

Title: Measurement as a shortcut to long-range entangled quantum matter

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Abstract: The preparation of long-range entangled states using unitary circuits is limited by Lieb-Robinson bounds, but circuits with projective measurements and feedback ("adaptive circuits") can evade such restrictions. We introduce three classes of local adaptive circuits that enable low-depth preparation of long-range entangled quantum matter characterized by gapped topological orders and conformal field theories (CFTs). The three classes are inspired by distinct physical insights, including tensor-network constructions, multiscale entanglement renormalization ansatz (MERA), and parton constructions. A large class of topological orders, including chiral topological order, can be prepared in constant depth or time, and one-dimensional CFT states and non-abelian topological orders with both solvable and non-solvable groups can be prepared in depth scaling logarithmically with system size. We also build on a recently discovered correspondence between symmetry-protected topological phases and long-range entanglement to derive efficient protocols for preparing symmetry-enriched topological order and arbitrary CSS (Calderbank-Shor-Steane) codes. Our work illustrates the practical and conceptual versatility of measurement for state preparation.

Measurement as a shortcut to long-range entangled quantum matter

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Long-Range Entangled Quantum Matter

Orders manifest in global structure of entanglement

Quantum “matters” — No classical description

Hosting various exotic physical phenomena

Quantum information

Robust quantum information storage and processing

Long-range entangled quantum matter

Example 1: Topological order

- Distinct from conventional SSB order
- Fractionalized excitations - Anyons
- Topology-dependent G.S degeneracy

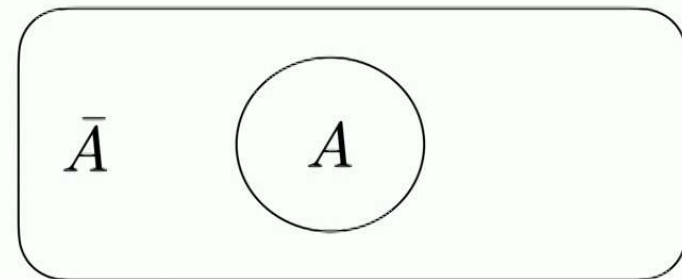
Long-range entangled quantum matter

Example 1: Topological order

Kitaev, Preskill;
Levin, Wen
(2006)

- Distinct from conventional SSB order
- Fractionalized excitations - Anyons
- Topology-dependent G.S degeneracy

Topological entanglement entropy



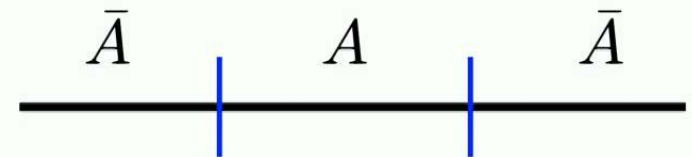
$$S_A = \alpha |\partial A| - \gamma$$

Long-range entangled quantum matter

Example 2: Gapless states described by CFT (conformal field theory)

- Quantum phase transitions
- Power-law long-range correlations

Entanglement in 1+1 D CFT



$$S_A \sim \frac{c}{3} \log L_A$$

Calabrese, Cardy (2004) **c**: central charge

Long-range entangled quantum matter

Question: how to prepare these non-trivial states?

Condensed matter

Exploration of novel physics

Quantum information

Application to quantum computing

Noisy intermediate-scale quantum (NISQ) era

John Preskill (2018)

Rapid progress on quantum device

Superconducting qubits

Trapped-ion quantum computer

Difficulty: limited qubit coherence time

Long-range entangled quantum matter

Question: how to prepare these non-trivial states **efficiently**?
e.g. in $O(1)$ time

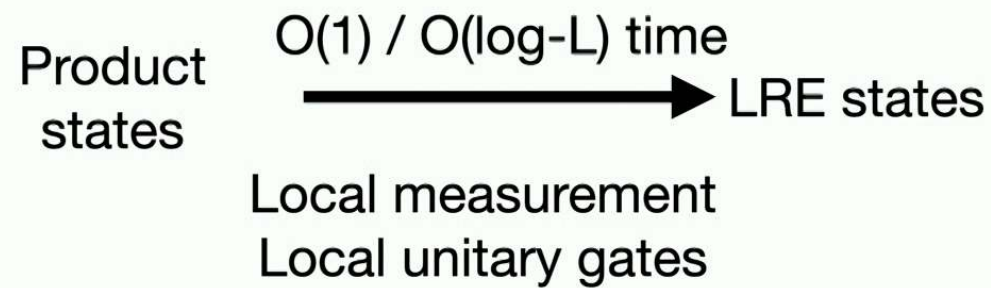


local, unitary protocol
of finite depth/time

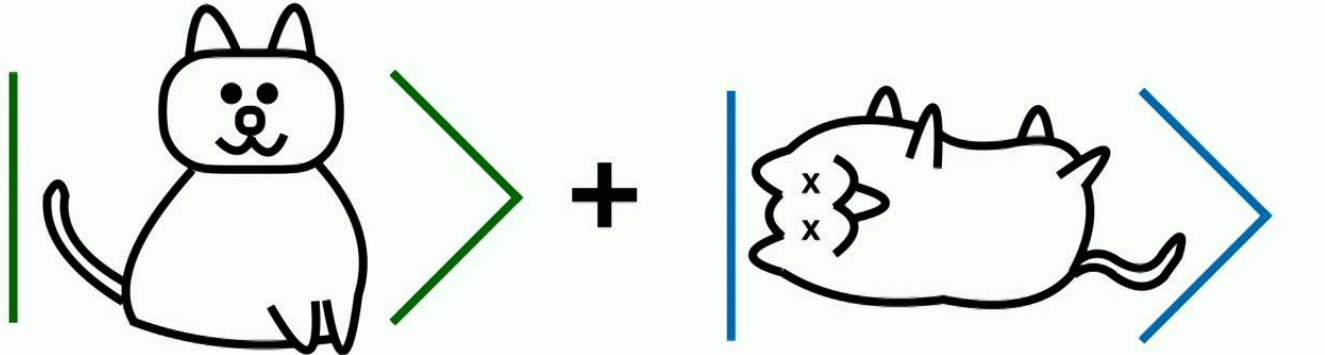
- Distinct quantum phases
- Existence of light cone (Lieb-Robinson bound, 1972)

This talk

Using **measurement** as an ingredient
in state preparation

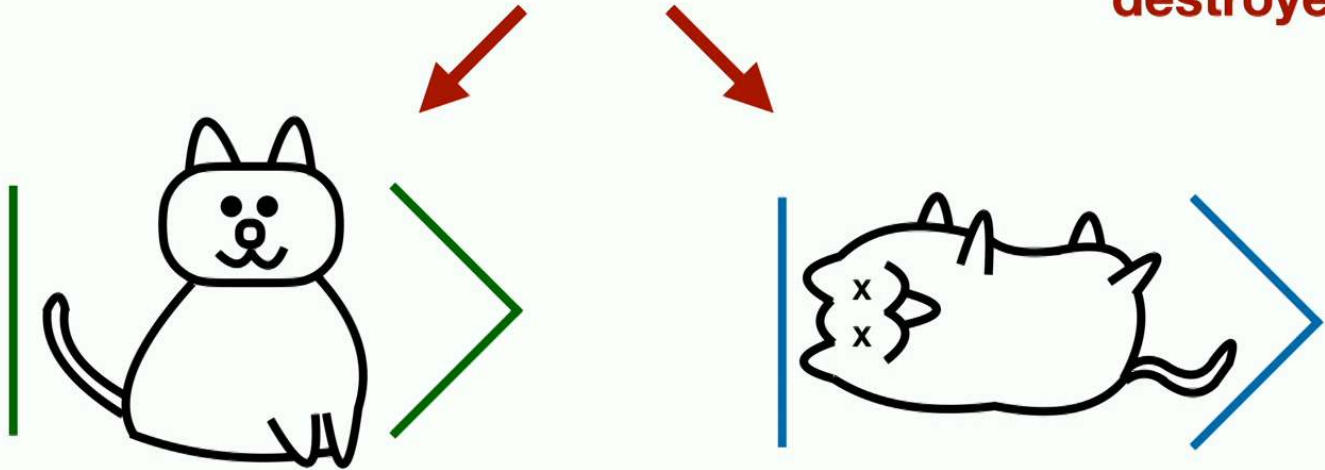


Measurement meets Schrodinger's cat

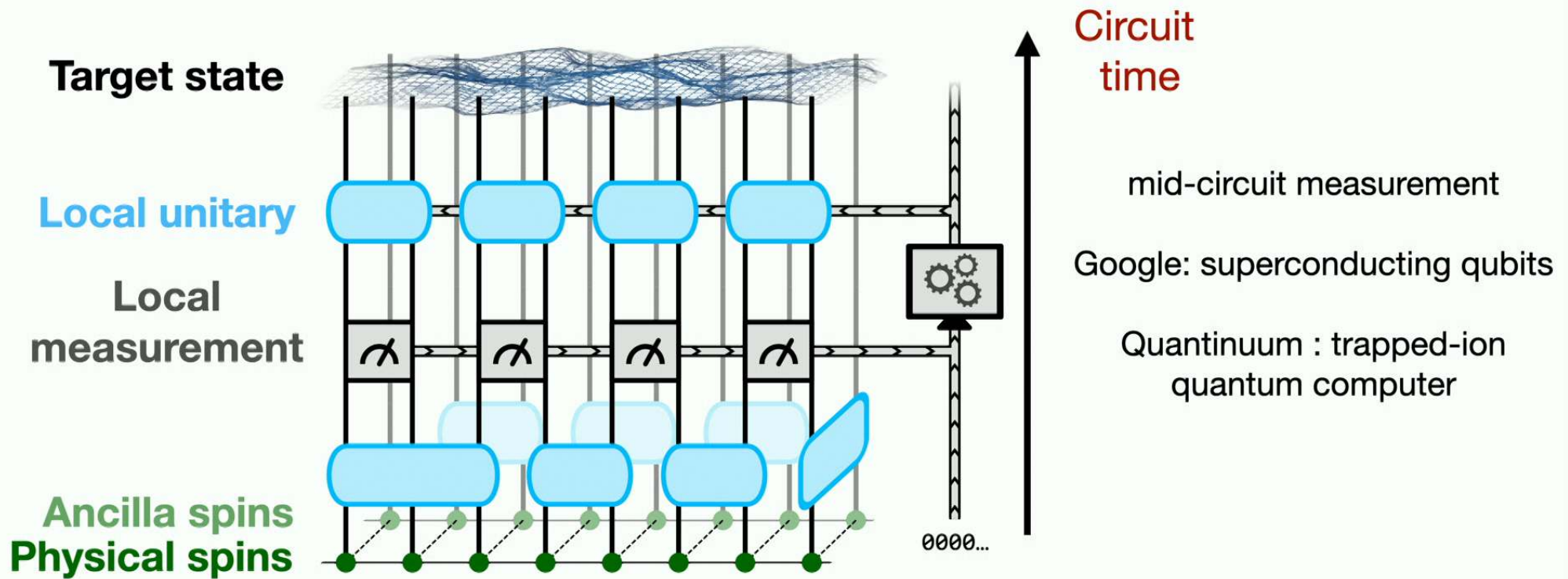


Measurement

Entanglement and coherence destroyed



Local adaptive circuits



Prior works: Piroli, Styliaris, Cirac (2021);
 Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021);
 Bravyi, Kim, Kliesch, Koenig, arXiv:2205.01933 (2022)

Local adaptive circuits

Three novel classes of circuits from three distinct physical insights

Tensor networks

Parton construction

MERA

(Multiscale Entanglement
Renormalization Ansatz)

$O(1)$ -depth/time preparation

(Chiral) (non-abelian) topological order

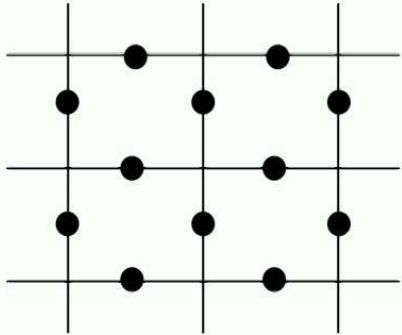
$O(\log L)$ -depth/time preparation

Non-abelian quantum double

Levin-Wen string net

Gapless CFT states

Z₂ topological order - 2d toric code Kitaev (2003)



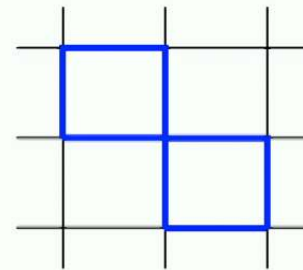
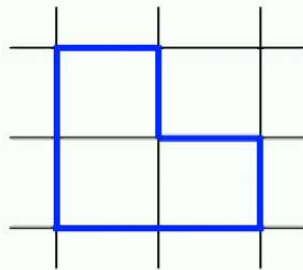
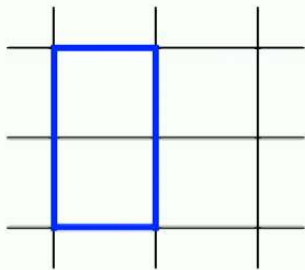
Superposition of loops

$$|\psi\rangle = \sum_c |\mathcal{C}\rangle \quad \mathcal{C} \text{ Loop (closed strings) configuration of spins}$$

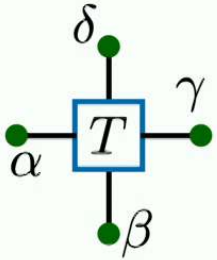
↑ (Spin up, no string)

↓ (Spin down, string)

Examples of loops



Adaptive circuits inspired by tensor networks



$$|T\rangle_v = \sum_{\alpha, \beta, \gamma, \delta} T_{\alpha\beta\gamma\delta} |\alpha, \beta, \gamma, \delta\rangle \quad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 & \text{if } \alpha\beta\gamma\delta = 1 \\ 0 & \text{if } \alpha\beta\gamma\delta = -1 \end{cases}$$

$$ZZZZ|T\rangle_v = |T\rangle_v \quad XX|T\rangle_v = |T\rangle_v$$

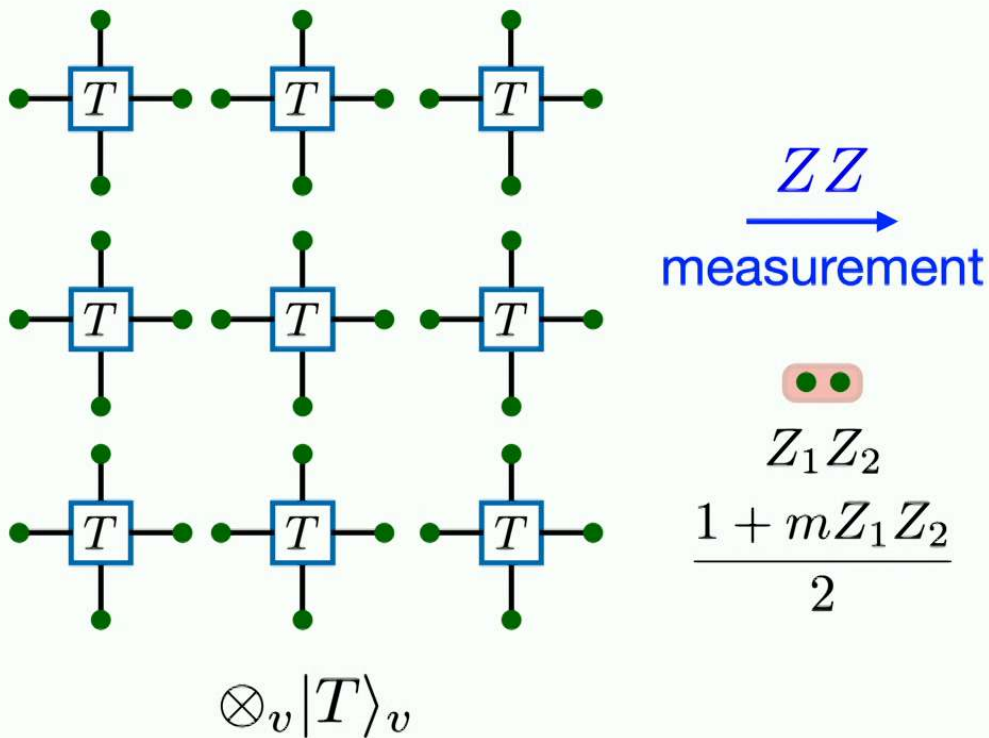
Superposition of even # spin-down

$$|T\rangle_v = \left| \begin{array}{c} \bullet \\ | \\ \text{---} T \text{---} \\ | \\ \bullet \end{array} \right\rangle + \left| \begin{array}{c} \bullet \\ | \\ \text{---} T \text{---} \\ | \\ \bullet \end{array} \right\rangle + \left| \begin{array}{c} \bullet \\ | \\ \text{---} T \text{---} \\ | \\ \bullet \end{array} \right\rangle + \dots$$

0 spin down
4 spin down
2 spin down (6 possible states)

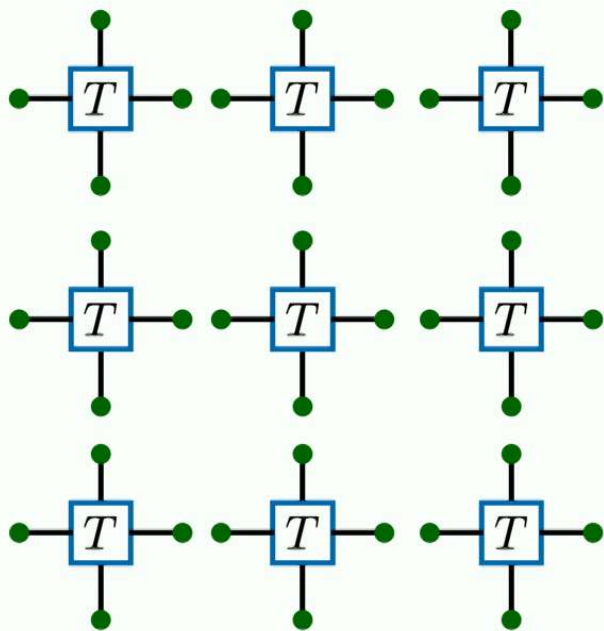
Adaptive circuits inspired by tensor networks

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Adaptive circuits inspired by tensor networks

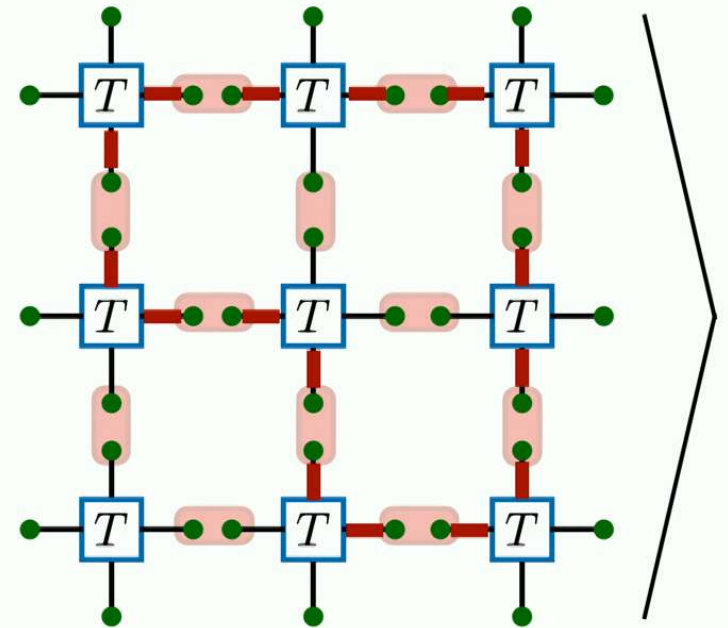
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$$\otimes_v |T\rangle_v$$

ZZ
measurement

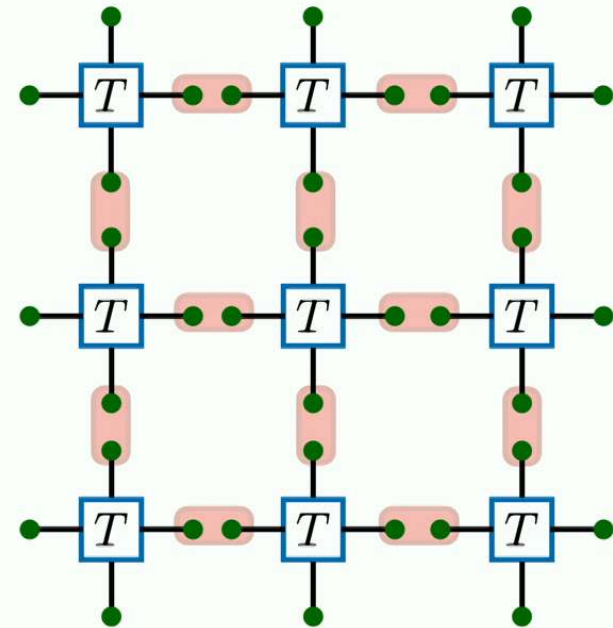
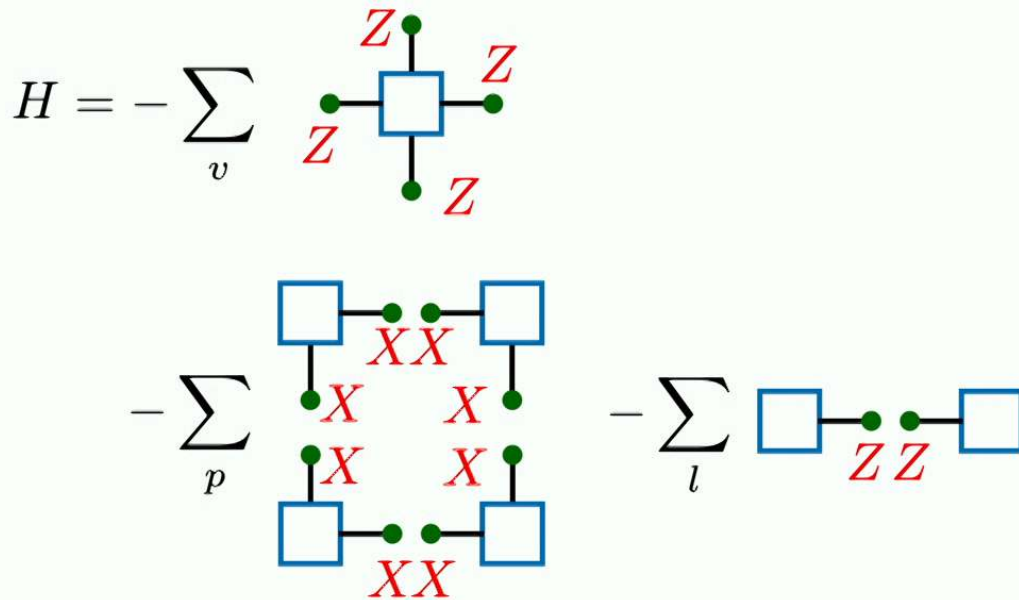
$$\frac{Z_1 Z_2}{1 + m Z_1 Z_2} \quad 2$$



Toric code order when outcome=1

Adaptive circuits inspired by tensor networks

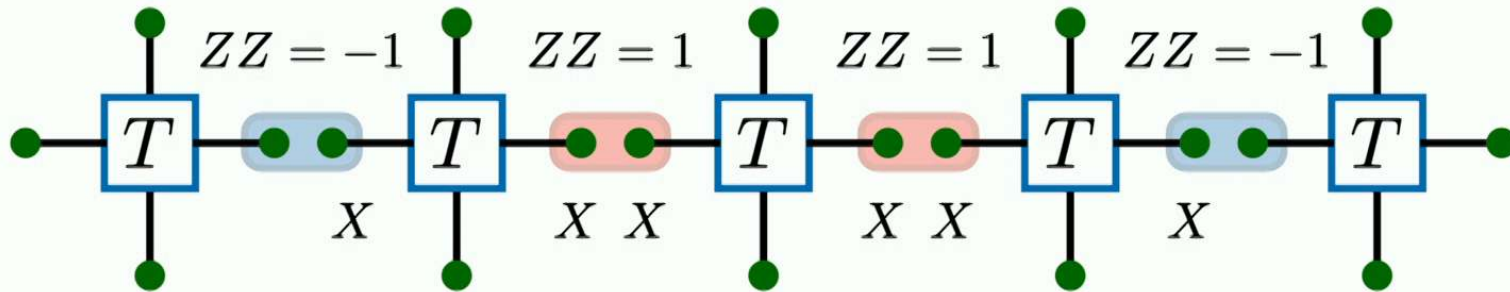
$$|T\rangle_v = \sum_{\alpha, \beta, \gamma, \delta} T_{\alpha\beta\gamma\delta} |\alpha, \beta, \gamma, \delta\rangle \quad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 & \text{if } \alpha\beta\gamma\delta = 1 \\ 0 & \text{if } \alpha\beta\gamma\delta = -1 \end{cases}$$



Toric code order when outcome=1

Adaptive circuits inspired by tensor networks

Measurement outcomes ($ZZ = -1$) come in pairs
as **anyon excitations**
correction using X-string operator



Adaptive circuits inspired by tensor networks

$O(1)$ -depth preparation for

Abelian quantum double

Kitaev (2003)

Double semion

Levin, Wen (2005)

Fracton topological order

Review by Nandkishore, Hermele (2019)
Pretko, Chen, You (2020)

Feasible for experimental realizations (a modular approach)

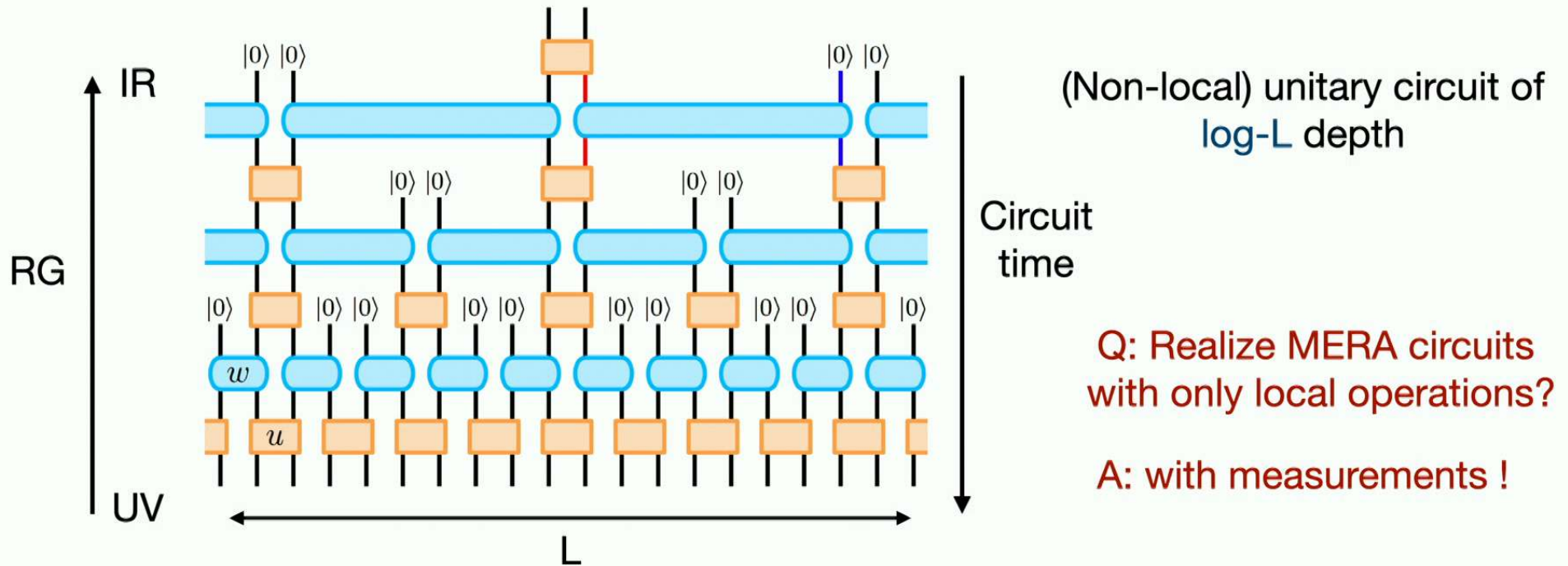
Open question

Non-abelian topological order?

Adaptive circuits via MERA

(Multiscale Entanglement Renormalisation Ansatz) Vidal (2007)

A systematic way of removing UV d.o.f. and short-distance entanglement



(Non-local) unitary circuit of $\log-L$ depth

Q: Realize MERA circuits with only local operations?

A: with measurements !

Adaptive circuits via MERA

Measurement-based teleportation

Bennett, Brassard, Crepeau,
Jozsa, Peres, Wootters (1993)



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad |\psi\rangle_1 \quad |\text{Bell}\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell pair measurement for qubit 1 and qubit 2

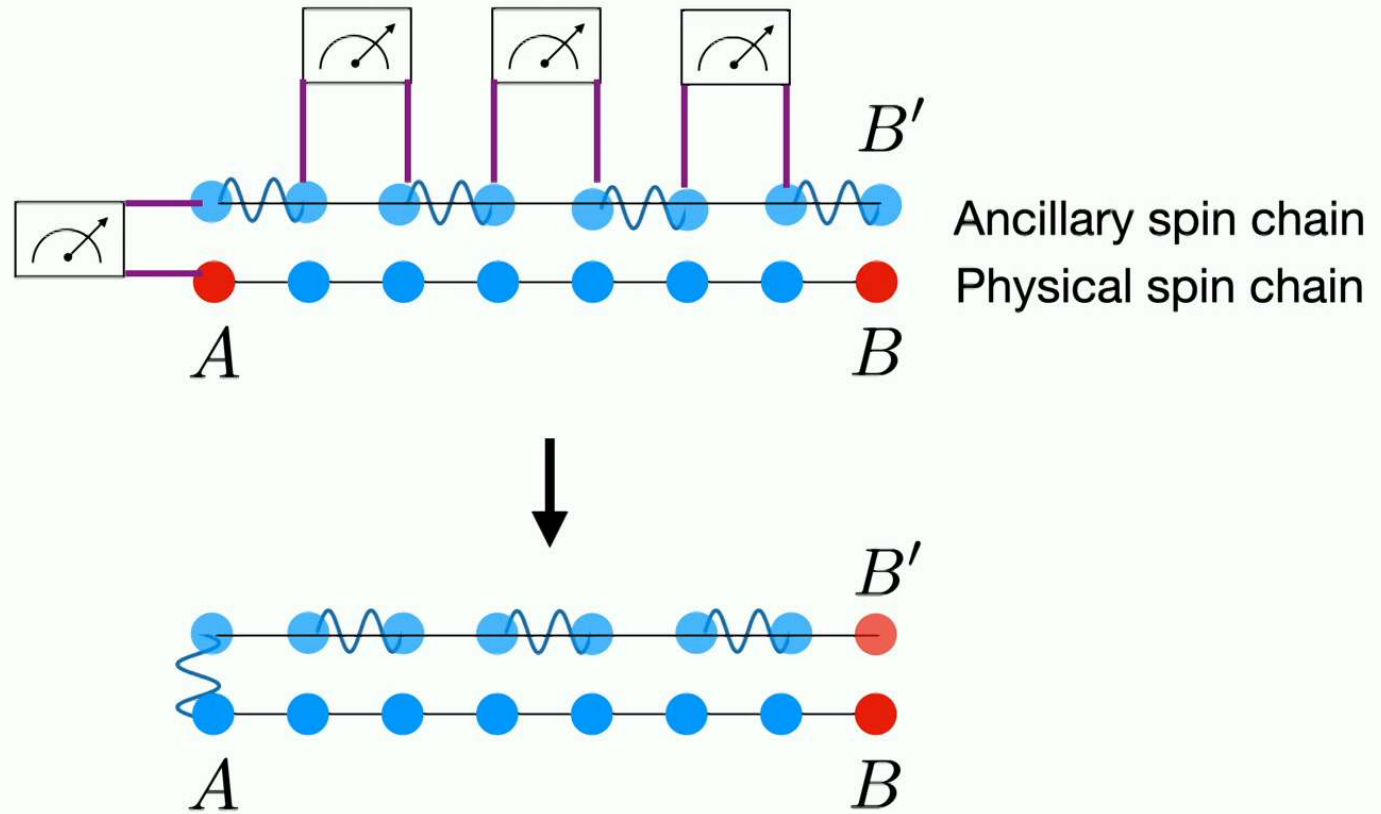
$$\begin{array}{ll} |00\rangle + |11\rangle & |01\rangle + |10\rangle \\ |00\rangle - |11\rangle & |01\rangle - |10\rangle \end{array}$$

$$\text{measurement outcome} = |00\rangle_{12} + |11\rangle_{12} \quad |\psi\rangle_3 = \alpha|0\rangle_3 + \beta|1\rangle_3$$

For other measurement outcomes,
apply a local operator on qubit 3 to correct error

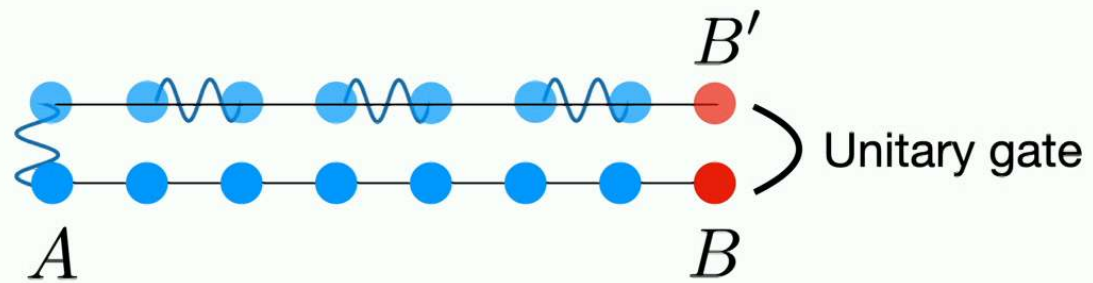
Adaptive circuits via MERA

Goal: apply a non-local gate acting on A and B



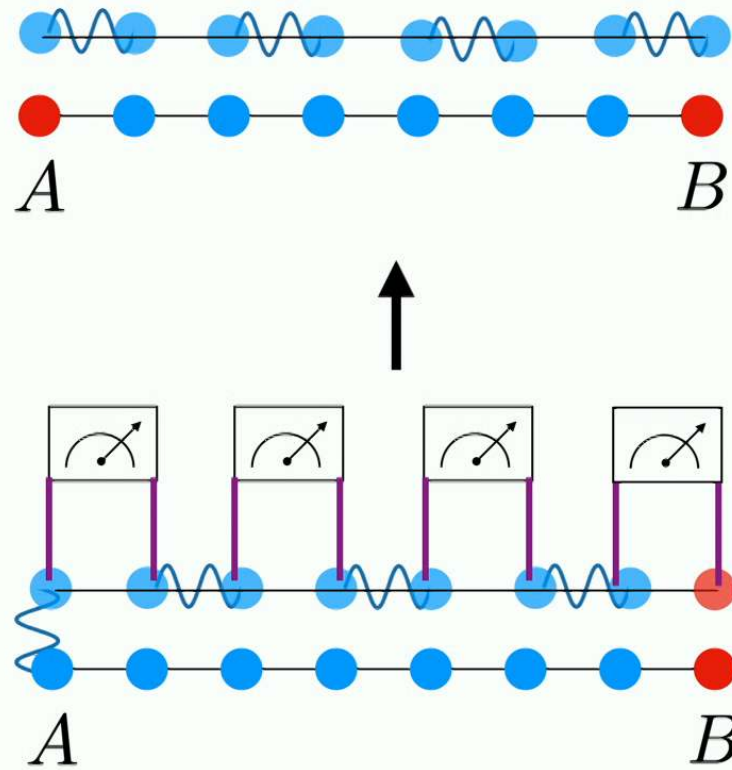
Adaptive circuits via MERA

Goal: apply a non-local gate acting on A and B



Adaptive circuits via MERA

Goal: apply a non-local gate acting on A and B



Adaptive circuits via MERA

MERA circuits using local unitary gates & measurements

Critical states & gapped topological order in $\log L$ depth

Quantum double (Aguado, Vidal, 2008)

Levin-Wen string nets (König, Reichardt, Vidal, 2009)

Note: in any **local** circuits

preparing 1d critical CFT states requires **depth** $\gtrsim O(\log L)$

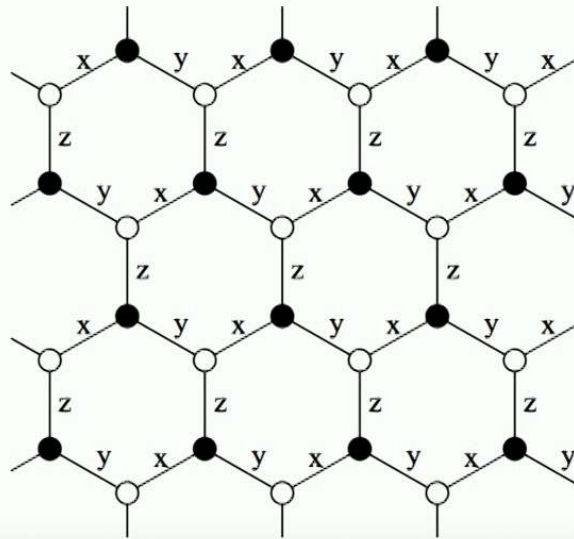
Fundamental obstruction from entanglement

Adaptive circuits via parton constructions

Highlight - **Chiral** non-abelian topological order in $O(1)$ time!

Key idea - using measurement to implement parton constraint

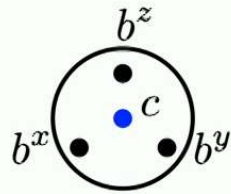
$$H = -J_x \sum_{x\text{-links}} X_j X_k - J_y \sum_{y\text{-links}} Y_j Y_k - J_z \sum_{z\text{-links}} Z_j Z_k$$



Kitaev honeycomb model
(2006)

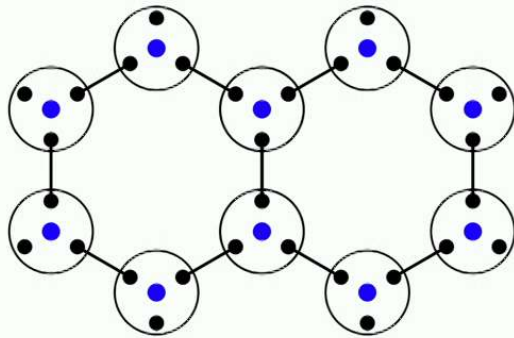
Adaptive circuits via parton constructions

Fractionalize a spin into **Majorana partons**



$$X_j = ib_j^x c_j \quad Y_j = ib_j^y c_j \quad Z_j = ib_j^z c_j$$

$$D_j = b_j^x b_j^y b_j^z c_j = 1$$



$$H = \frac{i}{4} \sum_{\langle jk \rangle} (2J_{jk}) u_{jk} c_j c_k$$

$$u_{jk} = ib_j^\alpha b_k^\alpha$$

$$|\psi\rangle = \prod_j \frac{1 + D_j}{2} |\psi_0\rangle$$

Projector

free-fermions
& Majorana dimers

Adaptive circuits via parton constructions

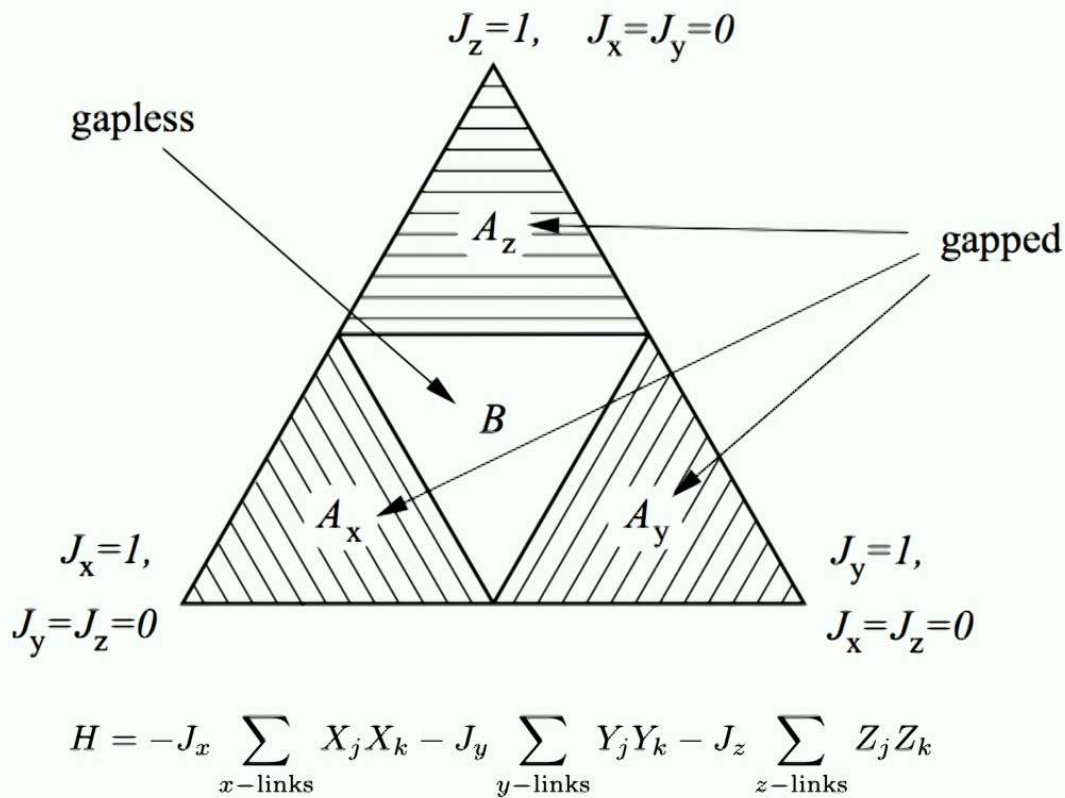
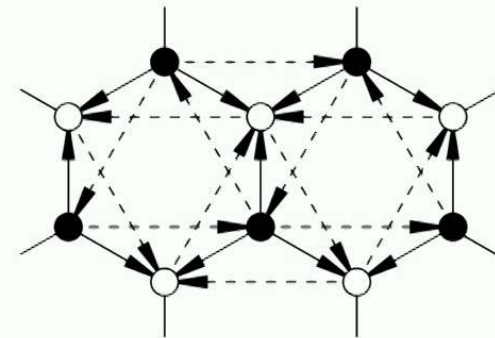


Fig. from Kitaev (2006)

B phase is gapped under a Mag. field



c Majoranas form a p+ip superconductor

Physical g.s $|\psi\rangle = \prod_j \frac{1 + D_j}{2} |\psi_0\rangle$

Chiral Ising-anyon topological order

Adaptive circuits via parton constructions

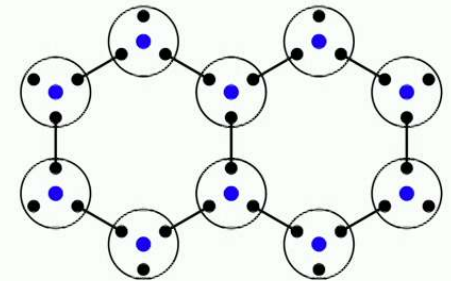
Adaptive protocol for preparing **Chiral Ising-anyon topological order**

(1) Preparing a free fermion state $|\psi_0\rangle =$ Majorana dimers + (p+ip) superconductor
 in $O(1)$ time

b Majoranas



c Majoranas ●

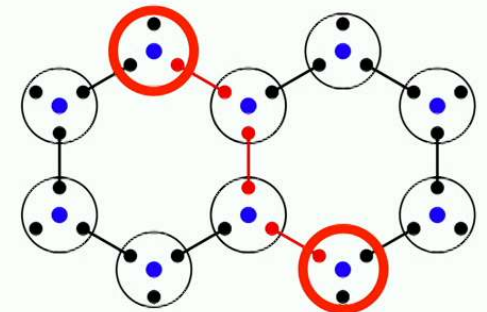


(2) Simultaneously measure $D_j = b_j^x b_j^y b_j^z c_j$ on each vertex

Outcome $D_j = 1 \longrightarrow |\psi\rangle = \prod_j \frac{1 + D_j}{2} |\psi_0\rangle$

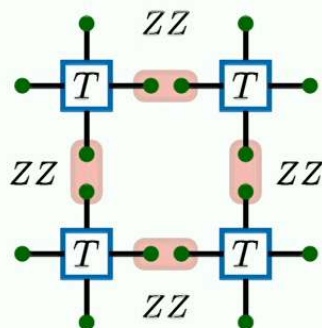
(3) Outcome $D_j = -1$ comes in pair

Corrected using a product of b-Majorana dimers



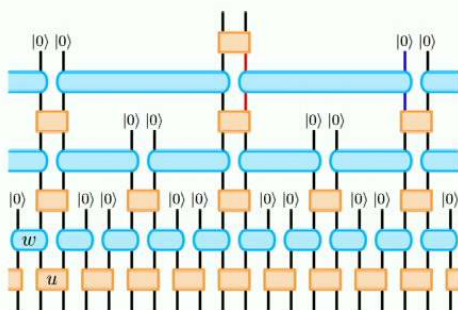
Measurement-assisted circuits (adaptive circuits)

Tensor-networks



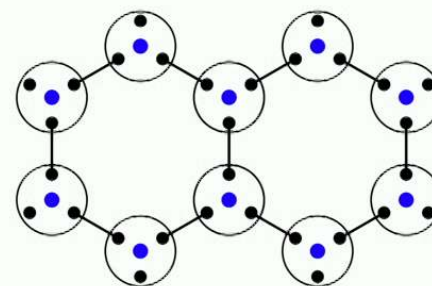
Non-abelian
topological order?

MERA



Higher dim CFT ?

Parton



More applications?
Other chiral spin liquids

Limitation of measurement-assisted (adaptive) circuits?

Preparing long-range entangled mixed states?

Measurement as a versatile ingredient in many-body physics