043067 (2021) PRL 124, 096603 (2020)

Metal

- Fractional number of electrons per unit cell per spin: metal, Fermi surface of gapless excitations.

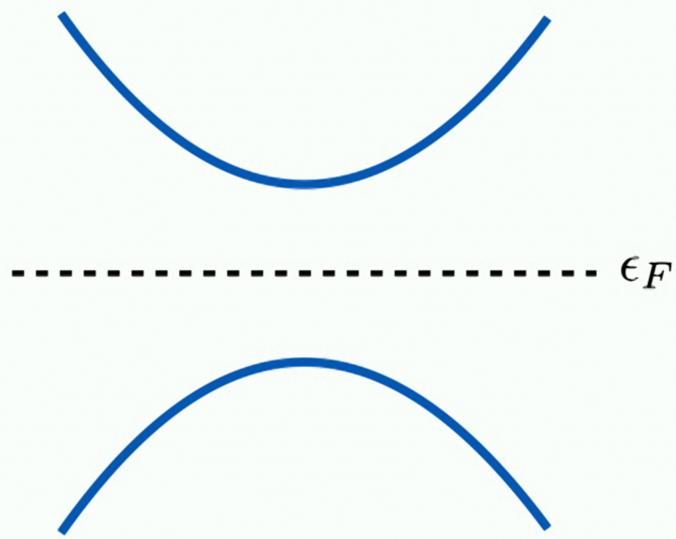


$$\frac{V_F}{(2\pi)^d/v_c} = \{\nu\}$$

Luttinger's theorem

Insulator

- Integer number of electrons per unit cell per spin: insulator, no gapless excitations.



Metal as intermediate phase between two insulators

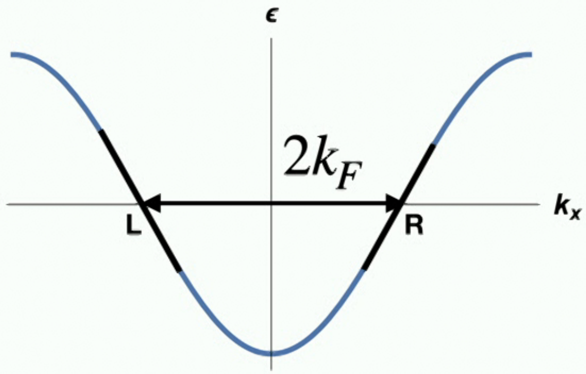
- Metal may be viewed as an intermediate phase between two insulators, characterized by integer values of ν.

- A noninteger ν requires gapless excitations or topological order.

Topological semimetals

- Bulk (semi)metals, which arise at integer ν as intermediate phases between insulators with different electronic structure topology.
- Magnetic Weyl semimetal: intermediate phase between ordinary and integer quantum Hall insulators in 3D.
- Nonmagnetic Weyl semimetal: intermediate phase between ordinary and a 3D TR-invariant TI.
- Type-I Dirac semimetal: intermediate phase between ordinary and a TCI, protected by rotational symmetry.
- Nodal line semimetal: intermediate phase between ordinary insulator and a TCI, protected by mirror symmetry.

Luttinger's theorem in a 1D metal



$$S = i \int d\tau\, dx\, \frac{\nu}{a} A_0$$

$\hbar = c = e = 1$ $k_F = \pi\nu$ dimensionless Fermi momentum

- May think of this as a topological term if we view the reciprocal lattice vector $2\pi/a$ as "elastic gauge field".

Elastic gauge fields

- A crystal may be described as intersections of planes of constant phase:

Nissinen & Volovik

$$\theta^i(\mathbf{x}, t) = 2\pi n^i$$

Song et al.

- In a perfect crystal we have:

$$\theta^i(\mathbf{x}, \mathbf{t}) = \mathbf{b}^i \cdot \mathbf{x} \qquad \mathbf{x} = \mathbf{R} = n^i \mathbf{a}_i \qquad \mathbf{b}^i \cdot \mathbf{a}_j = 2\pi\delta^i_j \qquad b^i_j = \partial_j \theta^i$$

- In a distorted crystal:

$$e^i_\mu = \frac{1}{2\pi}\partial_\mu \theta^i \qquad e^i = \frac{1}{2\pi} d\theta^i$$

Elastic gauge fields

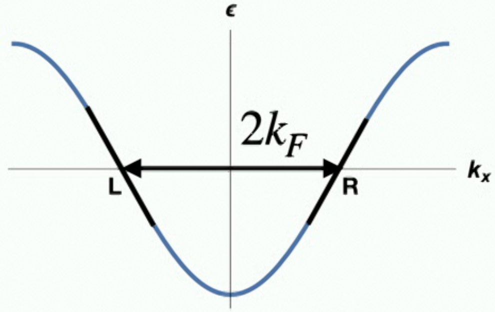
- In a distorted crystal:

$$e^i_\mu = \frac{1}{2\pi}\partial_\mu \theta^i \qquad e^i = \frac{1}{2\pi} d\theta^i$$

- Two-form $de^i = \frac{1}{2}(\partial_\mu e^i_\nu - \partial_\nu e^i_\mu) dx^\mu dx^\nu$ determines the dislocation density.

$$\oint_i e^i = N_i$$

Luttinger's theorem in a 1D metal



$$S = i\int d\tau\, dx\, \frac{\nu}{a} A_0$$

- Replace $\frac{1}{a} \rightarrow e^x$

$$S = i\nu \int e^x \wedge A$$

$$\nu = -i\frac{\delta S}{\delta e^x_x \delta A_0}$$

$$P = -i\oint_x \frac{\delta S}{\delta e^x_0} = \nu\Phi$$

Chiral anomaly

- Taken literally, this symmetry may not be realized on a 1D lattice, but may only appear on the edge of a 2D topological insulator.

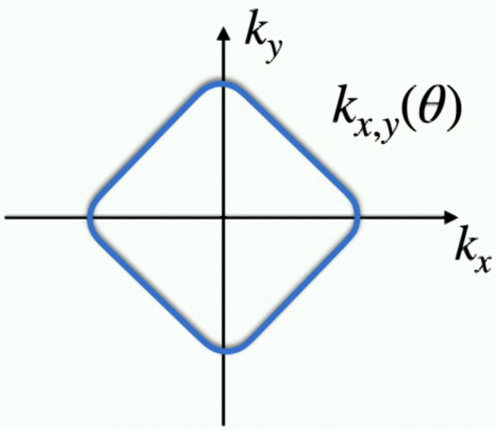
$$S_{2D} = \frac{i}{4\pi}\int (A_R \wedge dA_R - A_L \wedge dA_L)$$

$$A_R = A + k_F e^x \qquad A_L = A - k_F e^x \qquad k_F = \pi\nu$$

$$S_{2D} = i\nu \int e^x \wedge dA \longrightarrow S_{1D} = i\nu \int e^x \wedge A$$

Higher dimensions

- This reasoning may be generalized to higher dimensions.



$\theta \in [0,2\pi]$

Else, Thorngren, Senthil

Wang, Hickey, Ying, AAB

- May view θ as extra dimension.

- Emergent symmetry: separate particle number conservation at every point on the Fermi surface.

Luttinger's theorem in 2D Fermi liquid

- Emergent anomaly in 2+1+1+1=5 space-time dimensions.

$$S = \frac{i}{6(2\pi)^2} \int \mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A} \qquad \mathscr{A} = A + k_x(\theta)e^x + k_y(\theta)e^y$$

- Momentum-space components of A is the Berry connection.

Luttinger's theorem in 2D Fermi liquid

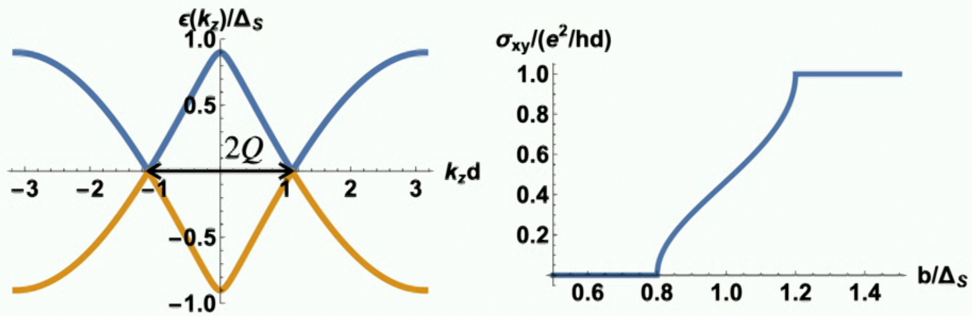
- Emergent anomaly in 2+1+1+1=5 space-time dimensions.

$$S = \frac{i}{6(2\pi)^2} \int \mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A} \qquad \mathscr{A} = A + k_x(\theta) e^x + k_y(\theta) e^y$$

- The nonvanishing part:

$$S = \frac{i}{2(2\pi)^2} \int (k_x e^x + k_y e^y) \wedge (k_x e^x + k_y e^y) \wedge dA$$

Magnetic Weyl semimetal



$$\sigma_{xy} = \frac{2Q}{2\pi}\frac{e^2}{h} = \frac{2Q}{2\pi}\frac{1}{2\pi}$$

Chiral anomaly in Weyl semimetals

- By same logic as before, have enlarged emergent symmetry $U(1)_L \times U(1)_R$ at the Weyl nodes.

- This may only be realized on the surface of a 4D TI:

$$S_{5D} = \frac{i}{6(2\pi)^2} \int (A_R \wedge dA_R \wedge dA_R - A_L \wedge dA_L \wedge dA_L)$$

$$A_R = A + Qe^z \qquad A_L = A - Qe^z$$

Chiral anomaly in Weyl semimetals

- May also be obtained from the 7-dimensional anomaly term of two Fermi surfaces, enclosing the Weyl nodes.

$$S_{7D} = \frac{i}{6(2\pi)^3} \mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A} \qquad \int_{FS} d\mathscr{A} = \pm 2\pi$$

$$\Downarrow$$

$$S_{5D} = \frac{i}{6(2\pi)^2} \int (A_R \wedge dA_R \wedge dA_R - A_L \wedge dA_L \wedge dA_L)$$

Chiral anomaly in Weyl semimetals

- This leads to:

$$S = i\frac{\lambda}{4\pi}\int e^z \wedge A \wedge dA \qquad \lambda = \frac{2Q}{2\pi}$$

- A noninteger λ requires gapless Weyl nodes.

- Expresses Hall conductance per atomic plane:

$$G_{xy} = \frac{\lambda}{2\pi}$$

Chiral anomaly in Weyl semimetals

$$S = i\frac{\lambda}{4\pi}\int e^z \wedge A \wedge dA \qquad \lambda = \frac{2Q}{2\pi}$$

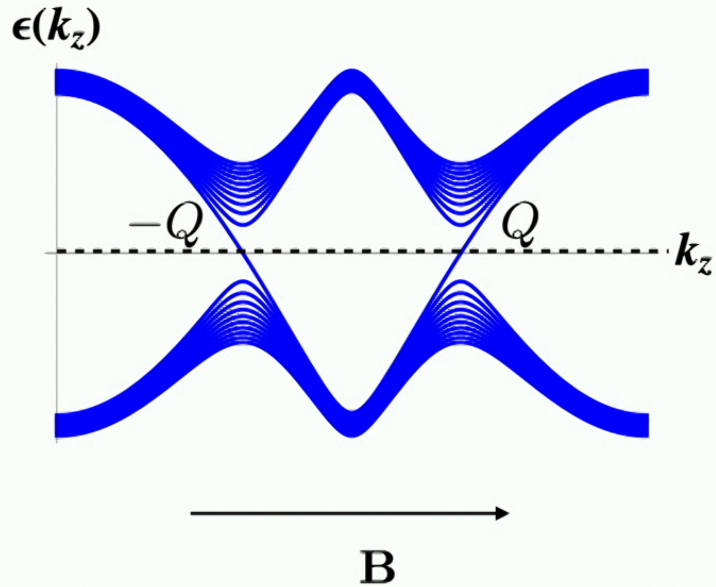
- Magnetic flux tube along the z-direction, carrying flux quantum $\Phi = 2\pi$ gives:

$$S_\Phi = i\lambda \int e^z \wedge A$$

- This corresponds to a 1D metal with charge λ per unit cell.

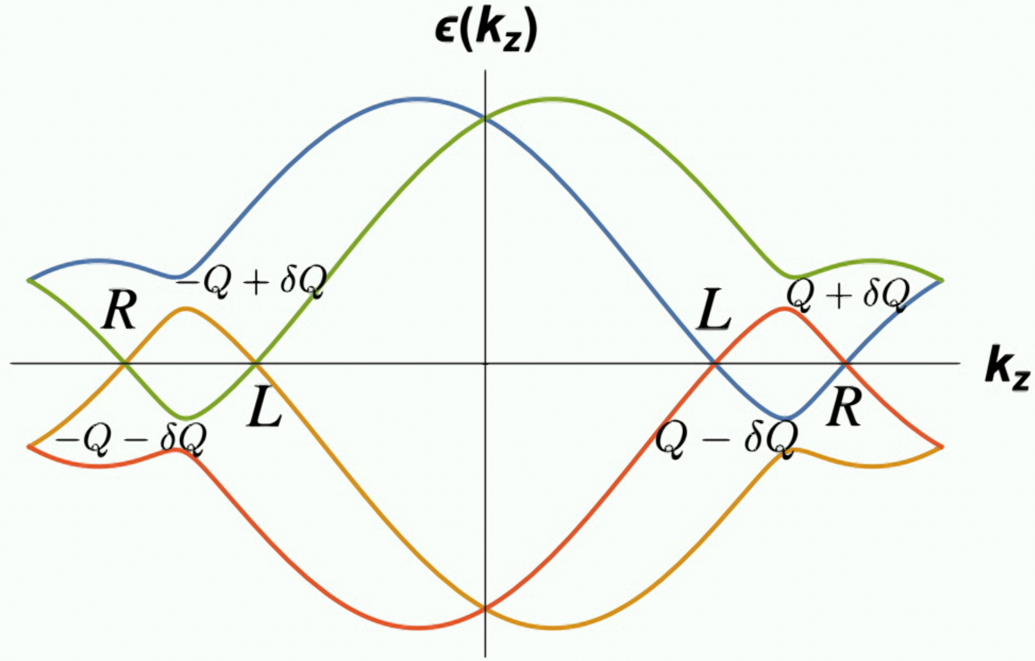
Chiral anomaly in Weyl semimetals

$$S_{\Phi} = i\lambda \int e^z \wedge A$$



TR-invariant Weyl semimetal

- Minimal model contains 4 Weyl nodes, can take them all to be on the z-axis for simplicity.



TR-invariant Weyl semimetal

- Topological response may be obtained using the same logic as before, i.e. invoking the chiral anomaly of the low-energy theory.

$$S_{4D} = \frac{i}{6(2\pi)^2}\int (A_{R+} \wedge dA_{R+} \wedge dA_{R+} + A_{R-} \wedge dA_{R-} \wedge dA_{R-}) - \frac{i}{6(2\pi)^2}\int (A_{L+} \wedge dA_{L+} \wedge dA_{L+} + A_{L-} \wedge dA_{L-} \wedge dA_{L-})$$

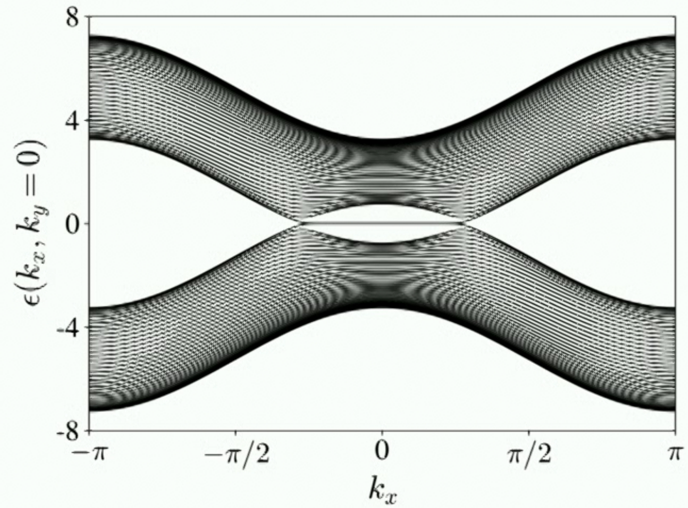
$$A_{R\pm} = A \pm (Q + \delta Q)e^z \qquad A_{L\pm} = A \pm (Q - \delta Q)e^z$$

$$S = \frac{i\lambda}{2}\int e^z \wedge de^z \wedge A \qquad \lambda = \frac{2Q\delta Q}{\pi^2}$$

- This describes charge λ per unit cell on a screw dislocation along the z-axis.

Nodal line semimetal

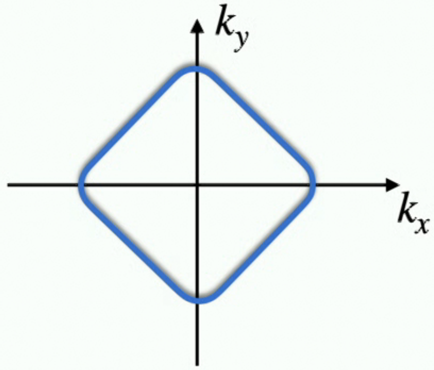
- Touching of a pair of bands along a line in 3D is protected by mirror symmetry with respect to the plane, containing the line.



- Drumhead surface state, filling the projection of the line onto the surface BZ.

Nodal line semimetal

- May use the same logic as before and look at anomaly of the emergent low-energy symmetry.



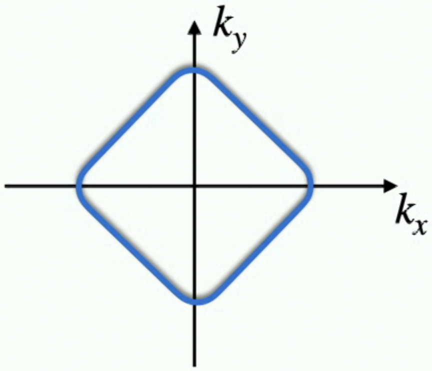
$$S = \pm \frac{i\pi}{6(2\pi)^3} \int d\mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A}$$

3+1+1+1=6-dimensional θ-term, with $\theta = \pi$.

$$\mathscr{A} = A + k_x(\phi)e^x + k_y(\phi)e^y \qquad \phi \in [0,2\pi]$$

- θ is quantized by mirror symmetry.

Nodal line semimetal



$$S = \pm \frac{i\pi}{6(2\pi)^3} \int d\mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A}$$

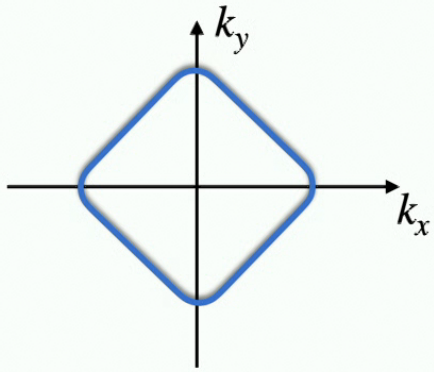
- May again be obtained from the 7D chiral anomaly term:

$$S_{7D} = \frac{i}{6(2\pi)^3} \mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A}$$

$$\int_0^{2\pi} d\phi \mathscr{A}_\phi = \pm \pi$$

Nodal line semimetal

- May use the same logic as before and look at anomaly of the emergent low-energy symmetry.



$$S = \pm \frac{i\pi}{6(2\pi)^3} \int d\mathscr{A} \wedge d\mathscr{A} \wedge d\mathscr{A}$$

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$$\mathscr{A} = A + k_x(\phi)e^x + k_y(\phi)e^y \qquad \phi \in [0,2\pi]$$

$$S = \pm \frac{iV_F}{8\pi^2} \int e^x \wedge e^y \wedge dA$$

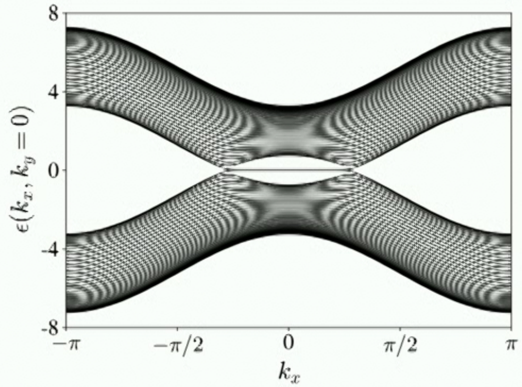
Nodal line semimetal

$$S = \pm \frac{iV_F}{8\pi^2} \int e^x \wedge e^y \wedge dA$$

- V_F is the area, enclosed by the nodal line.
- This describes spontaneous (i.e. existing without external electric field) electric polarization $P = \pm \frac{V_F}{8\pi^2}$.
- When $V_F = (2\pi)^2$, i.e. area of the BZ section, nodal line disappears and $P = \pm \frac{1}{2}$, which corresponds to a crystalline TI, protected by mirror symmetry.

Ramamurthy & Hughes   Wang, Hickey, Ying, AAB

Drumhead surface state



$$S = \pm \frac{iV_F}{8\pi^2} \int e^x \wedge e^y \wedge A$$

- The surface state may be viewed as a 2D Fermi liquid, which violates Luttinger's theorem:

$$\epsilon_F = 0+ \qquad \rho_+ = \frac{V_F}{4\pi^2} + P_+$$

$$P_\pm = \mp \frac{V_F}{8\pi^2}$$

$$\epsilon_F = 0- \qquad \rho_- = 0 + P_-$$

Strong interactions and anomalies

- Can ask: is there a fully gapped symmetric insulator with the same topological response?
- If it exists, it must have topological order.
- In the case of magnetic Weyl semimetal, this would be a 3D fractional quantum Hall liquid.

3D Fractional Quantum Hall Effect

PHYSICAL REVIEW LETTERS **124,** 096603 (2020)

Fractional Quantum Hall Effect in Weyl Semimetals

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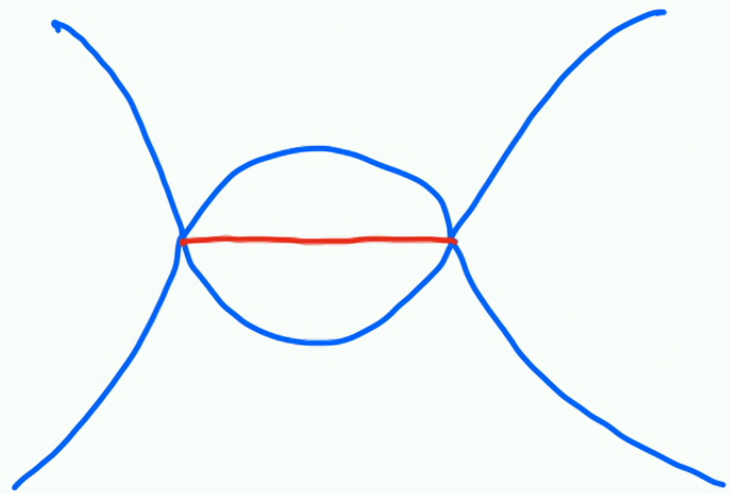
(Received 8 July 2019; accepted 11 February 2020; published 6 March 2020)

- Chiral anomaly turns out to be consistent with a gapped fractionalized state at one particular value of the Weyl node separation.

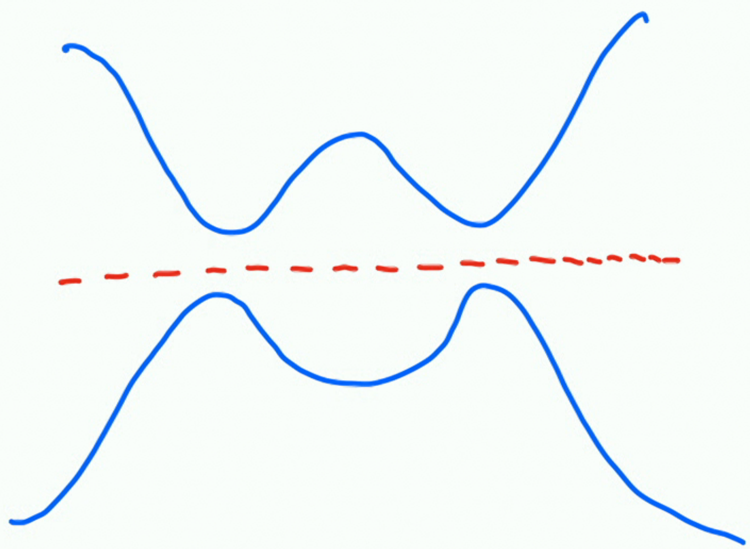
Gapping Weyl nodes and Fermi arcs

- Fermi arc must connect Weyl node projections:



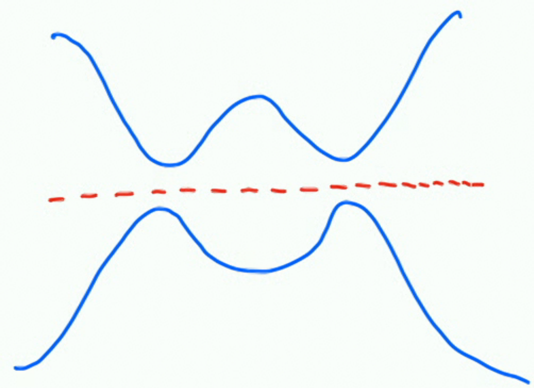
Gapping Weyl nodes and Fermi arcs

- The only thing that turns out to be possible is to split the Fermi arc into Majorana state, which spans the whole BZ:



Gapping Weyl nodes and Fermi arcs

- This only works when $2Q = G/2$ since Majorana is half a complex fermion.

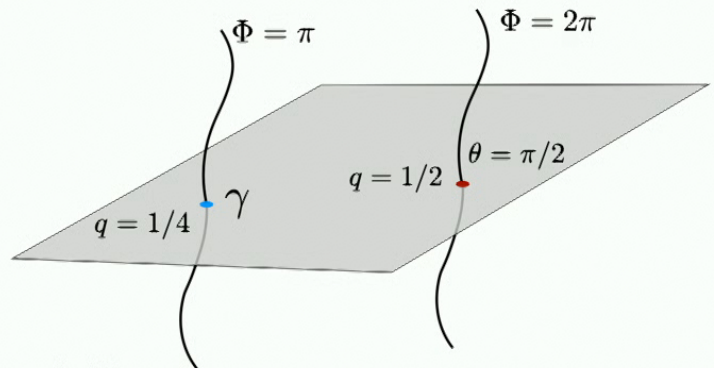


- Chiral Majorana surface sheet is like the surface state of a stack of p+ip topological superconductors.

Vortex condensation

- Flux $\pi = hc/2e$ vortices carry Majorana modes.

- Flux 2π vortices carry charge 1/2 per unit cell:

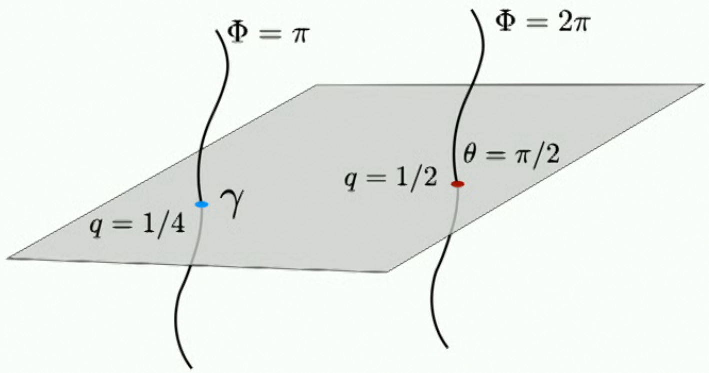


$$S_\Phi = i\lambda \int e^z \wedge A \qquad \lambda = 1/2$$

- The smallest "bosonic" vortex with no gapless modes in the core is flux 4π.

Vortex condensation

- The smallest "bosonic" vortex with no gapless modes in the core is flux 4π.



- This implies charge fractionalization:

$$Q\Phi = 2\pi$$

- The resulting insulator has $\mathbb{Z}_4$ topological order and is a generalization of nonabelian FQH states to 3D.

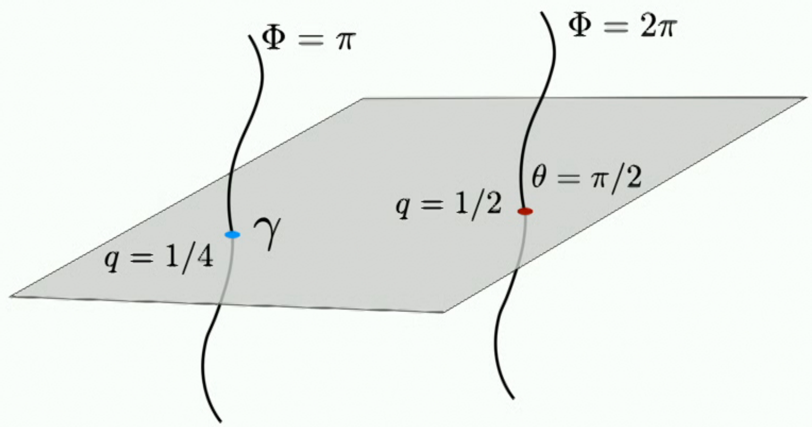
Anyons

- Existence of anyons is an essential feature of 2D FQHE.

- But anyons can only exist in 2D, which makes one doubt FQHE can be generalized to 3D.

Anyons in 3D

- Intersections of vortex loops with atomic planes are the anyons.



Conclusions

- Both ordinary metals and topological semimetals may be characterized by unquantized (i.e. having tunable coefficients) anomalies.
- The anomalies may be expressed as topological terms, involving electromagnetic as well as crystal symmetry gauge fields.
- These topological terms describe fractional electric charges, induced on symmetry defects, such as flux lines, dislocations and disclinations, or fractional polarization.
- The topological terms may be related to generalized chiral anomaly for nodes of odd codimension, or parity anomaly for nodes of even codimension.