Title: Geometric contribution to entanglement entropy and multipartite entanglement in two-dimensional chiral topological liquid

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Series: Quantum Matter

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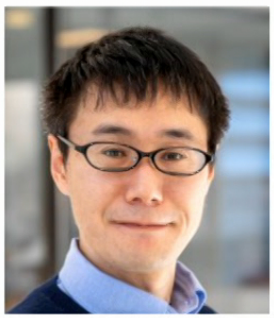
Abstract: The multipartite entanglement structure for the ground states of two dimensional topological phases is an interesting albeit not well understood question. Utilizing the bulk-boundary correspondence, the tripartite entanglement calculation of 2d topological phases can be reduced to that on the vertex state, defined by the boundary conditions at the interfaces between spatial regions. In this work, we use the conformal interface technique to calculate the entanglement measures of the vertex state, which include the area-law, geometrical and topological pieces, and the possible extra order one contribution. This explains our previous observation of Markov gap h=\frac{c}{3}\ln 2 in the 3-vertex state, and generalizes it to the p-vertex state as well as rational conformal field theory, and more general choices of subsystem. Finally, we support our prediction by numerical evidence.

Zoom link: https://pitp.zoom.us/j/93914854044?pwd=eWl3eGVLU25XUGhKbnFRSm5ab0JuUT09

Based on:

2110.14992 **YL**, Sohal, Kudler-Flam,Ryu
22**.***** **YL**, Kusuki, Sohal, Kudler-Flam,Ryu

Collaborators:



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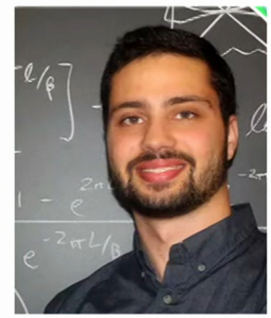
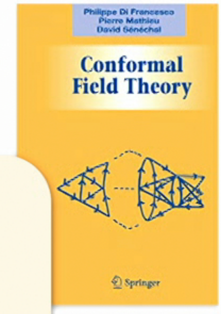


Jonah Kudler-Flam
(Chicago → IAS)

What Quantum Entanglement teaches us:

Tripartite entanglement?

Topological systems:
- Topological entanglement entropy
- Entanglement spectrum signature



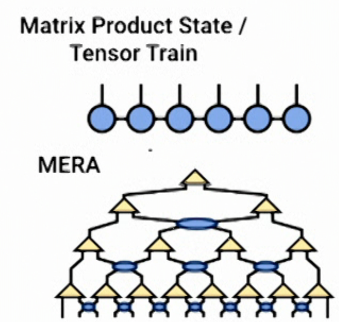
Holography

Quantum Entanglement

Phase transition
- Universal entanglement scaling

Entanglement transition:
- Measurement induced phase transition
- MBL to thermalization transition

Numerical methods:
- Tensor network



Little is known about tripartite entanglement…

Quantum Entanglement

Bipartite entanglement:

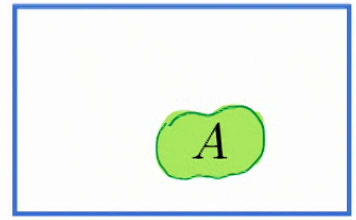
- Entanglement entropy

$$S = -\mathrm{tr}\rho_A \ln \rho_A$$

- Area-law: Number of Bell pairs
- Topological entanglement entropy

$$S_A = \alpha L - \gamma, \quad \gamma = \log \mathcal{D}$$

[Levin-Wen 08; Kitaev-Preskill 08]



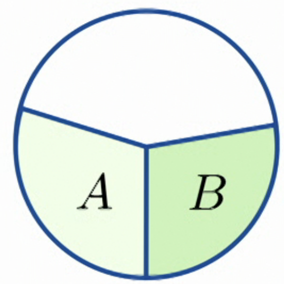
Bell pair

$|01\rangle + |10\rangle$

Tripartite entanglement:

?

- 3 qubits: GHZ, W
- beyond 3 qubits: ?
- Entanglement measures?
- Topological signature?

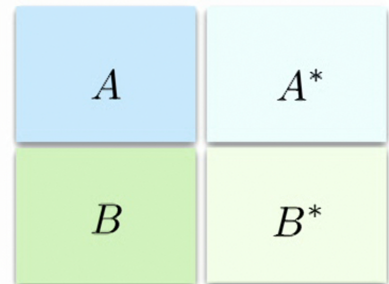


Other quantities:
mutual information
negativity, ...

Tripartite entanglement

- Reflected entropy

  - Definition: purify a reduced density matrix



$$\rho_{AB} = \sum_p \lambda_p |p\rangle\langle p| \quad \Rightarrow \quad |\sqrt{\rho_{AB}}\rangle\rangle = \sum_p \sqrt{\lambda_p}|p\rangle|p^*\rangle$$

like TFD state

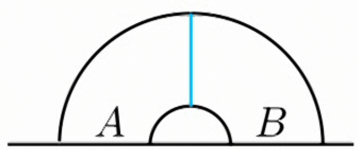
$$\rho_{AA^*} = \mathrm{tr}_{BB^*}|\sqrt{\rho_{AB}}\rangle\rangle\langle\langle\sqrt{\rho_{AB}}|$$

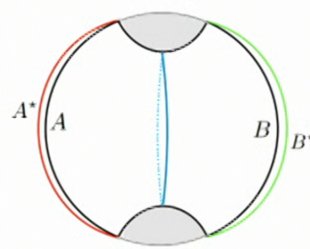
$$S_R(A:B) = S(\rho_{AA^*})$$

[Dutta-Faulkner 19]
[Akers-Rath 20]

  - Why reflected entropy?

    HEP: Holographic dual





Tripartite entanglement

- Why reflected entropy?

  CMT: Captures non-trivial tripartite entanglement

Markov gap $h = S_R(A:B) - I(A:B)$

$h = 0$:
- Bell pair
- GHZ state $|\mathrm{GHZ}\rangle = |000\rangle + |111\rangle$
- 1d Gapped GS
- Triangle state
- Stabilizer state
- $\dots$

1d critical GS: $h = \frac{c}{3} \ln 2$ ← Proven by CFT and replica trick

Holographic state: $h \sim \frac{1}{G_N}$

How about 2d?

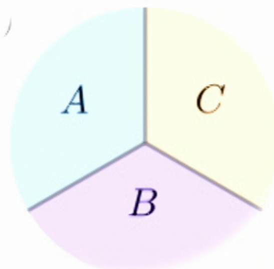
?

[Zou-Siva-Soejima-Mong-Zaletel 20]

Summary of results

- Setting: topologically ordered ground states in (2+1)d

- Tripartition



- Key result: Markov gap

$$h = \frac{c}{3} \ln 2$$

When:

(1) Deep inside the topological phase (large gap limit)

(2) Chiral phase

*[**YL**, Sohal, Kudler-Flam, Ryu 22]*
*[**YL**, Kusuki, Sohal, Kudler-Flam, Ryu 22 (to appear)]*

Importance of the results

- Markov gap captures information of topologically ordered ground states beyond the TEE

New universal form $h = \frac{c}{3}\ln 2$

insensitive to "corner"

- Conjecture:

  - If remove "deep inside topological phase" condition

$$h \geq \frac{c}{3}\ln 2$$

  - If remove "chiral phase" condition

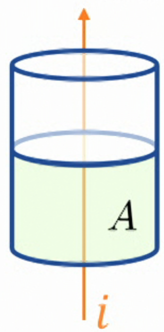
c : Ungappable central charge

- Technique: bulk-boundary correspondence

  - Direct calculation *[**YL**, Sohal, Kudler-Flam, Ryu 22]*
  - Conformal interface approach *[**YL**, Kusuki, Sohal, Kudler-Flam, Ryu 22 (to appear)]*

1. Universal TEE by boundary CFT

- **TEE in 2+1d**



$$|\mathrm{GS}\rangle = \sum_i c_i |\Phi_i\rangle$$

$$S_A = \alpha L - \gamma'$$

$$\gamma' = \ln \mathcal{D} + \sum_i |c_i|^2 \ln |c_i|^2 - \sum_i |c_i|^2 \ln d_i$$

[Levin-Wen 08; Kitaev-Preskill 08]

[Dong, et al 08]

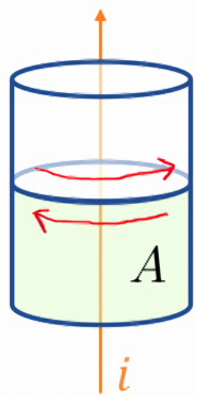
Left-right entanglement entropy(LREE) in 1+1d

boundary condition:

$$[T(\sigma) - \bar{T}(\sigma)]|B\rangle = 0$$

Solution: Ishibashi state:

$$|B_i\rangle = \sum_{N=0}^{\infty} \sum_{k=1}^{d_N} |h_i, N; k\rangle \otimes \overline{|h_i, N; k\rangle}$$



regularized state:

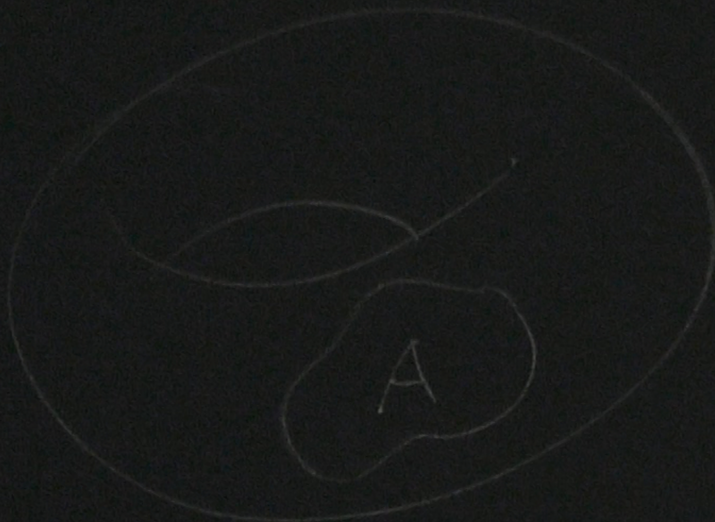
$$|\mathbf{B}\rangle = \sum_i \frac{c_i}{\sqrt{n_i}} e^{-\beta H_0} |B_i\rangle$$

trace out right moving part, compute entanglement

$$\gamma' = \ln \mathcal{D} + \sum_i |c_i|^2 \ln |c_i|^2 - \sum_i |c_i|^2 \ln d_i$$

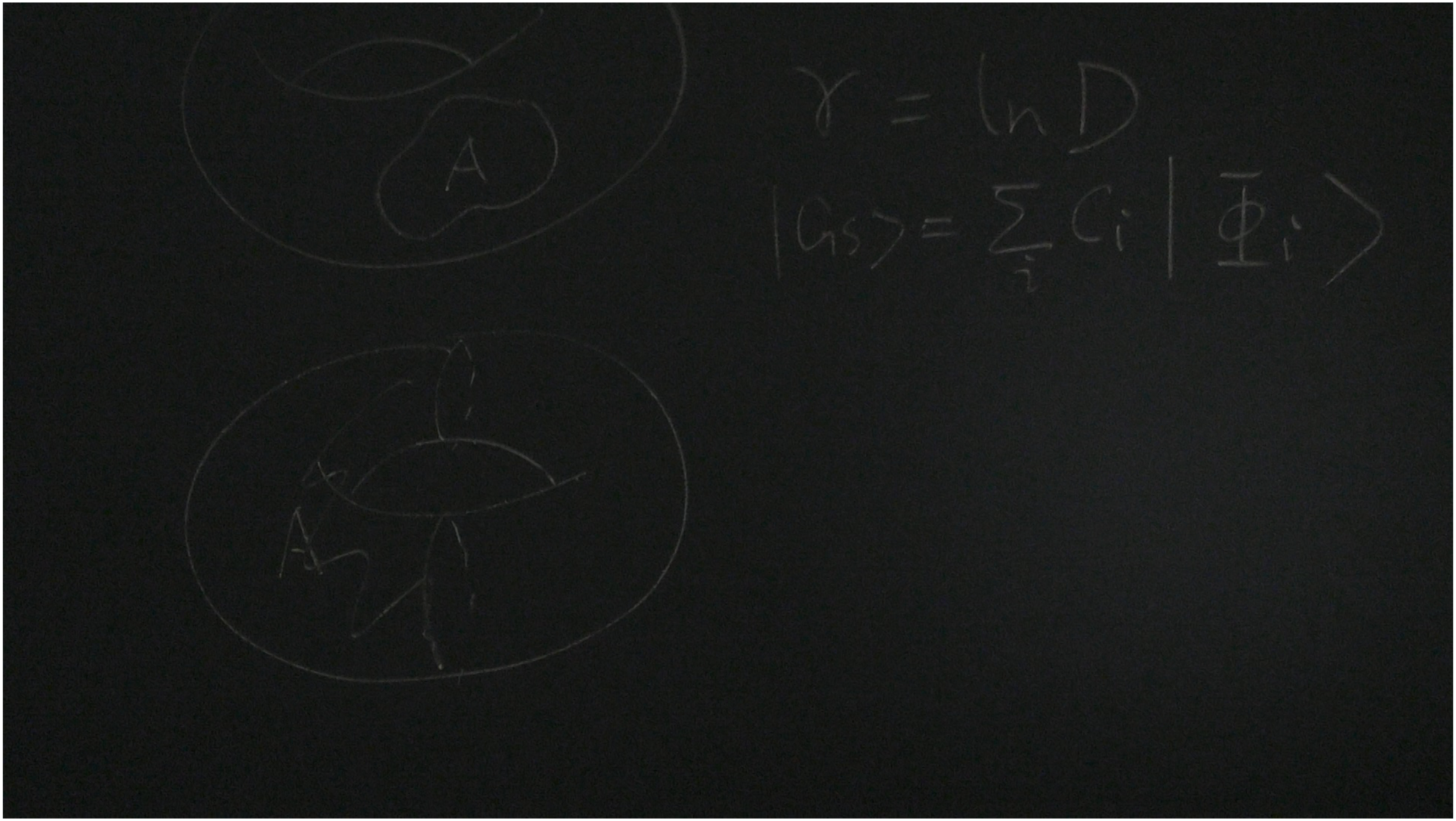
[Qi-Katsura-Ludwig 12]

[Wen-Matsuura-Ryu 16]



$$\gamma = \ln D$$

$$|GS\rangle = \sum_i c_i |\Phi_i\rangle$$

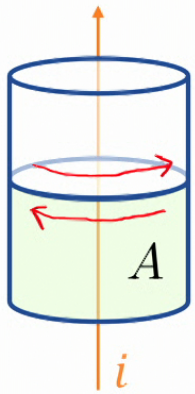


A
A
$\gamma = \ln D$
$|GS\rangle = \sum_i c_i |\Phi_i\rangle$

2.How about tripartition?

Two approaches: (1) bulk TQFT (2) boundary CFT

Insight: find a state mimic (2+1)d ground state near entangling boundary



$|\mathrm{GS}\rangle \sim |B\rangle$

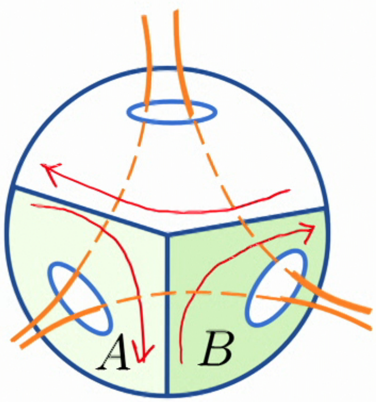
- Boundary state:

$$[T(\sigma) - \bar{T}(\sigma)]|B\rangle = 0$$

free fermion

$$[\psi^2(\sigma) - i\psi^1(L-\sigma)]|B\rangle = 0$$

$$\Rightarrow \quad |B\rangle = \exp\left(i \sum_{r \geq \frac{1}{2}} \psi^1_{-r}\psi^2_{-r}\right)|0\rangle$$



$|\mathrm{GS}\rangle \sim |V\rangle$

- Vertex state: 3 copies of edge theories

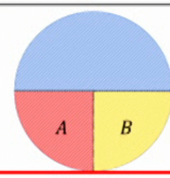
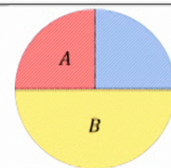
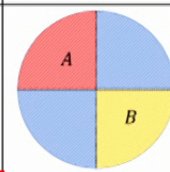
$$[T^{I+1}(\sigma) - T^I(L-\sigma)]|V\rangle = 0, \quad 0 \leq \sigma \leq L/2$$

free fermion

$$[\psi^{I+1}(\sigma) - i\psi^I(L-\sigma)]|V\rangle = 0, \quad 0 \leq \sigma \leq L/2.$$

2.2 Free fermion result

- Using free fermion vertex state (boundary of chiral superconductor)
- Numerical results show $h = \frac{c}{3} \ln 2$ *[**YL**, Sohal, Kudler-Flam,Ryu 21]*

partition	1	2	3
			
h (k)	$0.1155 (= \frac{c}{3} \log 2)$	$0.1155 (= \frac{c}{3} \log 2)$	0.1316
h (u)	0.1093	0.1043	0.1316

- Is this universal result? (for general TQFTs)

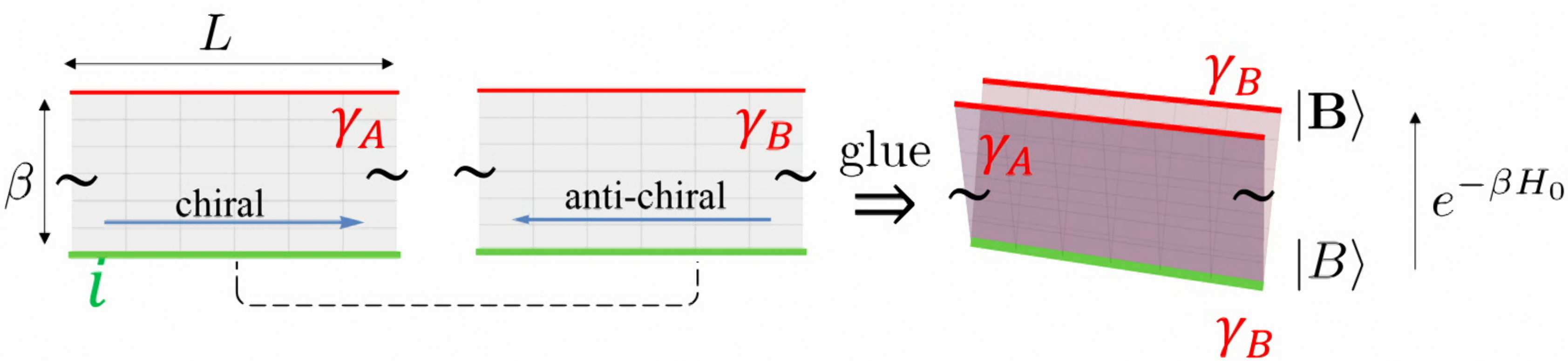
hard to obtain vertex state for general RCFT

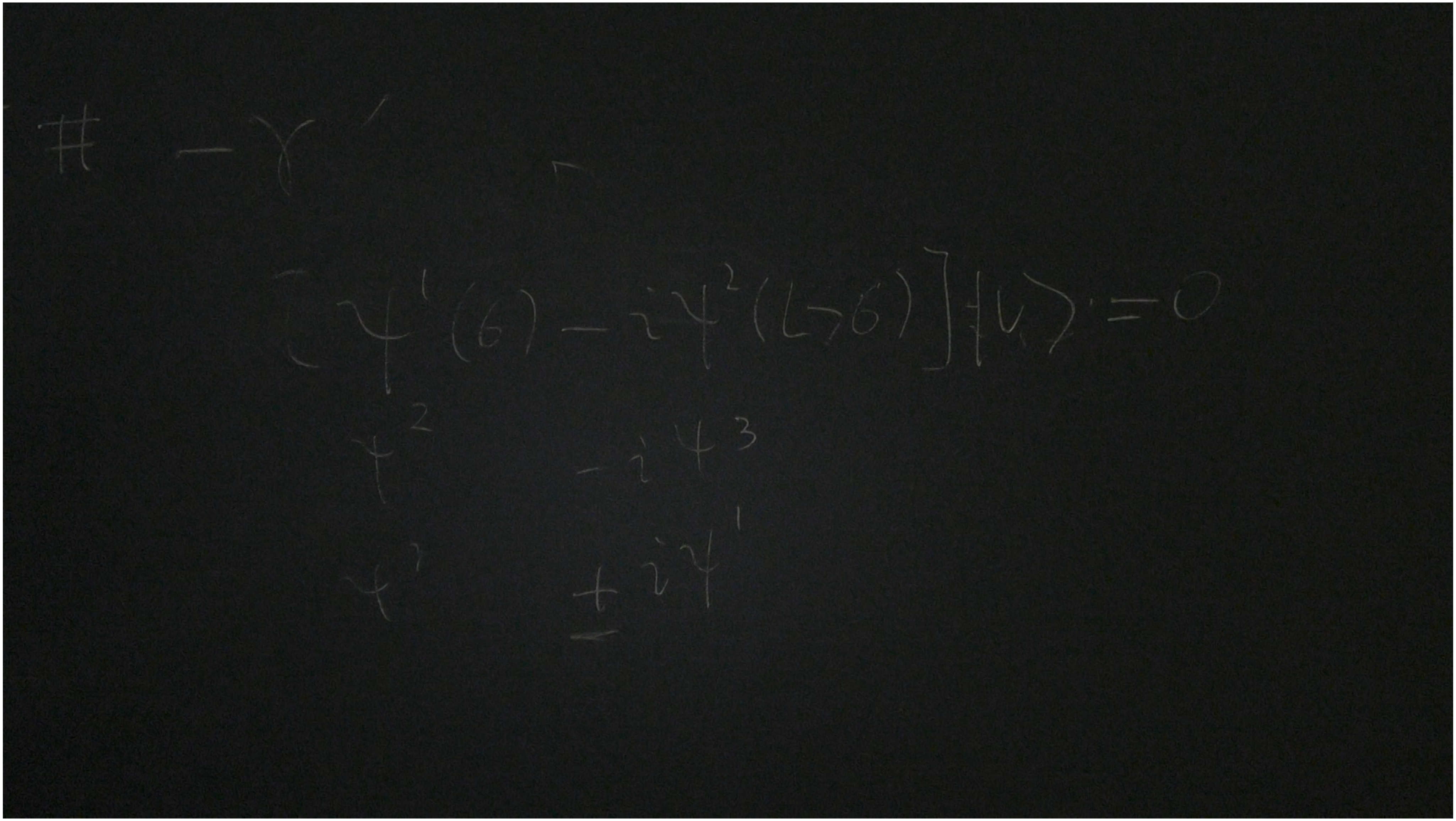
3.1 Boundary state revisit: Conformal interface

- Boundary state as path integral:

$$[T(\sigma) - \bar{T}(\sigma)]|B\rangle = 0$$

$$|\mathbf{B}\rangle = \sum_i \frac{c_i}{\sqrt{n_i}} e^{-\beta H_0} |B_i\rangle$$

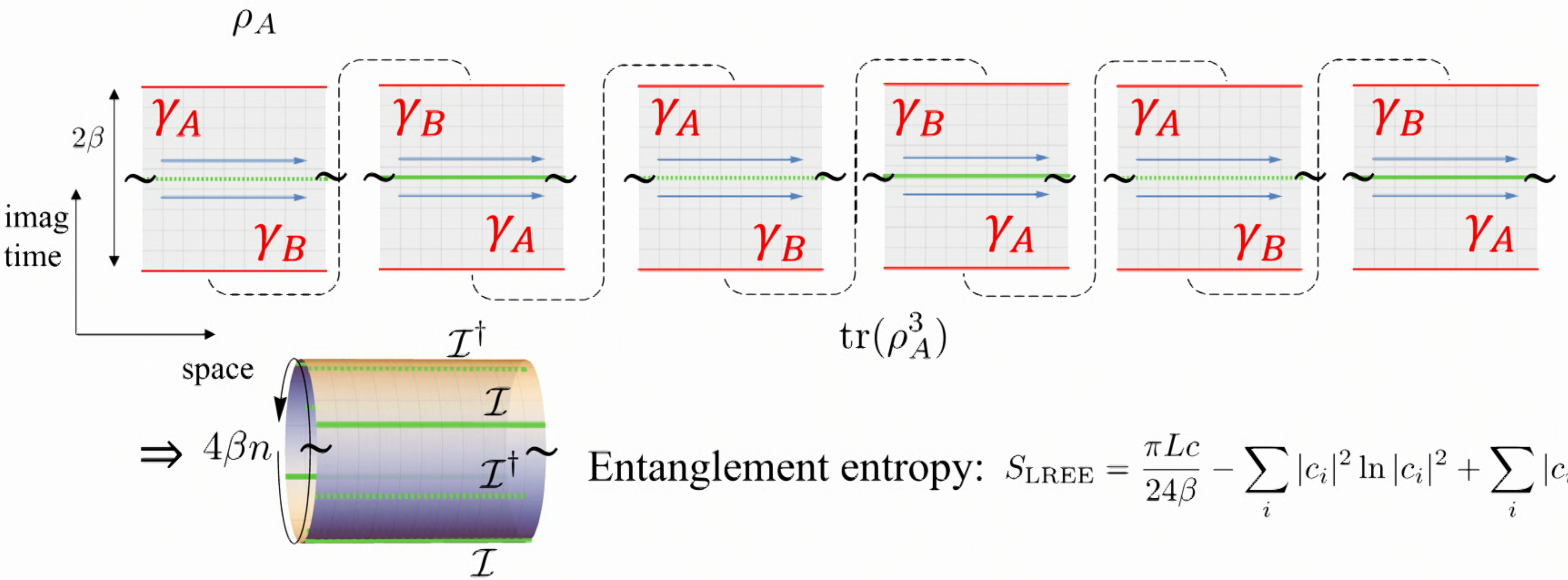



[ψ¹(σ) − iψ²(L−σ)] |ψ⟩ = 0
ψ² −iψ³
ψ² ±iψ¹

3.1 Boundary state revisit: Conformal interface

- Use path integral to evaluate S_n: $S_n = \frac{1}{1-n} \log \frac{\mathrm{tr}\rho_A^n}{(\mathrm{tr}\rho_A)^n}$

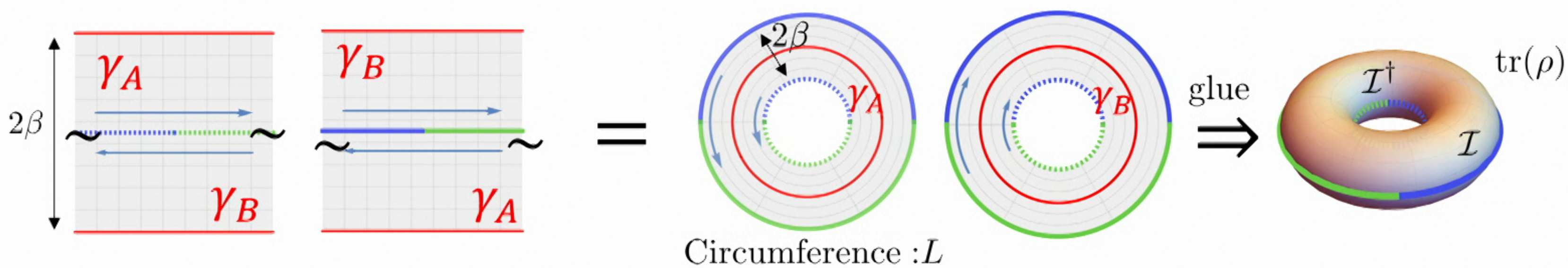
- Interface: "unfold" the boundary state → It is a projector! $\mathcal{I}_i = \sum_{N=0}^{\infty} \sum_{k=1}^{d_N} |h_i, N; k\rangle\langle h_i, N; k|$



⇒ Entanglement entropy: $S_{\mathrm{LREE}} = \frac{\pi L c}{24\beta} - \sum_i |c_i|^2 \ln |c_i|^2 + \sum_i |c_i|^2 \ln \mathcal{S}_{i0}.$

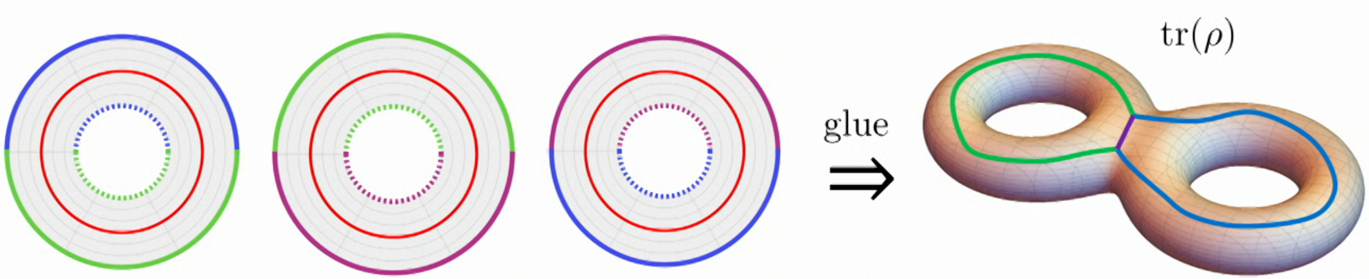
3.2 Universal tripartite entanglement

- Can we use conformal interface approach to show the universality?
- Recall $\mathrm{tr}(\rho)$ as path integral:



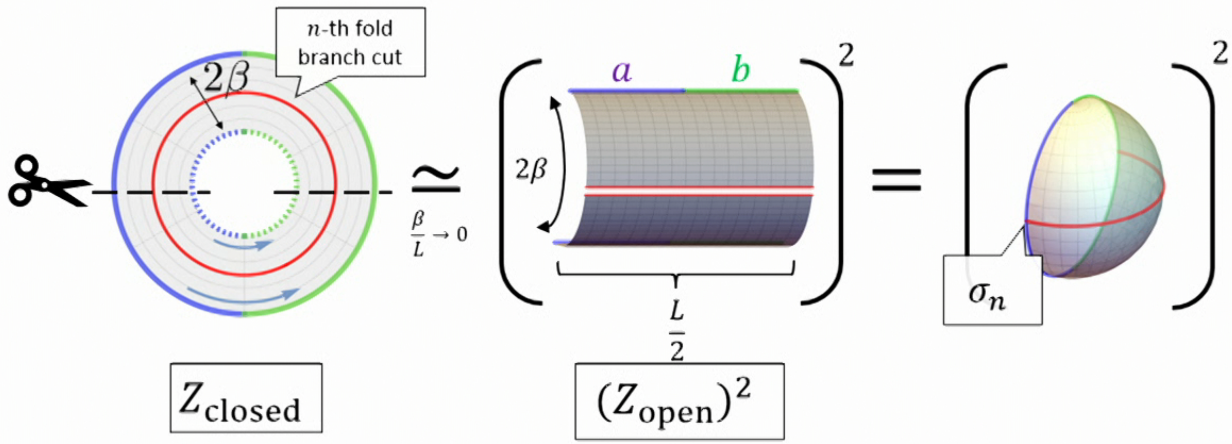
- For vertex state, $\mathrm{tr}(\rho)$ will be path integral on 2-genus:

still hard to evaluate path integral



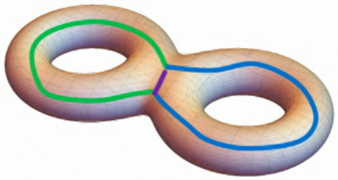
3.3 Path integral decomposition

- In the $\beta \ll L$ limit: cut an annulus into two strips

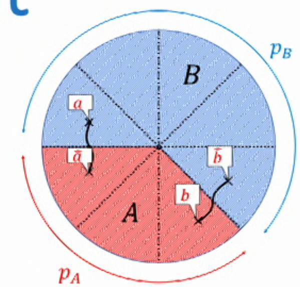


- Under this approximation:

$$Z_{\text{closed}} = \frac{1}{\mathcal{N}}(Z_{\text{open}})^2$$



3.4 Universal tripartite entanglement



$$p = p_A + p_B$$

- Entanglement entropy:

$$S_A = \frac{c\pi L}{24\beta} + \frac{c}{3}\ln\sin\frac{p_A\pi}{p} + \ln\frac{1}{\mathcal{D}}$$

area-law term → corner term → topological term

corner term agrees with free fermion numerics!

Mutual information: $$I(A:B) = \frac{c\pi L}{24\beta} + \frac{c}{3}\ln\frac{\tan\frac{\pi}{p}}{2} + \ln\frac{1}{\mathcal{D}}$$

[Sirois, et al. 20] Geometrical term in IQHE

- Reflected entropy

$$S_R(A,B) = \frac{c\pi L}{24\beta} + \frac{c}{3}\ln\frac{\tan\frac{\pi}{p}}{2} + \frac{c}{3}\ln 2 + \ln\frac{1}{\mathcal{D}}$$

area-law term, corner term, extra piece, topological term

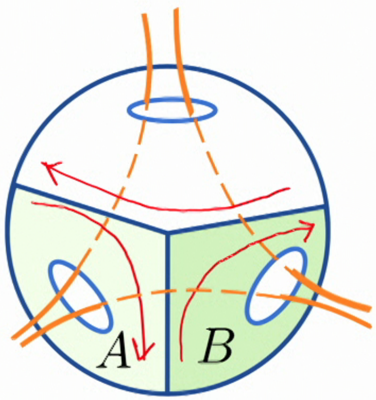
- Markov gap: $h = \frac{c}{3}\ln 2$

no "corner" contribution

*[**YL**, Kusuki, Sohal, Kudler-Flam, Ryu 22 (To appear)]*

Summary

- Tripartite entanglement unveils new information beyond bipartite entanglement
- We studied tripartite entanglement using bulk-boundary correspondence and introduce vertex state
- Vertex state can be evaluated (indirectly) using conformal interface approach
- Markov gap does not receive corner contribution



$|\mathrm{GS}\rangle \sim |V\rangle$