Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: November 22, 2022 - 2:30 PM

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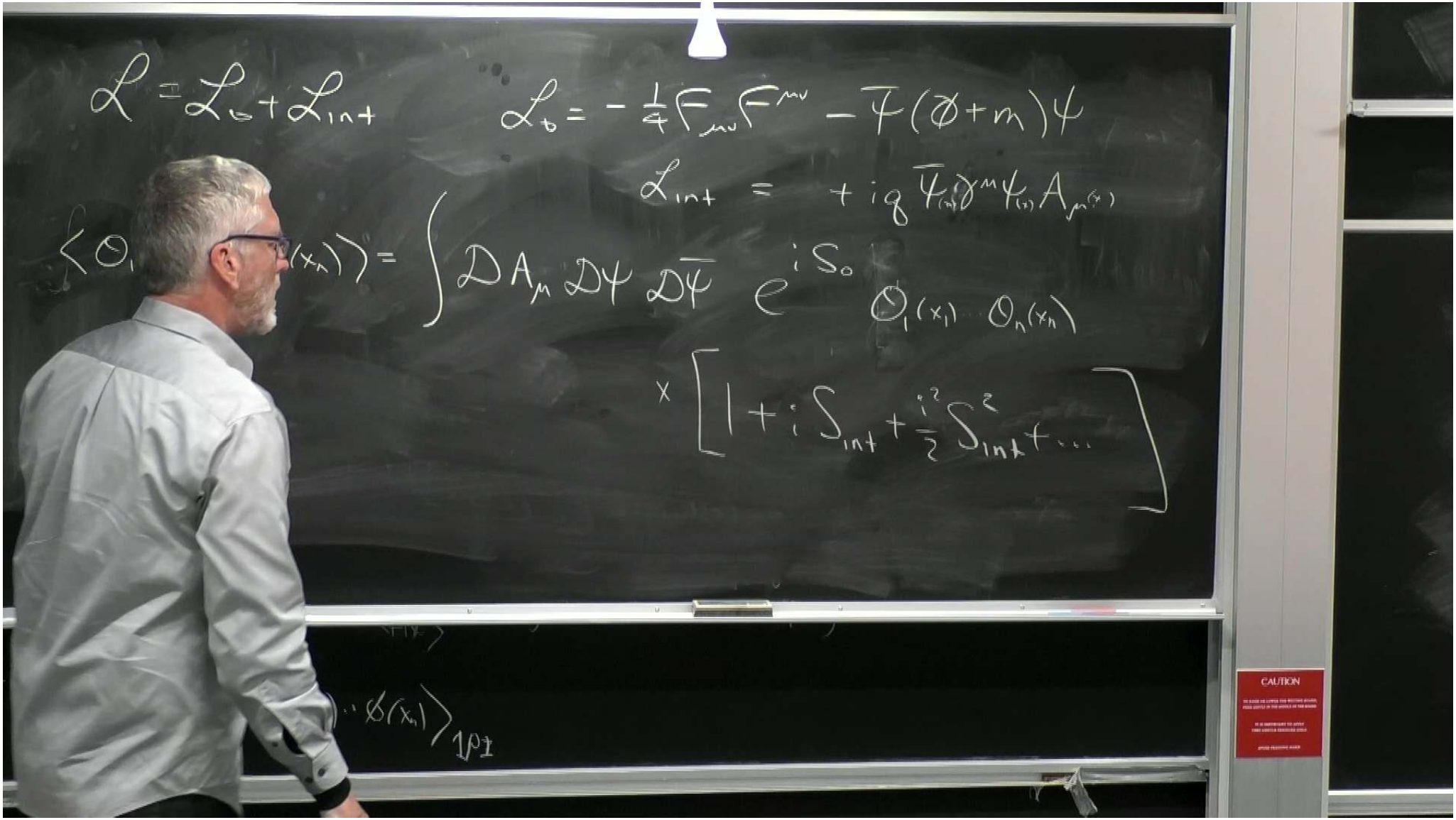
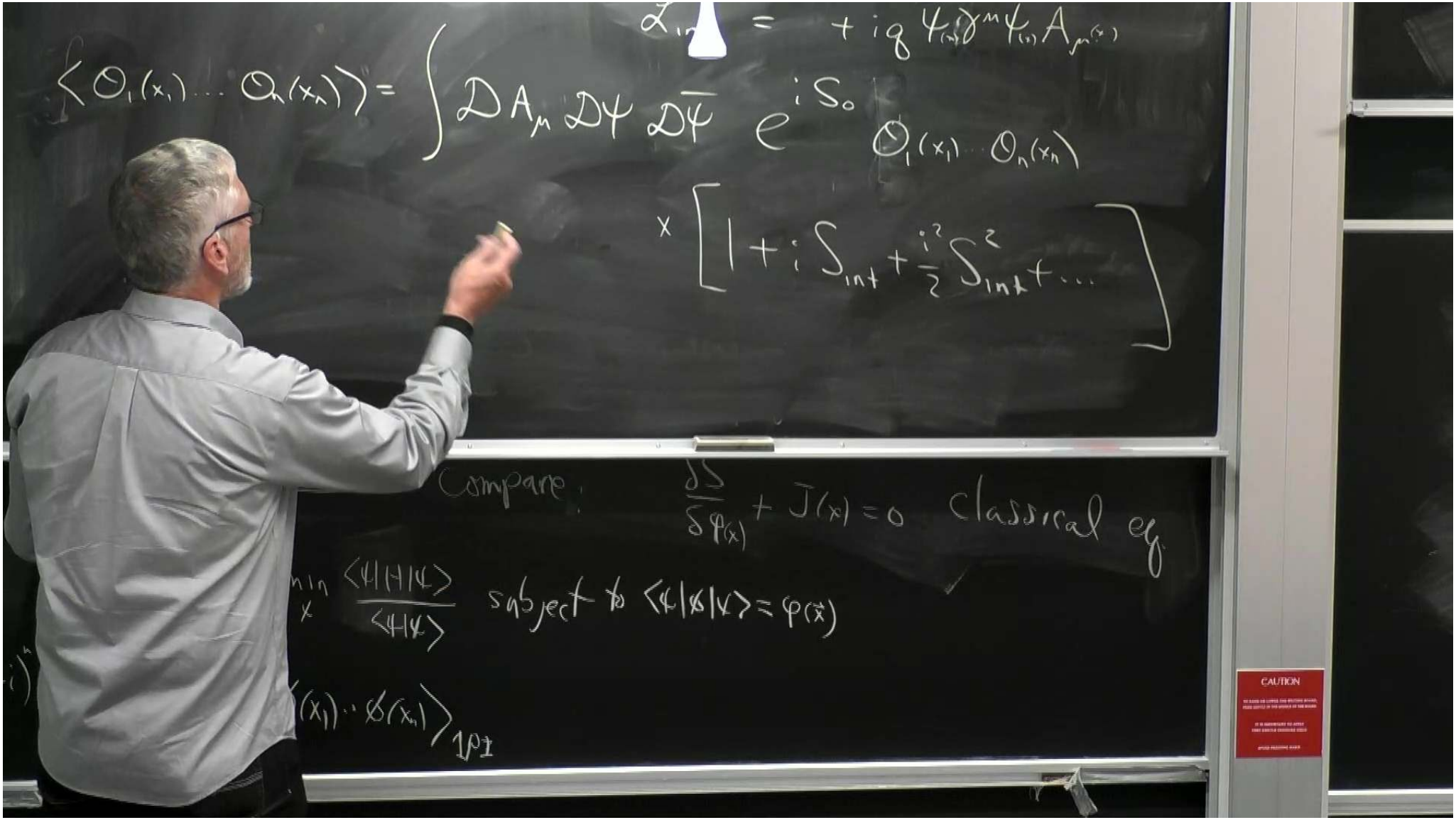
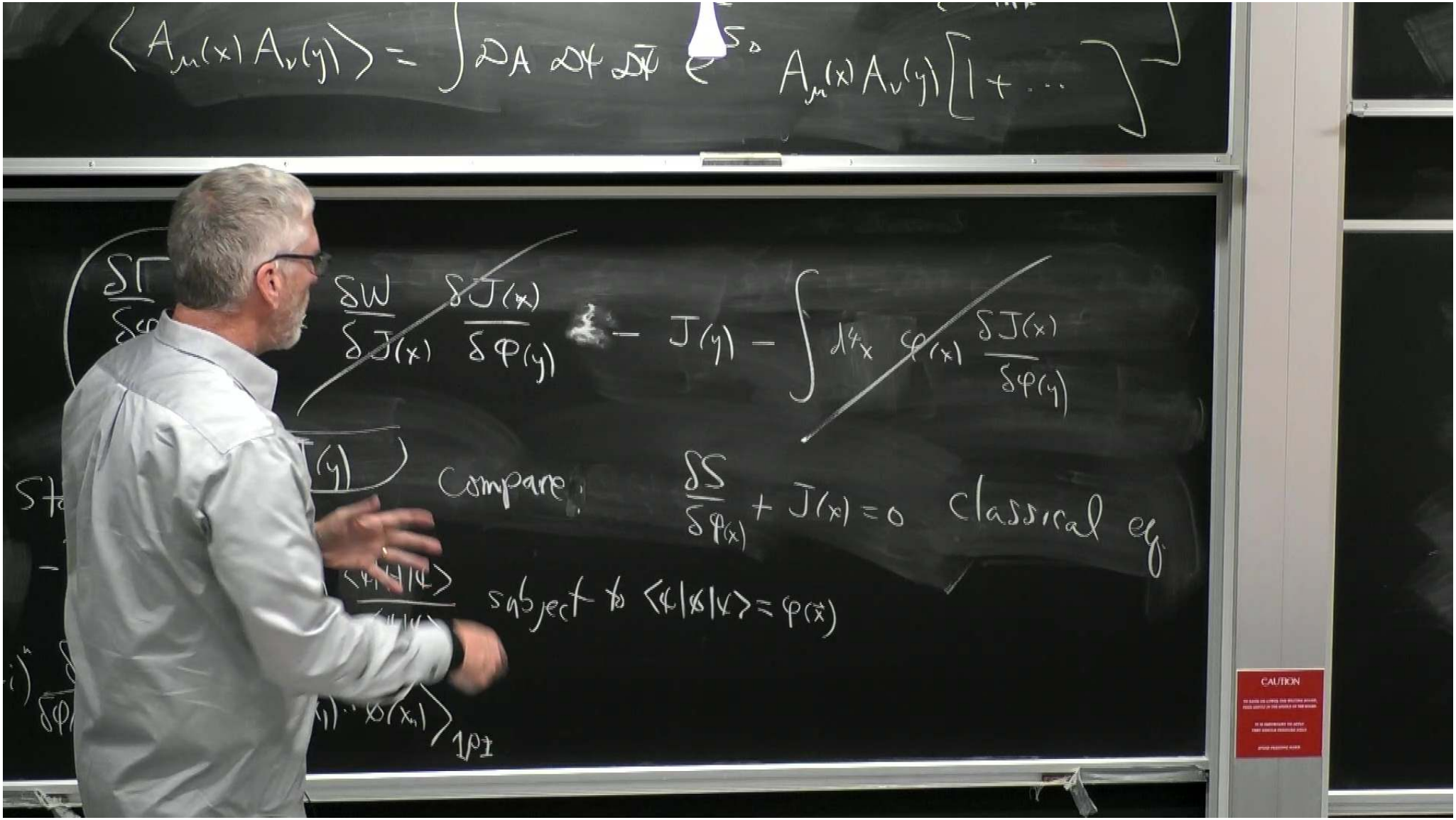
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi$
$\mathcal{L}_{int} = + i g \bar{\psi}_{(x)} \gamma^\mu \psi_{(x)} A_{\mu(x)}$
$\langle \mathcal{O}_1 \ldots \mathcal{O}_n(x_n) \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi}\ e^{iS_0}\ \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n)$
$\times \left[1 + iS_{int} + \frac{i^2}{2} S_{int}^2 + \ldots \right]$
CAUTION

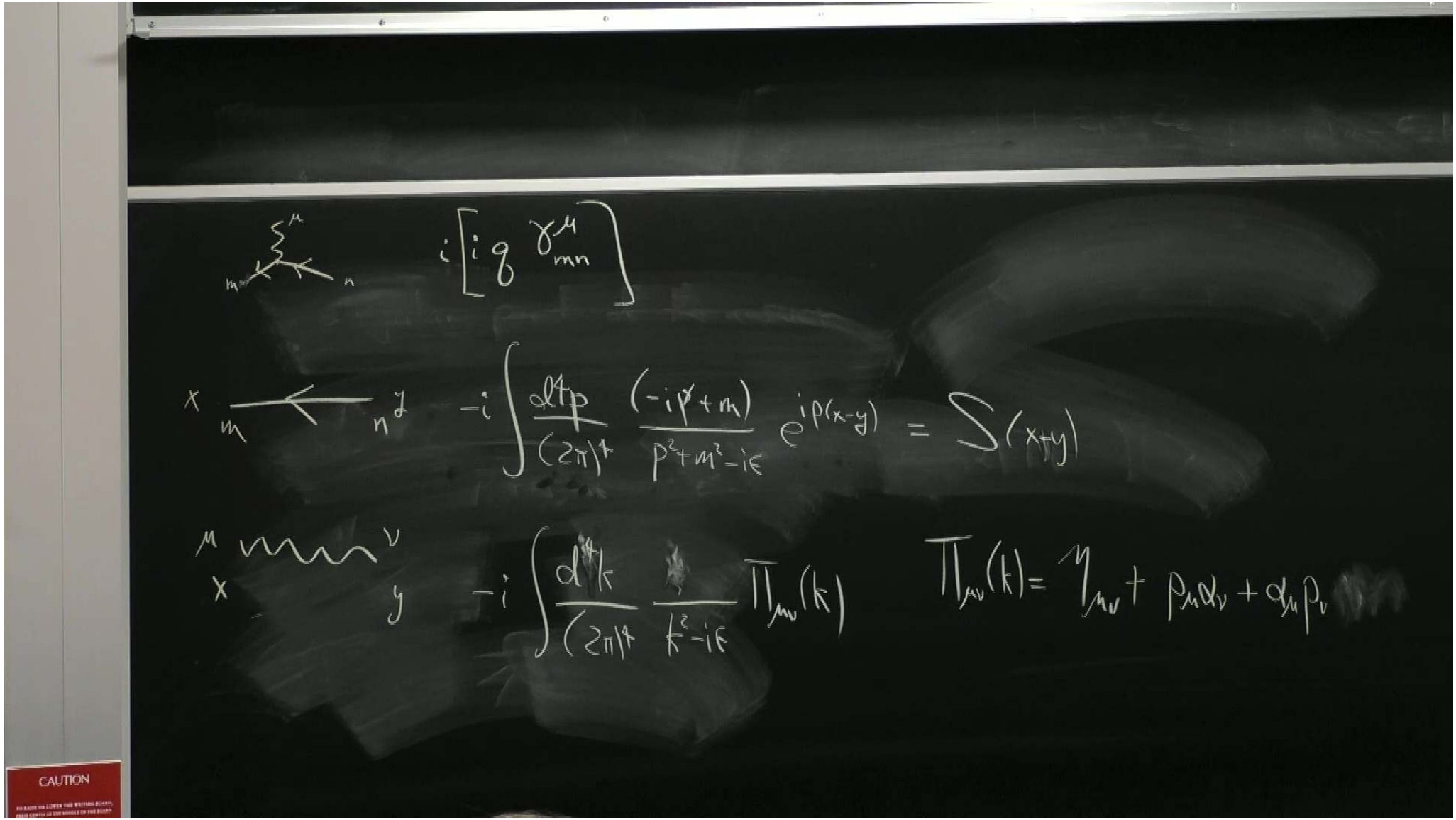

$\mathcal{L}_{int} = +iq\bar{\psi}_{(x)}\gamma^{\mu}\psi_{(x)}A_{\mu}(x)$
$\langle O_1(x_1)\cdots O_n(x_n)\rangle = \int \mathcal{D}A_\mu\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\; e^{iS_0}\, O_1(x_1)\cdots O_n(x_n)$
$\times\left[1+iS_{int}+\frac{i^2}{2}S_{int}^2+\cdots\right]$
Compare: $\frac{\delta S}{\delta\varphi(x)}+J(x)=0$ classical eq.
$\min_{\chi}\frac{\langle\chi|H|\chi\rangle}{\langle\chi|\chi\rangle}$ subject to $\langle\chi|\hat{\varphi}|\chi\rangle=\varphi(\vec{x})$
$\langle\hat{\varphi}(x_1)\cdots\hat{\varphi}(x_n)\rangle_{1PI}$
CAUTION

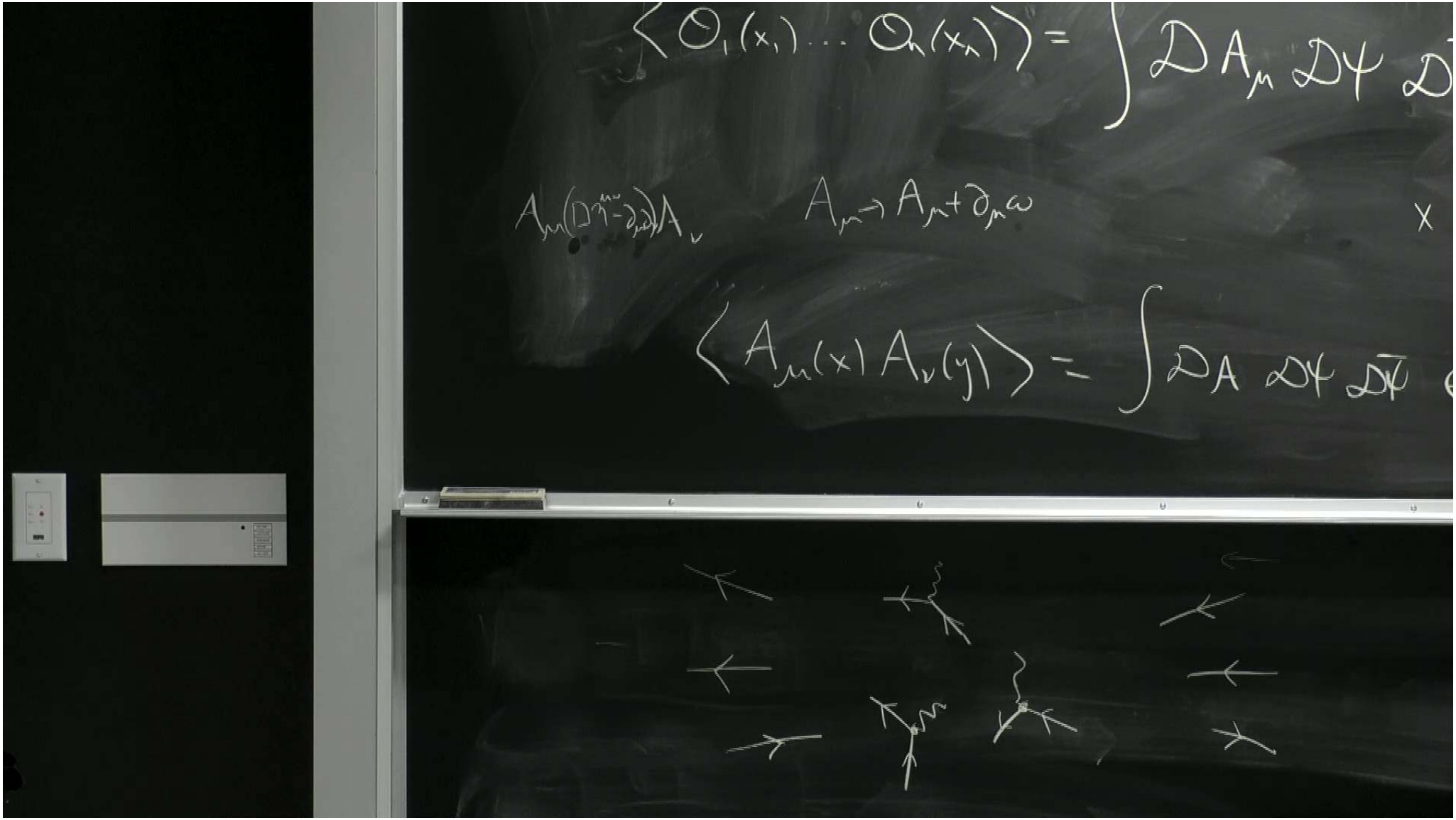

$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A \, \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, e^{iS_0} A_\mu(x) A_\nu(y) \left[1 + \dots \right]$
$\frac{\delta W}{\delta J(x)} \frac{\delta J(x)}{\delta \varphi(y)} - J(y) - \int d^4x \, \varphi(x) \frac{\delta J(x)}{\delta \varphi(y)}$
Compare!
$\frac{\delta S}{\delta \varphi(x)} + J(x) = 0$ classical eq
subject to $\langle \psi | \hat{\varphi} | \psi \rangle = \varphi(\vec{x})$
CAUTION

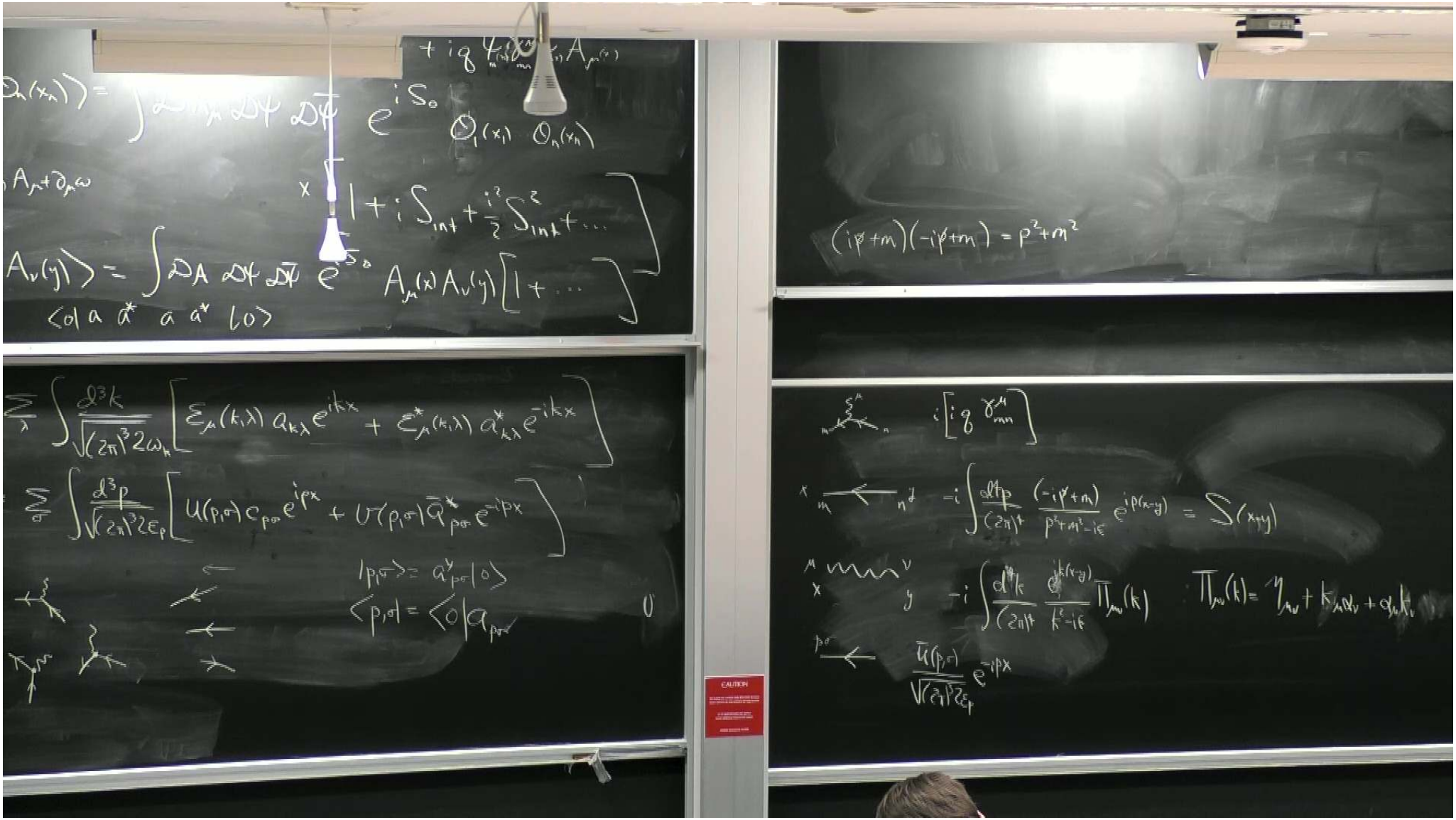
$$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, e^{iS_0} A_\mu(x) A_\nu(y) \left[1 + \dots \right]$$

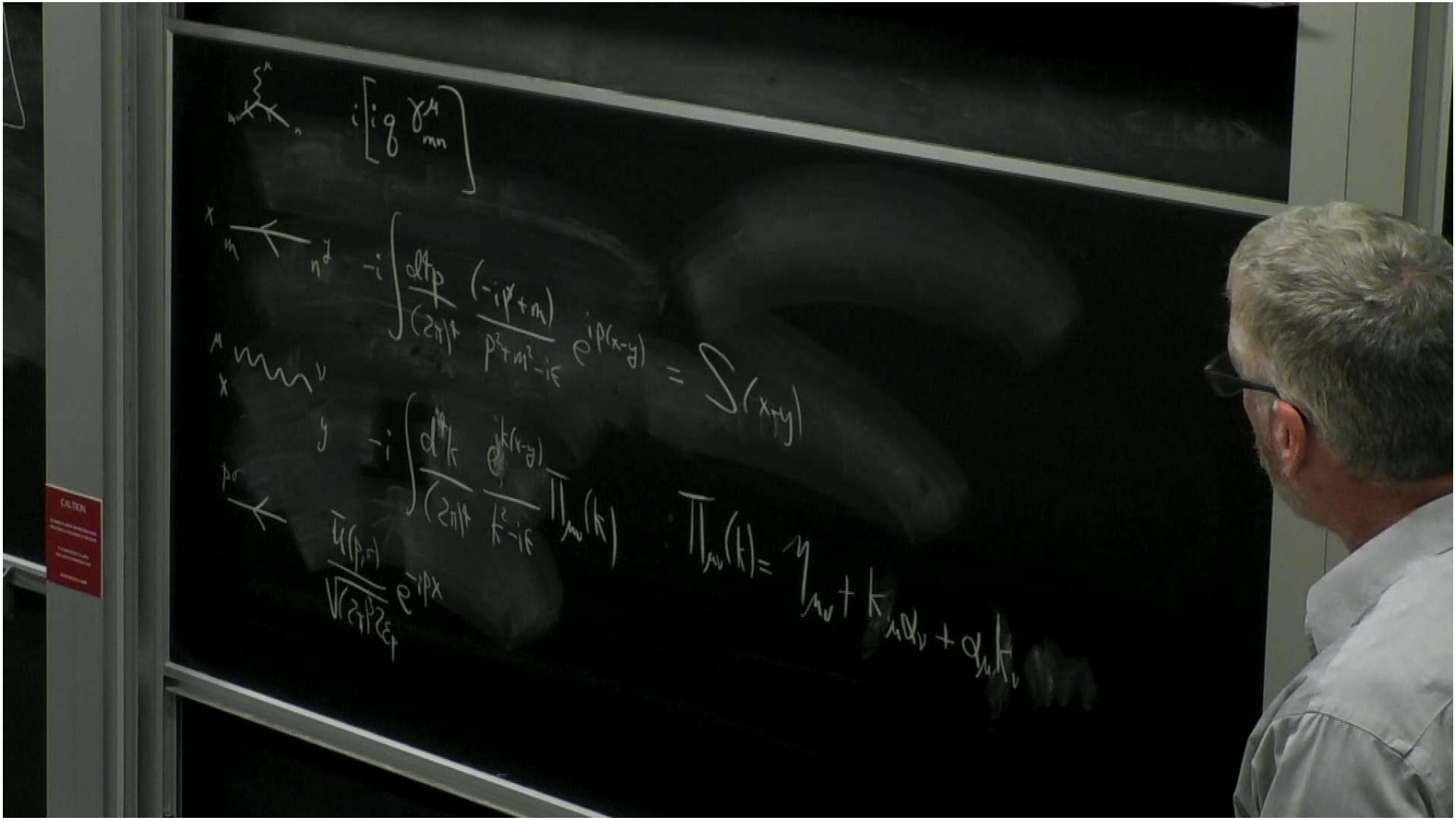
$$A_\mu(x) = \sum_\lambda \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left[\varepsilon_\mu(k,\lambda)\, a_{k\lambda}\, e^{ikx} + \varepsilon^*_\mu(k,\lambda)\, a^*_{k\lambda}\, e^{-ikx} \right]$$

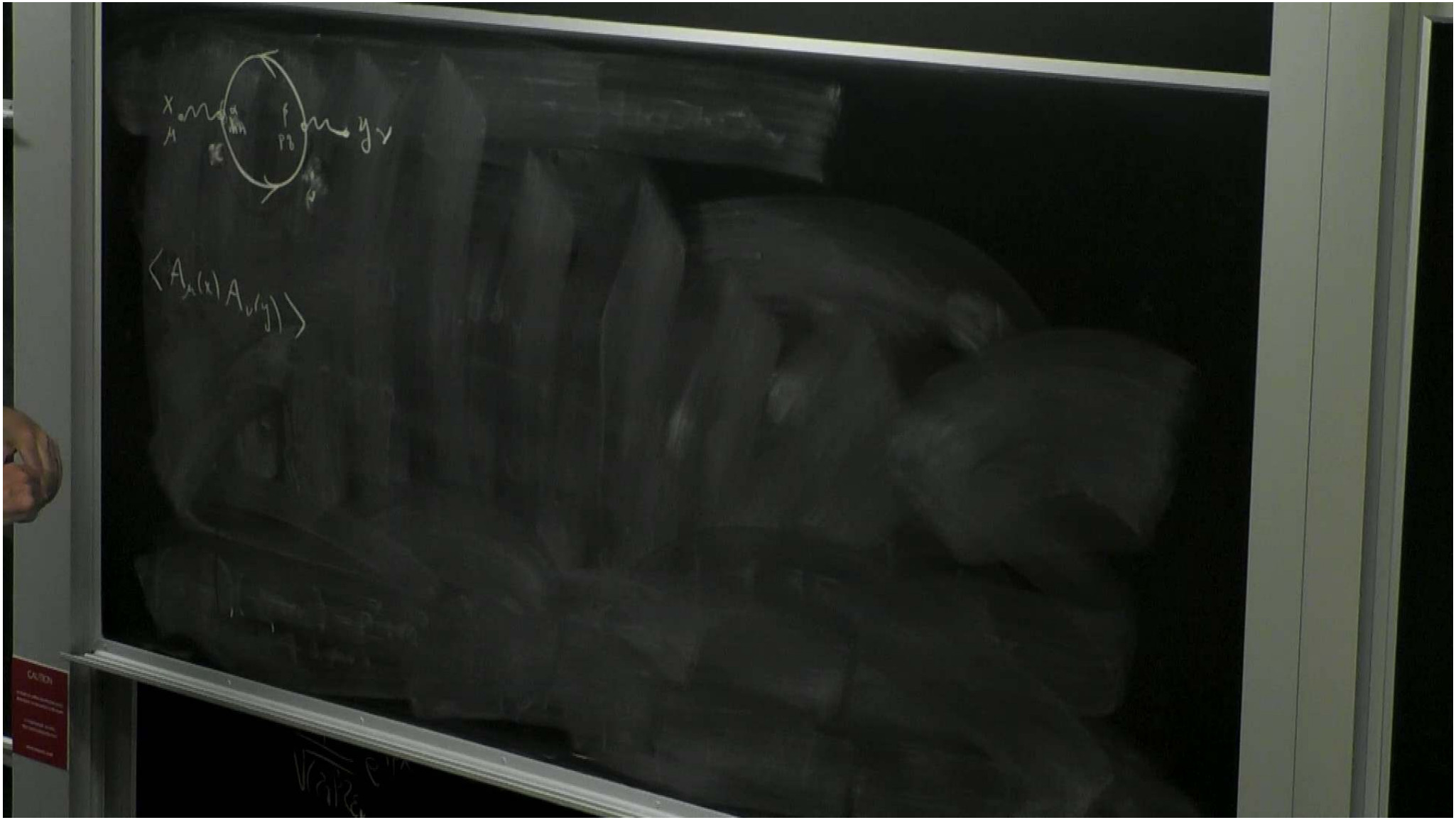
$$\psi(x) = \sum_\sigma \int \frac{d^3p}{\sqrt{(2\pi)^3 2\varepsilon_p}} \left[u(p,\sigma)\, c_{p\sigma}\, e^{ipx} + v(p,\sigma)\, \bar{a}^*_{p\sigma}\, e^{-ipx} \right]$$

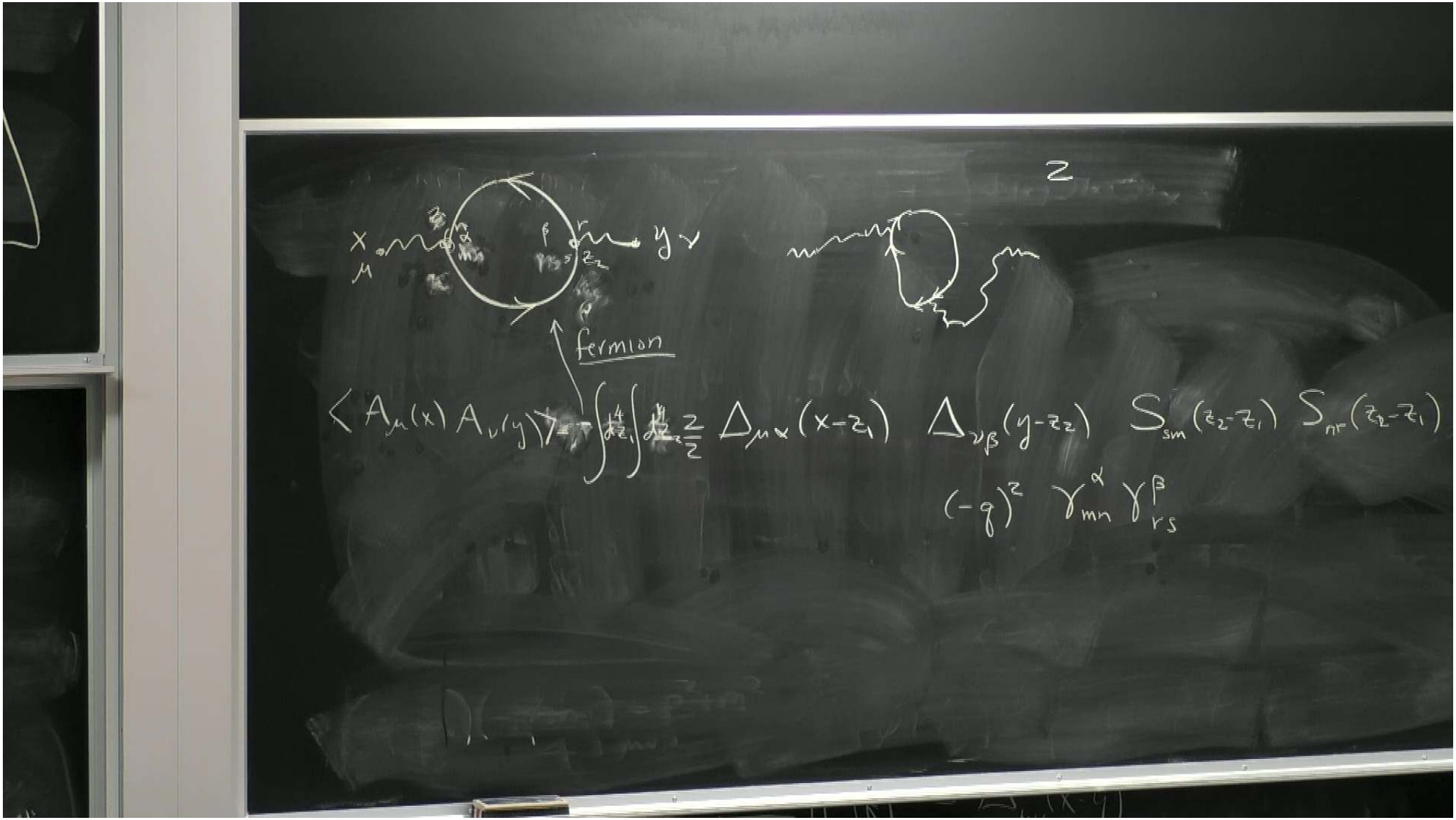

$i\left[ig\gamma^{\mu}_{mn}\right]$
x m n y
$-i\int \frac{d^4p}{(2\pi)^4} \frac{(-ip\!\!\!/+m)}{p^2+m^2-i\epsilon} e^{ip(x-y)} = S(x-y)$
μ ν x y
$-i\int \frac{d^4k}{(2\pi)^4} \frac{}{k^2-i\epsilon} \Pi_{\mu\nu}(k)$
$\Pi_{\mu\nu}(k) = \eta_{\mu\nu} + p_\mu q_\nu + q_\mu p_\nu$
CAUTION

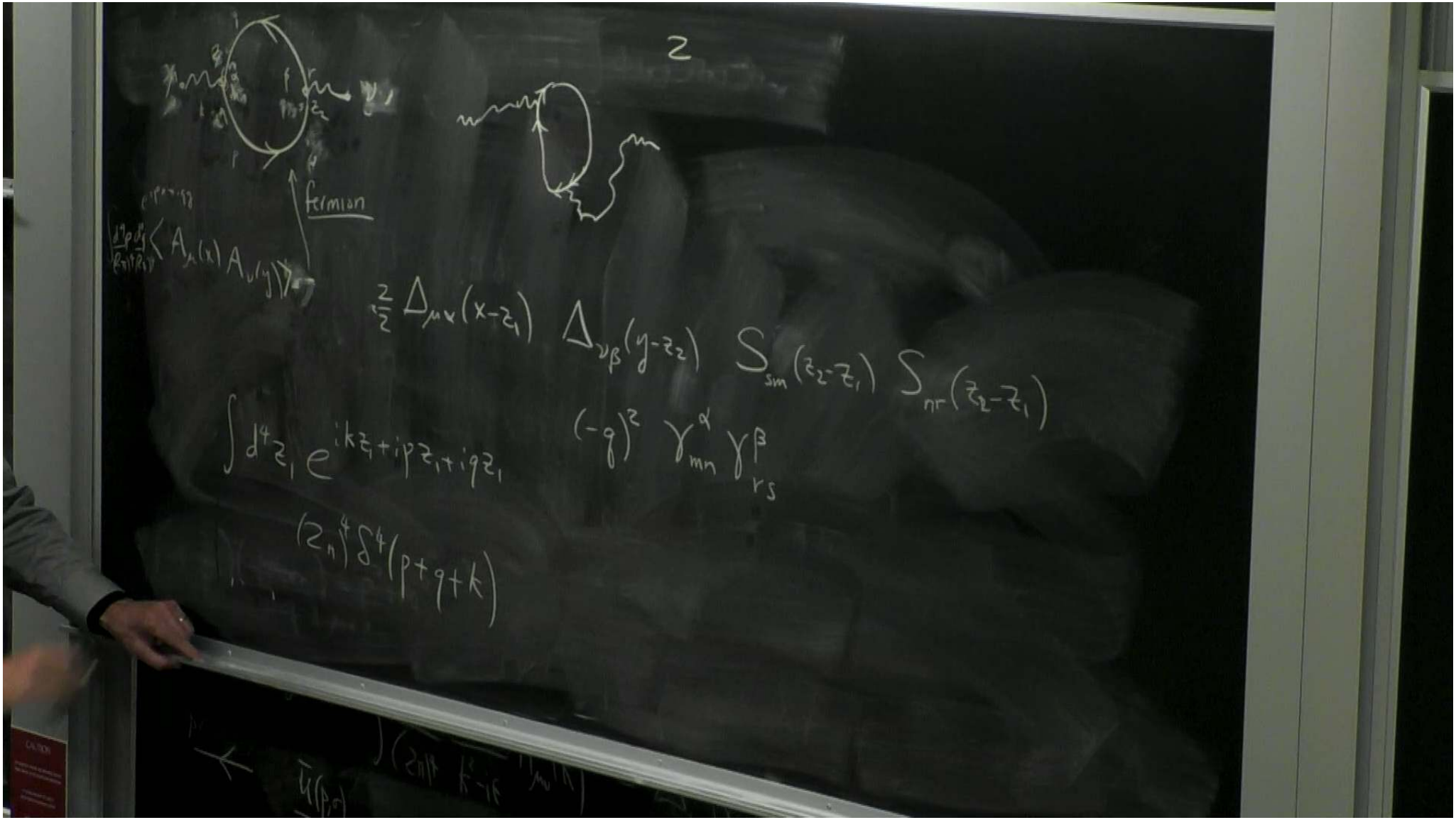

$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}$
$A_\mu (D\eta^{\mu\nu} - \partial^\mu\partial^\nu) A_\nu$
$A_\mu \to A_\mu + \partial_\mu \omega$
$\langle A_\mu(x) A_\nu(y) \rangle = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi}$

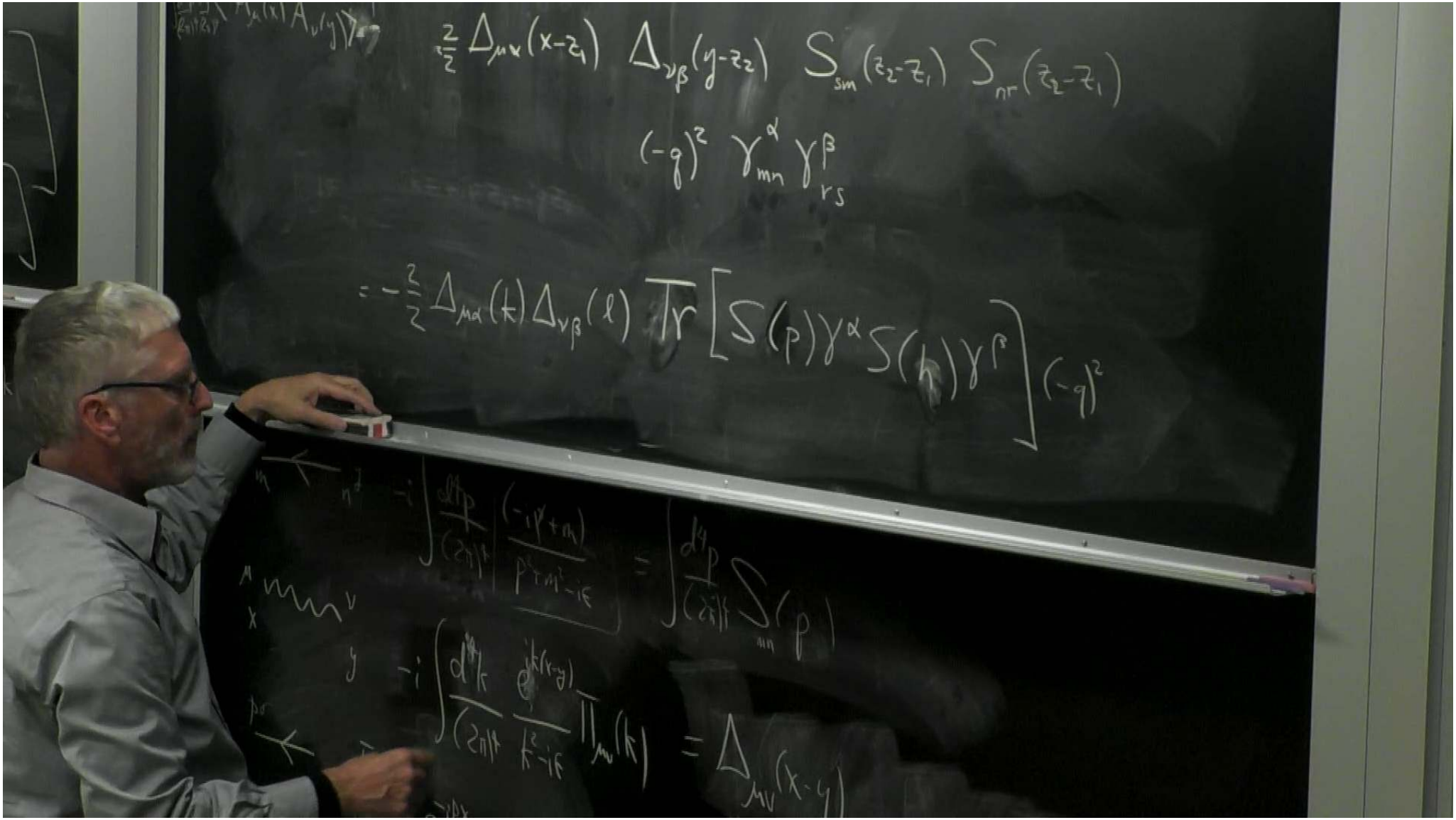

$(i\not{p}+m)(-i\not{p}+m) = p^2+m^2$
$|p,\sigma\rangle = a^\dagger_{p\sigma}|0\rangle$
$\langle p,\sigma| = \langle 0|a_{p\sigma}$
$\Pi_{\mu\nu}(k) = \eta_{\mu\nu} + k_\mu k_\nu + a_\mu k_\nu$
CAUTION

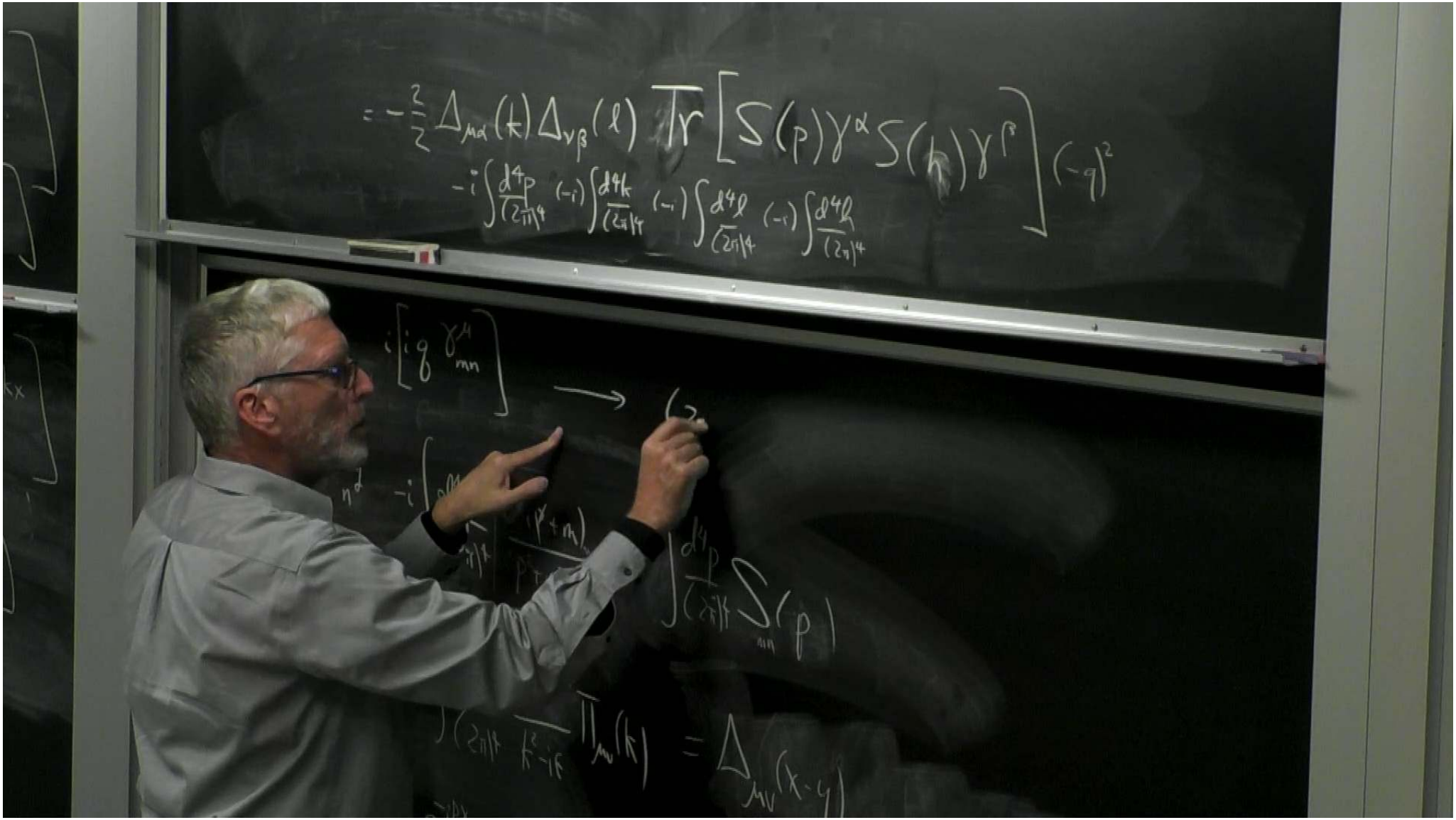

CAUTION

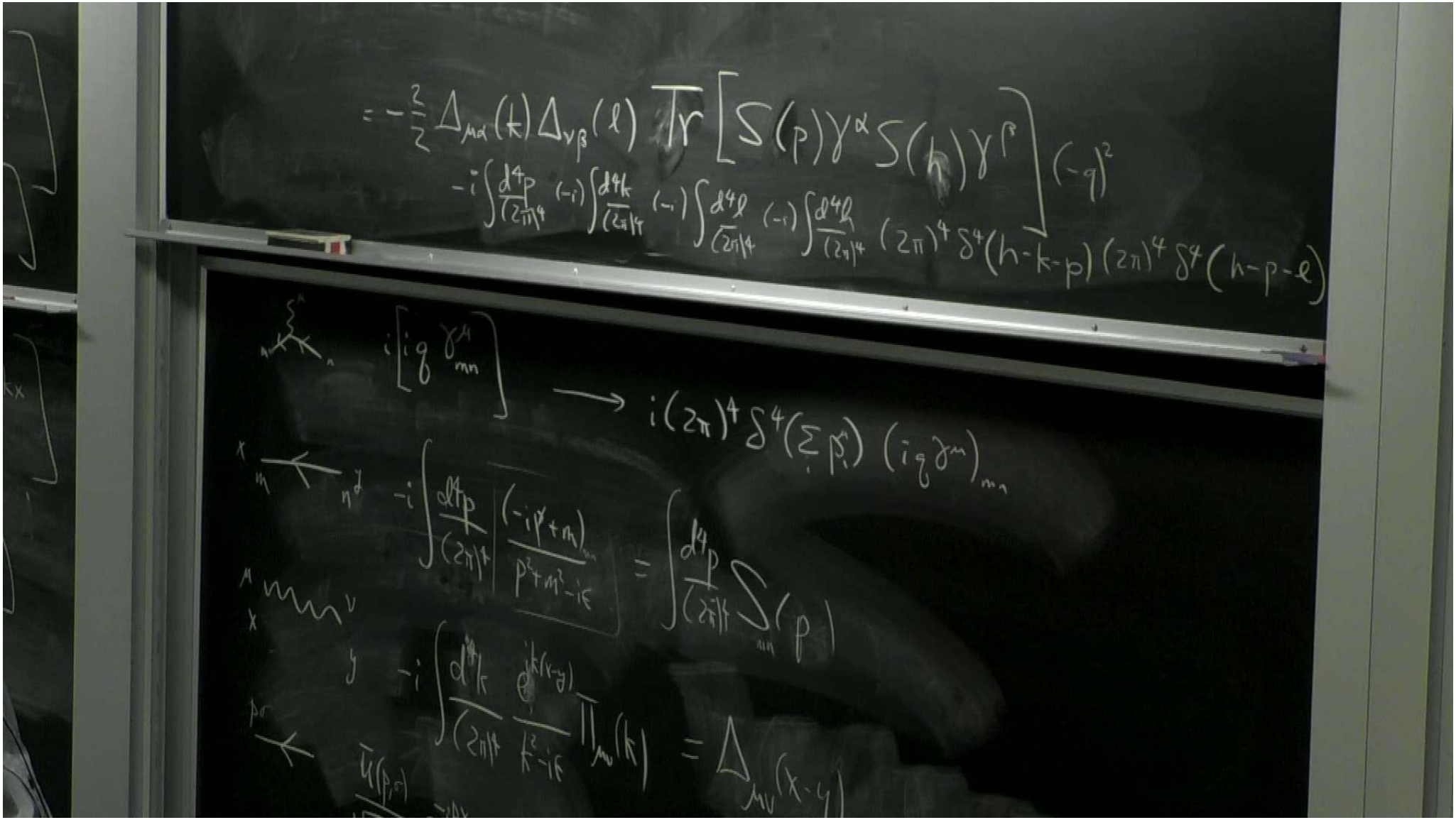

x
y
$\langle A_\mu(x) A_\nu(y) \rangle$

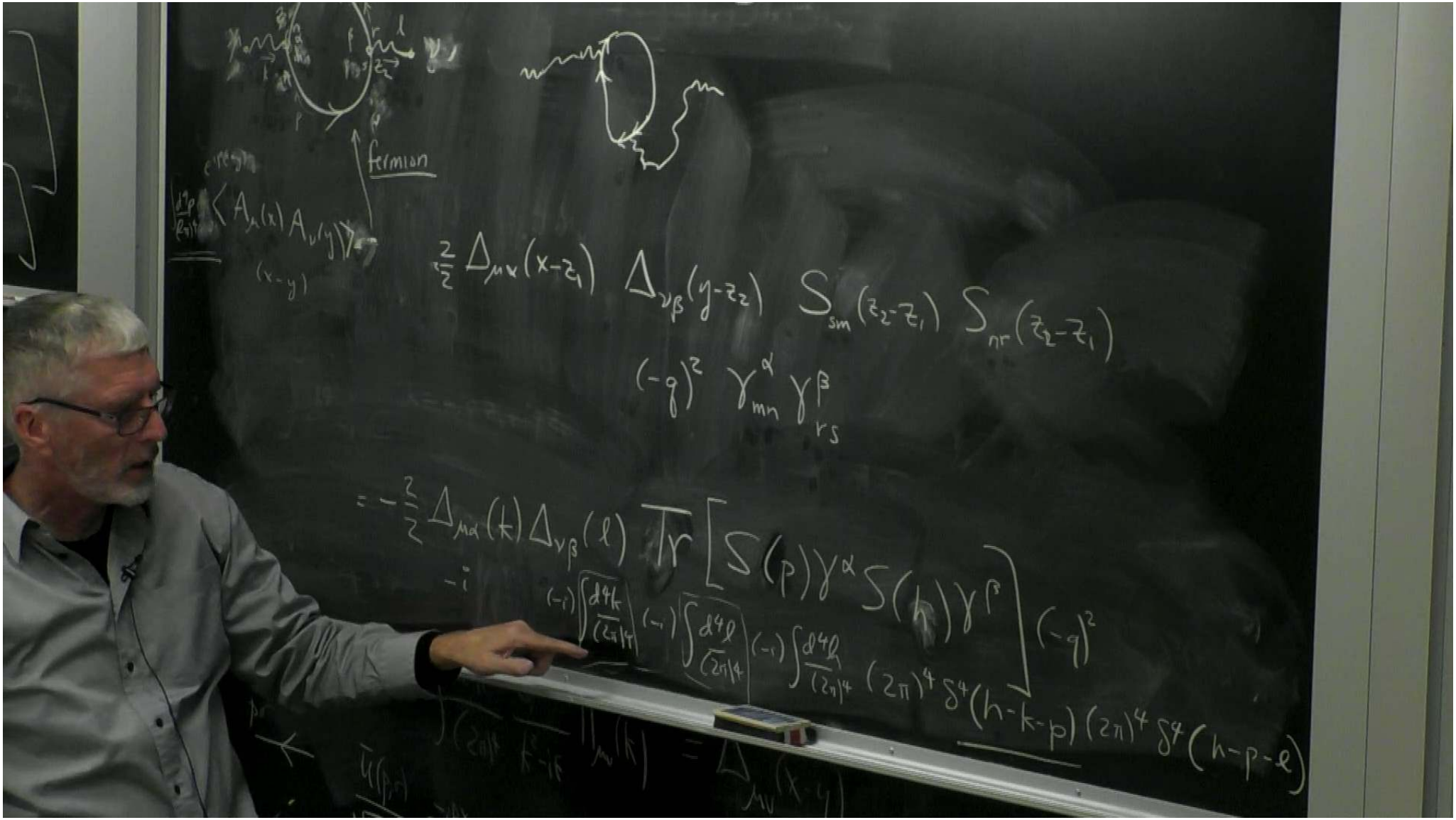

2
fermion
$\langle A_\mu(x) A_\nu(y) \rangle = \int d^4z_1 \int d^4z_2 \frac{z}{2} \Delta_{\mu\alpha}(x-z_1) \Delta_{\nu\beta}(y-z_2) S_{sm}(z_2-z_1) S_{nr}(z_2-z_1)$
$(-q)^2 \gamma^\alpha_{mn} \gamma^\beta_{rs}$


2
fermion
$\frac{z}{2}\Delta_{\mu\nu}(x-z_1)\,\Delta_{\alpha\beta}(y-z_2)\,S_{sm}(z_2-z_1)\,S_{nr}(z_2-z_1)$
$\int d^4z_1\, e^{ikz_1+ipz_1+iqz_1}$
$(-q)^2\,\gamma^{\alpha}_{mn}\,\gamma^{\beta}_{rs}$
$(2\pi)^4\delta^4(p+q+k)$


$\frac{2}{2}\Delta_{\mu\nu}(x-z_1)\ \Delta_{\nu\beta}(y-z_2)\ S_{sm}(z_2-z_1)\ S_{nr}(z_2-z_1)$
$(-q)^2\ \gamma^{\alpha}_{mn}\ \gamma^{\beta}_{rs}$
$= -\frac{2}{2}\Delta_{\mu\alpha}(k)\Delta_{\nu\beta}(l)\ \mathrm{Tr}\left[S(p)\gamma^{\alpha}S(k)\gamma^{\beta}\right](-q)^2$
$-i\int\frac{d^4p}{(2\pi)^4}\frac{(-i\not{p}+m)}{p^2+m^2-i\epsilon} = \int\frac{d^4p}{(2\pi)^4}S_{mn}(p)$
$-i\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2-i\epsilon}\Pi_{\mu\nu}(k) = \Delta_{\mu\nu}(x-y)$


$= -\frac{2}{2}\Delta_{\mu\alpha}(k)\Delta_{\nu\beta}(l)\,\mathrm{Tr}\left[S(p)\gamma^{\alpha}S(q)\gamma^{\beta}\right](-1)^2$
$-i\int\frac{d^4p}{(2\pi)^4}(-1)\int\frac{d^4k}{(2\pi)^4}(-1)\int\frac{d^4q}{(2\pi)^4}(-1)\int\frac{d^4l}{(2\pi)^4}$
$i\left[ig\,\gamma^{\mu}_{mn}\right]$
$\int\frac{d^4p}{(2\pi)^4}S_{mn}(p)$
$\Pi_{\mu\nu}(k) = \Delta_{\mu\nu}(x-y)$

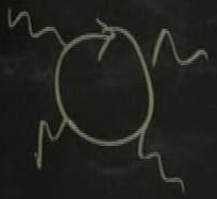

$= -\frac{2}{2}\Delta_{\mu\alpha}(k)\Delta_{\nu\beta}(l)\,\mathrm{Tr}\left[S(p)\gamma^{\alpha}S(n)\gamma^{\beta}\right](-q)^2$
$-i\int\frac{d^4p}{(2\pi)^4}(-i)\int\frac{d^4k}{(2\pi)^4}(-i)\int\frac{d^4q}{(2\pi)^4}(-i)\int\frac{d^4h}{(2\pi)^4}(2\pi)^4\delta^4(h-k-p)(2\pi)^4\delta^4(h-p-l)$
$i\left[ig\gamma^{\mu}_{mn}\right] \longrightarrow i(2\pi)^4\delta^4\left(\sum_i p_i^{\mu}\right)(ig\gamma_{\mu})_{mn}$
$-i\int\frac{d^4p}{(2\pi)^4}\frac{(-i\not{p}+m)_{mn}}{p^2+m^2-i\epsilon} = \int\frac{d^4p}{(2\pi)^4}S_{mn}(p)$
$-i\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2-i\epsilon}\Pi_{\mu\nu}(k) = \Delta_{\mu\nu}(x-y)$


fermion
$\frac{2}{2}\Delta_{\mu\nu}(x-z_1)\ \Delta_{\nu\beta}(y-z_2)\ S_{sm}(z_2-z_1)\ S_{nr}(z_2-z_1)$
$(-q)^2\ \gamma^{\alpha}_{mn}\ \gamma^{\beta}_{rs}$
$= -\frac{2}{2}\Delta_{\mu\alpha}(k)\Delta_{\nu\beta}(\ell)\ \mathrm{Tr}\left[S(p)\gamma^{\alpha}S(h)\gamma^{\beta}\right](-q)^2$
$(2\pi)^4\delta^4(h-k-p)(2\pi)^4\delta^4(h-p-\ell)$

I_e internal e lines

I_γ internal γ lines

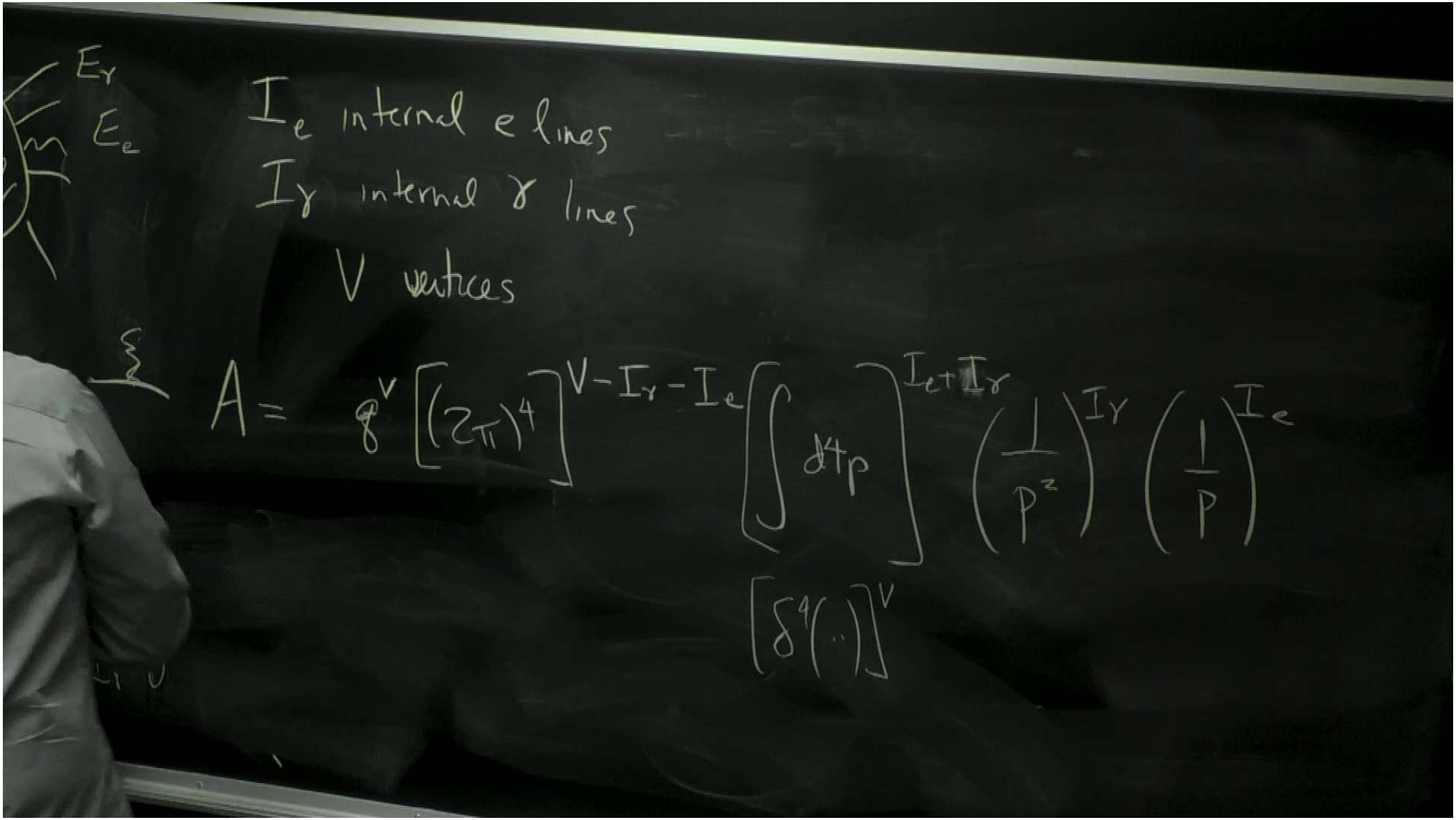
V vertices

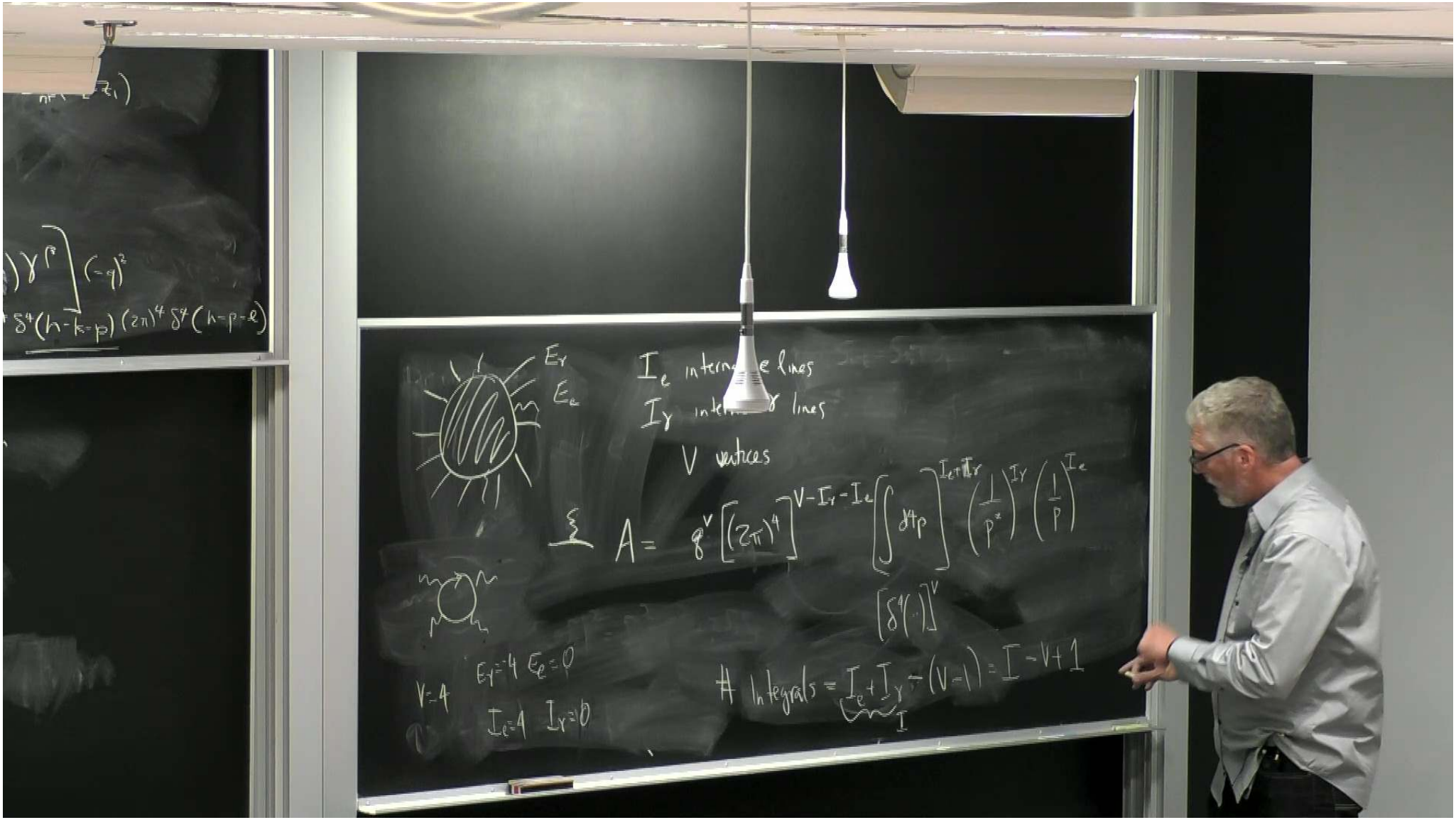


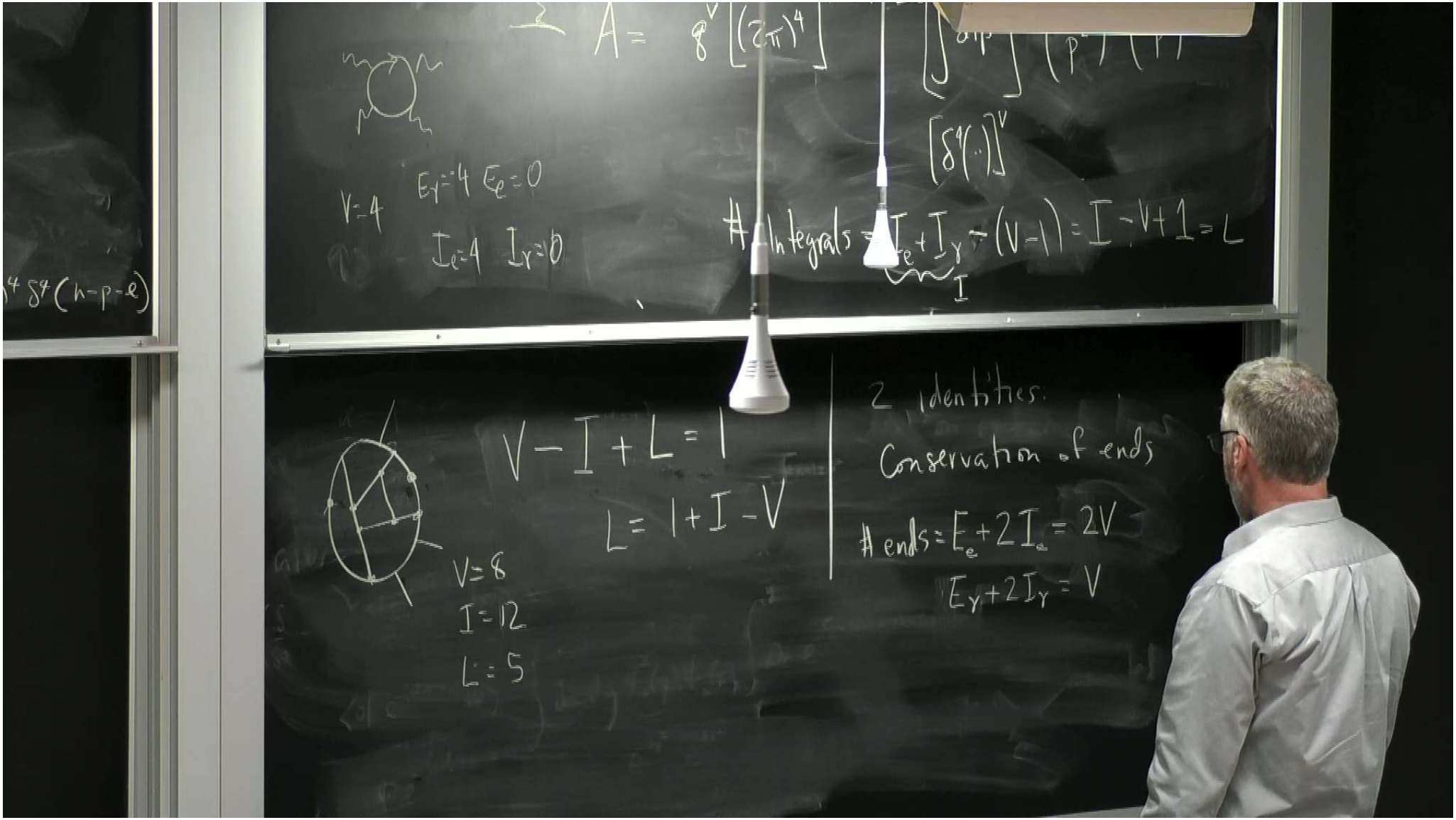
$$\sum A = g^V \left[(2\pi)^4\right]^{V - I_\gamma - I_e} \left[\int d^4p\right]^{I_e + I_\gamma}$$

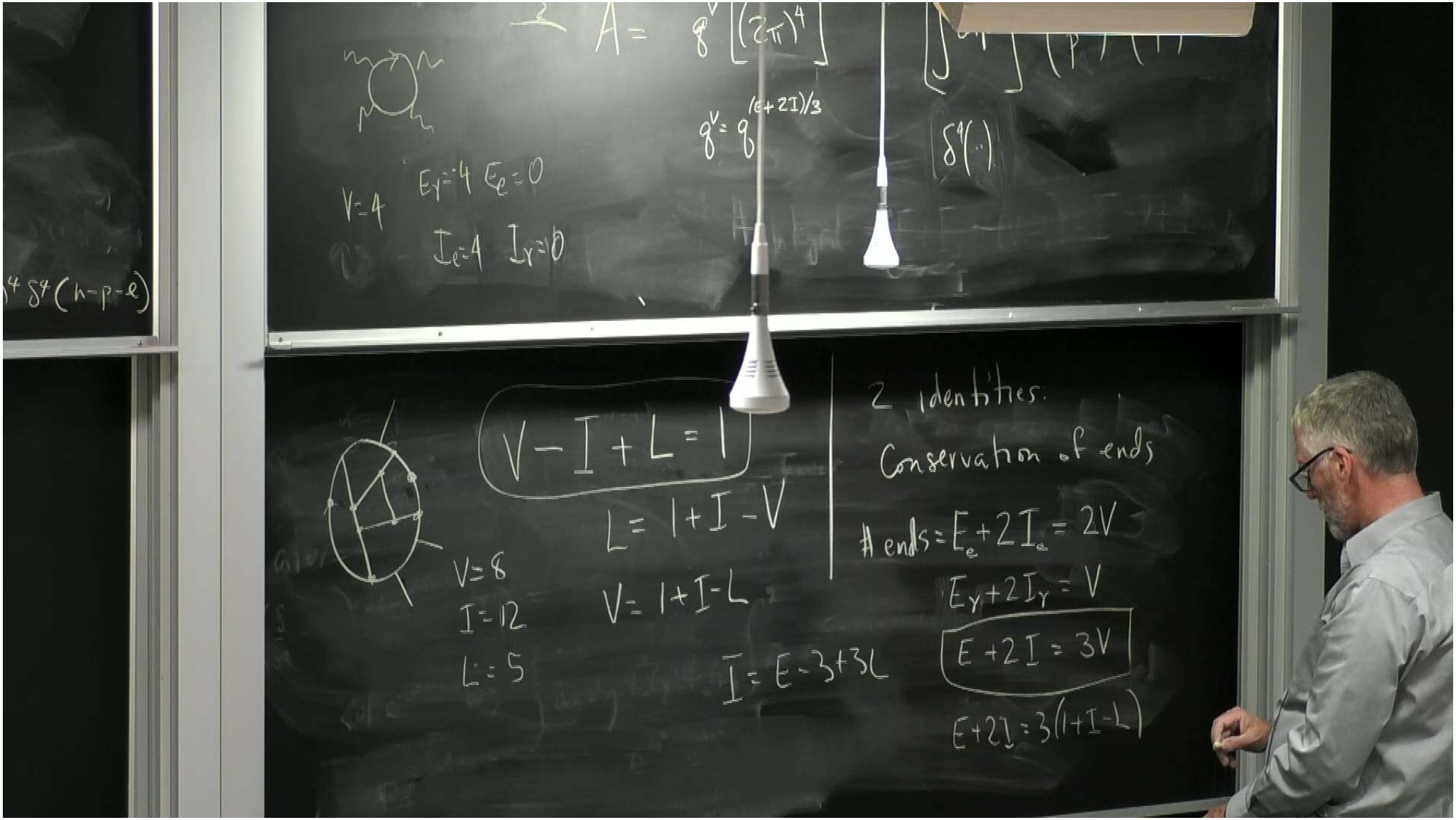
$V=4 \quad E_\gamma = 4 \quad E_e = 0$

$I_e = 4 \quad I_\gamma = 0$

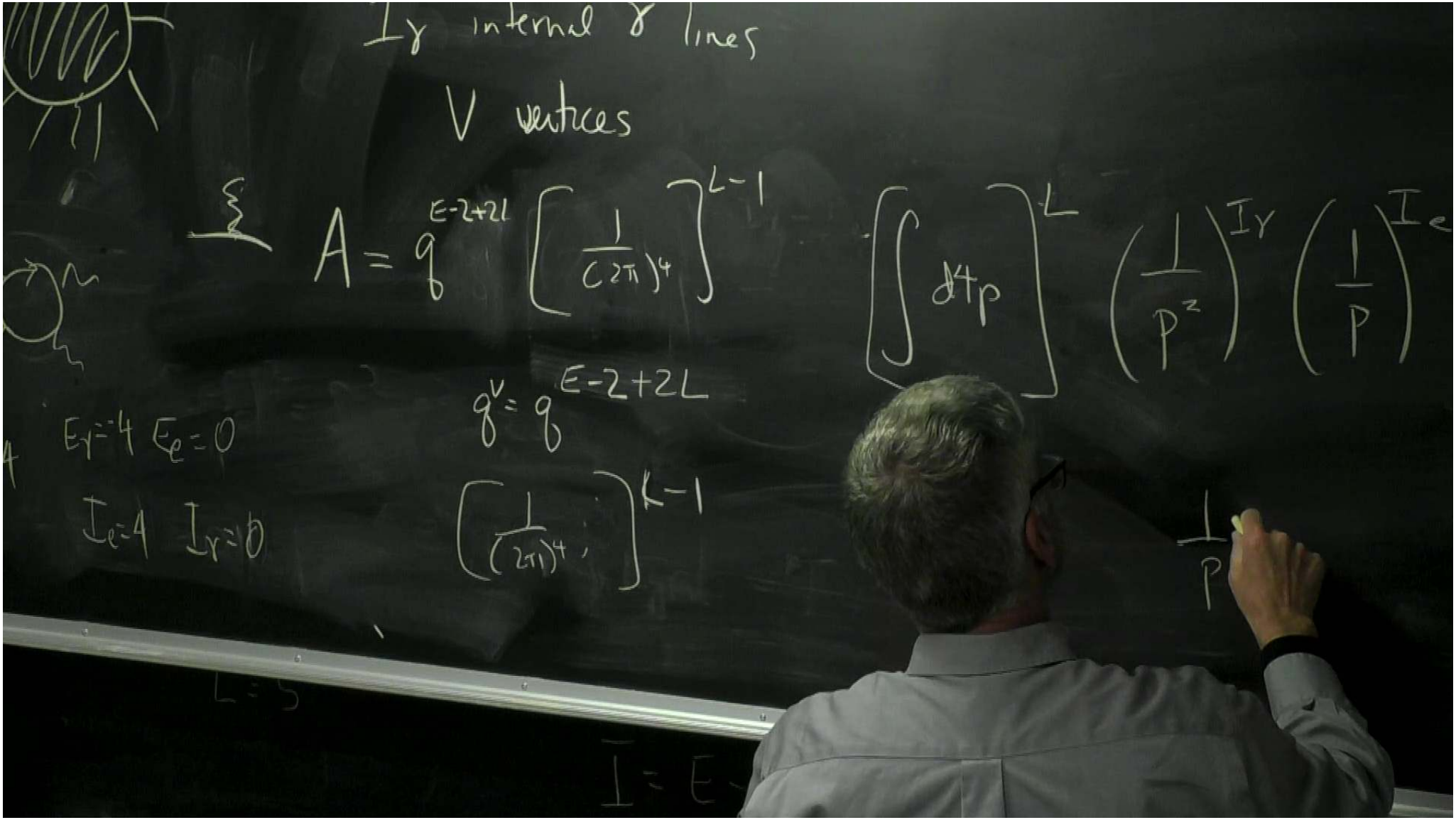

E_γ
E_e
I_e internal e lines
I_γ internal γ lines
V vertices
$A = g^V [(2\pi)^4]^{V - I_\gamma - I_e} \left[\int d^4p\right]^{I_e + I_\gamma} \left(\frac{1}{p^2}\right)^{I_\gamma} \left(\frac{1}{p}\right)^{I_e}$
$[\delta^4(\cdots)]^V$

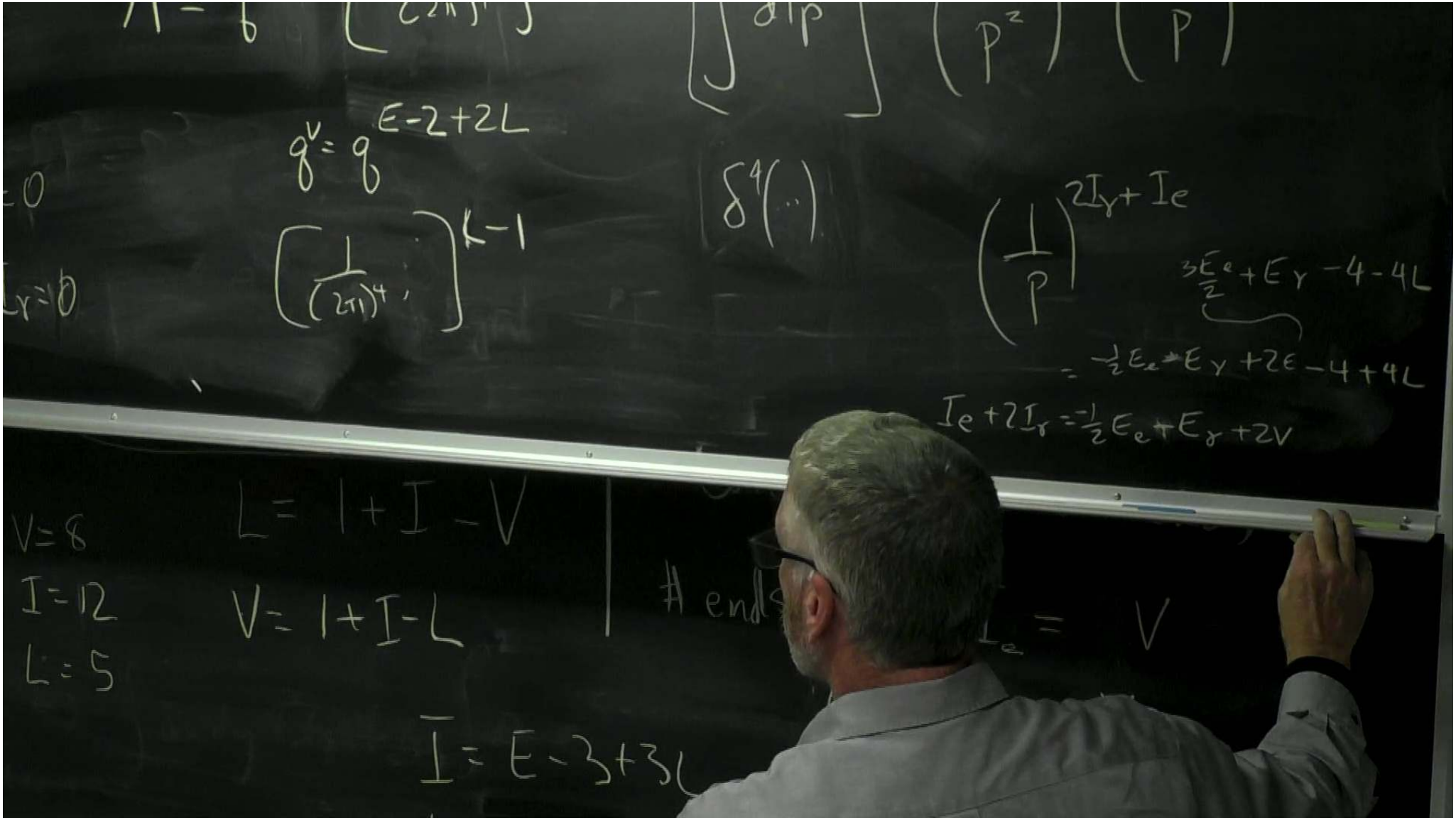

E_γ
E_e
I_e internal e lines
I_γ internal γ lines
V vertices
$\sum A = g^V \left[(2\pi)^4\right]^{V-I_\gamma-I_e} \left[\int d^4p\right]^{I_e+I_\gamma} \left(\frac{1}{p^2}\right)^{I_\gamma} \left(\frac{1}{p}\right)^{I_e}$
$\left[\delta^4(\cdots)\right]^V$
$V=4$ $E_\gamma=4$ $E_e=0$
$I_e=4$ $I_\gamma=0$
Integrals $= \underbrace{I_e+I_\gamma}_{I} - (V-1) = I-V+1$

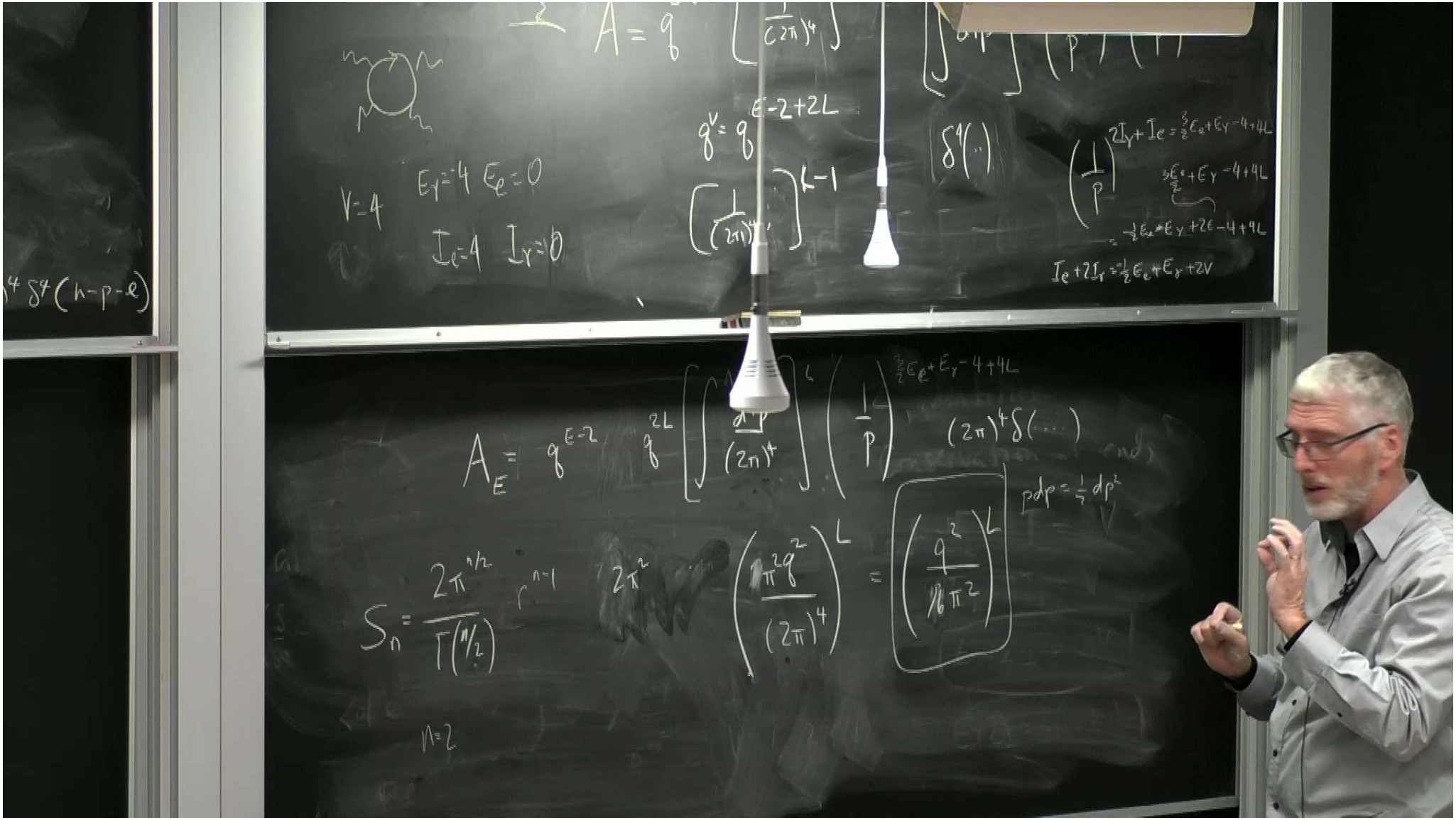

A =
V=4
E_γ=4 E_e=0
I_e=4 I_γ=0
# Integrals = I_e + I_γ − (V−1) = I − V + 1 = L
I
[δ⁴(·)]^V
V − I + L = 1
L = 1 + I − V
V=8
I=12
L=5
2 identities:
Conservation of ends
ends = E_e + 2I_e = 2V
E_γ + 2I_γ = V



$A = q^V [(2\pi)^4]$
$q^V = q^{(E+2I)/3}$
$\delta^4(\ldots)$
$V=4$
$E_\gamma=4$ $E_e=0$
$I_e=4$ $I_\gamma=0$
$V-I+L=1$
$L=1+I-V$
$V=1+I-L$
$V=8$
$I=12$
$L=5$
$I=E-3+3L$
2 identities:
Conservation of ends
$\#\text{ends}=E_e+2I_e=2V$
$E_\gamma+2I_\gamma=V$
$E+2I=3V$
$E+2I=3(1+I-L)$


I_γ internal γ lines
V vertices
$\sum$
$A = q^{E-2+2L} \left[\frac{1}{(2\pi)^4}\right]^{L-1} \left[\int d^4p\right]^L \left(\frac{1}{p^2}\right)^{I_\gamma} \left(\frac{1}{p}\right)^{I_e}$
$q^V = q^{E-2+2L}$
$\left[\frac{1}{(2\pi)^4}\right]^{V-1}$
$E_\gamma = 4$ $E_e = 0$
$I_e = 4$ $I_\gamma = 0$
$L = 5$
$\frac{1}{p}$

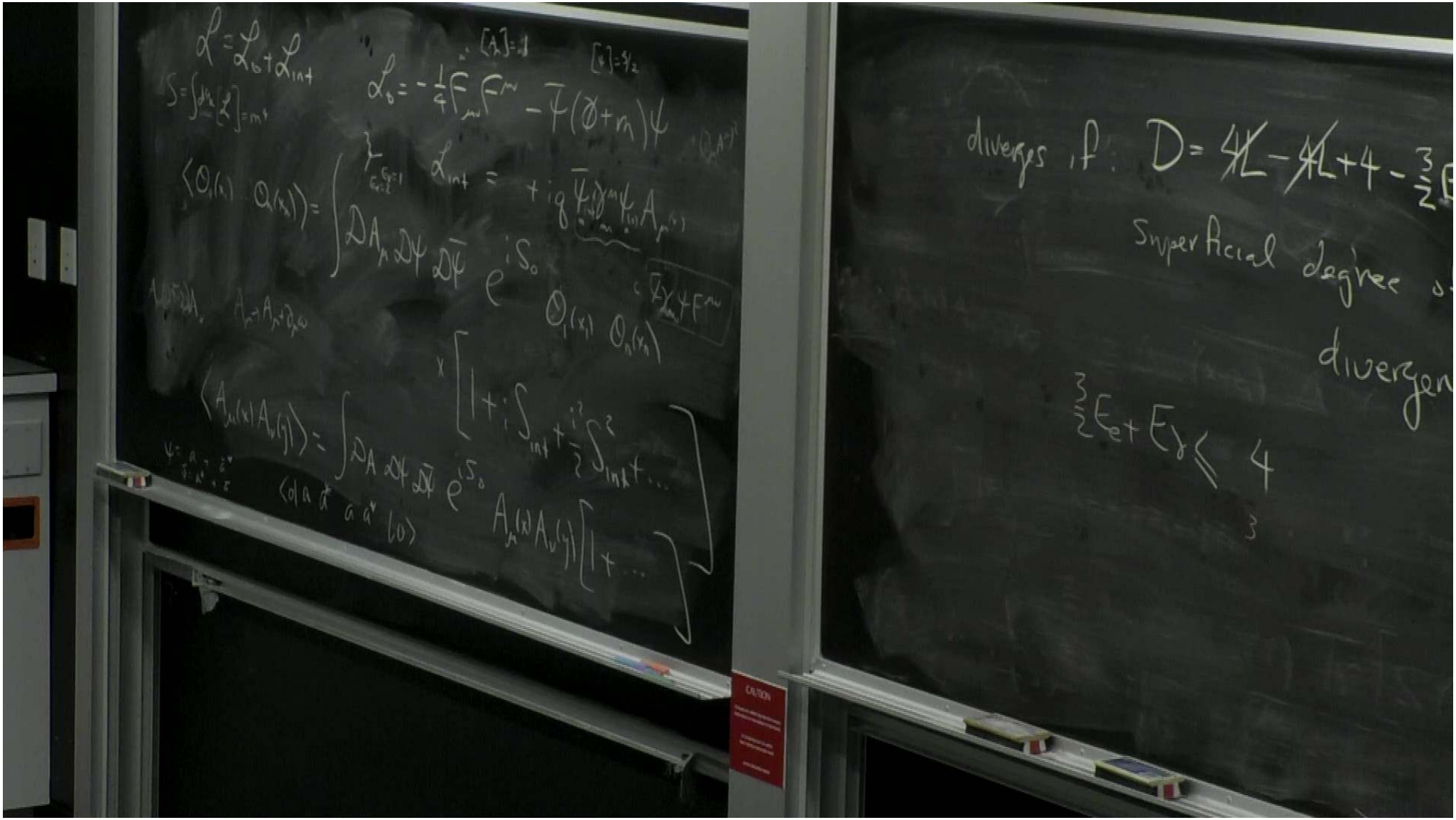

$q^V = q^{E-2+2L}$
$\left[\frac{1}{(2\pi)^4}\right]^{k-1}$
$\delta^4(\cdot)$
$\left(\frac{1}{p}\right)^{2I_\gamma + I_e}$
$\frac{3E_e}{2} + E_\gamma - 4 - 4L$
$= -\frac{1}{2}E_e + E_\gamma + 2\epsilon - 4 + 4L$
$I_e + 2I_\gamma = -\frac{1}{2}E_e + E_\gamma + 2V$
$V = 8$
$I = 12$
$L = 5$
$L = 1 + I - V$
$V = 1 + I - L$
$I = E - 3 + 3L$


V=4
Eγ=-4 Ee=0
Ie=4 Iγ=0
q^V = q^{E-2+2L}
δ^4(..)
Ie+2Iγ = -½Ee+Eγ+2V
A_E = q^{E-2} q^{2L}
(2π)^4 δ(...)
pdp = ½ dp²
S_n = 2π^{n/2} / Γ(n/2)
n=2

diverges if: $D = 4L - 4L + 4 - \frac{3}{2}E_e - E_\gamma \geq 0$

superficial degree of divergence

$\frac{3}{2}E_e + E_\gamma < 4$


diverges if: D = 4L − 4L + 4 − 3/2
Superficial degree
divergen
3/2 E_e + E_γ ≤ 4