Title: Functional renormalization group formalism for non-Fermi liquids and the antiferromagnetic quantum critical metal

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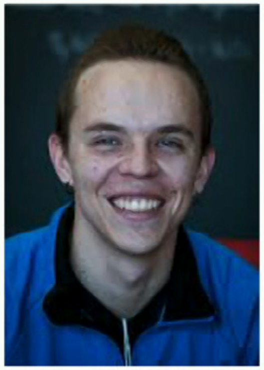
Abstract: We develop a field-theoretic functional renormalization group formalism for field theories of metals that include all gapless modes around the Fermi surface. Due to the presence of intrinsic scales (Fermi momenta), the usual notion of scale invariance and renormalizable field theory need to be generalized. The formalism is applied to the non-Fermi liquid that arises at the anti-ferromagnetic quantum critical point in two space dimensions. We identify the interacting non-Fermi liquid fixed point in the space of coupling functions, and extract the universal scaling behaviour of the normal state and the pathway to the superconducting state at low energies.

Francisco Borges

Anton Borissov

Ashu Singh

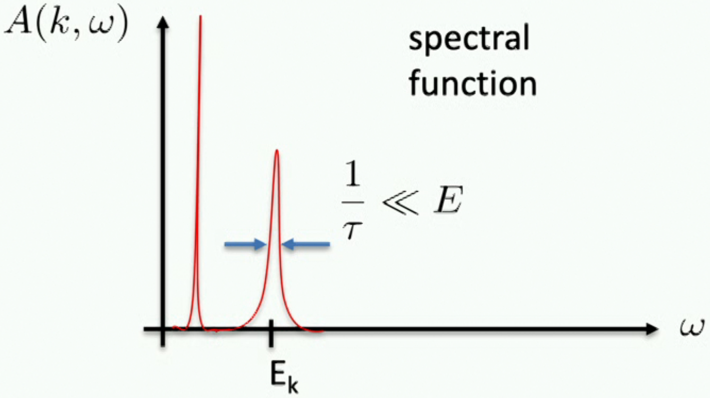


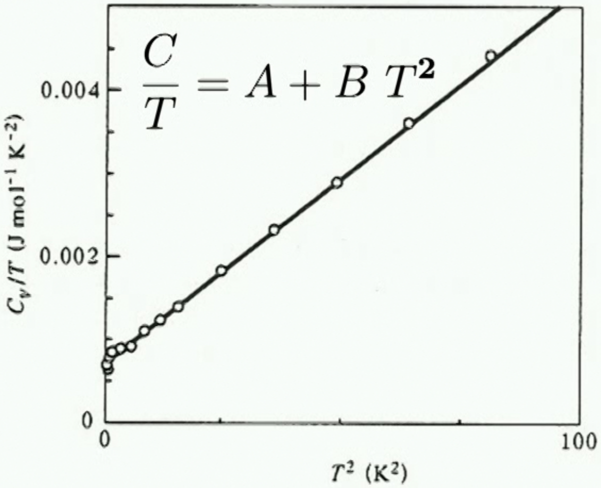
Andres Schlief

Borges, Borissov, Singh, Schlief, SL, arXiv : 2208.00730

Fermi liquids [Landau]

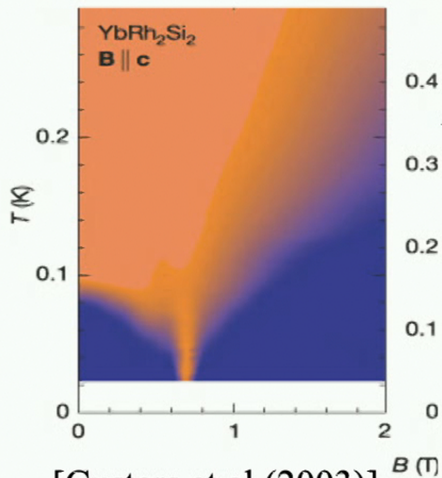
Copper



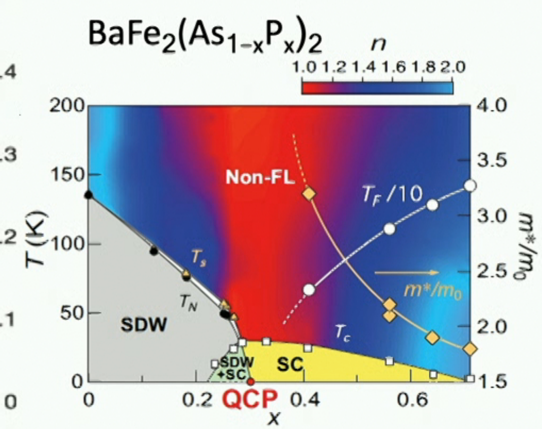


Non-Fermi Liquids

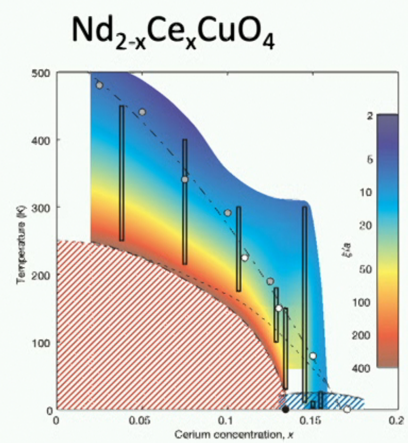
[heavy fermion; pnictides; cuprates]



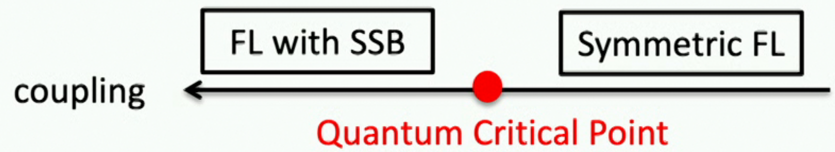
[Custers et al.(2003)]



[Hashimoto et al. (2012)]



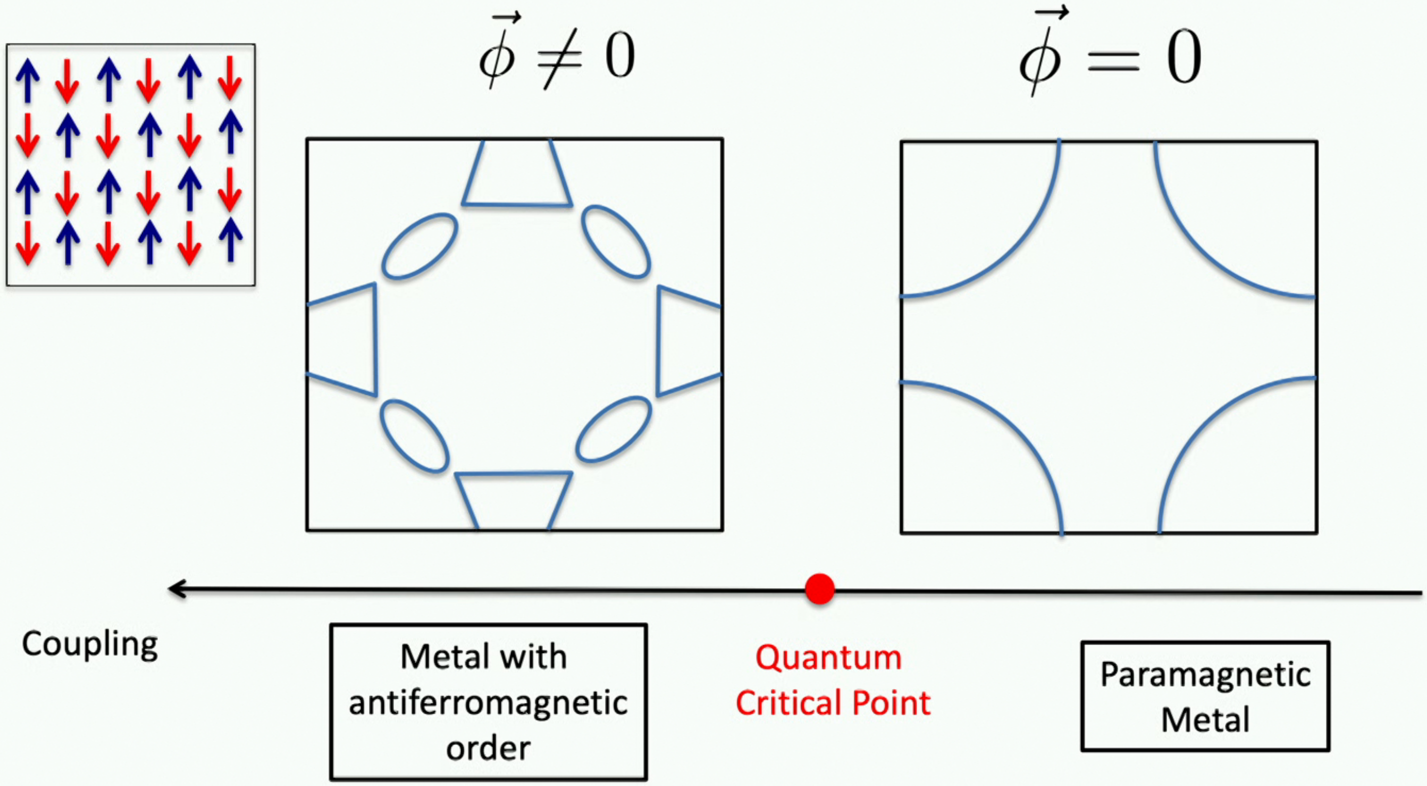
[Motoyama et al. (2012)]



- Despite recent progress, there are still outstanding questions
  - **Strong coupling problem in 2+1D**
  - **Low-energy field theories with infinitely many gapless modes**

Antiferromagnetic Quantum Critical Metal

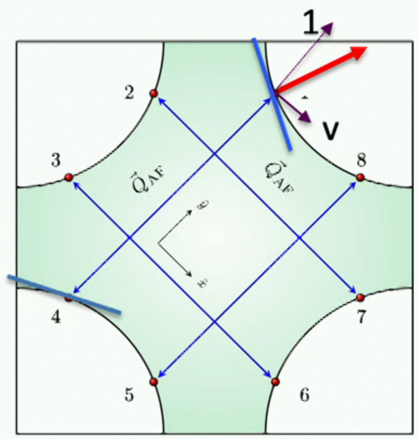
$$\vec{\phi}(\vec{r}) \sim \left\langle \vec{S}(\vec{r}) \right\rangle e^{i\vec{Q}_{AF}\cdot\vec{r}}$$



Patch theory for AF QCM : Hot Spot Theory

[Abanov, Chubukov, Schmalian; Metlitski, Sachdev; ..]

$$S = \sum_{N=1}^{8} \sum_{\sigma=1}^{N_c} \sum_{j=1}^{N_f} \int \mathrm{d}\mathbf{k}\ \psi^\dagger_{N,\sigma,j}(\mathbf{k}) \left[ik_0 + e_N(\vec{k};v) \right] \psi_{N,\sigma,j}(\mathbf{k})$$
$$+ \frac{1}{4} \int \mathrm{d}\mathbf{q}\ \left(q_0^2 + c_0^2 |\vec{q}|^2 \right) \mathrm{Tr}\left[\Phi(\mathbf{q}) \Phi(-\mathbf{q}) \right]$$
$$+ \frac{g}{\sqrt{N_f}} \sum_{N=1}^{8} \sum_{\sigma\sigma'=1}^{N_c} \sum_{j=1}^{N_f} \int \mathrm{d}\mathbf{k} \int \mathrm{d}\mathbf{q}\ \psi^\dagger_{N,\sigma',j}(\mathbf{k}+\mathbf{q}) \Phi_{\sigma'\sigma}(\mathbf{q}) \psi_{\bar{N},\sigma,j}(\mathbf{k})$$



$$e_1(\vec{k};v) = -e_5(\vec{k};v) = v k_x + k_y,$$
$$e_2(\vec{k};v) = -e_6(\vec{k};v) = -k_x - v k_y,$$
$$e_3(\vec{k};v) = -e_7(\vec{k};v) = -k_x + v k_y,$$
$$e_4(\vec{k};v) = -e_8(\vec{k};v) = v k_x - k_y.$$

- v : Fermi velocity perpendicular to Q_{AF}

Patch theory for AF QCM : Hot Spot Theory

[Abanov, Chubukov, Schmalian; Metlitski, Sachdev; ..]

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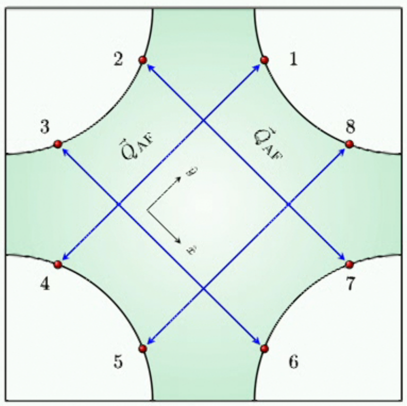


$$\Phi_{\sigma,\sigma'} = \sum_{a=1}^{N_c^2-1} \tau^a_{\sigma,\sigma'}\ \phi_a$$

- c : velocity of spin fluctuations

Patch theory for AF QCM : Hot Spot Theory

[Abanov, Chubukov, Schmalian; Metlitski, Sachdev; ..]

$$S = \sum_{N=1}^{8} \sum_{\sigma=1}^{N_c} \sum_{j=1}^{N_f} \int \mathrm{d}\mathbf{k} \; \psi^{\dagger}_{N,\sigma,j}(\mathbf{k}) \left[ik_0 + e_N(\vec{k}; v) \right] \psi_{N,\sigma,j}(\mathbf{k})$$
$$+ \frac{1}{4} \int \mathrm{d}\mathbf{q} \; \left(q_0^2 + c_0^2 |\vec{q}|^2 \right) \mathrm{Tr}\left[\Phi(\mathbf{q}) \Phi(-\mathbf{q}) \right]$$
$$+ \frac{g}{\sqrt{N_f}} \sum_{N=1}^{8} \sum_{\sigma\sigma'=1}^{N_c} \sum_{j=1}^{N_f} \int \mathrm{d}\mathbf{k} \int \mathrm{d}\mathbf{q} \; \psi^{\dagger}_{N,\sigma',j}(\mathbf{k}+\mathbf{q}) \Phi_{\sigma'\sigma}(\mathbf{q}) \psi_{\bar{N},\sigma,j}(\mathbf{k})$$

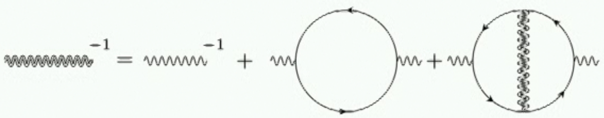
- g : coupling between electrons and spin fluctuations

- The Yukawa coupling is relevant and the theory flows to a strong coupling regime at low energies

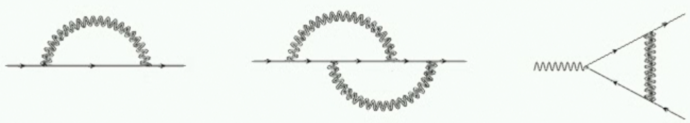
Solution of the patch theory in the small v limit

[Schlief, Lunts, SL (16)]

- In the limit that the nesting angle is small, all leading quantum corrections can be summed through the Schwinger-Dyson equation :



- Sub-leading corrections can be included order by order in the nesting angle. They make the nesting angle logarithmically flow to zero at low energies



  - the small nesting angle expansion becomes asymptotically exact in the low-energy limit

- Within the patch theory, the theory flows to an infrared fixed point with zero nesting angle

Limitation of the patch theory

- Four-fermion coupling, which has a negative scaling dimension under the patch scaling and thus not included in the patch theory, gives rise to strong interpatch scatterings that lead to superconducting instabilities
  - **Negative scaling dimension ≠ irrelevant**

Beyond the patch theory

- The full low-energy effective theory must include all gapless modes
  - coupling constants are promoted to functions of momentum

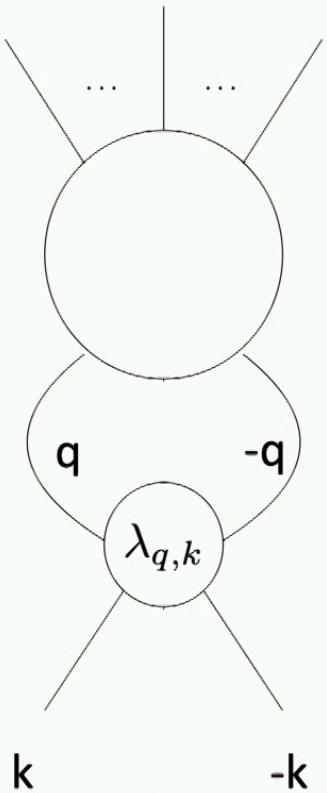
- A functional renormalization group is needed for the momentum dependent coupling functions [Polchinski(84), Wetterich(93)]. However, the RG equation for the full vertex function is hard to solve as it retains non-universal UV information [Metzner (12);..][J. Lee, Strack, Sachdev (13)]

- Wanted : **a framework of functional RG that only keeps the universal low-energy data near FS**

Field Theoretic Functional RG

- **Renormalizable theory**
  - It describes the IR physics of a universality class in the minimal way
  - It includes the minimal set of coupling functions in terms of which all low-energy observables at energy μ can be expressed within errors $(\mu/\Lambda)^a$, a>0
- In the presence of infinitely many gapless modes, a renormalizable theory can include couplings that have negative scaling dimensions
  - The 4-fermion coupling, which has dimension -1, should be included within the renormalizable theory because it gives rise to IR singularities

Superficially irrelevant couplings can give rise to IR divergences due to Fermi momentum



$$\int_{-k_F}^{k_F} dq$$

- Fermions can stay close to the Fermi surface irrespective of the momentum along the Fermi surface in the pairing channel
- The phase space of the low-energy fermion is controlled by k_F
- The phase space associated with momentum along FS promotes the four-fermion into a marginal coupling in the pairing channel

$$\left[\int_{-k_F}^{k_F} dq\ \lambda_{q,k}\right] = 0$$

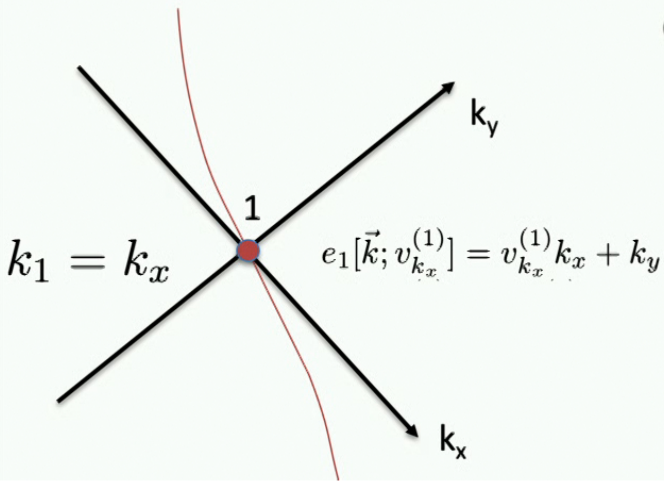
The full low-energy effective theory

$$S = \sum_{N,\sigma,j} \int \mathrm{d}\mathbf{k}\ \psi^{\dagger}_{N,\sigma,j}(\mathbf{k}) \left\{ ik_0 + V^{(N)}_{F,k_N} e_N[\vec{k}, v^{(N)}_{k_N}] \right\} \psi_{N,\sigma,j}(\mathbf{k})$$

$$+ \frac{1}{\sqrt{N_f}} \sum_{N,\sigma,j} \int \mathrm{d}\mathbf{k}\mathrm{d}\mathbf{q}\ g^{(N)}_{k_N+q_N,k_N} \psi^{\dagger}_{N,\sigma',j}(\mathbf{k}+\mathbf{q}) \Phi_{\sigma'\sigma}(\mathbf{q}) \psi_{\bar{N},\sigma,j}(\mathbf{k})$$

$$+ \frac{1}{4\mu} \sum_{N,\sigma,j} \int \prod_{i=1}^{4} \mathrm{d}\mathbf{k}_i\ \ \delta_{1+2,3+4} \lambda^{\begin{pmatrix} N_1 & N_2 \\ N_4 & N_3 \end{pmatrix};\begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_4 & \sigma_3 \end{pmatrix}}_{\begin{pmatrix} k_{1;N_1} & k_{2;N_2} \\ k_{4,N_4} & k_{3;N_3} \end{pmatrix}} \psi^{\dagger}_{N_1,\sigma_1,j_1}(\mathbf{k}_1) \psi^{\dagger}_{N_2,\sigma_2,j_2}(\mathbf{k}_2) \psi_{N_3,\sigma_3,j_2}(\mathbf{k}_3) \psi_{N_4,\sigma_4,j_1}(\mathbf{k}_4)$$

- The boson kinetic term is not included because it is irrelevant at low energies



Coupling functions :

$v^{(N)}_{k_N}$: FS shape

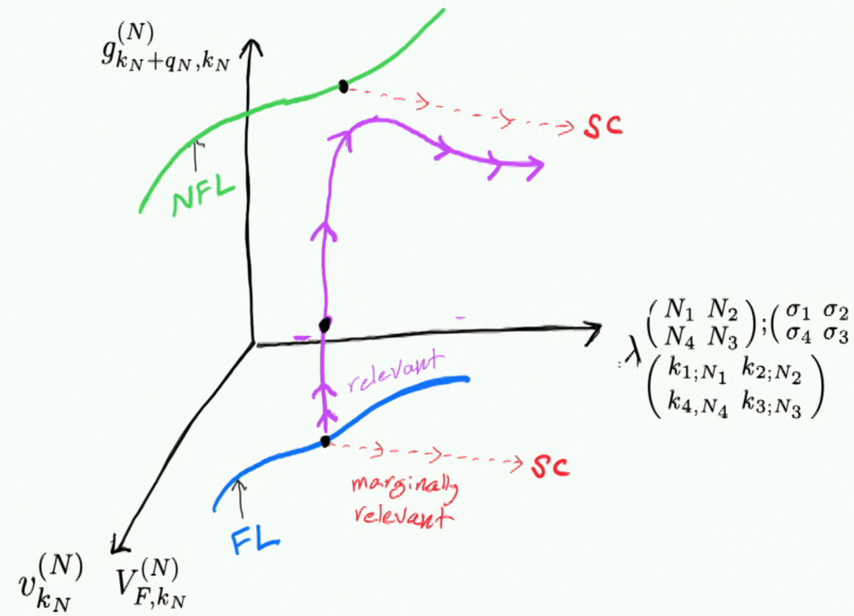
$V^{(N)}_{F,k_N}$: Fermi velocity

$g^{(N)}_{k_N+q_N,k_N}$: Yukawa coupling function

$\lambda^{\begin{pmatrix} N_1 & N_2 \\ N_4 & N_3 \end{pmatrix};\begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_4 & \sigma_3 \end{pmatrix}}_{\begin{pmatrix} k_{1;N_1} & k_{2;N_2} \\ k_{4,N_4} & k_{3;N_3} \end{pmatrix}}$: 4-fermion coupling function

Goals

- To characterize the space of low-energy fixed points in the space of coupling functions
- Understand universal low-energy physics of
  - the normal state
  - superconductivity



The full low-energy effective theory

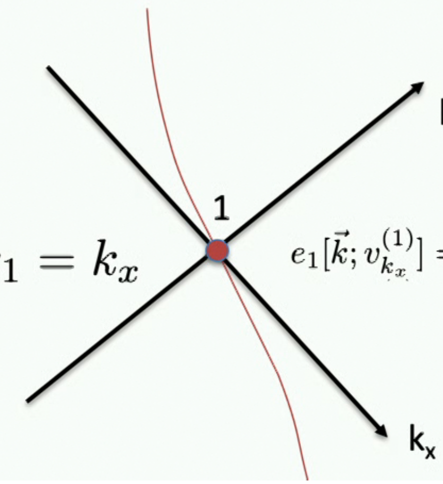
$$S=\sum_{N,\sigma,j}\int \mathrm{d}\mathbf{k}\,\psi^\dagger_{N,\sigma,j}(\mathbf{k})\left\{ik_0+V^{(N)}_{F,k_N}e_N[\vec{k},v^{(N)}_{k_N}]\right\}\psi_{N,\sigma,j}(\mathbf{k})$$

$$+\frac{1}{\sqrt{N_f}}\sum_{N,\sigma,j}\int \mathrm{d}\mathbf{k}\mathrm{d}\mathbf{q}\; g^{(N)}_{k_N+q_N,k_N}\psi^\dagger_{N,\sigma',j}(\mathbf{k}+\mathbf{q})\Phi_{\sigma'\sigma}(\mathbf{q})\psi_{\bar{N},\sigma,j}(\mathbf{k})$$

$$+\frac{1}{4\mu}\sum_{N,\sigma,j}\int\prod_{i=1}^{4}\mathrm{d}\mathbf{k}_i\;\;\delta_{1+2,3+4}\lambda^{\begin{pmatrix}N_1 & N_2\\ N_4 & N_3\end{pmatrix};\begin{pmatrix}\sigma_1 & \sigma_2\\ \sigma_4 & \sigma_3\end{pmatrix}}_{\begin{pmatrix}k_{1;N_1} & k_{2;N_2}\\ k_{4,N_4} & k_{3;N_3}\end{pmatrix}}\psi^\dagger_{N_1,\sigma_1,j_1}(\mathbf{k}_1)\psi^\dagger_{N_2,\sigma_2,j_2}(\mathbf{k}_2)\psi_{N_3,\sigma_3,j_2}(\mathbf{k}_3)\psi_{N_4,\sigma_4,j_1}(\mathbf{k}_4)$$

- The boson kinetic term is not included because it is irrelevant at low energies

$k_1 = k_x$



$e_1[\vec{k};v^{(1)}_{k_x}]=v^{(1)}_{k_x}k_x+k_y$

Coupling functions :

$v^{(N)}_{k_N}$: FS shape

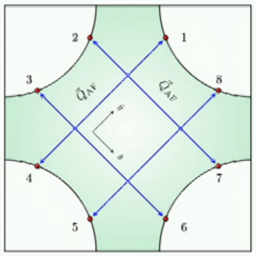
$V^{(N)}_{F,k_N}$: Fermi velocity

$g^{(N)}_{k_N+q_N,k_N}$: Yukawa coupling function

$\lambda^{\begin{pmatrix}N_1 & N_2\\ N_4 & N_3\end{pmatrix};\begin{pmatrix}\sigma_1 & \sigma_2\\ \sigma_4 & \sigma_3\end{pmatrix}}_{\begin{pmatrix}k_{1;N_1} & k_{2;N_2}\\ k_{4,N_4} & k_{3;N_3}\end{pmatrix}}$: 4-fermion coupling function

Beta functionals for the 4-fermion coupling functions

$$\frac{d}{d\ell}\hat{\lambda}^{\binom{1\ 5}{1\ 5};\binom{\sigma_1\ \sigma_2}{\sigma_4\ \sigma_3}}_{\binom{P\ -P}{K\ -K}} = -\left(1+3(z-1)+2\hat{\eta}^{(\psi)}_P+2\hat{\eta}^{(\psi)}_K+K\frac{\partial}{\partial K}+P\frac{\partial}{\partial P}\right)\hat{\lambda}^{\binom{1\ 5}{1\ 5};\binom{\sigma_1\ \sigma_2}{\sigma_4\ \sigma_3}}_{\binom{P\ -P}{K\ -K}}$$



$$-\int\frac{dQ}{2\pi\Lambda\hat{V}_{F,Q}}\Bigg\{-\frac{\mathsf{D}_\Lambda(P;Q)}{2\pi N_f}\mathsf{T}^{\sigma_1\sigma_2}_{\alpha\beta}\lambda^{\binom{4\ 8}{1\ 5};\binom{\alpha\ \ \beta}{\sigma_4\ \sigma_3}}_{\binom{Q\ -Q}{K\ -K}}-\frac{\mathsf{D}_\Lambda(Q;K)}{2\pi N_f}\lambda^{\binom{1\ 5}{4\ 8};\binom{\sigma_1\ \sigma_2}{\alpha\ \ \beta}}_{\binom{P\ -P}{Q\ -Q}}\mathsf{T}^{\alpha\beta}_{\sigma_4\sigma_3}$$

$$+\frac{1}{\pi N_f^2}\mathsf{T}^{\sigma_1\sigma_2}_{\alpha\beta}\mathsf{T}^{\alpha\beta}_{\sigma_4\sigma_3}\mathsf{D}_\Lambda(P;Q)\mathsf{D}_\mu(Q;K)$$

$$+\frac{1}{4\pi}\left(\lambda^{\binom{1\ 5}{1\ 5};\binom{\sigma_1\ \sigma_2}{\beta\ \ \alpha}}_{\binom{P\ -P}{Q\ -Q}}\lambda^{\binom{1\ 5}{1\ 5};\binom{\beta\ \ \alpha}{\sigma_4\ \sigma_3}}_{\binom{Q\ -Q}{K\ -K}}+\lambda^{\binom{1\ 5}{4\ 8};\binom{\sigma_1\ \sigma_2}{\beta\ \ \alpha}}_{\binom{P\ -P}{Q\ -Q}}\lambda^{\binom{4\ 8}{1\ 5};\binom{\beta\ \ \alpha}{\sigma_4\ \sigma_3}}_{\binom{Q\ -Q}{K\ -K}}\right)\Bigg\}.$$

z : dynamical scaling dimension

$$\hat{\eta}^{(\psi)}_K=\frac{N_c^2-1}{2\pi^2N_cN_f}\frac{\hat{g}_K^2}{c\hat{V}_{F,K}}\frac{\Lambda}{\Lambda+2c\hat{v}_K|K|_\Lambda}-(z-1)$$

: momentum-dependent anomalous dimension of electron

$$\mathsf{D}_\Lambda(Q;K)=\hat{g}^2_{Q,K}\frac{\Lambda}{\Lambda+c\left(|Q-K|_\Lambda+|\hat{v}_QQ+\hat{v}_KK|_\Lambda\right)}$$

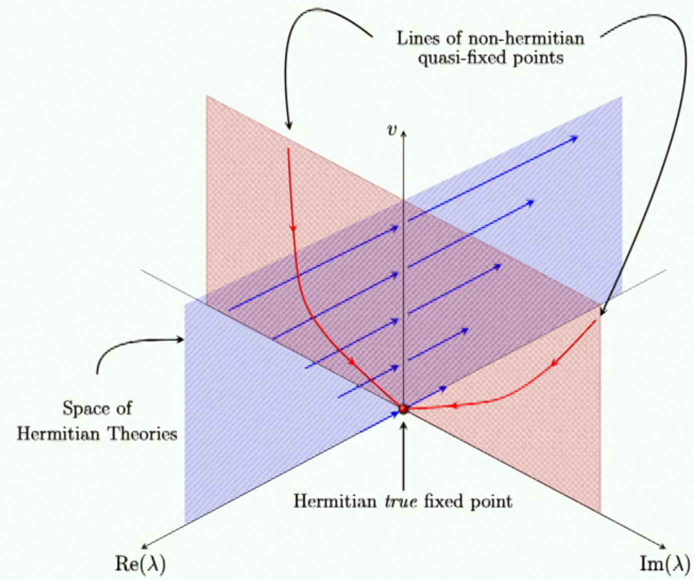
: momentum dependent mixing matrix $\left(|K|_\Lambda=\sqrt{K^2+\Lambda^2}\right)$

$$\mathsf{T}^{\sigma_1\sigma_2}_{\sigma_4\sigma_3}=2\left(\delta_{\sigma_1\sigma_3}\delta_{\sigma_2\sigma_4}-\frac{1}{N_c}\delta_{\sigma_1\sigma_4}\delta_{\sigma_2\sigma_3}\right)$$

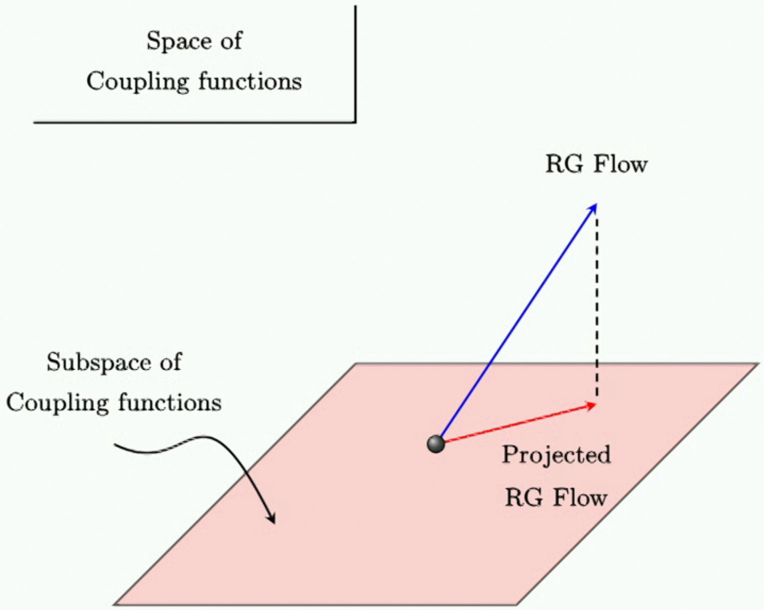
Summary of the functional RG flow

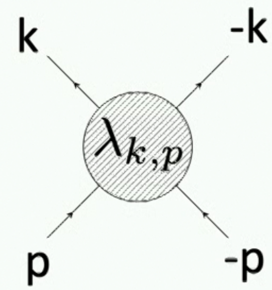
- Interacting fixed point at $v_q = 0$ with $\frac{g_{kp}^2}{v_q} = \frac{\pi}{2},\ \lambda = 0$ (pairing channel), z=1, $\eta_\phi = 1$ (exact)

- Away from the fixed point, v logarithmically flows toward zero at low energies. However, the 4-fermion coupling generated from the Yukawa coupling flows faster than v. Therefore, one can understand the RG flow within each `plane' with fixed v.
  - For a fixed v, there exist quasi-fixed points which are non-Hermitian away from the true fixed point
  - Hermitian theories with $v \neq 0$ inevitably flows to a superconducting state at low energies



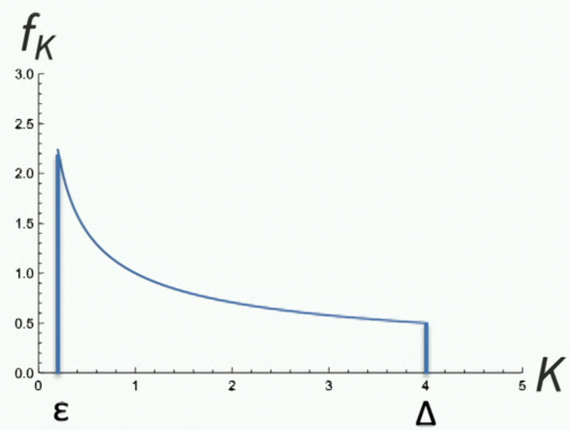
Projected RG flow



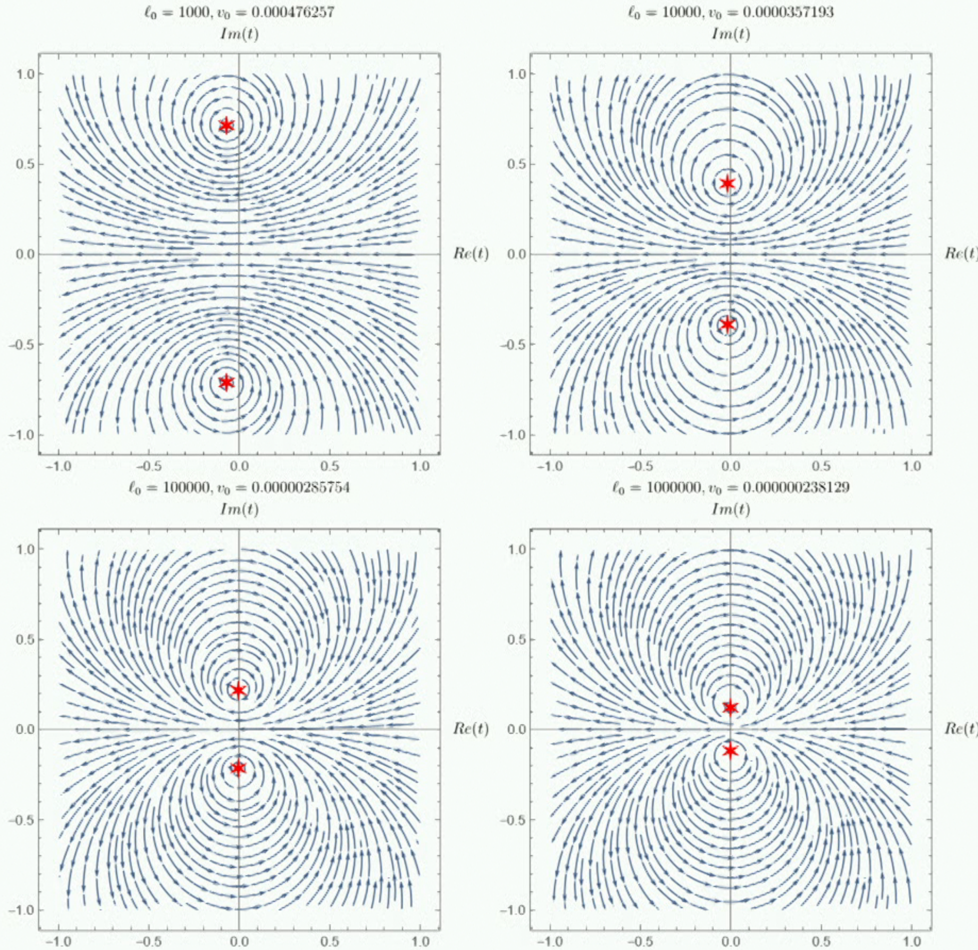


$$\lambda_{KP}(\ell) = f_K f_P^*$$

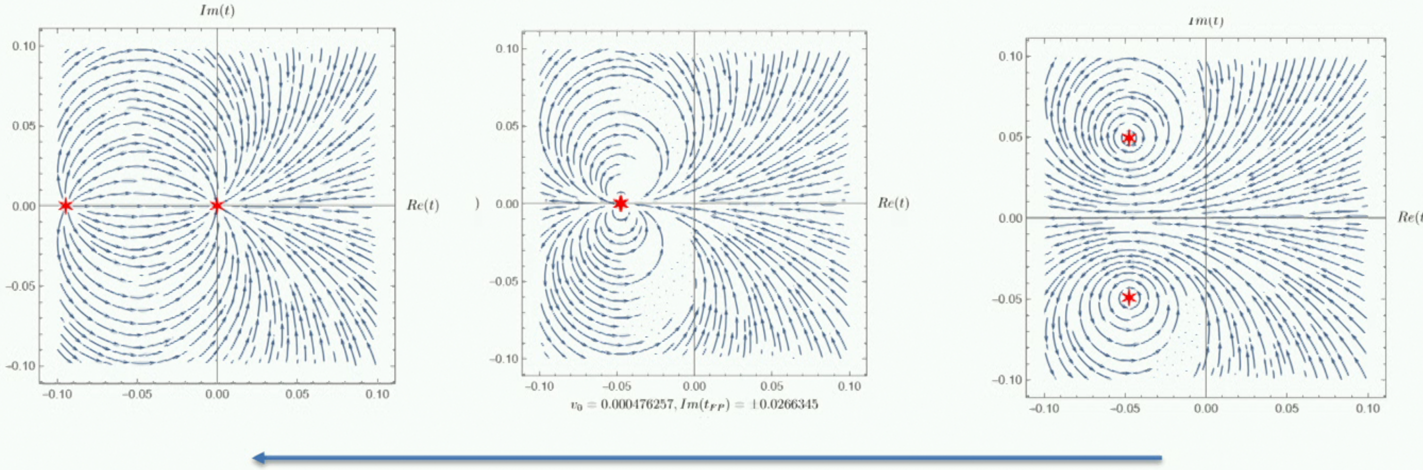
$$f_K = \sqrt{\frac{\Lambda}{|K|}}\Theta(|K| - \varepsilon)\Theta(\Delta - |K|)$$



Non-Hermitian fixed points of the projected RG flow at $v \neq 0$



Collision of fixed points of the projected RG flow



increasing weight of the hot electrons in the pairing channel

[Kaplan, Lee, Son, Stephanov (09)] [Veytsman (93); Glazek and Wilson (02)]

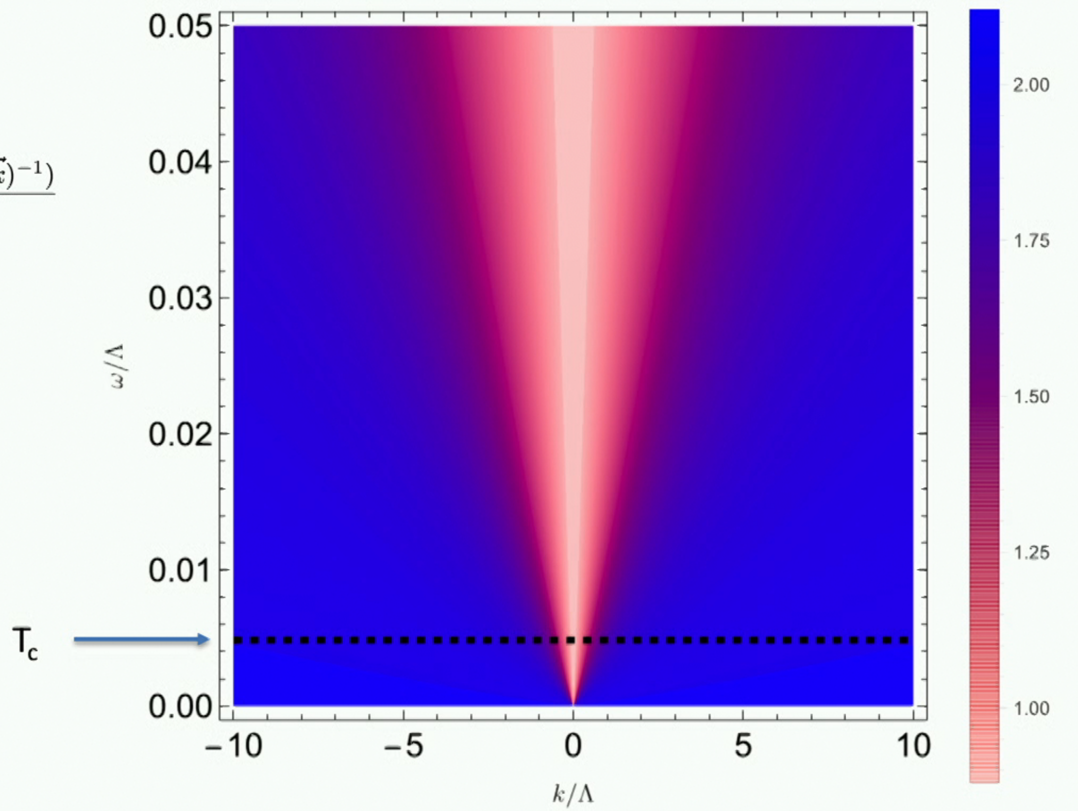
Superconductivity

- Theories whose bare interaction is weaker than the interaction mediated by the spin fluctuations stay near the bottleneck for a large scale which becomes large in limit that the nesting angle becomes small
  - SC instabilities at $T_c \sim \Lambda\, e^{-\frac{a}{\sqrt{v \log 1/v}}}$
  - Spin singlet d-wave channel with a quasi-universal Cooper pair wavefunction

Normal state (spectral function)

$ImG(\omega, k)^{-1} \sim \omega^{\alpha}$

$$\frac{\partial \log \mathrm{Im}(G_{\mathrm{R}}(\omega, \vec{k})^{-1})}{\partial \log \omega}$$



Summary

- Field theoretic functional RG for effective field theories for the entire Fermi surface
  - Due to Fermi momentum, the usual notion of renormalizable theory should be modified
  - The 4-fermion coupling, `irrelevant' by power-counting, can be actually relevant, invalidating the patch description at low energies
- Antiferromagnetic critical metal in 2+1D
  - Momentum dependent critical properties
  - Universal pathway from NFL to Superconductivity