Title: QFT2 - Quantum Electrodynamics - Afternoon Lecture

Speakers: Cliff Burgess

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

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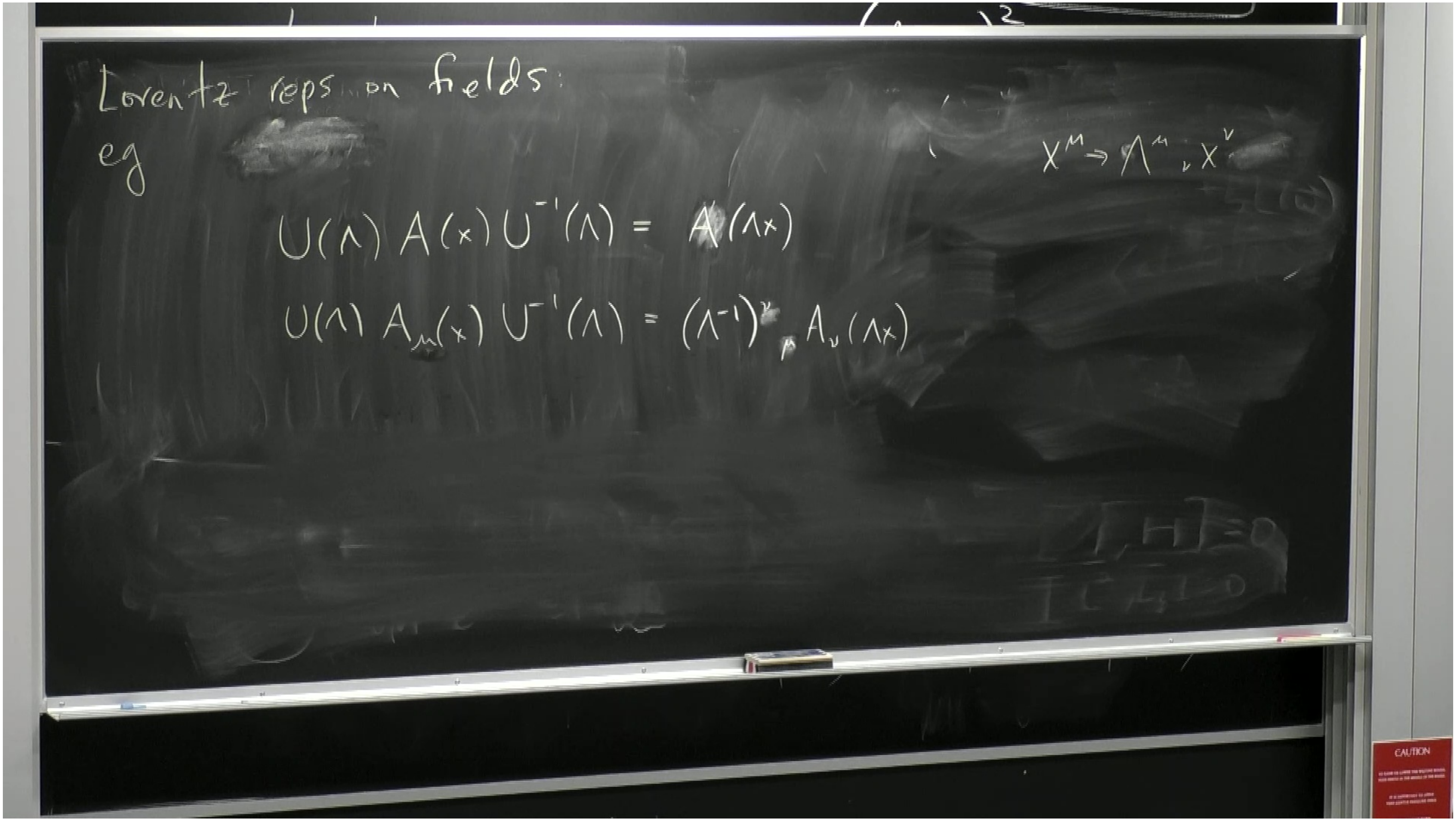
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


Lorentz reps on fields:
eg
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$U(\Lambda)\,A(x)\,U^{-1}(\Lambda) = A(\Lambda x)$
$U(\Lambda)\,A_\mu(x)\,U^{-1}(\Lambda) = (\Lambda^{-1})^\nu{}_\mu\,A_\nu(\Lambda x)$

Lorentz reps on fields:

eg

$$U(\Lambda)\, A(x)\, U^{-1}(\Lambda) = A(\Lambda x)$$

$$U(\Lambda)\, A_\mu(x)\, U^{-1}(\Lambda) = (\Lambda^{-1})^\nu{}_\mu\, A_\nu(\Lambda x)$$

$$= \Lambda_\mu{}^\nu A_\nu(\Lambda x)$$

$$x^\mu \to \Lambda^\mu{}_\nu\, x^\nu$$

$$\eta_{\alpha\beta}\Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = \eta_{\mu\nu}$$

$$(\Lambda^{-1})^\mu{}_\nu = \Lambda_\nu{}^\mu$$

Lorentz reps on fields:

eg

$$U(\Lambda)\, A(x)\, U^{-1}(\Lambda) = A(\Lambda x)$$

$$U(\Lambda)\, A_\mu(x)\, U^{-1}(\Lambda) = (\Lambda^{-1})^\nu{}_\mu\, A_\nu(\Lambda x)$$

$$= \Lambda_\mu{}^\nu A_\nu(\Lambda x)$$

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu$$

$$\eta_{\alpha\beta}\Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = \eta_{\mu\nu}$$

$$(\Lambda^{-1})^\mu{}_\nu = \Lambda_\nu{}^\mu$$

$$A_m(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}}\, U_m(p,\sigma)\, a_{p\sigma}\, e^{ip\cdot x}$$

Particle representations of Poincaré group.

$P^\mu P_\mu = -m^2$ sign p^0 if $p^2 \leq 0$

$P^\mu |p,\sigma\rangle = p^\mu |p,\sigma\rangle$

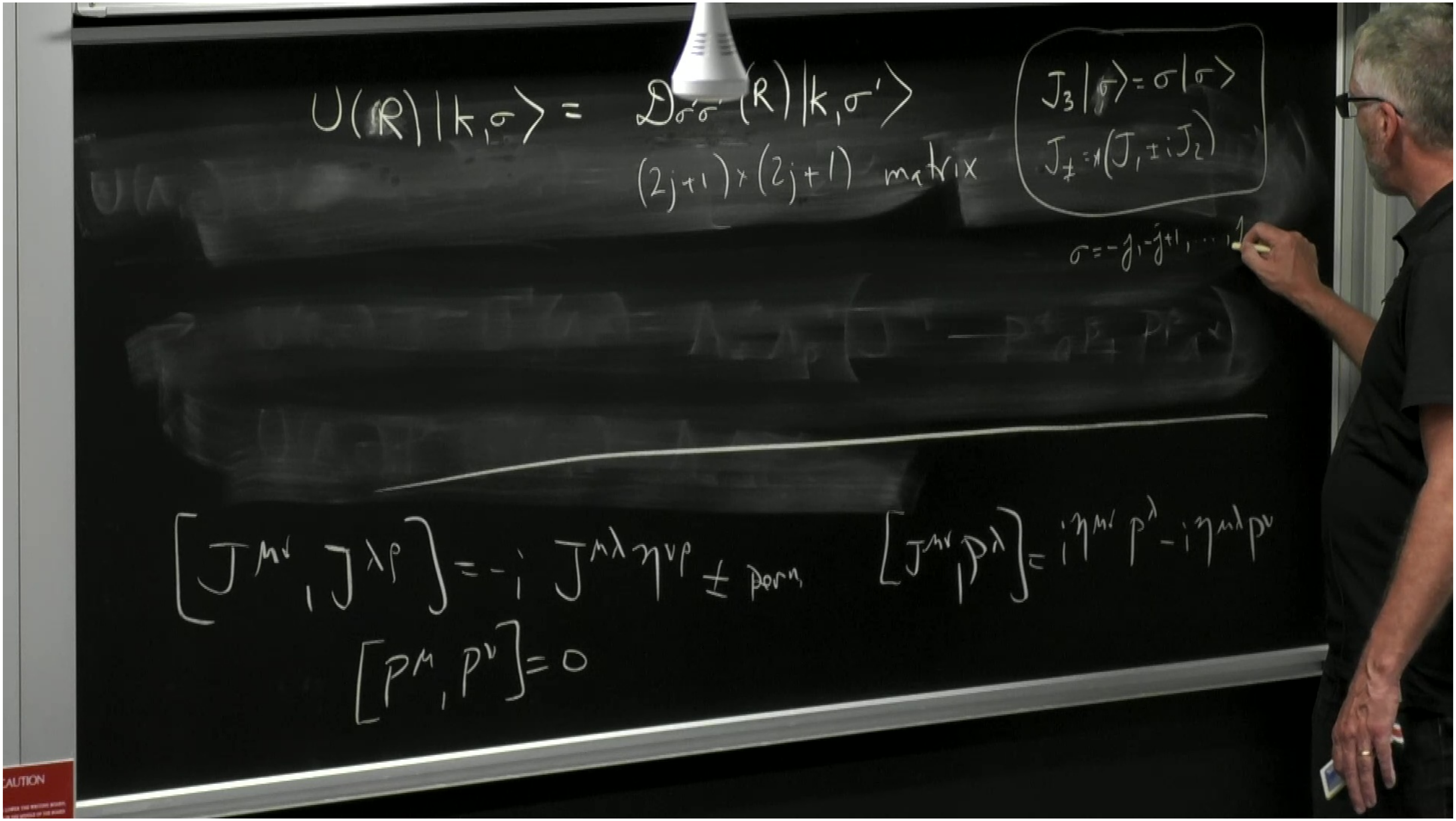
$E = \sqrt{\vec{p}^2 + m^2}$

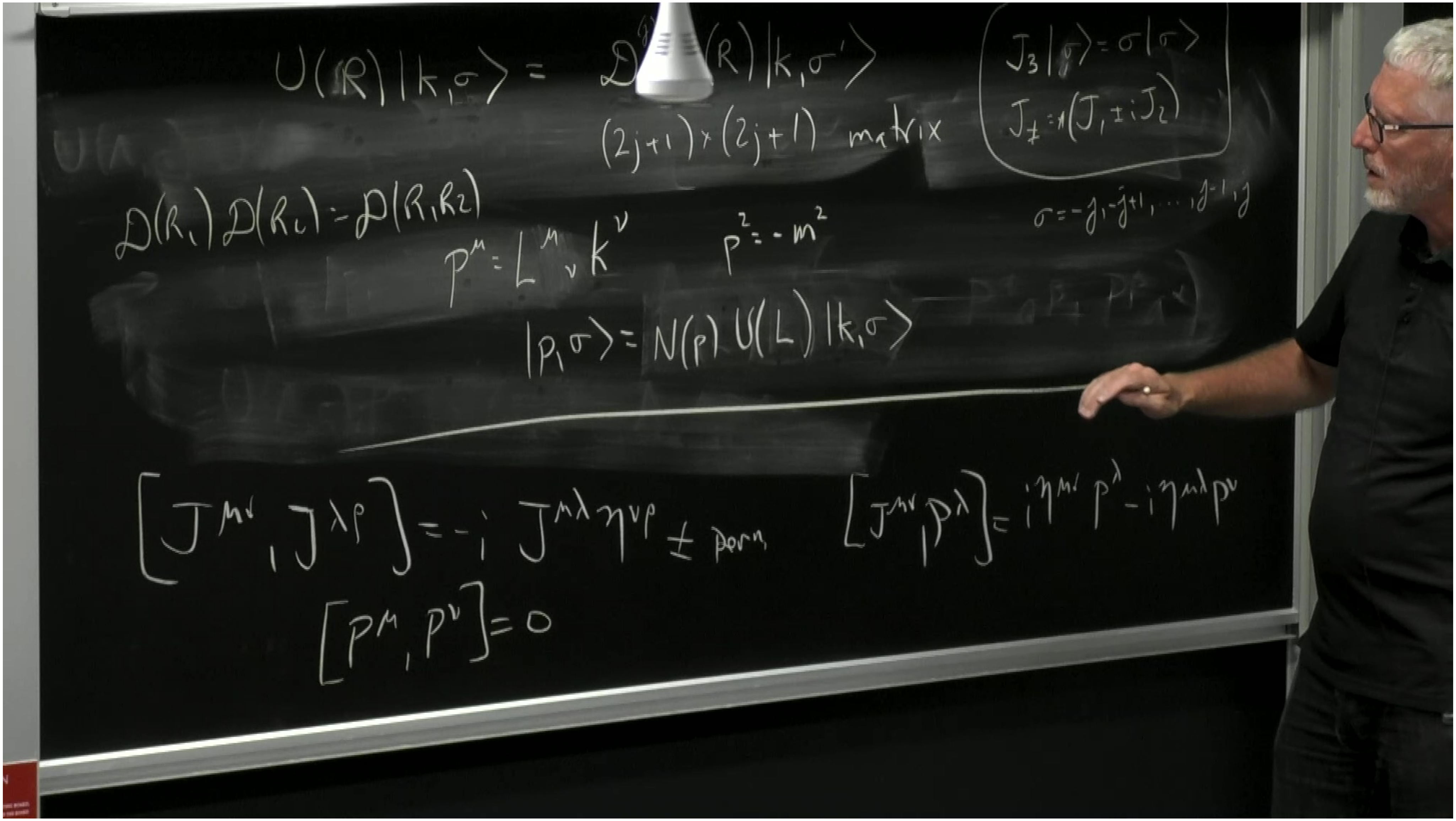
Little group is defined by

$\Lambda^\mu{}_\nu k^\nu = k^\mu$. $\Lambda = S(a,b) R(\theta)$

$$
\begin{cases}
p^\mu = 0 & U = 1 \\
p^2 = 0 \quad p^0 > 0 & k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \\
p^2 < 0 \quad p^0 > 0 & k^\mu = \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \Lambda^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & R^i{}_i & \\ 0 & & & \end{pmatrix}
\end{cases}
$$

$R^T R = 1$


$U(R)|k,\sigma\rangle = \mathcal{D}_{\sigma'\sigma}(R)|k,\sigma'\rangle$
$(2j+1)\times(2j+1)$ matrix
$J_3|\sigma\rangle = \sigma|\sigma\rangle$
$J_\pm = (J_1 \pm i J_2)$
$\sigma = -j, -j+1, \ldots, j$
$[J^{\mu\nu}, J^{\lambda\rho}] = -i J^{\mu\lambda}\eta^{\nu\rho} \pm$ perm
$[J^{\mu\nu}, P^\lambda] = i\eta^{\mu\nu}P^\lambda - i\eta^{\mu\lambda}P^\nu$
$[P^\mu, P^\nu] = 0$
CAUTION


$U(R)|k,\sigma\rangle = \mathcal{D}(R)|k,\sigma'\rangle$
$(2j+1)\times(2j+1)$ matrix
$J_3|\sigma\rangle = \sigma|\sigma\rangle$
$J_\pm = (J_1 \pm iJ_2)$
$\mathcal{D}(R_1)\mathcal{D}(R_2) = \mathcal{D}(R_1R_2)$
$\sigma = -j, -j+1, \ldots, j-1, j$
$p^\mu = L^\mu{}_\nu k^\nu$
$p^2 = -m^2$
$|p,\sigma\rangle = N(p)\,U(L)|k,\sigma\rangle$
$[J^{\mu\nu}, J^{\lambda\rho}] = -i J^{\mu\lambda}\eta^{\nu\rho} \pm$ perm
$[J^{\mu\nu}, P^\lambda] = i\eta^{\mu\nu}P^\lambda - i\eta^{\mu\lambda}P^\nu$
$[P^\mu, P^\nu] = 0$

$$U(R)|k,\sigma\rangle = D^{j}_{\sigma\sigma'}(R)|k,\sigma'\rangle$$

$(2j+1)\times(2j+1)$ matrix

$$\boxed{\begin{array}{l} J_3|\sigma\rangle = \sigma|\sigma\rangle \\ J_\pm = (J_1 \pm iJ_2) \end{array}}$$

$\sigma = -j, -j+1, \ldots, j-1, j$

$$D(R_1)D(R_2) = D(R_1 R_2)$$

$$p^\mu = L^\mu{}_\nu k^\nu \qquad p^2 = -m^2$$

$$|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\, U(L)|k,\sigma\rangle$$

$$[J^{\mu\nu}, J^{\lambda\rho}] = -i J^{\mu\lambda}\eta^{\nu\rho} \pm \text{perm.} \qquad [J^{\mu\nu}, P^\lambda] = i\eta^{\mu\nu}P^\lambda - i\eta^{\mu\lambda}P^\nu$$

$$[P^\mu, P^\nu] = 0$$

$$U(\Lambda)\,|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\; U(\Lambda)\, U[L(p)]\, |k,\sigma\rangle$$

$$U^{-1}(L(\Lambda p))\, U(L(\Lambda p))$$

$$= \sqrt{\frac{E(p)}{m}}\; U(\Lambda)\, U(L(p))\, U^{-1}(L(\Lambda p))\, |\Lambda p,\sigma\rangle$$

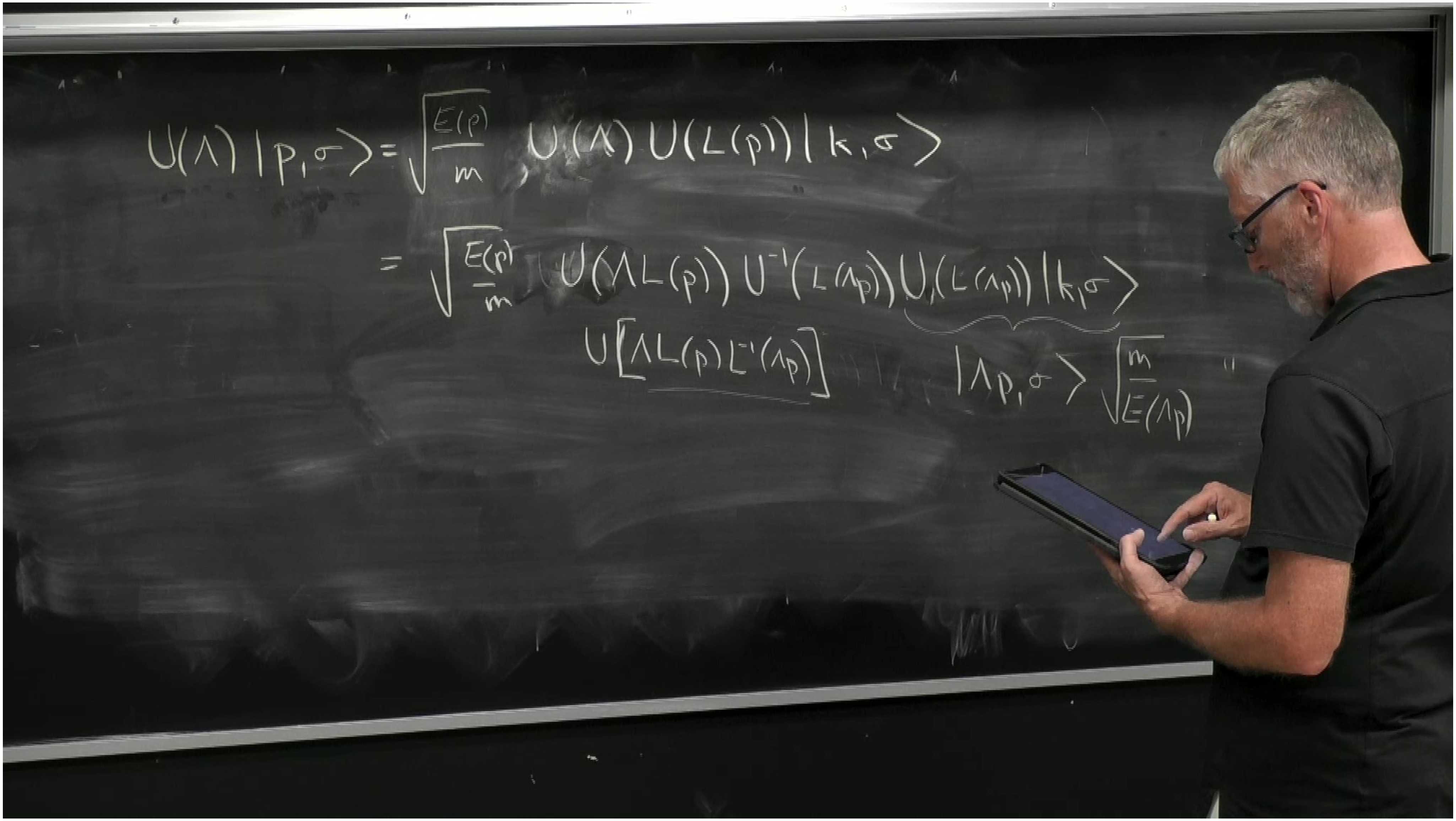
$$U(\Lambda L(p) L^{-1}(\Lambda p))$$

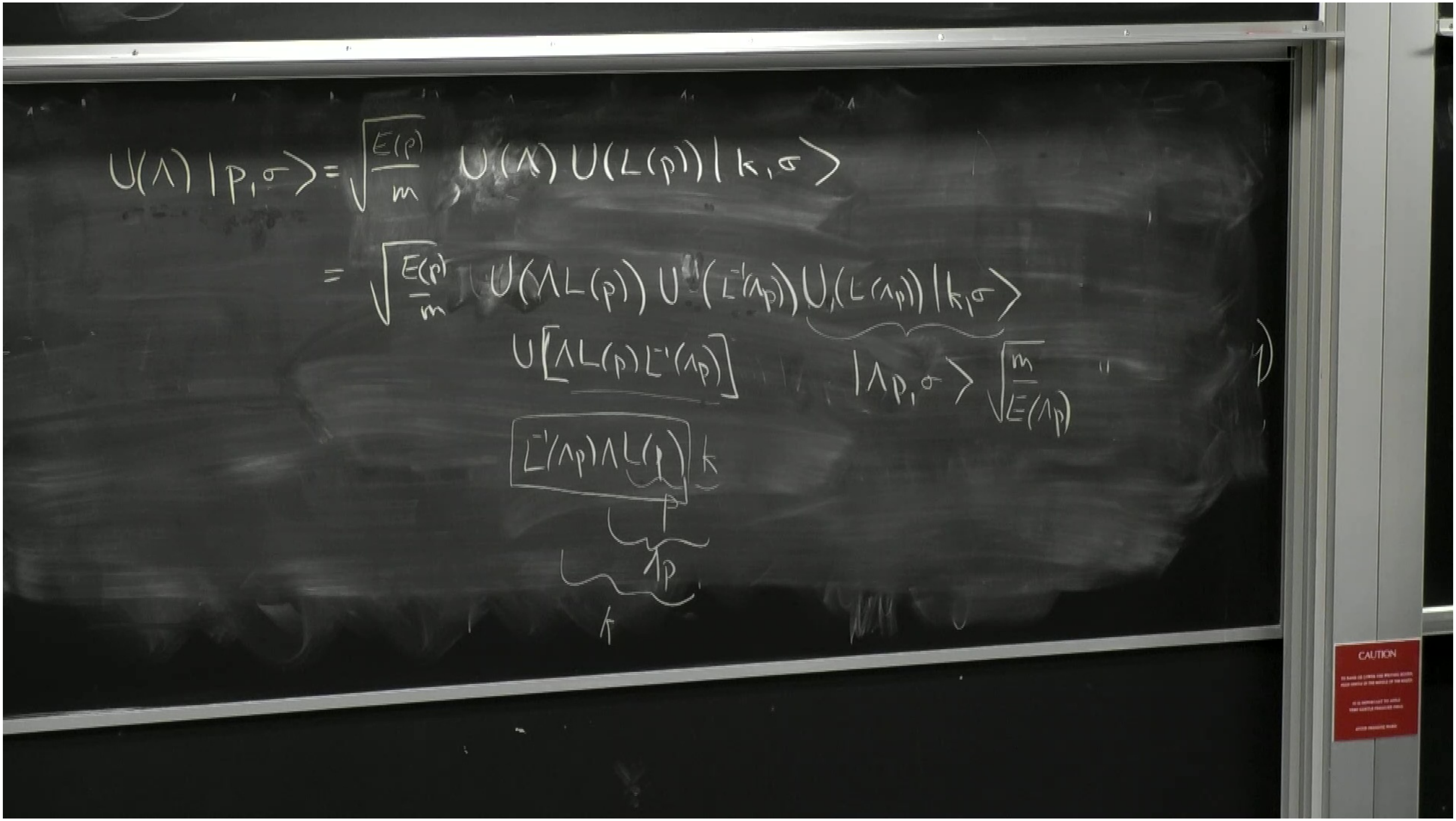
$$U(\Lambda)\,|p,\sigma\rangle = \sqrt{\frac{E(p)}{E(\Lambda p)}}\;\underbrace{U(L(\Lambda p))\;|k,\sigma\rangle}_{|\Lambda p,\sigma\rangle}$$

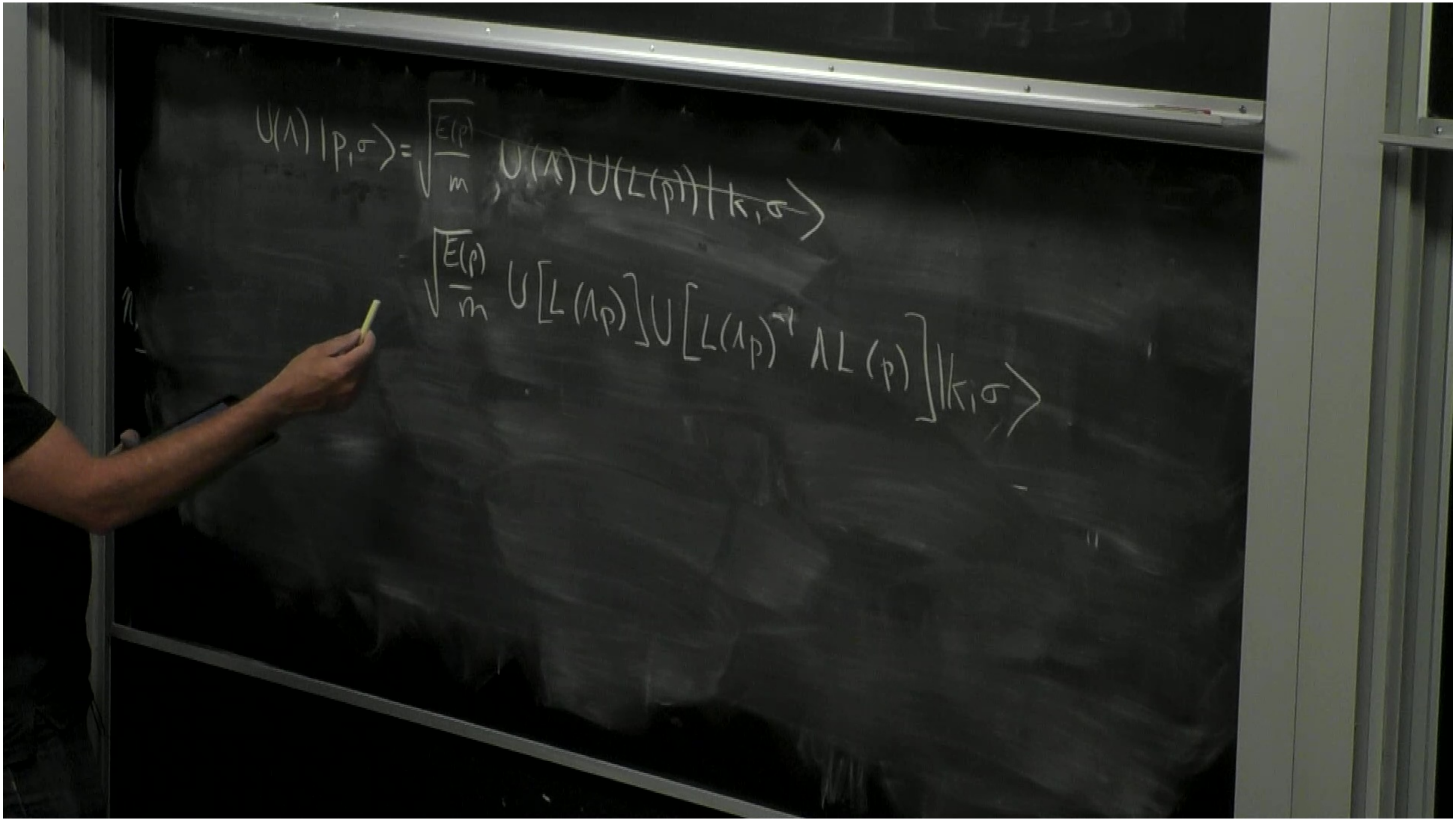
$$U^{-1}(L(p))\,U(L(p))$$

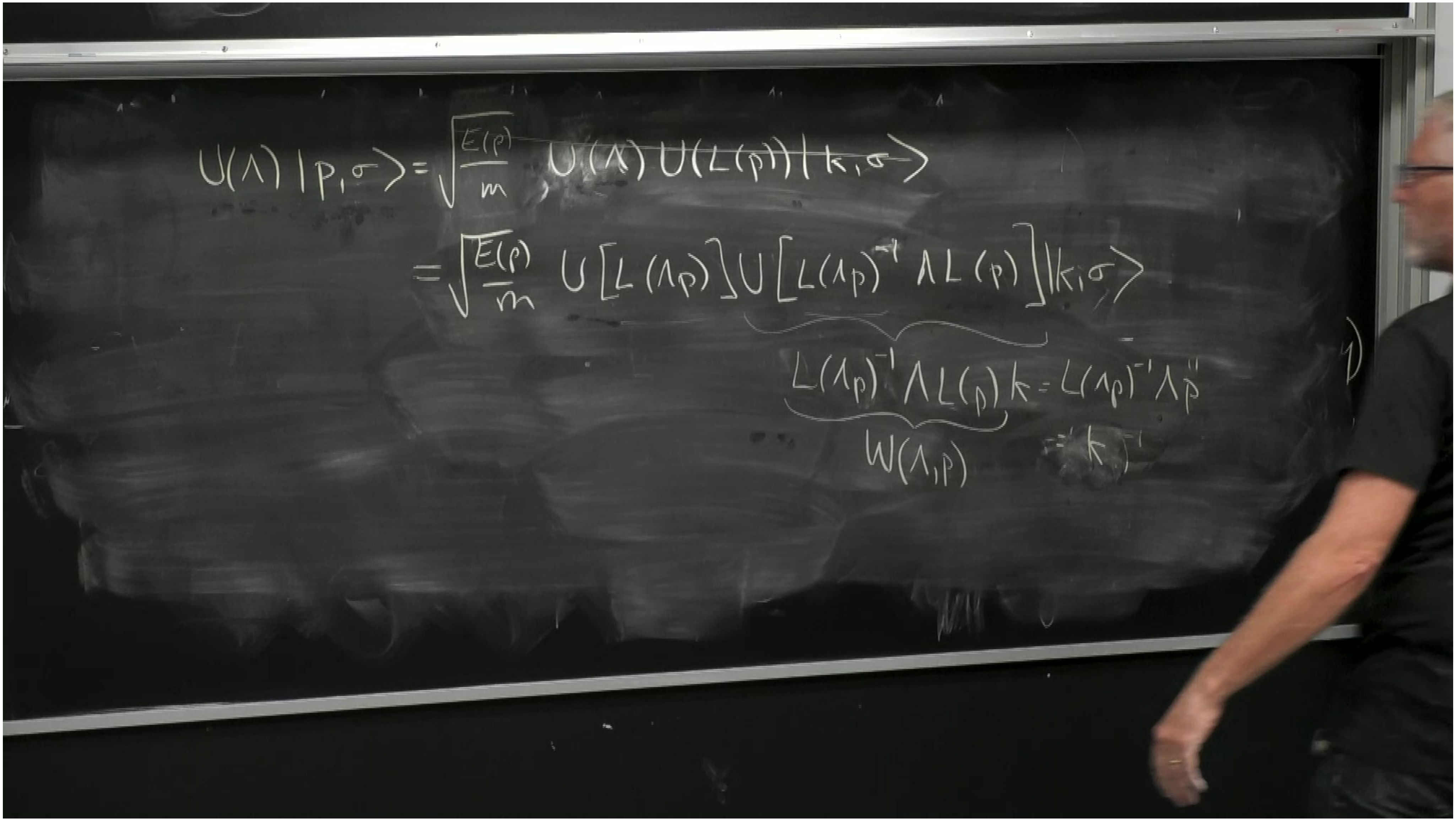
$$= \sqrt{\frac{E(p)}{m}}\;\;|\Lambda p,\sigma\rangle$$

$$U(\Lambda L(p) L^{-1}(\Lambda p))$$


$U(\Lambda)|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\, U(\Lambda)\, U(L(p))|k,\sigma\rangle$
$= \sqrt{\frac{E(p)}{m}}\, U(\Lambda L(p))\, U^{-1}(L(\Lambda p))\, \underbrace{U(L(\Lambda p))|k,\sigma\rangle}$
$U[\Lambda L(p) L^{-1}(\Lambda p)]$
$|\Lambda p,\sigma\rangle \sqrt{\frac{m}{E(\Lambda p)}}$


$U(\Lambda)|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\, U(\Lambda)\, U(L(p))\, |k,\sigma\rangle$
$= \sqrt{\frac{E(p)}{m}}\, U(\Lambda L(p))\, U(L^{-1}(\Lambda p))\, \underbrace{U(L(\Lambda p))\,|k,\sigma\rangle}_{|\Lambda p,\sigma\rangle \sqrt{\frac{m}{E(\Lambda p)}}}$
$U[\Lambda L(p) L^{-1}(\Lambda p)]$
$\boxed{L^{-1}(\Lambda p)\Lambda L(p)}\, k$


$U(\Lambda)|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\, U(\Lambda)\, U(L(p))\, |k,\sigma\rangle$
$= \sqrt{\frac{E(p)}{m}}\, U[L(\Lambda p)]\, U[L(\Lambda p)^{-1}\Lambda L(p)]\, |k,\sigma\rangle$

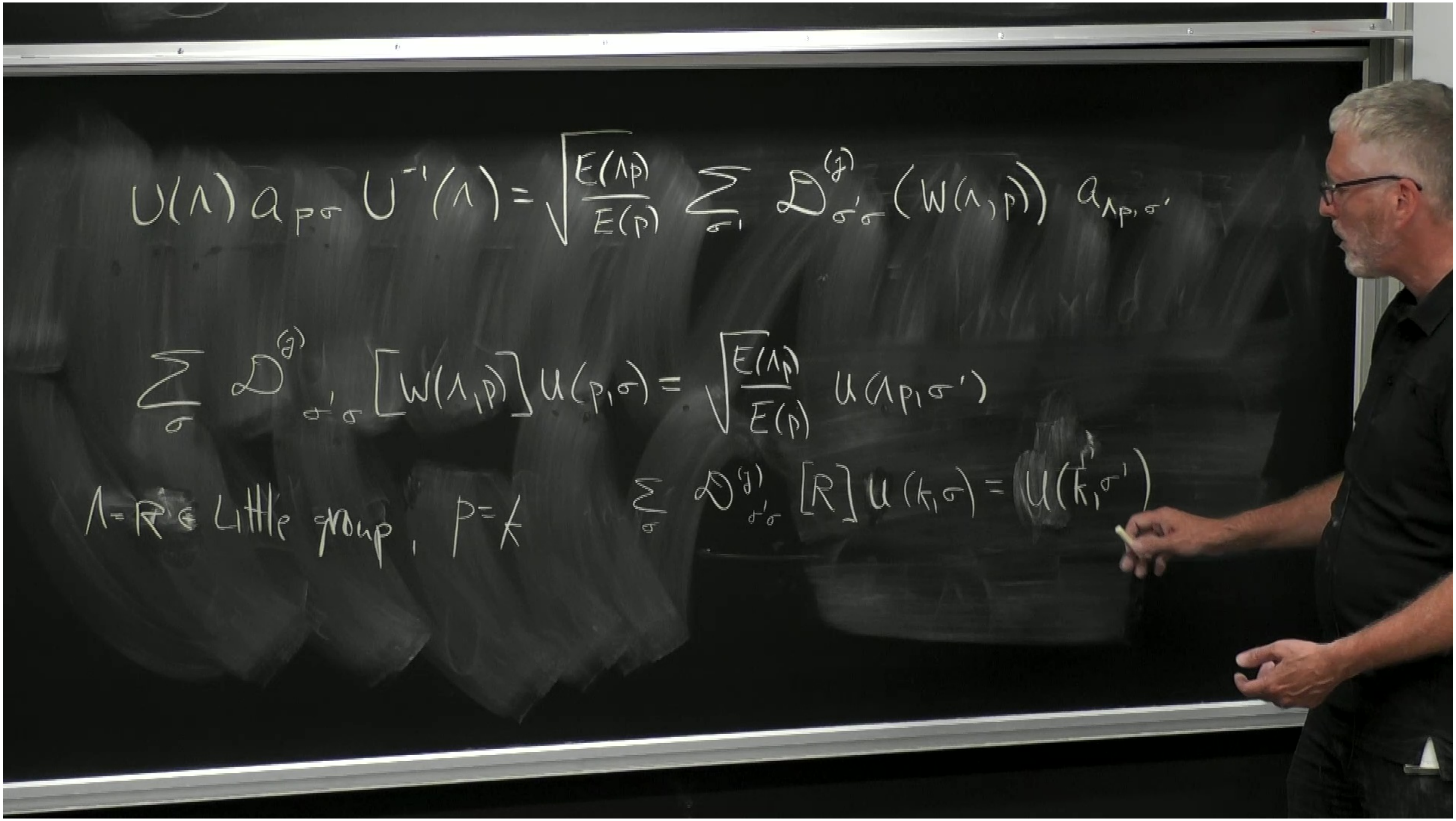

$U(\Lambda)|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\,\cancel{U(\Lambda)\,U(L(p))|k,\sigma\rangle}$
$= \sqrt{\frac{E(p)}{m}}\, U[L(\Lambda p)]\, U[\underbrace{L(\Lambda p)^{-1}\Lambda L(p)}]|k,\sigma\rangle$
$\underbrace{L(\Lambda p)^{-1}\Lambda L(p)}_{W(\Lambda,p)}\, k = L(\Lambda p)^{-1}\Lambda p$
$= k$

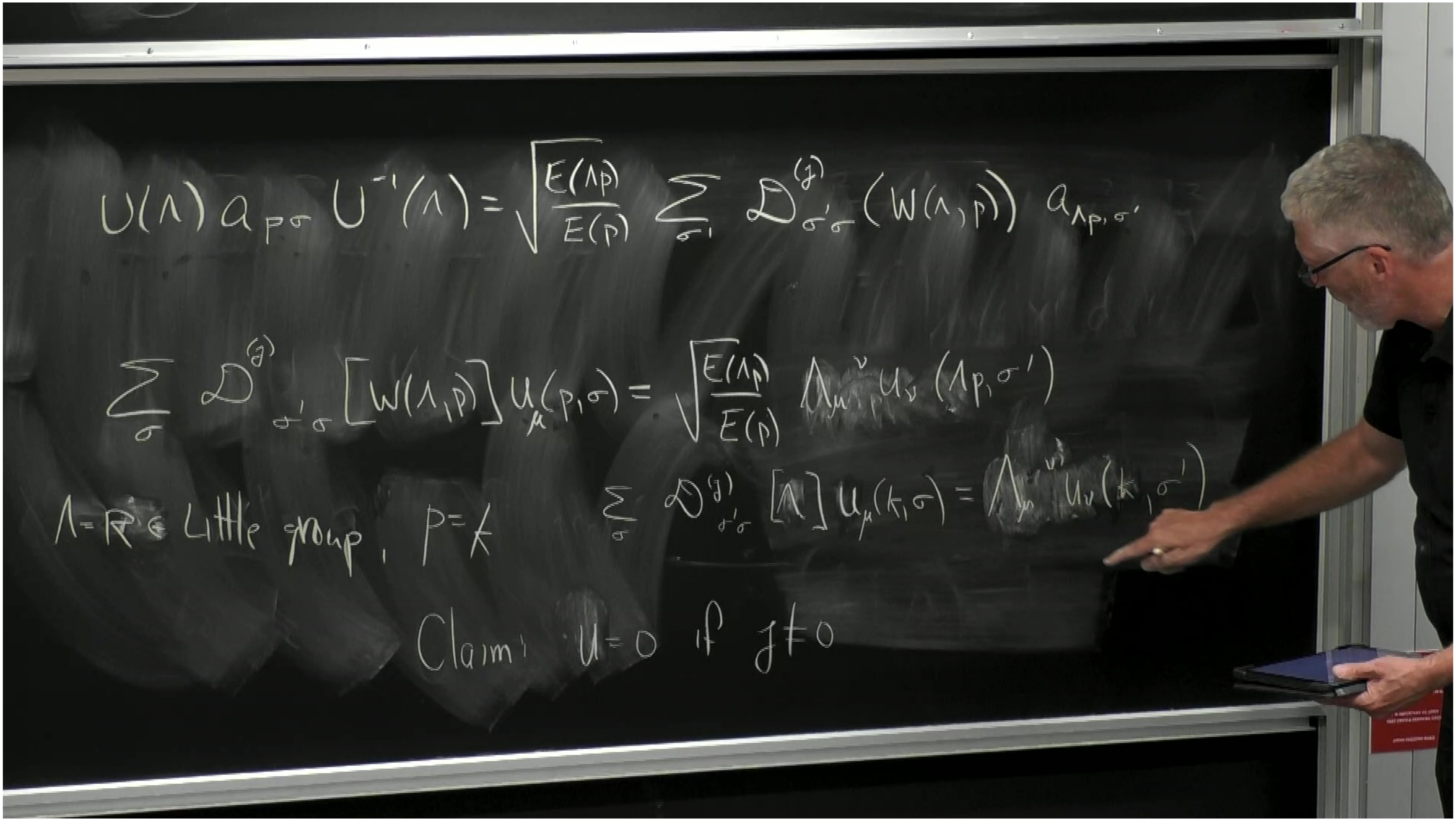
$$U(\Lambda)|p,\sigma\rangle = \sqrt{\frac{E(p)}{m}}\,\cancel{U(\Lambda)\,U(L(p))\,|k,\sigma\rangle}$$

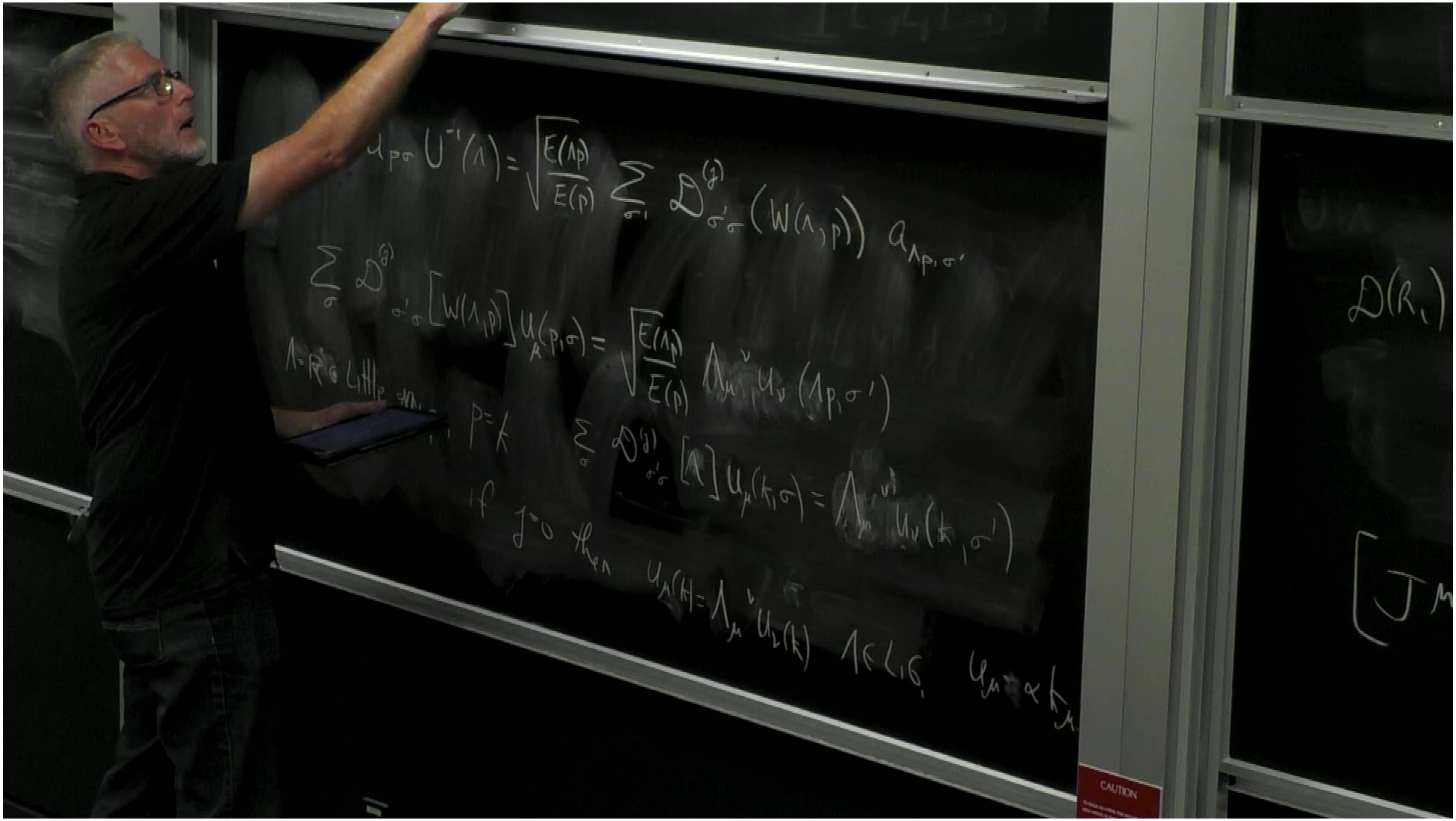
$$= \sqrt{\frac{E(p)}{m}}\, U[L(\Lambda p)]\,\underbrace{U[L(\Lambda p)^{-1}\Lambda L(p)]}_{\in \text{ Little group}}\,|k,\sigma\rangle$$

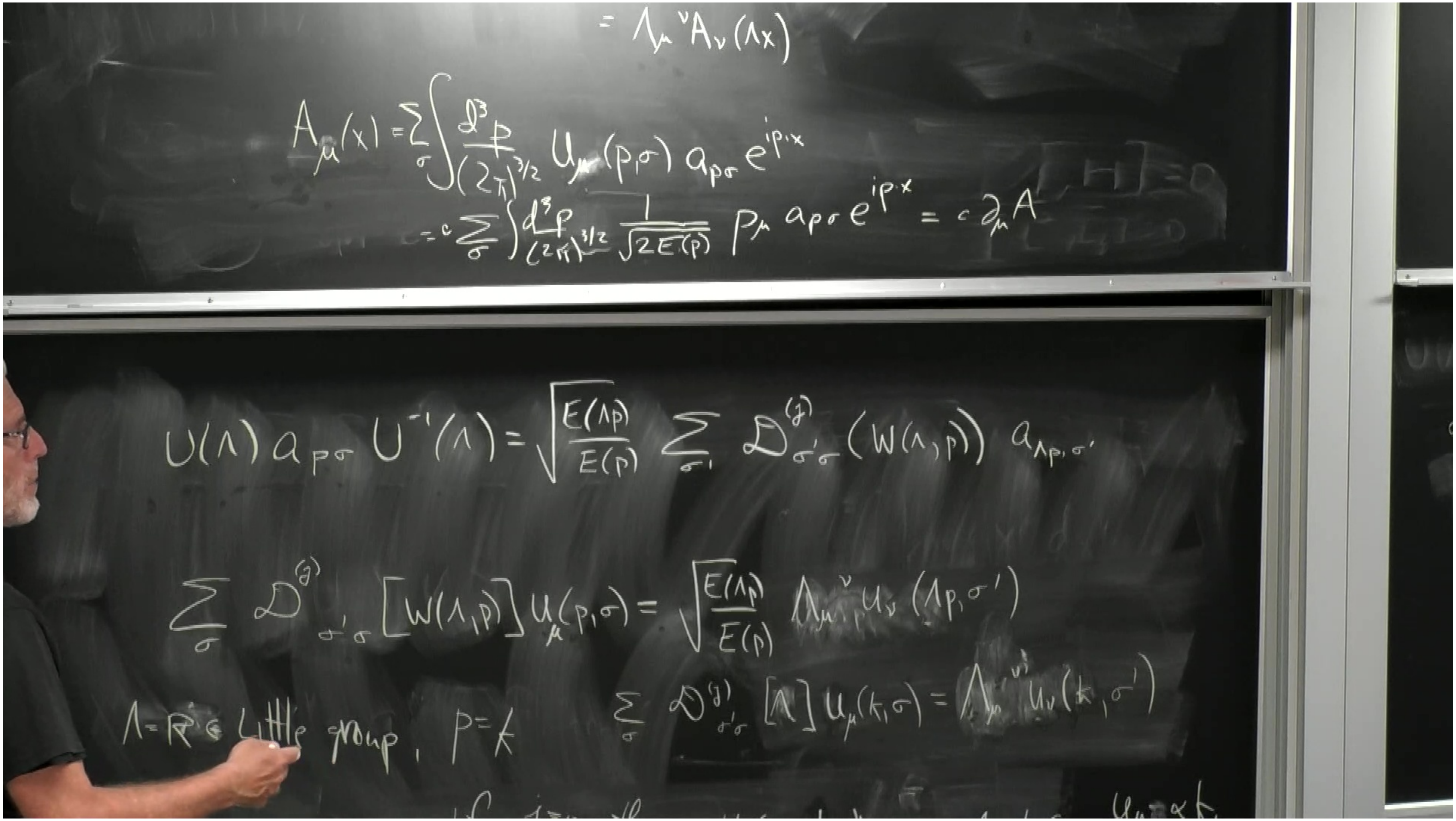
$$= \sqrt{\frac{E(p)}{m}} \sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(W)\,|\Lambda p,\sigma'\rangle$$

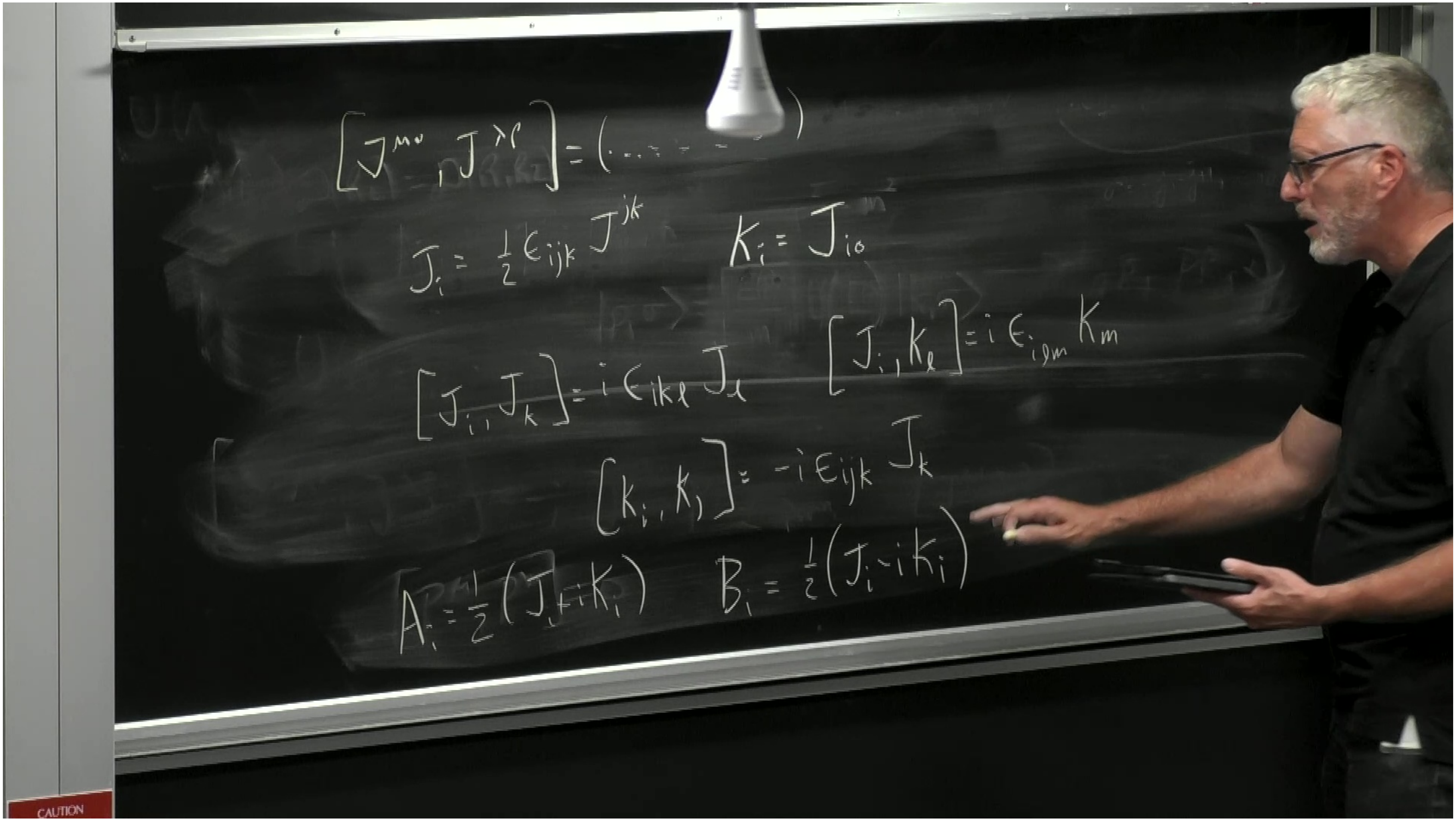
$$|p,\sigma\rangle = a^*_{p\sigma}|0\rangle \qquad U(\Lambda)|0\rangle = |0\rangle$$


$U(\Lambda)\, a_{p\sigma}\, U^{-1}(\Lambda) = \sqrt{\frac{E(\Lambda p)}{E(p)}} \sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(W(\Lambda,p))\, a_{\Lambda p,\sigma'}$
$\sum_{\sigma} D^{(j)}_{\sigma'\sigma}[W(\Lambda,p)]\, u(p,\sigma) = \sqrt{\frac{E(\Lambda p)}{E(p)}}\, u(\Lambda p,\sigma')$
$\Lambda = R \in$ Little group, $p = k$
$\sum_{\sigma} D^{(j)}_{\sigma'\sigma}[R]\, u(k,\sigma) = u(k,\sigma')$


$U(\Lambda)\, a_{p\sigma}\, U^{-1}(\Lambda) = \sqrt{\frac{E(\Lambda p)}{E(p)}} \sum_{\sigma'} \mathcal{D}^{(j)}_{\sigma\sigma'}(W(\Lambda,p))\, a_{\Lambda p,\sigma'}$
$\sum_{\sigma} \mathcal{D}^{(j)}_{\sigma'\sigma}[W(\Lambda,p)]\, u_\mu(p,\sigma) = \sqrt{\frac{E(\Lambda p)}{E(p)}}\, \Lambda_\mu{}^\nu u_\nu(\Lambda p,\sigma')$
$\Lambda = R \in$ Little group, $p = k$
$\sum_{\sigma} \mathcal{D}^{(j)}_{\sigma'\sigma}[R]\, u_\mu(k,\sigma) = R_\mu{}^\nu u_\nu(k,\sigma')$
Claim: $u = 0$ if $j \neq 0$




$= \Lambda_\mu{}^\nu A_\nu(\Lambda x)$
$A_\mu(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} u_\mu(p,\sigma)\, a_{p\sigma} e^{ip\cdot x}$
$= c \sum_\sigma \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(p)}} p_\mu a_{p\sigma} e^{ip\cdot x} = c\,\partial_\mu A$
$U(\Lambda)\, a_{p\sigma} U^{-1}(\Lambda) = \sqrt{\frac{E(\Lambda p)}{E(p)}} \sum_{\sigma'} D^{(j)}_{\sigma'\sigma}(W(\Lambda,p))\, a_{\Lambda p,\sigma'}$
$\sum_\sigma D^{(j)}_{\sigma'\sigma}[W(\Lambda,p)]\, u_\mu(p,\sigma) = \sqrt{\frac{E(\Lambda p)}{E(p)}} \Lambda_\mu{}^\nu u_\nu(\Lambda p,\sigma')$
$\Lambda = R \in$ little group, $p = k$
$\sum_\sigma D^{(j)}_{\sigma'\sigma}[\Lambda]\, u_\mu(k,\sigma) = \Lambda_\mu{}^\nu u_\nu(k,\sigma')$


$[J^{\mu\nu}, J^{\lambda\rho}] = (\ldots)$
$J_i = \frac{1}{2}\epsilon_{ijk} J^{jk}$
$K_i = J_{i0}$
$[J_i, J_k] = i\epsilon_{ik\ell} J_\ell$
$[J_i, K_\ell] = i\epsilon_{i\ell m} K_m$
$[K_i, K_j] = -i\epsilon_{ijk} J_k$
$A_i = \frac{1}{2}(J_i + iK_i)$
$B_i = \frac{1}{2}(J_i - iK_i)$
CAUTION

$$[J^{\mu\nu}, J^{\lambda\rho}] = (\ldots$$

$$J_i = \frac{1}{2}\epsilon_{ijk}J^{jk} \qquad K_i = J_{i0}$$

$$[J_i, J_k] = i\epsilon_{ikl}J_l \qquad [J_i, K_l] = i\epsilon_{ilm}K_m$$

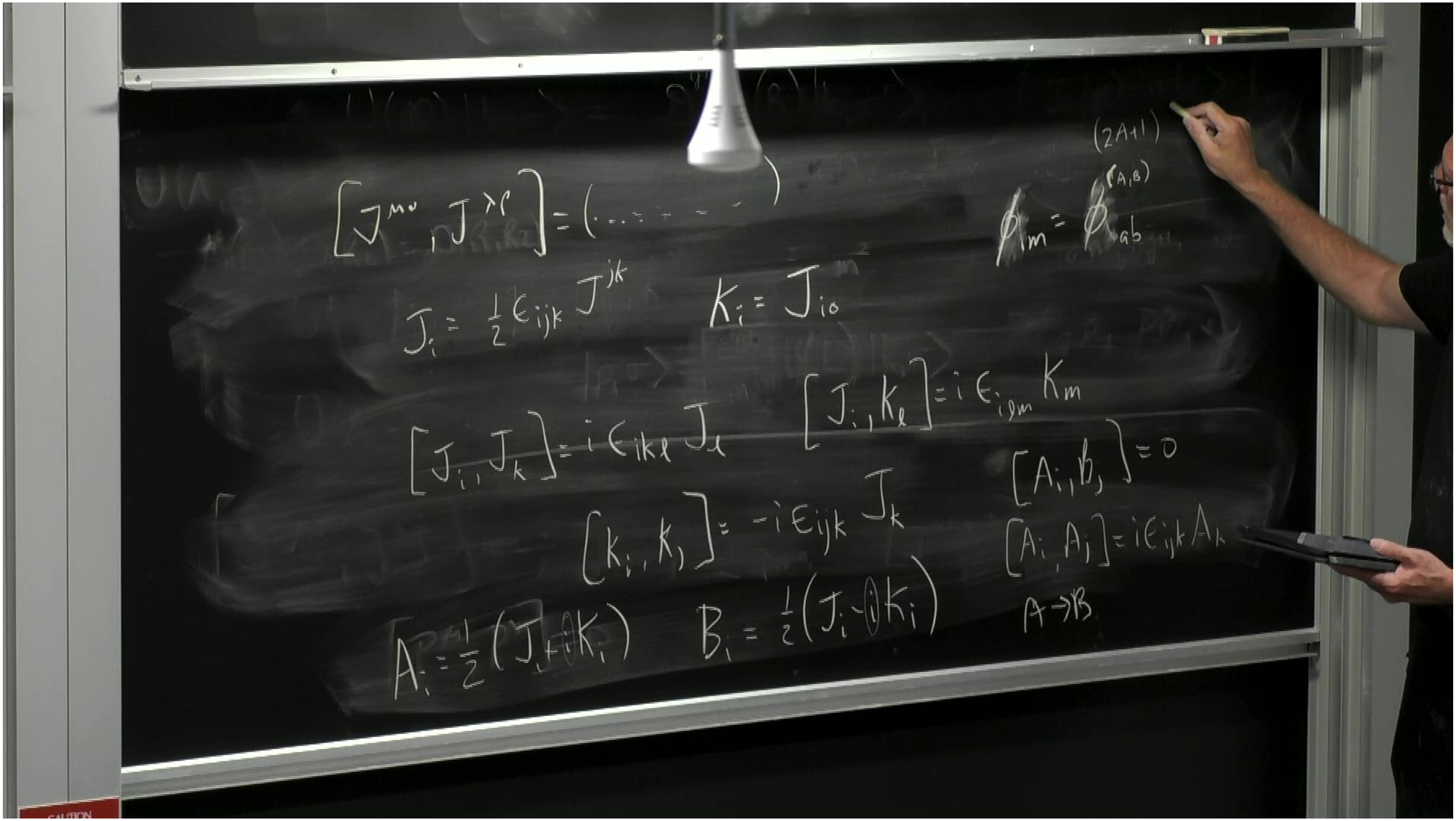
$$[K_i, K_j] = -i\epsilon_{ijk}J_k$$

$$[A_i, B_j] = 0$$

$$[A_i, A_j] = i\epsilon_{ijk}A_k$$

$$A_i = \frac{1}{2}(J_i + iK_i) \qquad B_i = \frac{1}{2}(J_i - iK_i)$$

$$A \to B$$


$[J^{\mu\nu}, J^{\lambda\rho}] = (\ldots)$
$(2A+1)$
$\phi_m = \phi^{(A,B)}_{ab}$
$J_i = \frac{1}{2}\epsilon_{ijk} J^{jk}$
$K_i = J_{i0}$
$[J_i, K_l] = i\epsilon_{ilm} K_m$
$[J_i, J_k] = i\epsilon_{ikl} J_l$
$[A_i, B_j] = 0$
$[K_i, K_j] = -i\epsilon_{ijk} J_k$
$[A_i, A_j] = i\epsilon_{ijk} A_k$
$A_i = \frac{1}{2}(J_i + iK_i)$
$B_i = \frac{1}{2}(J_i - iK_i)$
$A \to B$

$$[J^{\mu\nu}, J^{\lambda\rho}] = (\ldots)$$

$$\dim = (2A+1)(2B+1)$$

$$\phi_m = \phi_{ab}^{(A,B)}$$

$$J_i = \frac{1}{2}\epsilon_{ijk} J^{jk} \qquad K_i = J_{i0}$$

$$[J_i, K_l] = i\epsilon_{ilm} K_m$$

$$[J_i, J_k] = i\epsilon_{ikl} J_l$$

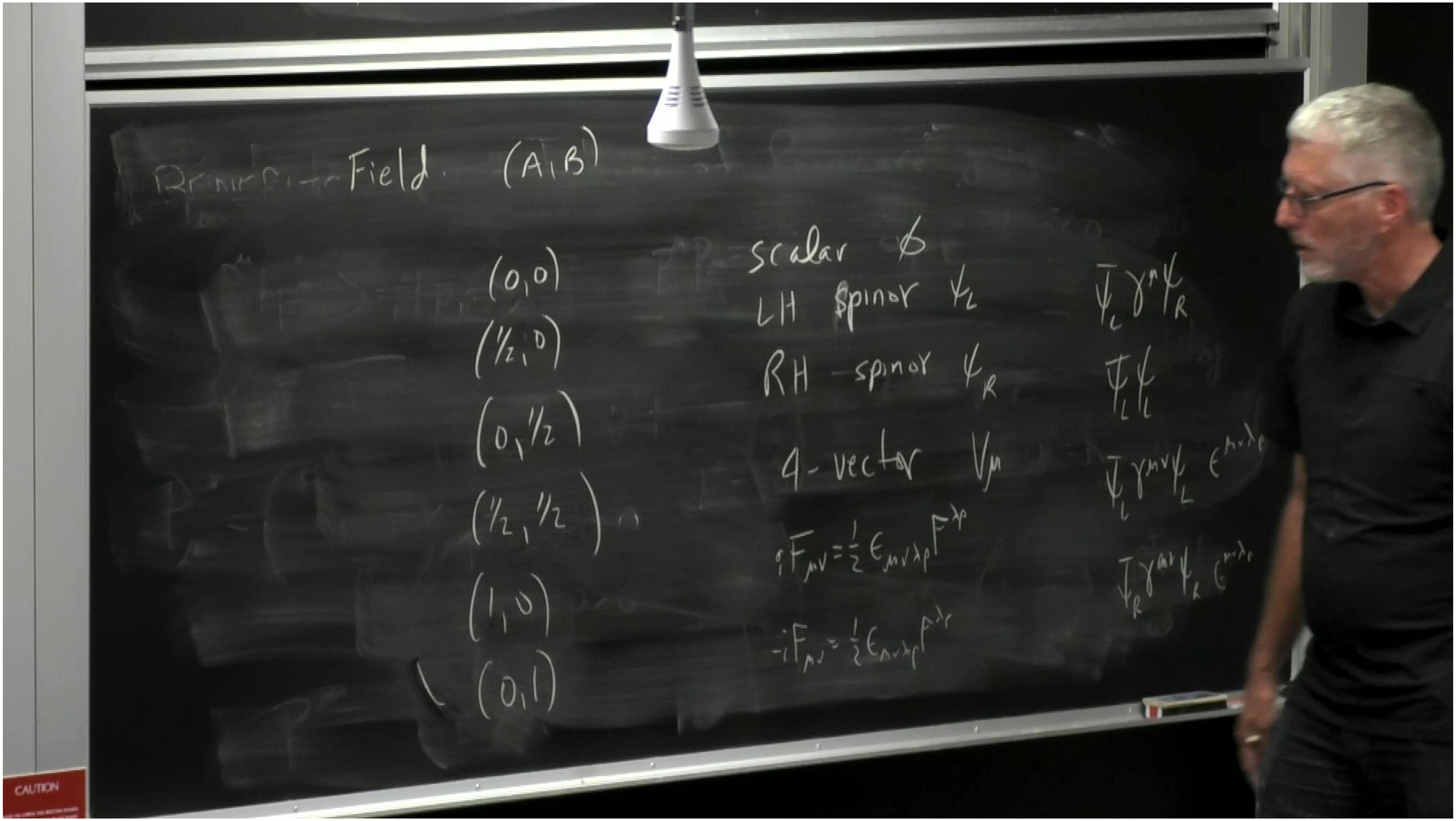
$$[A_i, B_j] = 0$$

$$[K_i, K_j] = -i\epsilon_{ijk} J_k$$

$$[A_i, A_j] = i\epsilon_{ijk} A_k$$

$$A_i = \frac{1}{2}(J_i + iK_i) \qquad B_i = \frac{1}{2}(J_i - iK_i)$$

$$A \to B$$


Field (A,B)
(0,0) scalar ϕ
(1/2,0) LH spinor ψ_L
(0,1/2) RH spinor ψ_R
(1/2,1/2) 4-vector V_μ
(1,0) $iF_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$
(0,1) $-iF_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$
$\bar{\psi}_L\gamma^\mu\psi_R$
$\bar{\psi}_L\psi_L$
$\bar{\psi}_L\gamma^{\mu\nu}\psi_L\,\epsilon^{\mu\nu\lambda\rho}$
$\bar{\psi}_R\gamma^{\mu\nu}\psi_R\,\epsilon^{\mu\nu\lambda\rho}$

Massive fields j

$\psi \in (A,B)$ can represent spin j massive particle

iff $j = |A-B|, |A-B|+1, \ldots, A+B$

massless fields

Massive fields $\quad j$

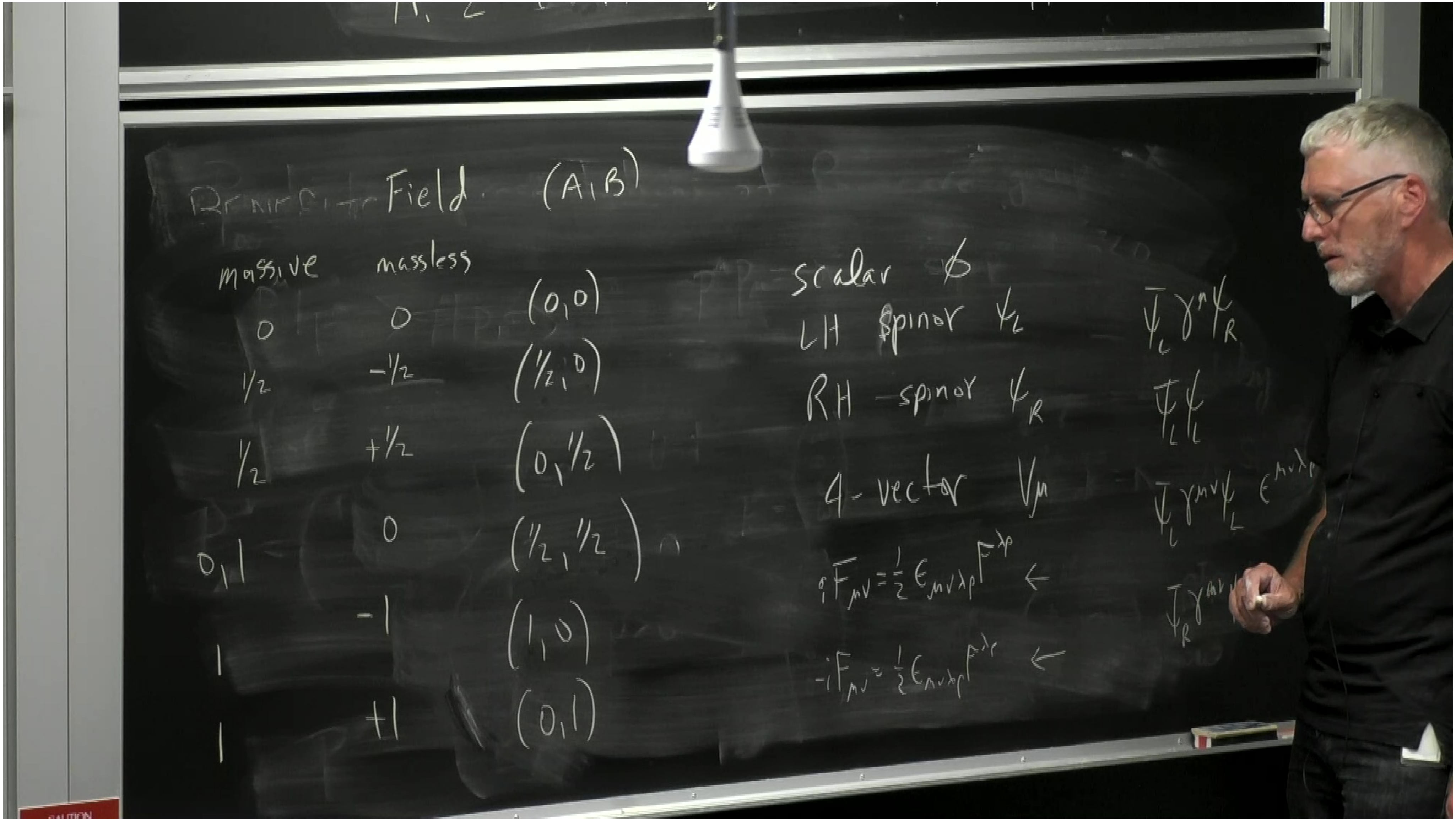
$\psi \in (A,B)$ can represent spin j massive particle

iff $\quad j = |A-B|, |A-B|+1, \ldots, A+B$

massless fields

$\psi \in (A,B)$ " " " massless " " " λ

iff $\lambda = B-A$


Field (A,B)
massive massless
0 0 (0,0)
1/2 -1/2 (1/2,0)
1/2 +1/2 (0,1/2)
0,1 0 (1/2,1/2)
1 -1 (1,0)
1 +1 (0,1)
scalar φ
LH spinor ψ_L
RH spinor ψ_R
4-vector V_μ
iF_μν = 1/2 ε_μνλρ F^λρ
-iF_μν = 1/2 ε_μνλρ F^λρ

$$\left[D(\Lambda_1^{-1})\, D(\Lambda_2^{-1})\right]_{mp} A_p \qquad D_{np}(\Lambda_2^{-1})\, A_p$$

$$\boxed{S(a,b)}\, R(\theta)$$

$$k^\mu = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} \qquad R = \begin{pmatrix} 1 & & & \\ & \cos\theta & \sin\theta & \\ & -\sin\theta & \cos\theta & \\ & & & 1 \end{pmatrix}$$

$$\mathcal{D}(R)\,|k,\lambda\rangle = e^{i\lambda\theta}\,|k,\lambda\rangle$$

helicity

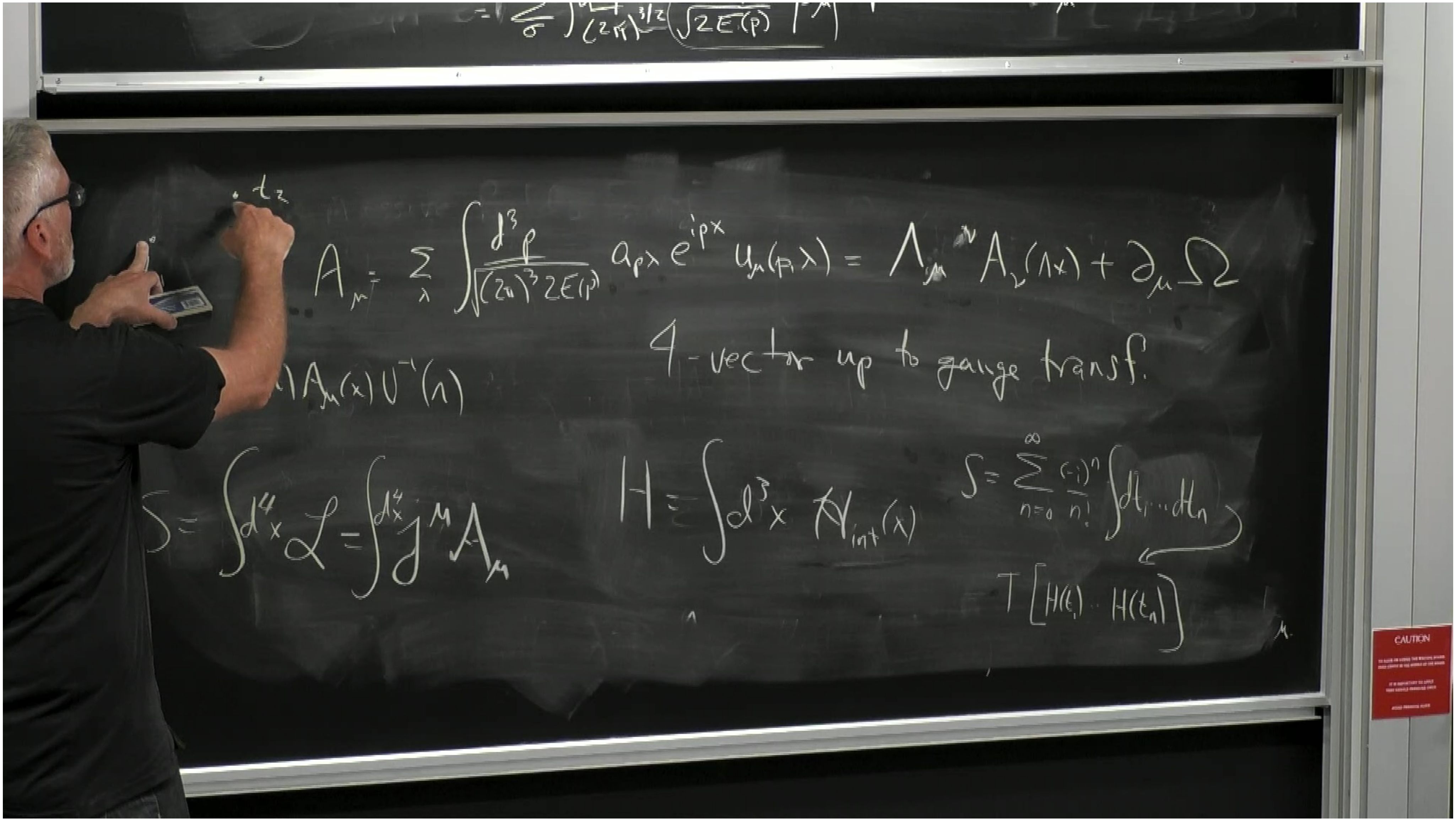
$$F_{\mu\nu} = \sum_\lambda \int \frac{d^3p}{(2\pi)^{3/2}\sqrt{2E(p)}} \left[a_{p,\lambda}\, e^{ipx} (p_\mu u_\nu - u_\mu p_\nu) \right] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$A_\mu = \sum_\lambda \int \frac{d^3p}{\sqrt{(2\pi)^3 2E(p)}}\, a_{p\lambda}\, e^{ipx}\, u_\mu(p,\lambda) = \Lambda_\mu{}^\nu A_\nu(\Lambda x) + \partial_\mu \Omega$$

4-vector up to gauge transf.

$$U(\Lambda) A_\mu(x) U^{-1}(\Lambda)$$

$$S = \int d^4x\, \mathcal{L} = \int d^4x\, J^\mu A_\mu$$



$A_\mu = \sum_\lambda \int \frac{d^3p}{\sqrt{(2\pi)^3 2E(p)}} a_{p\lambda} e^{ipx} u_\mu(p,\lambda) = \Lambda_\mu{}^\nu A_\nu(\Lambda x) + \partial_\mu \Omega$
4-vector up to gauge transf.
$A_\mu(x) U^{-1}(\Lambda)$
$S = \int d^4x \, \mathcal{L} = \int d^4x \, j^\mu A_\mu$
$H = \int d^3x \, \mathcal{H}_{int}(x)$
$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dt_1 \ldots dt_n$
$T\left[H(t_1) \ldots H(t_n)\right]$
$\frac{}{\sigma} \int \frac{}{(2\pi)^{3/2}} \sqrt{2E(p)}$