Title: Symmetries of Black Hole Perturbation Theory

Speakers: Adam Solomon

Series: Strong Gravity

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Abstract: I discuss novel symmetries of perturbation theory around rotating and non-rotating black holes in general relativity, and discuss their origins and implications for gravitational-wave astronomy. This is motivated by two special aspects of black hole perturbations in four dimensions: isospectrality of quasinormal modes and the vanishing of tidal Love numbers. There turn out to be off-shell symmetries underlying each of these phenomena. One is a duality, which on shell reproduces the famous Chandrasekhar duality and therefore underlies isospectrality, and can be thought of as an extension of electric-magnetic duality to black hole backgrounds. The other is a set of "ladder symmetries" relating modes of different angular momentum or spin, which imply the vanishing of Love numbers. This has a geometric origin in the conformal symmetry of low-frequency modes.

Zoom link: https://pitp.zoom.us/j/93633894223?pwd=cEFRYno5WSt5NUJlOWJLdVZHWjE1QT09

Symmetries of black hole perturbation theory

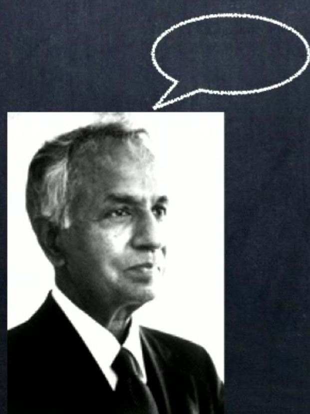
Adam Solomon
McMaster University

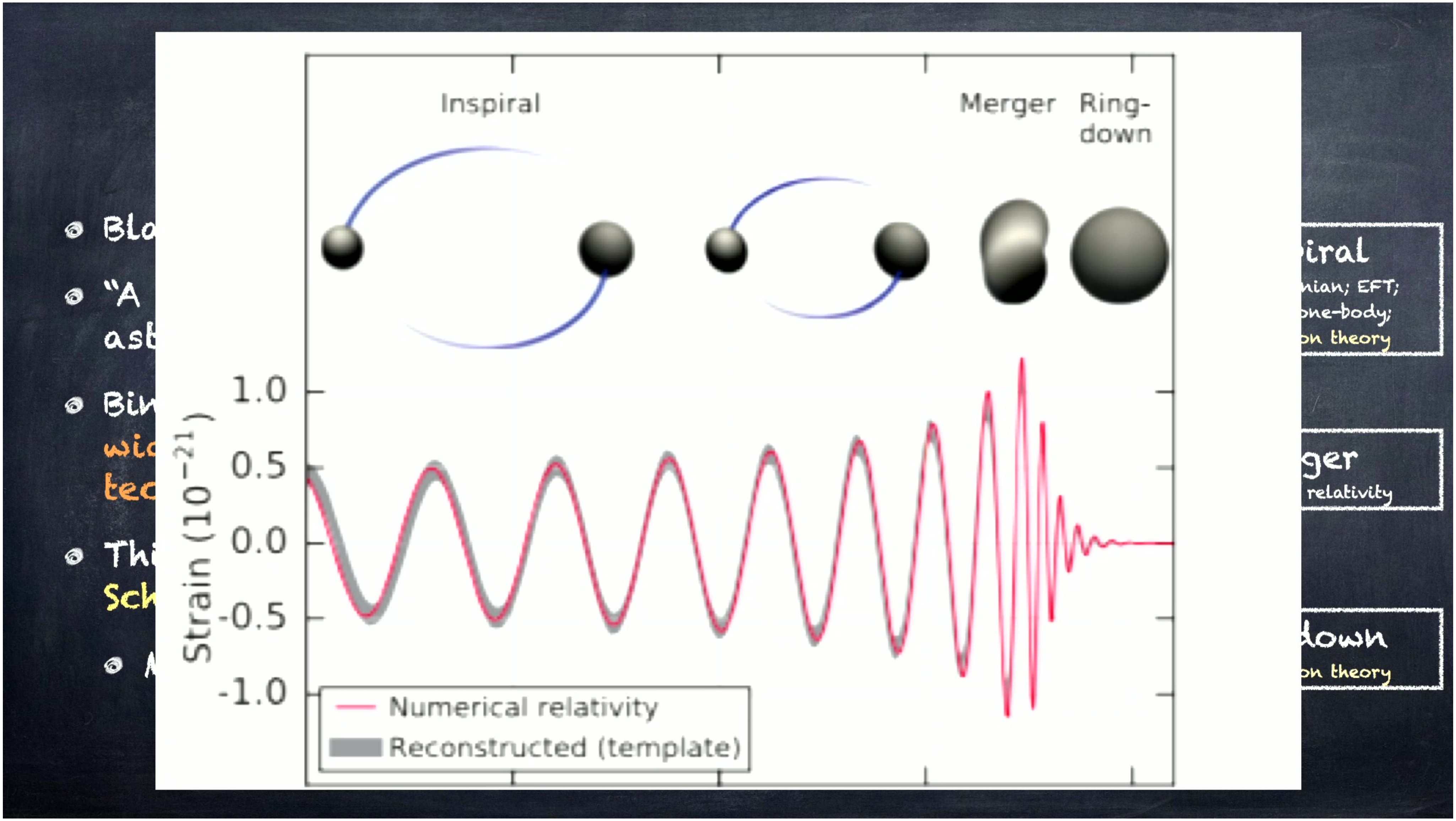
w/ Lam Hui (Columbia), Austin Joyce (Chicago), Riccardo Penco (CMU), & Luca Santoni (ICTP)

arXiv:2010.00593 (JCAP)
arXiv:2105.01069 (JCAP)
arXiv:2203.08832 (JHEP)

"The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time."

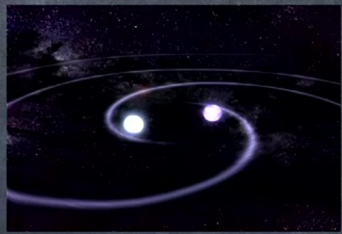
–Chandrasekhar



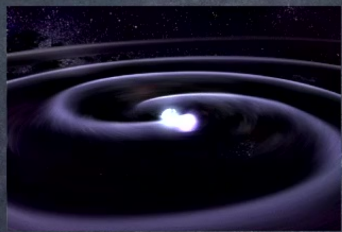

Inspiral
Merger
Ring-down
1.0
0.5
0.0
-0.5
-1.0
Strain (10^{-21})
Numerical relativity
Reconstructed (template)

Motivation

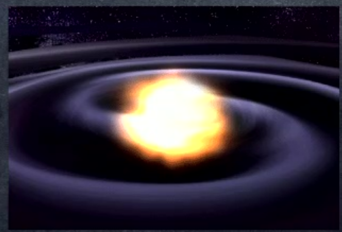
- Black holes are simple but mysterious
- "A new era of gravitational wave astronomy"™
- Binary BH mergers probe gravity in a wide variety of regimes and techniques
- This talk: linear perturbations; Schwarzschild for simplicity
  - Many/(all?) results apply to Kerr



Inspiral
Post-Newtonian; EFT; effective one-body; perturbation theory



Merger
Numerical relativity



Ringdown
Perturbation theory

Image: NASA

Hints of Symmetries
(in D=4 GR)

Tidal responses: Black hole Love numbers = 0

"Fine tuning": vanishing EFT coefficients

Isospectrality: Both GW polarizations have the same QNM spectrum

Due to a duality of the Einstein equations

Tidal Responses & Love Numbers

- During inspiral, companion object sources a tidal field
- Gravitational response to tidal deformation encoded in Love numbers
  - Static response ($\omega = 0$)
  - Observable at 5PN
  - Measures internal structure

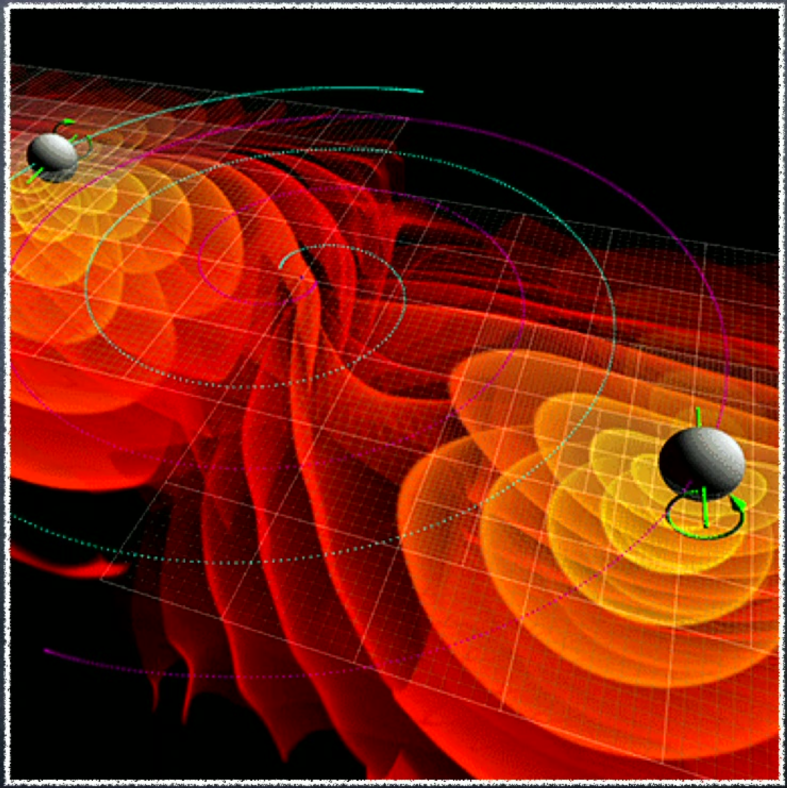


Image: C. Henze/NASA Ames Research Center

Vanishing Love

- Love numbers are Wilson coefficients of point-particle EFT:
  $$S_{\rm pp} = \int d\tau \left(-m + \frac{1}{2}\lambda_E E_{ij}^2 + \frac{1}{2}\lambda_B B_{ij}^2 + \cdots \right)$$
  - encode effects of finite size/structure
- Standard EFT logic: Wilson coefficients are O(1)
  - Small/vanishing coefficients signals fine-tuning unless protected by symmetry
  - Porto, 1606.08895

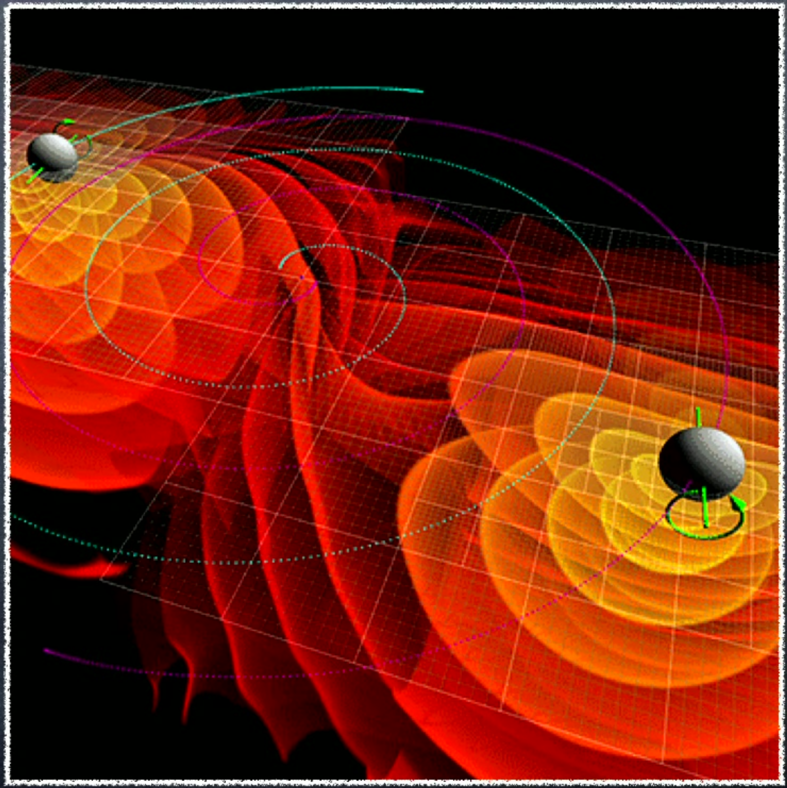


Image: C. Henze/NASA Ames Research Center

For Love numbers, it largely suffices to study spin-0 dynamics on Schwarzschild:

$$\Box\phi = 0 = \left[-\frac{r^4}{\Delta}\partial_t^2 + \partial_r\left(\Delta\partial_r\right) + \frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right)\right]\phi \qquad \Delta = r(r-r_s)$$

Separability: $$\phi = \sum_{\ell=0}^{\infty} e^{-i\omega t}\phi_\ell(r)Y_\ell(\theta)$$

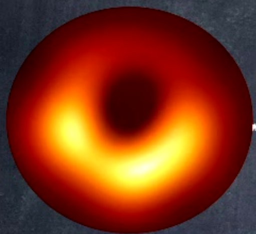
$Y_\ell(\theta)$: m=0 spherical harmonics

Radial equation: $$\left[\Delta\partial_r(\Delta\partial_r) - \ell(\ell+1)\Delta + \omega^2 r^4\right]\phi_\ell = 0$$

Vanishing Love in a nutshell

Static Klein-Gordon: $\left[\partial_r(\Delta\partial_r) - \ell(\ell+1)\right]\phi = 0$

Shares key features with Regge-Wheeler/Zerilli on Schwarzschild and Teukolsky on Kerr



Horizon		Infinity		
regular	$\phi \sim \text{const}\,.$	growing	$\phi \sim r^{\ell}$	(tidal field)
~~blowing~~ up	$\phi \sim \ln(1 - r_s/r)$	decaying	$\phi \sim \dfrac{1}{r^{\ell+1}}$	(static response)

Love number: coefficient of decaying term at infinity:

$$\phi \to \phi_\infty \left(r^{\ell} + \lambda_\ell r^{-\ell-1}\right)$$

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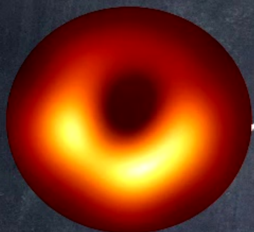

Horizon → **Infinity**

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Math: why do these diff eqs have this property?

Physics: why does D=4 GR give us diff eqs like this?

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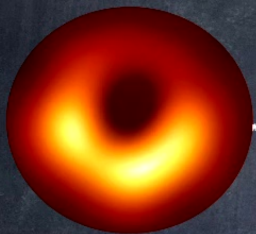

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Ladder operators

Static Klein-Gordon on Schwarzschild: $\sum_\ell H_\ell \phi_\ell = 0$

"Hamiltonian": $H_\ell = -\Delta\left[\partial_r(\Delta\partial_r) - \ell(\ell+1))\right]$ $\Delta(r) \equiv r(r - r_s)$

Admits raising and lowering operators: $D_\ell^+ \equiv -\Delta\partial_r - \frac{\ell+1}{2}\Delta'$

$$D_\ell^- \equiv \Delta\partial_r - \frac{\ell}{2}\Delta'$$

in the sense that $H_{\ell+1}(D_\ell^+\phi_\ell) = 0$

$$H_{\ell-1}(D_\ell^-\phi_\ell) = 0$$

Note: This example contains the salient features of spin-s on Kerr

Turning the Ladder Sideways

- The raising/lowering symmetry $D_\ell^\pm$ is unusual: it relates solutions at different levels ℓ
- Want: a symmetry for each level individually
- Strategy: lower to $\ell = 0$ and use accidental symmetry

  General spin: $\ell = s$
- Horizontal symmetry: $\delta\phi_\ell = Q_\ell \phi_\ell$
- Conserved charge: $P_\ell \equiv \Delta\partial_r(D_1^- D_2^- \cdots D_\ell^- \phi_\ell)$

$$P_0 = \Delta\partial_r\phi_0$$

$$Q_0 \equiv \Delta\partial_r$$

$$Q_1 \equiv D_0^+ Q_0 D_1^-$$

$$\vdots$$

$$Q_\ell \equiv D_{\ell-1}^+ Q_{\ell-1} D_\ell^-$$

See also Compton and Morrison (2003.08023)

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$$P_0 = \Delta\partial_r\phi_0$$

$$Q_0 \equiv \Delta\partial_r$$
$$Q_1 \equiv D^+_0 Q_0 D^-_1$$
$$\vdots$$
$$Q_\ell \equiv D^+_{\ell-1} Q_{\ell-1} D^-_\ell$$

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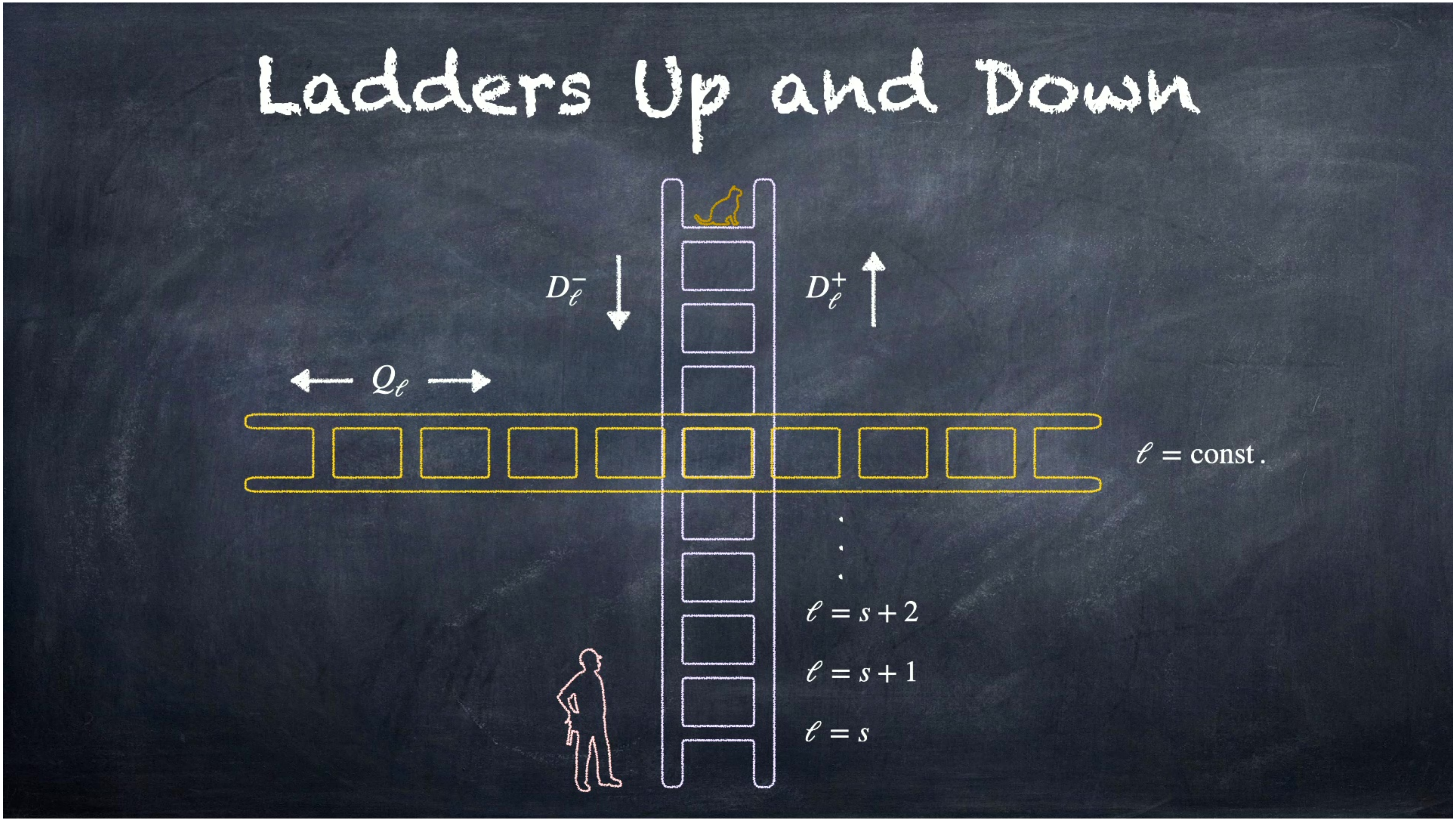

Ladders Up and Down
D_ℓ^-
D_ℓ^+
Q_ℓ
$\ell = \text{const}\,.$
$\ell = s+2$
$\ell = s+1$
$\ell = s$

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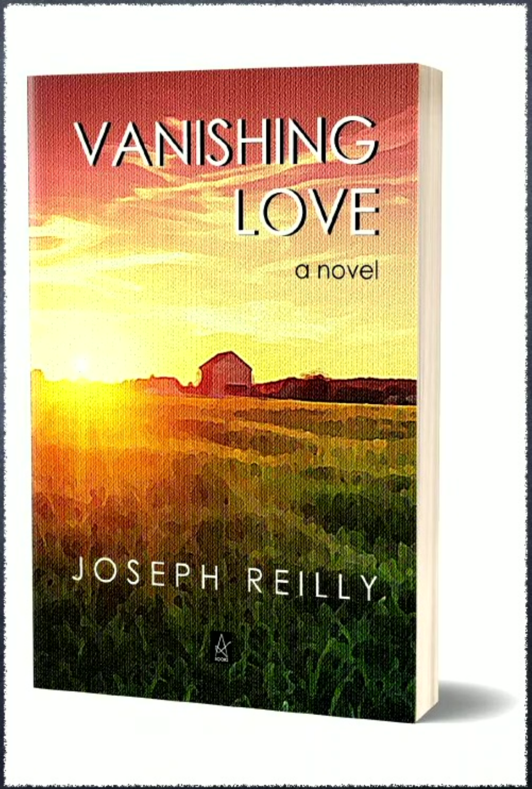
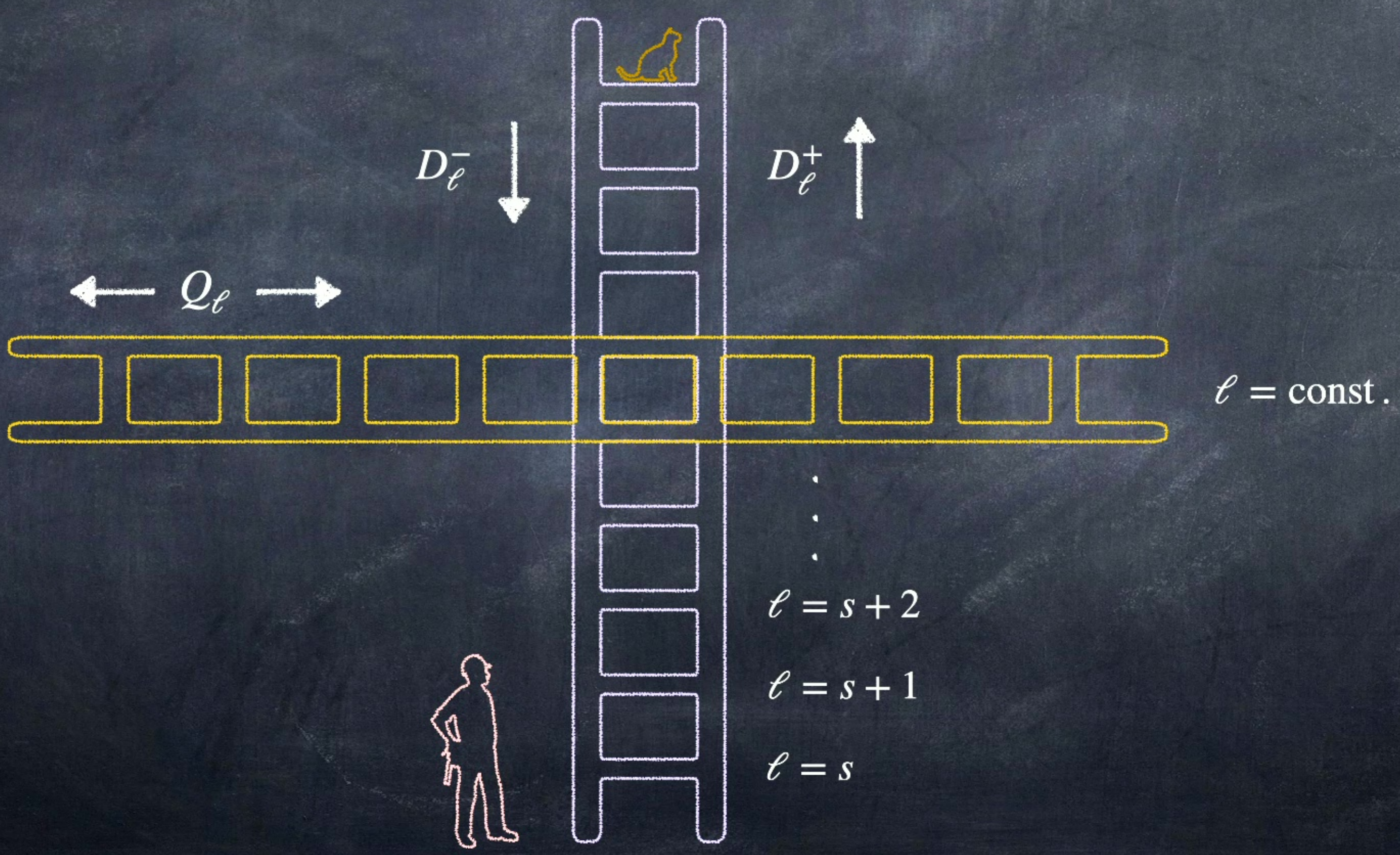
Vanishing Love

- Charge conservation implies Love = 0
  - $P_\ell = 0$ for growing (infinity) and constant (horizon) modes
  - $P_\ell \neq 0$ for decaying (infinity) and divergent (horizon) modes
- Conclusion: a decaying term diverges at the horizon
- Aside: spontaneous symmetry breaking: $Q_\ell \phi_\ell^{(\mathrm{g})} = 0, \;\; Q_\ell \phi_\ell^{(\mathrm{d})} \neq 0$



Ladders Up and Down



Towards Reality: Spin Ladder

Teukolsky equation:

$$\partial_r\left(\Delta\partial_r\phi_\ell^{(s)}\right)+s(2r-r_s)\partial_r\phi_\ell^{(s)}+\left(\frac{a^2m^2+is(2r-r_s)am}{\Delta}-(\ell-s)(\ell+s+1)\right)\phi_\ell^{(s)}=0$$

Admits ladders in ℓ and spin!

These extend the Teukolsky-Starobinsky identities for static perturbations

$$E^+\equiv\partial_r\,,\quad E_s^-\equiv\Delta\partial_r-s(r_++r_--2r)-2iam\frac{r_+-r_-}{r_+-r_-}$$

Relates solutions to Klein-Gordon, Maxwell, and Einstein

vanishing scalar Love ⟶ vanishing gravitational Love

NB: also can construct ladder for Regge-Wheeler

IR symmetries

Point-particle EFT:
treat BH as a point, encode structure in higher-order operators

γ: worldline einbein
g: (monopole) scalar charge
λ_ℓ: Love numbers

$$S = -\frac{1}{2}\int \mathrm{d}^4x(\partial\phi)^2 + \int \mathrm{d}\tau\gamma\left[\frac{1}{2}\gamma^{-2}\dot{x}^\mu\dot{x}_\mu - \frac{\mu^2}{2} - g\phi + \sum_{\ell=0}^{\infty}\frac{\lambda_\ell}{2\ell!}\left(\partial_{(a_1}\cdots\partial_{a_\ell)_T}\phi\right)^2\right]$$

UV symmetry in flat-space limit: $\delta\phi = r^2\cos\theta\partial_r\phi + r\partial_\theta(\sin\theta\phi)$

Punchline: only the bulk $(\partial\phi)^2$ term is invariant

Geometric Interpretation

From Schwarzschild to AdS

Consider a static scalar: $S = \frac{1}{2}\int \mathrm{d}^3x\sqrt{g}\phi\Box\phi$

and perform a conformal transformation:

$\tilde{g}_{ij} = \Omega^2 g_{ij}, \quad \tilde{\phi} = \Omega^{-1}\phi, \quad \Omega^2 \equiv L^2/\Delta$

L: arbitrary scale

so that

$$S = \frac{1}{2}\int \mathrm{d}^3x\sqrt{\tilde{g}}\left(\tilde{\phi}\tilde{\Box}\tilde{\phi} + \frac{r_s^2}{4L^4}\tilde{\phi}^2\right)$$

The metric $\tilde{g}_{ij}$ is nothing other than Euclidean AdS_3

Punchline: static massless scalar on Schwarzschild = massive scalar on $EAdS_3$

Beyond the static limit

Klein-Gordon: $\partial_r(\Delta \partial_r \phi) - \ell(\ell+1)\phi + \frac{\omega^2 r^4}{\Delta}\phi = 0$

Approximation: $\omega r, \omega r_s \ll 1$

Defines near zone: $\omega^{-1} \gg r \geq r_s$

Near zone includes near horizon, asymp. flat region

Identity:

$$\frac{\omega^2 r^4}{\Delta} = \omega^2 r^2 + \omega^2 r_s (r + r_s) + \frac{\omega^2 r_s^3}{r} + \frac{\omega^2 r_s^4}{\Delta} \approx \frac{\omega^2 r_s^4}{\Delta}$$

Effective near-zone metric

- Near-zone Klein-Gordon equation:

$$\partial_r(\Delta\partial_r\phi) - \ell(\ell+1)\phi + \frac{\omega^2 r_s^4}{\Delta}\phi = 0$$

- This is $\Box_{\rm NZ}\phi = 0$ in an effective near-zone metric:

$$ds^2_{\rm NZ} = -\frac{\Delta}{r_s^2}dt^2 + \frac{r_s^2}{\Delta}dr^2 + r_s^2 d\Omega^2$$

- This metric is $AdS_2 \times S^2$
  - Possesses a very rich symmetry structure
  - Same near-zone geometry for Kerr (suitable def of near-zone)

Beyond the static limit

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Properties of the near-zone metric

$$\mathrm{d}s^2_{\mathrm{NZ}} = -\frac{\Delta}{r_s^2}\mathrm{d}t^2 + \frac{r_s^2}{\Delta}\mathrm{d}r^2 + r_s^2\mathrm{d}\Omega^2$$

- Conformally flat ($C_{\mu\nu\alpha\beta} = 0$)
  - 6 Killing vectors, 9 conformal Killing vectors
- Ricci scalar vanishes ($R = 0, R_{\mu\nu} \neq 0$)
  - Massless scalar is conformally coupled
  - All 15 (C)KVs generate symmetries of near-zone action/equations of motion

Near-zone symmetries

Killing:

$$T = 2r_s\,\partial_t,$$
$$L_\pm = e^{\pm t/2r_s}(2r_s\,\partial_r\sqrt{\Delta}\partial_t \mp \sqrt{\Delta}\partial_r),$$
$$J_{23} = \partial_\varphi,$$
$$J_{12} = \cos\varphi\,\partial_\theta - \cot\theta\sin\varphi\,\partial_\varphi,$$
$$J_{13} = \sin\varphi\,\partial_\theta + \cot\theta\cos\varphi\,\partial_\varphi.$$

Conformal Killing:

Love symmetry

$$J_{01} = -\tfrac{2\Delta}{r_s}\cos\theta\,\partial_r - \tfrac{\partial_r\Delta}{r_s}\sin\theta\,\partial_\theta, \quad (5a$$
$$J_{02} = -\cos\varphi\left[\tfrac{2\Delta}{r_s}\sin\theta\,\partial_r + \tfrac{\partial_r\Delta}{r_s}\left(\tfrac{\tan\varphi}{\sin\theta}\partial_\varphi - \cos\theta\partial_\theta\right)\right] \quad (5b$$
$$J_{03} = -\sin\varphi\left[\tfrac{2\Delta}{r_s}\sin\theta\,\partial_r - \tfrac{\partial_r\Delta}{r_s}\left(\tfrac{\cot\varphi}{\sin\theta}\partial_\varphi + \cos\theta\partial_\theta\right)\right], \quad (5c$$
$$K_\pm = e^{\pm t/2r_s}\tfrac{\sqrt{\Delta}}{r_s}\cos\theta\left(\tfrac{r_s^3}{\Delta}\partial_t \mp \partial_r\Delta\partial_r \mp 2\tan\theta\partial_\theta\right), \quad (5d$$
$$M_\pm = e^{\pm t/2r_s}\cos\varphi\left[\tfrac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \tfrac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \tfrac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \mp \tfrac{2\sqrt{\Delta}}{r_s}\tfrac{\tan\varphi}{\sin\theta}\partial_\varphi\right], \quad (5e$$
$$N_\pm = e^{\pm t/2r_s}\sin\varphi\left[\tfrac{r_s^2}{\sqrt{\Delta}}\sin\theta\partial_t \mp \tfrac{\sqrt{\Delta}\partial_r\Delta\sin\theta}{r_s}\partial_r \pm \tfrac{2\sqrt{\Delta}}{r_s}\cos\theta\partial_\theta \pm \tfrac{2\sqrt{\Delta}}{r_s}\tfrac{\cot\varphi}{\sin\theta}\partial_\varphi\right].$$

Acting J_{01} on a single multipole yields our ladder operators

full algebra: so(4,2)
sl(2) subalgebras: T and $L_\pm$, $K_\pm$, $M_\pm$, $N_\pm$

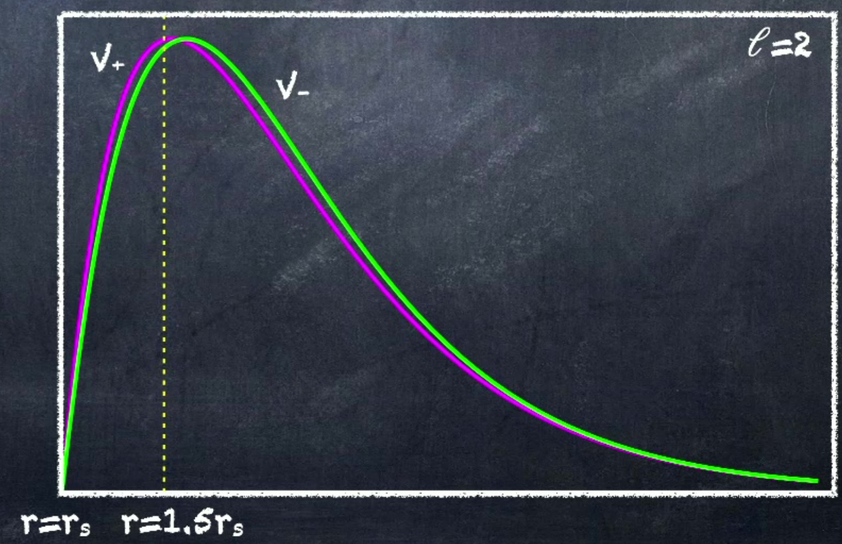
Regge-Wheeler and Zerilli

- Encode dynamical d.o.f. in **master variables**:
  - Even: Zerilli Ψ_+
  - Odd: Regge-Wheeler Ψ_-
- These obey simple **Schrödinger-like** equations
- Tortoise coordinate:

$$\mathrm{d}r \equiv \left(1 - \frac{r_s}{r}\right) \mathrm{d}r_\star$$

Master equation:

$$\frac{\partial^2 \Psi_\pm(t,r)}{\partial t^2} - \frac{\partial^2 \Psi_\pm(t,r)}{\partial r_\star^2} + V_\pm(r)\Psi_\pm(t,r) = 0$$



Chandrasekhar's Duality

- Secret link between Regge-Wheeler and Zerilli potentials:

$$V_{\pm}(r) = W^2(r) \mp \frac{\mathrm{d}W(r)}{\mathrm{d}r_\star} + \beta$$

with W(r) the superpotential and β a constant

- This directly implies isospectrality

Chandrasekhar (1980s)

$$V_+ = \frac{1-\frac{r_s}{r}}{r^3}\frac{9r_s^3 + 12\lambda^2 r_s r^2 + 8\lambda^2(1+\lambda)r^3 + 18\lambda r_s^2 r}{(2\lambda r + 3r_s)^2}$$

$$V_- = \left(1-\frac{r_s}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{3r_s}{r^3}\right)$$

$2\lambda \equiv (\ell-1)(\ell+2)$

Linearized Einstein-Hilbert

To study symmetries, we want to work at the level of the action:

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R\bigg|_{g_{\mu\nu}=\bar{g}_{\mu\nu}+\frac{1}{M_{\rm Pl}}h_{\mu\nu}}$$

Why? Allows us to calculate e.g. Noether currents

Procedure

1. Fix gauge
2. Integrate out non-dynamical (auxiliary) dofs
3. Canonically normalize: rescale field and coordinates

parity and spherical harmonics decoupling:

$$S = \sum_{\pm}\sum_{\ell}\int \mathrm{d}t\mathrm{d}r\mathscr{L}_{\pm}^{\ell}(t,r)$$

Linearized Einstein-Hilbert

Canonical normalization

- Coordinate choice: $r \to r_\star$
- Field redefinition: $h_{\mu\nu}^{\pm} \to \Psi_\pm$

End result

$$S = \frac{1}{2}\sum_{\ell=2}^{\infty}\int \mathrm{d}t\,\mathrm{d}r_\star \sum_{\pm}\left[\left(\frac{\partial \Psi_\pm}{\partial t}\right)^2 - \left(\frac{\partial \Psi_\pm}{\partial r_\star}\right)^2 - V_\pm(r)\Psi_\pm^2\right]$$

Application: Tidal Response

- Noether current for static solutions:

$$J^{r_\star} = \partial_{r_\star}\Psi_+\partial_{r_\star}\Psi_- + W(\Psi_+\partial_{r_\star}\Psi_- - \Psi_-\partial_{r_\star}\Psi_+) - (W^2+\beta)\Psi_+\Psi_- = \text{const}\,.$$

- Regularity at the horizon: $J^{r_\star} = 0$
- At infinity: $\Psi_\pm \propto r^{\ell+1} + \lambda_\pm r^{-\ell}$: $J^{r_\star} \propto (\lambda_+ - \lambda_-)$
- Duality implies equal Love numbers
- Ladder symmetry for RW but not Zerilli -> even Love = 0

Minkowski Limit

Electric-magnetic duality

Duality in the flat limit: SO(2)

$$\delta\Psi_+ = -\Psi_-$$
$$\delta\Psi_- = \Psi_+$$

implies, on-shell,

Gravitational electric-magnetic duality:

$$\delta R_{\mu\nu\alpha\beta} = \tilde{R}_{\mu\nu\alpha\beta}$$
$$\delta \tilde{R}_{\mu\nu\alpha\beta} = -R_{\mu\nu\alpha\beta}$$

$$\tilde{R}_{\mu\nu\alpha\beta} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}$$

EM duality previously known symmetry of Einstein-Hilbert around Minkowski, (A)dS

Summary

- Vanishing Love numbers and isospectral QNMs both indicate hidden symmetries of GR (/massless fields on Schwarzschild)
- We find symmetries of Einstein-Hilbert underlying these:
  - Love = 0: ladder symmetries/conformal symmetry
  - Isospectrality: EM duality on Schwarzschild