Title: Z2 spin liquids in spin-S Kitaev honeycomb model via parton construction

Speakers: Han Ma

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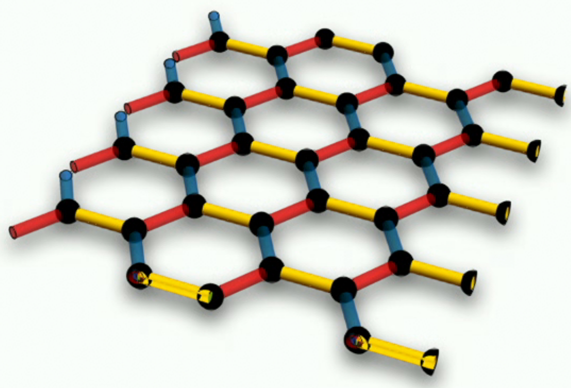
Abstract: Unlike the half spin Kitaev honeycomb model which can be solved by an exact parton construction, the higher spin analogue of it is not solvable and it is still controversial if it exhibits a quantum spin liquid phase. In this talk, I will present a generalized parton construction where each spin-S is represented by 8S Majorana fermions. This framework naturally leads to a Z2 spin liquid when S is a half integer and gives a trivial ground state when S is an integer. Particularly, in the Z2 spin liquid, the Z2 charge is carried by a product of 2S Majorana fermions. In the anisotropic limit, say the interaction on the z bond is much stronger than others, the charges are gapped and the higher spin Kitaev model is reduced to a Wen-plaquette model exhibiting Z2 topological order. However, it is expected that at certain interaction strength on the x,y,z bond, the charges become gapless which results in a gapless Z2 spin liquid.

$\mathbb{Z}_2$ spin liquid in spin-S Kitaev honeycomb model via parton construction



Han Ma
Perimeter Institute

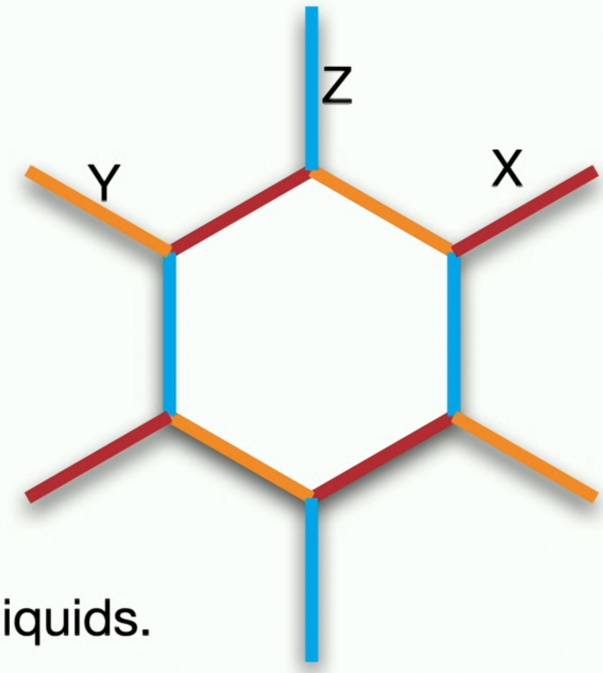
PERIMETER INSTITUTE

Quantum matter workshop Nov.14

Kitaev honeycomb model

$$H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu}$$

Kitaev (2006)



- We study the higher spin version of it for possible spin liquids.
- Motivation I: It has spin liquid phases.
- Motivation II: Candidate materials have been proposed.

Baskaran, Sen, Shankar (2008)
Lee, Kawashima, Kim (2020)
Lee, Suzuki, Kim, Kawashima (2021)
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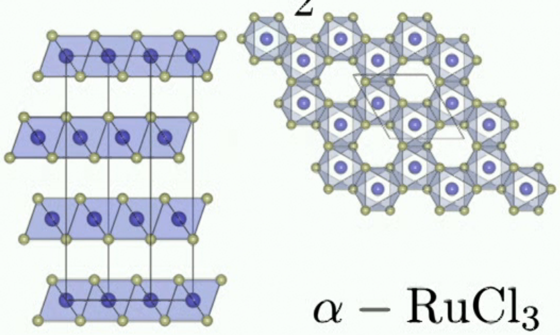
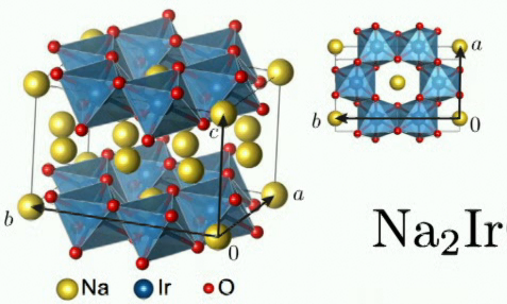
Candidate Kitaev Materials

Jackli, Khaliullin (2009)
Trebst, Hickey (2022)

$S = \frac{1}{2}$

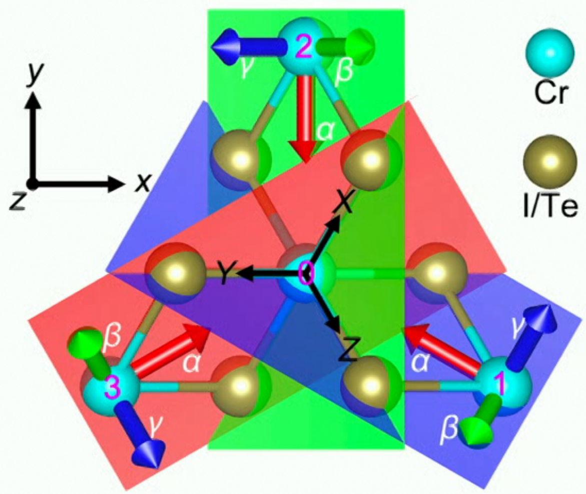


$\alpha - \mathrm{RuCl_3}$



$\mathrm{Na_2IrO_3}$

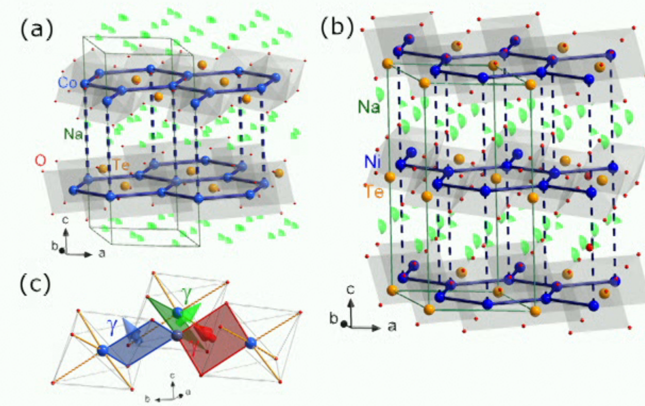
honeycomb iridates



$S = \frac{3}{2}$

$\mathrm{CrI_3}$
$\mathrm{CrGeTe_3}$

Xu, Feng, Xiang, Bellaiche (2018)
Stavropoulos, Pereira, and Kee (2019)
Samarakoon, Chen, Zhou, Garlea (2021)



$S = 1$

$\mathrm{Na_2Ni_2TeO_6}$

Kitaev honeycomb model

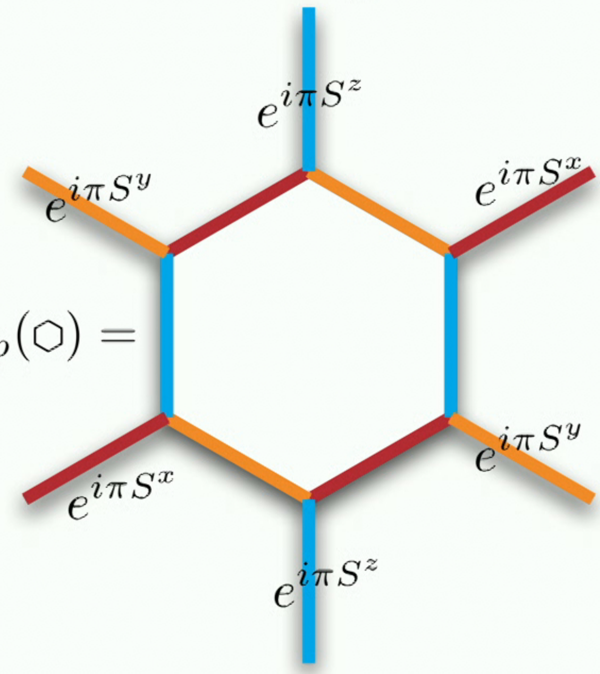
$$H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu}$$

- Motivation III:
  there are extensive local conserved fluxes.

$$S \geq \frac{1}{2}$$

1. The model of spin-1/2 is solvable
2. Higher spin model is ***not*** solvable
   due to larger Hilbert space.
3. $W_p(\hexagon)$ is $\mathbb{Z}_2$ gauge flux in spin-1/2 model
4. Can we understand the $W_p(\hexagon)$ the same way
   for higher spin model?

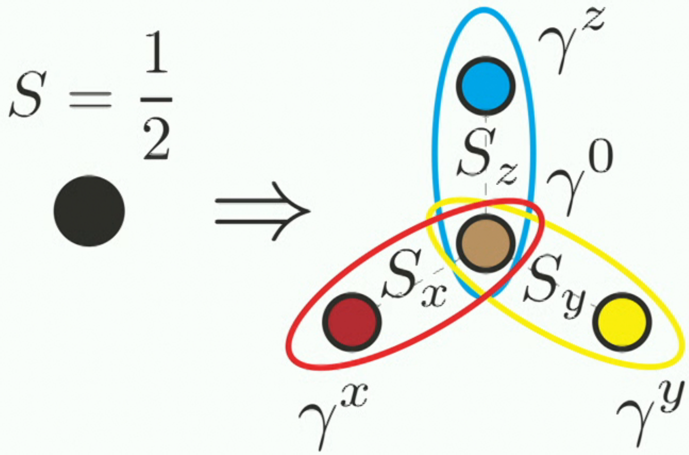
$W_p(\hexagon) =$



Spin-1/2 Kitaev model

Review the spin-1/2 Kitaev model and its flux operators

Parton construction — spin-1/2



$$\gamma^\alpha \gamma^\beta + \gamma^\beta \gamma^\alpha = 2\delta_{\alpha\beta} \qquad \alpha, \beta = 0, \mu$$

$$2S^\mu = i\gamma^0 \gamma^\mu \qquad \mu = x, y, z$$

Local constraint: $\gamma^0 \gamma^x \gamma^y \gamma^z = 1$

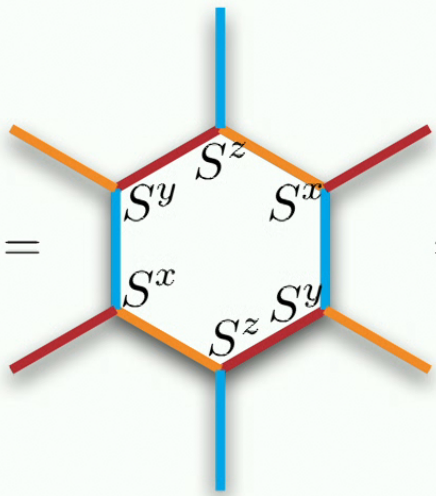
$\mathbb{Z}_2$ gauge redundancy: $\gamma^{0,\mu} \to -\gamma^{0,\mu}$

Parton construction — spin-1/2

- Hamiltonian: $H = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} S_i^{\mu} S_j^{\mu} = -\sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} \gamma_i^0 \gamma_i^{\mu} \gamma_j^{\mu} \gamma_j^0$

- $\mathbb{Z}_2$ gauge field: $u_{ij}^{\mu} = i\gamma_i^{\mu}\gamma_j^{\mu}$ commutes with Hamiltonian

- $\mathbb{Z}_2$ Flux operator: $W_p(\hexagon) =$



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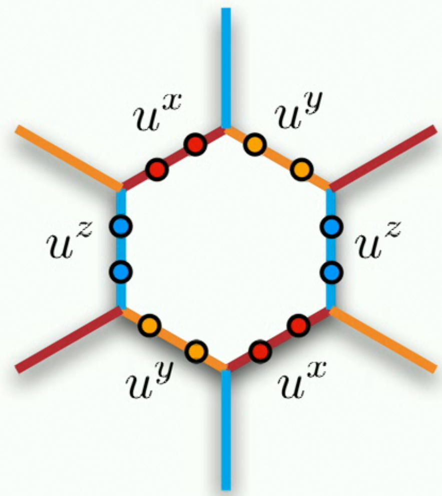
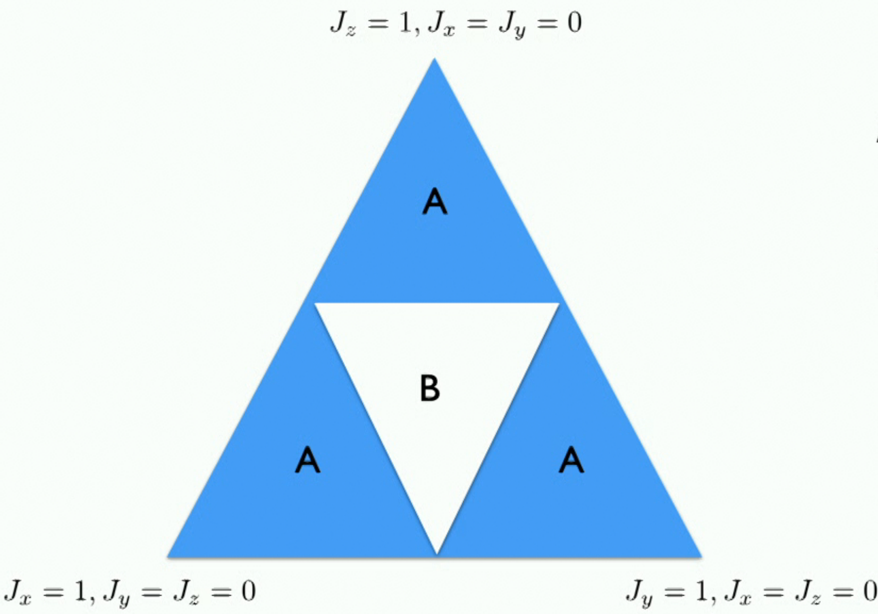


Phase diagram of spin-1/2 Kitaev model

$$H = i \sum_{\mu} J_{\mu} \sum_{\langle ij \rangle \in \mu} \gamma_i^0 u_{ij}^{\mu} \gamma_j^0$$



Phases are determined by the physics of γ^0

A: Majorana fermion γ^0 is gapped.
This phase is a $\mathbb{Z}_2$ topological order.

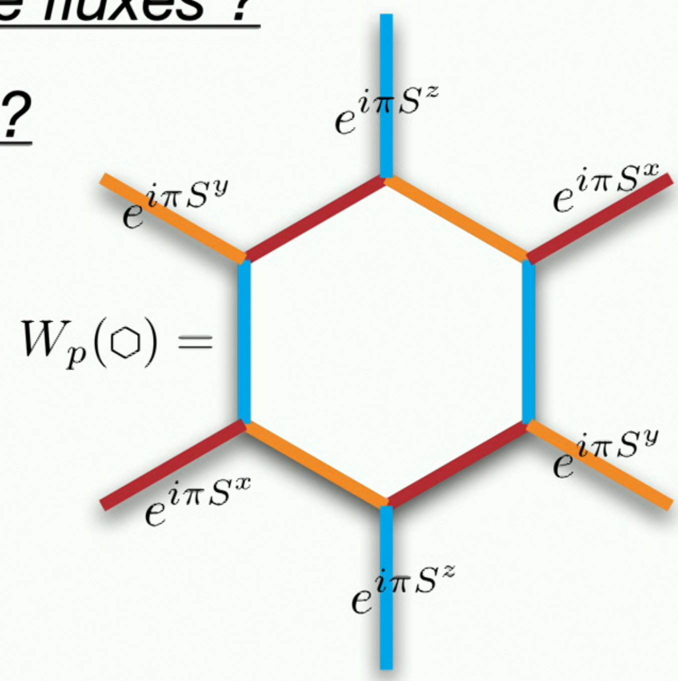
B: Majorana fermion γ^0 is gapless and is coupled to a $\mathbb{Z}_2$ gauge field.

Spin-1/2 -> Higher spin Kitaev model

Are the conserved quantities still $\mathbb{Z}_2$ gauge fluxes ?

Can we see this from Parton construction?

What are the possible phases?



Parton construction — higher spin

$$\gamma_a^\alpha \gamma_b^\beta + \gamma_b^\beta \gamma_a^\alpha = 2\delta_{\alpha\beta}\delta_{ab} \qquad \begin{array}{c} \alpha,\beta = 0,\mu \\ a,b = 1,\ldots,2S \end{array}$$

$$2S^\mu = i\sum_{a=1}^{2S}(\gamma_a^0\gamma_a^\mu) = i\left[\gamma_1^0, \gamma_2^0, \ldots, \gamma_{2S}^0\right] \mathbb{1} \begin{bmatrix} \gamma_1^\mu \\ \gamma_2^\mu \\ \vdots \\ \gamma_{2S}^\mu \end{bmatrix}$$

O(2S) gauge redundancy: $U^T U = \mathbb{1}$

Local constraints:

$$\sum_\mu \Big(\sum_{a=1}^{2S} \gamma_a^0 \gamma_a^\mu\Big)^2 = S(S+1)$$

$$\gamma_a^0 \gamma_a^x \gamma_a^y \gamma_a^z = 1$$

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1. $S^\mu = \sum_{a=1}^{2S} S_a^\mu$ — Local constraint: $\sum_\mu \Big(\sum_{a=1}^{2S} S_a^\mu\Big)^2 = S(S+1)$

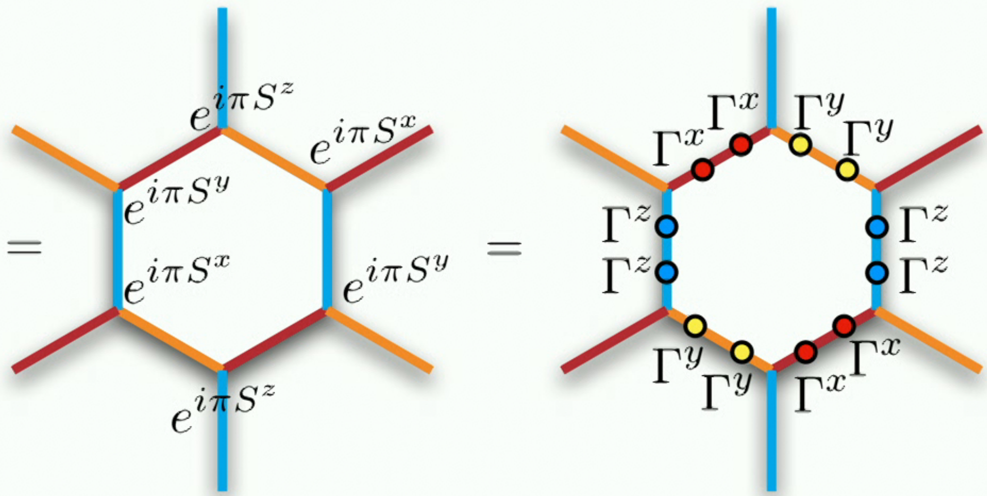
2. $2S_a^\mu = i\gamma_a^0\gamma_a^\mu$ — Local constraints: $\gamma_a^0 \gamma_a^x \gamma_a^y \gamma_a^z = 1$

Giant Parton

$$\Gamma^{0,\mu} = (-1)^{\frac{S(2S-1)}{2}} \prod_{a=1}^{2S} \gamma_a^{0,\mu} = \frac{(-1)^{\frac{S(2S-1)}{2}}}{(2S)!} \epsilon_{a_1,a_2,\ldots,a_{2S}} \gamma_{a_1}^{0,\mu} \gamma_{a_2}^{0,\mu} \cdots \gamma_{a_{2S}}^{0,\mu}$$

$$(\Gamma^{0,\mu})^2 = 1$$

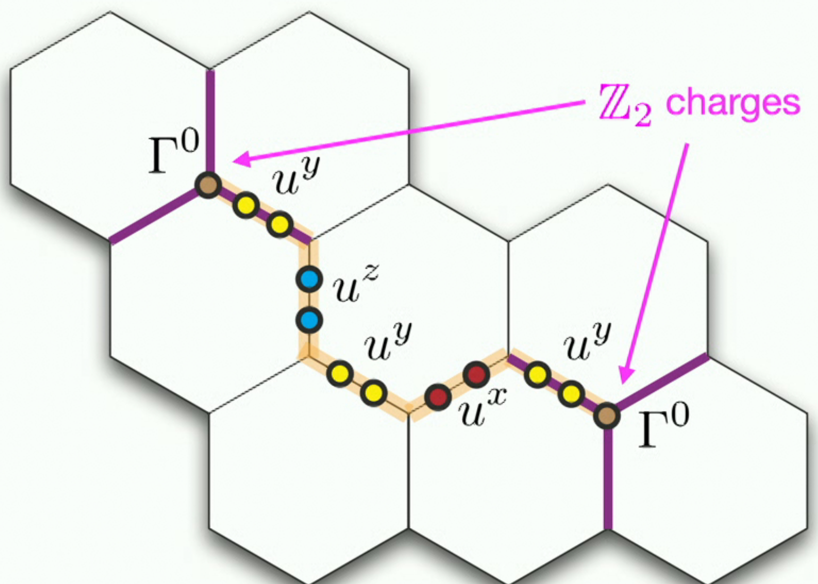
- $\mathbb{Z}_2$ Flux operator: $W_p(\hexagon) =$

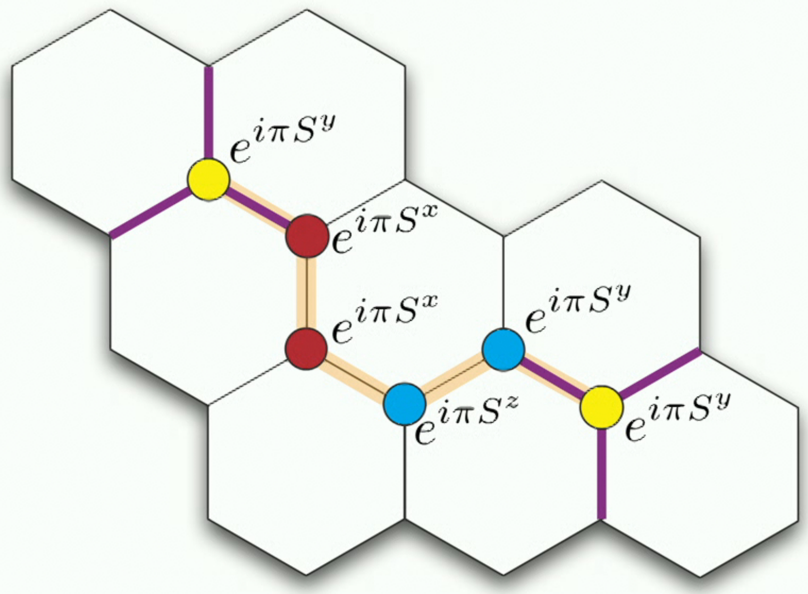


- $\mathbb{Z}_2$ gauge field: $u_{ij}^{\mu} = \Gamma_i^{\mu} \Gamma_j^{\mu}$ commutes with Hamiltonian

Giant string operator

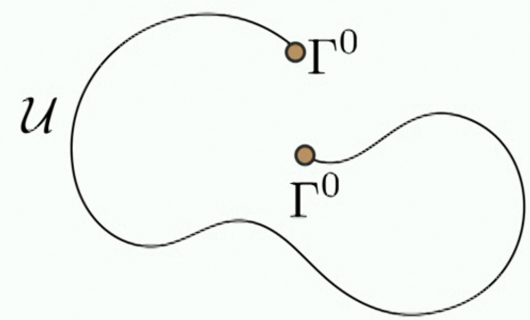
- The $\mathbb{Z}_2$ charge is attached to a tensionless string operator.
- The string operator commutes with Hamiltonian except the two end points.





Giant string operator

- The tensionless string operator $\mathcal{U}$ indicates that the charges Γ^0 are either deconfined or condensed.



- condensed charges $H(\mathcal{U}|\mathrm{gs}\rangle) = E_0|\mathrm{gs}\rangle$

- deconfined charges $H(\mathcal{U}|\mathrm{gs}\rangle) = (E_0 + 2\delta E)|\mathrm{gs} + 2\Gamma^0\rangle$

$\delta E \sim \mathcal{O}(1)$ and is independent of the length of string

Giant charge

- $\mathbb{Z}_2$ gauge charge: $\Gamma^0 = \frac{(-1)^{\frac{S(2S-1)}{2}}}{(2S)!}\epsilon_{a_1,a_2,\ldots,a_{2S}}\gamma^0_{a_1}\gamma^0_{a_2}\cdots\gamma^0_{a_{2S}}$

  - It is a SO(2S) singlet.
  - It carries the charge of improper $\mathbb{Z}_2$ of O(2S).

  - It is a boson when S is an integer.
  - It is a fermion when S is a half-integer.

Fate of Giant charge

If the string operators $\mathcal{U}$ for charge Γ^0 are tensionless.

- Fermionic charges Γ^0

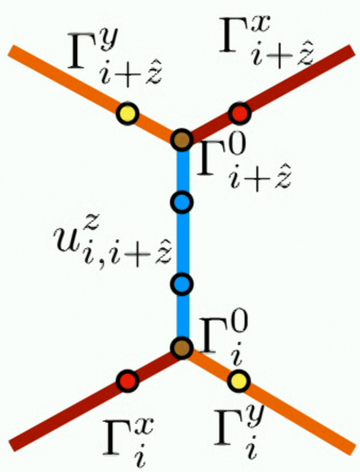
  They can only be deconfined.

- Bosonic charges Γ^0

  They can be condensed or deconfined depending on dynamics.

Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$



$$S^z_{i+\hat{z}} S^z_i = S^2$$

Extensive degeneracy

$$\Gamma^0_{i+\hat{z}} \Gamma^z_{i+\hat{z}} \Gamma^z_i \Gamma^0_i = 1$$

$$\Gamma^y_{i+\hat{z}} \Gamma^x_{i+\hat{z}} \Gamma^x_i \Gamma^y_i = 1$$

Higher spin Kitaev model

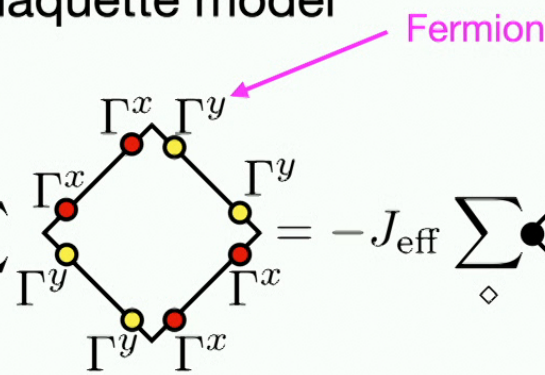
In the anisotropic limit $J_z \gg J_x \sim J_y$

Lee, Suzuki, Kim, Kawashima (2021)

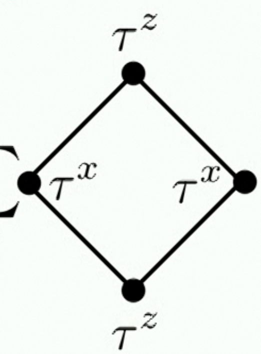
Half-integer spin model: Wen-plaquette model

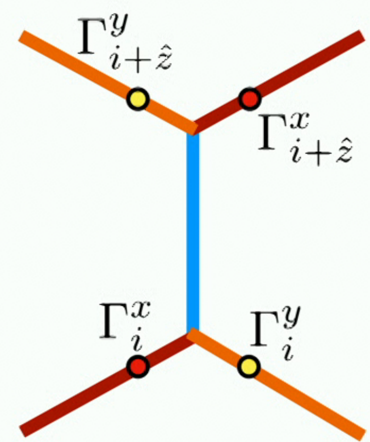
$\sim 8S\text{th}$

$$H_{\text{eff}}^{(2\mathbb{Z}+1)/2} = -J_{\text{eff}} \sum_{\diamond}$$



$$= -J_{\text{eff}} \sum_{\diamond}$$





The effective Hamiltonian is the same as the half spin model.

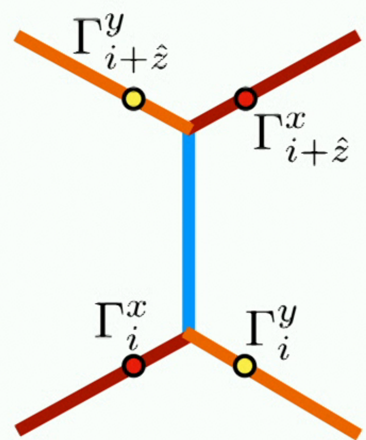
Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$

Lee, Suzuki, Kim, Kawashima (2021)

Half-integer spin model: Wen-plaquette model

Ground state is the $\mathbb{Z}_2$ topological order.



Excitations: $\Gamma_i^{0,x,y,z} \sim \varepsilon, e \times m$

$$i\Gamma_i^x \Gamma_i^y \sim e \times e$$

$$i\Gamma_i^x \Gamma_{i+\hat{z}}^y \sim m \times m$$

Higher spin Kitaev model

In the anisotropic limit $J_z \gg J_x \sim J_y$

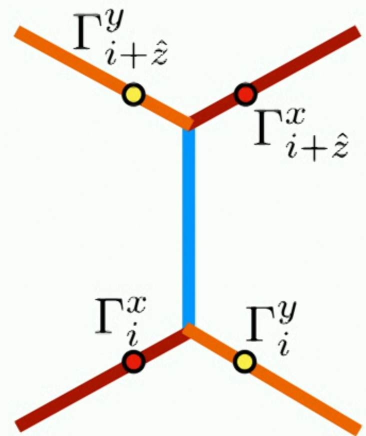
Lee, Suzuki, Kim, Kawashima (2021)

Integer spin model: trivial

Boson

$$\sim 4S\text{th}$$

$$H_{\text{eff}}^{\mathbb{Z}} = -J_{\text{eff}} \sum_i \Gamma_i^x \Gamma_{i+\hat{z}}^y = -J_{\text{eff}} \sum_{\ell \in z} \tau_\ell^x$$

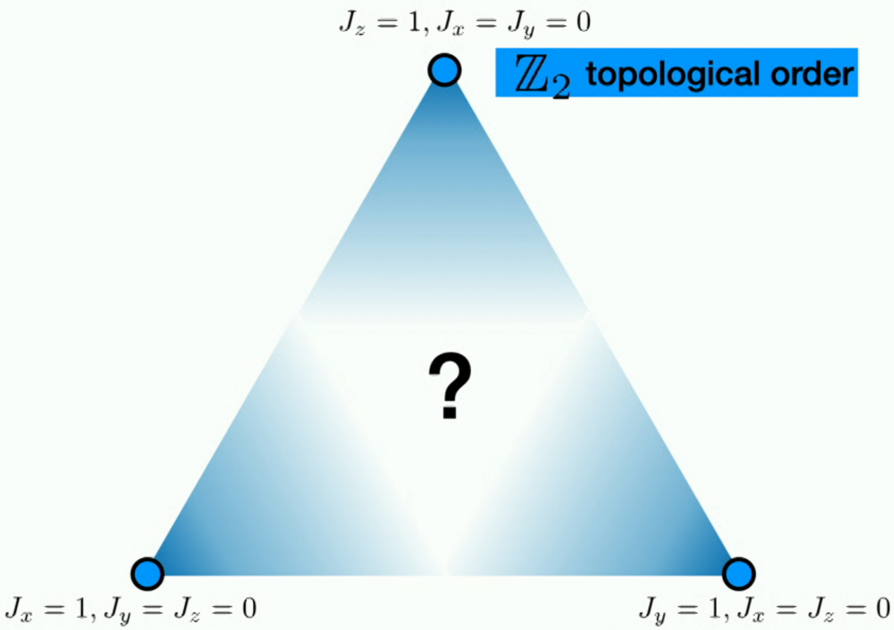


The effective Hamiltonian is trivial.

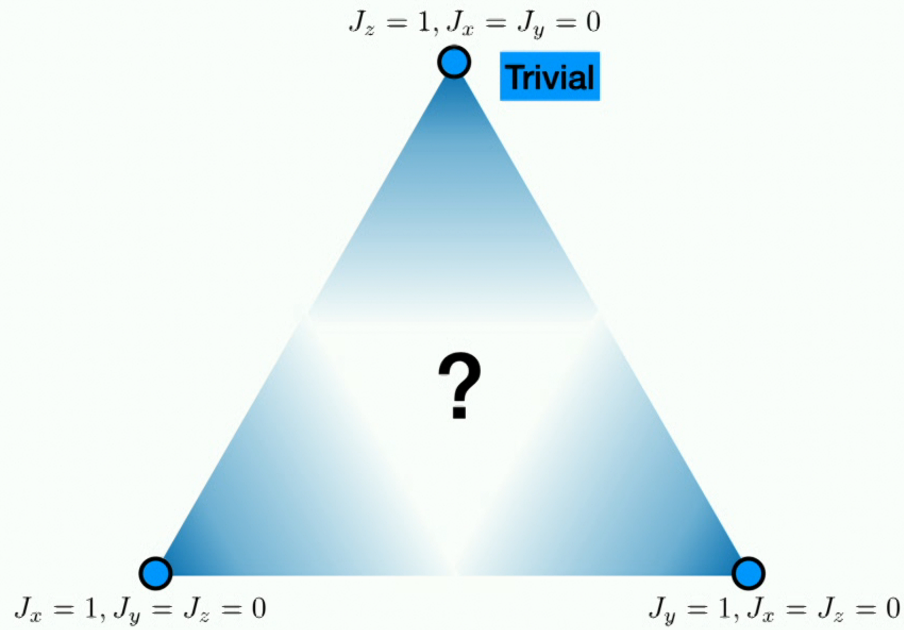
Ground state is the trivial paramagnet with $\Gamma_i^x = \Gamma_i^y = \Gamma_{i+\hat{z}}^x = \Gamma_{i+\hat{z}}^y$

Any string operator is 1. Since $H(\Gamma^0|\text{gs}\rangle) = E_0\ (\Gamma^0|\text{gs}\rangle)$, the boson Γ^0 is condensed.

Higher spin Kitaev model



Half-integer spin

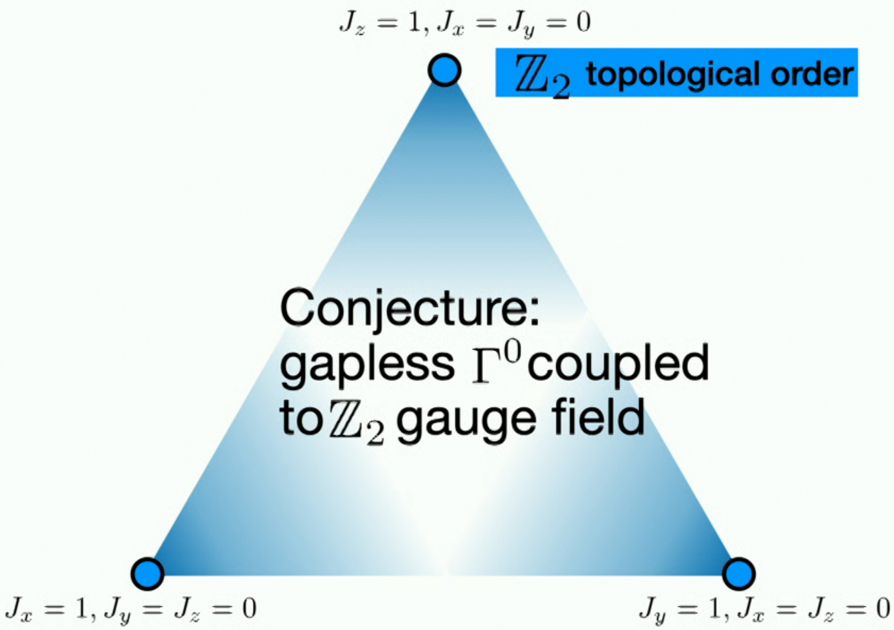


Integer spin

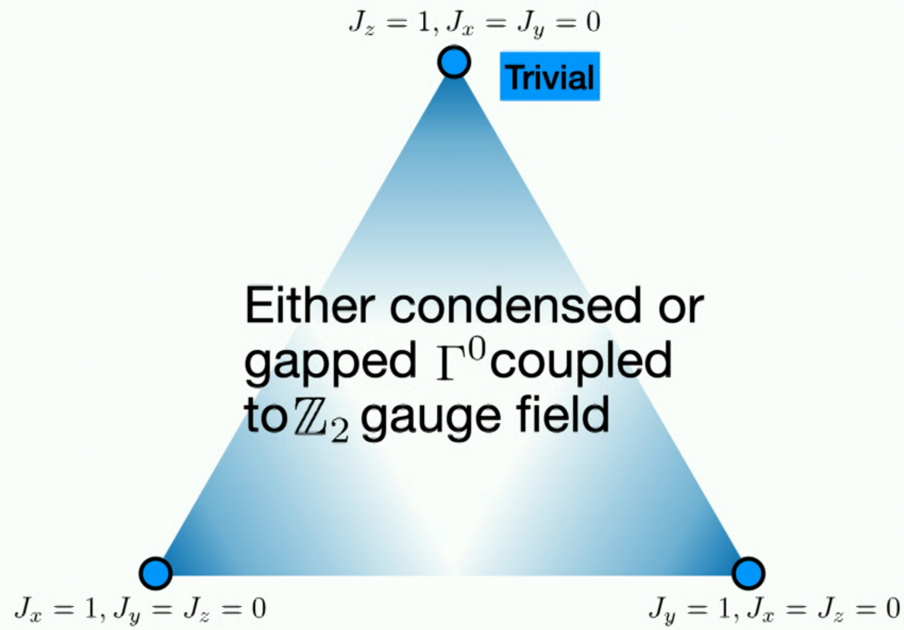
Higher spin Kitaev model

General claim for the isotropic model ?

Higher spin Kitaev model



Half-integer spin



Integer spin

Take home message

Half-integer spin Kitaev model

There always exist deconfined fermionic excitations coupled to $\mathbb{Z}_2$ gauge field.

Integer spin Kitaev model

Trivial state is possible.

Open questions

- Is the full SO(2S) confined?
- Are bosonic charges condensed in the isotropic model?
- Fundamental reason, e.g. anomaly, for this difference between integer and half-integer spin Kitaev models?
- How does the solvability depend on the number of local conserved quantities and the dimension of Hilbert space?
- …….

Summary

- In the higher spin Kitaev model, local Flux operators are Z2 fluxes of the giant Partons.
- The giant Parton is a Majorana fermion in the half-integer spin model. The system always has deconfined excitations and is expected to form a spin liquid phase.
- The giant parton is a boson in the integer spin model. The system can be trivial when the bosons condense.