Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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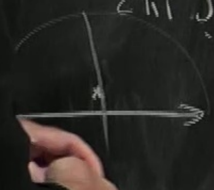
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.

$$\langle 0|\partial_\mu \partial_{\nu'} T[\phi(x)\phi(x')]|0\rangle \quad \text{covariant}$$

$$\langle 0| T[\partial_\mu \phi(x)\, \partial_\nu \phi(x')]|0\rangle \quad \text{not covariant}$$

$$\langle 0|T[\phi(x)\phi(x')]|0\rangle = \theta(x^0 - x^{0'})\langle 0|\phi(x)\phi(x')|0\rangle + (x \Leftrightarrow x')$$

$$\theta(x) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{i\omega x}}{\omega + i\delta} = \frac{1}{2\pi i}\int \frac{e^{i\omega x}}{\omega+i\delta}$$



$$= i\,\theta(x^0 - x^{0'})\int \frac{d^3p}{(2\pi)^3 2\varepsilon_p} e^{ip\cdot(x-x')} + (x \Leftrightarrow x') = \int \frac{d^4p}{(2\pi)^4} \frac{p_\mu p_\nu e^{ip\cdot(x-x')}}{p^2+m^2-i\delta}$$

$$p^0 = \varepsilon_p = \sqrt{\vec{p}^2+m^2}$$

$$\varepsilon(p) \qquad p^0 = \omega$$

$$p^0 = \varepsilon_p = \sqrt{p^2+m^2}$$

$$\int \frac{\cdots}{(2\pi)^4} \frac{\cdots}{p^2+m^2-i\epsilon}$$

$$\mathcal{H} = J^\mu A_\mu + \frac{1}{8\pi}\int d^3x\, d^3y\, \frac{J^0(x,t)\, J^0(y,t)}{|\vec{x}-\vec{y}|}$$

$$\langle 0|\, S\, |0\rangle = 1 - i\int \mathcal{H}\, d^4x + \frac{(-i)^2}{2}\iint d^4x\, d^4x'\, T\left[\mathcal{H}(x)\,\mathcal{H}(x')\right] + \dots$$

$$\langle 0|\; \quad \; |0\rangle$$

$$\langle 0|\, T\, A_\mu(x)\, A_\nu(x')\,|0\rangle$$

$$A_\mu(x) = \sum_{\lambda=\pm 1} \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[a_{k\lambda}\, \epsilon_\mu(k,\lambda)\, e^{ik\cdot x} + a^*_{k\lambda}\, \epsilon^*_\mu(k,\lambda)\, e^{-ik\cdot x} \right]$$

in special frame: $k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \begin{matrix} t \\ x \\ y \\ z \end{matrix}$

$$\epsilon_\mu(k,\lambda=x) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_\mu(k,\lambda=\pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$$

$$\epsilon_\mu(k,\lambda=y) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle 0 | T\left[A_\mu(x) A_\nu(y)\right] | 0 \rangle = \theta(x^0 - y^0) \sum_\lambda \int \frac{d^3k}{(2\pi)^3 2\omega_k}\, \epsilon_\mu(k,\lambda)\, \epsilon^*_\nu(k,\lambda)\, e^{ik(x-y)}$$

$$+ (x \leftrightarrow y,\ \mu \leftrightarrow \nu)$$

In sp. frame $\sum_{\lambda} \epsilon_\mu(k,\lambda)\epsilon^*_\nu(k,\lambda) = \Pi_{\mu\nu}(\hat{k})$

$$\Pi_{ij}(\hat{k}) = \delta_{ij} - \hat{k}_i \hat{k}_j \qquad \Pi_{0i}(k) = \Pi_{00} = 0$$

$$n^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$$

$$\Pi_{ij}(k) = \delta_{ij} - \hat{k}_i \hat{k}_j \qquad \Pi_{0i}(k) = \Pi_{00} = 0.$$

$$n^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$$

$$\Pi_{\mu\nu}(p) = \eta_{\mu\nu} + \frac{(n_\mu p_\nu + p_\mu n_\nu)\,p^0 - p_\mu p_\nu}{\vec{p}^{\,2}} + \frac{(n_\mu n_\nu)\,p^\lambda p_\lambda}{\vec{p}^{\,2}}$$

$$p^\mu = \begin{pmatrix} p^0 \\ \vec{p} \end{pmatrix} \qquad p^0 = \text{anything}$$

$$A_\mu(x) = \sum_{\lambda=\pm 1} \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[a_{k\lambda}\, \epsilon_\mu(k,\lambda)\, e^{ik\cdot x} + a^*_{k\lambda}\, \epsilon^*_\mu(k,\lambda)\, e^{-ik\cdot x} \right]$$

in special frame: $k^\mu = \begin{array}{c} \\ \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \end{array}\begin{array}{l} t \\ x \\ y \\ z \end{array}$ $\qquad \epsilon_\mu(k, \lambda = x) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_\mu(k, \lambda = \pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$

$$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{ip(x-y)}}{p^2-i\delta}\,\Pi_{\mu\nu}(\hat{p})$$

$$\langle 0|S_2|0\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)\, J^\nu(y)\, \langle 0|T\, A_\mu(x)A_\nu(y)|0\rangle$$

$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$

$$\supset -\frac{1}{2}\int d^4x\, d^4y\; J^0(x,t)\, J^0(y,t)\; i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{ip(x-y)}}{p^2-i\delta}\,\frac{p^\lambda p_\lambda}{\vec{p}^{\,2}}$$

$$\langle 0|T\left[A_\mu(x)A_\nu(y)\right]|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\,\Pi_{\mu\nu}(\hat{p})$$

$$\langle 0|S_2|0\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)J^\nu(y)\langle 0|T A_\mu(x)A_\nu(y)|0\rangle$$

$$\nabla^2\left(\frac{1}{4\pi|\vec{x}-\vec{y}|}\right) = \delta^3(x-y)$$

$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$

$$\rightarrow -\frac{1}{2}\int d^4x\, d^4y\; J^0(x,t)J^0(y,t)\; i\int\frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\,\cancel{\frac{1}{p^2-i\delta}}\,\frac{\cancel{p^\lambda p_\lambda}}{\vec{p}^2}$$

$$\frac{1}{p^2} = \frac{1}{-\nabla^2}$$

$$= +\frac{i}{8\pi}\int d^4x\, d^3y\;\frac{J^0(\vec{x},t)J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}\;\underbrace{\int\frac{dp^0}{2\pi}\,e^{-ip^0(x^0-y^0)}}_{}\quad \delta(x^0-y^0)$$

$$\mathcal{H} = J^{\mu} A_{\mu} + \frac{1}{8\pi}\int d^3x\, d^3y\, \frac{J^0(x,t)\, J^0(y,t)}{|\vec{x}-\vec{y}|}$$

$$\langle 0| S |0\rangle = \underbrace{1 - i\int \mathcal{H}\, d^4x}_{\langle 0| \qquad |0\rangle} + \frac{(-i)^2}{2}\iint d^4x\, d^4x'\, T\left[\mathcal{H}(x)\,\mathcal{H}(x')\right] + \dots$$

$$\frac{-i}{8\pi^2}\int d^4x\, \frac{J^0(\vec{x},t)\, J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}$$

$$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\,\Pi_{\mu\nu}(\hat{p})$$

$$\langle 0|S_2|0\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)J^\nu(y)\langle 0|T A_\mu(x)A_\nu(y)|0\rangle$$

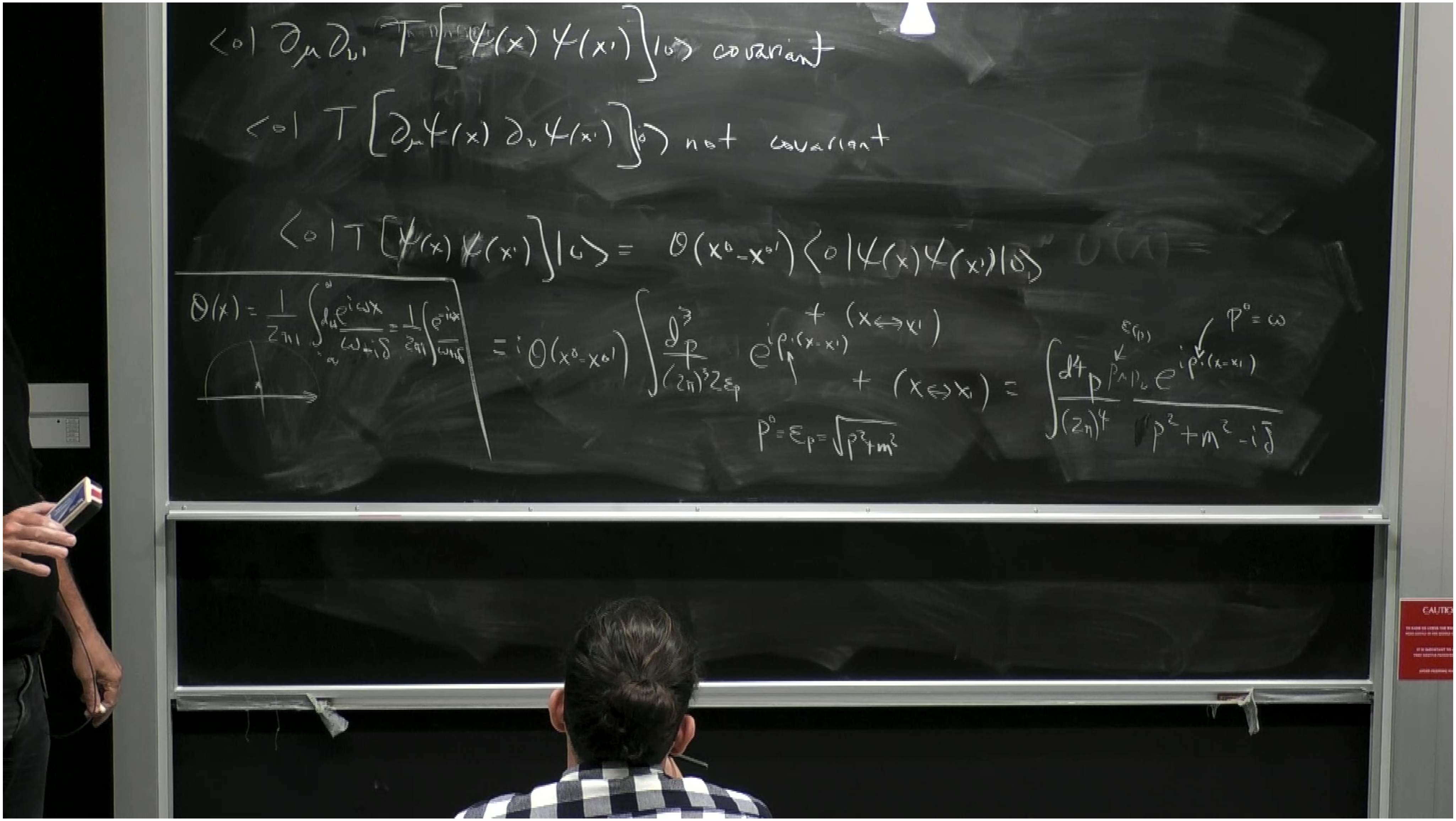
$$\nabla^2\left(\frac{1}{4\pi|\vec{x}-\vec{y}|}\right) = \delta^3(x-y)$$

$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$

$$> -\frac{1}{2}\int d^4x\, d^4y\; J^0(x,t)J^0(y,t)\; i\int\frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\,\frac{\cancel{p^\lambda p_\lambda}}{\cancel{p^2-i\delta}\;\vec{p}^2}$$

$$\frac{1}{p^2} = \frac{1}{-\nabla^2}$$

$$= +\frac{i}{8\pi}\int d^4x\, d^4y\;\frac{J^0(\vec{x},t)J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}\int\frac{dp^0}{2\pi}\, e^{-ip^0(x^0-y^0)}\quad \cancel{\delta(x^0-y^0)}$$


$\langle 0| \partial_\mu \partial_{\nu'} T\left[\psi(x)\psi(x')\right]|0\rangle$ covariant
$\langle 0| T\left[\partial_\mu \psi(x)\, \partial_\nu \psi(x')\right]|0\rangle$ not covariant
$\langle 0| T\left[\psi(x)\psi(x')\right]|0\rangle = \theta(x^0 - x^{0'})\langle 0|\psi(x)\psi(x')|0\rangle + (x \leftrightarrow x')$
$\theta(x) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{e^{i\omega x}}{\omega - i\delta} = \frac{1}{2\pi i}\int \frac{e^{-i\omega x}}{\omega + i\delta}$
$= i\,\theta(x^0 - x^{0'})\int \frac{d^3p}{(2\pi)^3 2\epsilon_p} e^{ip\cdot(x-x')} + (x \leftrightarrow x') = \int \frac{d^4p}{(2\pi)^4} \frac{p_\mu p_\nu e^{ip\cdot(x-x')}}{p^2 + m^2 - i\delta}$
$p^0 = \epsilon_p = \sqrt{p^2 + m^2}$
$\epsilon(p)$
$p^0 = \omega$

Canonical quantization is useful because it automatically generates non-covariant terms in $\mathcal{H}$ needed for covariance of S.

$$\varphi(x) \qquad \dot{\varphi} = \partial_t \varphi$$

$$S = \int dt\, L(\varphi, \partial\varphi) = \int d^4x\, \mathcal{L}$$

$$\pi = \frac{\delta S}{\delta \dot{\varphi}} = \pi(\varphi, \nabla\varphi, \partial_t \varphi)$$

$$\mathcal{H} = \sum_a \pi_a \dot{\varphi}_a - \mathcal{L} = \mathcal{H}(\pi, \nabla\varphi, \varphi)$$

$$[\varphi(x,t), \pi(y,t)] = i\delta(x-y)$$

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{m^2}{2}\psi^2 - c = +\frac{1}{2}\dot{\psi}^2 - \frac{1}{2}(\nabla\psi)^2 - \frac{m^2}{2}\psi^2 - c$$

$$\mathcal{L}_{int} = -W^\mu\partial_\mu\psi = +W^0\dot{\psi} - \vec{W}\cdot\nabla\psi$$

$$\Pi = \frac{\delta S}{\delta\dot{\psi}} = \dot{\psi} - W^0 \qquad \dot{\psi} = \Pi + W^0.$$

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{1}{2}m^2\psi^2 + c}_{H_0}$$

$$\mathcal{H} = \Pi\dot{\psi} - \mathcal{L}$$
$$= \Pi(\Pi + W^0) - \frac{1}{2}(\Pi + W^0)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 + c$$
$$+ W^0(\Pi + W^0) + \vec{W}\cdot\nabla\psi$$

$$= \mathcal{H}_0 + \Pi W^0 + \frac{1}{2}(W^0)^2 + \vec{W}\cdot\nabla\psi$$

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\mu\psi\,\partial^\mu\psi - \frac{m^2}{2}\psi^2 - c \;=\; +\frac{1}{2}\dot{\psi}^2 - \frac{1}{2}(\nabla\psi)^2 - \frac{m^2}{2}\psi^2 - c$$

$$\mathcal{L}_{int} = -W^\mu\partial_\mu\psi = +W^0\dot{\psi} - \vec{W}\cdot\nabla\psi$$

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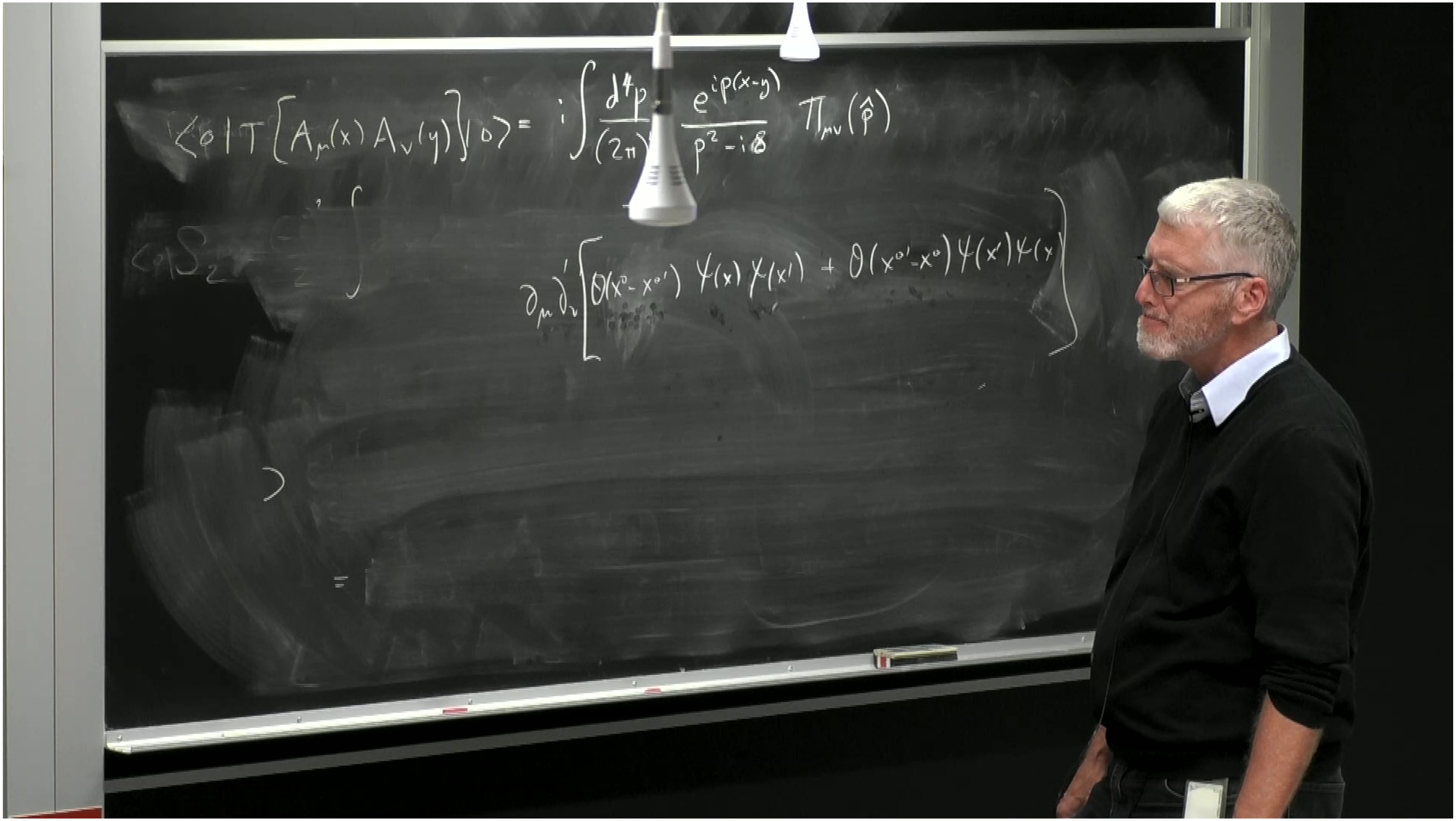
$$\mathcal{H} = \pi\dot{\psi} - \mathcal{L}$$
$$= \pi(\pi+W^0) - \frac{1}{2}(\pi+W^0)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 + c$$
$$+W^0(\pi+W^0) + \vec{W}\cdot\nabla\psi$$
$$= \mathcal{H}_0 + \pi W^0 + \frac{1}{2}(W^0)^2 + \vec{W}\cdot\nabla\psi$$

$$S = 1 - i\int \mathcal{H}_{int}(x)\, d^4x - \frac{1}{2}\int d^4x\, d^4y\; T\left[\mathcal{H}_{int}(x)\,\mathcal{H}_{int}(y)\right] + \cdots$$

$$= 1 - \frac{i}{2}\int d^4x \left[(W^0)^2 + \cdots\right] - \frac{1}{2}\int d^4x\, d^4y\; T\left[\left(W^0\pi + \vec{W}\cdot\nabla\psi\right)\left(W^0\pi + \vec{W}\cdot\nabla\psi\right)\right] + \cdots$$

$$i\,\delta(x-y)\,(W^0)^2$$


$\langle 0 | T [A_\mu(x) A_\nu(y)] | 0 \rangle = i \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - i\delta} \Pi_{\mu\nu}(\hat{p})$
$\partial_\mu \partial_\nu' [\theta(x^0 - x^{0\prime}) \psi(x) \psi(x') + \theta(x^{0\prime} - x^0) \psi(x') \psi(x)]$

$$\underbrace{+\tfrac{1}{2}(\nabla\psi)^2+\tfrac{1}{2}m^2\psi^2+c}_{H_0}$$

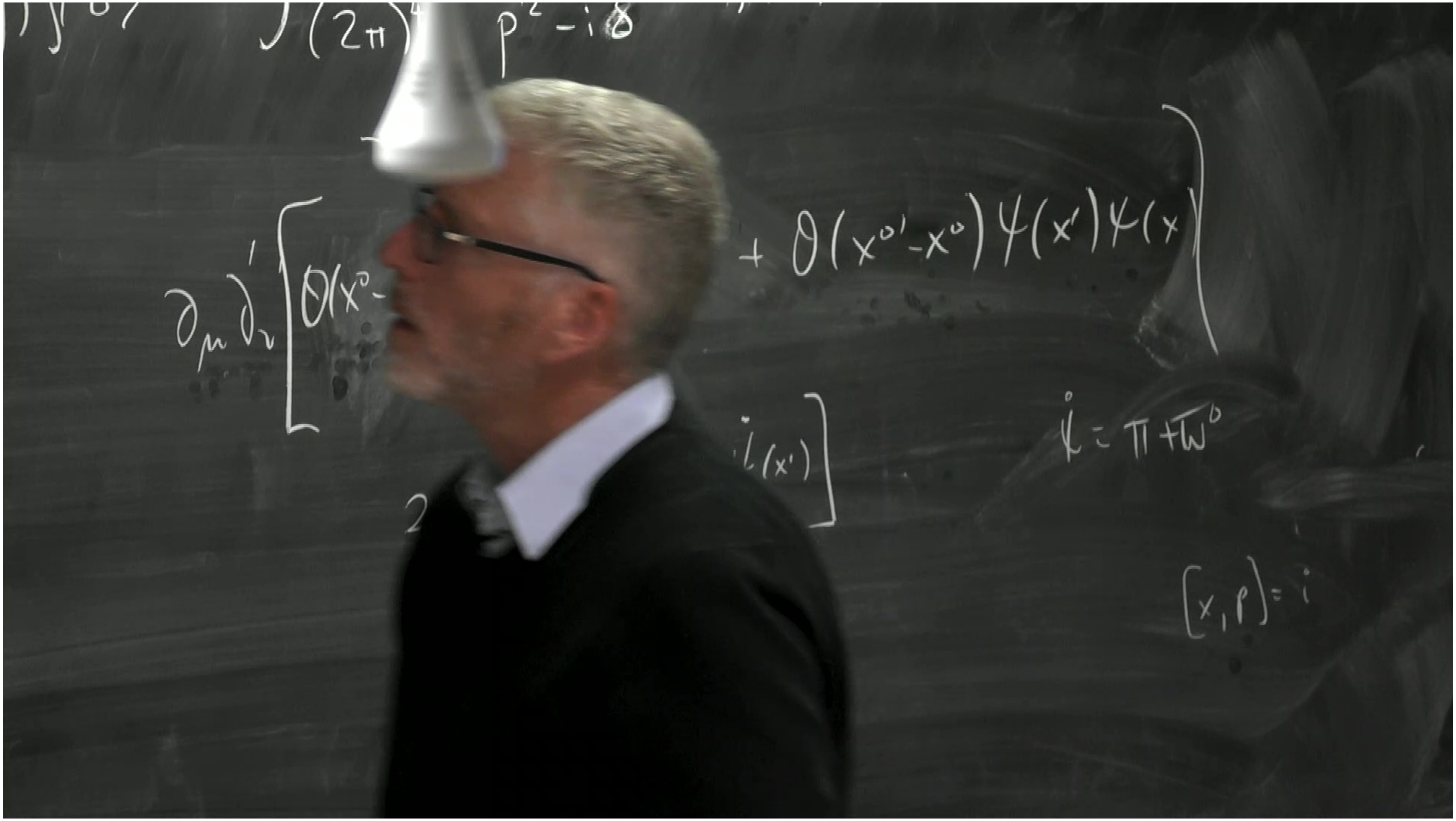
$$+W^0(\pi+W^0)+\vec{W}\cdot\nabla\psi$$

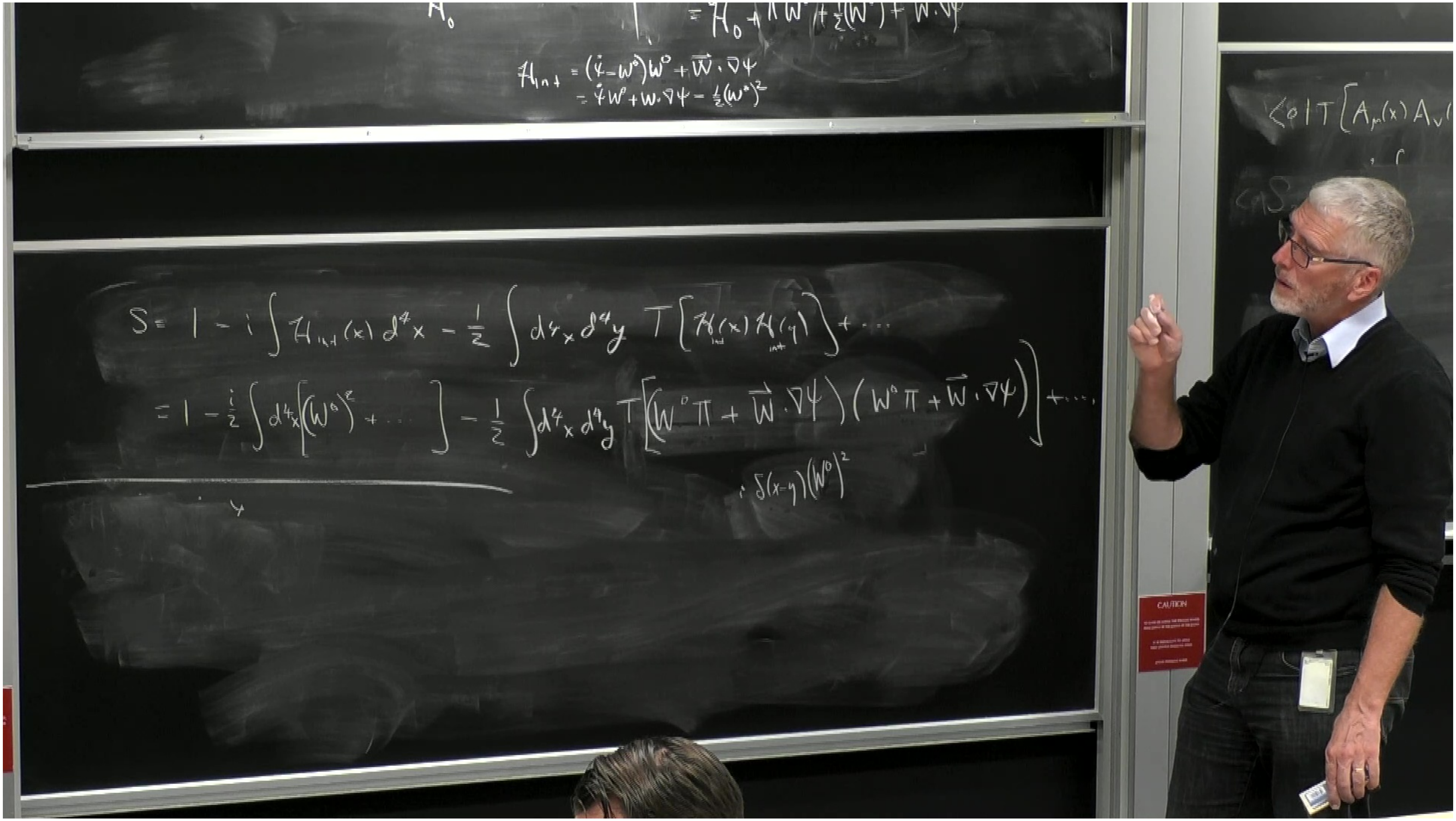
$$=\mathcal{H}_0+\pi W^0+\tfrac{1}{2}(W^0)^2+\vec{W}\cdot\nabla\psi$$

$$\mathcal{H}_{int}=(\dot\psi-W^0)W^0+\vec{W}\cdot\nabla\psi$$

$$=\dot\psi W^0+W\cdot\nabla\psi-\tfrac{1}{2}(W^0)^2$$

$$\mathcal{H}_{int}(x)\,d^4x-\frac{1}{2}\int d^4x\,d^4y\;T\left[\mathcal{H}_{int}(x)\,\mathcal{H}_{int}(y)\right]+\cdots$$


[x, p] = i


$\mathcal{H}_{int} = (\dot\psi - W^0)W^0 + \vec{W}\cdot\nabla\psi$
$= \dot\psi W^0 + W\cdot\nabla\psi - \frac{1}{2}(W^0)^2$
$S = 1 - i\int \mathcal{H}_{int}(x)\, d^4x - \frac{1}{2}\int d^4x\, d^4y\, T\left[\mathcal{H}_{int}(x)\mathcal{H}_{int}(y)\right] + \ldots$
$= 1 - \frac{i}{2}\int d^4x\left[(W^0)^2 + \ldots\right] - \frac{1}{2}\int d^4x\, d^4y\, T\left[(W^0\pi + \vec{W}\cdot\nabla\psi)(W^0\pi + \vec{W}\cdot\nabla\psi)\right] + \ldots$
$\delta(x-y)(W^0)^2$
CAUTION

Canonical quantization is useful because it automatically generates non-covariant terms in $\mathcal{H}$ needed for covariance of S.

$$S = \int dt \, L(\psi, \partial\psi) \qquad \Pi = \frac{\delta S}{\delta \dot\psi} = \Pi(\psi, \nabla\psi, \partial_t\psi)$$

$$= \int d^4x \, \mathcal{L} \qquad \mathcal{H} = \sum_a \Pi_a \dot\psi_a - \mathcal{L} = \mathcal{H}(\Pi, \nabla\psi, \psi)$$

$\psi(x) \quad \dot\psi = \partial_t \psi$

$[\psi(x,t), \Pi(y,t)] = i\delta(x-y)$