Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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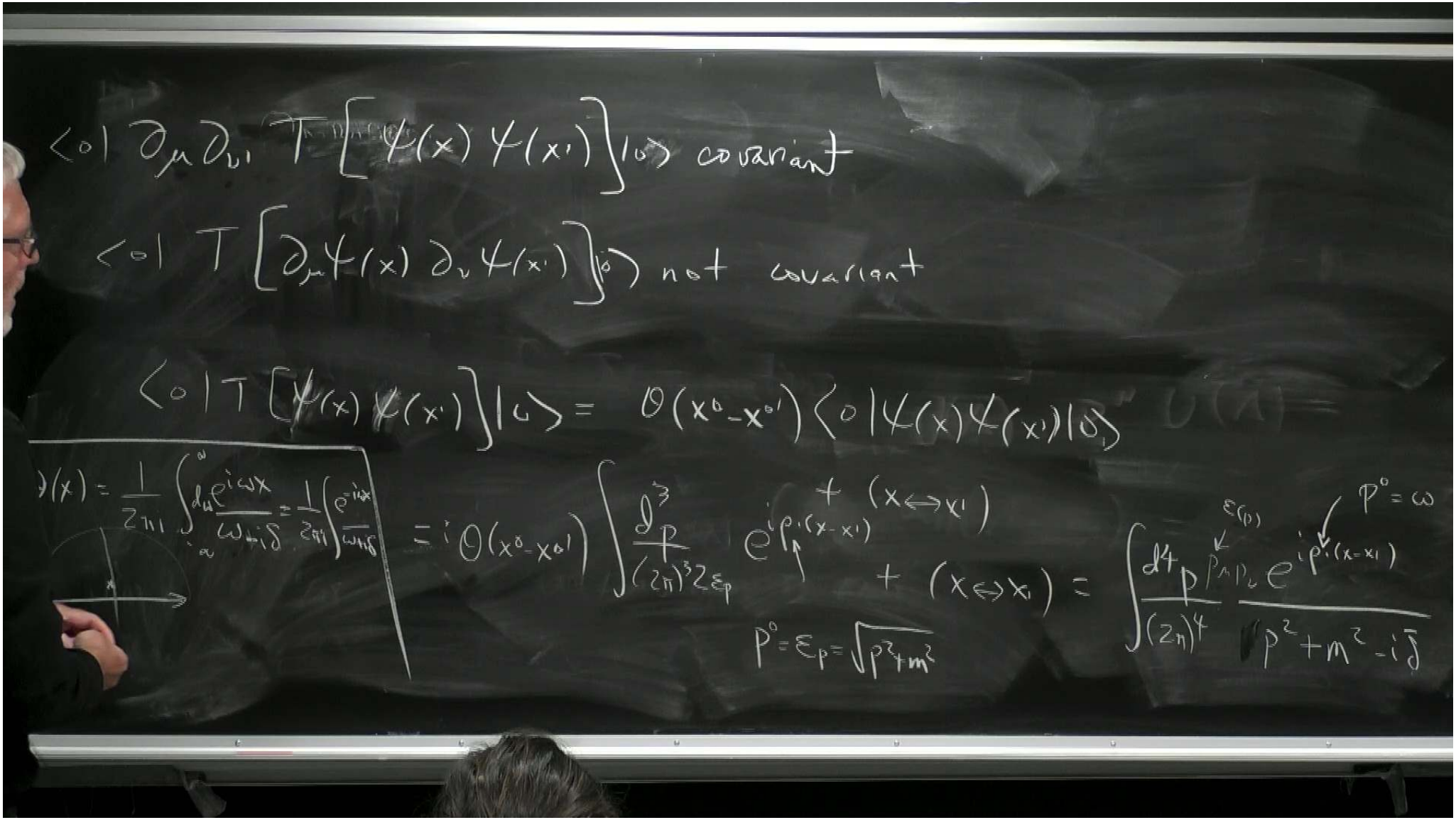
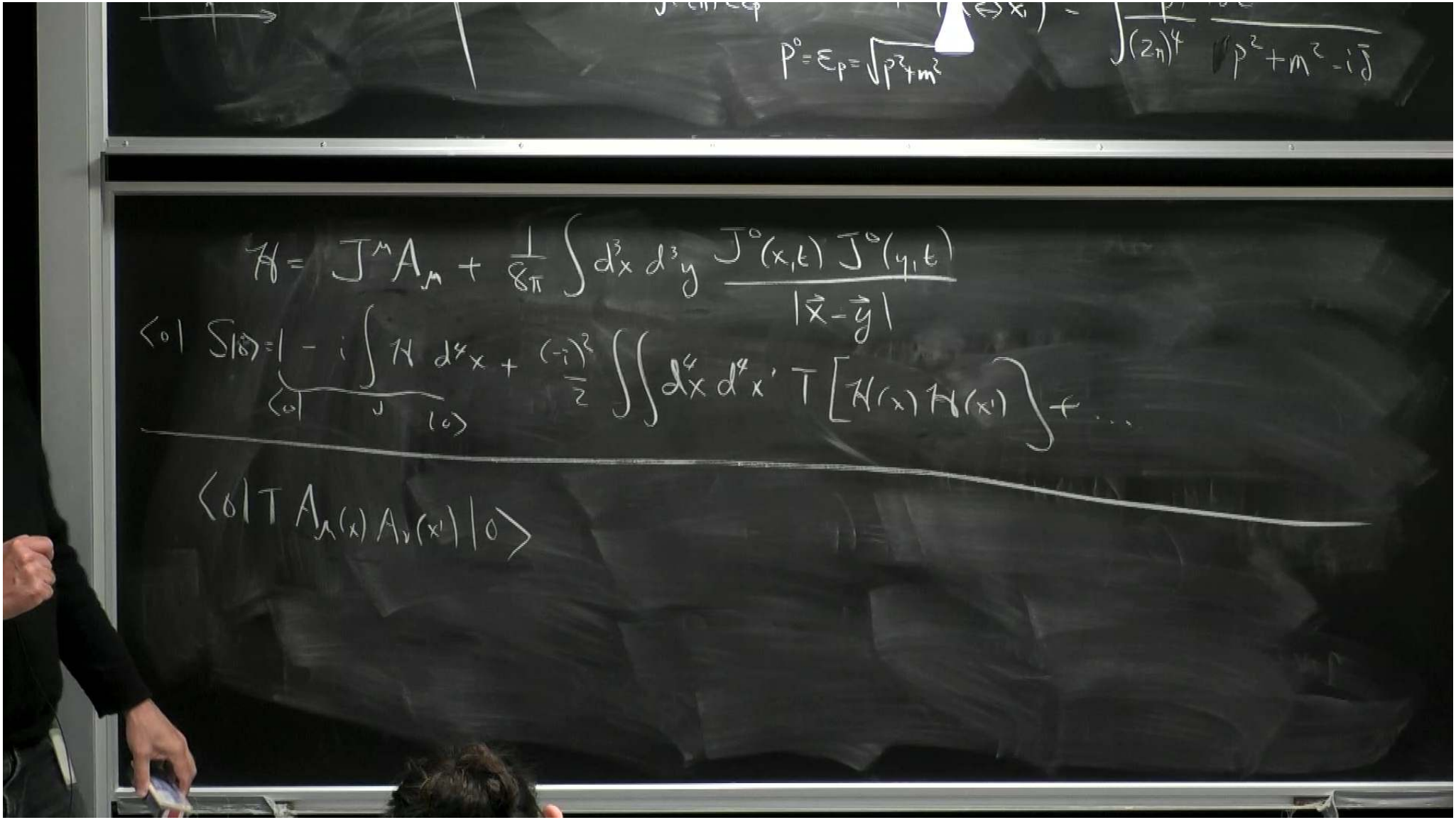
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


⟨0| ∂μ ∂ν T[ψ(x) ψ(x')] |0⟩ covariant
⟨0| T[∂μ ψ(x) ∂ν ψ(x')] |0⟩ not covariant
⟨0| T[ψ(x) ψ(x')] |0⟩ = θ(x⁰ − x'⁰) ⟨0|ψ(x)ψ(x')|0⟩
= iθ(x⁰ − x'⁰) ∫ d³p/(2π)³ 2εp e^{ip·(x−x')} + (x ⇔ x')
+ (x ⇔ x') = ∫ d⁴p/(2π)⁴ pμ pν e^{ip·(x−x')} / (p² + m² − iδ)
p⁰ = εp = √(p² + m²)
p⁰ = ω
ε(p)


$p^0 = \varepsilon_p = \sqrt{p^2 + m^2}$
$\frac{1}{(2\pi)^4} \frac{1}{p^2 + m^2 - i\delta}$
$\mathcal{H} = J^\mu A_\mu + \frac{1}{8\pi} \int d^3x\, d^3y \frac{J^0(x,t)\, J^0(y,t)}{|\vec{x} - \vec{y}|}$
$\langle 0 | S | 0 \rangle = 1 - i \int \mathcal{H}\, d^4x + \frac{(-i)^2}{2} \iint d^4x\, d^4x'\, T\left[\mathcal{H}(x)\, \mathcal{H}(x')\right] + \ldots$
$\langle 0 | J | 0 \rangle$
$\langle 0 | T\, A_\mu(x)\, A_\nu(x') | 0 \rangle$

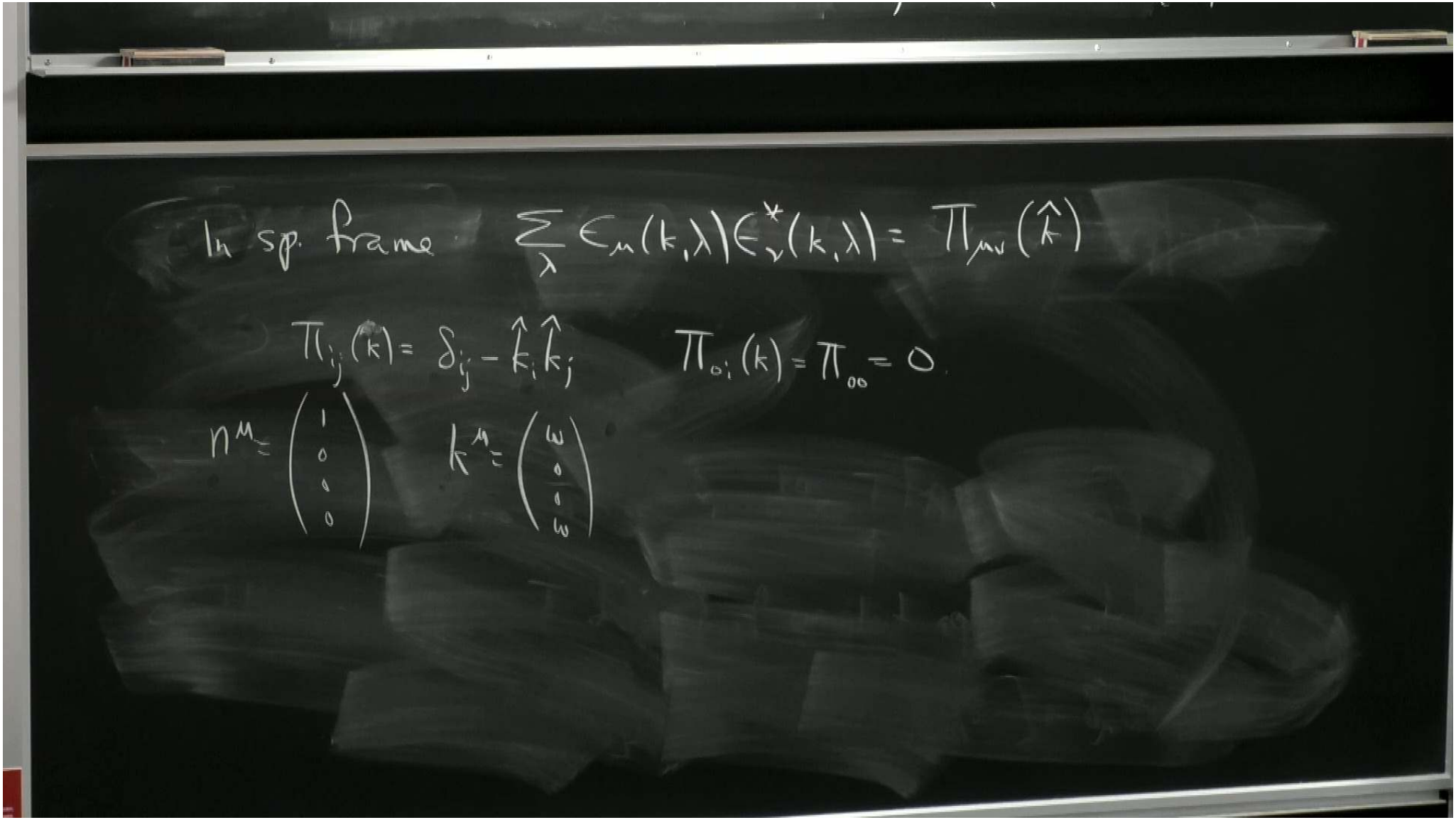
$$A_\mu(x) = \sum_{\lambda=\pm1} \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[a_{k\lambda}\, \epsilon_\mu(k,\lambda)\, e^{ik\cdot x} + a^*_{k\lambda}\, \epsilon^*_\mu(k,\lambda)\, e^{-ik\cdot x} \right]$$

in special frame: $k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$ $\begin{matrix} t \\ x \\ y \\ z \end{matrix}$

$$\epsilon_\mu(k,\lambda=x) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_\mu(k,\lambda=\pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \mp i \\ 0 \end{pmatrix}$$

$$\epsilon_\mu(k,\lambda=y) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle 0 | T\left[A_\mu(x) A_\nu(y) \right] | 0 \rangle = \theta(x^0-y^0) \sum_\lambda \int \frac{d^3k}{(2\pi)^3 2\omega_k} \epsilon_\mu(k,\lambda)\, \epsilon^*_\nu(k,\lambda)\, e^{ik(x-y)} + (x \leftrightarrow y,\ \mu \leftrightarrow \nu)$$


In sp. frame $\sum_\lambda \epsilon_\mu(k,\lambda)\epsilon^*_\nu(k,\lambda) = \Pi_{\mu\nu}(\hat{k})$
$\Pi_{ij}(\hat{k}) = \delta_{ij} - \hat{k}_i\hat{k}_j$ $\Pi_{0i}(k) = \Pi_{00} = 0$
$n^\mu = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $k^\mu = \begin{pmatrix}\omega\\0\\0\\\omega\end{pmatrix}$

$$\Pi_{ij}(k) = \delta_{ij} - \hat{k}_i \hat{k}_j \qquad \Pi_{0i}(k) = \Pi_{00} = 0$$

$$n^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$$

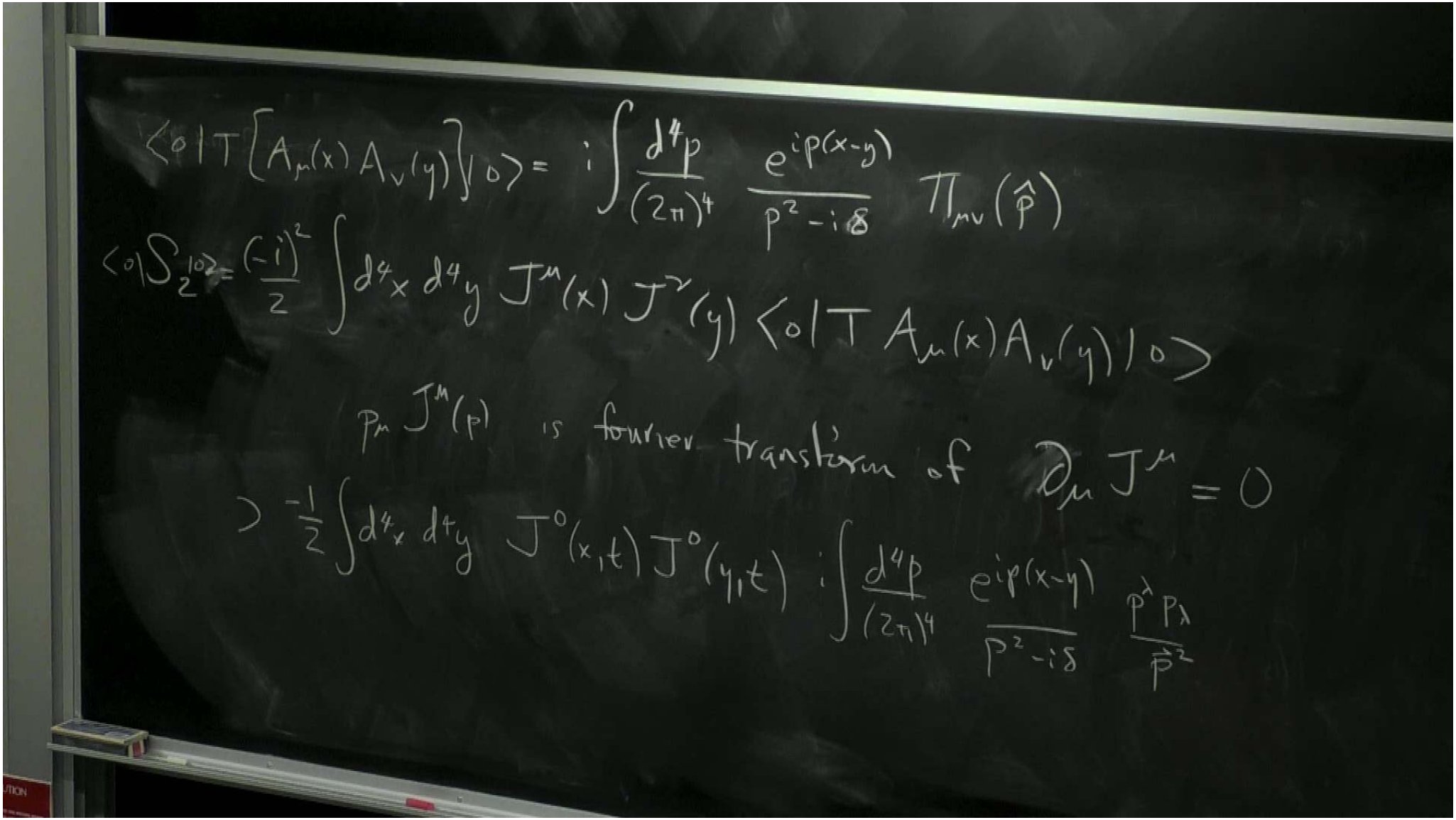
$$\Pi_{\mu\nu}(p) = \eta_{\mu\nu} + \frac{(n_\mu p_\nu + p_\mu n_\nu)\, p\cdot n - p_\mu p_\nu}{\vec{p}^{\,2}} + \frac{(n_\mu n_\nu)\, p^\lambda p_\lambda}{\vec{p}^{\,2}}$$

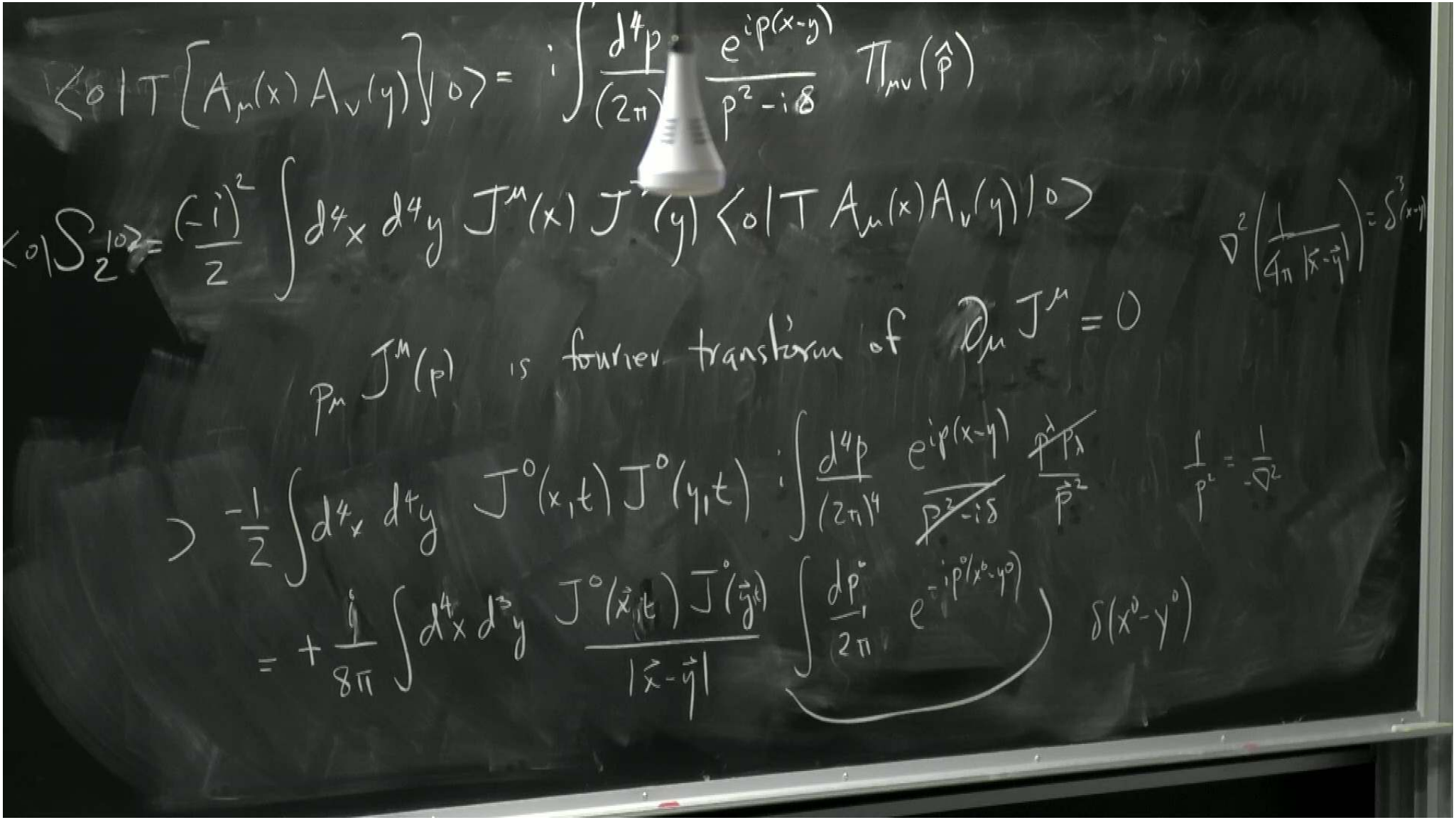
$$p^\mu = \begin{pmatrix} p^0 \\ \vec{p} \end{pmatrix} \qquad p^0 = \text{anything}$$

$$A_\mu(x) = \sum_{\lambda=\pm 1} \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[a_{k\lambda}\, \epsilon_\mu(k,\lambda)\, e^{ik\cdot x} + a^*_{k\lambda}\, \epsilon^*_\mu(k,\lambda)\, e^{-ik\cdot x} \right]$$

in special frame: $k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix}$ $\begin{matrix} t \\ x \\ y \\ z \end{matrix}$ $\qquad \epsilon_\mu(k,\lambda=x) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_\mu(k,\lambda=\pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$

$\epsilon_\mu(k,\lambda=y) = \begin{pmatrix} 0 \\ \end{pmatrix}$

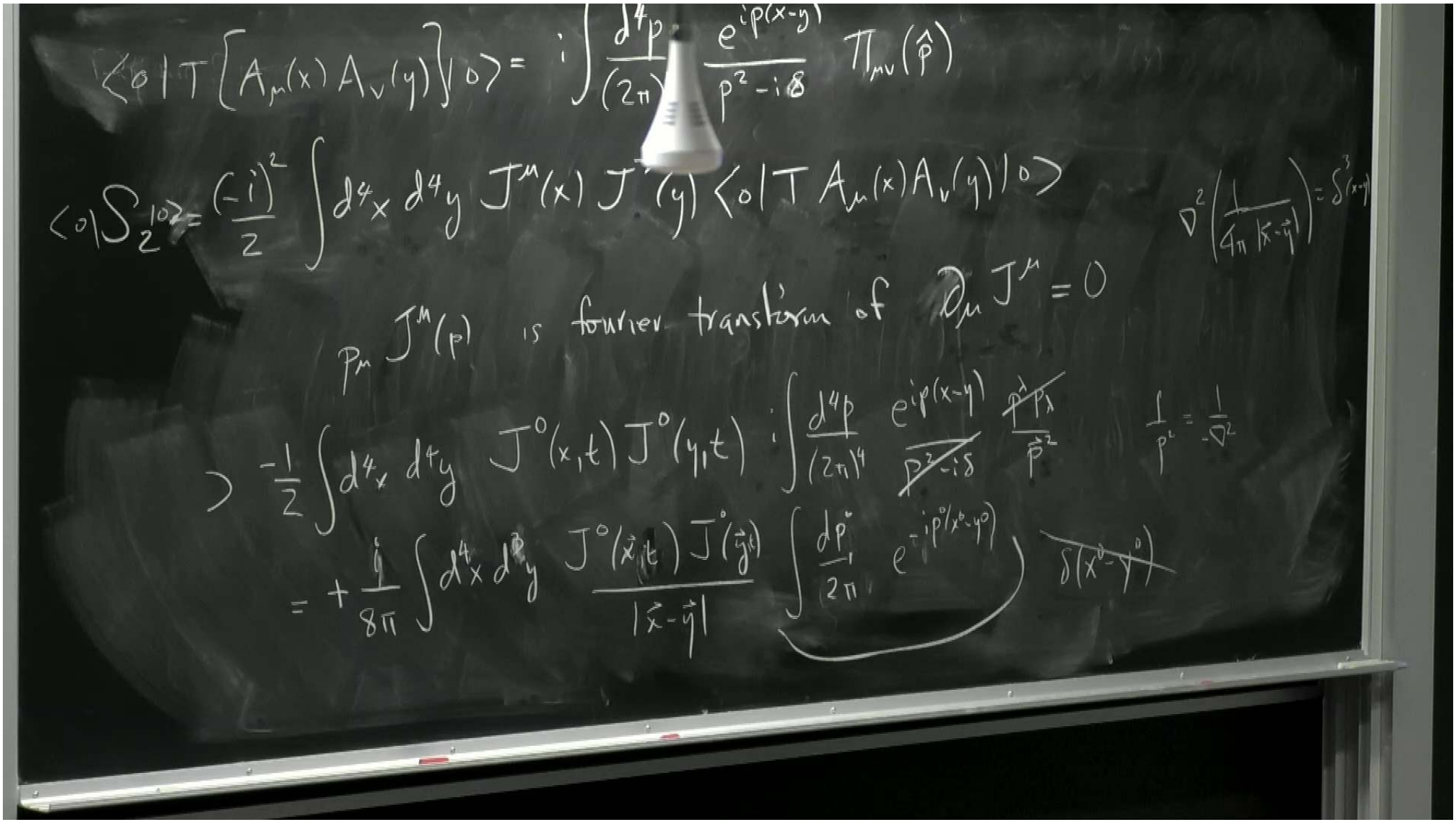

$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\Pi_{\mu\nu}(\hat{p})$
$\langle 0|S_2^{10}|2\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)J^\nu(y)\langle 0|T\,A_\mu(x)A_\nu(y)|0\rangle$
$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$
$= -\frac{1}{2}\int d^4x\, d^4y\, J^0(x,t)J^0(y,t)\, i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\frac{\vec{p}^2 P_\lambda}{\vec{p}^2}$

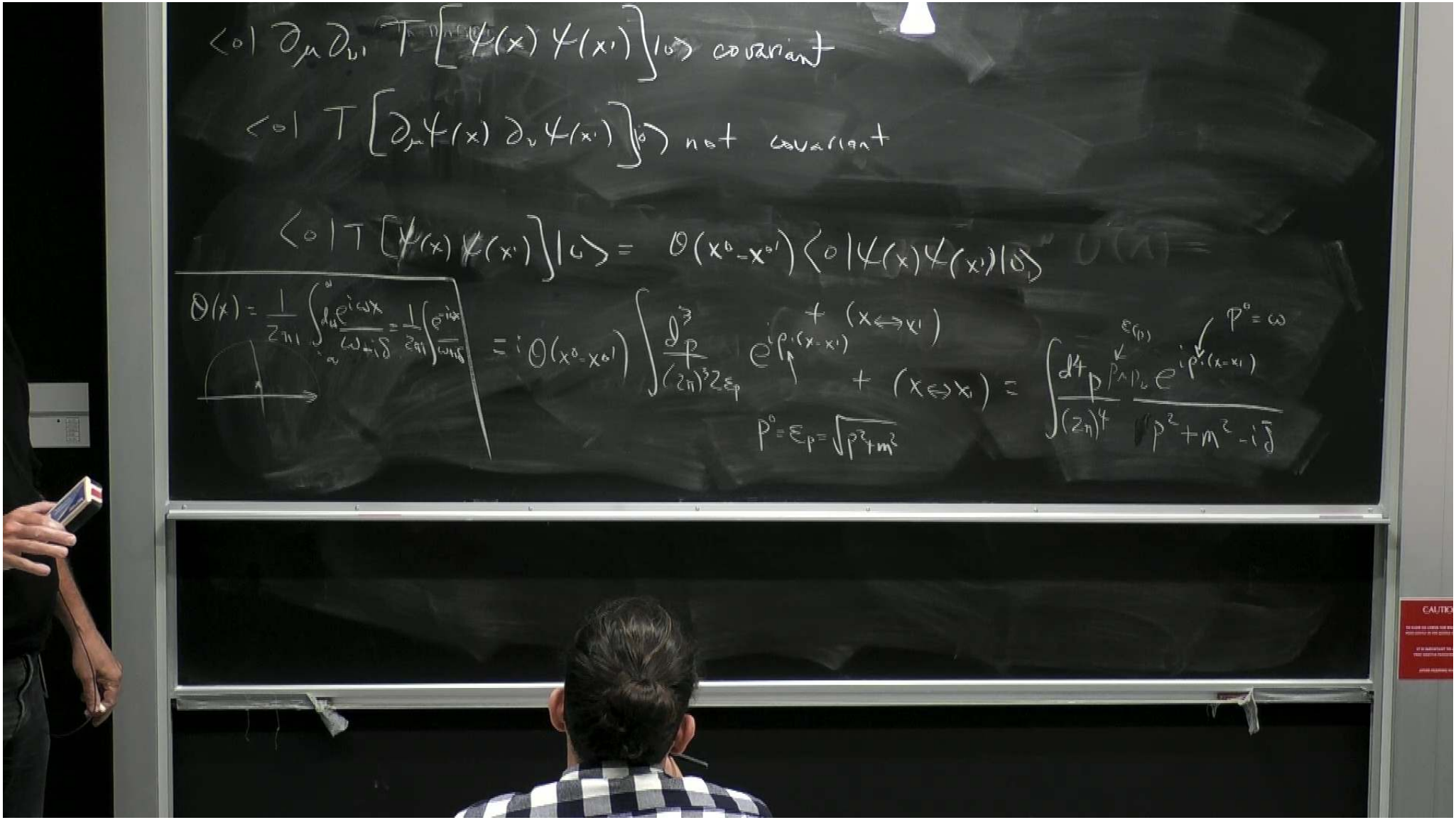

$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\Pi_{\mu\nu}(\hat{p})$
$\langle 0|S_2^{(2)}|0\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)J^\nu(y)\langle 0|T\,A_\mu(x)A_\nu(y)|0\rangle$
$\nabla^2\left(\frac{1}{4\pi|\vec{x}-\vec{y}|}\right) = \delta^3(x-y)$
$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$
$= -\frac{1}{2}\int d^4x\, d^4y\; J^0(x,t)J^0(y,t)\; i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\frac{p^1p^1}{\vec{p}^2}$
$\frac{1}{p^2} = -\frac{1}{\vec{p}^2}$
$= +\frac{i}{8\pi}\int d^4x\, d^3y\; \frac{J^0(\vec{x},t)J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}\int \frac{dp^0}{2\pi}\, e^{-ip^0(x^0-y^0)}$
$\delta(x^0-y^0)$

$$\mathcal{H} = J^\mu A_\mu + \frac{1}{8\pi}\int d^3x\, d^3y\, \frac{J^0(x,t)\, J^0(y,t)}{|\vec{x}-\vec{y}|}$$

$$\langle 0|\, S\,|0\rangle = \underbrace{\langle 0|\, 1 - i\int \mathcal{H}\, d^4x \,|0\rangle}_{\langle 0|\ U\ |0\rangle} + \frac{(-i)^2}{2}\iint d^4x\, d^4x'\, T\left[\mathcal{H}(x)\,\mathcal{H}(x')\right] + \dots$$

$$\frac{-i}{8\pi}\int d^4x\, \frac{J^0(\vec{x},t)\, J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}$$


$\langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\Pi_{\mu\nu}(\hat{p})$
$\langle 0|S_2^{10}|0\rangle = \frac{(-i)^2}{2}\int d^4x\, d^4y\, J^\mu(x)J^\nu(y)\langle 0|T\,A_\mu(x)A_\nu(y)|0\rangle$
$\nabla^2\left(\frac{1}{4\pi|\vec{x}-\vec{y}|}\right) = \delta^3(x-y)$
$p_\mu J^\mu(p)$ is fourier transform of $\partial_\mu J^\mu = 0$
$\frac{1}{p^2} = \frac{1}{-\vec{p}^2}$
$= \frac{-1}{2}\int d^4x\, d^4y\; J^0(x,t)J^0(y,t)\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2-i\delta}\frac{p^2}{\vec{p}^2}$
$= +\frac{i}{8\pi}\int d^4x\, d^4y\; \frac{J^0(\vec{x},t)J^0(\vec{y},t)}{|\vec{x}-\vec{y}|}\int\frac{dp^0}{2\pi}e^{-ip^0(x^0-y^0)}$
$\delta(x^0-y^0)$


$\langle 0| \partial_\mu \partial_\nu T[\psi(x)\psi(x')]|0\rangle$ covariant
$\langle 0| T[\partial_\mu \psi(x)\, \partial_\nu \psi(x')]|0\rangle$ not covariant
$\langle 0|T[\psi(x)\psi(x')]|0\rangle = \theta(x^0-x'^0)\langle 0|\psi(x)\psi(x')|0\rangle$
$\theta(x) = \frac{1}{2\pi i}\int d\omega \frac{e^{i\omega x}}{\omega+i\delta} = \frac{1}{2\pi i}\int \frac{e^{i\omega x}}{\omega+i\delta}$
$= i\theta(x^0-x'^0)\int \frac{d^3p}{(2\pi)^3 2\varepsilon_p} e^{ip\cdot(x-x')} + (x\leftrightarrow x') = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip\cdot(x-x')}}{p^2+m^2-i\delta}$
$p^0 = \varepsilon_p = \sqrt{p^2+m^2}$
$\varepsilon(p)$
$p^0 = \omega$

Canonical quantization is useful because it automatically generates non-covariant terms in $\mathcal{H}$ needed for covariance of S.

$$S = \int dt\, L(\psi, \partial\psi) = \int d^4x\, \mathcal{L}$$

$\psi(x)$ $\dot{\psi} = \partial_t \psi$

$$\pi = \frac{\delta S}{\delta \dot{\psi}} = \pi(\psi, \nabla\psi, \partial_t\psi)$$

$$\mathcal{H} = \sum_q \pi_q \dot{\psi}_q - \mathcal{L} = \mathcal{H}(\pi, \nabla\psi, \psi)$$

$$[\psi(x,t), \pi(y,t)] = i\,\delta(x-y)$$

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{m^2}{2}\psi^2 - c = +\frac{1}{2}\dot{\psi}^2 - \frac{1}{2}(\nabla\psi)^2 - \frac{m^2}{2}\psi^2 - c$$

$$\mathcal{L}_{int} = -W^\mu\partial_\mu\psi = +W^0\dot{\psi} - \vec{W}\cdot\nabla\psi$$

$$\pi = \frac{\delta S}{\delta\dot{\psi}} = \dot{\psi} - W^0 \qquad \dot{\psi} = \pi + W^0$$

$$\mathcal{H}_0 = \underbrace{\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{1}{2}m^2\psi^2 + c}_{H_0}$$

$$\mathcal{H} = \pi\dot{\psi} - \mathcal{L}$$

$$= \pi(\pi + W^0) - \frac{1}{2}(\pi + W^0)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 + c$$

$$+ W^0(\pi + W^0) + \vec{W}\cdot\nabla\psi$$

$$= \mathcal{H}_0 + \pi W^0 + \frac{1}{2}(W^0)^2 + \vec{W}\cdot\nabla\psi$$

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{m^2}{2}\psi^2 - c = +\frac{1}{2}\dot{\psi}^2 - \frac{1}{2}(\nabla\psi)^2 - \frac{m^2}{2}\psi^2 - c$$

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$$= \pi(\pi + W^0) - \frac{1}{2}(\pi + W^0)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 + c$$

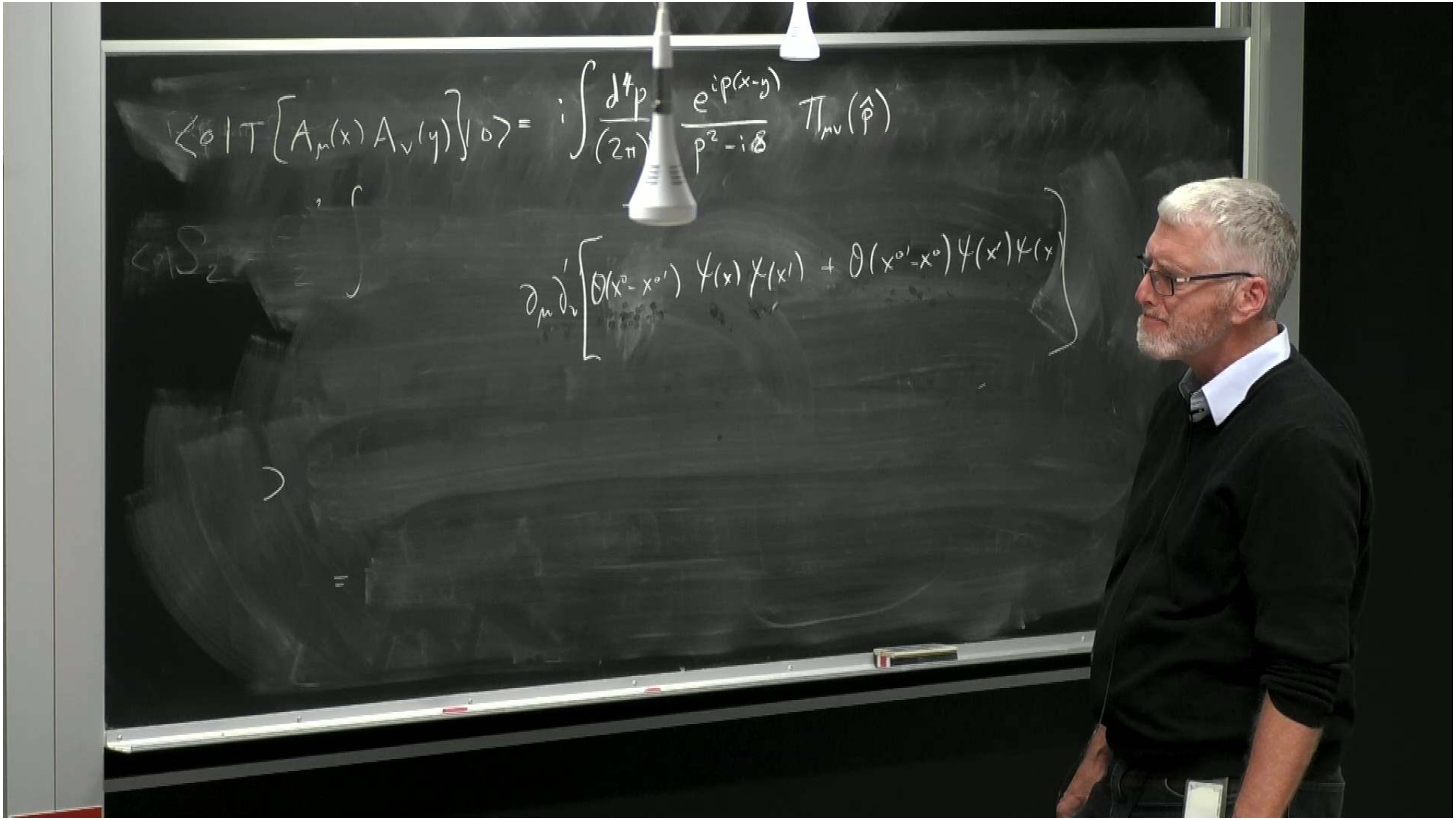
$$+ W^0(\pi + W^0) + \vec{W}\cdot\nabla\psi$$

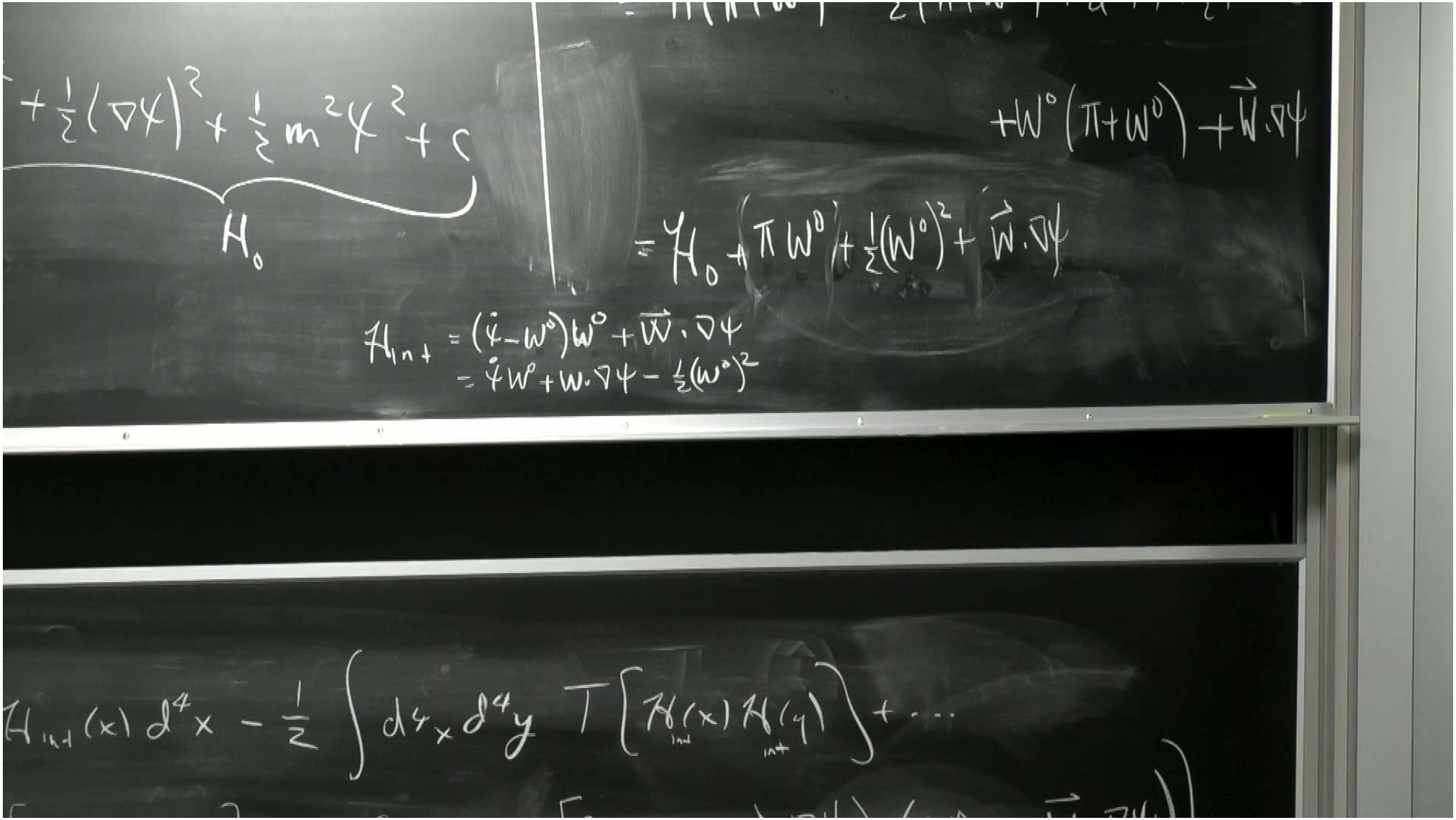
$$= \mathcal{H}_0 + \pi W^0 + \frac{1}{2}(W^0)^2 + \vec{W}\cdot\nabla\psi$$

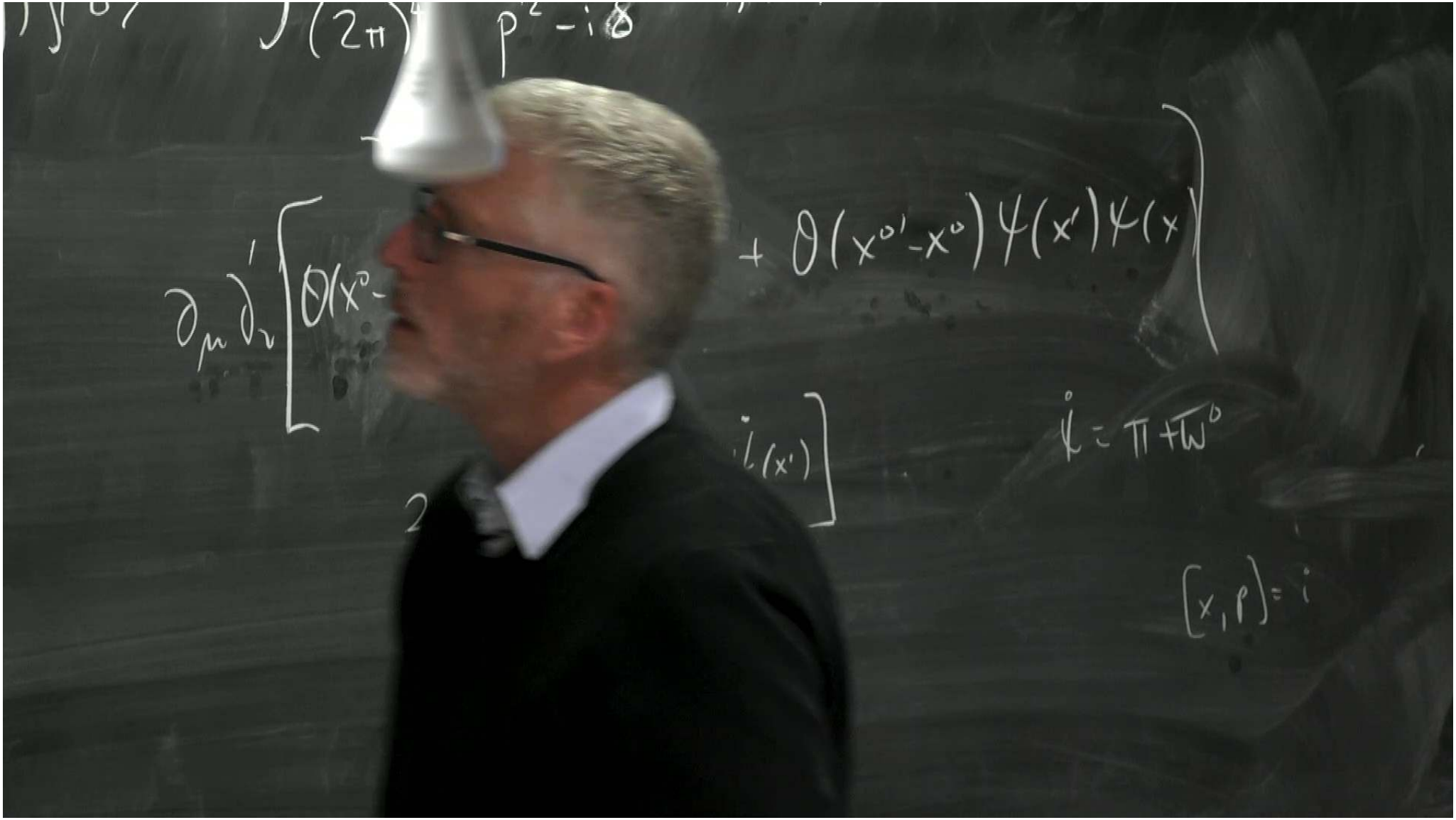
$$S = 1 - i\int \mathcal{H}_{int}(x)\, d^4x - \frac{1}{2}\int d^4x\, d^4y\; T\left[\mathcal{H}_{int}(x)\mathcal{H}_{int}(y)\right] + \cdots$$

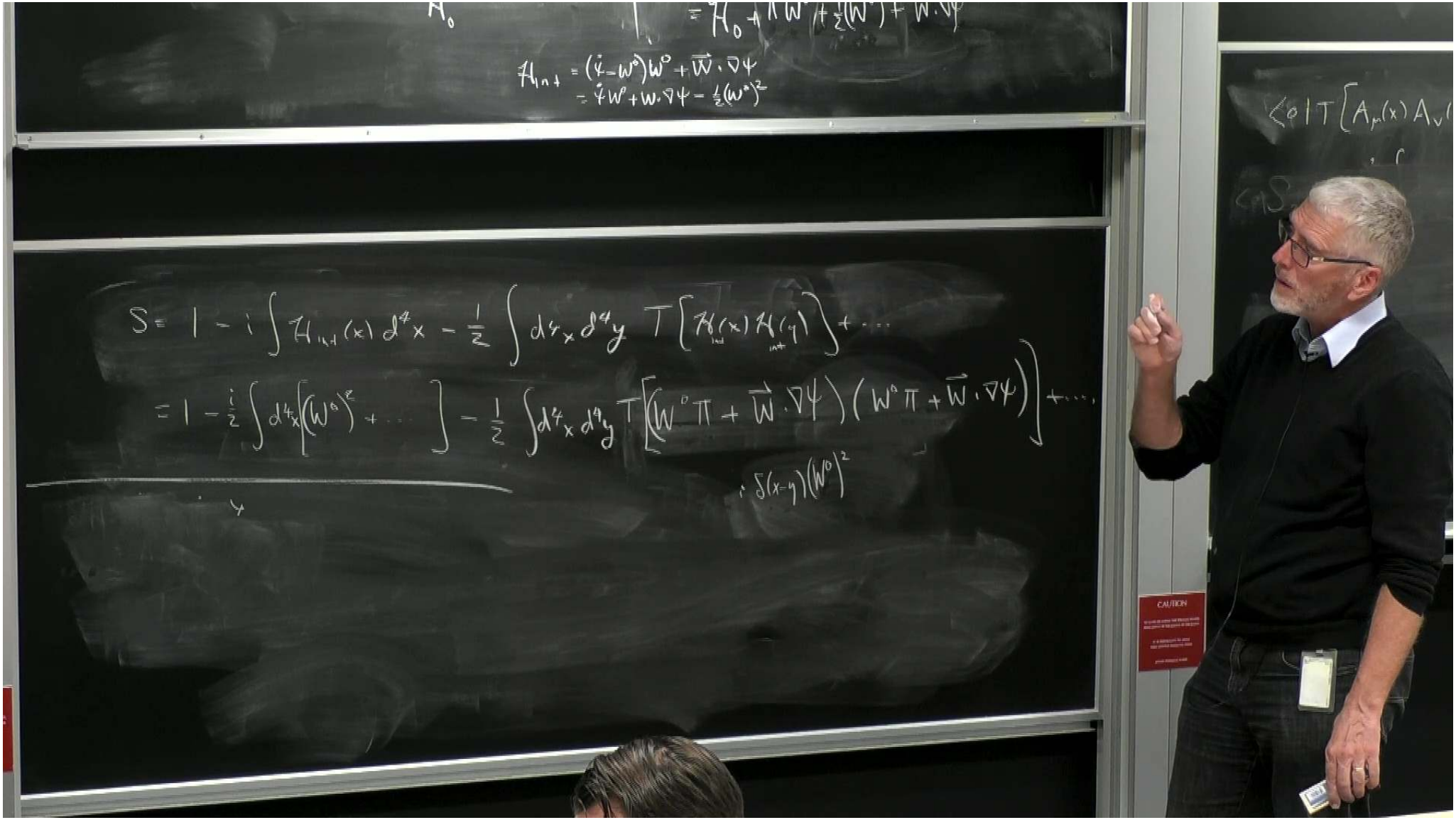
$$= 1 - \frac{i}{2}\int d^4x \left[(W^0)^2 + \cdots\right] - \frac{1}{2}\int d^4x\, d^4y\; T\left[\left(W^0\pi + \vec{W}\cdot\nabla\psi\right)\left(W^0\pi + \vec{W}\cdot\nabla\psi\right)\right] + \cdots$$

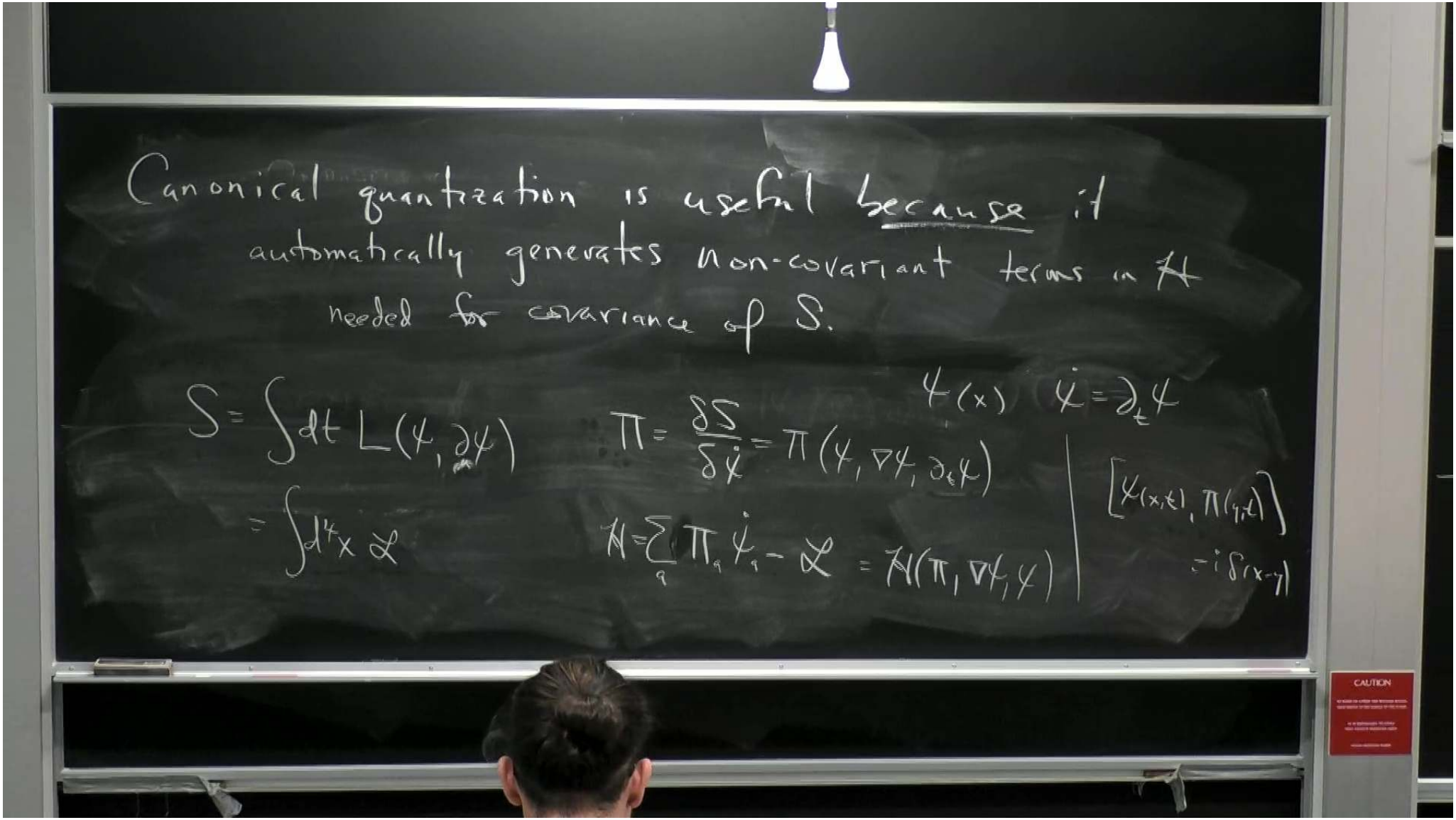
$$\therefore \delta(x-y)(W^0)^2$$


$\langle \Omega | T\{A_\mu(x) A_\nu(y)\} | \Omega \rangle = i \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - i\delta} \pi_{\mu\nu}(\hat{p})$
$\partial_\mu \partial'_\nu \{ \theta(x^0 - x^{0\prime}) \psi(x) \bar{\psi}(x') + \theta(x^{0\prime} - x^0) \psi(x') \bar{\psi}(x$


$+\frac{1}{2}(\nabla\psi)^2+\frac{1}{2}m^2\psi^2+c$
H_0
$+W^0(\pi+W^0)+\vec{W}\cdot\nabla\psi$
$=\mathcal{H}_0+\pi W^0+\frac{1}{2}(W^0)^2+\vec{W}\cdot\nabla\psi$
$\mathcal{H}_{int}=(\dot{\psi}-W^0)W^0+\vec{W}\cdot\nabla\psi$
$=\dot{\psi}W^0+W\cdot\nabla\psi-\frac{1}{2}(W^0)^2$
$\mathcal{H}_{int}(x)\,d^4x-\frac{1}{2}\int d^4x\,d^4y\;T\left[\mathcal{H}_{int}(x)\mathcal{H}_{int}(y)\right]+\cdots$


(2π)
$p^2 - i\delta$
$\partial_\mu \partial_\nu$
$\theta(x^0 -$
$+ \theta(x^{0\prime} - x^0)\psi(x')\psi(x$
$i(x')$
$\dot{\psi} = \pi + \nabla^2$
$[x, p] = i$


$\mathcal{H}_{int} = (\dot{\psi} - w^0)\pi^0 + \vec{w}\cdot\nabla\psi$
$= \dot{\psi}\pi + w\cdot\nabla\psi - \frac{1}{2}(w^0)^2$
$S = 1 - i\int \mathcal{H}_{int}(x)\,d^4x - \frac{1}{2}\int d^4x\,d^4y\; T\left[\mathcal{H}_{int}(x)\mathcal{H}_{int}(y)\right] + \dots$
$= 1 - \frac{i}{2}\int d^4x\left[(w^0)^2 + \dots\right] - \frac{1}{2}\int d^4x\,d^4y\; T\left[\left(w^0\pi + \vec{w}\cdot\nabla\psi\right)\left(w^0\pi + \vec{w}\cdot\nabla\psi\right)\right] + \dots$
$+ \delta(x-y)(w^0)^2$
CAUTION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$\mathcal{H} = \sum_q \pi_q \dot{\psi}_q - \mathcal{L} = \mathcal{H}(\pi, \nabla\psi, \psi)$
$[\psi(x,t), \pi(y,t)]$
$= i\delta(x-y)$
CAUTION