Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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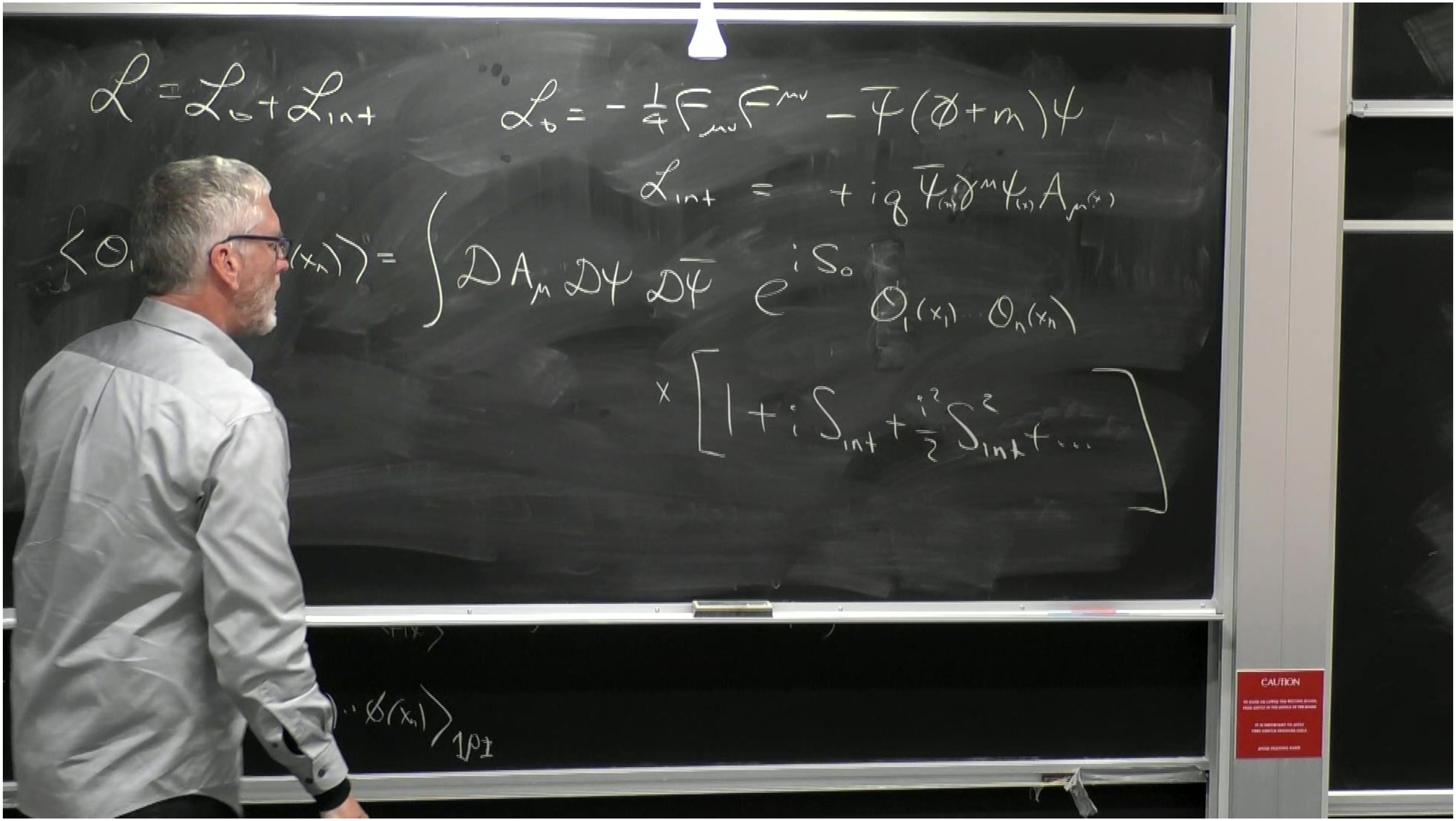
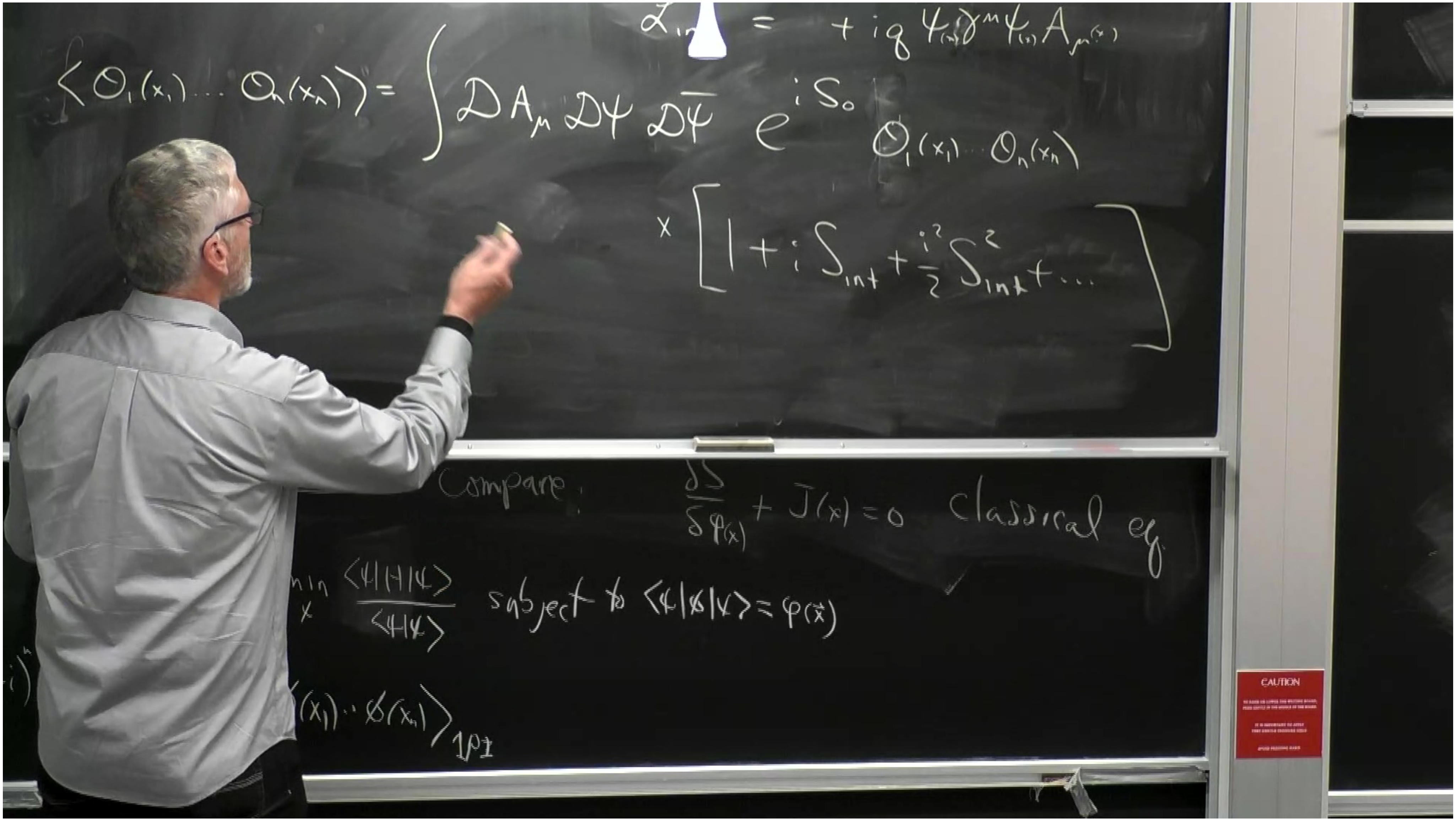
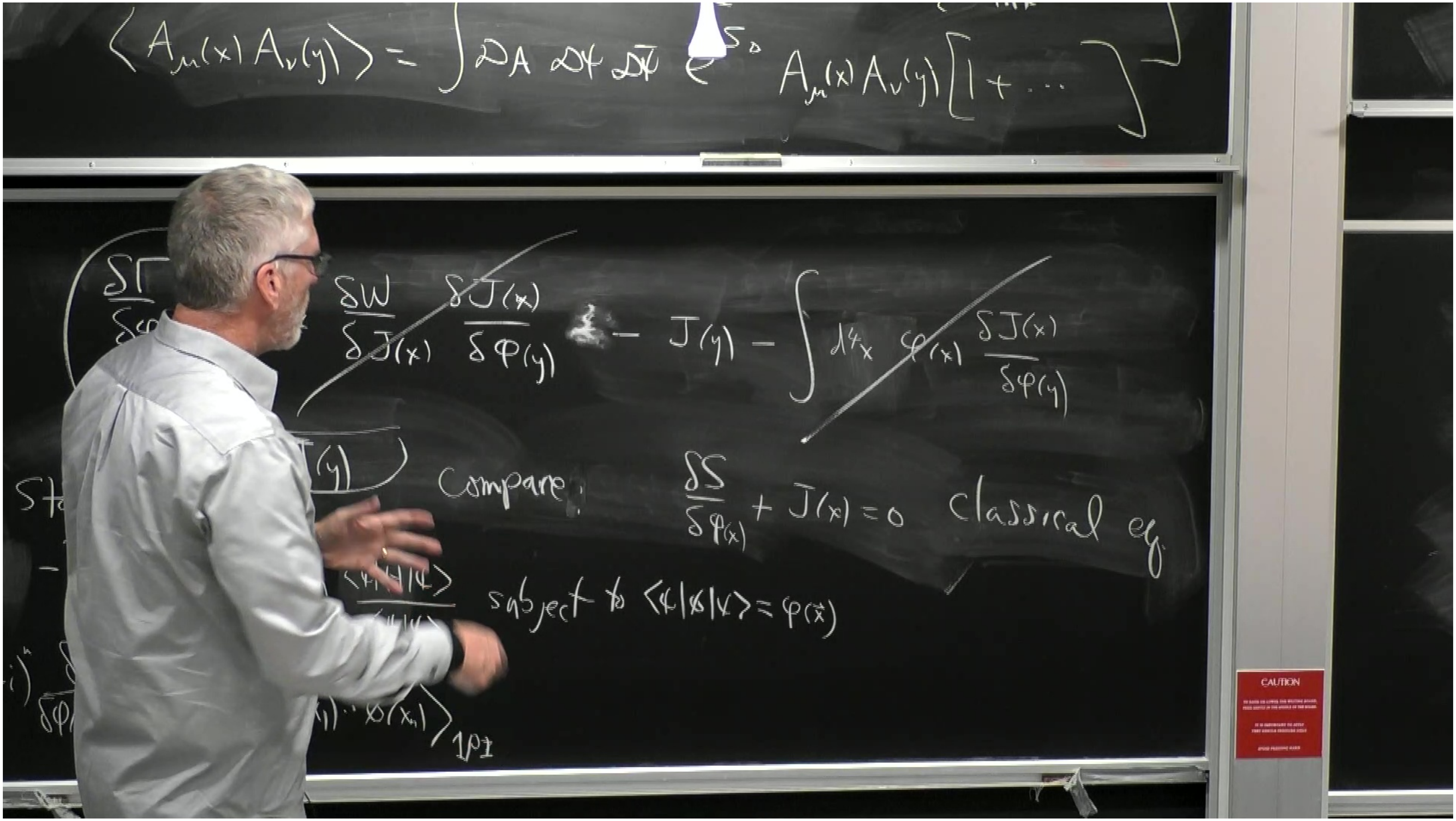
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi$
$\mathcal{L}_{int} = + iq \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu(x)$
$= \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{iS_0}\, \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n)$
$\times \left[1 + iS_{int} + \frac{i^2}{2} S_{int}^2 + \ldots\right]$
$\cdots \phi(x_n) \rangle_{1PI}$


$\mathcal{L}_{int} = +iq\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$
$\langle O_1(x_1)\dots O_n(x_n)\rangle = \int \mathcal{D}A_\mu\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\; e^{iS_0}\, O_1(x_1)\cdots O_n(x_n)$
$\times\left[1 + iS_{int} + \frac{i^2}{2}S_{int}^2 + \dots\right]$
Compare: $\frac{\delta S}{\delta \varphi(x)} + J(x) = 0$ classical eq.
$\min_\psi \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$ subject to $\langle\psi|\phi|\psi\rangle = \varphi(\vec{x})$
$(x_1)\cdots\phi(x_n)\rangle_{1PI}$
CAUTION

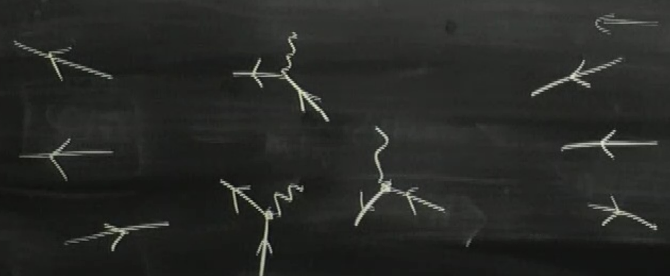

Compare
classical eq
subject to
CAUTION

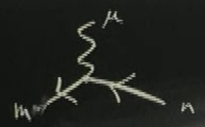
$$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\; e^{S_0}\, A_\mu(x) A_\nu(y) \left[1 + \dots \right]$$

$\psi \quad \bar{\psi} \quad \bar{\psi}$

$$A_\mu(x) = \sum_\lambda \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left[\varepsilon_\mu(k,\lambda)\, a_{k\lambda}\, e^{ikx} + \varepsilon^*_\mu(k,\lambda)\, a^*_{k\lambda}\, e^{-ikx} \right]$$

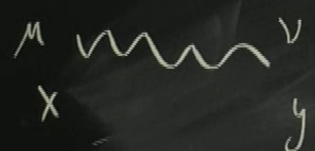
$$\psi(x) = \sum_\sigma \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[u(p,\sigma)\, c_{p\sigma}\, e^{ipx} + v(p,\sigma)\, \bar{d}^*_{p\sigma}\, e^{-ipx} \right]$$





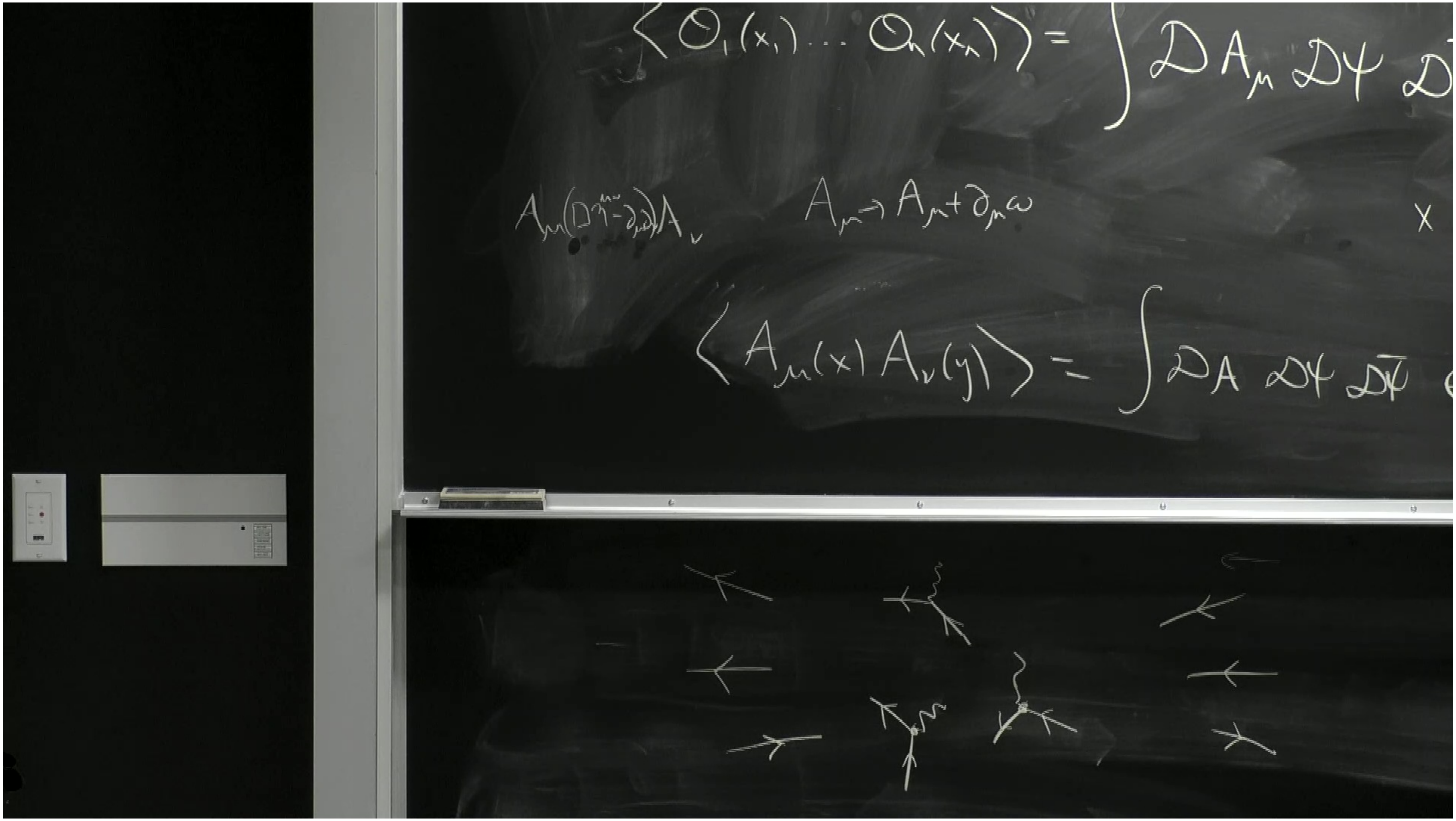
$$i\left[iq\,\gamma^{\mu}_{mn}\right]$$

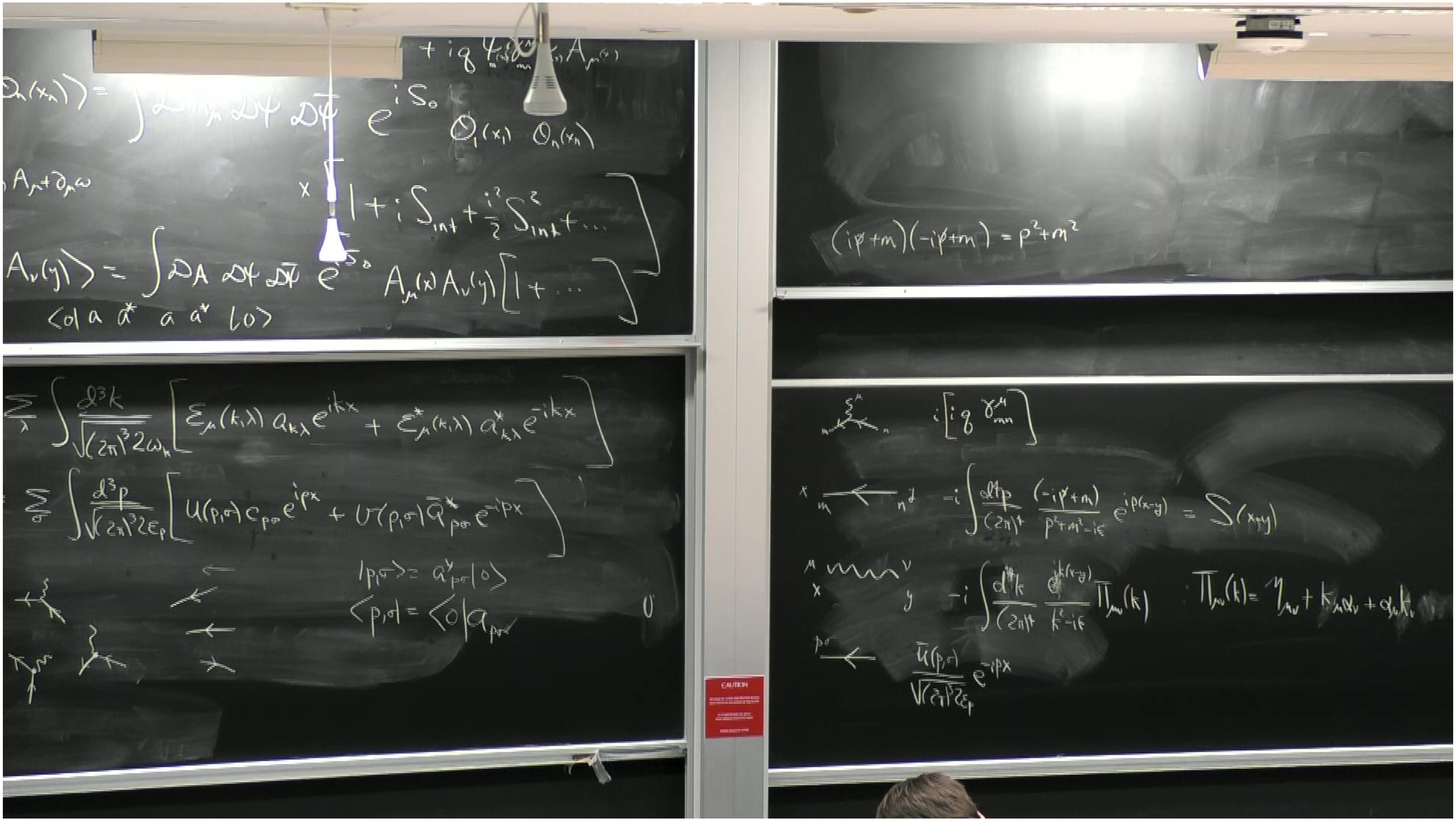
$$-i\int\frac{d^4p}{(2\pi)^4}\,\frac{(-i\not{p}+m)}{p^2+m^2-i\epsilon}\,e^{ip(x-y)} = S(x-y)$$

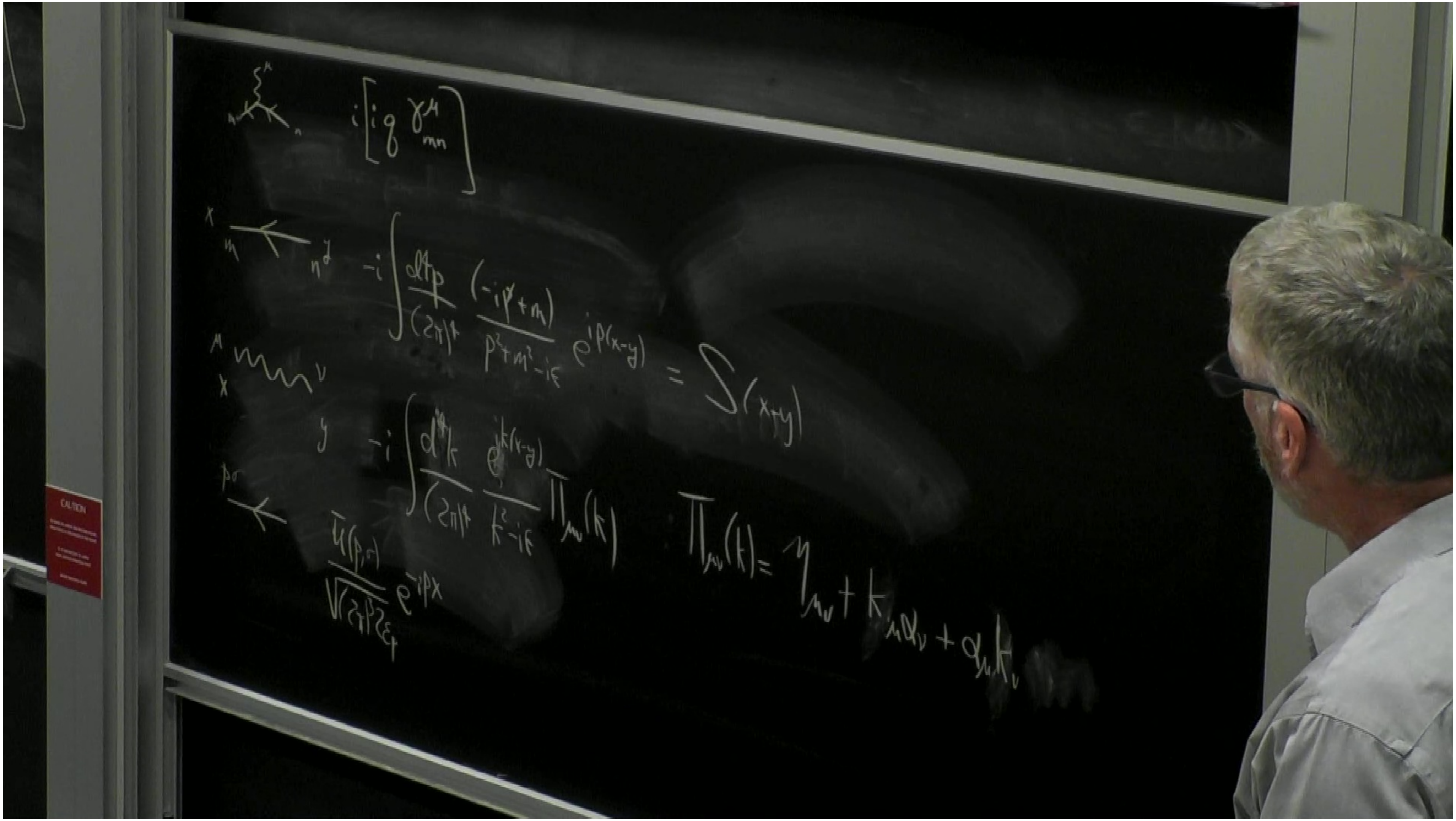


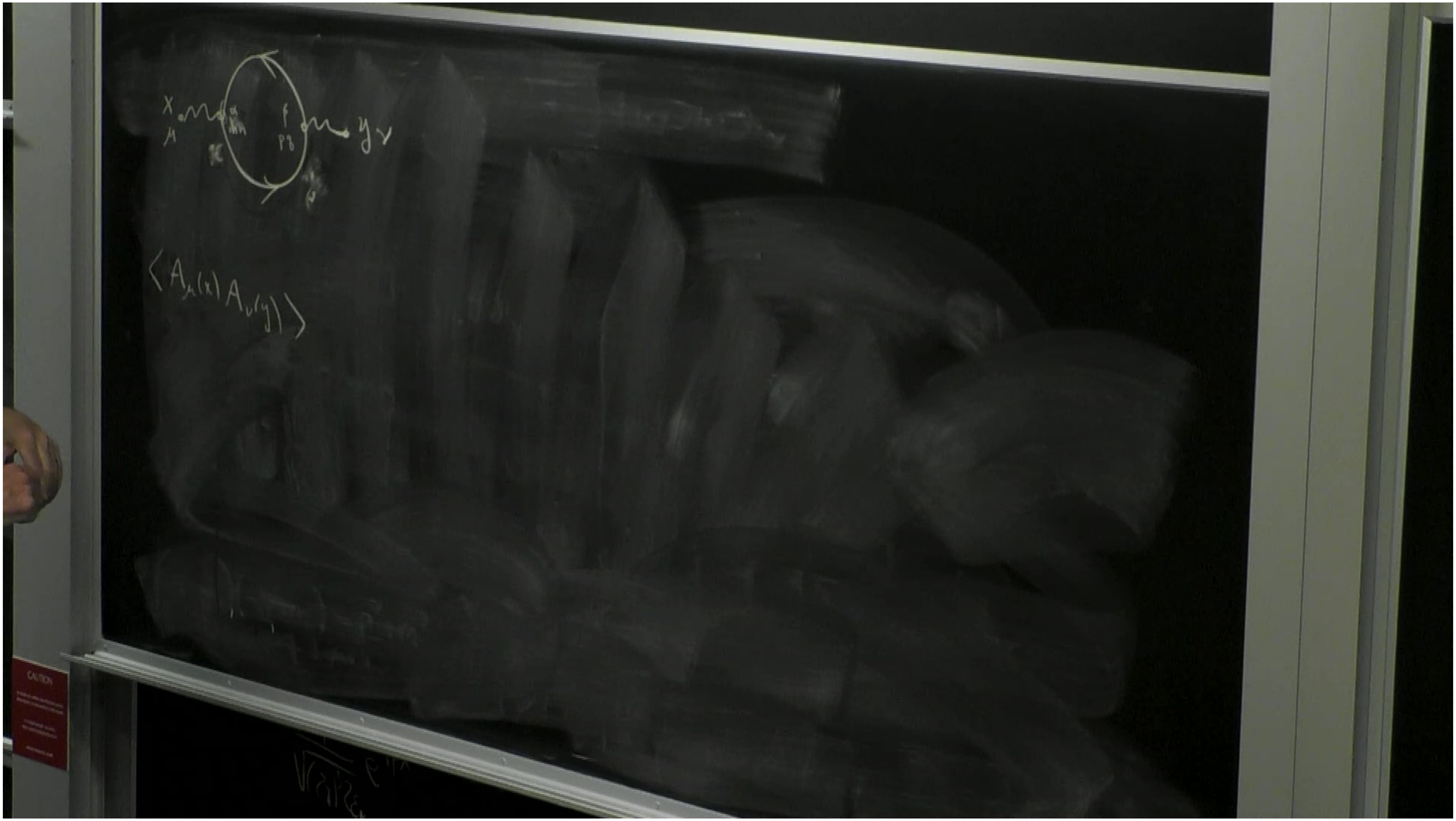
$$-i\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-i\epsilon}\,\Pi_{\mu\nu}(k)$$

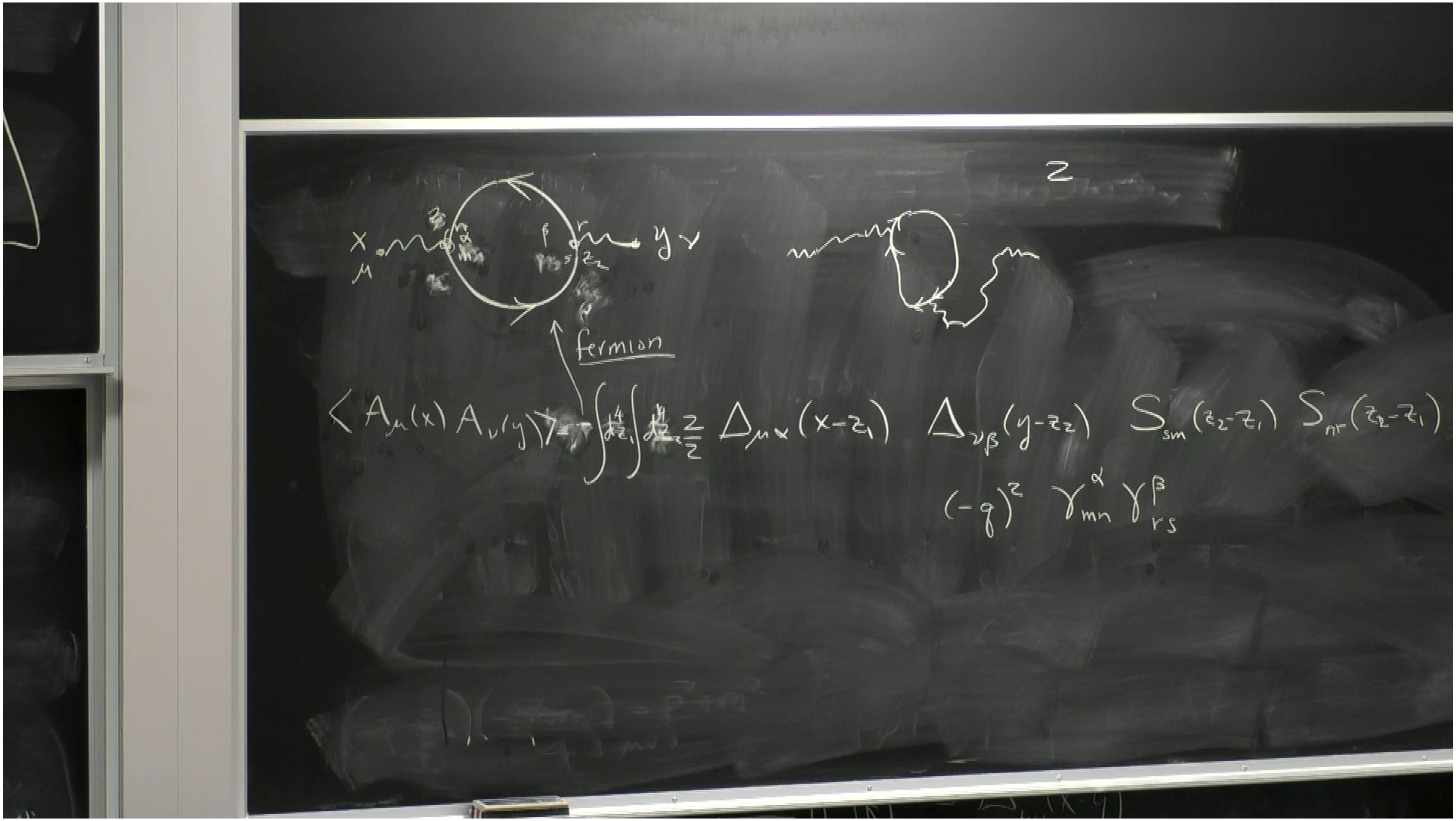
$$\Pi_{\mu\nu}(k) = \eta_{\mu\nu} + p_\mu\alpha_\nu + \alpha_\mu p_\nu$$

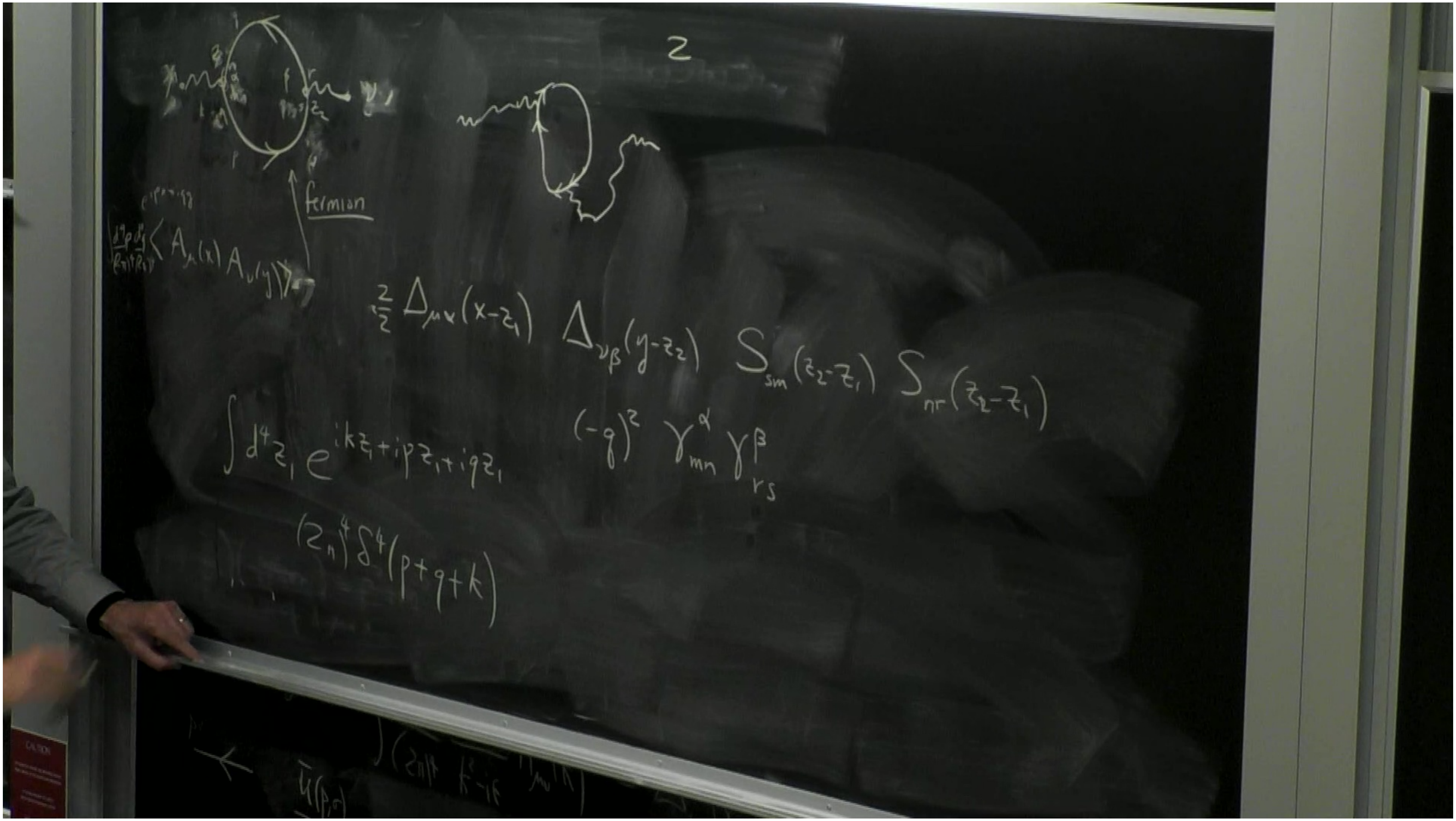

$\langle O_1(x_1) \ldots O_n(x_n) \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}$
$A_\mu(\Box \eta^{\mu\nu} - \partial_\mu \partial_\nu) A_\nu$
$A_\mu \to A_\mu + \partial_\mu \omega$
X
$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A \, \mathcal{D}\psi \, \mathcal{D}\bar{\psi}$

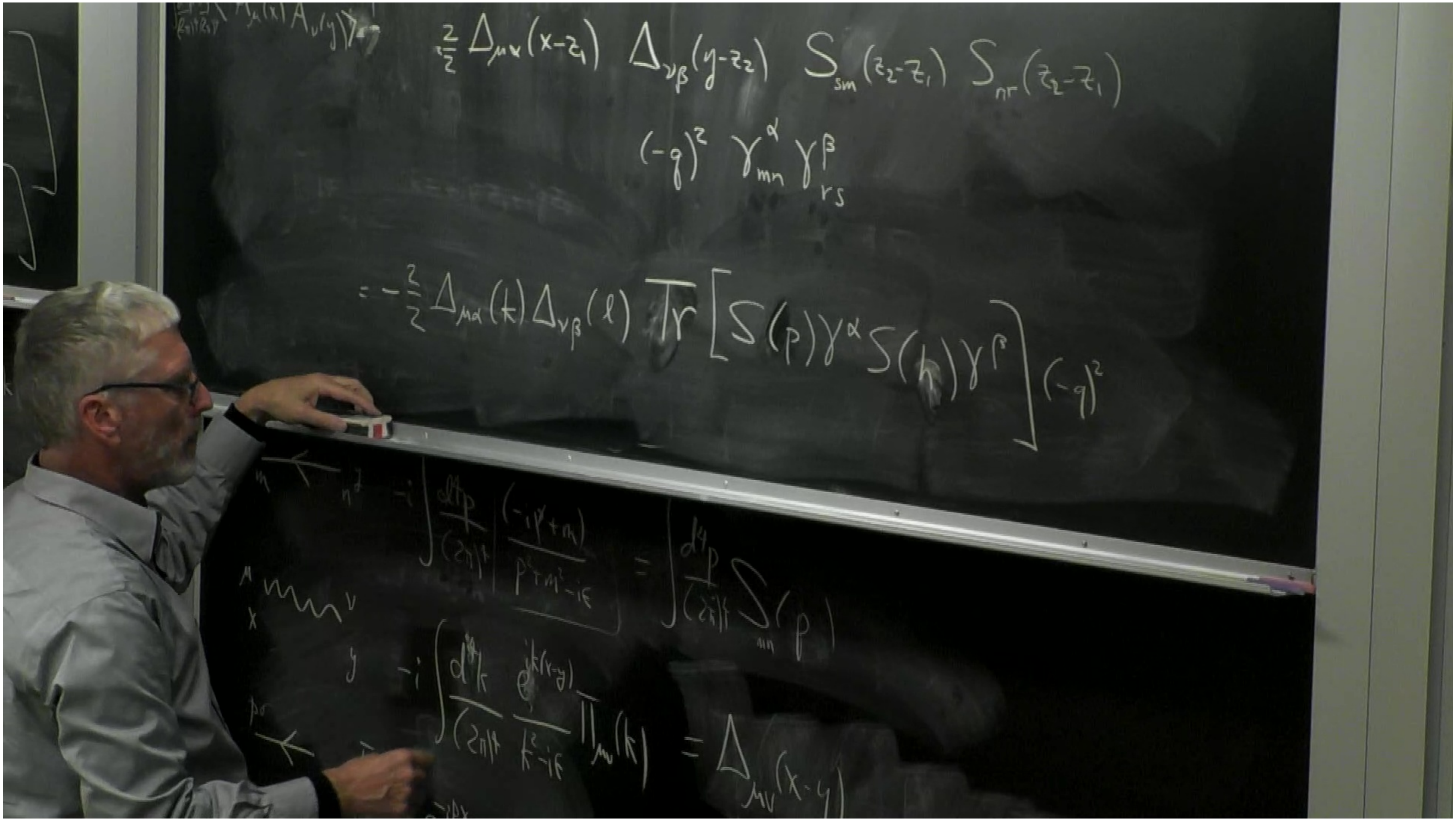

$A_\mu + \partial_\mu \omega$
$1 + i S_{int} + \frac{i^2}{2} S_{int}^2 + \dots$
$\langle 0| a\ a^*\ a\ a^* |0\rangle$
$(i\not p + m)(-i\not p + m) = p^2 + m^2$
$|p,\sigma\rangle = a^\dagger_{p\sigma}|0\rangle$
$\langle p,\sigma| = \langle 0|a_{p\sigma}$
$\Pi_{\mu\nu}(k) = \eta_{\mu\nu} + k_\mu \alpha_\nu + \alpha_\mu k_\nu$

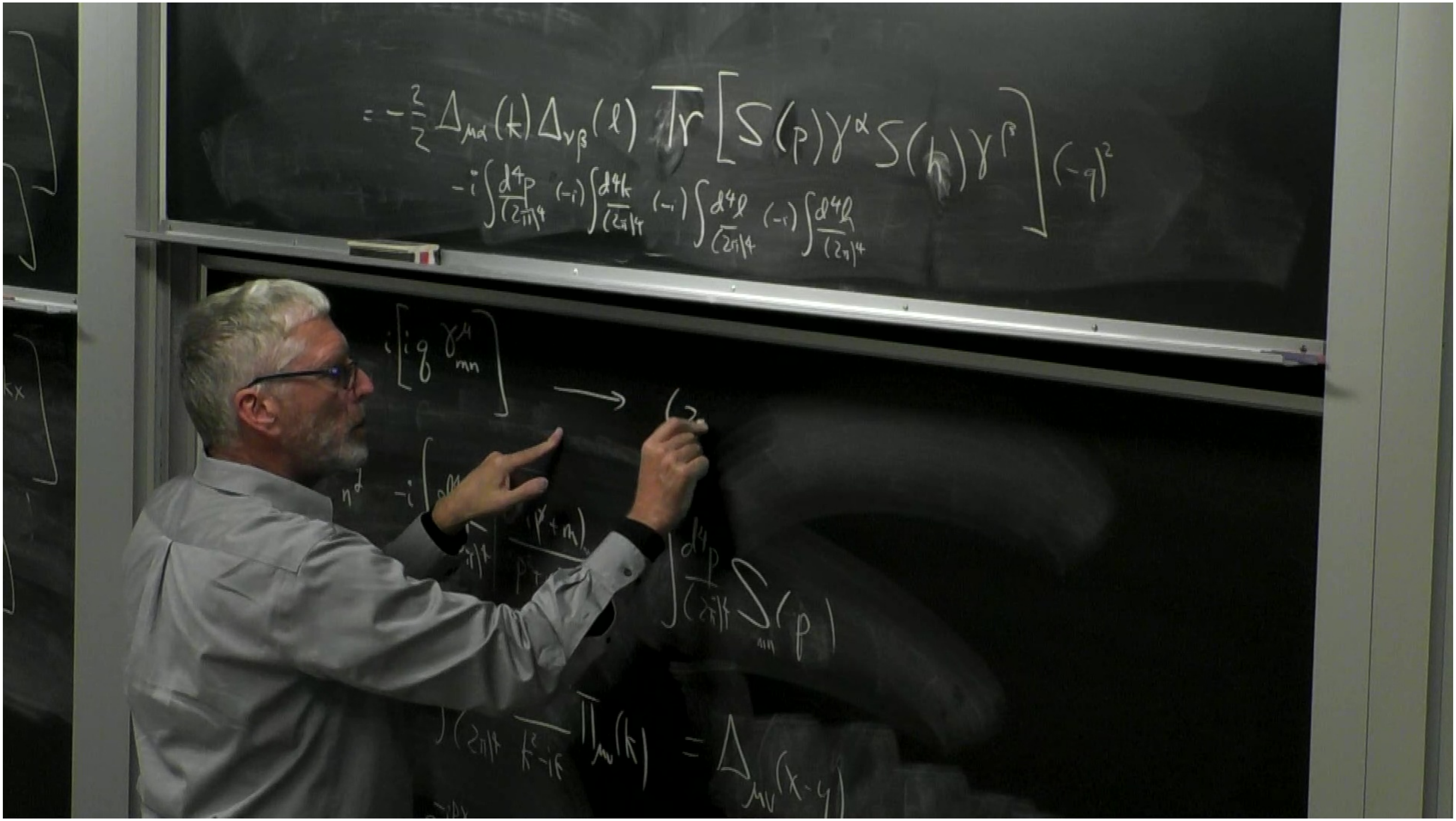

$i\left[iq\,\gamma^{\mu}_{mn}\right]$
$-i\int\frac{d^4p}{(2\pi)^4}\frac{(-i\not{p}+m)}{p^2+m^2-i\epsilon}e^{ip(x-y)} = S(x-y)$
$-i\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2-i\epsilon}\Pi_{\mu\nu}(k)$
$\Pi_{\mu\nu}(k) = \eta_{\mu\nu} + k_{\mu}\alpha_{\nu} + \alpha_{\mu}k_{\nu}$
$\frac{\bar{u}(p,\sigma)}{\sqrt{(2\pi)^3 2E_p}}e^{-ipx}$


x
μ
y ν
$\langle A_\mu(x)\, A_\nu(y)\rangle$


z
x μ
y ν
fermion
$\langle A_\mu(x) A_\nu(y) \rangle = \int d^4z_1 \int d^4z_2 \, \Delta_{\mu\alpha}(x-z_1) \, \Delta_{\nu\beta}(y-z_2) \, S_{sm}(z_2-z_1) \, S_{nr}(z_2-z_1)$
$(-q)^2 \, \gamma^\alpha_{mn} \gamma^\beta_{rs}$


z
fermion
$\langle A_\mu(x) A_\nu(y) \rangle$
$\frac{z}{2} \Delta_{\mu\alpha}(x-z_1) \Delta_{\nu\beta}(y-z_2) S_{sm}(z_2-z_1) S_{nr}(z_2-z_1)$
$(-q)^2 \gamma^\alpha_{mn} \gamma^\beta_{rs}$
$\int d^4 z_1 e^{ikz_1+ipz_1+iqz_1}$
$(2\pi)^4 \delta^4(p+q+k)$


$\frac{2}{2} \Delta_{\mu\alpha}(x-z_1)\ \Delta_{\nu\beta}(y-z_2)\ S_{sm}(z_2-z_1)\ S_{nr}(z_2-z_1)$
$(-q)^2\ \gamma^{\alpha}_{mn}\ \gamma^{\beta}_{rs}$
$= -\frac{2}{2} \Delta_{\mu\alpha}(k) \Delta_{\nu\beta}(\ell)\ \mathrm{Tr}\left[S(p)\gamma^{\alpha} S(h)\gamma^{\beta}\right] (-q)^2$
$-i\int \frac{d^4p}{(2\pi)^4} \frac{(-i\not p + m)}{p^2+m^2-i\epsilon} = \int \frac{d^4p}{(2\pi)^4} S_{mn}(p)$
$-i\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2-i\epsilon} \Pi_{\mu\nu}(k) = \Delta_{\mu\nu}(x-y)$


$= -\frac{2}{2} \Delta_{\mu\alpha}(k) \Delta_{\nu\beta}(\ell) \,\mathrm{Tr}\left[S(p)\gamma^\alpha S(h)\gamma^\beta \right] (-q)^2$
$-i\int \frac{d^4p}{(2\pi)^4} (-i)\int \frac{d^4k}{(2\pi)^4} (-i)\int \frac{d^4\ell}{(2\pi)^4} (-i)\int \frac{d^4h}{(2\pi)^4}$
$i\left[iq\,\gamma^\mu_{mn} \right]$
$\int \frac{d^4p}{(2\pi)^4} S_{mn}(p)$
$= \Delta_{\mu\nu}(x-y)$

$$= -\frac{2}{2}\,\Delta_{\mu\alpha}(k)\,\Delta_{\nu\beta}(\ell)\,\mathrm{Tr}\left[S(p)\gamma^{\alpha}S(h)\gamma^{\beta}\right](-q)^2$$

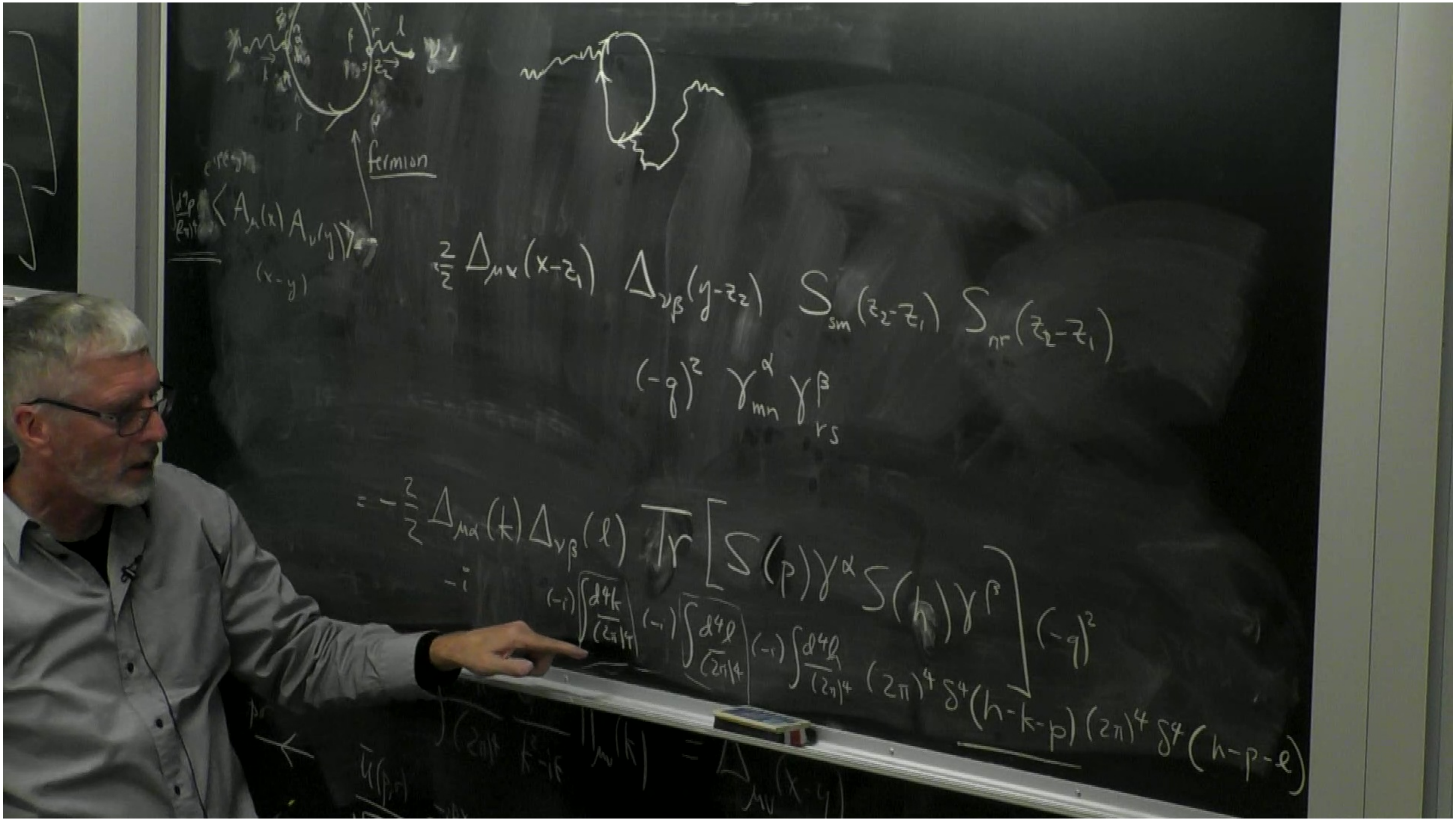
$$-i\int\frac{d^4p}{(2\pi)^4}\,(-i)\int\frac{d^4k}{(2\pi)^4}\,(-i)\int\frac{d^4\ell}{(2\pi)^4}\,(-i)\int\frac{d^4h}{(2\pi)^4}\,(2\pi)^4\delta^4(h-k-p)\,(2\pi)^4\delta^4(h-p-\ell)$$

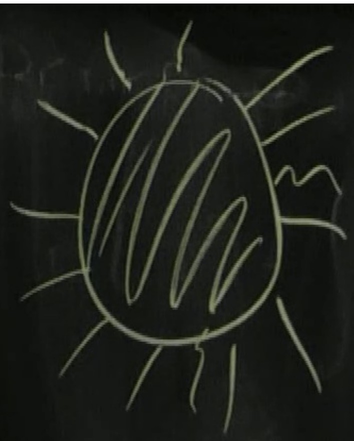
$$i\left[iq\,\gamma^{\mu}_{mn}\right] \longrightarrow i(2\pi)^4\delta^4\Big(\sum_i p_i\Big)\,(iq\gamma^{\mu})_{mn}$$

$$-i\int\frac{d^4p}{(2\pi)^4}\,\frac{(-i\not p+m)_{mn}}{p^2+m^2-i\epsilon} = \int\frac{d^4p}{(2\pi)^4}\,S_{mn}(p)$$

$$-i\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{ik(x-y)}}{k^2-i\epsilon}\,\Pi_{\mu\nu}(k) = \Delta_{\mu\nu}(x-y)$$

$\bar{u}(p,\sigma)$


fermion
⟨A_μ(x) A_ν(y)⟩
(x-y)
Δ_μα(x-z_1) Δ_νβ(y-z_2) S_sm(z_2-z_1) S_nr(z_2-z_1)
(-q)^2 γ^α_mn γ^β_rs
Δ_μα(k) Δ_νβ(ℓ) Tr [S(p) γ^α S(h) γ^β] (-q)^2
(2π)^4 δ^4(h-k-p) (2π)^4 δ^4(h-p-ℓ)



I_e internal e lines

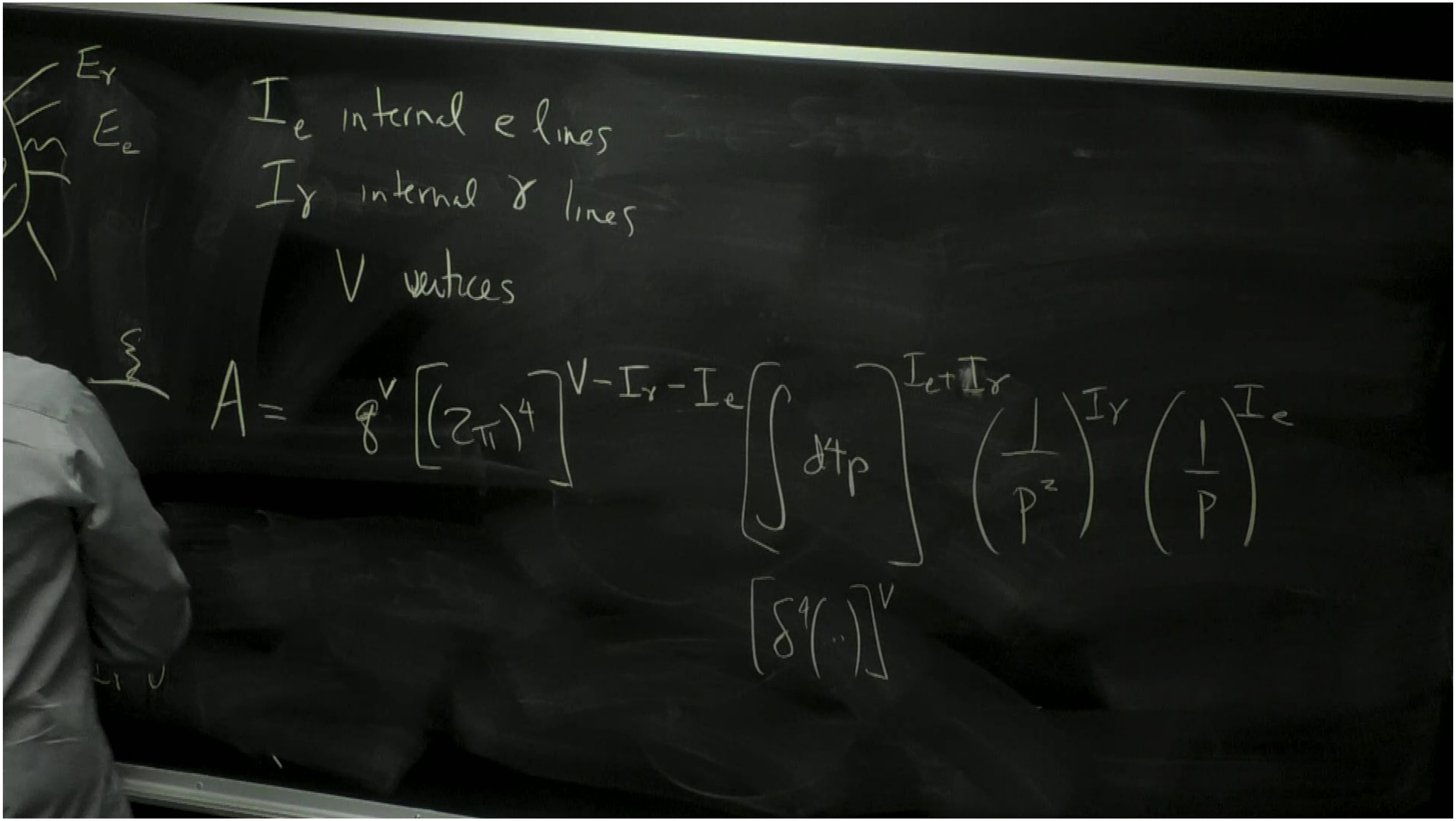
I_γ internal γ lines

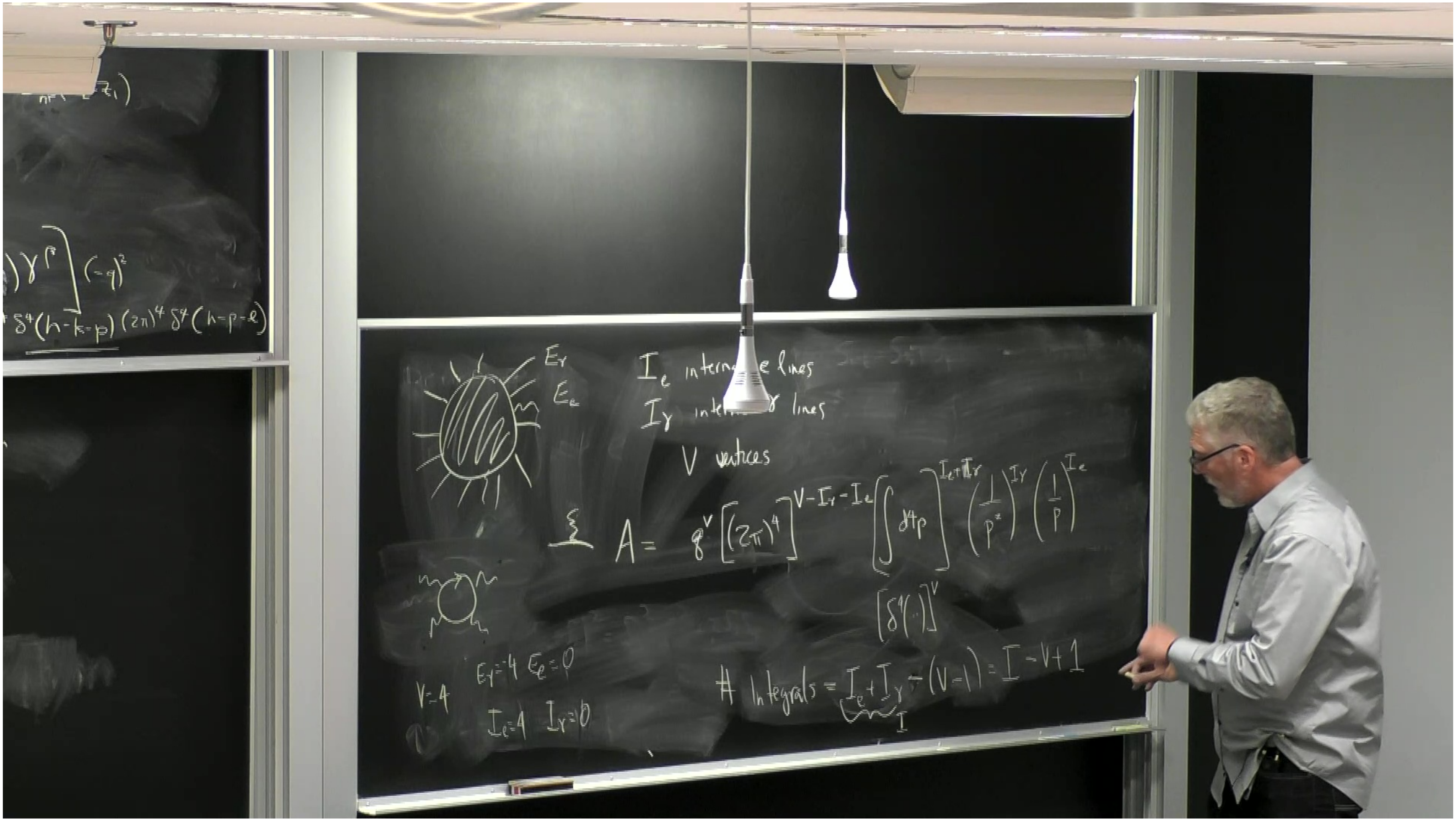
V vertices

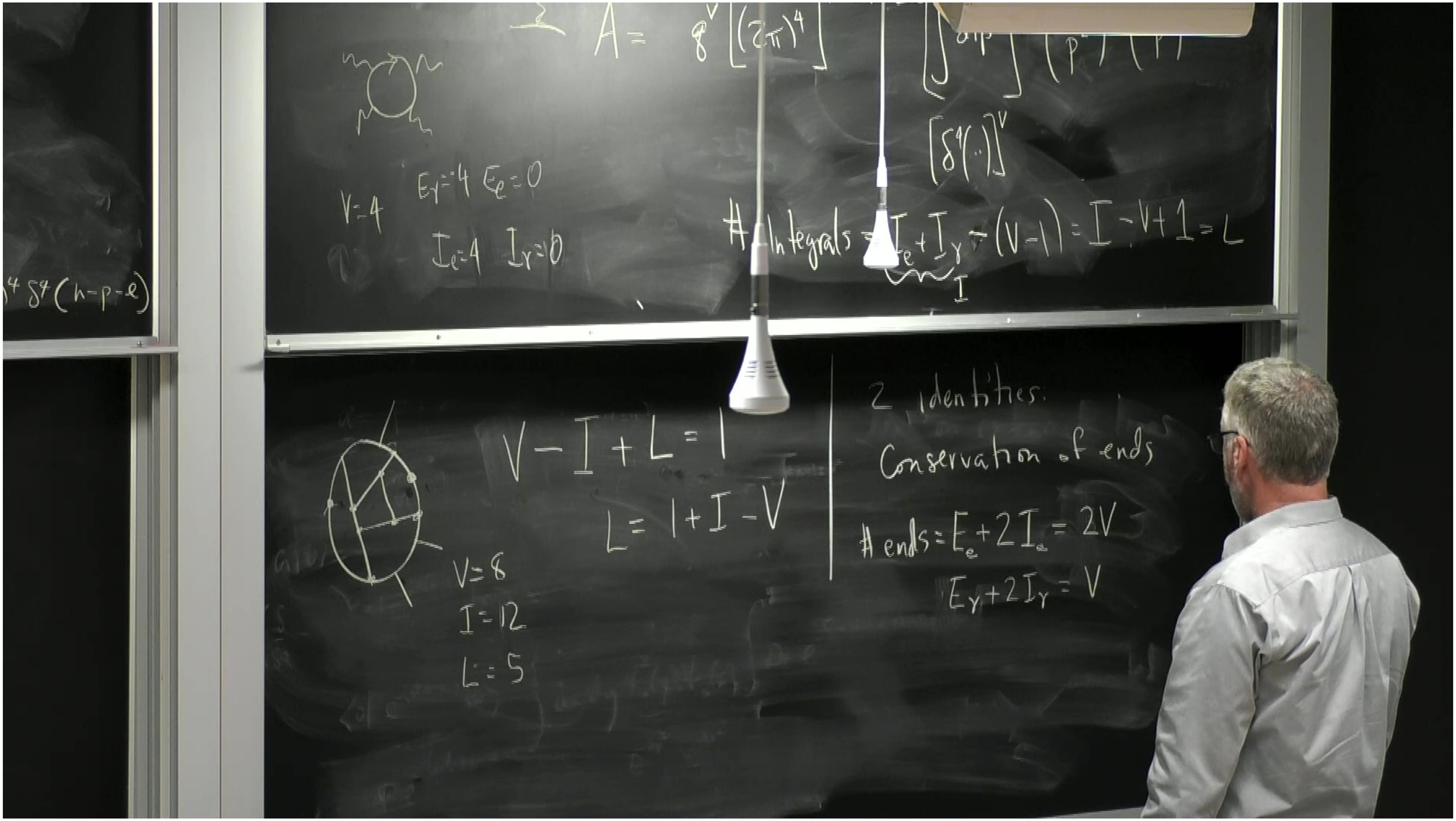
$$A = g^V \left[(2\pi)^4\right]^{V - I_\gamma - I_e} \left[\int d^4p\right]^{I_e + I_\gamma}$$

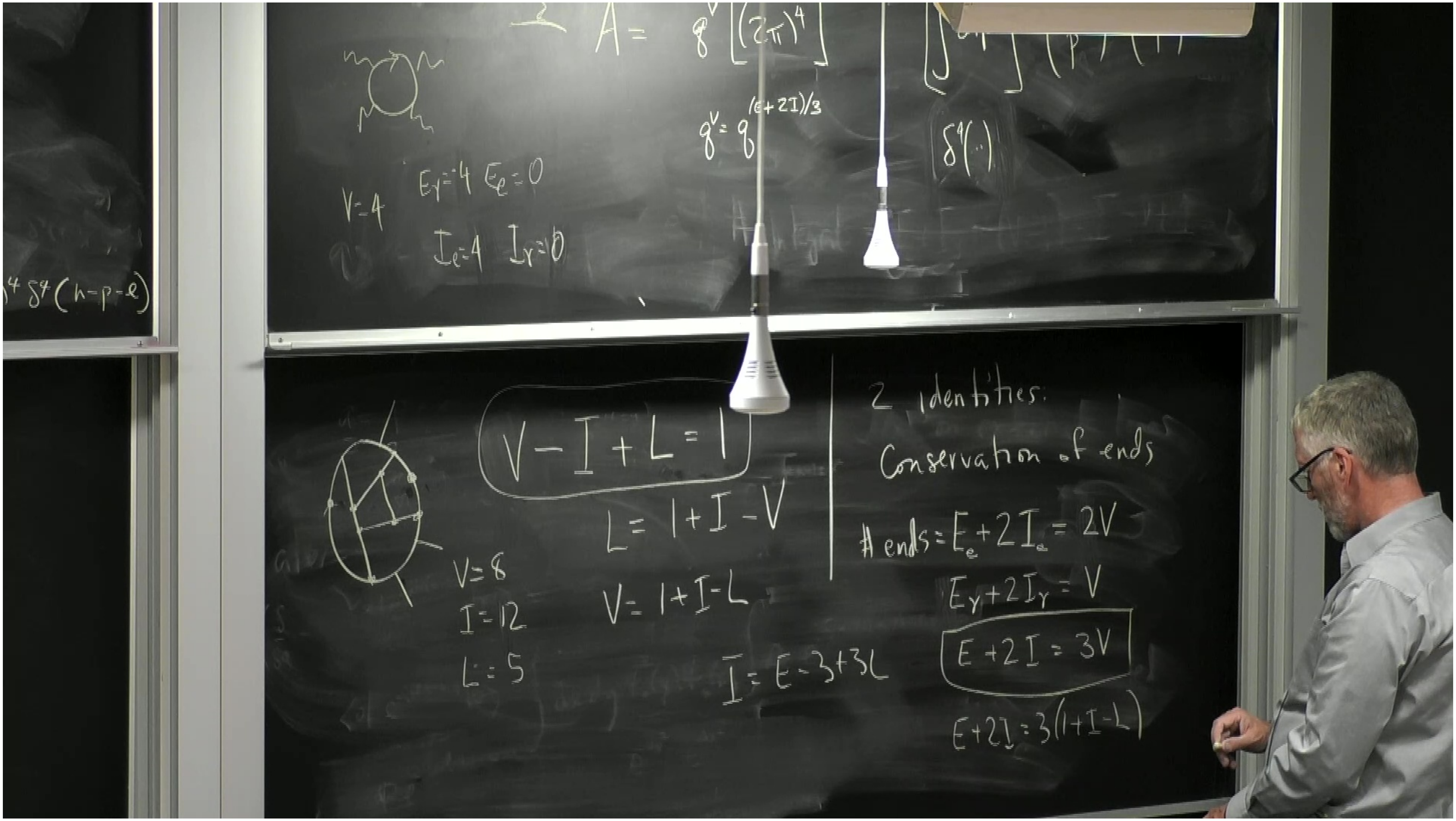
$V = 4$ $E_\gamma = 4$ $E_e = 0$

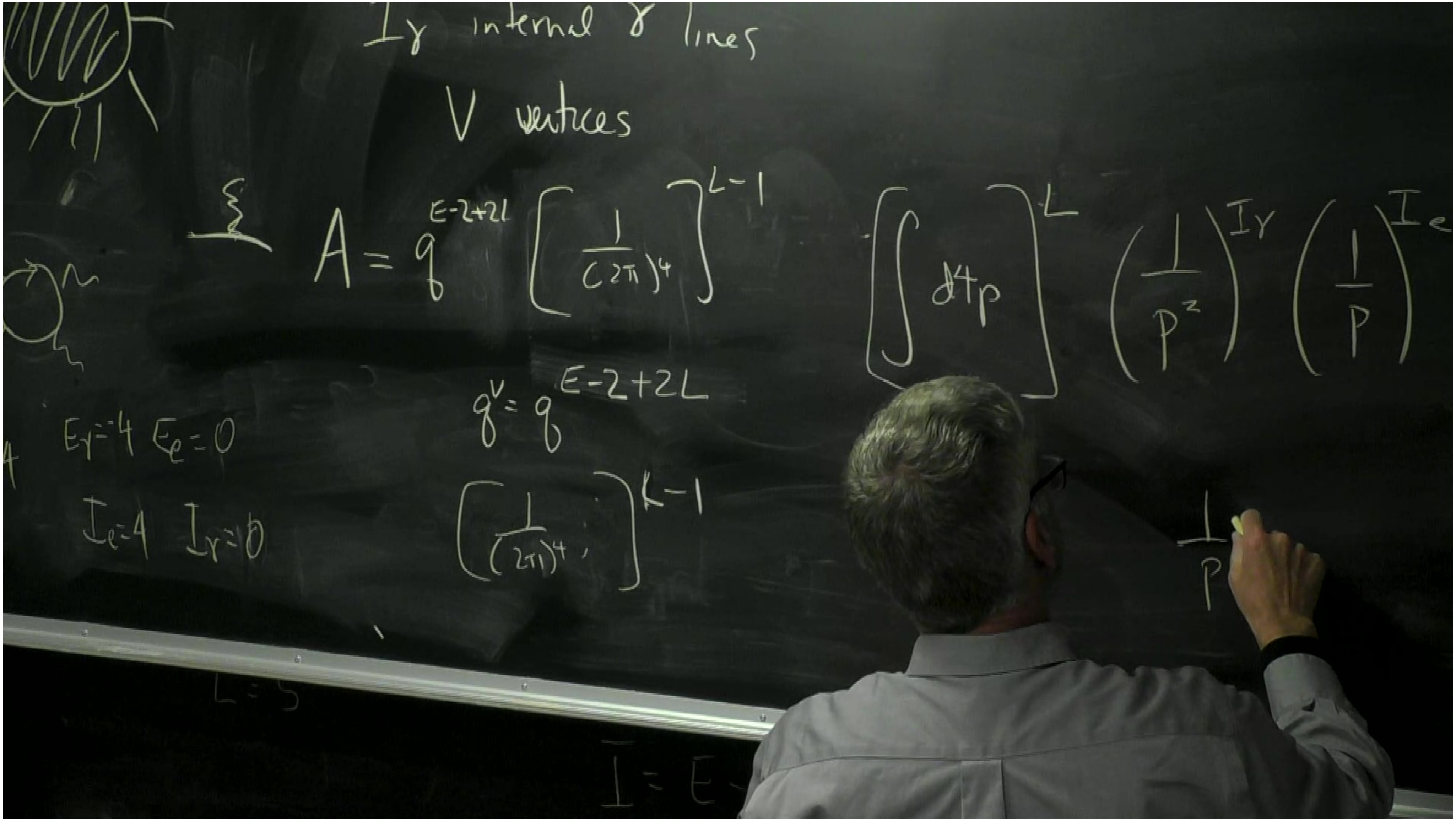
$I_e = 4$ $I_\gamma = 0$

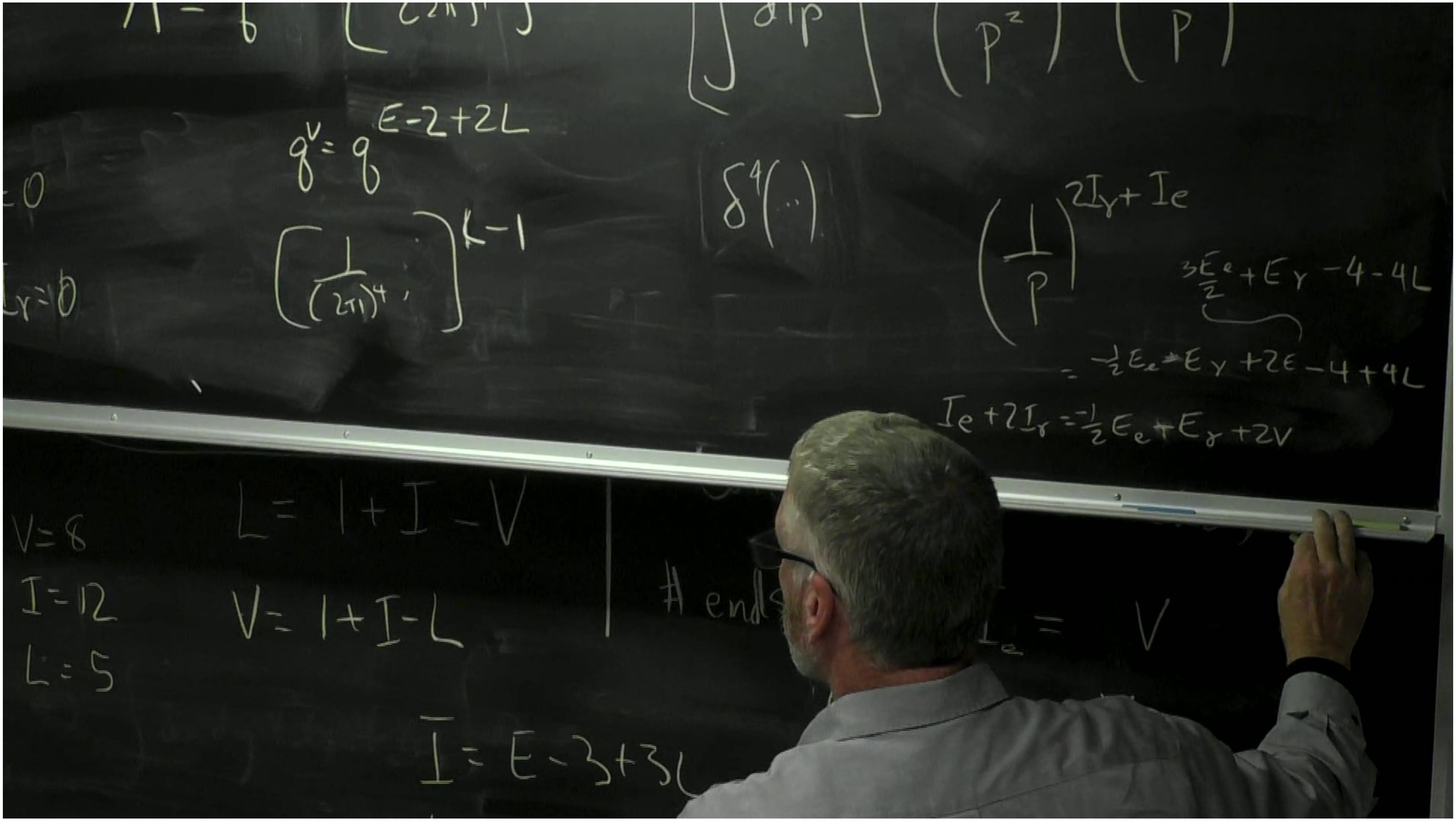

E_γ
E_e
I_e internal e lines
I_γ internal γ lines
V vertices
$A = g^V \left[(2\pi)^4\right]^{V-I_\gamma-I_e} \left[\int d^4p\right]^{I_e+I_\gamma} \left(\frac{1}{p^2}\right)^{I_\gamma} \left(\frac{1}{p}\right)^{I_e}$
$\left[\delta^4(\ldots)\right]^V$


E_γ
E_e
I_e intern e lines
I_γ inte γ lines
V vertices
A = g^V [(2π)^4]^(V-I_γ-I_e) [∫d^4p]^(I_e+I_γ) (1/p^2)^(I_γ) (1/p)^(I_e)
[δ^4(..)]^V
# Integrals = I_e + I_γ - (V-1) = I - V + 1
V=4
E_γ=4 E_e=0
I_e=4 I_γ=0

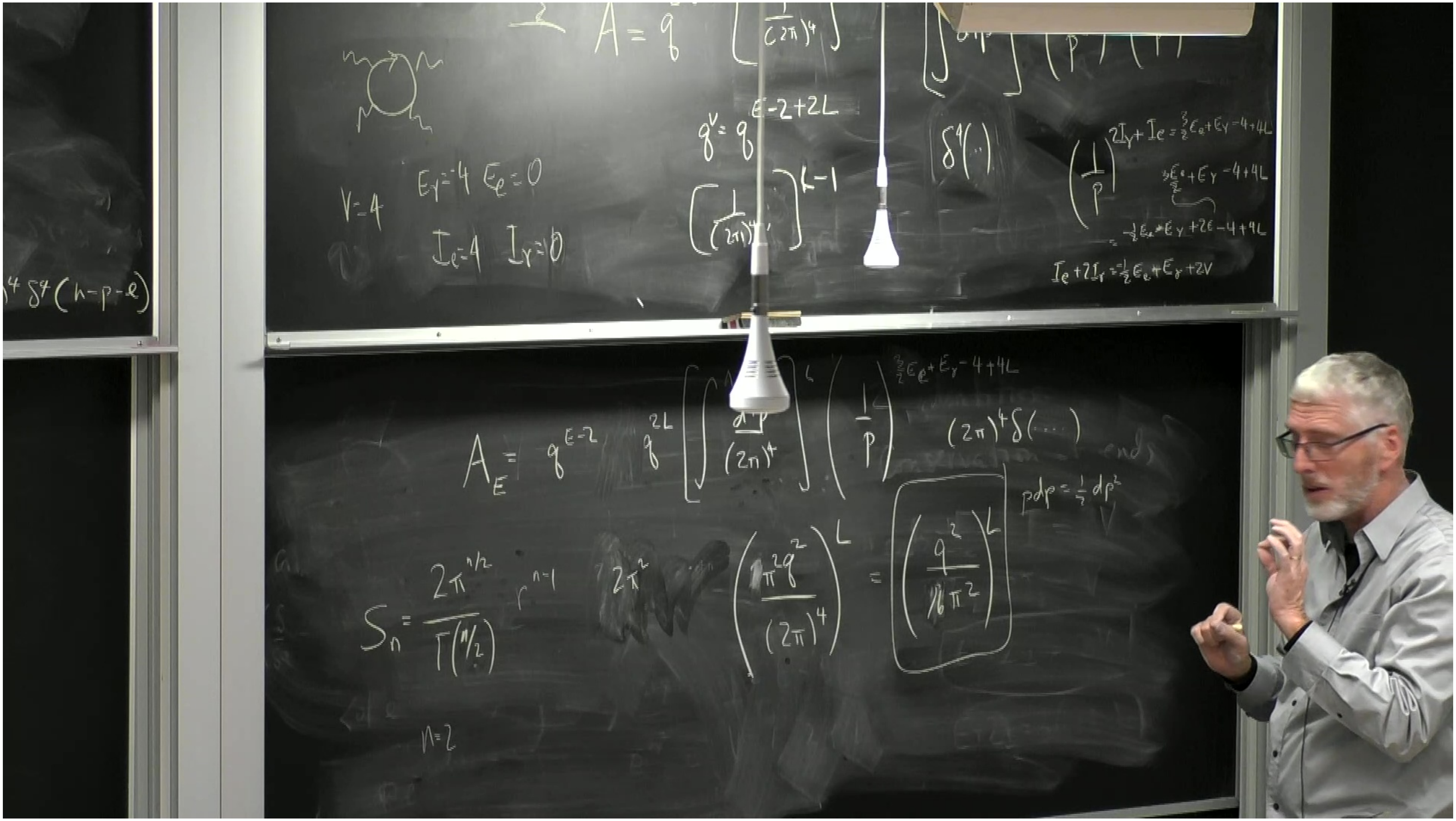

A = g^V [(2π)^4]
[δ^4(..)]^V
V=4   E_γ=4 E_e=0
I_e=4 I_γ=0
# Integrals = I_e + I_γ − (V−1) = I − V + 1 = L
V − I + L = 1
L = 1 + I − V
V=8
I=12
L=5
2 identities:
Conservation of ends
ends = E_e + 2I_e = 2V
E_γ + 2I_γ = V


A = g^V [(2π)^4]
g^V = g^(E+2I)/3
δ^4(..)
V=4
Eγ=4 Ee=0
Ie=4 Iγ=0
V - I + L = 1
L = 1 + I - V
V = 1 + I - L
I = E - 3 + 3L
V=8
I=12
L=5
2 identities:
Conservation of ends
ends = Ee + 2Ie = 2V
Eγ + 2Iγ = V
E + 2I = 3V
E+2I = 3(1+I-L)


Iγ internal γ lines
V vertices
A = q^(E-2+2L) [1/(2π)^4]^(L-1) [∫d4p]^L (1/p²)^Iγ (1/p)^Ie
q^V = q^(E-2+2L)
[1/(2π)^4]^(L-1)
Eγ=4 Ee=0
Ie=4 Iγ=0
1/p



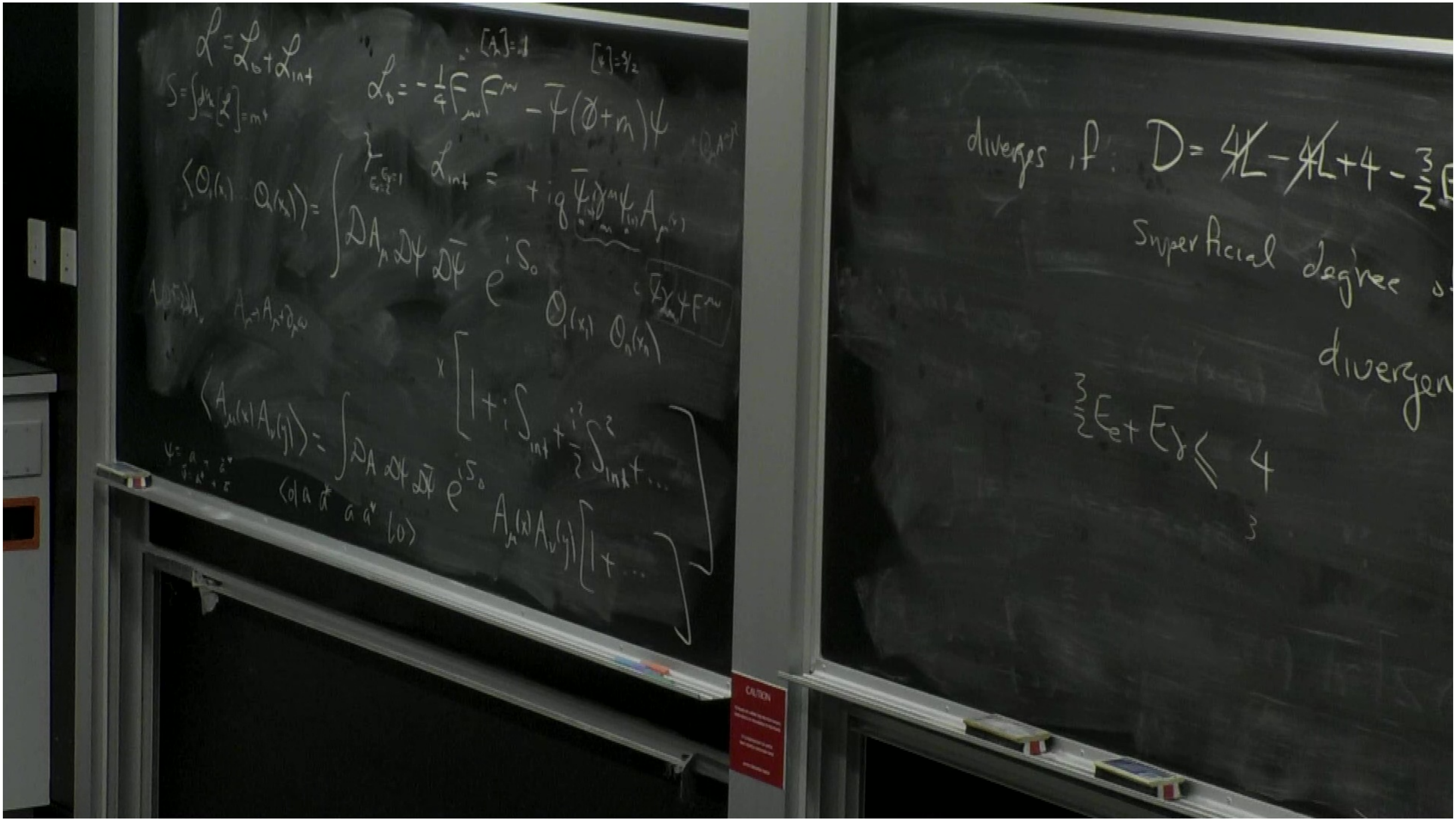
q^V = q^{E-2+2L}
δ^4(...)
(1/p)^{2I_γ + I_e}
3E_e/2 + E_γ - 4 - 4L
= -1/2 E_e + E_γ + 2E - 4 + 4L
I_e + 2I_γ = -1/2 E_e + E_γ + 2V
V = 8
I = 12
L = 5
L = 1 + I - V
V = 1 + I - L
# ends
I_e = V


V=4
E_γ=4 E_e=0
I_e=4 I_γ=0
g^V = g^{E-2+2L}
δ^4(...)
(2π)^4 δ(....)
pdp = ½ dp²
S_n = 2π^{n/2} / Γ(n/2) r^{n-1}
n=2

diverges if: $D = \cancel{4L} - \cancel{4L} + 4 - \frac{3}{2}E_e - E_\gamma \geq 0$

superficial degree of divergence

$\frac{3}{2}E_e + E_\gamma < 4$



$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$S = \int d^4x \; [\mathcal{L}] = m^4$
$[A_\mu] = 1$
$[\psi] = 3/2$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi$
$\mathcal{L}_{int} = + i q \bar{\psi} \gamma^\mu \psi A_\mu$
$\langle O_1(x_1) \ldots O_n(x_n) \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} \, e^{iS_0} \, O_1(x_1) \ldots O_n(x_n)$
$\times \left[1 + i S_{int} + \frac{i^2}{2} S_{int}^2 + \ldots \right]$
$A_\mu \to A_\mu + \partial_\mu \omega$
$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \, e^{iS_0} A_\mu(x) A_\nu(y) [1 + \ldots]$
diverges if: $D = 4L - 4L + 4 - \frac{3}{2}E$
Superficial degree of
divergen
$\frac{3}{2}E_e + E_\gamma \leq 4$