Title: Measurement as a shortcut to long-range entangled quantum matter

Speakers: Peter Lu

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Abstract: The preparation of long-range entangled states using unitary circuits is limited by Lieb-Robinson bounds, but circuits with projective measurements and feedback (``adaptive circuits") can evade such restrictions. We introduce three classes of local adaptive circuits that enable low-depth preparation of long-range entangled quantum matter characterized by gapped topological orders and conformal field theories (CFTs). The three classes are inspired by distinct physical insights, including tensor-network constructions, multiscale entanglement renormalization ansatz (MERA), and parton constructions. A large class of topological orders, including chiral topological order, can be prepared in constant depth or time, and one-dimensional CFT states and non-abelian topological orders with both solvable and non-solvable groups can be prepared in depth scaling logarithmically with system size. We also build on a recently discovered correspondence between symmetry-protected topological phases and long-range entanglement to derive efficient protocols for preparing symmetry-enriched topological order and arbitrary CSS (Calderbank-Shor-Steane) codes. Our work illustrates the practical and conceptual versatility of measurement for state preparation.

# Measurement as a shortcut to long-range entangled quantum matter

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# Long-Range Entangled Quantum Matter

Orders manifest in global structure of entanglement

Quantum "matters" — No classical description

Hosting various exotic physical phenomena

Quantum information

Robust quantum information storage and processing

# Long-range entangled quantum matter

## Example 1:
## Topological order

- Distinct from conventional SSB order
- Fractionalized excitations - Anyons
- Topology-dependent G.S degeneracy

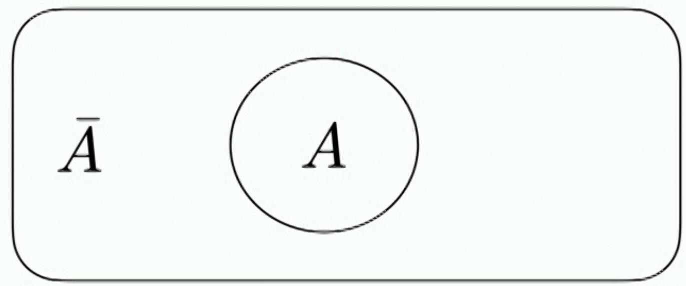
# Long-range entangled quantum matter

## Example 1:
## Topological order

Kitaev, Preskill;
Levin, Wen
(2006)

- Distinct from conventional SSB order
- Fractionalized excitations - Anyons
- Topology-dependent G.S degeneracy

Topological entanglement entropy



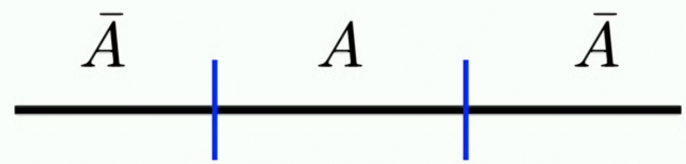

$$S_A = \alpha|\partial A| - \gamma$$

# Long-range entangled quantum matter

## Example 2:
## Gapless states described by CFT (conformal field theory)

- Quantum phase transitions
- Power-law long-range correlations

Entanglement in 1+1 D CFT




$$S_A \sim \frac{c}{3} \log L_A$$

Calabrese, Cardy (2004) c: central charge

# Long-range entangled quantum matter

**Question: how to prepare these non-trivial states?**

| Condensed matter | Quantum information |
| --- | --- |
| Exploration of novel physics | Application to quantum computing |

**Noisy intermediate-scale quantum (NISQ) era**

John Preskill (2018)

Rapid progress on quantum device

Superconducting qubits

Trapped-ion quantum computer

Difficulty: limited qubit coherence time

# Long-range entangled quantum matter

**Question: how to prepare these non-trivial states efficiently?**

e.g. in O(1) time

Product states


LRE states

local, unitary protocol
of finite depth/time

- Distinct quantum phases
- Existence of light cone
  (Lieb-Robinson bound, 1972)

## This talk

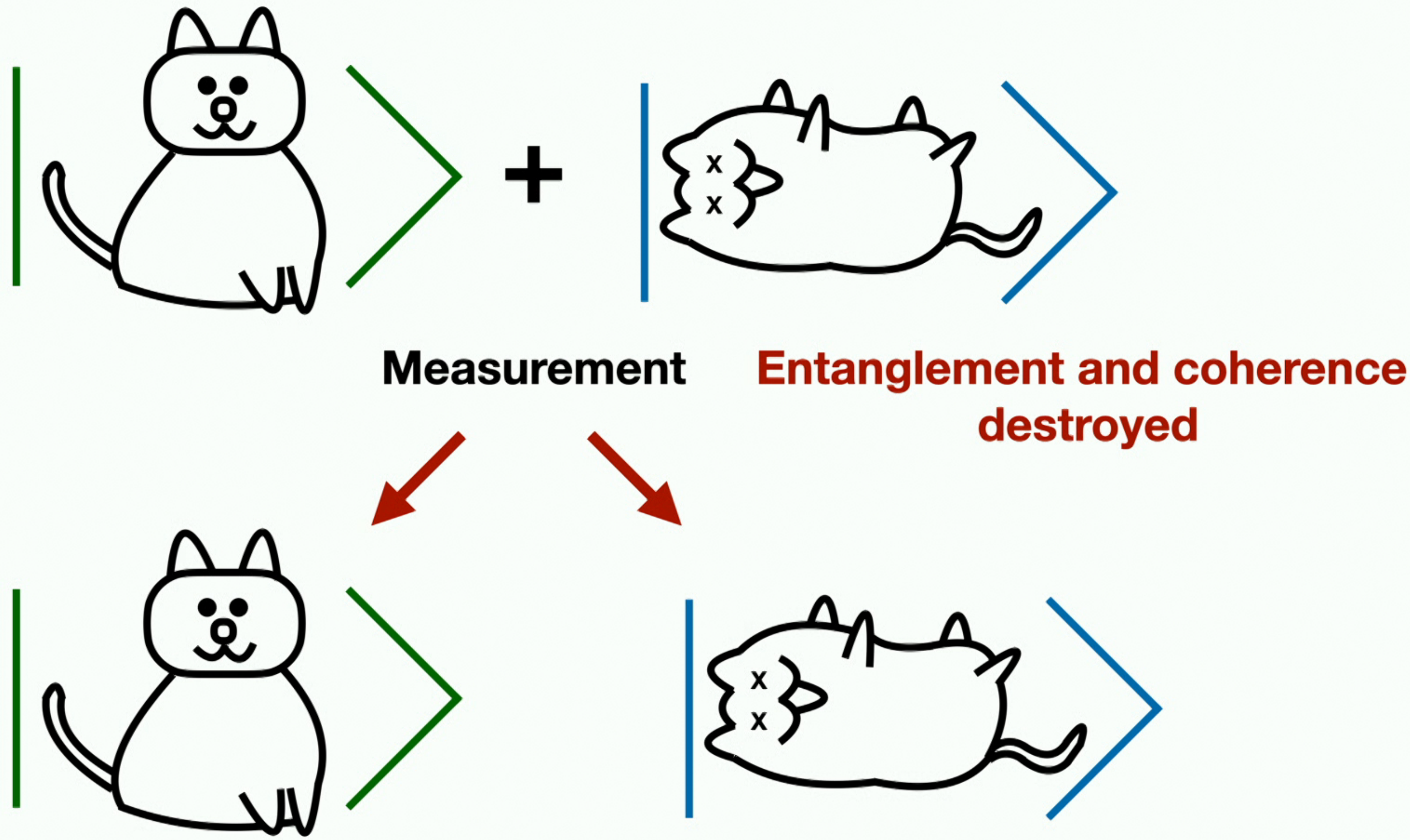
**Using measurement as an ingredient in state preparation**

Product states → LRE states

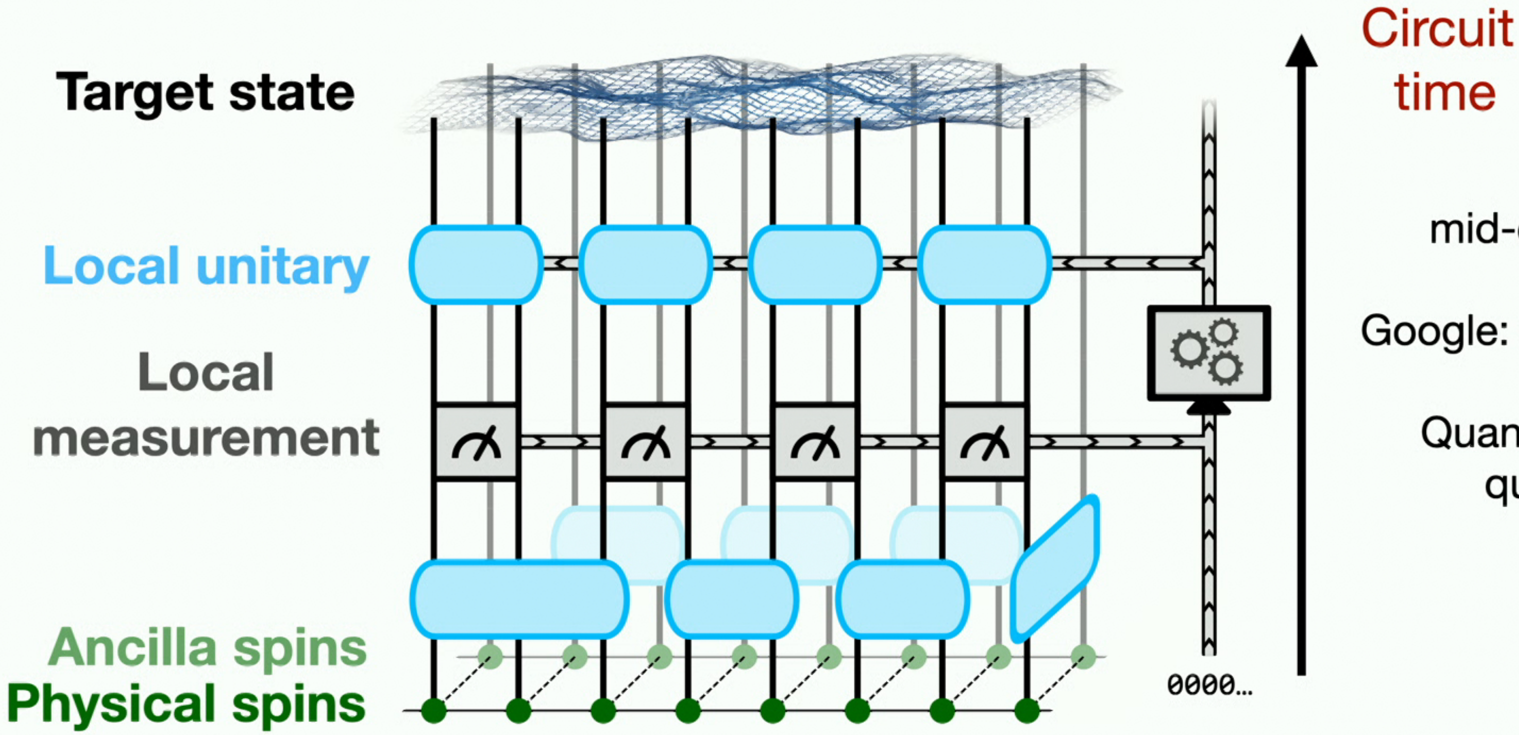
O(1) / O(log-L) time

Local measurement
Local unitary gates

# Measurement meets Schrodinger's cat



# Local adaptive circuits




mid-circuit measurement

Google: superconducting qubits

Quantinuum : trapped-ion quantum computer

Prior works: Piroli, Styliaris, Cirac (2021);
Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021);
Bravyi, Kim, Kliesch, Koenig, arXiv:2205.01933 (2022)

## Local adaptive circuits

Three novel classes of circuits from three distinct physical insights

Tensor networks

Parton construction

MERA
(Multiscale Entanglement Renormalization Ansatz)

**O(1)-depth/time preparation**

(Chiral) (non-abelian) topological order

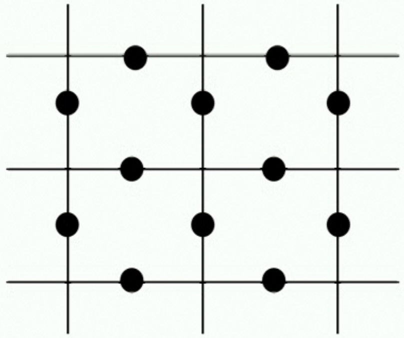
**O(log L)-depth/time preparation**

Non-abelian quantum double

Levin-Wen string net

Gapless CFT states

# $Z_2$ topological order - 2d toric code Kitaev (2003)



Superposition of loops

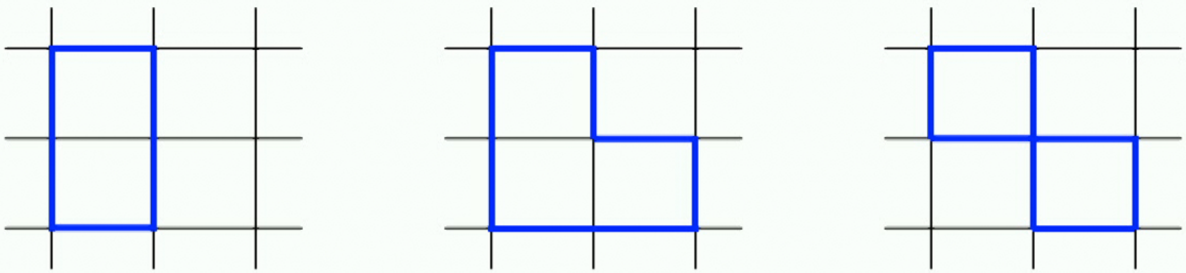
$$|\psi\rangle = \sum_{\mathcal{C}} |\mathcal{C}\rangle$$

$\mathcal{C}$ Loop (closed strings) configuration of spins

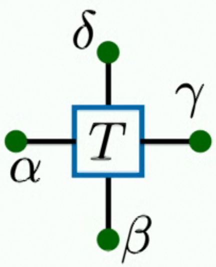
↑ (Spin up, no string)

↓ (Spin down, string)

Examples of loops



# Adaptive circuits inspired by tensor networks

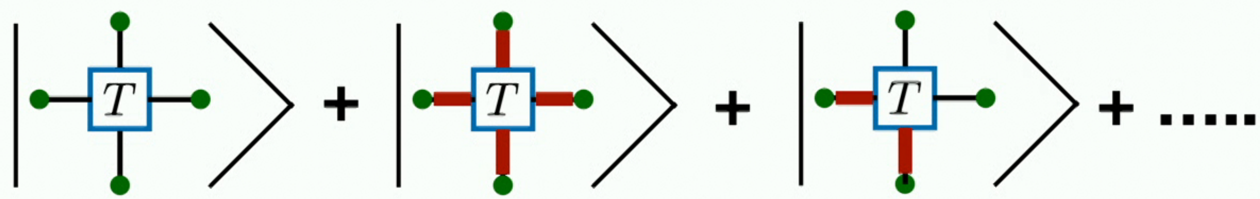



$$|T\rangle_v = \sum_{\alpha,\beta,\gamma,\delta} T_{\alpha\beta\gamma\delta}|\alpha,\beta,\gamma,\delta\rangle \qquad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 \text{ if } \alpha\beta\gamma\delta = 1 \\ 0 \text{ if } \alpha\beta\gamma\delta = -1 \end{cases}$$

$$ZZZZ|T\rangle_v = |T\rangle_v \quad XX|T\rangle_v = |T\rangle_v$$

Superposition of even # spin-down

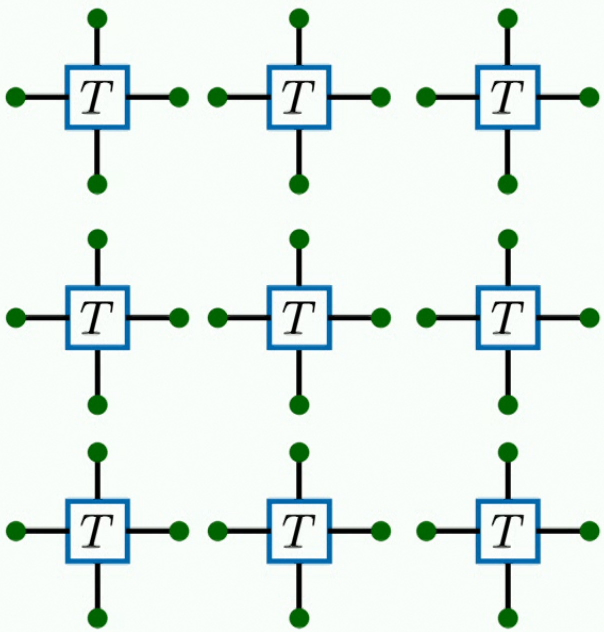
$|T\rangle_v =$




0 spin down 4 spin down 2 spin down (6 possible states)

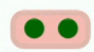
# Adaptive circuits inspired by tensor networks

$$|T\rangle_v = \sum_{\alpha,\beta,\gamma,\delta} T_{\alpha\beta\gamma\delta}|\alpha,\beta,\gamma,\delta\rangle \qquad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 \text{ if } \alpha\beta\gamma\delta = 1 \\ 0 \text{ if } \alpha\beta\gamma\delta = -1 \end{cases}$$




$\otimes_v |T\rangle_v$

$ZZ$ measurement

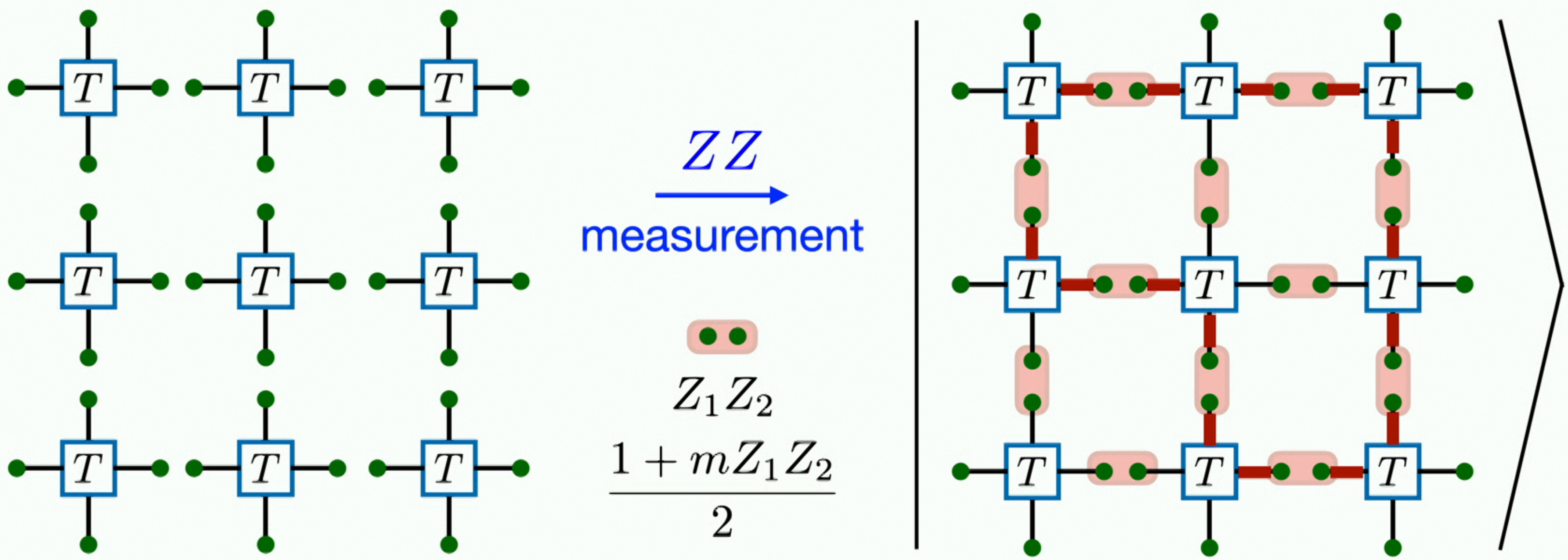


$Z_1 Z_2$

$$\frac{1 + mZ_1Z_2}{2}$$

# Adaptive circuits inspired by tensor networks

$$|T\rangle_v = \sum_{\alpha,\beta,\gamma,\delta} T_{\alpha\beta\gamma\delta} |\alpha,\beta,\gamma,\delta\rangle \qquad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 \text{ if } \alpha\beta\gamma\delta = 1 \\ 0 \text{ if } \alpha\beta\gamma\delta = -1 \end{cases}$$

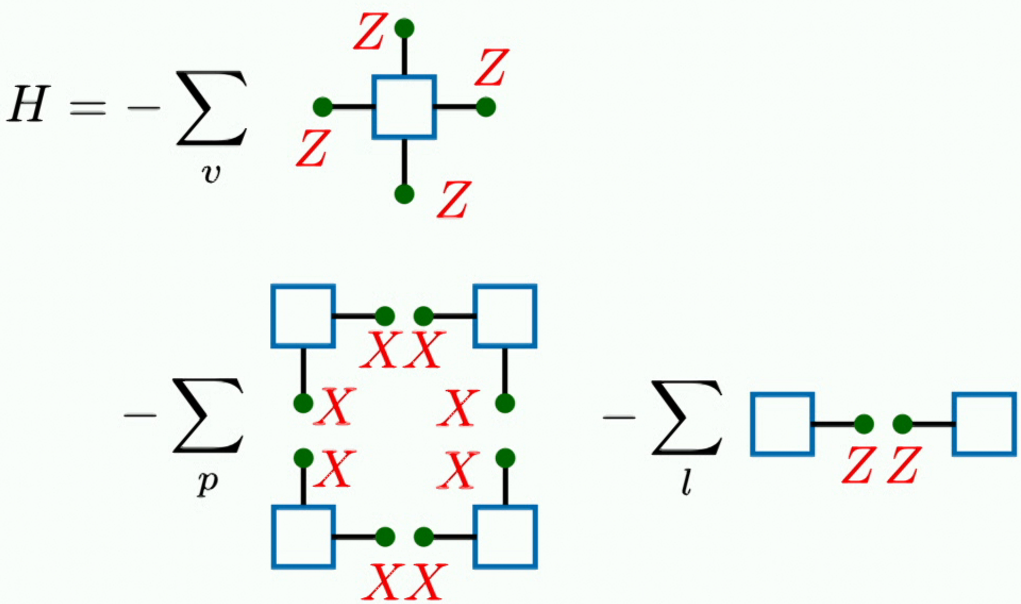


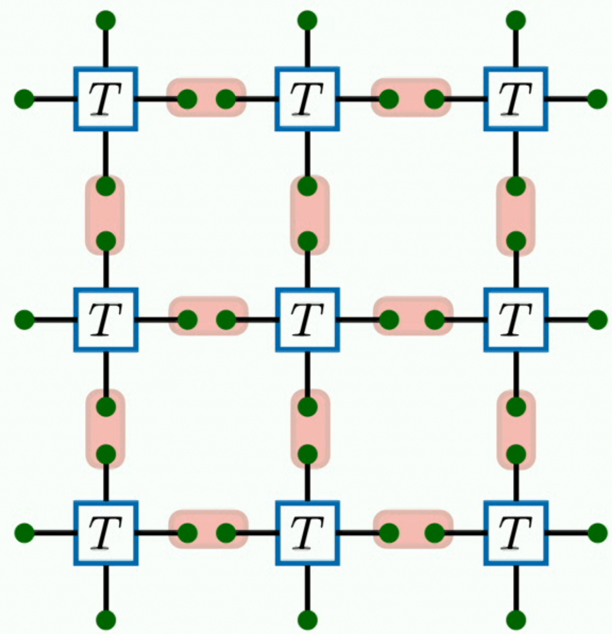

$\otimes_v |T\rangle_v$

Toric code order when outcome=1

# Adaptive circuits inspired by tensor networks

$$|T\rangle_v = \sum_{\alpha,\beta,\gamma,\delta} T_{\alpha\beta\gamma\delta} |\alpha,\beta,\gamma,\delta\rangle \qquad T_{\alpha\beta\gamma\delta} = \begin{cases} 1 \text{ if } \alpha\beta\gamma\delta = 1 \\ 0 \text{ if } \alpha\beta\gamma\delta = -1 \end{cases}$$

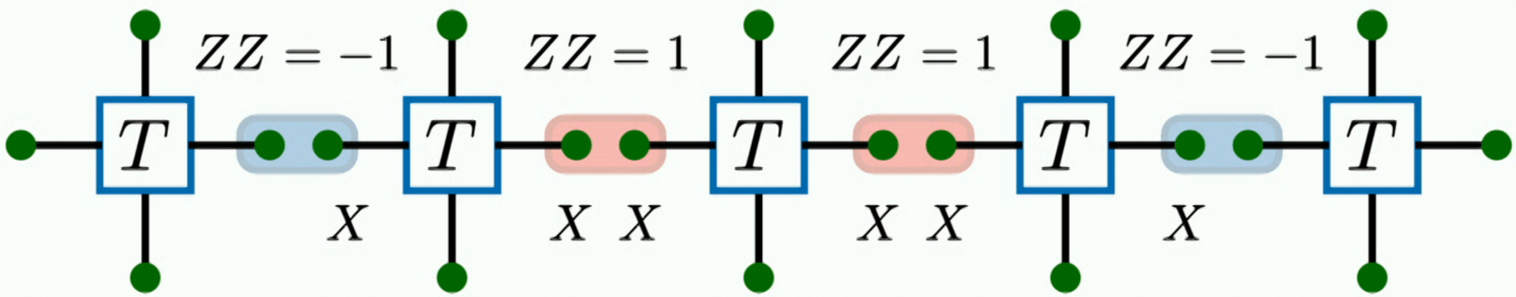



Toric code order when outcome=1

# Adaptive circuits inspired by tensor networks

Measurement outcomes (ZZ = -1) come in pairs
as anyon excitations
correction using X-string operator



# Adaptive circuits inspired by tensor networks

O(1)-depth preparation for

**Abelian quantum double**
Kitaev (2003)

**Double semion**
Levin, Wen (2005)

**Fracton topological order**
Review by Nandkishore, Hermele (2019)
Pretko, Chen, You (2020)

Feasible for experimental realizations (a modular approach)

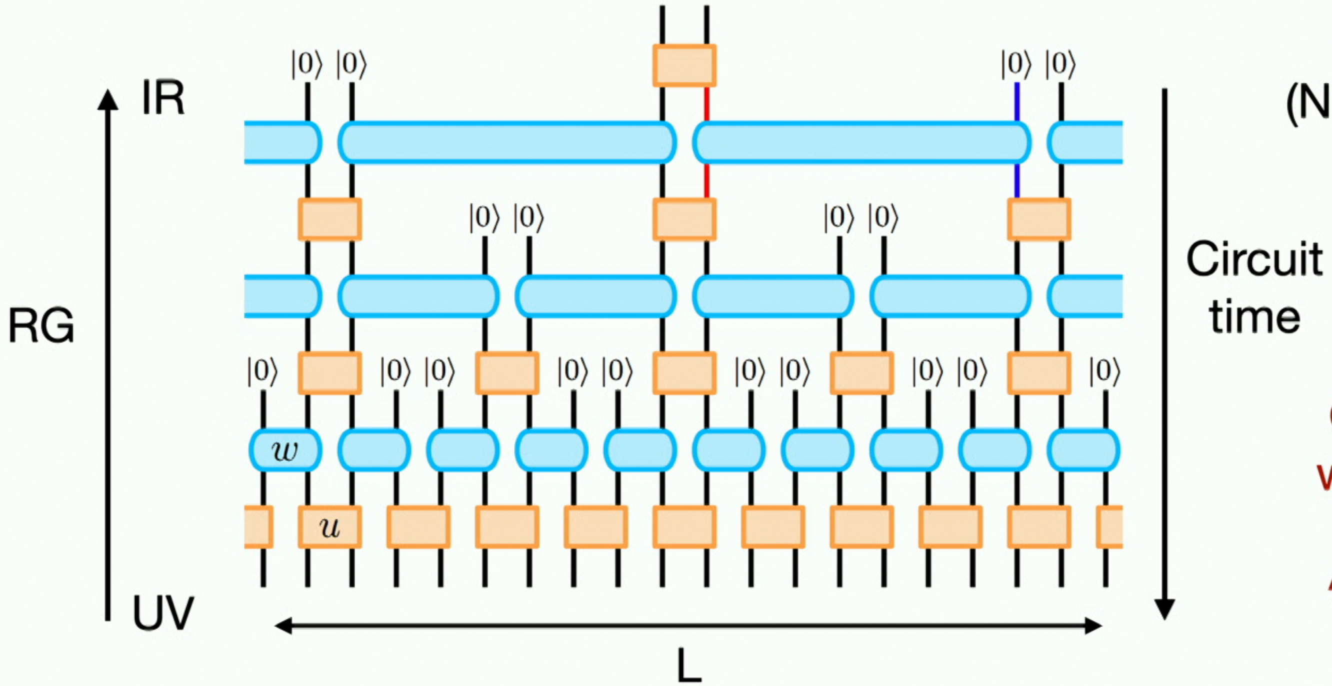
**Open question**

Non-abelian topological order?

# Adaptive circuits via MERA

(Multiscale Entanglement Renormalisation Ansatz) Vidal (2007)

A systematic way of removing UV d.o.f. and short-distance entanglement




(Non-local) unitary circuit of log-L depth

Q: Realize MERA circuits with only local operations?

A: with measurements !

# Adaptive circuits via MERA

## Measurement-based teleportation

Bennett, Brassard, Crepeau, Jozsa, Peres, Wootters (1993)


$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\psi\rangle_1 \qquad |\text{Bell}\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Bell pair measurement for qubit 1 and qubit 2

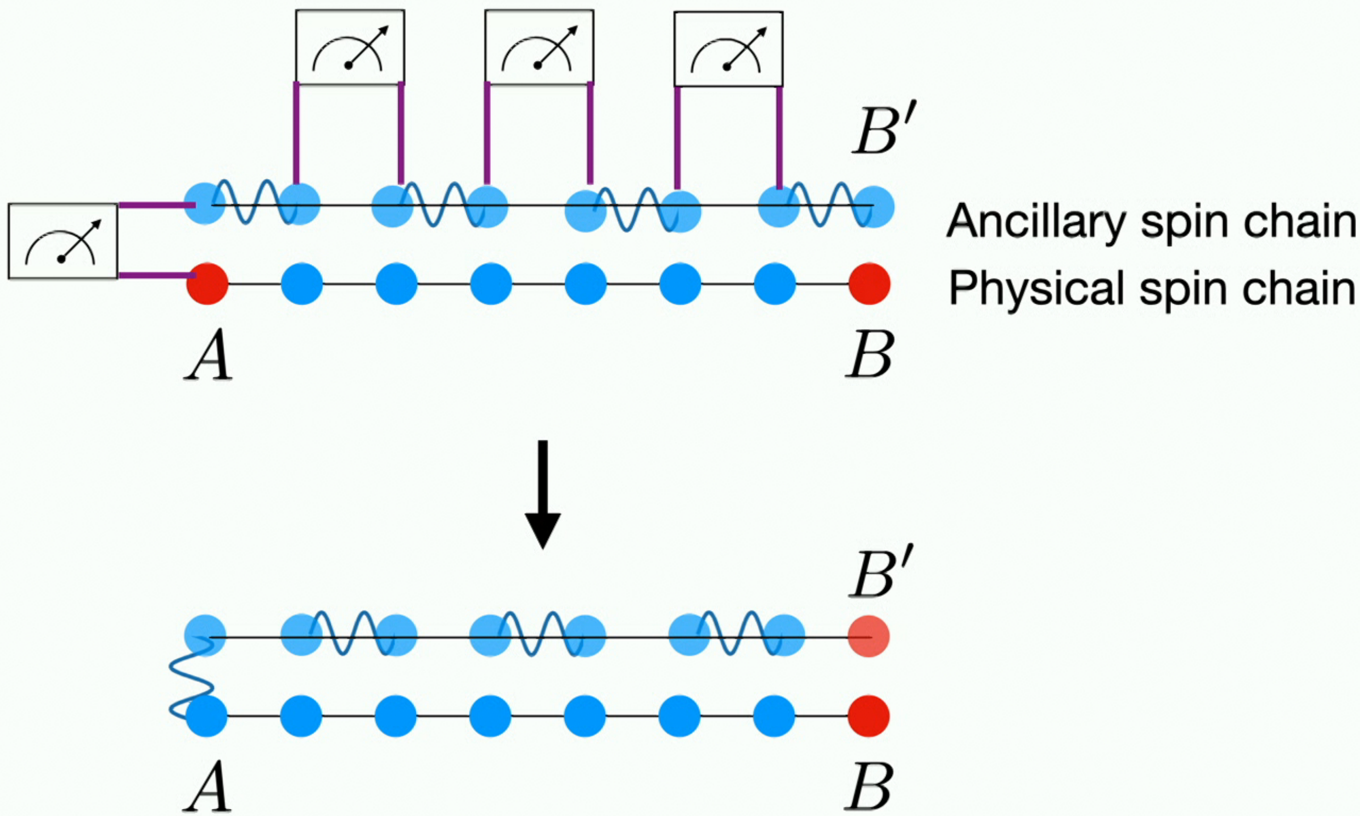
$$|00\rangle + |11\rangle \qquad |01\rangle + |10\rangle$$
$$|00\rangle - |11\rangle \qquad |01\rangle - |10\rangle$$

measurement outcome = $|00\rangle_{12} + |11\rangle_{12}$ $\quad |\psi\rangle_3 = \alpha|0\rangle_3 + \beta|1\rangle_3$

For other measurement outcomes,
apply a local operator on qubit 3 to correct error

# Adaptive circuits via MERA

Goal: apply a non-local gate acting on A and B



# Adaptive circuits via MERA

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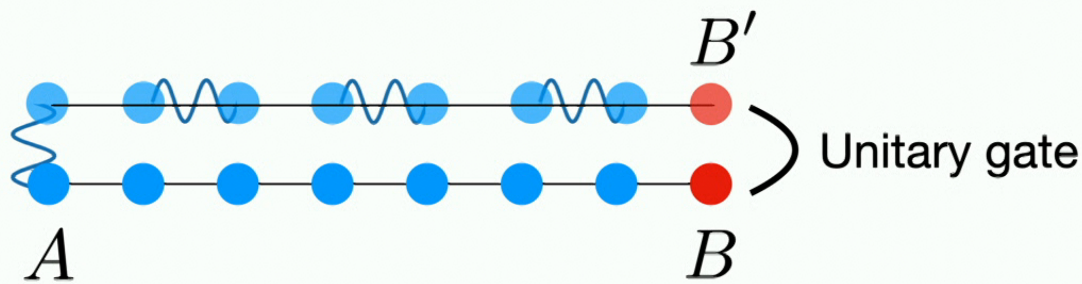
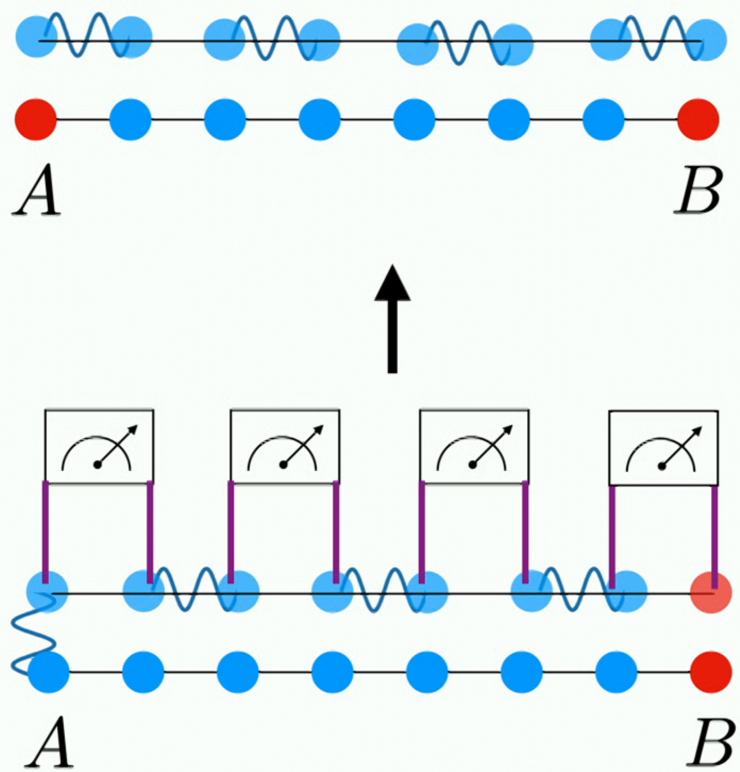


# Adaptive circuits via MERA

Goal: apply a non-local gate acting on A and B



# Adaptive circuits via MERA

MERA circuits using local unitary gates & measurements

**Critical states & gapped topological order in log L depth**

Quantum double (Aguado, Vidal, 2008)

Levin-Wen string nets (König, Reichardt, Vidal, 2009)

Note: in any local circuits

preparing 1d critical CFT states requires depth $\gtrsim$ O(Log L)

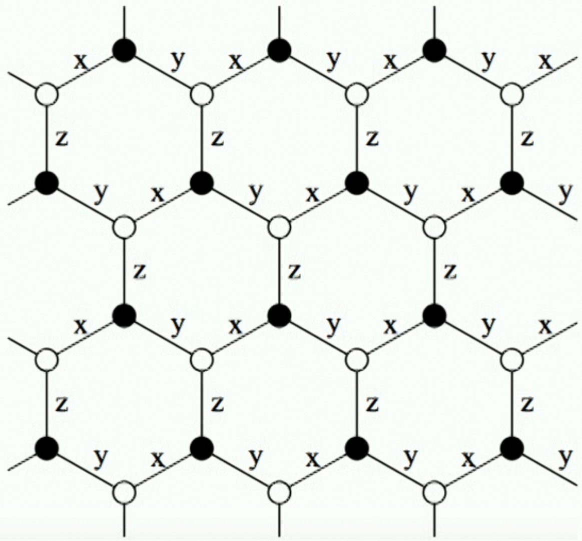
Fundamental obstruction from entanglement

# Adaptive circuits via parton constructions

Highlight - Chiral non-abelian topological order in O(1) time!

Key idea - using measurement to implement parton constraint

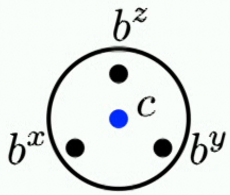
$$H = -J_x \sum_{x-\text{links}} X_j X_k - J_y \sum_{y-\text{links}} Y_j Y_k - J_z \sum_{z-\text{links}} Z_j Z_k$$




Kitaev honeycomb model
(2006)

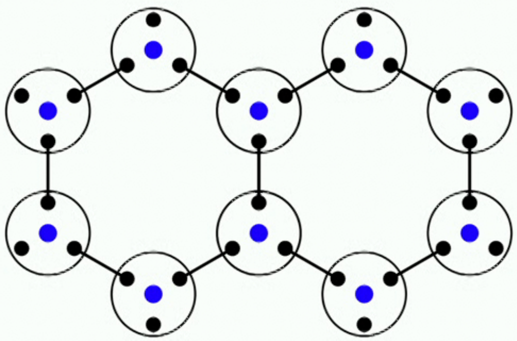
# Adaptive circuits via parton constructions

Fractionalize a spin into Majorana partons




$$X_j = i b_j^x c_j \quad Y_j = i b_j^y c_j \quad Z_j = i b_j^z c_j$$

$$D_j = b_j^x b_j^y b_j^z c_j = 1$$



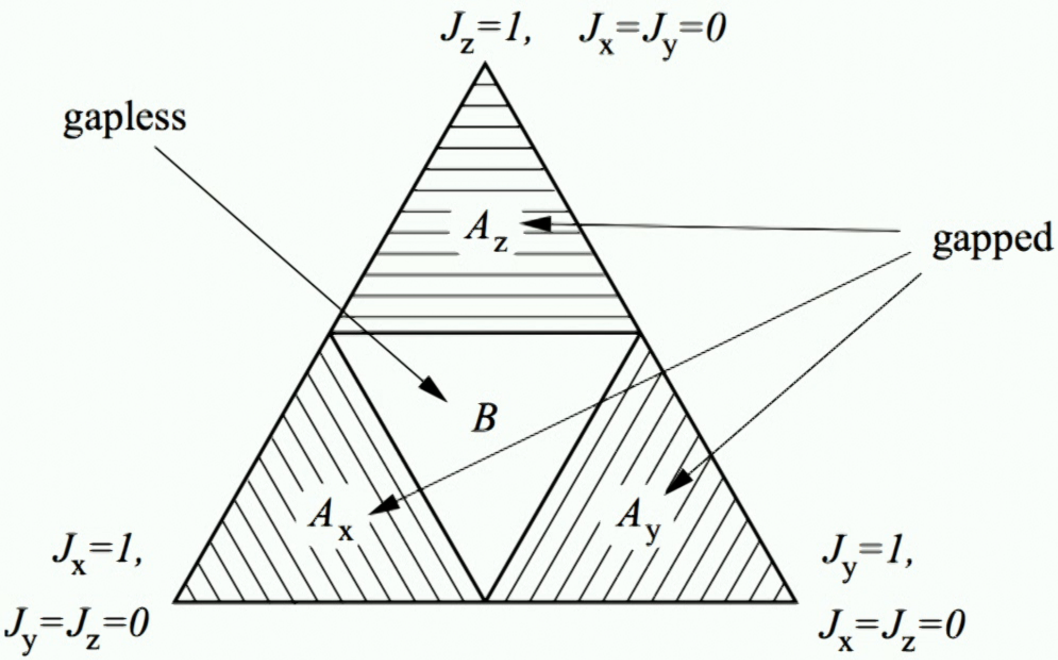
$$H = \frac{i}{4} \sum_{\langle jk \rangle} (2J_{jk}) u_{jk} c_j c_k \qquad u_{jk} = i b_j^\alpha b_k^\alpha$$

$$|\psi\rangle = \prod_j \frac{1 + D_j}{2} |\psi_0\rangle$$

Projector

free-fermions
& Majorana dimers

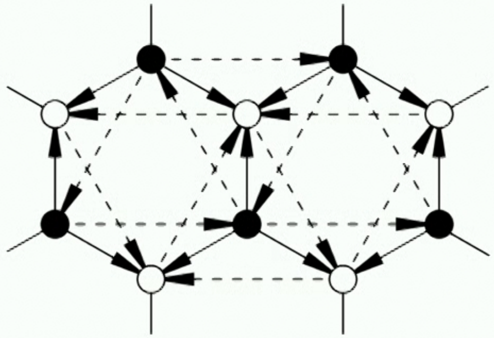
# Adaptive circuits via parton constructions




$$H = -J_x \sum_{x-\text{links}} X_j X_k - J_y \sum_{y-\text{links}} Y_j Y_k - J_z \sum_{z-\text{links}} Z_j Z_k$$

Fig. from Kitaev (2006)

B phase is gapped under a Mag. field



c Majoranas form a p+ip superconductor

Physical g.s $|\psi\rangle = \prod_j \frac{1+D_j}{2} |\psi_0\rangle$

Chiral Ising-anyon topological order

# Adaptive circuits via parton constructions

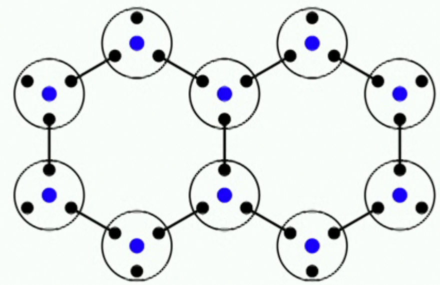
Adaptive protocol for preparing Chiral Ising-anyon topological order

(1) Preparing a free fermion state $|\psi_0\rangle$ = Majorana dimers + (p+ip) superconductor

in O(1) time — b Majoranas — c Majoranas

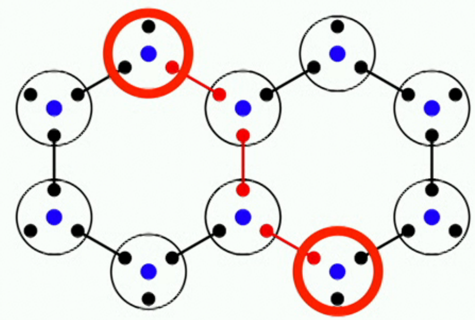
(2) Simultaneously measure $D_j = b_j^x b_j^y b_j^z c_j$ on each vertex

Outcome $D_j = 1$ $\longrightarrow$ $|\psi\rangle = \prod_j \frac{1+D_j}{2} |\psi_0\rangle$



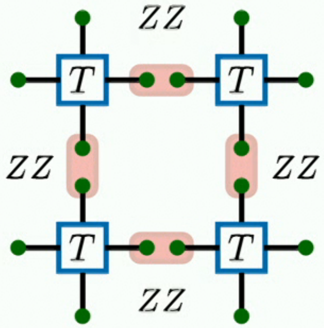
(3) Outcome $D_j = -1$ comes in pair

Corrected using a product of b-Majorana dimers



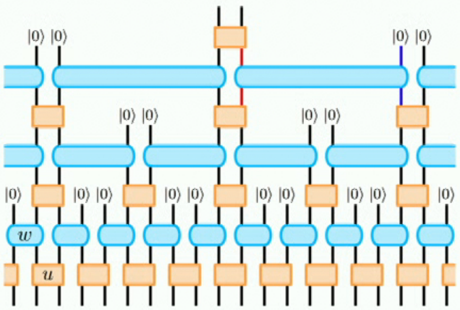
# Measurement-assisted circuits (adaptive circuits)

**Tensor-networks**



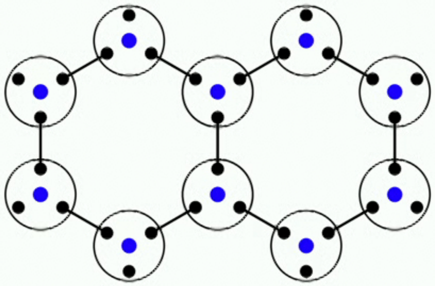

Non-abelian topological order?

**MERA**



Higher dim CFT ?

**Parton**



More applications?
Other chiral spin liquids

Limitation of measurement-assisted (adaptive) circuits?

Preparing long-range entangled mixed states?

**Measurement as a versatile ingredient in many-body physics**