Title: Entanglement entropy in (1+1)-d with defects

Speakers: Fei Yan

Series: Quantum Matter

Date: November 01, 2022 - 3:30 PM

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Abstract: In this talk I will explore perspectives of quantum entanglement in (1+1)-d systems in presence of defects. The talk consists of two parts. In the first part, I will talk about the ground state entanglement entropy for 1d quantum spin chains with defects, using the transverse field Ising model and the three-state Potts model as examples. In the second part, I will describe the field theoretical replica trick approach to study entanglement entropy in such systems. This talk consists of some reviews about existing work, as well as work in progress with Linnea Grans-Samuelsson, Ananda Roy, Hubert Saleur, and Yifan Wang.

Zoom link: https://pitp.zoom.us/j/93049110080?pwd=QmpneGs2QlpZMlBROUlvU3VzaGtsZz09

Entanglement Entropy in (1+1)-d with Defects

Fei Yan

Rutgers University

Quantum Matter Seminar
Perimeter Institute

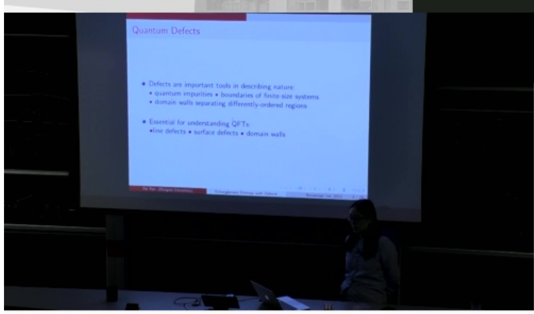
November 1st, 2022

Quantum Entanglement

- Quantum information and quantum computing: quantum entanglement as a resource for manipulating computational tasks.
  [Preskill],[Vedral],[Ohya],[Eisert],[Horodecki-Horodecki-Horodecki-Horodecki],[Nielsen-Chuang]
- Condensed matter physics:
  - ⋆ Diagnose quantum critical phenomena
    [Calabrese-Cardy],[Jin-Korepin],[Vidal-Latorre-Rico-Kitaev], [Li-Chen-Fischer], [Skinner-Ruhman-Nahum], [Gullans-Huse], [Sang-Hsieh], [Serwatka-Melko-Burkov-Roy]...
  - ⋆ Dynamics of strongly correlated quantum systems
    [Calabrese-Cardy],[Eisler-Peschel],...
  - ⋆ Characterize quantum phases of matter
    [Kitaev-Preskill],[Levin-Wen], [Lu-Hsieh-Grover], [Szasz-Wang-He],...
- Quantum field theories:
  - ⋆ Probe of phase transitions, e.g. confinement/deconfinement
    [Klebanov-Kutasov-Murugan],[Nishioka-Takayanagi,][Pakman-Parnachev]
  - ⋆ Measures of degrees of freedoms under RG flows in QFTs
    [Zamolodchikov],[Cardy],[Affleck-Ludwig],[Friedan-Konechny],[Komargodski-Schwimmer],[Myers-Sinha] [Jafferis-Klebanov-Pufu-Safdi],[Casini-Huerta],[Liu-Mezei],[Cuomo,Komargodski,Raviv-Moshe],[Wang],...

Quantum Defects

- Defects are important tools in describing nature:
  - quantum impurities • boundaries of finite-size systems
  - domain walls separating differently-ordered regions
- Essential for understanding QFTs:
  •line defects • surface defects • domain walls



Outlook

Entanglement entropy in (1+1)-d with defects:

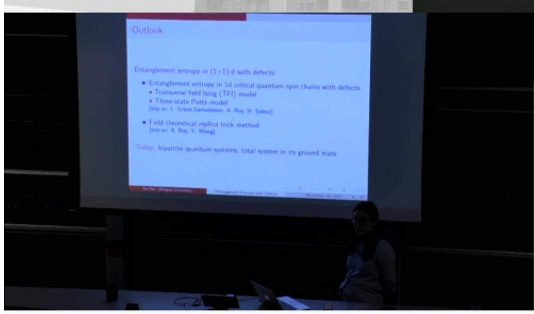
- Entanglement entropy in 1d critical quantum spin chains with defects:
  - Transverse field Ising (TFI) model
  - Three-state Potts model

  [wip w/ L. Grans-Samuelsson, A. Roy, H. Saleur]

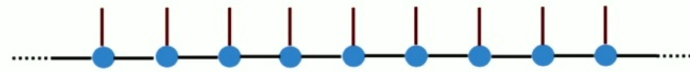
- Field theoretical replica trick method

  [wip w/ A. Roy, Y. Wang]

Today: bipartite quantum systems, total system in its ground state



Transverse Field Ising model

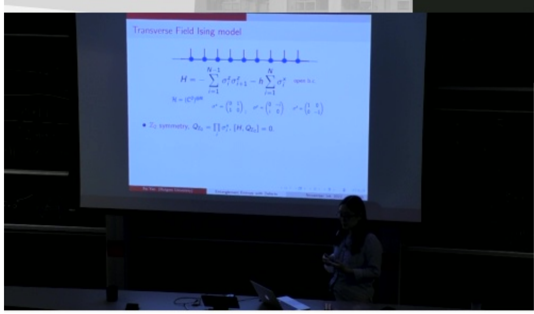


$$H = -\sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z - h \sum_{i=1}^{N} \sigma_i^x \quad \text{open b.c.}$$

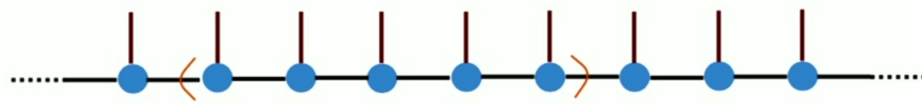
$\mathcal{H} = (\mathbb{C}^2)^{\otimes N}$

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- $\mathbb{Z}_2$ symmetry, $Q_{\mathbb{Z}_2} = \prod_i \sigma_i^x$, $[H, Q_{\mathbb{Z}_2}] = 0$.

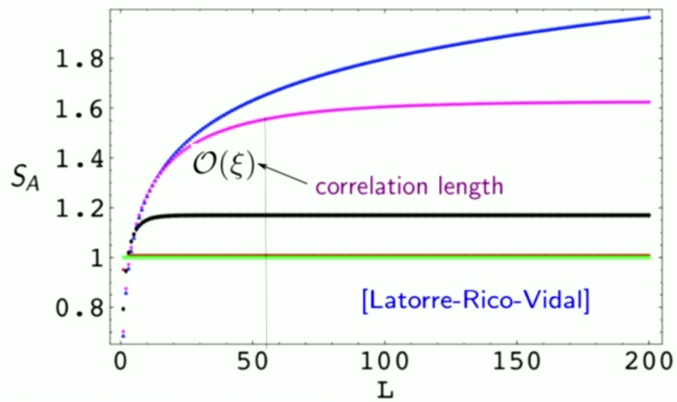


Ground State Entanglement Entropy in TFI



$\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$

$S_A := -\text{Tr}[\rho_A \log \rho_A]$ $\quad \rho_A := \text{Tr}_{A^c} |\Psi_g\rangle\langle\Psi_g|$ ← ground state



$h = 1$

$S_A = \frac{1}{6}\log L + S_0$

$c/3$

[Holzhey-Larsen-Wilczek],[Calabrese-Cardy]

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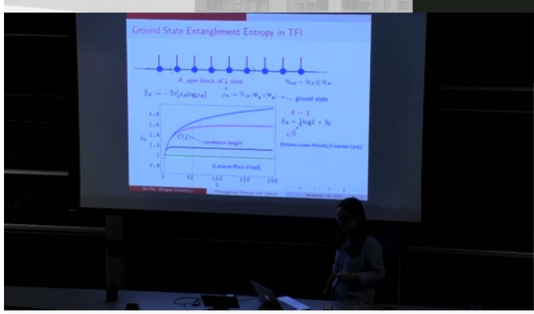
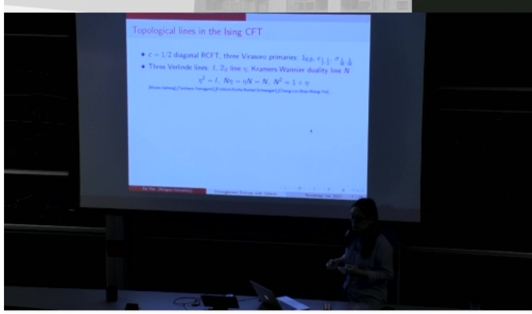


Topological lines in the Ising CFT

- $c = 1/2$ diagonal RCFT, three Virasoro primaries: $1_{0,0}$, $\epsilon_{\frac{1}{2},\frac{1}{2}}$, $\sigma_{\frac{1}{16},\frac{1}{16}}$
- Three Verlinde lines: I, $\mathbb{Z}_2$ line η, Kramers-Wannier duality line N

$$\eta^2 = I,\ N\eta = \eta N = N,\ N^2 = 1 + \eta$$

[Moore-Seiberg],[Tambara-Yamagami],[Frohlich-Fuchs-Runkel-Schweigert],[Chang-Lin-Shao-Wang-Yin]...



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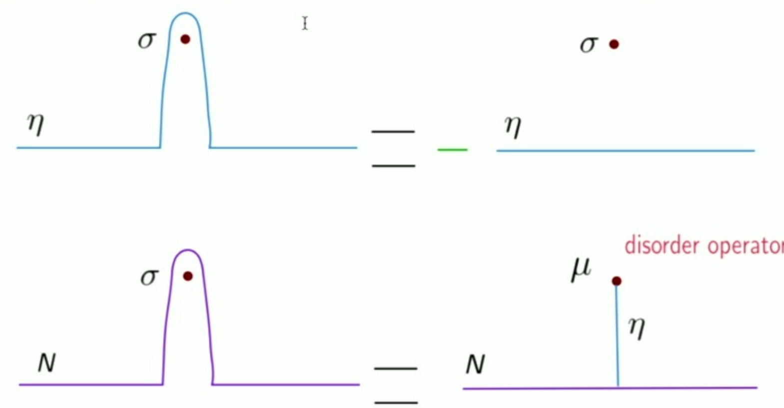
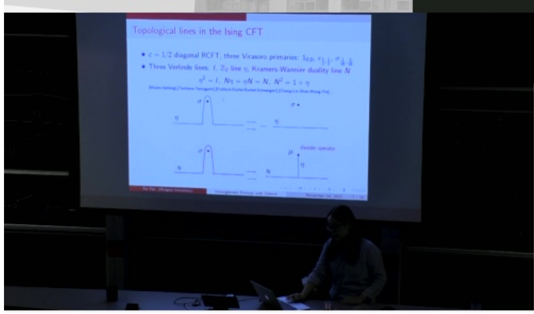


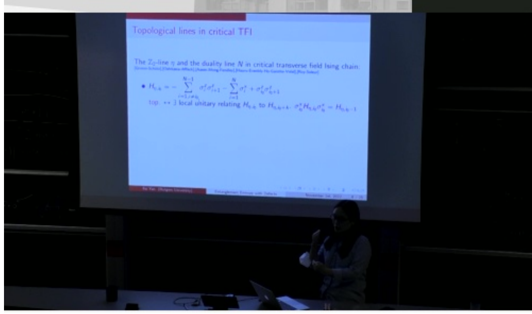


Topological lines in critical TFI

The $\mathbb{Z}_2$-line η and the duality line N in critical transverse field Ising chain:
[Grimm-Schütz],[Oshikawa-Affleck],[Aasen-Mong-Fendley],[Hauru-Evenbly-Ho-Gaiotto-Vidal],[Roy-Saleur]

- $H_{\eta,i_0} = -\sum_{i=1,i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1}^{N} \sigma_i^x + \sigma_{i_0}^z \sigma_{i_0+1}^z$

  top. $\leftrightarrow$ $\exists$ local unitary relating H_{η,i_0} to H_{η,i_0+k}. $\sigma_{i_0}^x H_{\eta,i_0} \sigma_{i_0}^x = H_{\eta,i_0-1}$



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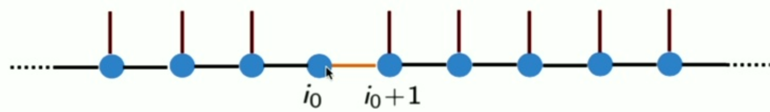
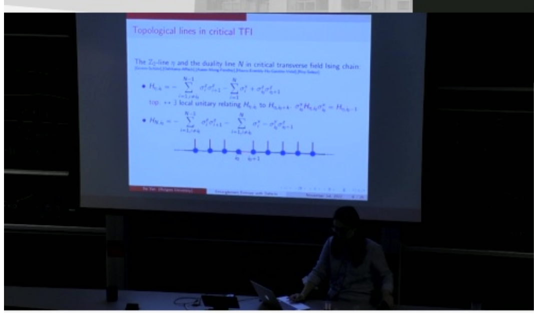
- $H_{N,i_0} = -\sum_{i=1,i\neq i_0}^{N-1} \sigma_i^z\sigma_{i+1}^z - \sum_{i=1,i\neq i_0}^{N} \sigma_i^x - \sigma_{i_0}^y\sigma_{i_0+1}^z$





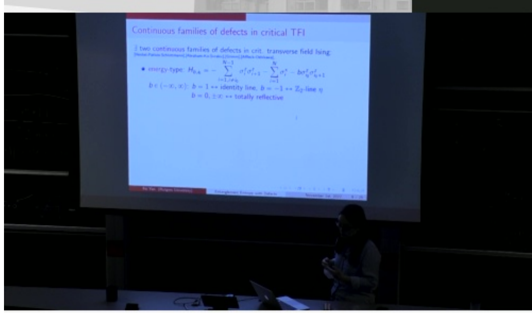
Continuous families of defects in critical TFI

∃ two continuous families of defects in crit. transverse field Ising:
[Henkel-Patkós-Schlottmann],[Abraham-Ko-Svrakic],[Grimm],[Affleck-Oshikawa],...

- energy-type: $H_{b,i_0} = -\sum_{i=1, i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1}^{N} \sigma_i^x - b\sigma_{i_0}^z \sigma_{i_0+1}^z$

  $b \in (-\infty, \infty)$: $b = 1 \leftrightarrow$ identity line, $b = -1 \leftrightarrow \mathbb{Z}_2$-line η

  $b = 0, \pm\infty \leftrightarrow$ totally reflective



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- duality-type: $H_{\tilde{b},i_0} = -\sum_{i=1,i\neq i_0}^{N-1} \sigma_i^z \sigma_{i+1}^z - \sum_{i=1,i\neq i_0}^{N} \sigma_i^x - \tilde{b}\sigma_{i_0}^y \sigma_{i_0+1}^z$

  $\tilde{b} \in [0, \infty)$: $\tilde{b} = 1 \leftrightarrow$ KW duality line N, $b = 0, \infty \leftrightarrow$ totally reflective

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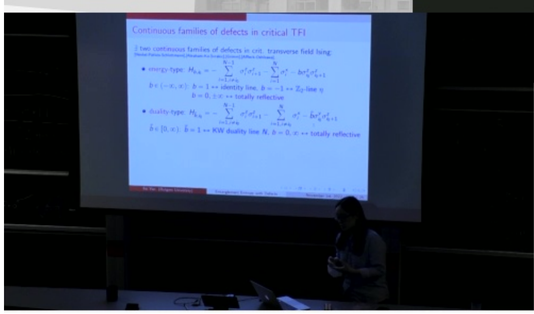

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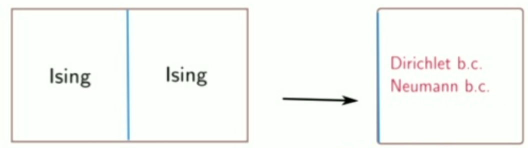
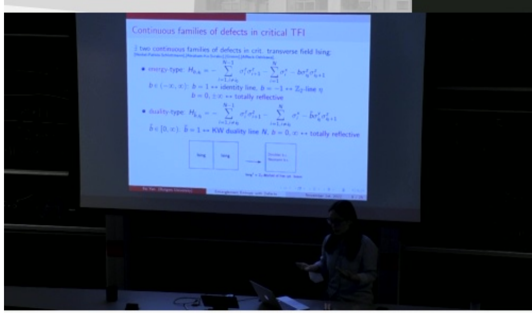
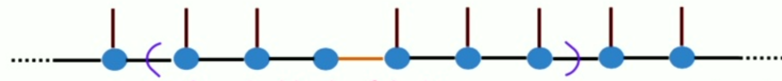


$\text{Ising}^2 \approx \mathbb{Z}_2$-orbifold of free cpt. boson



Ground state EE in critical TFI with defects

[Affleck-Oshikawa],[Saleur],[Sakai-Satoh],[Bachas-Brunner-Roggenkamp],[Herzog-Nishioka],[Brehm-Brunner],[Eisler-Peschel,] [Gutperle-Miller],[Calabrese-Mintchev-Vicari],[Klich-Vaman-Wong],[Roy-Saleur],[Rogerson-Pollmann-Roy],...



A: spin block of L sites

$$S_{A,D} = \frac{c}{3}\log L + S_D^0$$

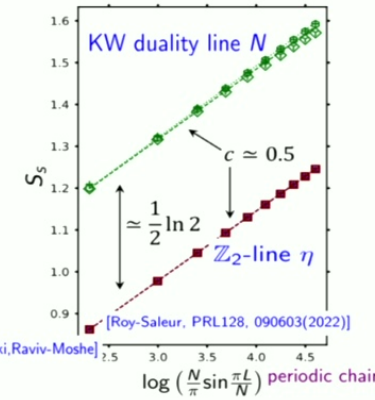
$$S_D^0 = \delta + \log(g_D)$$

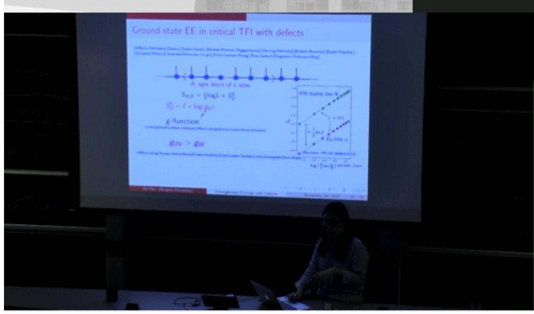
g-function

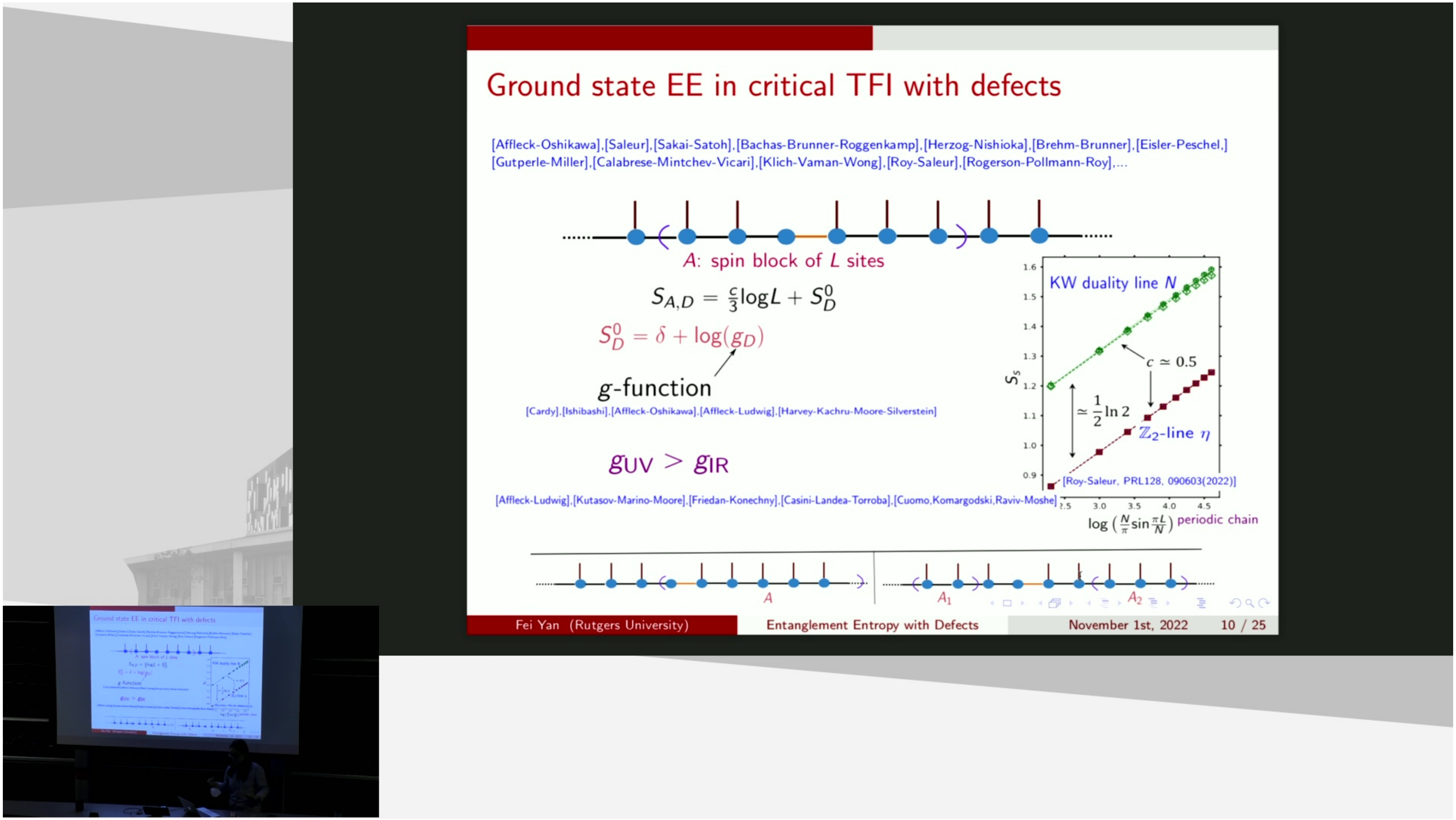
[Cardy],[Ishibashi],[Affleck-Oshikawa],[Affleck-Ludwig],[Harvey-Kachru-Moore-Silverstein]

$$g_{\mathrm{UV}} > g_{\mathrm{IR}}$$

[Affleck-Ludwig],[Kutasov-Marino-Moore],[Friedan-Konechny],[Casini-Landea-Torroba],[Cuomo,Komargodski,Raviv-Moshe]







Ground state EE in critical TFI with defects
[Affleck-Oshikawa],[Saleur],[Sakai-Satoh],[Bachas-Brunner-Roggenkamp],[Herzog-Nishioka],[Brehm-Brunner],[Eisler-Peschel,]
[Gutperle-Miller],[Calabrese-Mintchev-Vicari],[Klich-Vaman-Wong],[Roy-Saleur],[Rogerson-Pollmann-Roy],...
A: spin block of L sites
$S_{A,D} = \frac{c}{3}\log L + S_D^0$
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[Cardy],[Ishibashi],[Affleck-Oshikawa],[Affleck-Ludwig],[Harvey-Kachru-Moore-Silverstein]
$g_{UV} > g_{IR}$
[Affleck-Ludwig],[Kutasov-Marino-Moore],[Friedan-Konechny],[Casini-Landea-Torroba],[Cuomo,Komargodski,Raviv-Moshe]
KW duality line N
$c \simeq 0.5$
$\simeq \frac{1}{2}\ln 2$
$\mathbb{Z}_2$-line η
[Roy-Saleur, PRL128, 090603(2022)]
S_S
$\log\left(\frac{N}{\pi}\sin\frac{\pi L}{N}\right)$ periodic chain
A
A_1
A_2
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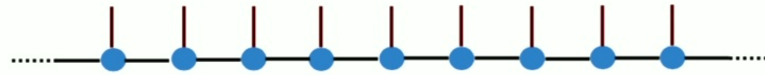
The three-state Potts model



$$H = -\sum_{i=1}^{N-1}\left(\sigma_i^\dagger \sigma_{i+1} + \sigma_{i+1}^\dagger \sigma_i\right) - h\sum_{i=1}^{N}\left(\tau_i + \tau_i^\dagger\right) \quad \text{open b.c.} \qquad \mathcal{H} = (\mathbb{C}^3)^{\otimes N}$$

measure spin

$$\sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w^2 \end{pmatrix}, \quad \tau = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad w = e^{2\pi i/3}$$

shift spin

$$\sigma_i^3 = \tau_i^3 = 1, \sigma_i \tau_i = w \tau_i \sigma_i$$

$S_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_2$

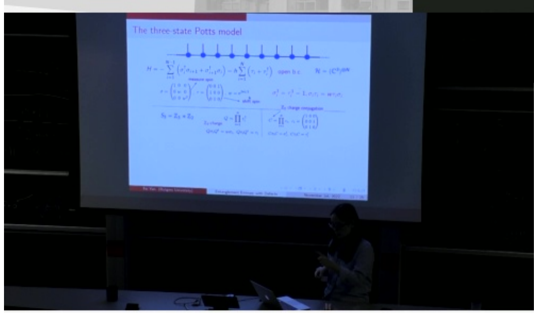
$\mathbb{Z}_3$-charge $\quad Q = \prod_{i=1}^{N} \tau_i^\dagger$

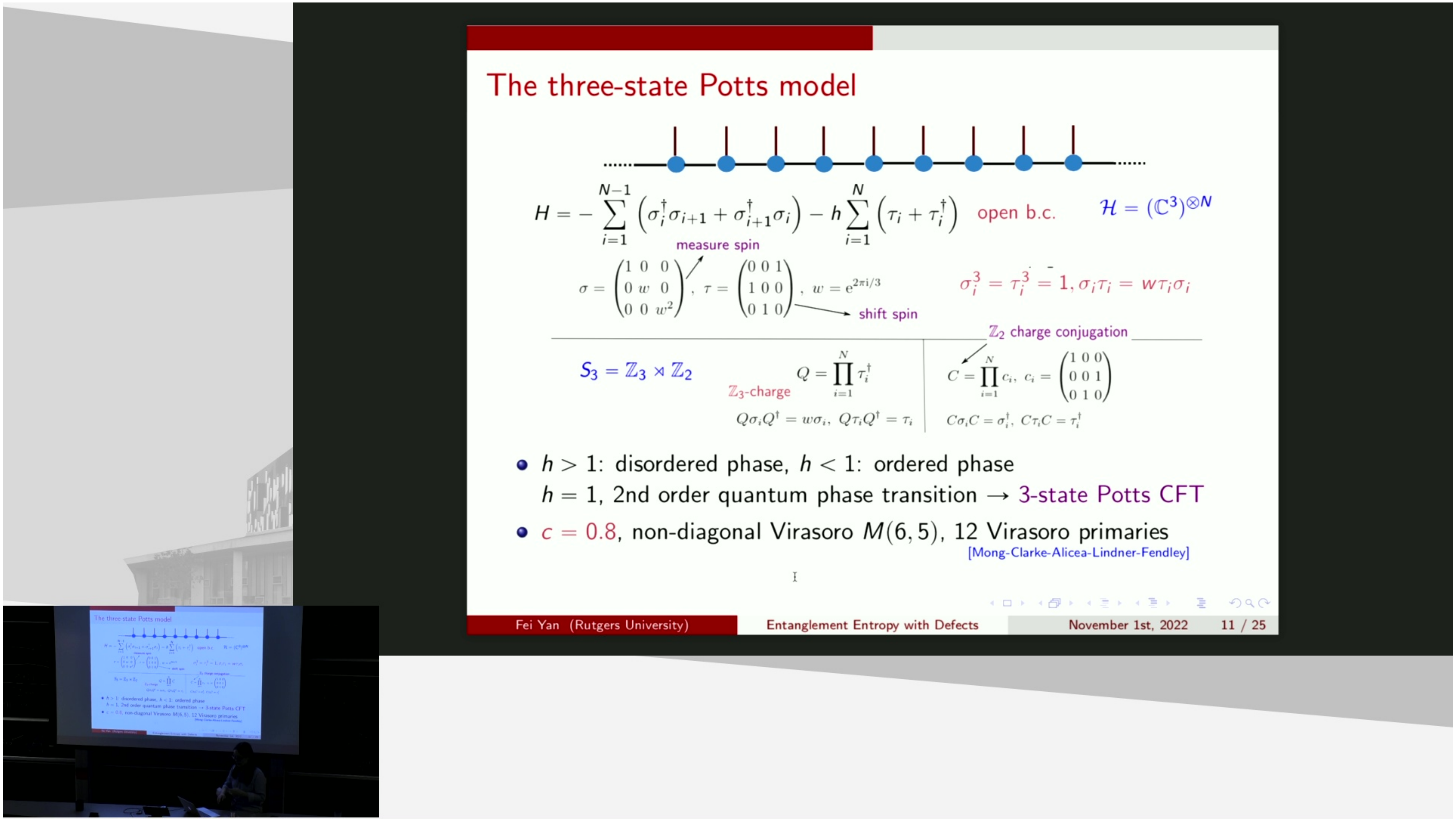
$$Q\sigma_i Q^\dagger = w\sigma_i, \; Q\tau_i Q^\dagger = \tau_i$$

$\mathbb{Z}_2$ charge conjugation

$$C = \prod_{i=1}^{N} c_i, \quad c_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$C\sigma_i C = \sigma_i^\dagger, \; C\tau_i C = \tau_i^\dagger$$




The three-state Potts model
$H = -\sum_{i=1}^{N-1}\left(\sigma_i^\dagger\sigma_{i+1} + \sigma_{i+1}^\dagger\sigma_i\right) - h\sum_{i=1}^{N}\left(\tau_i + \tau_i^\dagger\right)$
open b.c.
$\mathcal{H} = (\mathbb{C}^3)^{\otimes N}$
measure spin
$\sigma = \begin{pmatrix}1&0&0\\0&w&0\\0&0&w^2\end{pmatrix}, \ \tau = \begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}, \ w = e^{2\pi i/3}$
shift spin
$\sigma_i^3 = \tau_i^3 = 1, \sigma_i\tau_i = w\tau_i\sigma_i$
$\mathbb{Z}_2$ charge conjugation
$S_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_2$
$\mathbb{Z}_3$-charge
$Q = \prod_{i=1}^{N}\tau_i^\dagger$
$Q\sigma_i Q^\dagger = w\sigma_i, \ Q\tau_i Q^\dagger = \tau_i$
$C = \prod_{i=1}^{N} c_i, \ c_i = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$
$C\sigma_i C = \sigma_i^\dagger, \ C\tau_i C = \tau_i^\dagger$
• $h > 1$: disordered phase, $h < 1$: ordered phase
$h = 1$, 2nd order quantum phase transition → 3-state Potts CFT
• $c = 0.8$, non-diagonal Virasoro $M(6,5)$, 12 Virasoro primaries
[Mong-Clarke-Alicea-Lindner-Fendley]
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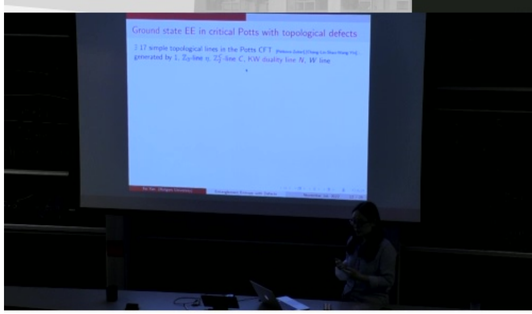
Ground state EE in critical Potts with topological defects

$\exists$ 17 simple topological lines in the Potts CFT [Petkova-Zuber],[Chang-Lin-Shao-Wang-Yin],...
generated by 1, $\mathbb{Z}_3$-line η, $\mathbb{Z}_2^C$-line C, KW duality line N, W line



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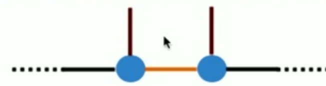
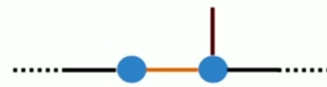
[wip w/ Grans-Samuelsson, Roy, Saleur]



$\mathbb{Z}_3$-line η $\quad e^{2\pi i/3}\sigma_{i_0}^\dagger \sigma_{i_0+1} + \text{h.c.}$

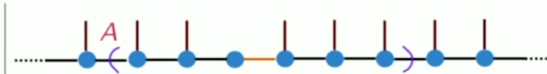
$\mathbb{Z}_2^C$-line C $\quad \sigma_{i_0}^\dagger \sigma_{i_0+1}^\dagger + \text{h.c.}$



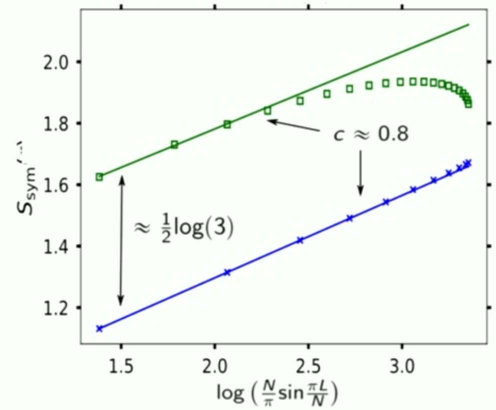
KW duality line N

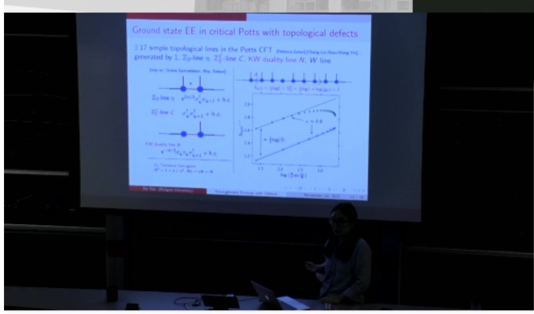
$$e^{-i\pi/3}\sigma_{i_0}\tau_{i_0}\sigma_{i_0+1}^\dagger + \text{h.c.}$$

$\mathbb{Z}_3$ Tambara-Yamagami
$N^2 = 1 + \eta + \eta^2,\ N\eta = \eta N = N$



$S_{A,D} = \frac{c}{3}\log L + S_D^0 = \frac{c}{3}\log L + \log(g_D) + \delta$

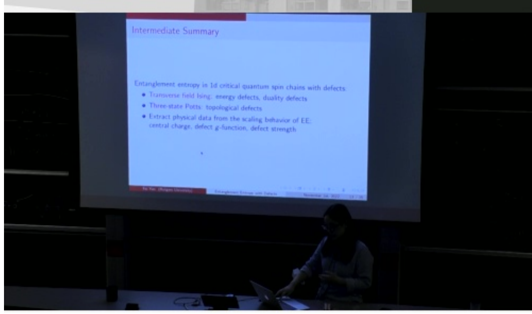




Intermediate Summary

Entanglement entropy in 1d critical quantum spin chains with defects:

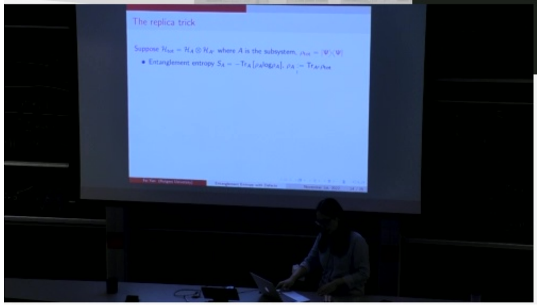
- Transverse field Ising: energy defects, duality defects
- Three-state Potts: topological defects
- Extract physical data from the scaling behavior of EE:
  central charge, defect g-function, defect strength



The replica trick

Suppose $\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_{A^c}$ where A is the subsystem, $\rho_{\text{tot}} = |\Psi\rangle\langle\Psi|$

- Entanglement entropy $S_A = -\text{Tr}_A\left[\rho_A \log \rho_A\right]$, $\rho_A := \text{Tr}_{A^c}\rho_{\text{tot}}$



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- $S_A = -\lim_{n\to 1} \partial_n \log \text{Tr}_A\left(\rho_A^n\right) = \lim_{n\to 1} S_A^{(n)}$

  Rényi entropy $S_A^{(n)} = \frac{1}{1-n}\log \text{Tr}_A\left(\rho_A^n\right)$

  $S_A^{(n)}$ defined for $n \in \mathbb{Z}_+$, analytic continuation to $\mathbb{R}_+$ in taking $n \to 1$.

[Cardy-Calabrese],[Ohmori-Tachikawa],...

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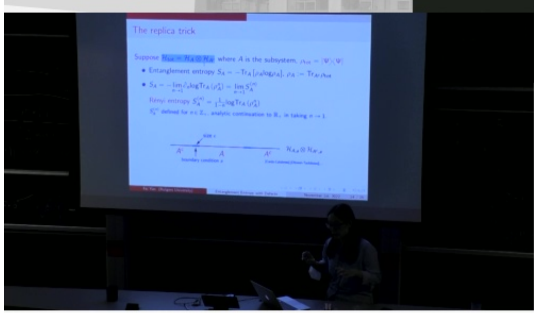
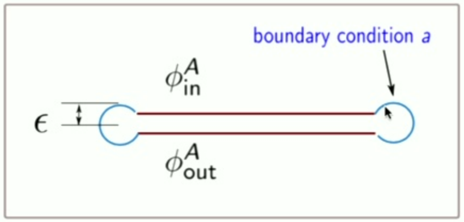


Path integral formulation of the replica trick

[Calabrese-Cardy],[Holzhey-Larsen-Wilczek],...,[Ohmori-Tachikawa],...

$$\rho_A := \mathrm{Tr}_{A^c} |\Psi_g\rangle\langle\Psi_g|$$

$$\langle\phi^A_{\mathrm{out}}|\rho_A|\phi^A_{\mathrm{in}}\rangle = \frac{1}{Z} \times$$



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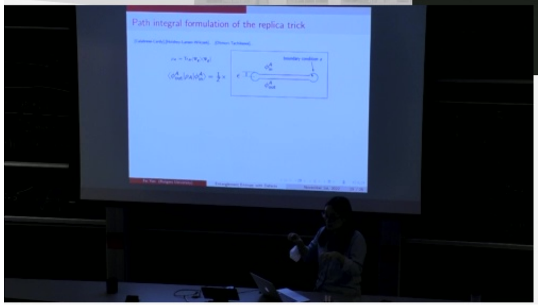
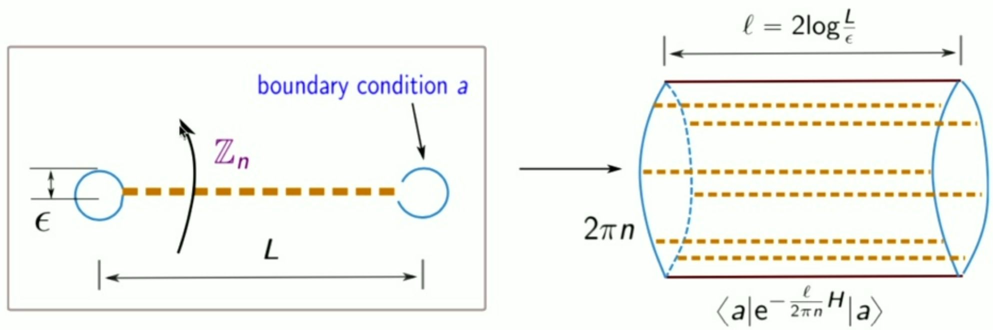
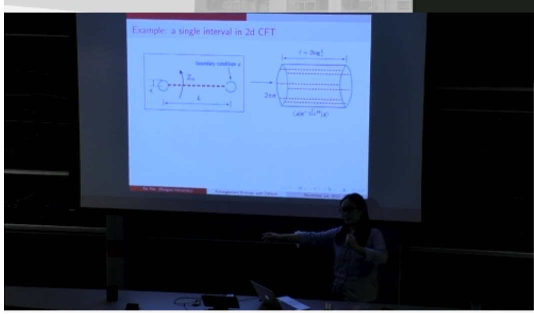
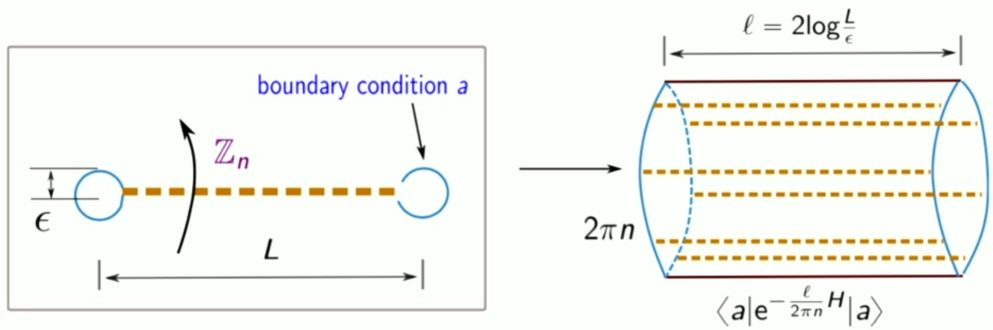


Example: a single interval in 2d CFT





Example: a single interval in 2d CFT

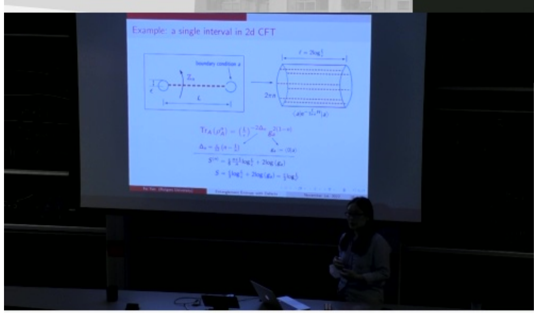


$$\mathrm{Tr}_A\left(\rho_A^n\right) = \left(\frac{L}{\epsilon}\right)^{-2\Delta_n} g_a^{2(1-n)}$$

$$\Delta_n = \frac{c}{12}\left(n - \frac{1}{n}\right) \qquad g_a := \langle 0|a\rangle$$

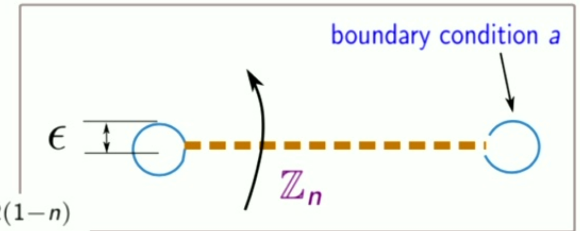
$$S^{(n)} = \frac{c}{6}\frac{n+1}{n}\log\frac{L}{\epsilon} + 2\log\left(g_a\right)$$

$$S = \frac{c}{3}\log\frac{L}{\epsilon} + 2\log\left(g_a\right) = \frac{c}{3}\log\frac{L}{\epsilon'}$$



The twist fields

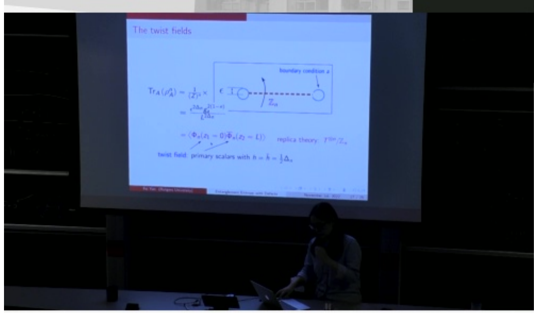
$$\mathrm{Tr}_A(\rho_A^n) = \frac{1}{(Z)^n} \times$$



$$= \frac{\epsilon^{2\Delta_n} g_a^{2(1-n)}}{L^{2\Delta_n}}$$

$$= \langle \Phi_n(z_1 = 0)\overline{\Phi}_n(z_2 = L)\rangle$$ replica theory: $T^{\otimes n}/\mathbb{Z}_n$

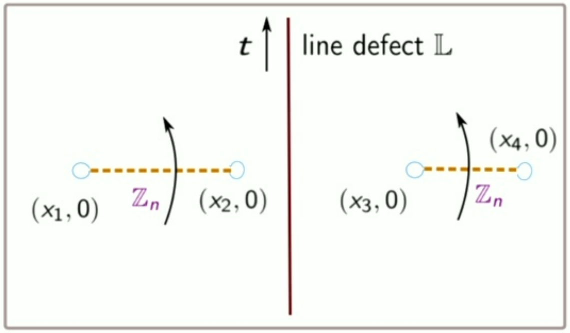
twist field: primary scalars with $h = \bar{h} = \frac{1}{2}\Delta_n$



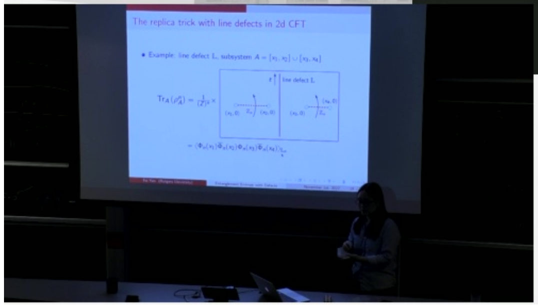
The replica trick with line defects in 2d CFT

- Example: line defect $\mathbb{L}$, subsystem $A = [x_1, x_2] \cup [x_3, x_4]$

$$\mathrm{Tr}_A\left(\rho_A^n\right) = \frac{1}{(Z)^n} \times$$



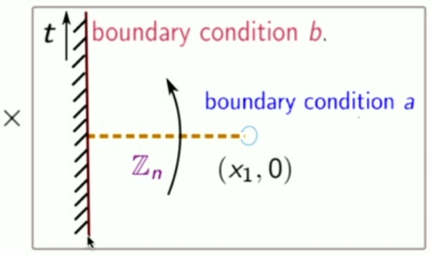
$$= \langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\Phi_n(x_3)\overline{\Phi}_n(x_4)\rangle_{\mathbb{L}_n}$$



BCFT: an interval touching the boundary

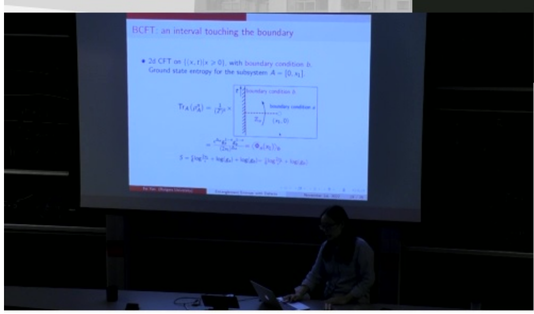
- 2d CFT on $\{(x,t)|x \geqslant 0\}$, with boundary condition b. Ground state entropy for the subsystem $A = [0, x_1]$.

$$\mathrm{Tr}_A\left(\rho_A^n\right) = \frac{1}{(Z)^n} \times$$



$$= \frac{\epsilon^{\Delta_n} g_a^{1-n} g_b^{1-n}}{(2x_1)^{\Delta_n}} = \langle \overline{\Phi}_n(x_1) \rangle_b$$

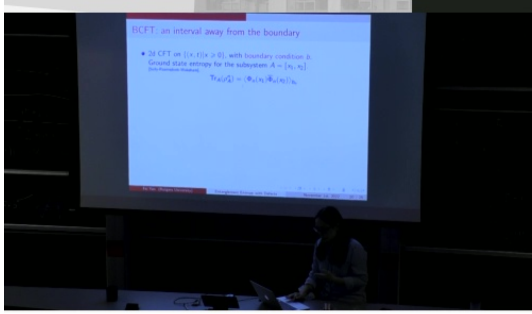
$$S = \frac{c}{6} \log \frac{2x_1}{\epsilon} + \log(g_a) + \log(g_b) = \frac{c}{6} \log \frac{2x_1}{\epsilon'} + \log(g_b)$$



BCFT: an interval away from the boundary

- 2d CFT on $\{(x,t)|x \geqslant 0\}$, with boundary condition b.
  Ground state entropy for the subsystem $A = [x_1, x_2]$.
  [Sully-Raamsdonk-Wakeham],...

$$\mathrm{Tr}_A(\rho_A^n) = \langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\rangle_{b_n}$$



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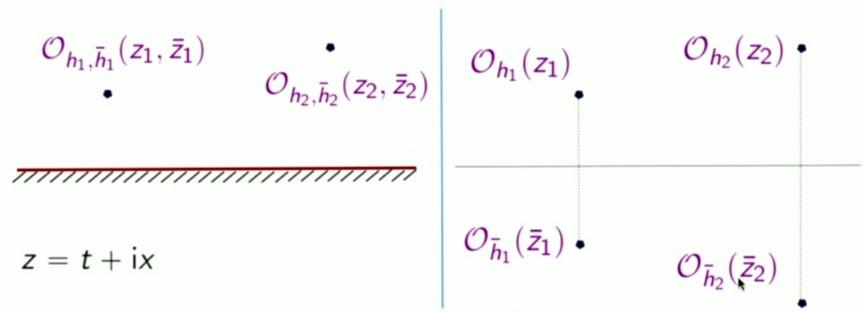
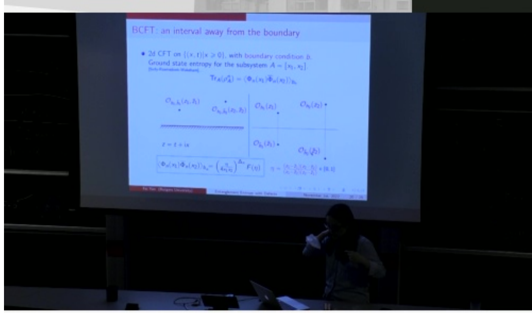
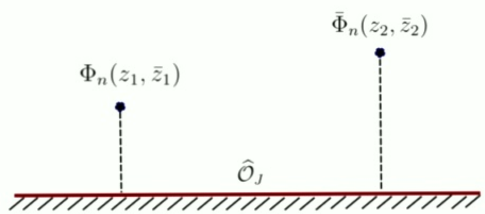


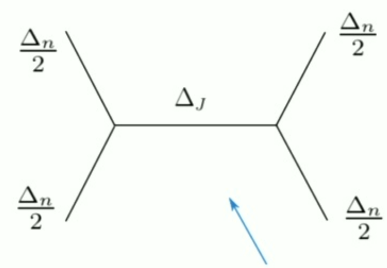
$$\langle \Phi_n(x_1)\overline{\Phi}_n(x_2)\rangle_{b_n} = \left(\frac{\eta}{4x_1x_2}\right)^{\Delta_n} F(\eta)$$

$$\eta = \frac{(z_1-\bar{z}_1)(z_2-\bar{z}_2)}{(z_1-\bar{z}_2)(z_2-\bar{z}_1)} \in [0,1]$$



The boundary channel



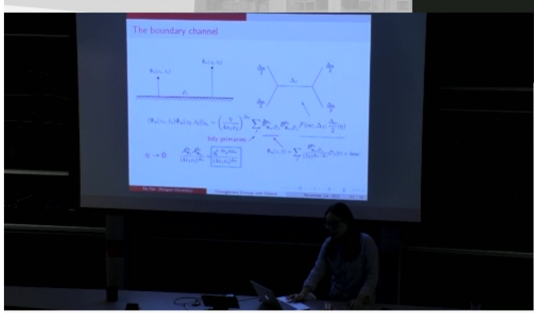


$$\langle \Phi_n(z_1,\bar z_1)\bar\Phi_n(z_2,\bar z_2)\rangle_{b_n} = \left(\frac{\eta}{4x_1x_2}\right)^{\Delta_n}\sum_J \mathcal{B}^{b_n}_{\Phi_n,\hat{\mathcal{O}}_J}\mathcal{B}^{b_n}_{\bar\Phi_n,\hat{\mathcal{O}}_J}\mathcal{F}(nc,\Delta_J;\frac{\Delta_n}{2}|\eta)$$

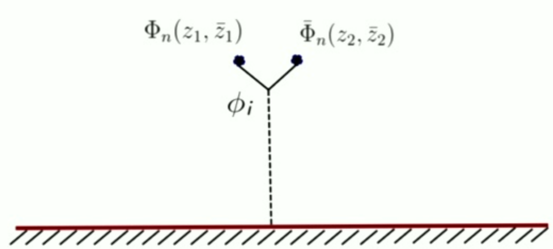
bdy primaries

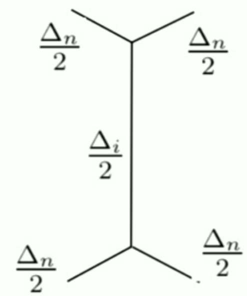
$$\Phi_n(z,\bar z) = \sum_J \frac{\mathcal{B}^{b_n}_{\Phi_n,\hat{\mathcal{O}}_J}}{(2x)^{\Delta_n-\Delta_J}}\hat{\mathcal{O}}_J(t) + \text{desc}$$

$\eta \to 0$: $$\frac{\mathcal{A}^{b_n}_{\Phi_n}\mathcal{A}^{b_n}_{\bar\Phi_n}}{(4x_1x_2)^{\Delta_n}} = \boxed{\frac{g_b^{2-2n}\epsilon'^{2\Delta_n}}{(4x_1x_2)^{\Delta_n}}}$$



The bulk channel





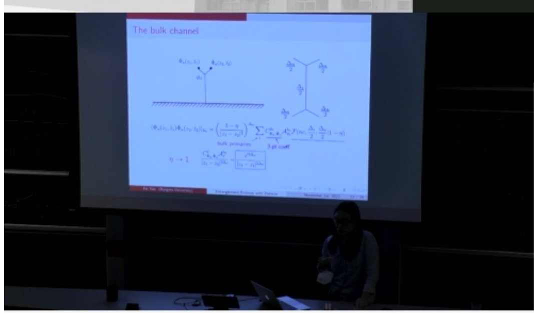
$$\langle \Phi_n(z_1, \bar{z}_1) \bar{\Phi}_n(z_2, \bar{z}_2) \rangle_{b_n} = \left(\frac{1-\eta}{|z_1 - z_2|^2} \right)^{\Delta_n} \sum_i C^{\phi_i}_{\Phi_n, \bar{\Phi}_n} \mathcal{A}^{b_n}_{\phi_i} \mathcal{F}(nc, \frac{\Delta_i}{2}; \frac{\Delta_n}{2} | 1-\eta)$$

bulk primaries

3-pt coeff.

$\eta \to 1$

$$\frac{C^I_{\Phi_n, \bar{\Phi}_n} \mathcal{A}^{b_n}_I}{|z_1 - z_2|^{2\Delta_n}} = \boxed{\frac{\epsilon'^{2\Delta_n}}{|z_1 - z_2|^{2\Delta_n}}}$$



BCFT entropy: special limits

[Sully-Raamsdonk-Wakeham]

Rényi entropy

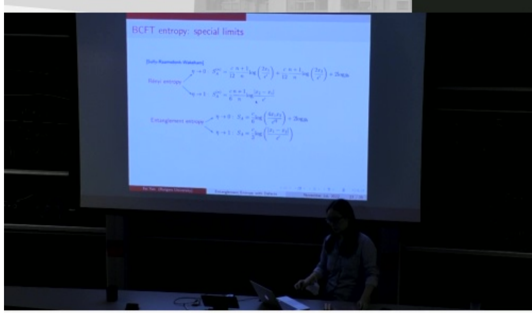
$$\eta \to 0: \; S_A^{(n)} = \frac{c}{12}\frac{n+1}{n}\log\left(\frac{2x_1}{\epsilon'}\right) + \frac{c}{12}\frac{n+1}{n}\log\left(\frac{2x_2}{\epsilon'}\right) + 2\log g_b$$

$$\eta \to 1: \; S_A^{(n)} = \frac{c}{6}\frac{n+1}{n}\log\frac{|x_2 - x_1|}{\epsilon'}$$

Entanglement entropy

$$\eta \to 0: \; S_A = \frac{c}{6}\log\left(\frac{4x_1x_2}{\epsilon'^2}\right) + 2\log g_b$$

$$\eta \to 1: \; S_A = \frac{c}{3}\log\left(\frac{|x_1 - x_2|}{\epsilon'}\right)$$



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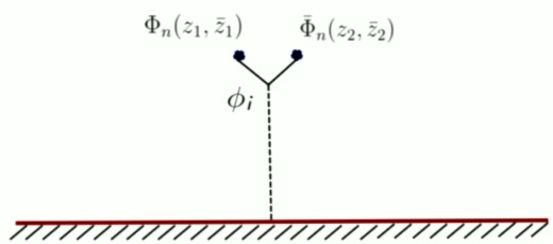

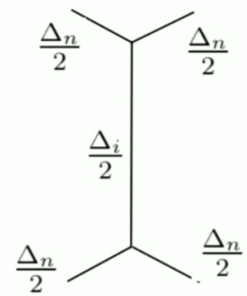

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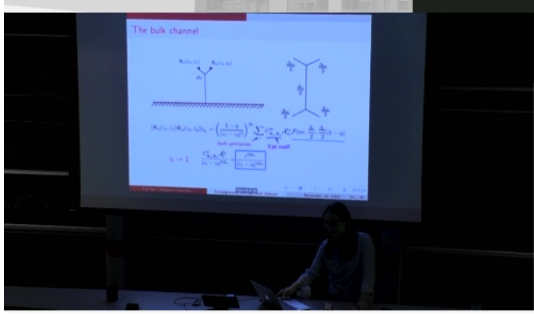
bulk primaries

3-pt coeff.

$\eta \to 1$

$$\frac{C^I_{\Phi_n,\bar\Phi_n}\mathcal{A}^{b_n}_I}{|z_1-z_2|^{2\Delta_n}} = \boxed{\frac{\epsilon'^{2\Delta_n}}{|z_1-z_2|^{2\Delta_n}}}$$



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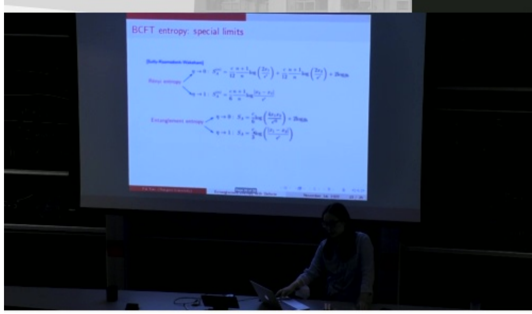

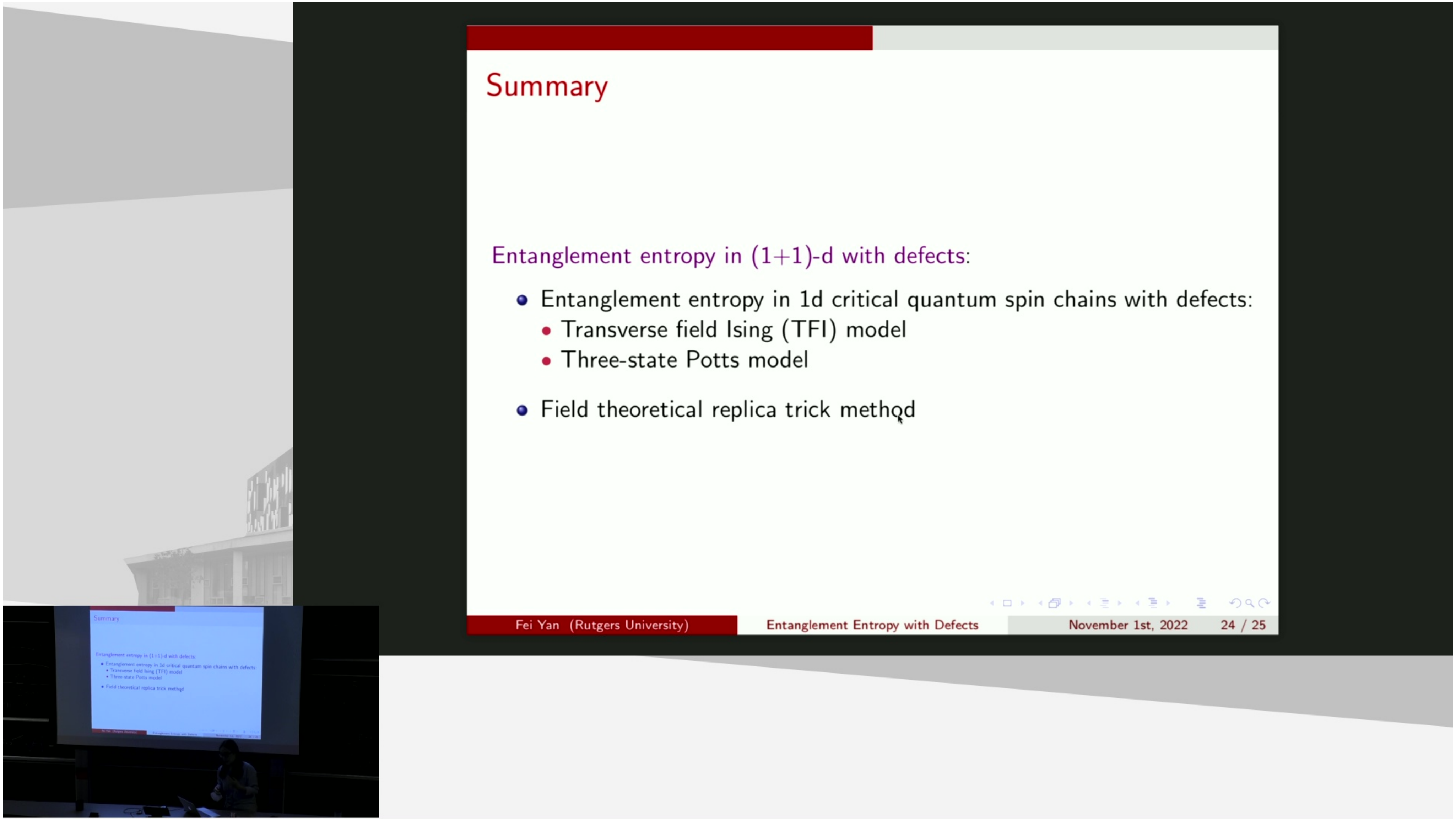
Summary
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Entanglement entropy in 1d critical quantum spin chains with defects:
Transverse field Ising (TFI) model
Three-state Potts model
Field theoretical replica trick method
Fei Yan (Rutgers University)
Entanglement Entropy with Defects
November 1st, 2022
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