Title: Learning in the quantum universe Speakers: Hsin-Yuan Huang Series: Colloquium Date: November 23, 2022 - 2:00 PM

URL: https://pirsa.org/22110052

Abstract: I will present recent progress in building a rigorous theory to understand how scientists, machines, and future quantum computers could learn models of our quantum universe. The talk will begin with an experimentally feasible procedure for converting a quantum many-body system into a succinct classical description of the system, its classical shadow. Classical shadows can be applied to efficiently predict many properties of interest, including expectation values of local observables and few-body correlation functions. I will then build on the classical shadow formalism to answer two fundamental questions at the intersection of machine learning and quantum physics: Can classical machines learn to solve challenging problems in quantum physics? And can quantum machines learn exponentially faster than classical machines?

Zoom link: https://pitp.zoom.us/j/97994359596?pwd=UlBwc2hoSkNzWlZvM1o1RWErU1U2QT09

Learning in the quantum universe

Presenter: Hsin-Yuan Huang (Robert)

Collaborators: Richard Kueng, Giacomo Torlai, Victor Albert, John Preskill,

Sitan Chen, Jordan Cotler, Jerry Li, Michael Broughton, Jarrod McClean, and more





• A central goal of science is to learn how our universe operates.



Examples of scientific disciplines

- A central goal of science is to learn how our universe operates.
- Because our universe is **inherently quantum**, the ability to efficiently learn in the quantum world could lead to many advances.







Examples of scientific disciplines

Image credits: (Top left) https://www.energy.gov/science/doe-explainscatalysta (Top right) https://theconversation.com/aspharmaceutical-use-continues-to-rise-side-effects-are-becoming-a-costly-health-issue-105499 (Boottom left) https://www.mit.edu/2019/ ultra-quantum-matter-ugm-research-given-Bm-boost-0529 (Bottom right) https://www.nature.com/article/41158-012-03213-z

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- Because our universe is **inherently quantum**, the ability to efficiently learn in the quantum world could lead to many advances.



Examples of scientific disciplines



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• To accelerate and automate the development of (quantum) science, it is important to understand how to design better algorithms to learn in the quantum universe.



A cartoon depiction of learning



Image credits: (Top left) <u>https://www.energy.gov/science/doe-explainscatalyst</u> (Top right) <u>https://theconversation.com/as-</u> pharmaceutical-use-continues-to-rise-side-effects-are-becoming-a-costly-health-issue-105494 (Bottom left) <u>https://news.mit.edu/2019/</u> ultra-quantum-matter-ugm-research-given-Bm-boost-05292 (Bottom right) <u>https://www.neture.com/article/</u>411586-019-03213-z

Overview

How to efficiently learn in the quantum universe?



Learning with quantum machines

Can quantum machines learn faster and/or predict more accurately than classical machines?



Overview



Predicting many properties of a quantum system from very few measurements, Nature Physics
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• How can classical machines "see" quantum many-body systems?



- What do we mean by "seeing" a quantum system?
- Converting the quantum system to a classical form that accurately captures many properties of the quantum system.



- Why do we want to construct classical representations of quantum systems?
 - ✦ We often want to know what the quantum system is.
 - Many quantum applications require an interface between the classical and the quantum realm (e.g., variational algorithms).



Standard approach

• Quantum state tomography:

Learn a <u>complete representation</u> of an *n*-qubit quantum state. $(d \times d \text{ matrix}, d = 2^n)$

• Sample-optimal protocol (Haah et al.; O'Donnel, Wright):

Sample complexity: $\Theta(2^{2n})$

Quantum resource: $\Theta(n2^{2n})$ qubits + exponentially long circuits

- Classical storage: $\Omega(2^{2n})$
- Classical post-processing: $\Omega(2^{2n})$

Theorem 1 [HKP20]

There exists procedure that guarantees the following.

1. Given $B, M, \epsilon > 0$, the procedure learns a classical representation of an unknown quantum state ρ from

 $T = \mathcal{O}(B \log(M)/\epsilon^2)$ measurements.

2. Subsequently, given any $O_1, ..., O_M$ with $B \ge \max \|O_i\|_{shadow}^2$, the procedure can use the classical representation to predict $\hat{o}_1, ..., \hat{o}_M$, where $|\hat{o}_i - tr(O_i\rho)| < \epsilon$, for all i.

For example:

- $M = 10^6$, B = 1, then naively we need $10^6/\epsilon^2$ measurements.
- This theorem shows that we only need $6 \log(10)/\epsilon^2$ measurements. Furthermore, we don't need to know O_1, \ldots, O_M in advance.

Repeat the following T times:



Data Acquisition Phase



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• Sample a random unitary U_i to rotate the quantum system.





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- Sample a random unitary U_i to rotate the quantum system.
- Measure the system in the computational basis $|b_i\rangle \in \{0,1\}^n$.
- Store the "classical shadow": $|s_i\rangle = U_i^{\dagger}|b_i\rangle$.

E.g., measure each qubit in a random basis

Few Repetitions



Data Acquisition Phase



The Procedure: Prediction Phase

Given $S(\rho) = \{|s_1\rangle, ..., |s_T\rangle\}$ (the classical shadow),

how to predict properties of the quantum state ρ ?

★ $\mathbb{E}[|s_i X |] = \mathcal{M}(\rho)$. (\mathcal{M} : some CPTP map)

$$\Rightarrow \rho = \mathbb{E}[\mathscr{M}^{-1}(|s_i X s_i|)] \implies \rho \approx \mathscr{M}^{-1}(|s_i X s_i|).$$

Algorithm for predicting $tr(O\rho)$: (median-of-means)

Compute
$$X_i = \operatorname{tr}(O\mathcal{M}^{-1}(|s_i| \langle s_i|)), \forall i = 1, ..., T.$$

Predict
$$\hat{o}$$
 = median $\left(\frac{1}{T/K}\sum_{i=1}^{T/K}X_i, \dots, \frac{1}{T/K}\sum_{i=T-T/K+1}^NX_i\right)$.

Repeat the following T times:

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Classical ML for quantum problems

- Can classical machines learn to solve challenging problems in quantum physics?
- And can they yield better solutions than non-ML algorithms?



- Given $x \in [-1,1]^m$ that describes an *n*-qubit Hamiltonian H(x), the machine predicts a classical representation (e.g., classical shadow) of the ground state $\rho(x)$ of H(x).
- Vector x specifies laser intensities, few-body interactions, magnetic fields, etc.





Computational hardness

- This problem is *extremely* hard!
- Consider a smooth class of *n*-qubit 2D Hamiltonians H(x) with spectral gap 1, and the machine only predicts 1-body observable O in ground state $\rho(x)$.
- Furthermore, we only care about average prediction error.



1**D**



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1D

Proposition 1 Assuming RP \neq NP, then no randomized classical algorithm can achieve an average prediction error $\leq 1/4$ within poly(*n*) time. RP \neq NP: NP-complete problems cannot be solved in randomized polynomial time.

• Can classical ML algorithms do something useful for this challenging problem?











Proposition 1

Assuming RP \neq NP, then no randomized classical algorithm can achieve an average prediction error $\leq 1/4$ within poly(n) time.

Classical algorithm

2D spectral gap 1 1-body observable average prediction error 1/4













Classical algorithm

2D spectral gap 1 1-body observable average prediction error 1/4

Classical ML (trained with data)

any constant dimension any constant spectral gap any local observable any average prediction error $\epsilon = O(1)$

We proved that a poly-time classical ML algorithm (w/ data) can predict **much better** than **any** poly-time classical algorithm.



The question : Why ML can be more useful than non-ML algorithms?



The answer 😒: Generalizing from data can be easier than computing everything






2D random Heisenberg model

We consider training data size N = 100, T = 500 randomized measurements for constructing classical shadows. The best ML model is chosen from Gaussian kernel method, infinite-width neural networks, and l_2 -Dirichlet kernel.



[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

Classifying quantum phases: Theorem

Theorem 2

If there exists a **nonlinear** function of **few-body** reduced density matrices for classifying the phases, then the classical ML algorithm can **efficiently** learn to classify these phases.

• The assumption is believed to hold for gapped quantum systems.



[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

1D Symmetry protected topological phases

We consider T = 500 randomized measurements to construct classical shadows for each state. The classical **unsupervised** ML model is a kernel PCA using the shadow kernel.



[HKT+21] Huang, Kueng, Torlai, Albert, Preskill. Provably efficient machine learning for quantum many-body problems, Science, 2022.

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[HKP21] Huang, Kueng, Preskill. Information-theoretic bounds on quantum advantage in machine learning, *Physical Review Letters*, 2021. [CCHL21] Chen, Cotler, Huang, Li. Exponential separations in learning with and without quantum memory, *FOCS*, 2021. [HBC+] Huang, Broughton, Cotler, Chen, Li, Mohseni, Neven, Babbush, Kueng, Preskill, McClean. Quantum advantage in learning from experiments, *Science*, 2022.

Classical vs Quantum

- What are the advantage of a quantum agent over a classical agent?
- Could quantum technology fundamentally alter our ability to learn about the physical world?



Learning a state

- Assume the only unknown in the entire universe is an *n*-qubit state ρ .
- Classical agent can perform any measurement on ρ in each experiment.
- Quantum agent can obtain and store ρ coherently from each experiment.



Quantum advantage in predicting Pauli observables

- The classical/quantum agent learns about the unknown *n*-qubit state ρ .
- Subsequently, the agent predicts $Tr(P\rho)$ for any observable $P \in \{I, X, Y, Z\}^{\otimes n}$.

Theorem

Classical agent needs $\Omega(2^n)$ experiments to predict an adversarially chosen P, but quantum agent only needs $\mathcal{O}(n)$ experiments to predict all 4^n observables.

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Exponential quantum advantage is present even when the state ρ is a classical distribution over product states (no entanglement!).

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Uncertainty principle significantly hinders the learning ability of classical agents, but surprisingly not the ability of a quantum agent.

Predicting many incompatible observables

To predict non-commuting observables $O_1, O_2, ...,$ classical agent suffers from uncertainty principle, quantum agent does not.



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Performing quantum PCA

To estimate property of principal component, classical agent needs exponential time, quantum agent needs polynomial.



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Uncovering symmetry in dynamics

Classifying dynamics with or without time-reversal symmetry



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Performing quantum PCA

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Learning physical dynamics

To learn a polynomial-time quantum process, a classical agent requires exponential experiments, a quantum agent requires polynomial.



Demonstration on Sycamore: Quantum advantage in learning states



Utilizing a total of 40 qubits

[HFP22] Huang, Flammia, Preskill. Foundations for learning from noisy quantum experiments, QIP, 2022.

[HBC+] Huang, Broughton, Cotler, Chen, Li, Mohseni, Neven, Babbush, Kueng, Preskill, McClean. Quantum advantage in learning from experiments, Science, 2022.

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Conclusion

- Significant progress in understanding how to learn in the quantum universe. But most on lower-level tasks (predicting properties, classifying phases, etc.).
- How to create rigorous ML algorithms for higher-level tasks: designing quantum circuits / protocols / algorithms, discovering new physics?



Long-term ambitions

- Develop our understanding of learning to accelerate/automate scientific development (and, perhaps, give rise to an algorithmic theory of science).
- Build a quantum machine capable of learning and discovering new facets of our universe beyond humans and classical machines.



Al imagination of itself learning and discovering new facets of our quantum universe (Credit: DALL·E)