Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

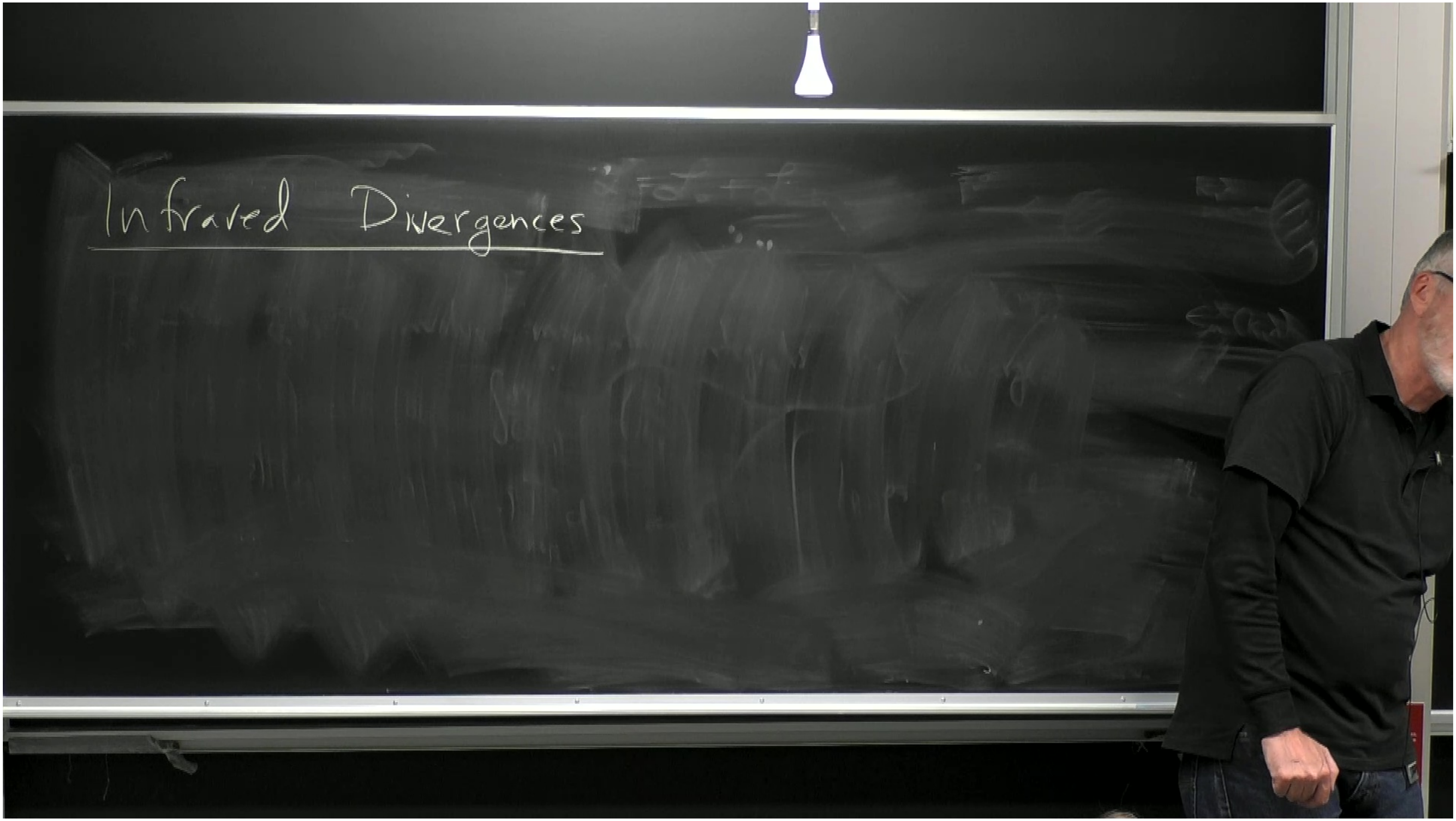
Speakers: Cliff Burgess

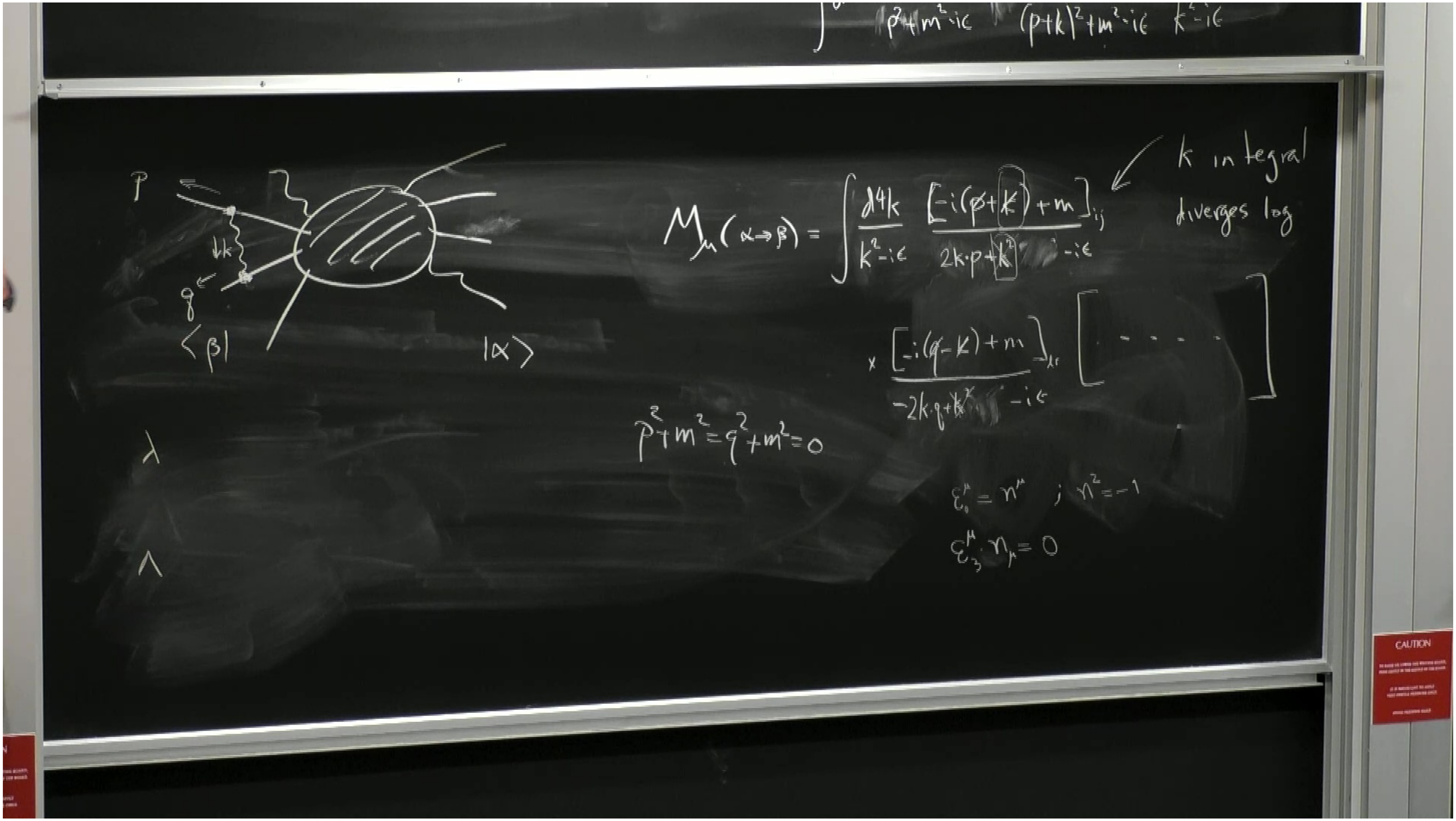
Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: November 29, 2022 - 2:30 PM

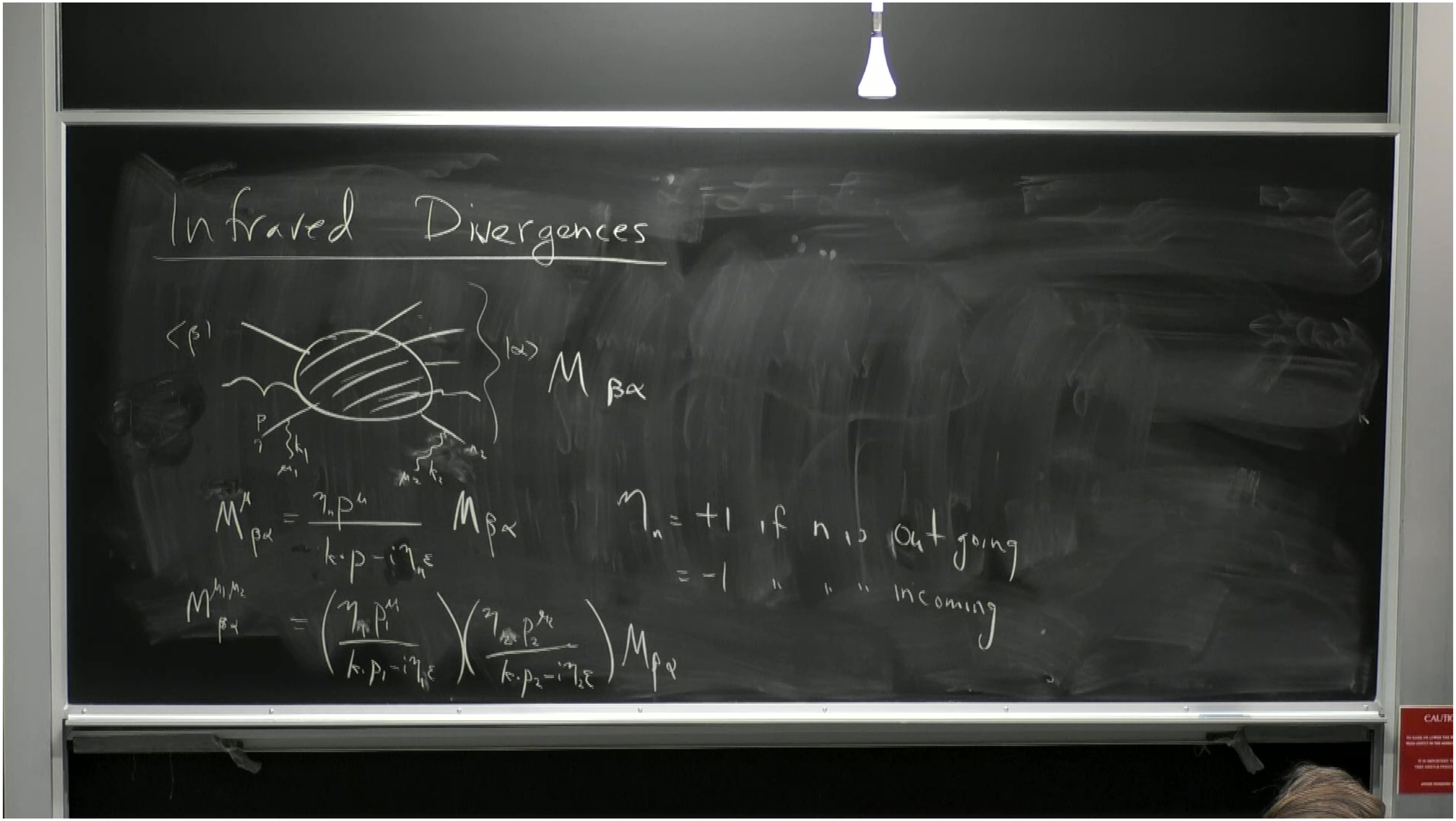
URL: https://pirsa.org/22110042

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


Infrared Divergences



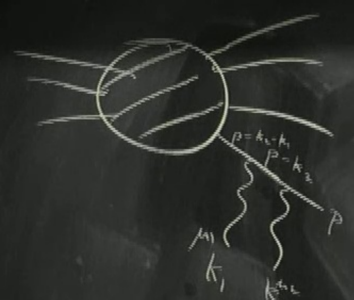
$p^2+m^2-i\epsilon$ $(p+k)^2+m^2-i\epsilon$ $k^2-i\epsilon$
p
k
q
$\langle\beta|$
$|\alpha\rangle$
λ
Λ
$M_\mu(\alpha\to\beta) = \int \frac{d^4k}{k^2-i\epsilon} \frac{[-i(\not p+\not k)+m]_{ij}}{2k\cdot p+k^2-i\epsilon}$
$\times \frac{[-i(\not q-\not k)+m]_{\lambda r}}{-2k\cdot q+k^2-i\epsilon} [\ \cdots\ ]$
k integral diverges log
$p^2+m^2=q^2+m^2=0$
$\varepsilon_0^\mu = n^\mu$; $n^2=-1$
$\varepsilon_3^\mu n_\mu = 0$
CAUTION


Infrared Divergences
$\langle\beta|$
$|\alpha\rangle$ $M_{\beta\alpha}$
p η k_1 μ_1 μ_2 k_2 n_2
$M^{\mu}_{\beta\alpha} = \frac{\eta_n p^{\mu}}{k\cdot p - i\eta_n\varepsilon} M_{\beta\alpha}$
$\eta_n = +1$ if n is outgoing
$= -1$ " " " incoming
$M^{\mu_1\mu_2}_{\beta\alpha} = \left(\frac{\eta_1 p_1^{\mu_1}}{k\cdot p_1 - i\eta_1\varepsilon}\right)\left(\frac{\eta_2 p_2^{\mu_2}}{k\cdot p_2 - i\eta_2\varepsilon}\right) M_{\beta\alpha}$

$$k\cdot p - i\eta_n \varepsilon$$

$= -1$ " " incoming

$$M^{\mu_1\mu_2}_{\beta\alpha} = \left(\frac{\eta_1 p_1^{\mu_1}}{k_1\cdot p_1 - i\eta_1\varepsilon}\right)\left(\frac{\eta_2 p_2^{\mu_2}}{k_2\cdot p_2 - i\eta_2\varepsilon}\right) M_{\beta\alpha}$$



$$M^{\mu_1\mu_2}_{\beta\alpha} = \frac{\eta p^{\mu_1}}{k_2\cdot p - i\eta\varepsilon}\,\frac{\eta p^{\mu_2}}{(k_1+k_2)\cdot p - i\eta\varepsilon} M_{\beta\alpha}$$

$$+ \frac{p^{\mu_1} p^{\mu_2}}{(k_1\cdot p - i\eta\varepsilon)((k_1+k_2)\cdot p - i\eta\varepsilon)} M_{\beta\alpha}$$

$$\frac{1}{k_2\cdot p} + \frac{1}{k_1\cdot p} = \frac{(k_1+k_2)\cdot p}{(k_1\cdot p)(k_2\cdot p)}$$

$$\cdots \; p^2+m^2-i\epsilon \quad (p+k)^2+m^2-i\epsilon \quad k^2-i\epsilon$$

$$M^{\mu_1\mu_2}_{\beta\alpha} = \frac{\eta^2 p^{\mu_1} p^{\mu_2}}{(k_1\cdot p - i\eta\epsilon)(k_2\cdot p - i\eta\epsilon)} M_{\beta\alpha}$$

$$M^{\mu_1\cdots\mu_N}_{\beta\alpha} = \left[\prod_{i=1}^{N} \frac{\eta_{n_i} p_{n_i}^{\mu_i}}{k_i\cdot p_{n_i} - i\eta_{n_i}\epsilon}\right] M_{\beta\alpha}$$

pair off ends with photon propagator, + do the loops

$$\tilde{M}_{\beta\alpha} = \sum_{R=0}^{\infty} \left\{ \frac{1}{2^R R!} \left[-i \int_{\lambda}^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{\eta_{\mu\nu}}{k^2 - i\varepsilon} \sum_{nm} \frac{\eta_n Q_n p_n^{\mu}}{p_n \cdot k - i\eta_n \varepsilon} \frac{\eta_m Q_m p_m^{\nu}}{(-p_m \cdot k - i\eta_m \varepsilon)} \right]^R \right\}$$

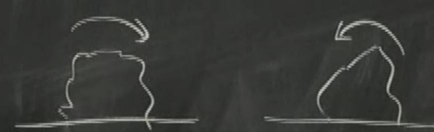
$2^R!$

pair off ends with photon propagator, + do the loops

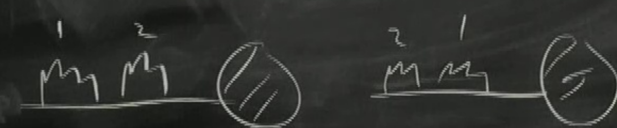
$$\tilde{M}_{\beta\alpha} = \sum_{R=0}^{\infty} \left\{ \frac{1}{2^R R!} \left[-i \int_{\lambda}^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{\eta_{\mu\nu}}{k^2 - i\varepsilon} \sum_{nm} \frac{\eta_n Q_n p_n^\mu}{p_n \cdot k - i\eta_n \varepsilon} \frac{\eta_m Q_m p_m^\nu}{(-p_m \cdot k - i\eta_m \varepsilon)} \right]^R \right\} M_{\beta\alpha}$$

Q_n = charge

2^R :



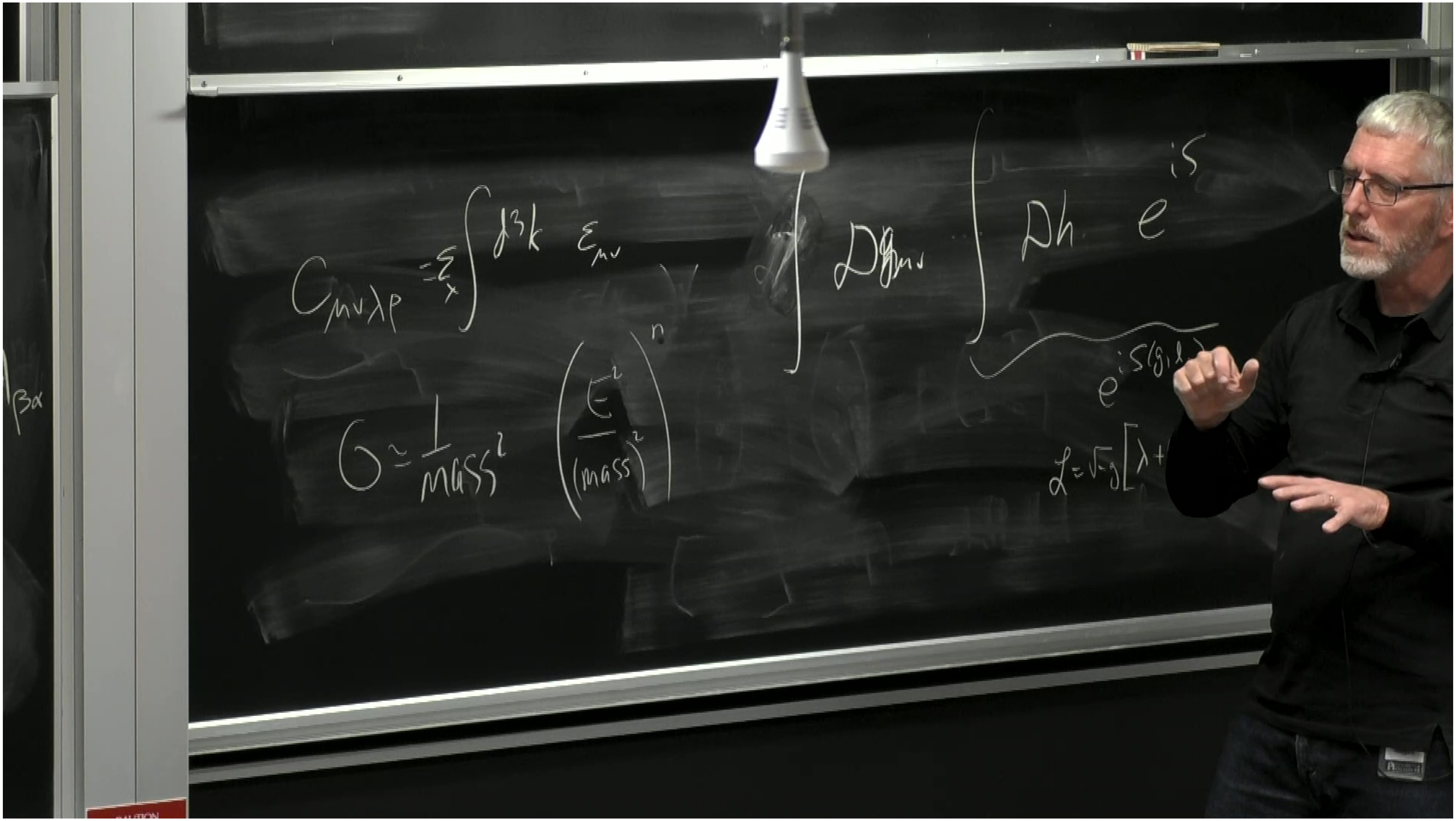
$R!$:



$$\tilde{M}_{\beta\alpha} = \mathcal{F}(\lambda, \Lambda) M_{\beta\alpha}(\Lambda)$$

$$\mathcal{F} = \exp\left[\frac{1}{2} F\right]$$

$$F = -i \int_{\lambda}^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{\eta_{\mu\nu}}{k^2 - i\epsilon} \sum_{nm} \Big[\qquad \Big]$$



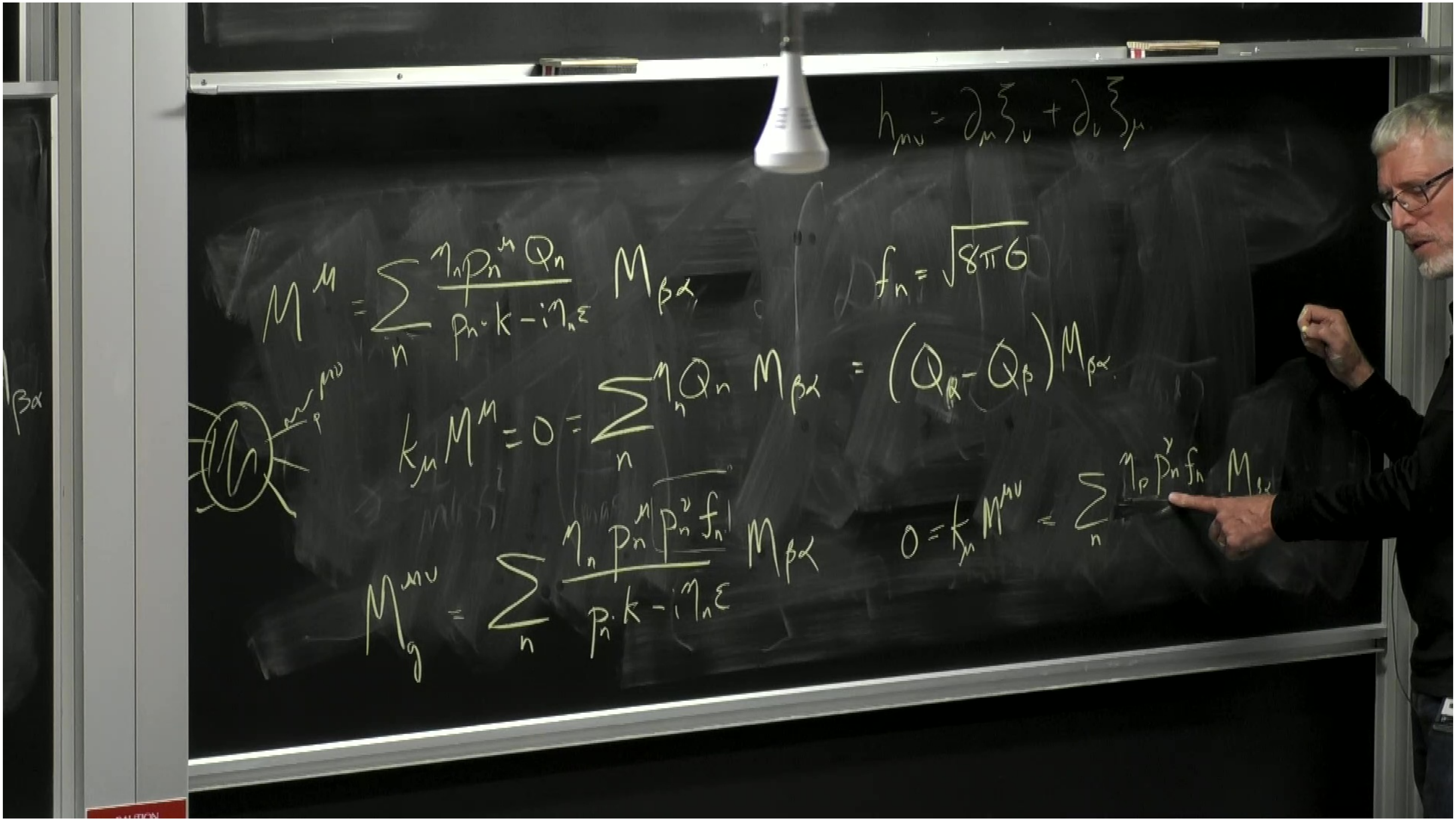
$$C_{\mu\nu\lambda\rho} = \sum_{\lambda} \int d^3k \; \varepsilon_{\mu\nu}$$

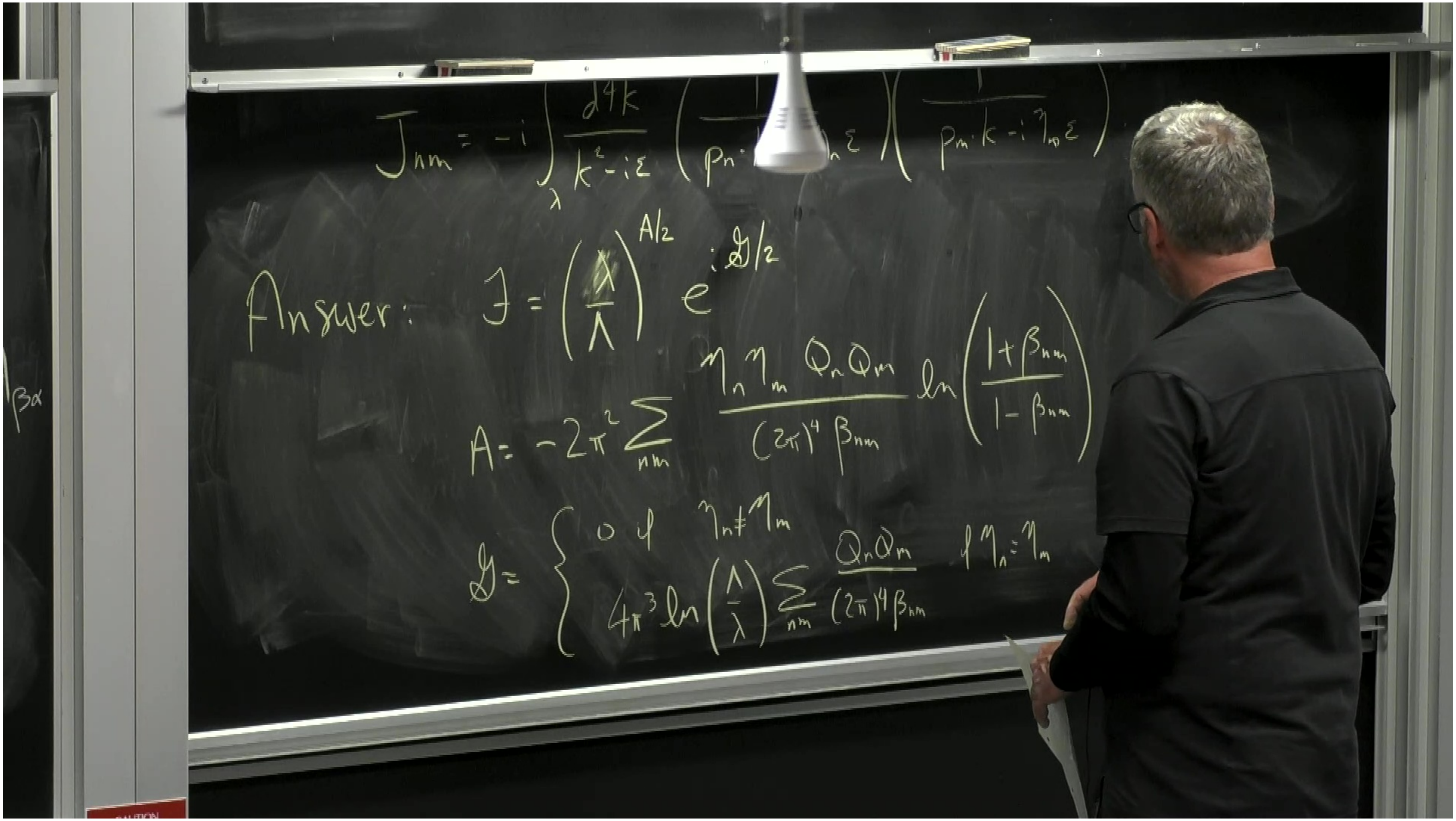
$$\int \mathcal{D}g_{\mu\nu} \quad \int \mathcal{D}h \; e^{iS}$$

$$e^{iS(g,\ell)}$$

$$G \simeq \frac{1}{\text{mass}^2} \left(\frac{E^2}{(\text{mass})^2} \right)^n$$

$$\mathcal{L} = \sqrt{-g}\left[\lambda + M_p^2 R + c R^2 + \cdots\right]$$


$h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
$M^\mu = \sum_n \frac{\eta_n p_n^\mu Q_n}{p_n \cdot k - i\eta_n \varepsilon} M_{\beta\alpha}$
$f_n = \sqrt{8\pi G}$
$k_\mu M^\mu = 0 = \sum_n \eta_n Q_n M_{\beta\alpha} = (Q_\alpha - Q_\beta) M_{\beta\alpha}$
$M_g^{\mu\nu} = \sum_n \frac{\eta_n p_n^\mu \boxed{p_n^\nu f_n}}{p_n \cdot k - i\eta_n \varepsilon} M_{\beta\alpha}$
$0 = k_\mu M^{\mu\nu} = \sum_n \eta_n p_n^\nu f_n M_{\beta\alpha}$


$J_{nm} = -i \int_\lambda \frac{d^4k}{k^2 - i\varepsilon}$
Answer: $\mathcal{J} = \left(\frac{\lambda}{\Lambda}\right)^{A/2} e^{i\mathcal{G}/2}$
$A = -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$
$\mathcal{G} = \begin{cases} 0 & \text{if } \eta_n \neq \eta_m \\ 4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}} & \text{if } \eta_n = \eta_m \end{cases}$

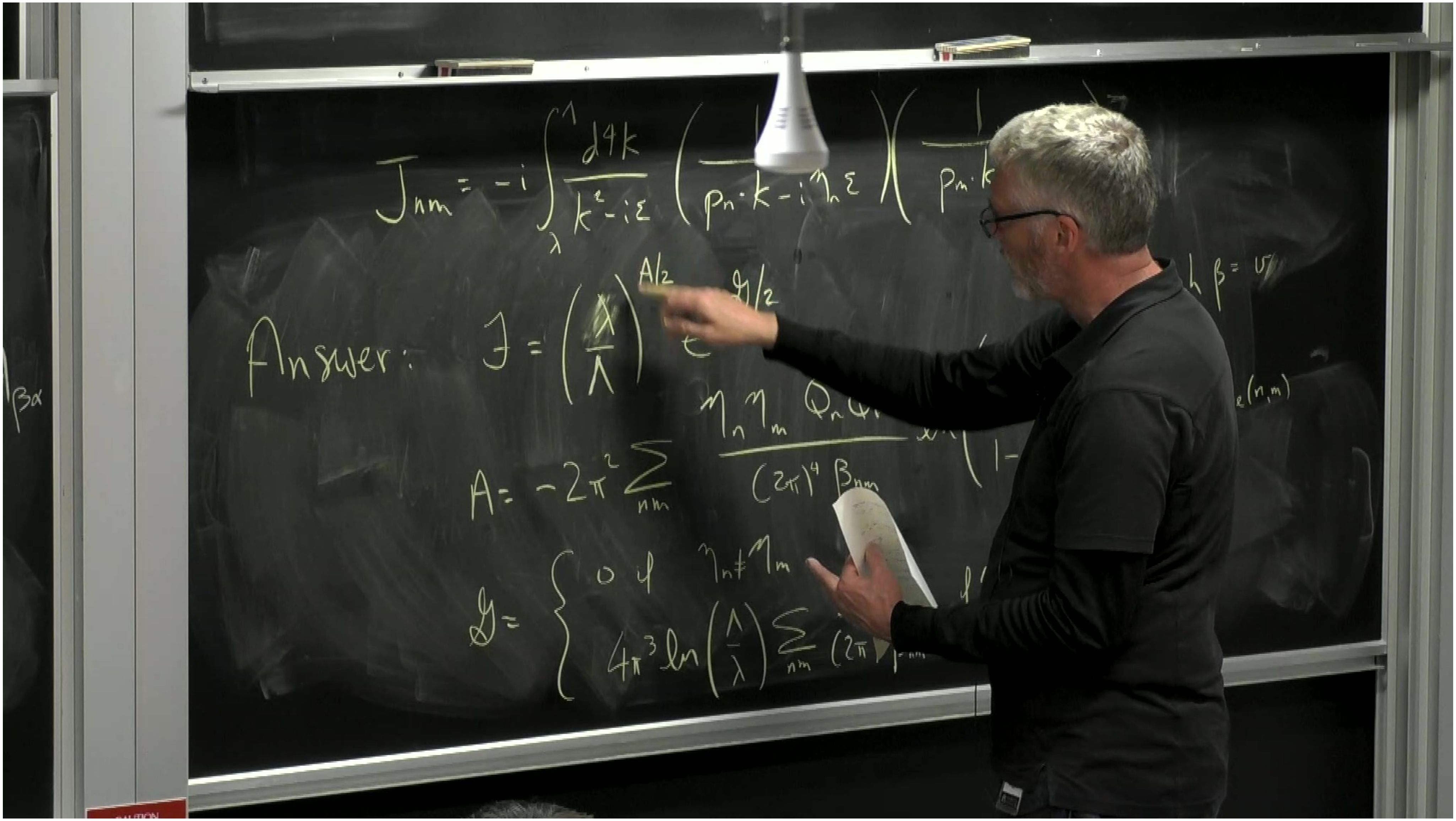
Answer: $\mathcal{I} = \left(\frac{\lambda}{\Lambda}\right)^{A/2} e^{i\mathcal{G}/2}$

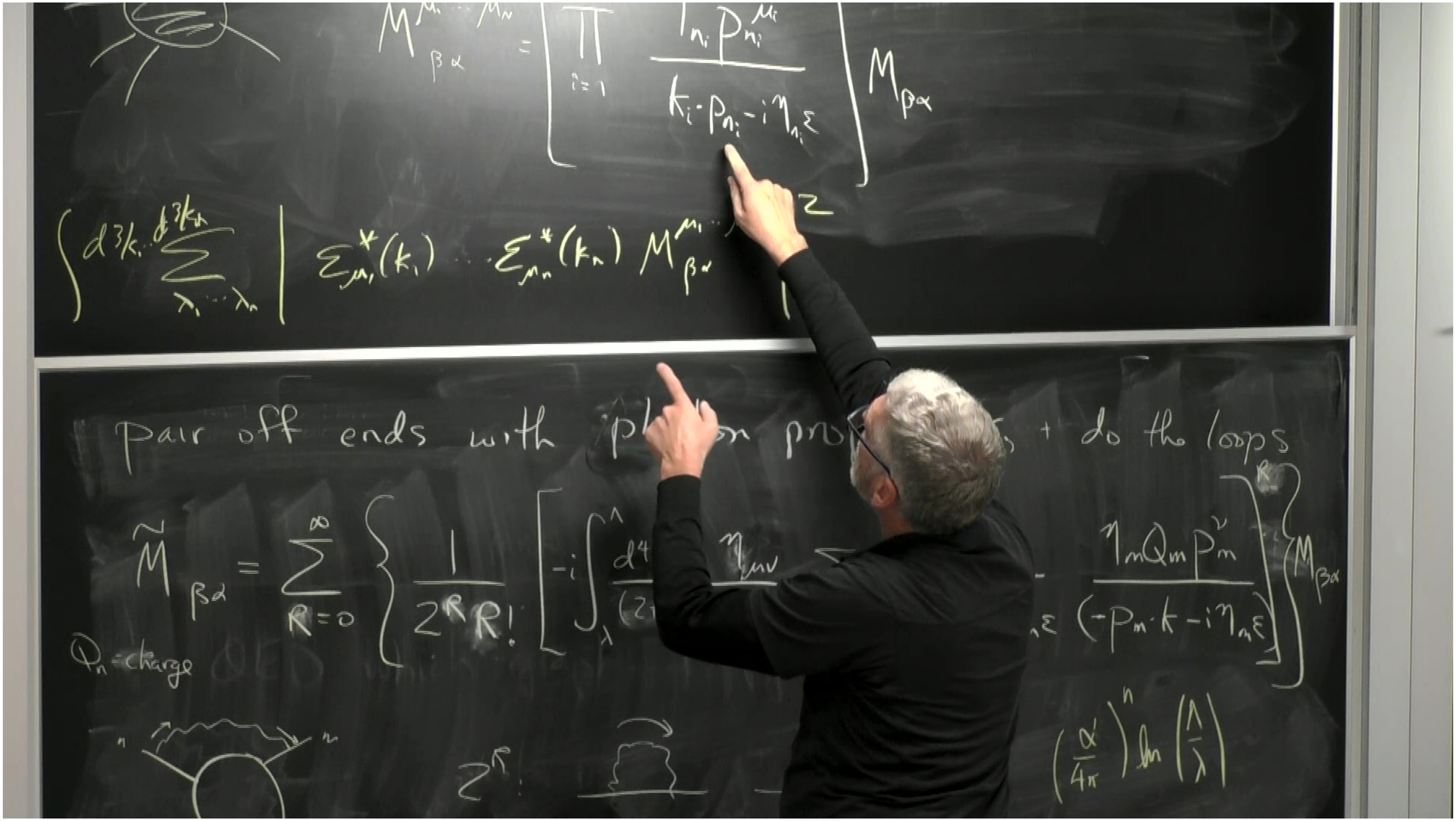
$\tanh\beta = v$

$$A = -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$$

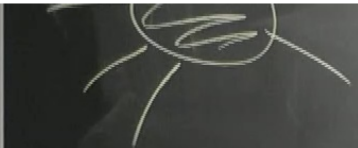
$\beta_{nm} = v_{rel}(n,m)$

$$\mathcal{G} = \begin{cases} 0 & \text{if } \eta_n \neq \eta_m \\ 4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}} & \text{if } \eta_n = \eta_m \end{cases}$$


Answer:



pair off ends with
do the loops
Q_n = charge



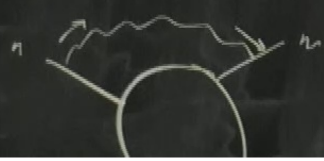
$$M^{\mu_1\cdots\mu_N}_{\beta\alpha} = \left[\prod_{i=1} \frac{\eta_{n_i} p^{\mu_i}_{n_i}}{k_i\cdot p_{n_i} - i\eta_{n_i}\varepsilon}\right] M_{\beta\alpha}$$

$$\int \frac{d^3k_1}{\omega_{k_1}} \cdots d^3k_n \sum_{\lambda_1\cdots\lambda_n} \left| \varepsilon^*_{\mu_1}(k_1)\cdots\varepsilon^*_{\mu_n}(k_n)\, M^{\mu_1\cdots\mu_n}_{\beta\alpha}\right|^2$$

pair off ends with photon propagator, + do the loops

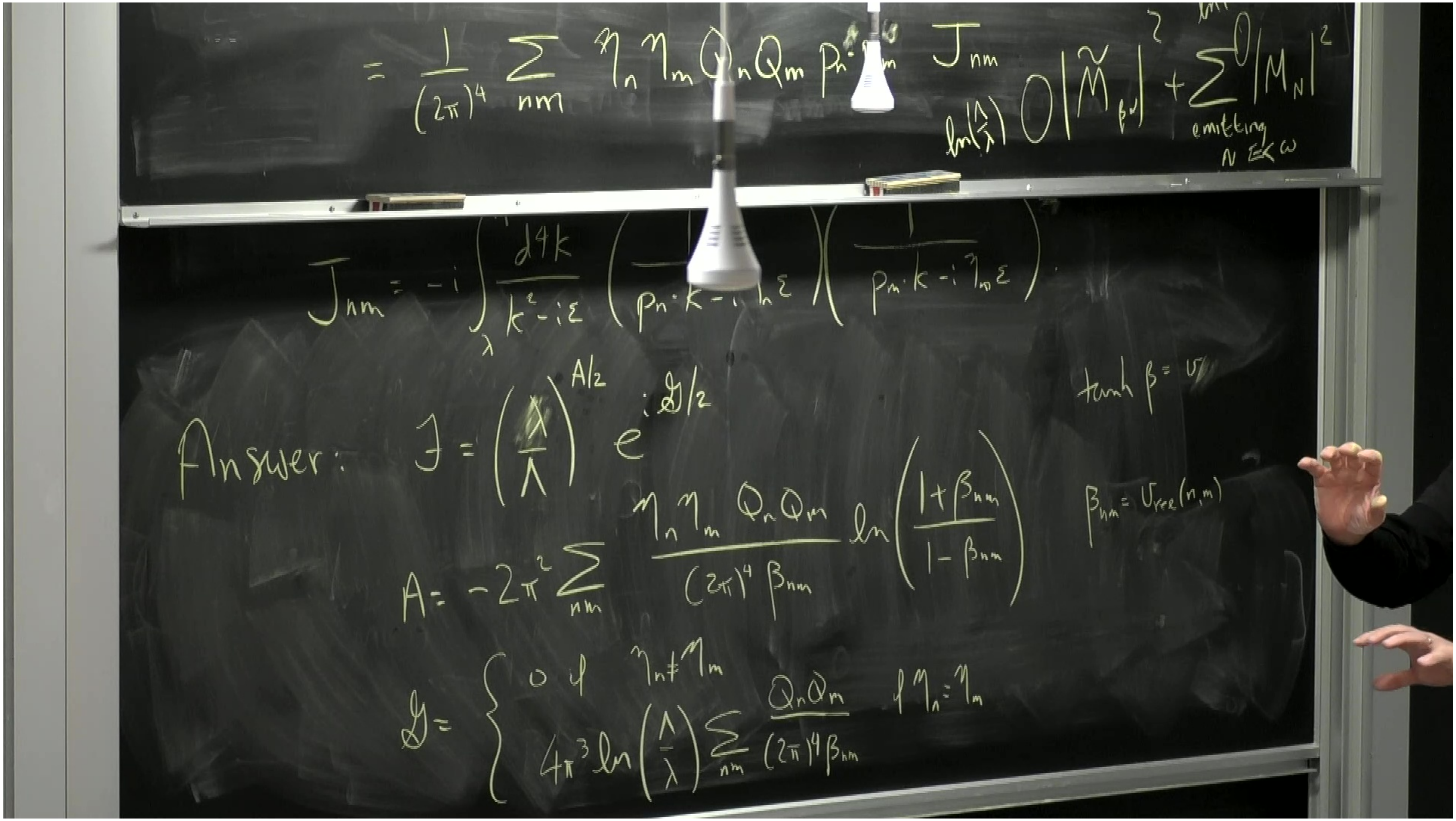
$$\tilde M_{\beta\alpha} = \sum_{R=0}^{\infty} \left\{ \frac{1}{2^R R!}\left[-i\int_\lambda^\Lambda \frac{d^4k}{(2\pi)^4}\,\frac{\eta_{\mu\nu}}{k^2-i\varepsilon}\sum_{nm}\frac{\eta_n Q_n p_n^\mu}{p_n\cdot k - i\eta_n\varepsilon}\,\frac{\eta_m Q_m p_m^\nu}{(-p_m\cdot k - i\eta_m\varepsilon)}\right]^R\right\} M_{\beta\alpha}$$

Q_n = charge



2^R:

$$\left(\frac{\alpha}{4\pi}\right)^n \ln\left(\frac{\Lambda}{\lambda}\right)$$


$= \frac{1}{(2\pi)^4} \sum_{nm} \eta_n \eta_m Q_n Q_m p_n \cdot p_m \, J_{nm} \quad \ln\left(\frac{\Lambda}{\lambda}\right) O |\tilde{M}_{\beta\nu}|^2 + \sum_{\substack{\text{emitting} \\ N \ll \omega}} O |M_N|^2$
$J_{nm} = -i \int_{\lambda} \frac{d^4k}{k^2 - i\varepsilon} \left(\frac{1}{p_n \cdot k - i \eta_n \varepsilon} \right) \left(\frac{1}{p_m \cdot k - i \eta_m \varepsilon} \right)$
$\tanh \beta = v$
Answer: $\mathcal{J} = \left(\frac{\lambda}{\Lambda} \right)^{A/2} e^{i \mathcal{G}/2}$
$\beta_{nm} = v_{rel}(n,m)$
$A = -2\pi^2 \sum_{nm} \frac{\eta_n \eta_m Q_n Q_m}{(2\pi)^4 \beta_{nm}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}} \right)$
$\mathcal{G} = \begin{cases} 0 & \text{if } \eta_n \neq \eta_m \\ 4\pi^3 \ln\left(\frac{\Lambda}{\lambda}\right) \sum_{nm} \frac{Q_n Q_m}{(2\pi)^4 \beta_{nm}} & \text{if } \eta_n = \eta_m \end{cases}$