Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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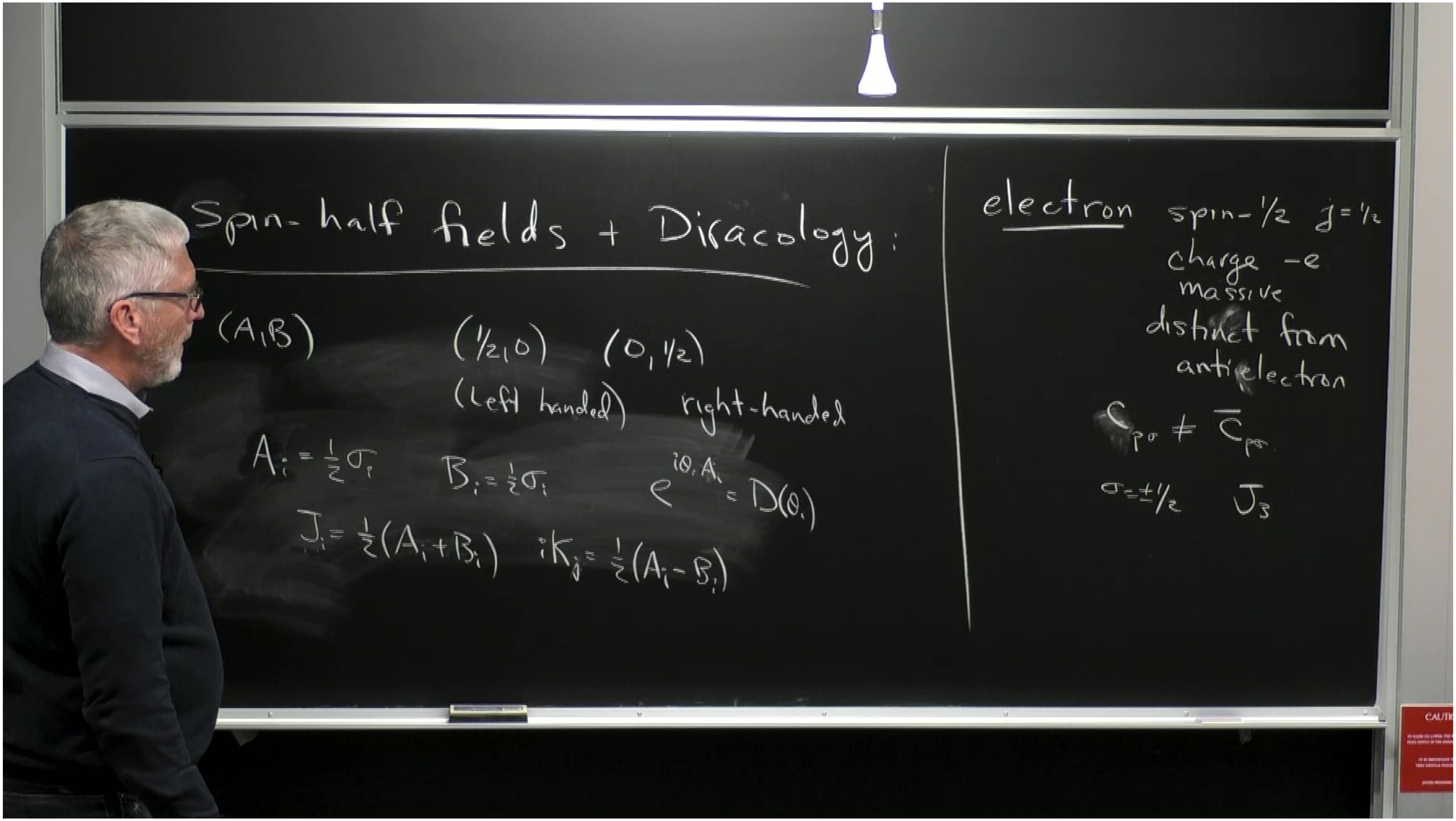
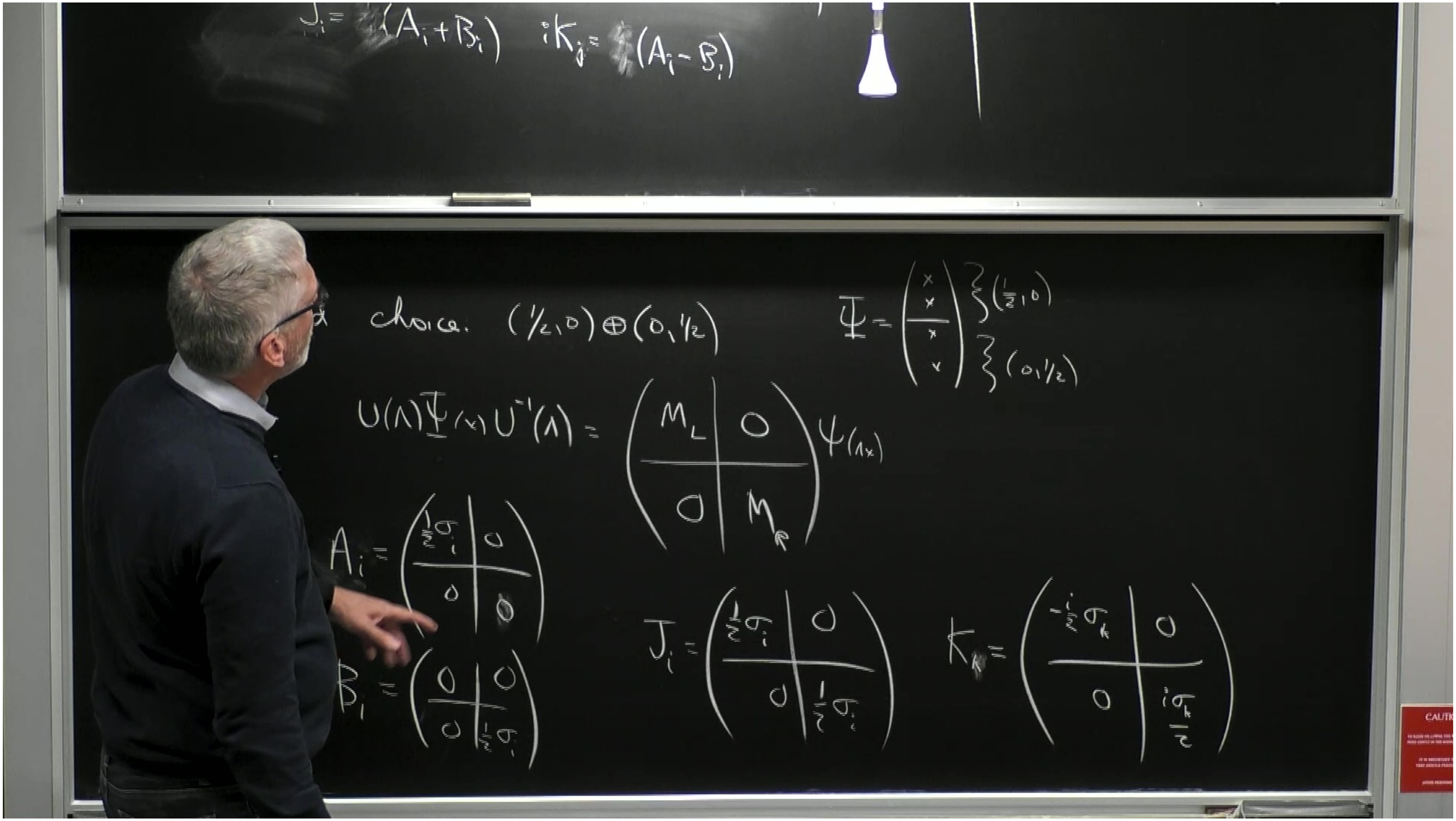
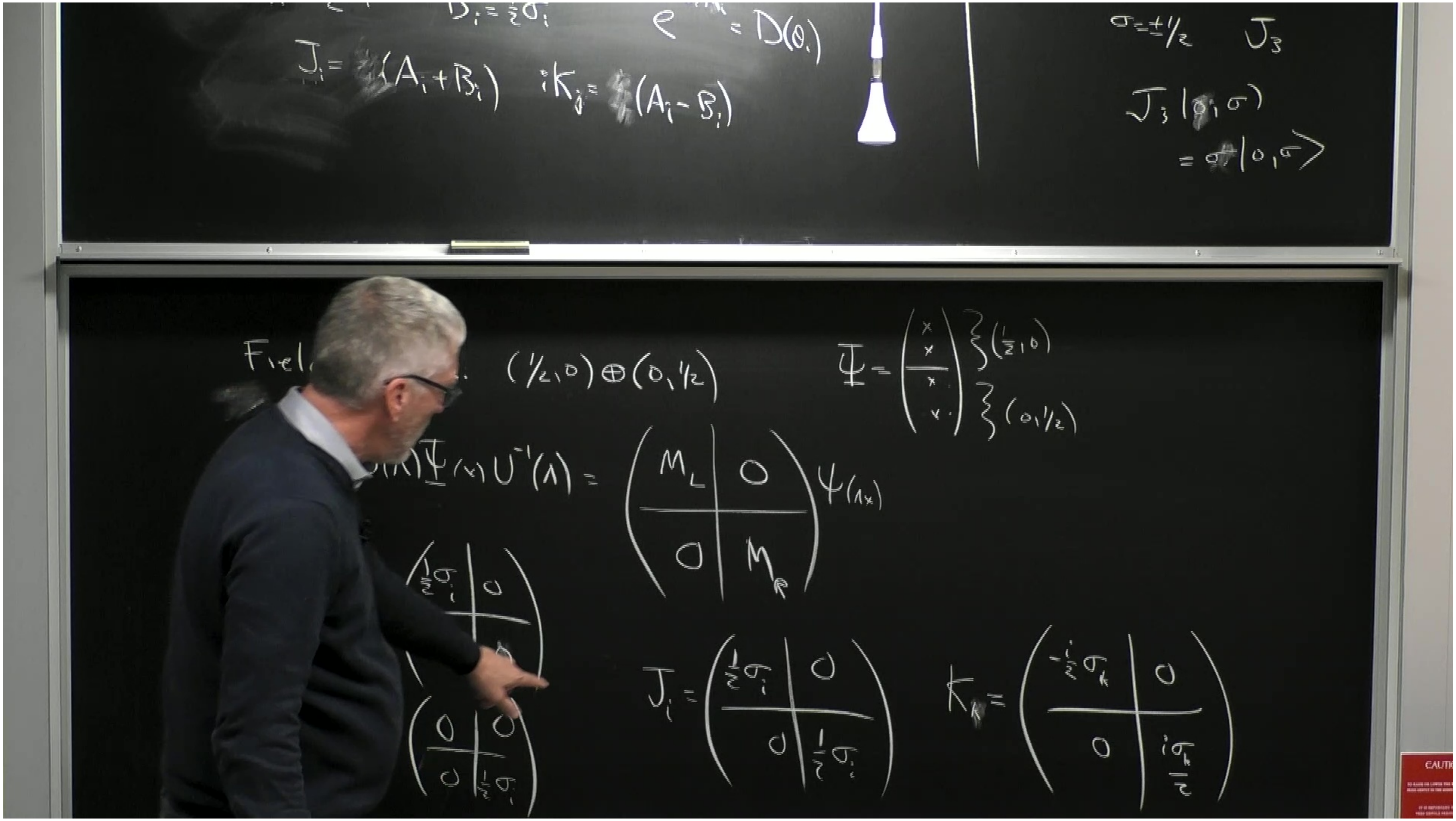
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


Spin-half fields + Diracology:
(A,B)
$(1/2,0)$ $(0,1/2)$
(Left handed) right-handed
$A_i = \frac{1}{2}\sigma_i$ $B_i = \frac{1}{2}\sigma_i$ $e^{i\theta_i A_i} = D(\theta_i)$
$J_i = \frac{1}{2}(A_i + B_i)$ $iK_j = \frac{1}{2}(A_i - B_i)$
electron spin-1/2 $j = 1/2$
charge $-e$
massive
distinct from antielectron
$c_{p\sigma} \neq \bar{c}_{p\sigma}$
$\sigma = \pm 1/2$ J_3


$J_i = \frac{1}{2}(A_i + B_i)$ $iK_j = \frac{1}{2}(A_j - B_j)$
choice: $(1/2, 0) \oplus (0, 1/2)$
$\Psi = \begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix}$ $(\frac{1}{2}, 0)$ $(0, 1/2)$
$U(\Lambda)\Psi(x)U^{-1}(\Lambda) = \begin{pmatrix} M_L & 0 \\ 0 & M_R \end{pmatrix} \Psi(\Lambda x)$
$A_i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & 0 \end{pmatrix}$
$B_i = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\sigma_i \end{pmatrix}$
$J_i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & \frac{1}{2}\sigma_i \end{pmatrix}$
$K_k = \begin{pmatrix} -\frac{i}{2}\sigma_k & 0 \\ 0 & \frac{i\sigma_k}{2} \end{pmatrix}$


σ = ±½ J3
J3 |0, σ⟩ = σ |0, σ⟩
Ji = (Ai + Bi)
iKj = (Ai − Bi)
(½, 0) ⊕ (0, ½)
(½, 0)
(0, ½)
ML
MR
Ψ(Λx)
Ji =
Kk =

$B_i = \frac{i}{2}\sigma_i$ $\qquad e^{\ldots} = D(\theta_i)$

$J_i = \ (A_i + B_i) \qquad iK_j = \ (A_j - B_j)$

$\sigma = \pm 1/2 \qquad J_3$

$J_3|0,\sigma\rangle = \sigma|0,\sigma\rangle$

Field choice. $(1/2,0)\oplus(0,1/2)$

$$\Psi = \begin{pmatrix} x \\ x \\ \hline x \\ x \end{pmatrix} \begin{matrix} \}\,(\tfrac{1}{2},0) \\ \\ \}\,(0,\tfrac{1}{2}) \end{matrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$U(\Lambda)\Psi(x)U^{-1}(\Lambda) = \begin{pmatrix} M_L & 0 \\ 0 & M_R \end{pmatrix}\Psi(\Lambda x)$$

$$A_i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & 0 \end{pmatrix} \qquad B_i = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\sigma_i \end{pmatrix}$$

$$J_i = \begin{pmatrix} \frac{1}{2}\sigma_i & 0 \\ 0 & \frac{1}{2}\sigma_i \end{pmatrix} \qquad K_k = \begin{pmatrix} -\frac{i}{2}\sigma_k & 0 \\ 0 & \frac{i\sigma_k}{2} \end{pmatrix}$$

$$\Psi_a(x) = \sum_{\sigma=\pm 1/2} \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}} \left[c_{p\sigma}\, e^{ipx}\, u_a(p,\sigma) + \bar{c}^*_{p\sigma}\, e^{-ipx}\, v_a(p,\sigma) \right]$$

We build $u(p,\sigma)$ by boosting $u(0,\sigma)$

and little group's action is a constraint of $u(0,\sigma)$

$$J_3\, u(0,\sigma) = \sigma\, u(0,\sigma) \qquad u(0,\sigma=+1/2) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad u(0,\sigma=-1/2) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

and little group's action is a constraint of $u(0,\sigma)$

$$J_3 u(0,\sigma) = \sigma\, u(0,\sigma) \qquad u(0,\sigma=+\tfrac{1}{2}) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad u(0,\sigma=-\tfrac{1}{2}) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$$

$$u(0,\sigma) = \beta\, u(0,\sigma)$$

$$\beta = \left(\begin{array}{c|c} 0 & I \\ \hline I & 0 \end{array}\right)$$

$$\beta = \left(\begin{array}{c|c} 0 & I \\ \hline I & 0 \end{array}\right)$$

In general frame $D(L(p))$

$$u(p,\sigma) = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} \sqrt{E_p+m} - \hat{p}\cdot\vec{\sigma}\sqrt{E_p-m} & 0 \\ \hline 0 & \sqrt{E_p+m} + \hat{p}\cdot\vec{\sigma}\sqrt{E_p-m} \end{array}\right) u(0,\sigma)$$

$$u(0,\sigma) = \beta\, u(0,\sigma)$$

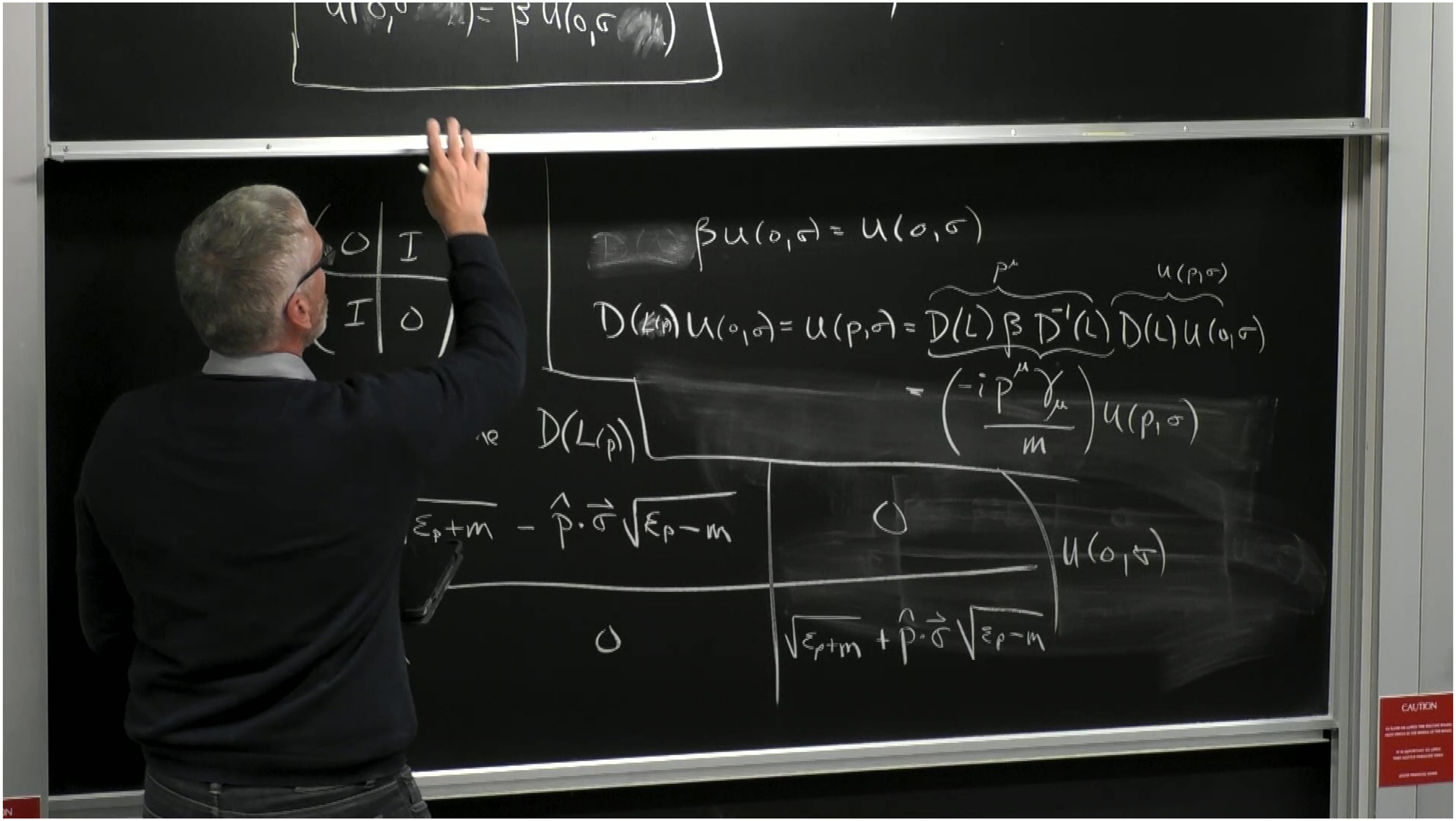
$$\beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\beta\, u(0,\sigma) = u(0,\sigma)$$

$$D(L(p))\, u(0,\sigma) = u(p,\sigma) = \overbrace{D(L)\,\beta\, D^{-1}(L)}^{p^{\mu}}\ \overbrace{D(L)\, u(0,\sigma)}^{u(p,\sigma)}$$

In general frame $D(L(p))$

$$u(p,\sigma) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\varepsilon_p + m} - \hat{p}\cdot\vec{\sigma}\sqrt{\varepsilon_p - m} & 0 \\ 0 & \sqrt{\varepsilon_p + m} + \hat{p}\cdot\vec{\sigma}\sqrt{\varepsilon_p - m} \end{pmatrix} u(0,\sigma)$$


β u(0,σ) = u(0,σ)
D(L(p)) u(0,σ) = u(p,σ) = D(L) β D⁻¹(L) D(L) u(0,σ)
p^μ
u(p,σ)
= (−i p^μ γ_μ / m) u(p,σ)
D(L(p))
√(E_p+m) − p̂·σ⃗ √(E_p−m)
0
u(0,σ)
0
√(E_p+m) + p̂·σ⃗ √(E_p−m)
CAUTION

$$-i\gamma_0 = \beta \qquad \gamma_0 = i\beta = \begin{pmatrix} 0 & \\ i & 0 \end{pmatrix}$$

$$u = \left(\frac{-ip^\mu\gamma_\mu}{m}\right)u$$

$$\gamma_k = \left(\begin{array}{c|c} 0 & -i\sigma_k \\ \hline i\sigma_k & 0 \end{array}\right)$$

$(ip^\mu\gamma_\mu + m)u = 0$ Dirac eq

$$(\gamma^\mu\partial_\mu + m)\Psi(x) = 0$$

$$v(p,\sigma) = (-)^{2j}(-)^{j-\sigma}u(p,-\sigma)$$

$$\gamma_5 = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}\right) \qquad [\gamma_5, D(\Lambda)] = 0$$

$$v(0,\sigma=+1/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \qquad v(0,\sigma=-1/2) = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\beta v(0,\sigma) = -v(0,\sigma)$$

$$\beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

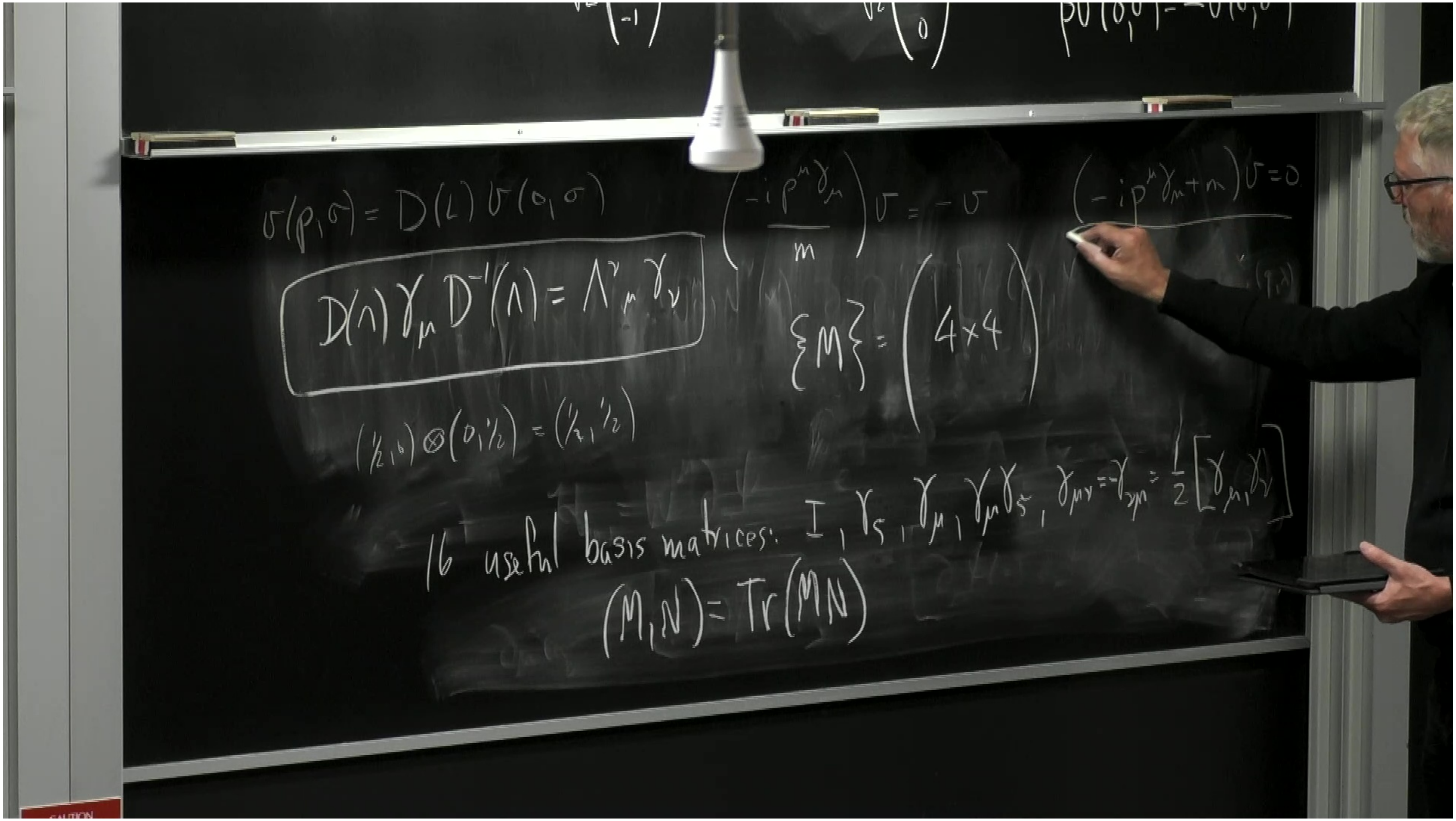
$$\beta u(0,\sigma) = u(0,\sigma)$$

$$D(L(p))\, u(0,\sigma) = u(p,\sigma) = \overbrace{D(L)\,\beta\, D^{-1}(L)}^{p^\mu}\ \overbrace{D(L)\, u(0,\sigma)}^{u(p,\sigma)}$$

$$= \left(\frac{-i p^\mu \gamma_\mu}{m}\right) u(p,\sigma)$$

In general frame $D(L(p))$

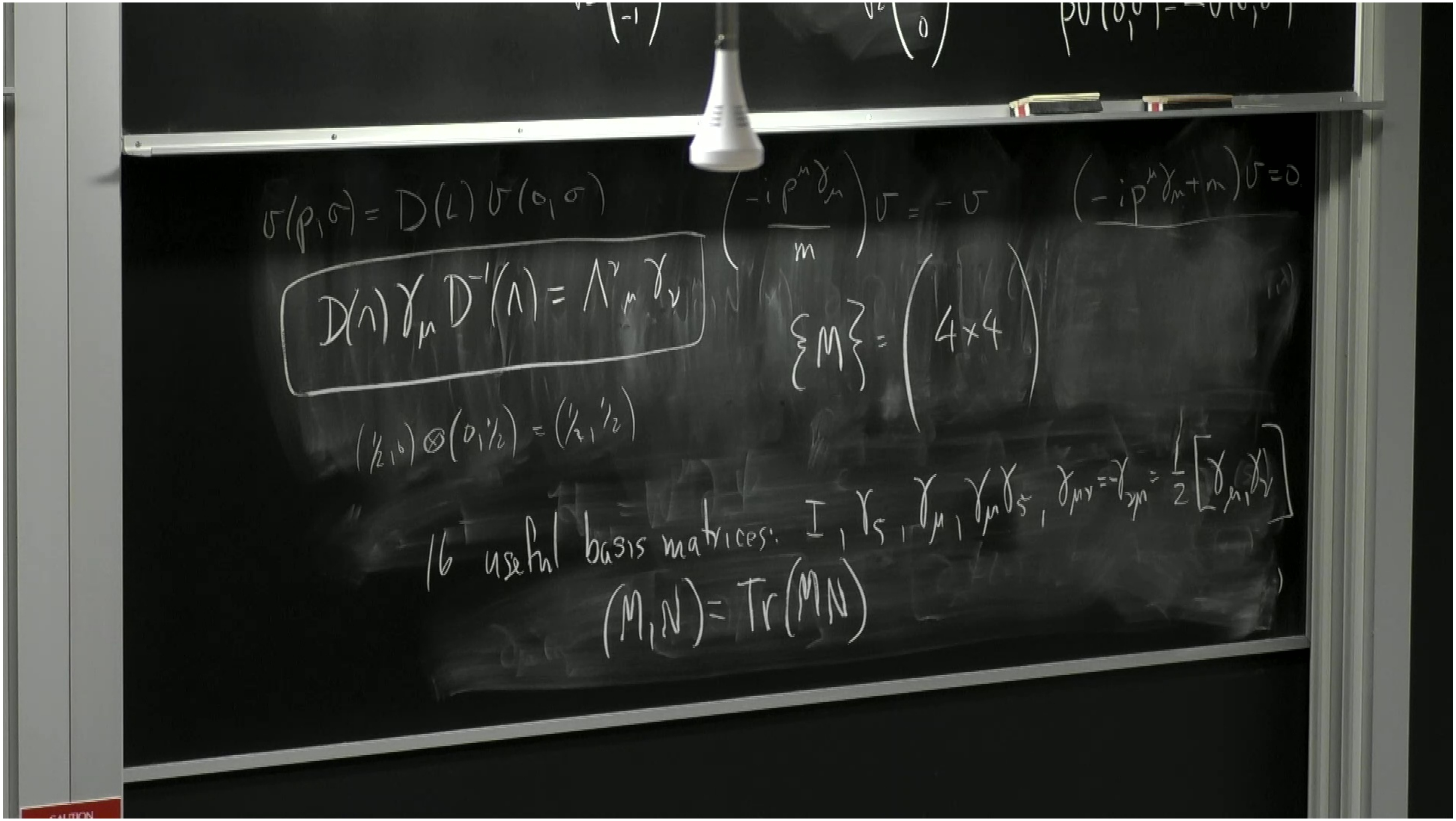
$$u(p,\sigma) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{E_p+m} - \hat{p}\cdot\vec{\sigma}\sqrt{E_p-m} & 0 \\ 0 & \sqrt{E_p+m} + \hat{p}\cdot\vec{\sigma}\sqrt{E_p-m} \end{pmatrix} u(0,\sigma)$$


v(p,σ) = D(L) v(0,σ)
D(Λ) γ_μ D^-1(Λ) = Λ^ν_μ γ_ν
(1/2,0) ⊗ (0,1/2) = (1/2,1/2)
(-i p^μ γ_μ / m) v = -v
(-i p^μ γ_μ + m) v = 0
{M} = (4×4)
16 useful basis matrices: I, γ_5, γ_μ, γ_μ γ_5, γ_μν = -γ_νμ = 1/2 [γ_μ, γ_ν]
(M,N) = Tr(MN)

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$$

$$C = c_1 I + c_5 \gamma_5 + c^\mu \gamma_\mu + c_5^\mu \gamma_\mu \gamma_5 + c^{\mu\nu} \gamma_{\mu\nu}$$

$$\mathrm{Tr}(\gamma_5 C) = 4 c_5$$


$v(p,\sigma) = D(L)\, v(0,\sigma)$
$\boxed{D(\Lambda)\gamma_\mu D^{-1}(\Lambda) = \Lambda^\nu{}_\mu \gamma_\nu}$
$(\frac{1}{2},0)\otimes(0,\frac{1}{2}) = (\frac{1}{2},\frac{1}{2})$
$\left(\frac{-ip^\mu\gamma_\mu}{m}\right) v = -v$
$(-ip^\mu\gamma_\mu + m)\, v = 0$
$\{M\} = \left(4\times 4\right)$
16 useful basis matrices: $I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \gamma_{\mu\nu} = -\gamma_{\nu\mu} = \frac{1}{2}[\gamma_\mu,\gamma_\nu]$
$(M,N) = Tr(MN)$

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$$

$$\psi \to D(\Lambda)\psi$$

$$\psi^+ \to \psi^+ D^+(\Lambda) \neq \psi^+ D^{-1}(\Lambda)$$

$$C = c_1 I + c_5\gamma_5 + c^\mu\gamma_\mu + c_5^\mu\gamma_\mu\gamma_5 + c^{\mu\nu}\gamma_{\mu\nu}$$

$$\mathrm{Tr}(\gamma_5 C) = 4c_5$$

$$\boxed{\sigma_k^* = \varepsilon\sigma_k\varepsilon}$$

$$\bar{\Psi} := \psi^+\beta$$

$$D^+(\Lambda)\beta = \beta D^{-1}(\Lambda)$$

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$$

$$\psi \to D(\Lambda)\psi$$

$$\psi^\dagger \to \psi^\dagger D^\dagger(\Lambda) \neq \psi^\dagger D^{-1}(\Lambda)$$

$$C = c_1 I + c_5\gamma_5 + c^\mu\gamma_\mu + c_5^\mu\gamma_\mu\gamma_5 + c^{\mu\nu}\gamma_{\mu\nu}$$

$$\mathrm{Tr}(\gamma_5 C) = 4c_5$$

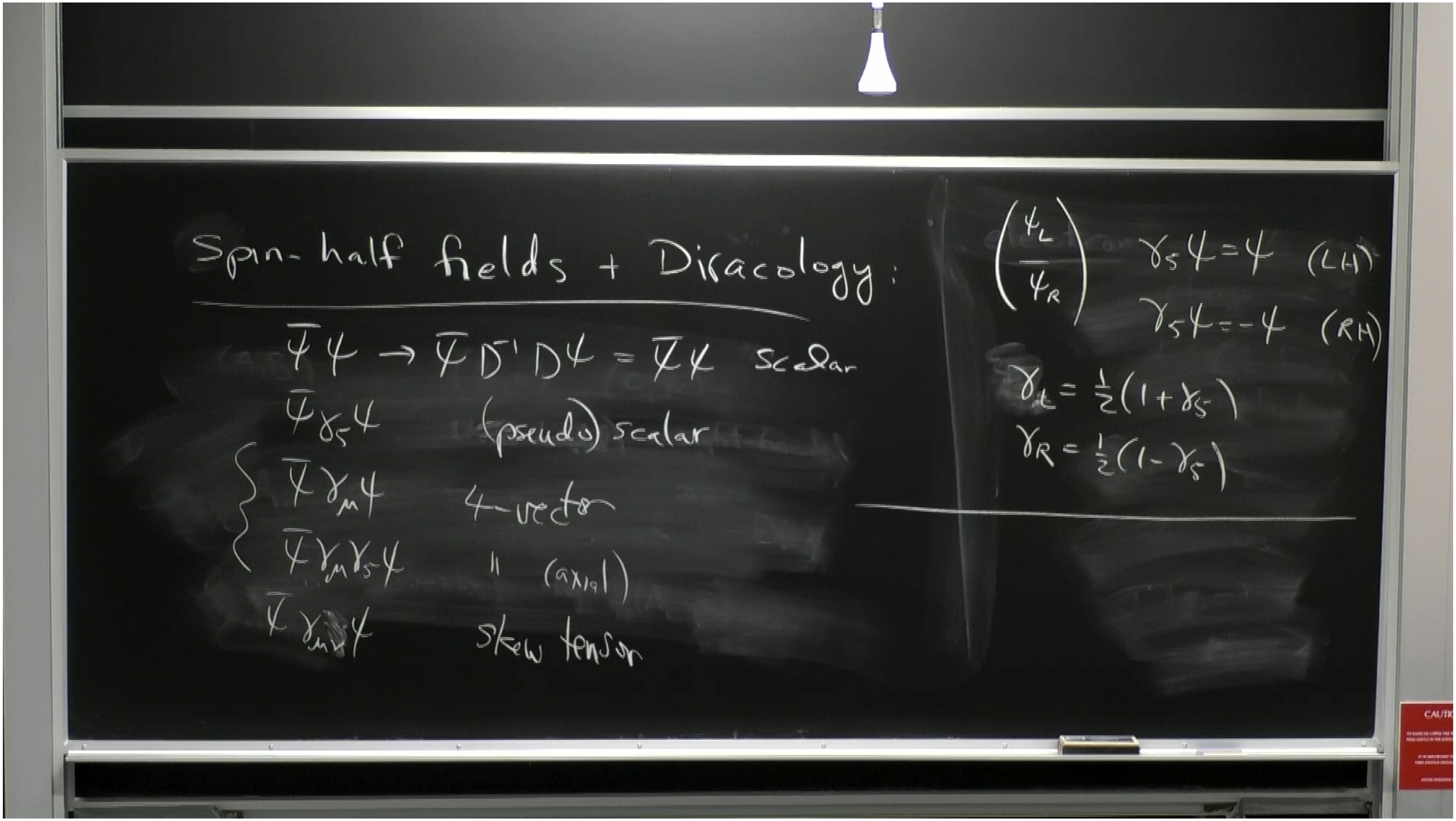
$$\boxed{\sigma_k^* = \varepsilon\,\sigma_k\,\varepsilon}$$

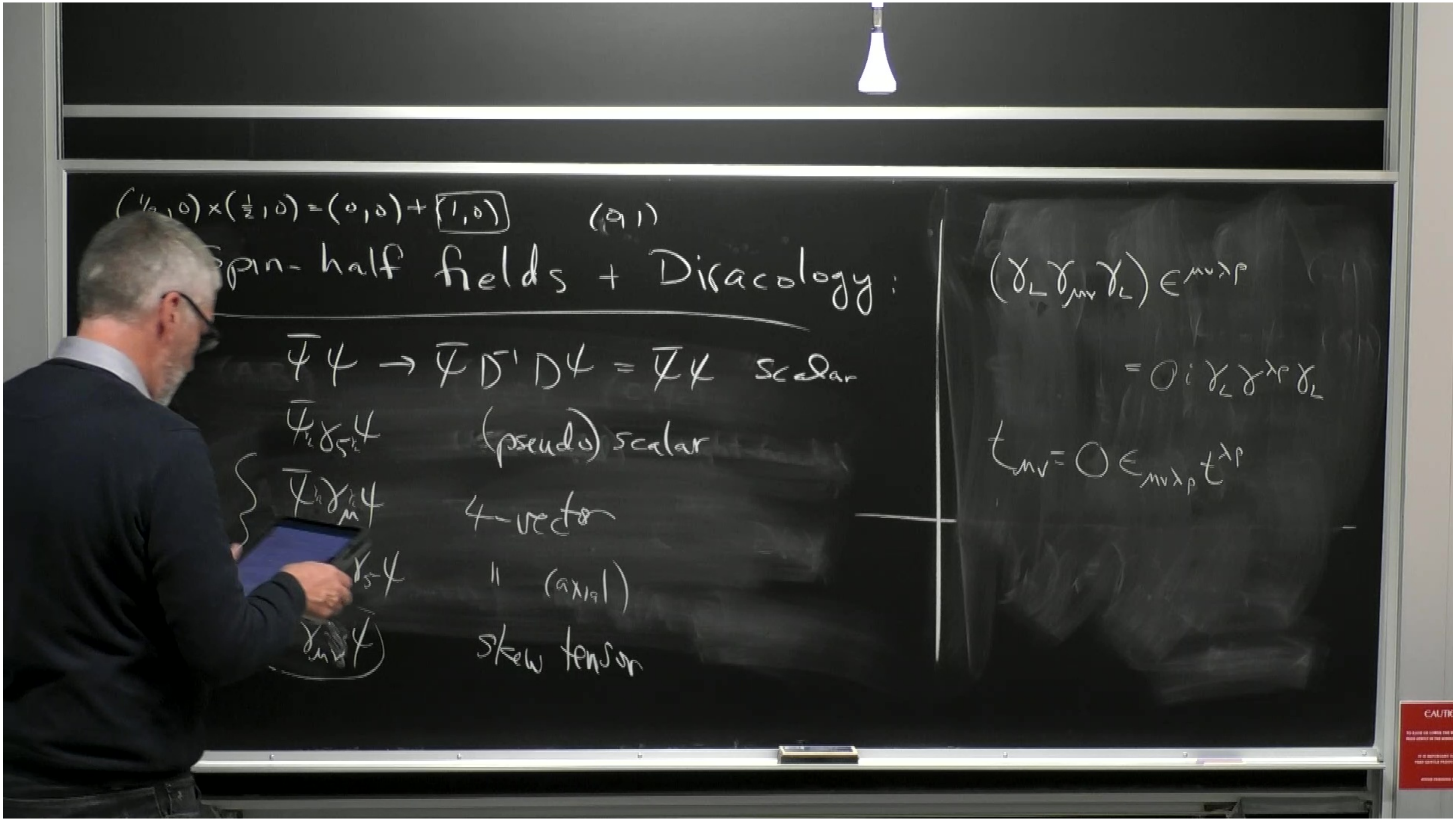
$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2$$

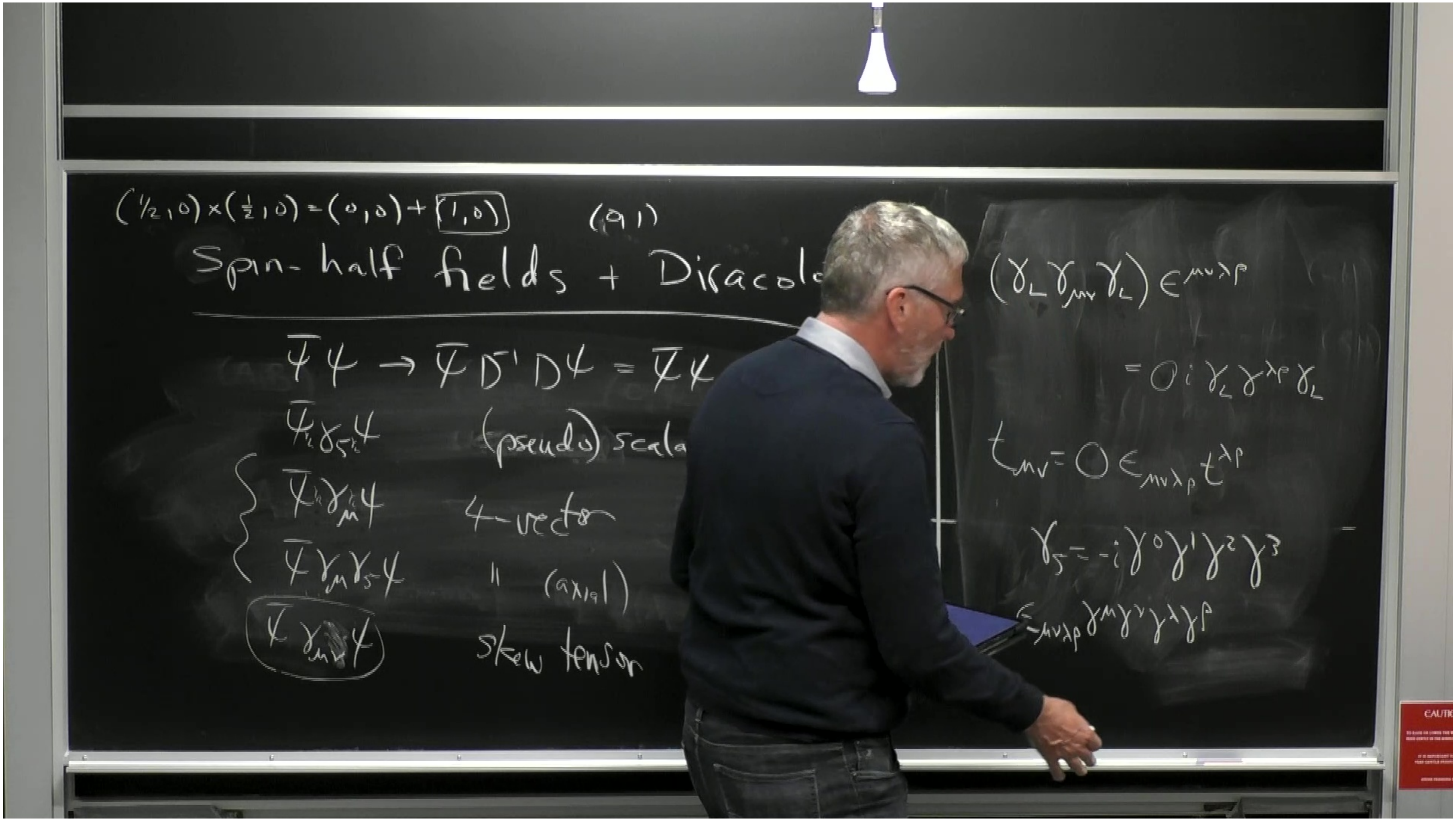
$$\boxed{\bar\psi := \psi^\dagger\beta = i\psi^\dagger\gamma^0 = -i\psi^\dagger\gamma_0}$$

$$D^\dagger(\Lambda)\beta = \beta D^{-1}(\Lambda)$$

$$\bar\psi \to \bar\psi D^{-1}(\Lambda)$$


Spin-half fields + Diracology:
$\bar\psi\psi \to \bar\psi D^{-1} D\psi = \bar\psi\psi$ scalar
$\bar\psi\gamma_5\psi$ (pseudo) scalar
$\bar\psi\gamma_\mu\psi$ 4-vector
$\bar\psi\gamma_\mu\gamma_5\psi$ " (axial)
$\bar\psi\gamma_{\mu\nu}\psi$ skew tensor
$\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}$
$\gamma_5\psi = \psi$ (LH)
$\gamma_5\psi = -\psi$ (RH)
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\gamma_R = \frac{1}{2}(1-\gamma_5)$


$(\frac{1}{2},0)\times(\frac{1}{2},0)=(0,0)+\boxed{(1,0)}$ $(0,1)$
Spin-half fields + Diracology:
$\bar\psi\psi \to \bar\psi D^{-1} D\psi = \bar\psi\psi$ scalar
$\bar\psi\gamma_5\psi$ (pseudo) scalar
$\bar\psi\gamma_\mu\psi$ 4-vector
$\bar\psi\gamma_\mu\gamma_5\psi$ " (axial)
$\bar\psi\gamma_{\mu\nu}\psi$ skew tensor
$(\gamma_L\gamma_{\mu\nu}\gamma_L)\,\epsilon^{\mu\nu\lambda\rho}$
$= \bigcirc\, i\,\gamma_L\gamma^{\lambda\rho}\gamma_L$
$t_{\mu\nu} = \bigcirc\,\epsilon_{\mu\nu\lambda\rho}t^{\lambda\rho}$


$(\frac{1}{2},0)\times(\frac{1}{2},0)=(0,0)+\boxed{(1,0)}$
$(0,1)$
Spin-half fields + Diracol
$\bar\Psi\Psi \to \bar\Psi D^{-1} D\Psi = \bar\Psi\Psi$
$\bar\Psi\gamma_5\Psi$ (pseudo) scala
$\bar\Psi\gamma_\mu\Psi$ 4-vector
$\bar\Psi\gamma_\mu\gamma_5\Psi$ " (axial)
$\bar\Psi\gamma_{\mu\nu}\Psi$ skew tensor
$(\gamma_L\gamma_{\mu\nu}\gamma_L)\epsilon^{\mu\nu\lambda\rho}$
$= \bigcirc\, i\gamma_L\gamma^{\lambda\rho}\gamma_L$
$t_{\mu\nu} = \bigcirc\,\epsilon_{\mu\nu\lambda\rho}t^{\lambda\rho}$
$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$
$\epsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho$

$\bar\psi \gamma_\mu \gamma_5 \psi$ (axial)

$\bar\psi \gamma_{\mu\nu} \psi$ skew tensor

$\frac{i}{4!} \epsilon_{\mu\nu\lambda\rho} \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho$

$$\psi = \left(\frac{\psi_L}{\psi_R} \right)$$

$$\psi_L = \gamma_L \psi = \tfrac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

$$\psi_R = \gamma_R \psi = \tfrac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

$(1/2, 0) \longleftrightarrow (0, 1/2)$

under complex conjugation

"Weyl Spinor"

$\bar\psi \gamma_\mu \gamma_5 \psi$ (axial)

$\bar\psi \gamma_{\mu\nu} \psi$ skew tensor

$\frac{i}{4!}\epsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho$

$$\psi = \begin{pmatrix} \psi_L \\ \hline \psi_R \end{pmatrix}$$

$$\psi_L = \gamma_L \psi = \frac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

$$\psi_R = \gamma_R \psi = \frac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

$(\frac{1}{2}, 0) \longleftrightarrow (0, \frac{1}{2})$

under complex conjugation

"Weyl Spinor"

If $u_L \to D_L(\Lambda) u_L$

then εu_L^* transforms like u_R

"Majorana" spinor: $\psi = \begin{pmatrix} \psi_L \\ -\varepsilon \psi_L^* \end{pmatrix} = \underbrace{\left(\begin{array}{c|c} 0 & \varepsilon \\ \hline -\varepsilon & 0 \end{array} \right)}_{C} \begin{pmatrix} \psi_L \\ -\varepsilon \psi_L^* \end{pmatrix}^*$

$\psi^* = C\psi$ is Lorentz inv.

$C^2 = 1$

$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = i\sigma_2$

$\bar\psi \gamma_\mu \gamma_5 \psi$ = (axial)

$\bar\psi \gamma_{\mu\nu} \psi$ skew tensor

$\frac{i}{4!}\epsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho$

$$\psi = \left(\frac{\psi_L}{\psi_R}\right) \qquad \begin{matrix} u^\alpha \\ u^{\dot\alpha} \end{matrix}$$

$$\psi_L = \gamma_L \psi = \tfrac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

$$\psi_R = \gamma_R \psi = \tfrac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

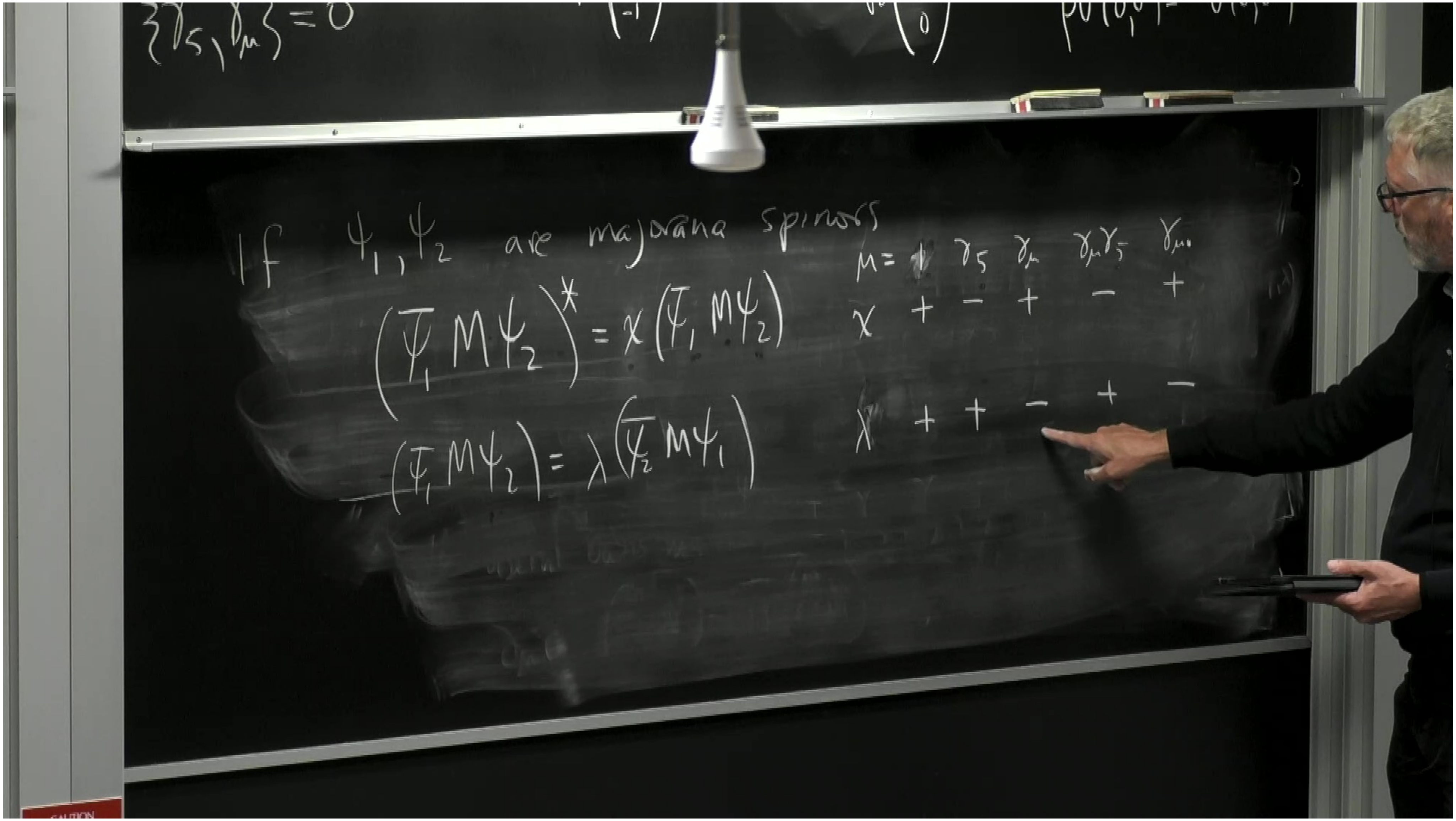
$(\tfrac{1}{2}, 0) \longleftrightarrow (0, \tfrac{1}{2})$

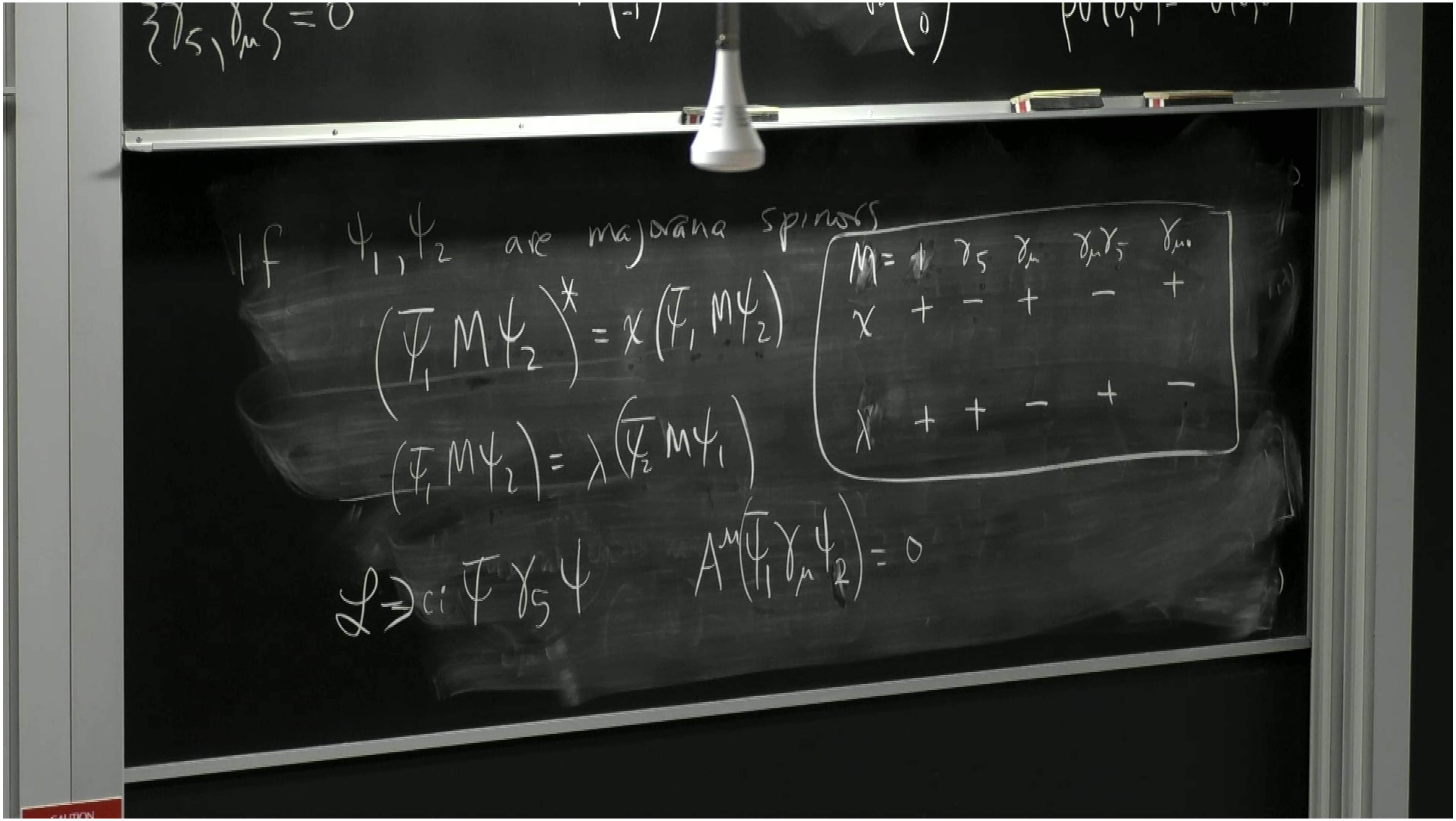
under complex conjugation

"Weyl spinor"

If $u_L \to D_L(\Lambda)\, u_L$

then ϵu_L^* transforms like u_R


$\{\gamma_5, \gamma_\mu\} = 0$
If ψ_1, ψ_2 are majorana spinors
$M = 1 \quad \gamma_5 \quad \gamma_\mu \quad \gamma_\mu\gamma_5 \quad \gamma_{\mu\nu}$
$\chi \quad + \quad - \quad + \quad - \quad +$
$(\bar\psi_1 M \psi_2)^* = \chi (\bar\psi_1 M \psi_2)$
$\lambda \quad + \quad + \quad - \quad + \quad -$
$(\bar\psi_1 M \psi_2) = \lambda (\bar\psi_2 M \psi_1)$


$\{\gamma_5, \gamma_\mu\} = 0$
If ψ_1, ψ_2 are majorana spinors
$(\bar\psi_1 M \psi_2)^* = \chi (\bar\psi_1 M \psi_2)$
$(\bar\psi_1 M \psi_2) = \lambda (\bar\psi_2 M \psi_1)$
$M =$ 1 γ_5 γ_μ $\gamma_\mu\gamma_5$ $\gamma_{\mu\nu}$
χ + − + − +
λ + + − + −
$\mathcal{L} \ni ci \bar\psi \gamma_5 \psi$
$A^\mu (\bar\psi_1 \gamma_\mu \psi_2) = 0$