Title: QFT2 - Quantum Electrodynamics - Morning Lecture

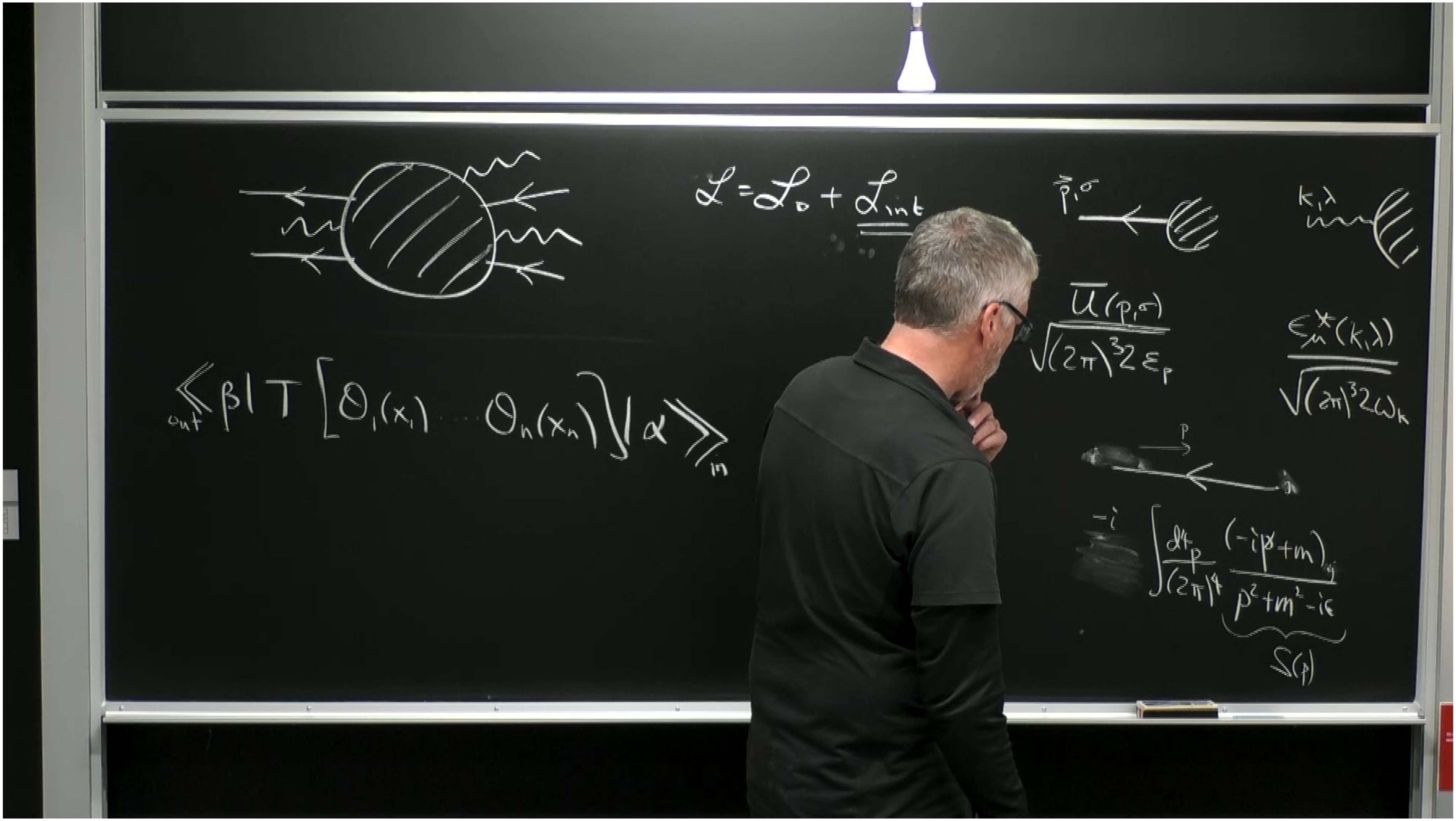
Speakers:

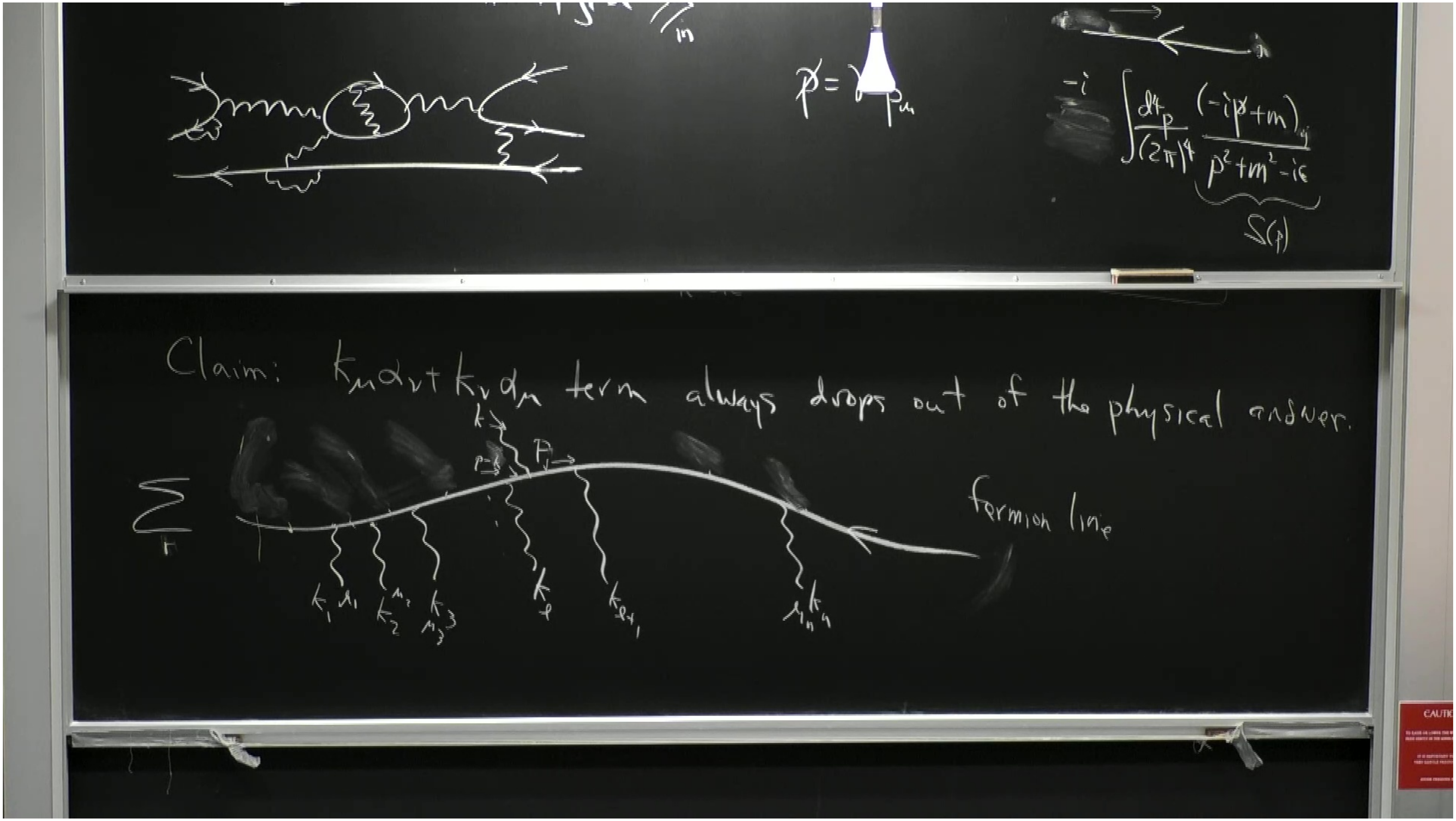
Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: November 29, 2022 - 10:00 AM

URL: https://pirsa.org/22110037

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$\vec{p}, \sigma$
k, λ
$\frac{\bar{u}(p,\sigma)}{\sqrt{(2\pi)^3 2E_p}}$
$\frac{\epsilon^*_\mu(k,\lambda)}{\sqrt{(2\pi)^3 2\omega_k}}$
${}_{out}\langle\langle \beta | T[O_1(x_1) \cdots O_n(x_n)] | \alpha \rangle\rangle_{in}$
p
$-i \int \frac{d^4p}{(2\pi)^4} \underbrace{\frac{(-i\not{p}+m)_{ij}}{p^2+m^2-i\epsilon}}_{S(p)}$


$\not{p} = \gamma^\mu p_\mu$
$-i \int \frac{d^4p}{(2\pi)^4} \underbrace{\frac{(-i\not{p}+m)_{ij}}{p^2+m^2-i\epsilon}}_{S(p)}$
Claim: $k_\mu \alpha_\nu + k_\nu \alpha_\mu$ term always drops out of the physical answer.
$\sum_{\pi}$
fermion line
k_1 μ_1 k_2 μ_2 k_3 μ_3 k_ℓ $k_{\ell+1}$ k_n μ_n
$p-k$ p k

$$\boxed{\text{possible external line}}\left[\;\cdots\cdots\;\right]\frac{-i(\not p-\not k)+m}{(p-k)^2+m^2-i\epsilon}\,(e\not k)\,\frac{-i\not p+m}{p^2+m^2-i\epsilon}\left[\;\cdots\;\right]\left[\text{possible ext. line}\right]$$

$$-i\not k=\left[+i(\not p-\not k)+m\right]-\left[+i\not p+m\right]$$

$$\left[i\not p+m\right]\left[-i\not p+m\right]=m^2+(\not p)^2$$

$$=p^2+m^2$$

$$\not p^2=p^\mu p^\nu\gamma_\mu\gamma_\nu=p^2$$

possible external line $\left[\;\cdots\cdots\;\right]$ $\dfrac{-i(\not p-\not k)+m}{(p-k)^2+m^2-i\epsilon}\,(e\not k)\,\dfrac{-i\not p+m}{p^2+m^2-i\epsilon}$ $\left[\;\cdots\;\right]$ possible ext. line

$$-i\not k = \left[+i(\not p-\not k)+m\right]-\left[+i\not p+m\right]$$

$$\boxed{\not k = i\,S^{-1}(p-k) - i\,S^{-1}(p)}$$

(Ward Identity)

$$\langle \psi J^\mu \psi\rangle \propto \bar u(p')\Gamma^\mu(p',p)u(p) \qquad (p'_\mu - p_\mu)\Gamma^\mu(p',p) = (\ )\left[S^{-1}(p') - S^{-1}(p)\right]$$

$$\Big[\;\cdots\cdots\;\Big]\Big\{ S(p)\,\not{k}\,S(p+k)\,\not{K}_{l+1}\,S(p+k-k_{l+1})$$

$$+\,S(p)\,\not{K}_{l+1}\,S(p-k_{l+1})\,\not{k}\,S(p-k_{l+1}+k)\Big\}\Big[\;\cdots\cdots\;\Big]$$

$$\propto\; S(p+k)\,\not{K}_{l+1}\,S(p+k-k_{l+1}) - S(p)\,\cancel{\not{K}_{l+1}\,S(p+k-k_{l+1})}$$

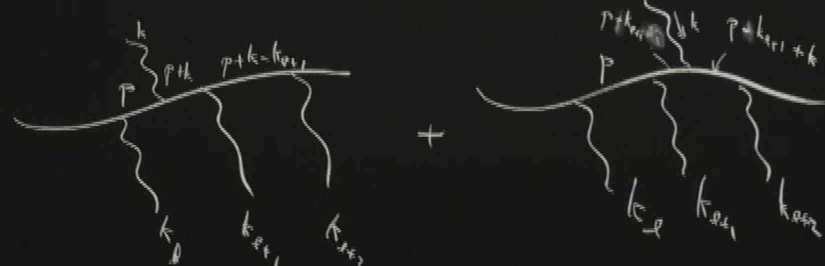
$$+\,\cancel{S(p)\,\not{K}_{l+1}}\,S(p-k_{l+1}+k) - S(p)\,\not{K}_{l+1}\,S(p-k_{l+1})\Big\}$$

(Ward Identity)

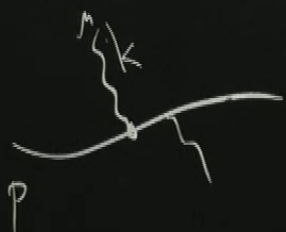
$$\langle \psi J^\mu \psi \rangle \propto \bar{u}(p')\Gamma^\mu(p',p)\,u(p) \qquad (p'_\mu - p_\mu)\Gamma^\mu(p',p) = (\;)\left[S^{-1}(p') - S^{-1}(p)\right]$$

$$\boxed{S(p-k)\not{k}\,S(p)} = iS(p) - iS(p-k)$$

Sum over insertion point



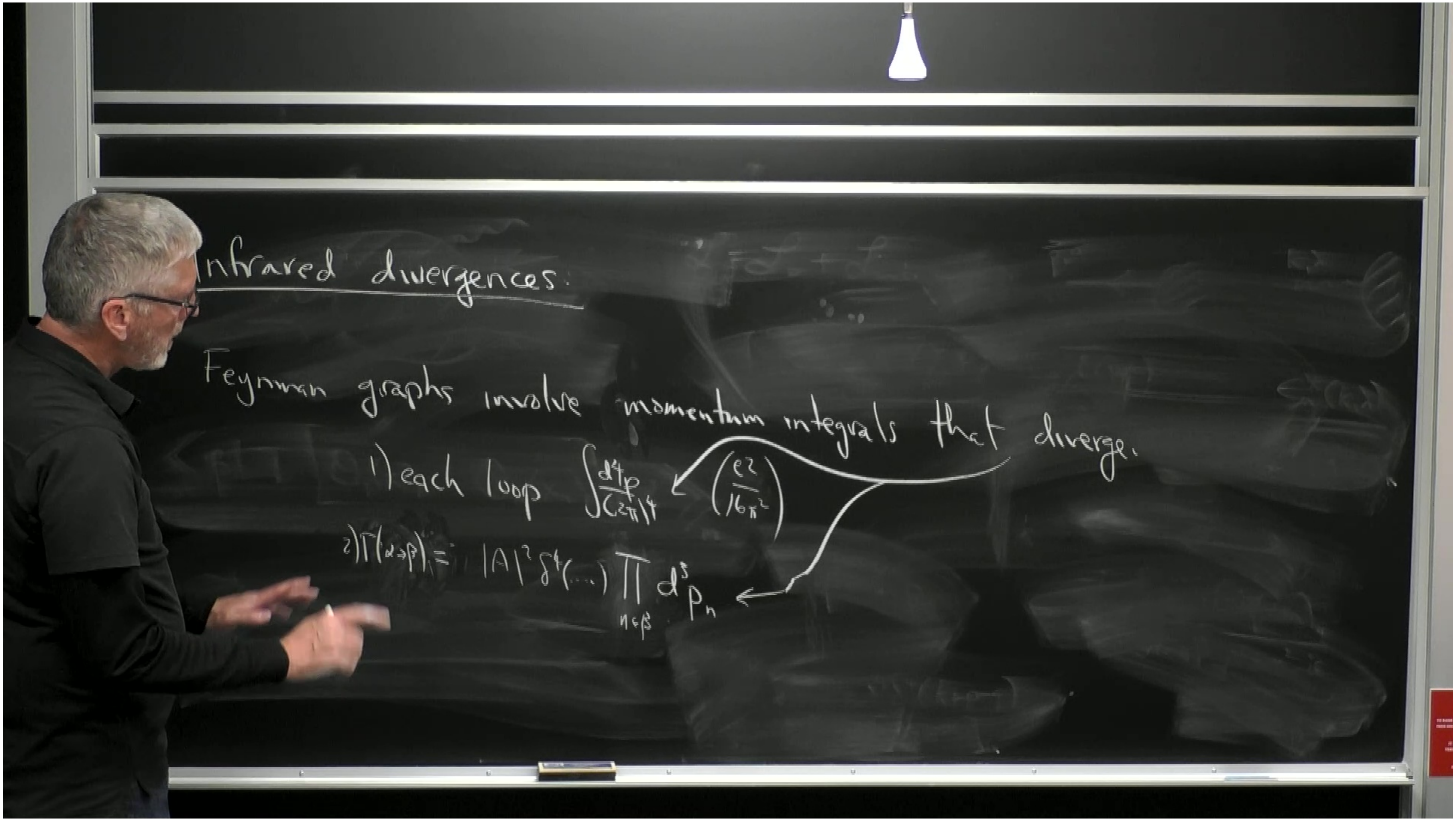
on far left $\quad \bar{u}(p)\not{k}\left[\cdots\right] = \boxed{\bar{u}(p)(i\not{p}+m)}\left[\cdots\right]$ + stuff that cancels

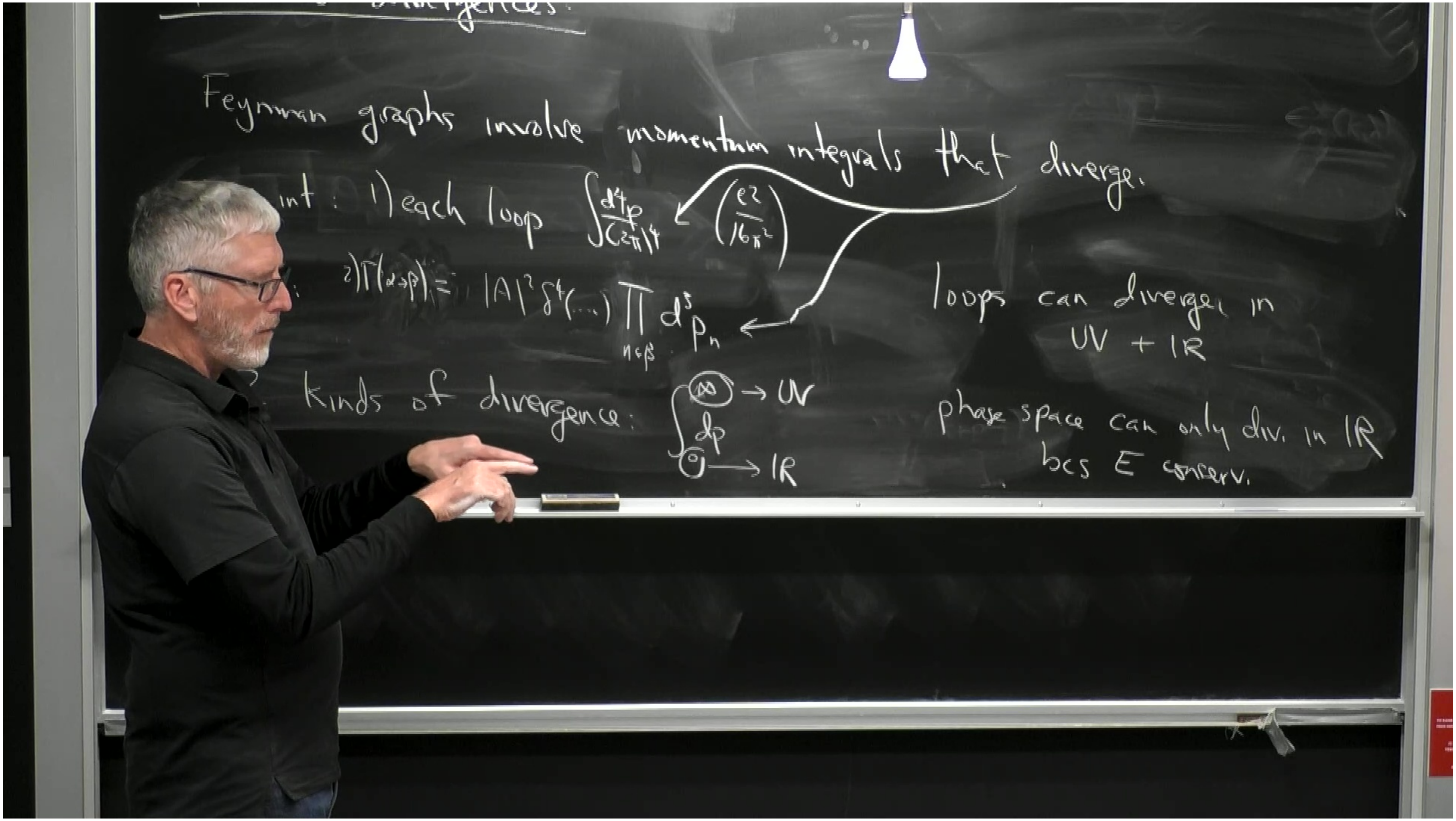


$$(\not{\partial}+m)\psi = 0 \Leftrightarrow (i\not{p}+m)u = 0$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \beta \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\left[\cdots\right]\not{k}\,u(p) = \text{stuff that cancels} + \underbrace{(i\not{p}+m)u}_{=0}$$


Infrared divergences:
Feynman graphs involve momentum integrals that diverge.
1) each loop $\int \frac{d^4p}{(2\pi)^4}$ $\left(\frac{e^2}{16\pi^2}\right)$
2) $\Gamma(\alpha\to\beta) = |A|^2 \delta^4(\dots) \prod_{n\in\beta} d^3p_n$


Feynman graphs involve momentum integrals that diverge.
int: 1) each loop $\int \frac{d^4p}{(2\pi)^4}$
$\left(\frac{e^2}{16\pi^2}\right)$
2) $\Gamma(\alpha\to\beta) = |A|^2 \delta^4(\ldots) \prod_{n\in\beta} d^3p_n$
kinds of divergence: $\int dp$
$\infty \to$ UV
$0 \to$ IR
loops can diverge in
UV + IR
phase space can only div. in IR
bcs E conserv.

2 kinds of divergence: $\int^{(\infty) \to UV}_{(0) \to IR} dp$

phase space can only div. in IR bcs E conserv.

<u>Moral</u>: UV divergences are all <u>local</u> + so can be absorbed into couplings ~~for~~ local interactions in $\mathcal{L}_{int}$

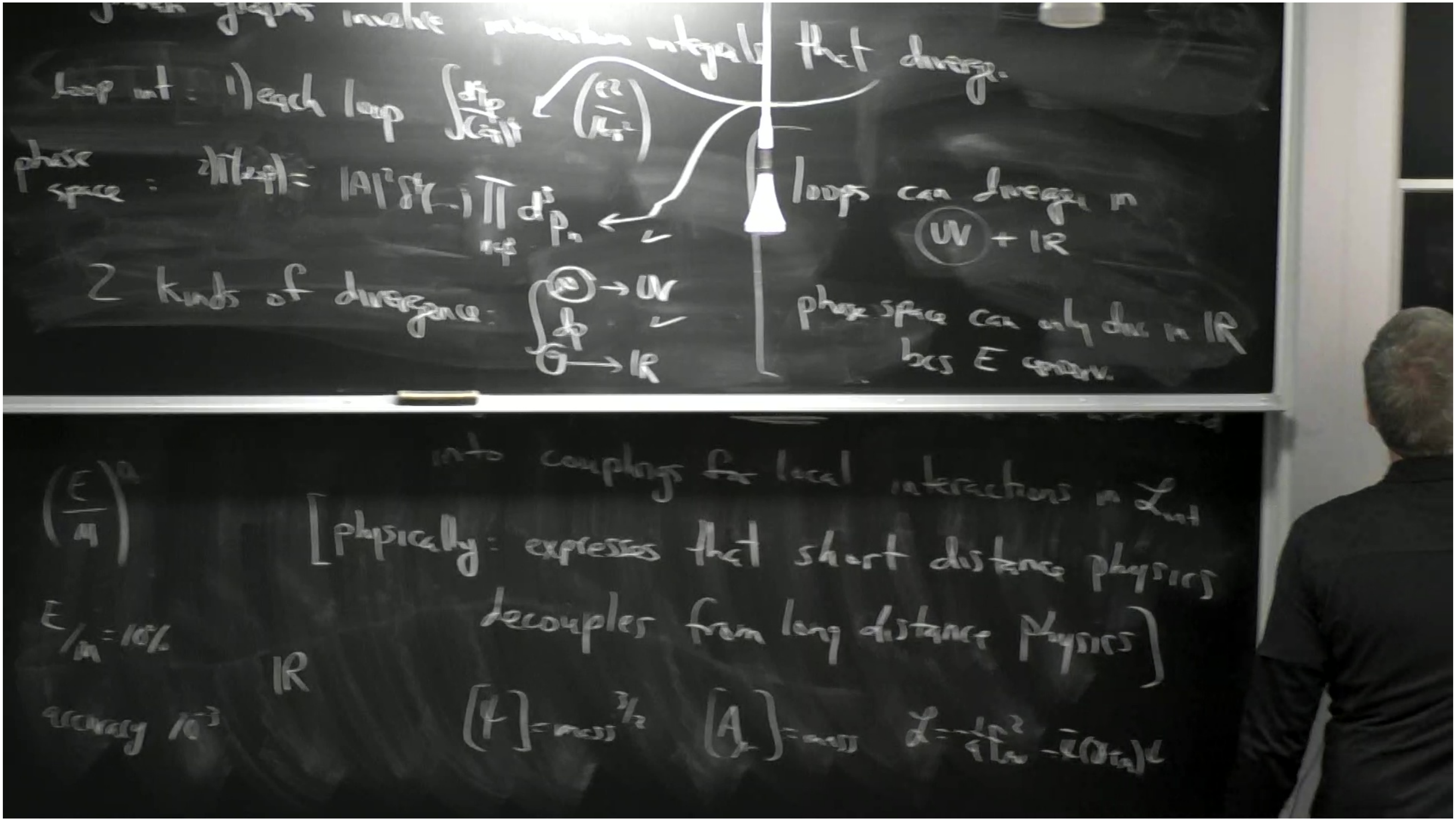
[physically: expresses that short distance physics decouples from long distance physics]

$\left(\frac{E}{M}\right)^n$

$E/M = 10\%$

IR

accuracy 10^{-3}



involve momentum integrals that diverge.
loop int: 1) each loop
phase space:
loops can diverge in
UV + IR
2 kinds of divergence:
→ UV
→ IR
phase space can only div. in IR
bcs E conserv.
into couplings for local interactions in
[physically: expresses that short distance physics
decouples from long distance physics]
E/M = 10%
IR
accuracy 10⁻³

phase space : $2\Gamma(\alpha\to\beta) = |A|^2 \delta^4(\ldots) \prod_{n\in\beta} d^3 p_n$

loops can diverge in (UV) + IR

2 kinds of divergence: $\int_{0}^{\infty} dp$ — $\infty \to$ UV, $0 \to$ IR

phase space can only div. in IR bcs E conserv.

into couplings for local interactions in $\mathcal{L}_{int}$

$\left(\frac{E}{M}\right)^n$

[physically: expresses that short distance physics decouples from long distance physics]

$E/M = 10\%$

IR

accuracy 10^{-3}

$[\psi] = \text{mass}^{3/2}$ $\quad [A_\mu] = \text{mass}$ $\quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(\gamma + m)\psi$

IR divergences: the thing you tried to compute didn't make sense so start again after rethinking.

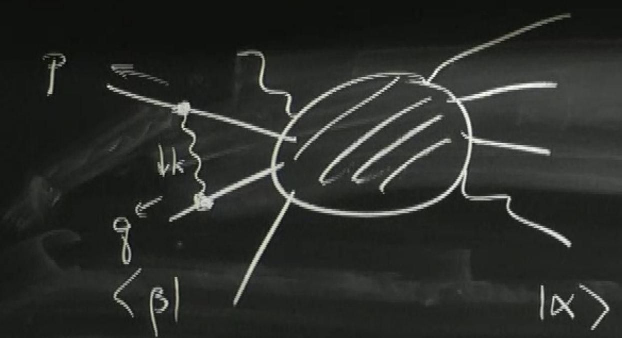
In QED which graphs diverge in the $\underline{IR}$?

photon line: $\int \frac{d^4k}{k^2} \int d^4k \left[S(p)\gamma_\mu S(p+k) \frac{1}{k^2-i\epsilon} \right]$

safe provided $p^2+m^2 \neq 0$

$$\int d^4k \, \frac{-i\not{p}+m}{p^2+m^2-i\epsilon} \gamma_\mu \frac{-i(\not{p}+\not{k})+m}{(p+k)^2+m^2-i\epsilon} \frac{1}{k^2-i\epsilon}$$

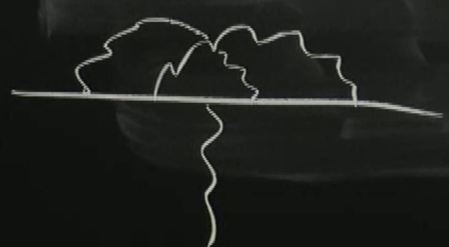
$$\int d^4k \; \frac{\cdots}{p^2+m^2-i\epsilon}\,\gamma_\mu\,\frac{\cdots(p+k)\cdots}{(p+k)^2+m^2-i\epsilon}\;\frac{1}{k^2-i\epsilon}$$



$$M_\mu(\alpha\to\beta) = \int \frac{d^4k}{k^2-i\epsilon}\,\frac{[-i(\not p+\not k)+m]_{ij}}{2k\cdot p+k^2-i\epsilon} \times \frac{[-i(\not q-\not k)+m]_{kl}}{-2k\cdot q+k^2-i\epsilon}\Big[\cdots\cdots\Big]$$

k integral diverges log

$$p^2+m^2=q^2+m^2=0$$



$$\not k\, u(p) = \text{stuff that cancels} + (i\not p + m)u$$

$$\bar u(p)\gamma_\mu \frac{\left[-i\not p + m\right]}{2k\cdot p - i\epsilon}\left[\;\cdots\;\right]$$

$$\bar u(p)\left[i\not p + m\right] = 0$$

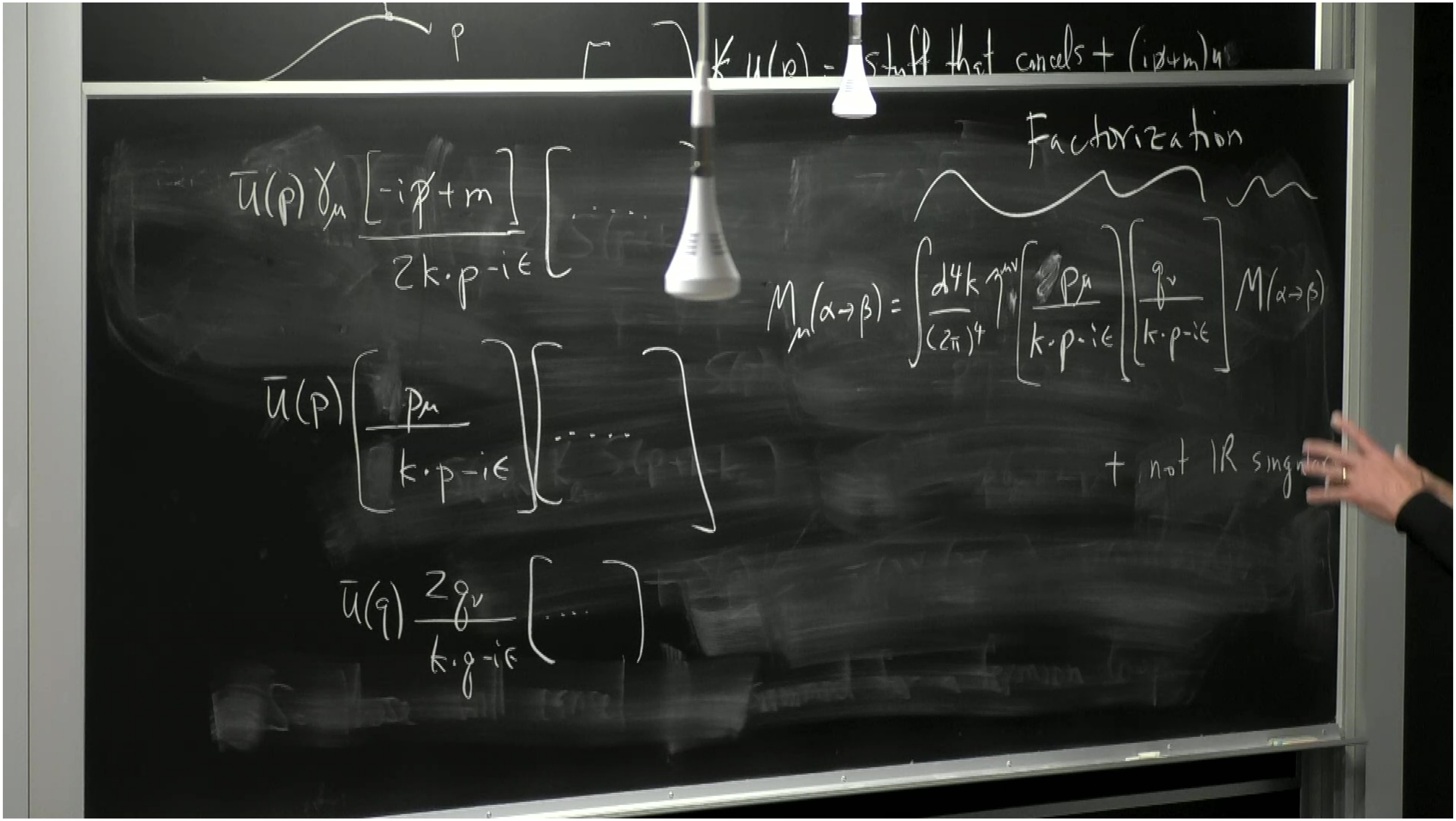
$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$$

$$\begin{aligned}\gamma_\mu \not p &= p^\lambda \gamma_\mu \gamma_\lambda \\ &= p^\lambda\left[-\gamma_\lambda\gamma_\mu + 2\eta_{\mu\lambda}\right] \\ &= -\not p\gamma_\mu + 2p_\mu\end{aligned}$$

$$\bar u(p)\left(\frac{p_\mu}{k\cdot p - i\epsilon}\right)\left[\;\cdots\;\right]$$

$$\bar u(p)\gamma_\mu(i\not p + m) = \bar u(p)\left[\cancel{(i\not p + m)}\gamma_\mu + 2p_\mu\right]$$

$$\bar u(q)\frac{2q_\nu}{k\cdot q - i\epsilon}\left(\;\cdots\;\right)$$


stuff that cancels + (ip̸+m)u
Factorization
$\bar{u}(p)\gamma_\mu \frac{[-i\not{p}+m]}{2k\cdot p-i\epsilon}[\ldots]$
$\bar{u}(p)\left(\frac{p_\mu}{k\cdot p-i\epsilon}\right)[\ldots]$
$\bar{u}(q)\frac{2q_\nu}{k\cdot q-i\epsilon}(\ldots)$
$M_\mu(\alpha\to\beta)=\int\frac{d^4k}{(2\pi)^4}\eta^{\mu\nu}\left[\frac{p_\mu}{k\cdot p-i\epsilon}\right]\left[\frac{q_\nu}{k\cdot p-i\epsilon}\right]M(\alpha\to\beta)$
+ not IR singular