Title: QFT2 - Quantum Electrodynamics - Morning Lecture

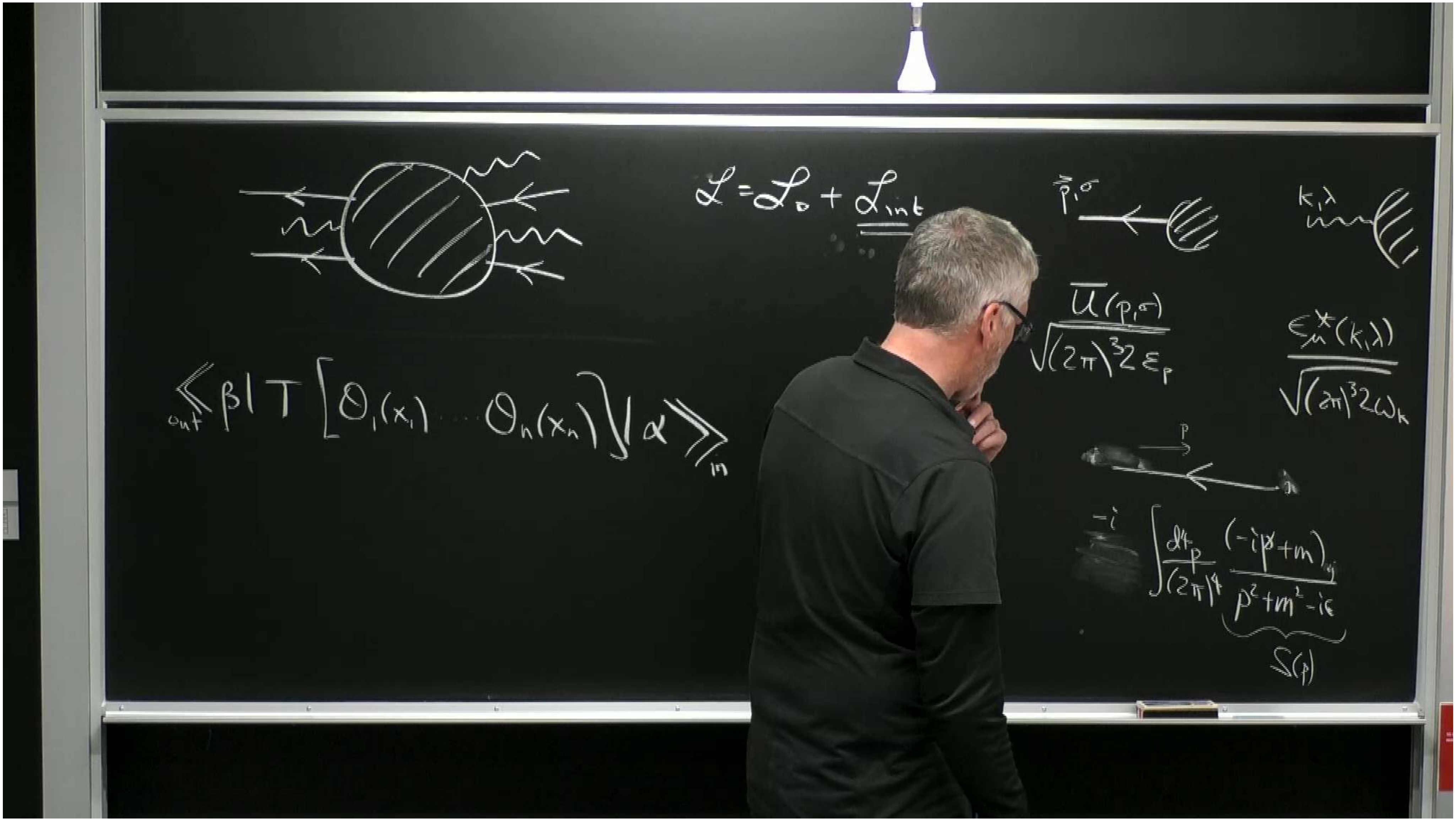
Speakers:

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

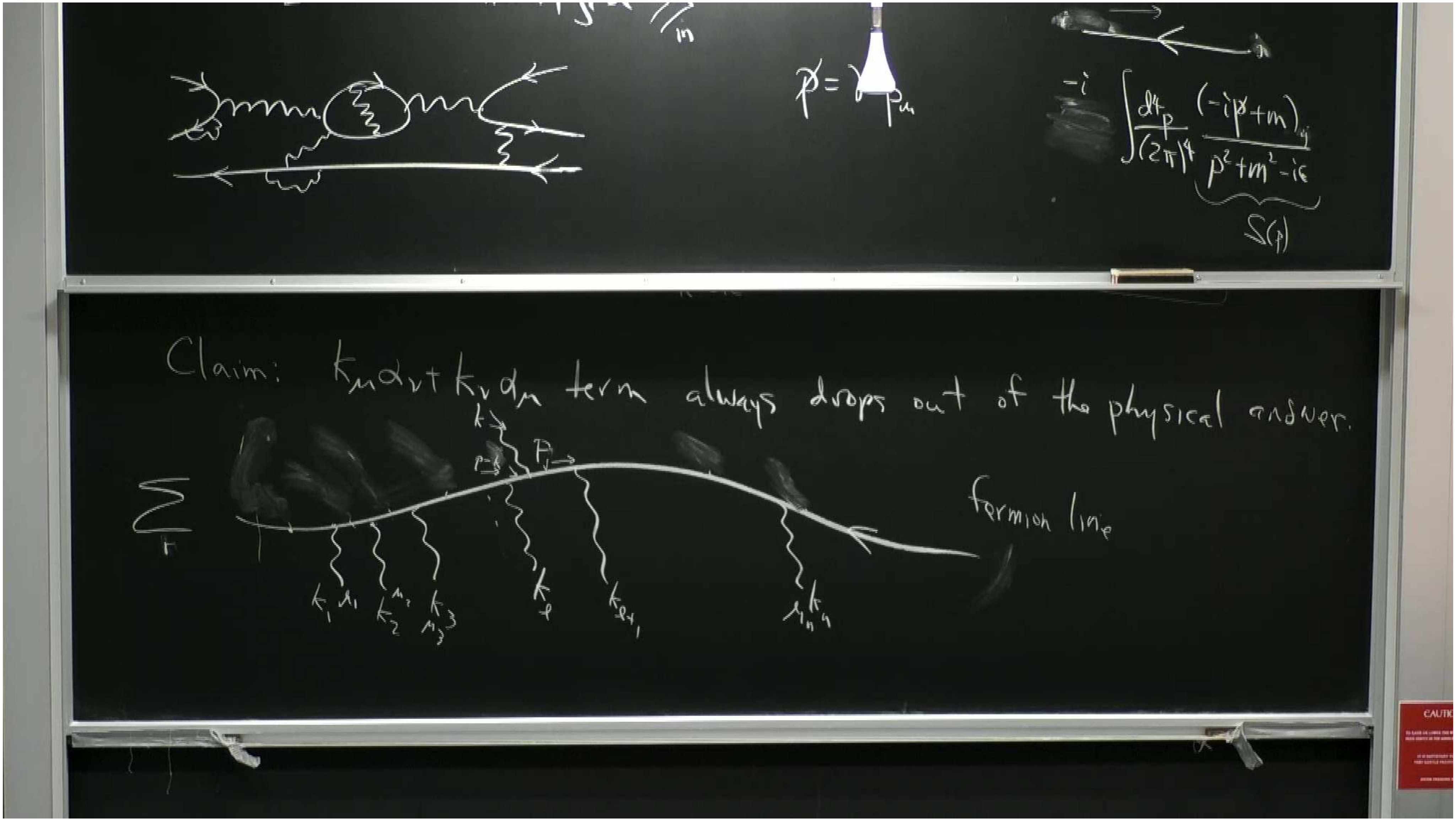
Date: November 29, 2022 - 10:00 AM

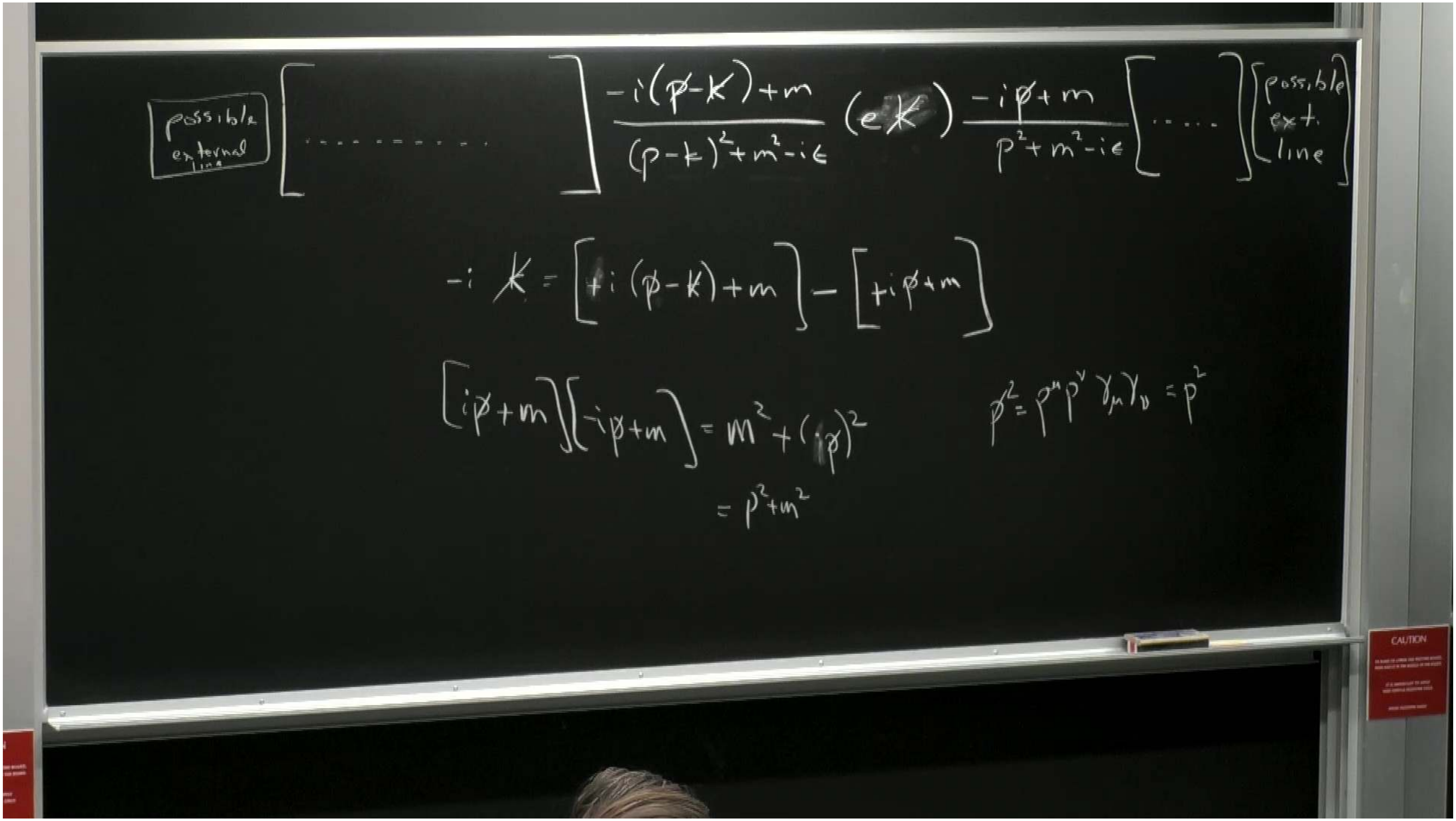
URL: https://pirsa.org/22110037

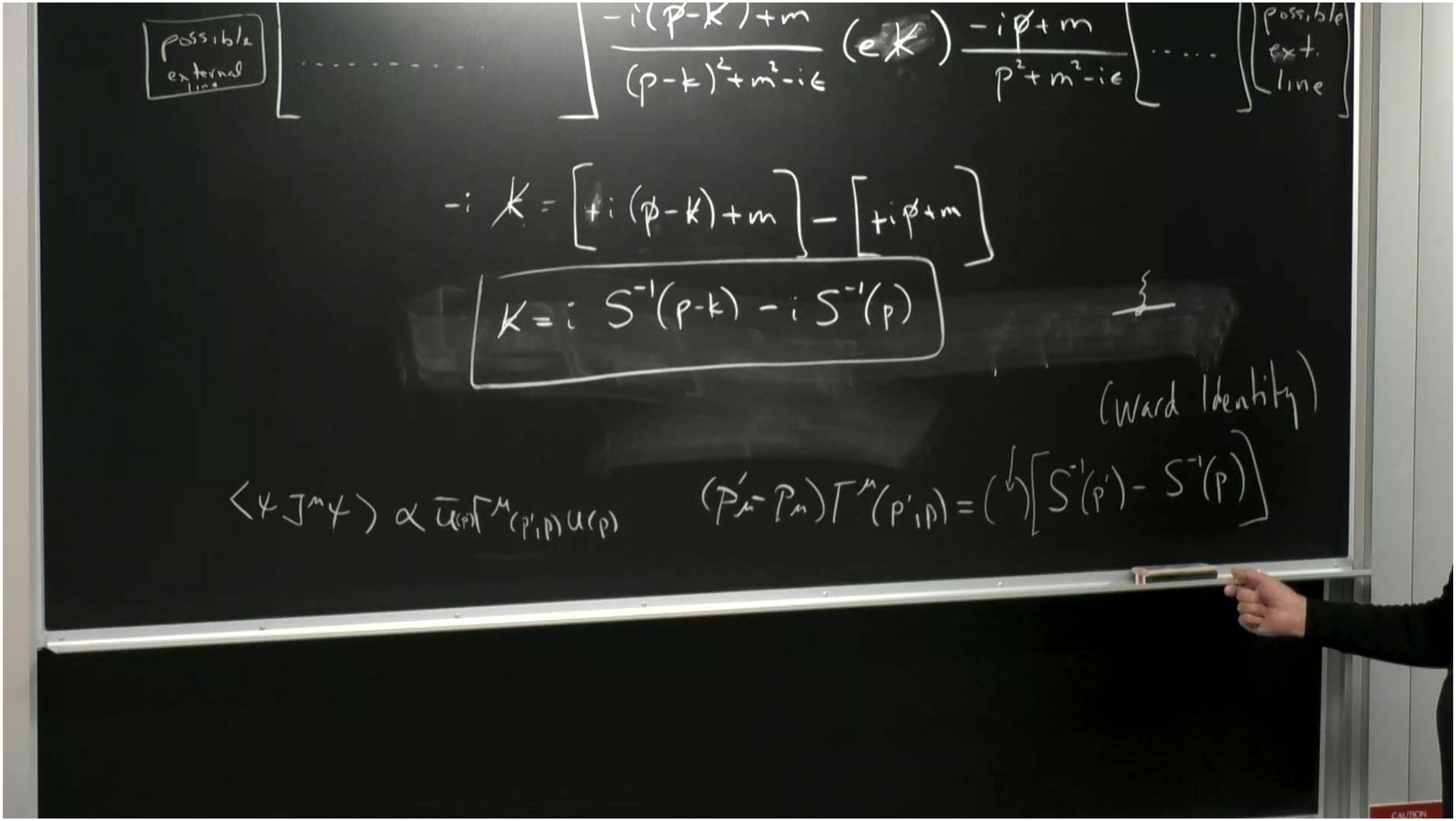
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.

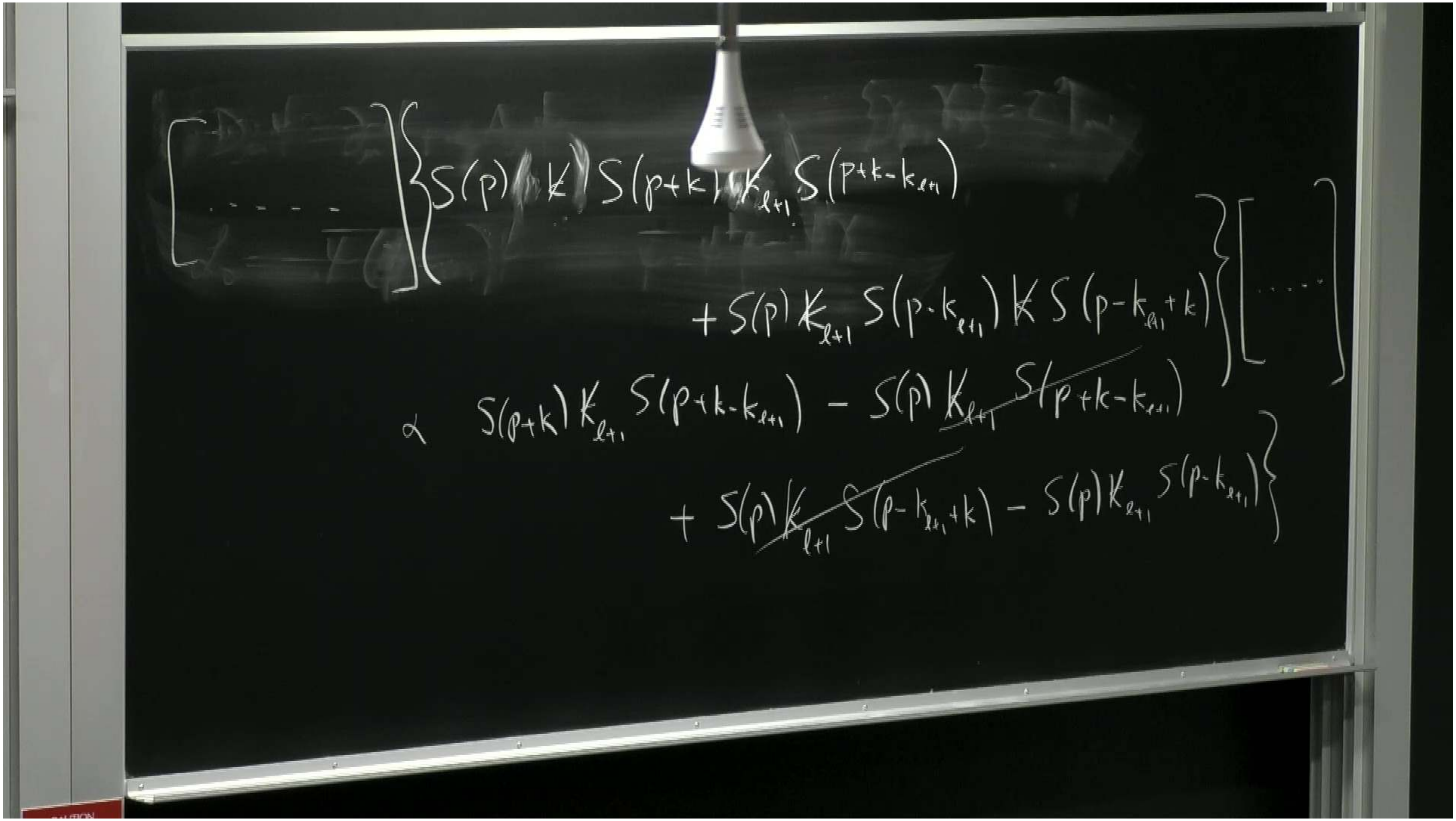


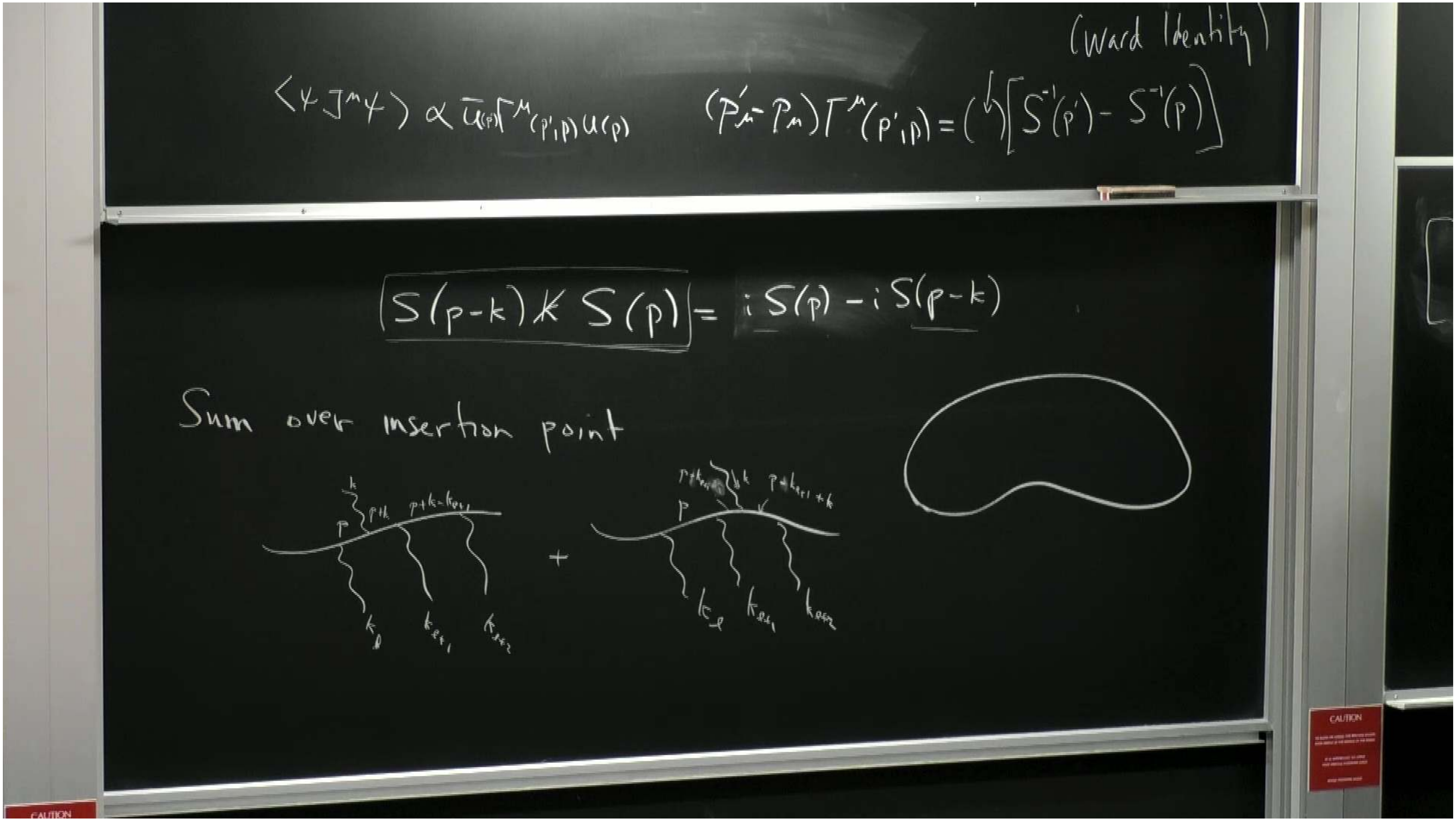
L = L0 + Lint
p, σ
k, λ
ū(p, σ) / √((2π)³ 2E_p)
ε*μ(k, λ) / √((2π)³ 2ω_k)
out⟨⟨β| T [O1(x1) ··· On(xn)] |α⟩⟩in
p
−i ∫ d⁴p / (2π)⁴ (−ip̸ + m) / (p² + m² − iε)
S(p)

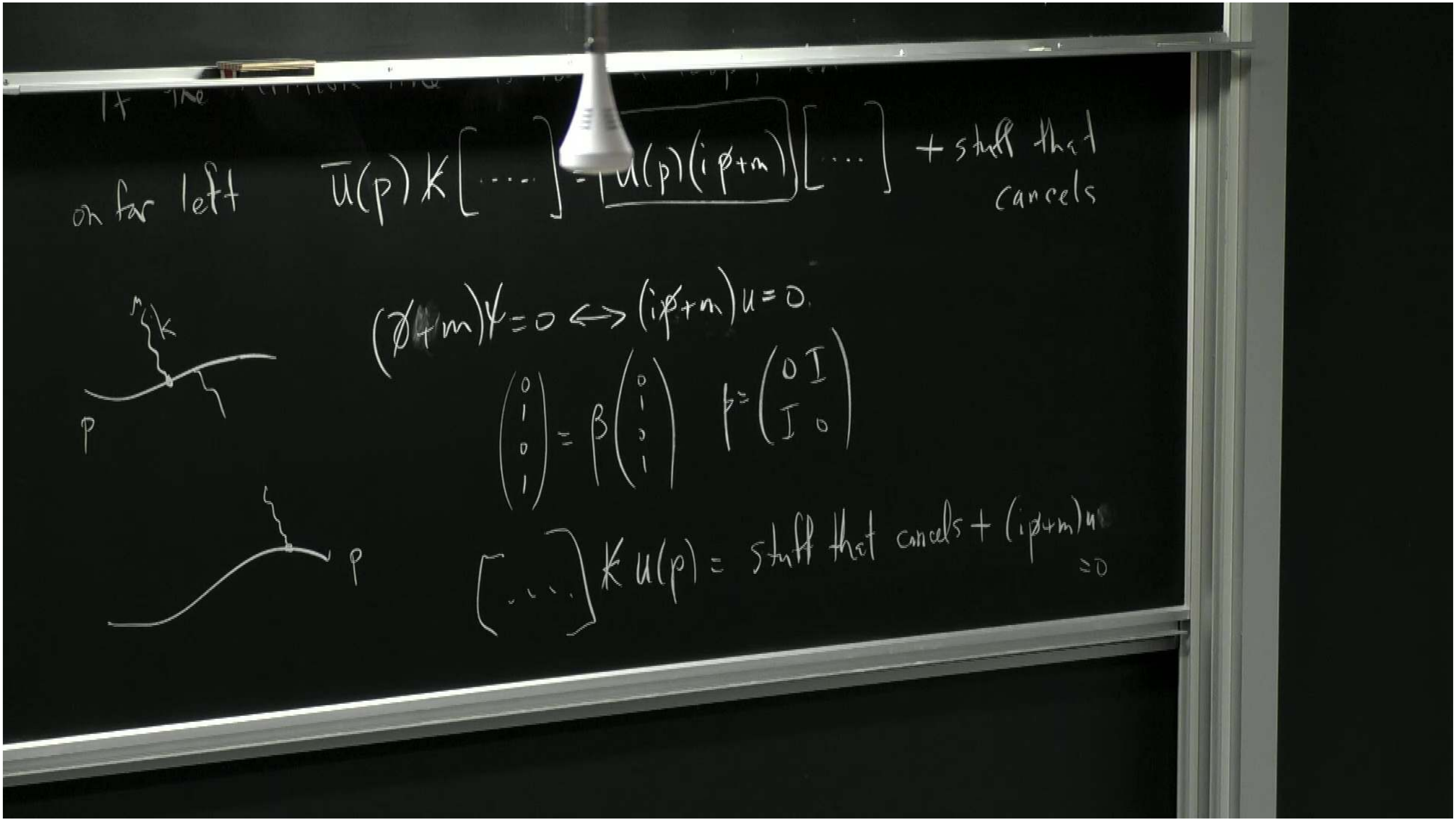

Claim: kμαν + kναμ term always drops out of the physical answer.
fermion line
S(p)

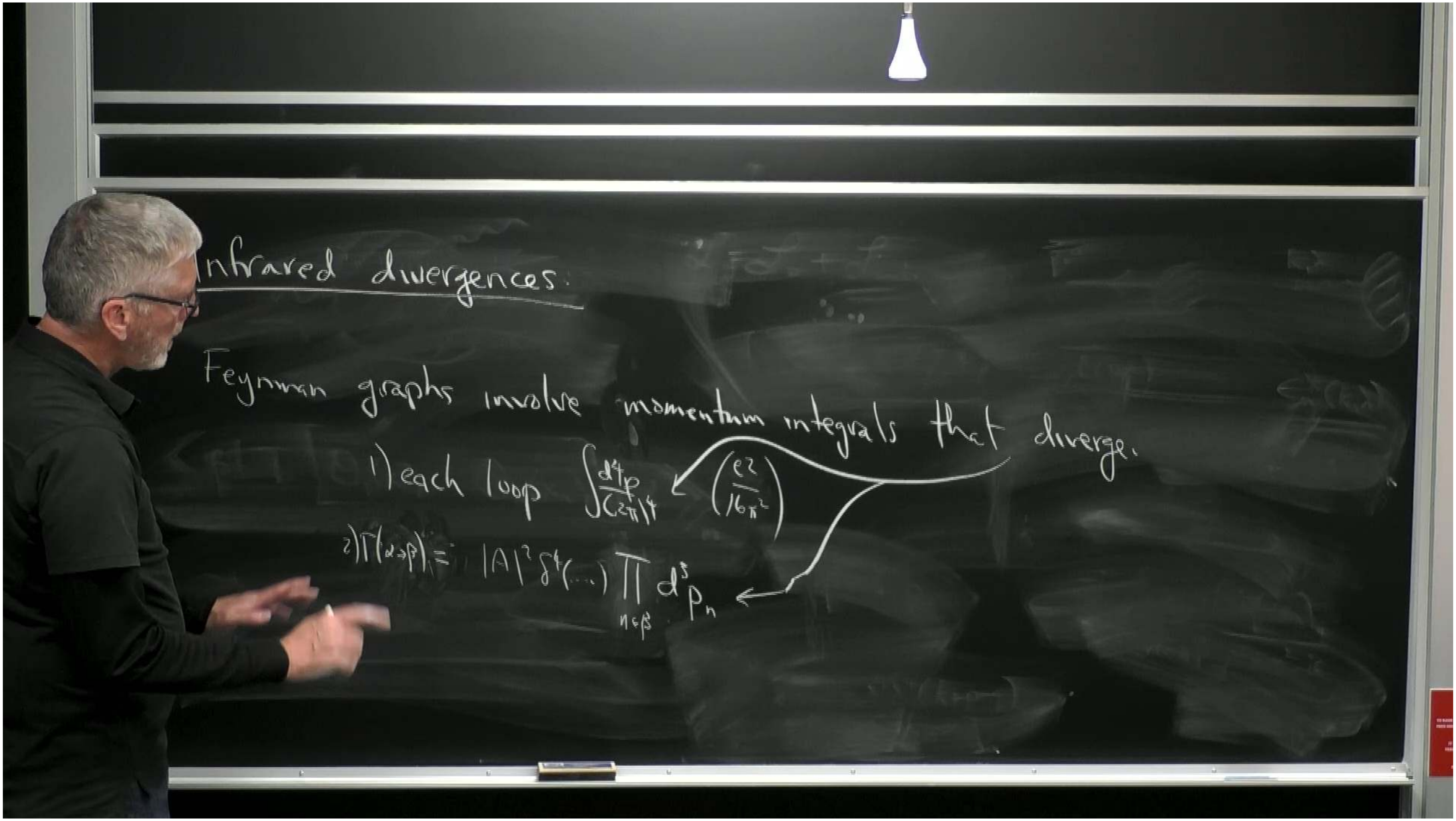

possible external line
$\left[\cdots \right] \frac{-i(\not{p}-\not{k})+m}{(p-k)^2+m^2-i\epsilon} (e\not{k}) \frac{-i\not{p}+m}{p^2+m^2-i\epsilon} \left[\cdots \right]$
possible ext. line
$-i\not{k} = \left[+i(\not{p}-\not{k})+m\right] - \left[+i\not{p}+m\right]$
$\left[i\not{p}+m\right]\left[-i\not{p}+m\right] = m^2 + (\not{p})^2$
$= p^2+m^2$
$\not{p}^2 = p^\mu p^\nu \gamma_\mu \gamma_\nu = p^2$

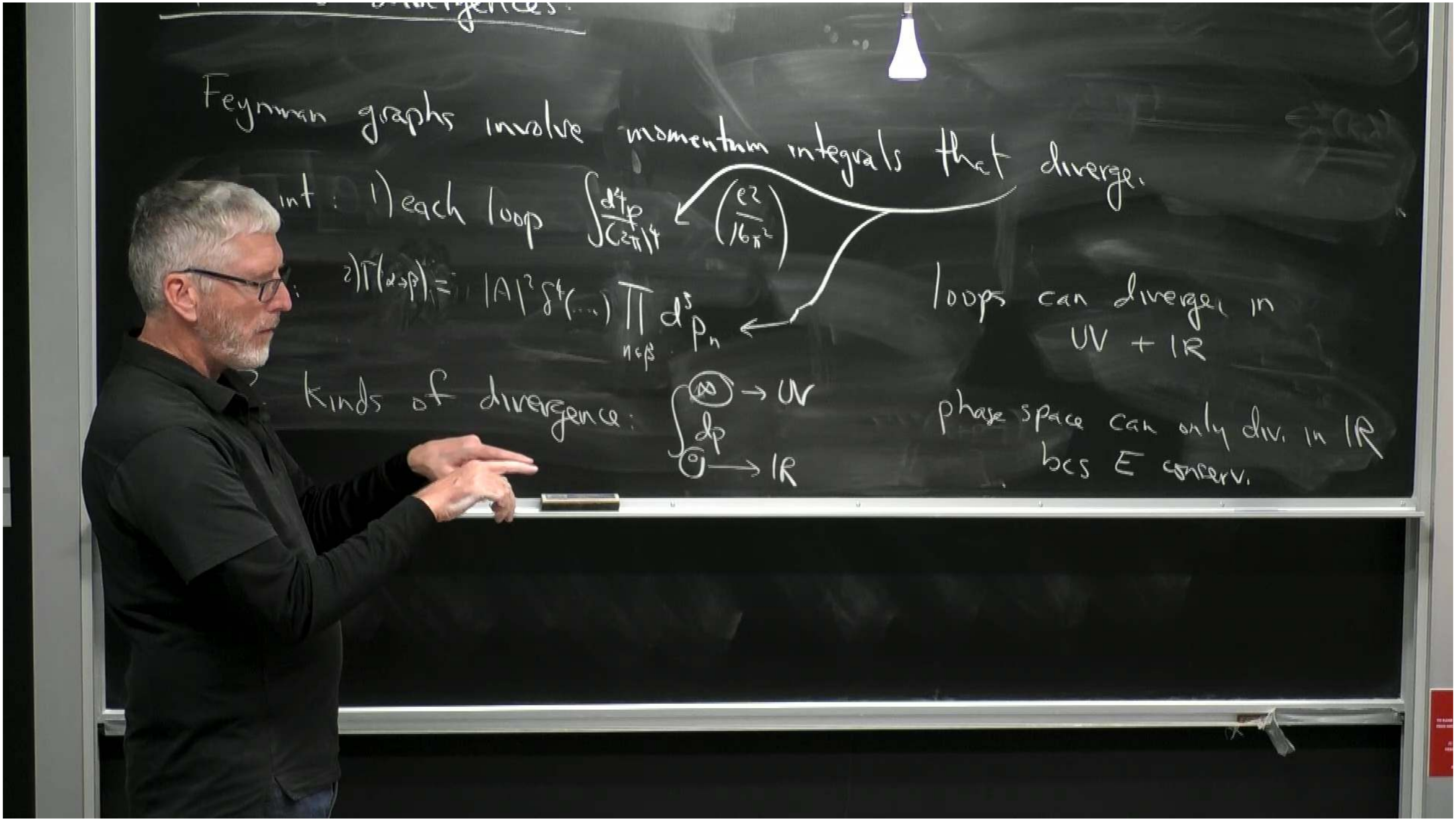

possible external line
$\left[\cdots \right] \frac{-i(\not p - \not k)+m}{(p-k)^2+m^2-i\epsilon} (e \not k) \frac{-i\not p+m}{p^2+m^2-i\epsilon} \left[\cdots \right]$
possible ext. line
$-i \not k = [+i(\not p - \not k)+m] - [+i\not p + m]$
$\not k = i S^{-1}(p-k) - i S^{-1}(p)$
(Ward Identity)
$\langle \psi J^\mu \psi \rangle \propto \bar u(p') \Gamma^\mu(p',p) u(p)$
$(p'_\mu - p_\mu)\Gamma^\mu(p',p) = [S^{-1}(p') - S^{-1}(p)]$


$S(p) \not{k} S(p+k) \not{k}_{l+1} S(p+k-k_{l+1})$
$+ S(p) \not{k}_{l+1} S(p-k_{l+1}) \not{k} S(p-k_{l+1}+k)$
$\propto S(p+k) \not{k}_{l+1} S(p+k-k_{l+1}) - S(p) \not{k}_{l+1} S(p+k-k_{l+1})$
$+ S(p) \not{k}_{l+1} S(p-k_{l+1}+k) - S(p) \not{k}_{l+1} S(p-k_{l+1})$


(Ward Identity)
$\langle \psi J^\mu \psi \rangle \propto \bar{u}(p')\Gamma^\mu(p',p)u(p)$
$(p'_\mu - p_\mu)\Gamma^\mu(p',p) = (\ )[S^{-1}(p') - S^{-1}(p)]$
$S(p-k)\not{k}S(p) = iS(p) - iS(p-k)$
Sum over insertion point


on far left
ū(p) k̸ [....] = ū(p)(ip̸+m) [...] + stuff that cancels
(∂̸+m)ψ = 0 ⟺ (ip̸+m)u = 0
[...] k̸ u(p) = stuff that cancels + (ip̸+m)u
= 0


Infrared divergences:
Feynman graphs involve momentum integrals that diverge.
1) each loop $\int \frac{d^4p}{(2\pi)^4}$ ← $\left(\frac{e^2}{16\pi^2}\right)$
2) $\Gamma(\alpha\to\beta) = |A|^2 \delta^4(\ldots) \prod_{n\in\beta} d^3p_n$


Feynman graphs involve momentum integrals that diverge.
int: 1) each loop ∫ d⁴p/(2π)⁴ ← (e²/16π²)
2) Γ(α→β) = |A|² δ⁴(...) ∏ d³pₙ
Kinds of divergence: ∫dp (∞) → UV, (0) → IR
loops can diverge in UV + IR
phase space can only div. in IR bcs E conserv.

2 kinds of divergence: $\int dp$ (∞) → UV, (0) → IR

phase space can only div. in IR bcs E conserv.

Moral: UV divergences are all **local** + so can be absorbed into couplings ~~for~~ local interactions in $\mathcal{L}_{int}$

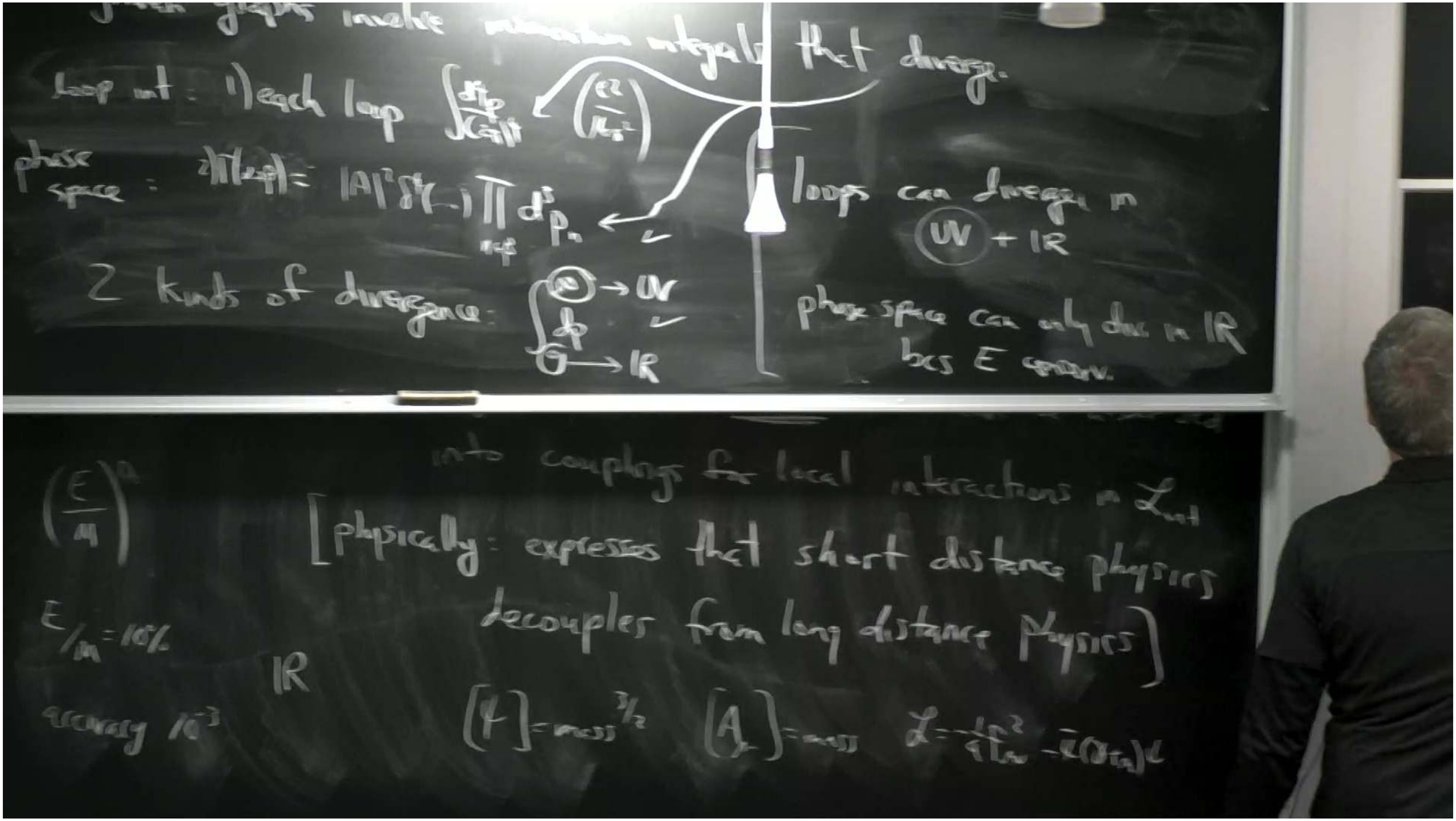
[physically: expresses that short distance physics decouples from long distance physics]

$\left(\frac{E}{M}\right)^n$

$E/M = 10\%$

accuracy 10^{-3}

IR



graphs involve integrals that diverge.
loop int: 1) each loop
phase space:
2 kinds of divergence:
→ UV
→ IR
loops can diverge in
UV + IR
phase space can only div. in IR
bcs E conserv.
into couplings for local interactions in
[physically: expresses that short distance physics
decouples from long distance physics]
IR
accuracy

phase space : $2) \Gamma(\alpha \to \beta) = |A|^2 \delta^4(\dots) \prod_{n \in \beta} d^3 p_n$

2 kinds of divergence: $\int_0^\infty dp$ — $\infty \to$ UV, $0 \to$ IR

loops can diverge in (UV) + IR

phase space can only div. in IR bcs E conserv.

into couplings for local interactions in $\mathcal{L}_{int}$

[physically: expresses that short distance physics decouples from long distance physics]

$\left(\frac{E}{M}\right)^n$

$E/M = 10\%$

accuracy 10^{-3}

IR

$[\psi] = \text{mass}^{3/2}$ $\quad [A_\mu] = \text{mass}$ $\quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}(\gamma \partial + m)\psi$

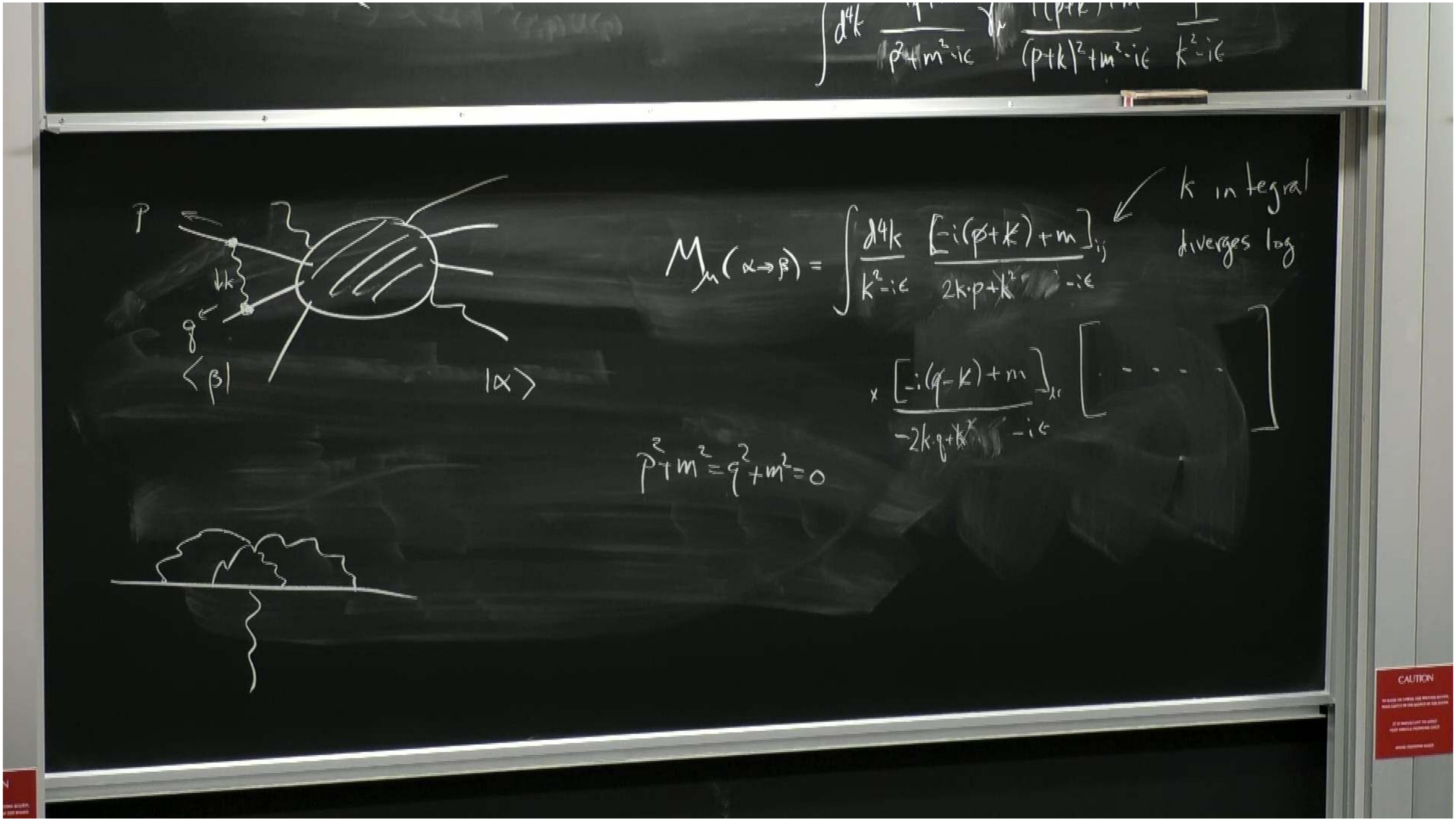
IR divergences: the thing you tried to compute didn't make sense so start again after rethinking.

In QED which graphs diverge in the IR?

photon line: $\int \frac{d^4k}{k^2} \int d^4k \left[S(p)\gamma_\mu S(p+k) \frac{1}{k^2-i\epsilon} \right]$

safe provided $p^2+m^2 \neq 0$

$$\int d^4k \, \frac{-i\not{p}+m}{p^2+m^2-i\epsilon} \gamma_\mu \frac{-i(\not{p}+\not{k})+m}{(p+k)^2+m^2-i\epsilon} \frac{1}{k^2-i\epsilon}$$


$M_\mu(\alpha\to\beta) = \int \frac{d^4k}{k^2 - i\epsilon} \frac{[-i(\not p + \not k) + m]_{ij}}{2k\cdot p + k^2 - i\epsilon}$
$\times \frac{[-i(\not q - \not k) + m]_{kl}}{-2k\cdot q + k^2 - i\epsilon}$
k integral
diverges log
$p^2 + m^2 = q^2 + m^2 = 0$
$\langle\beta|$
$|\alpha\rangle$

$\not{\varepsilon}\, u(p) = $ stuff that cancels $+ (i\not{p}+m)u$

$$\bar{u}(p)\gamma_\mu \frac{[-i\not{p}+m]}{2k\cdot p - i\epsilon}\left[\;\ldots\ldots\;\right]$$

$$\bar{u}(p)\left(\frac{p_\mu}{k\cdot p - i\epsilon}\right)\left[\;\ldots\ldots\;\right]$$

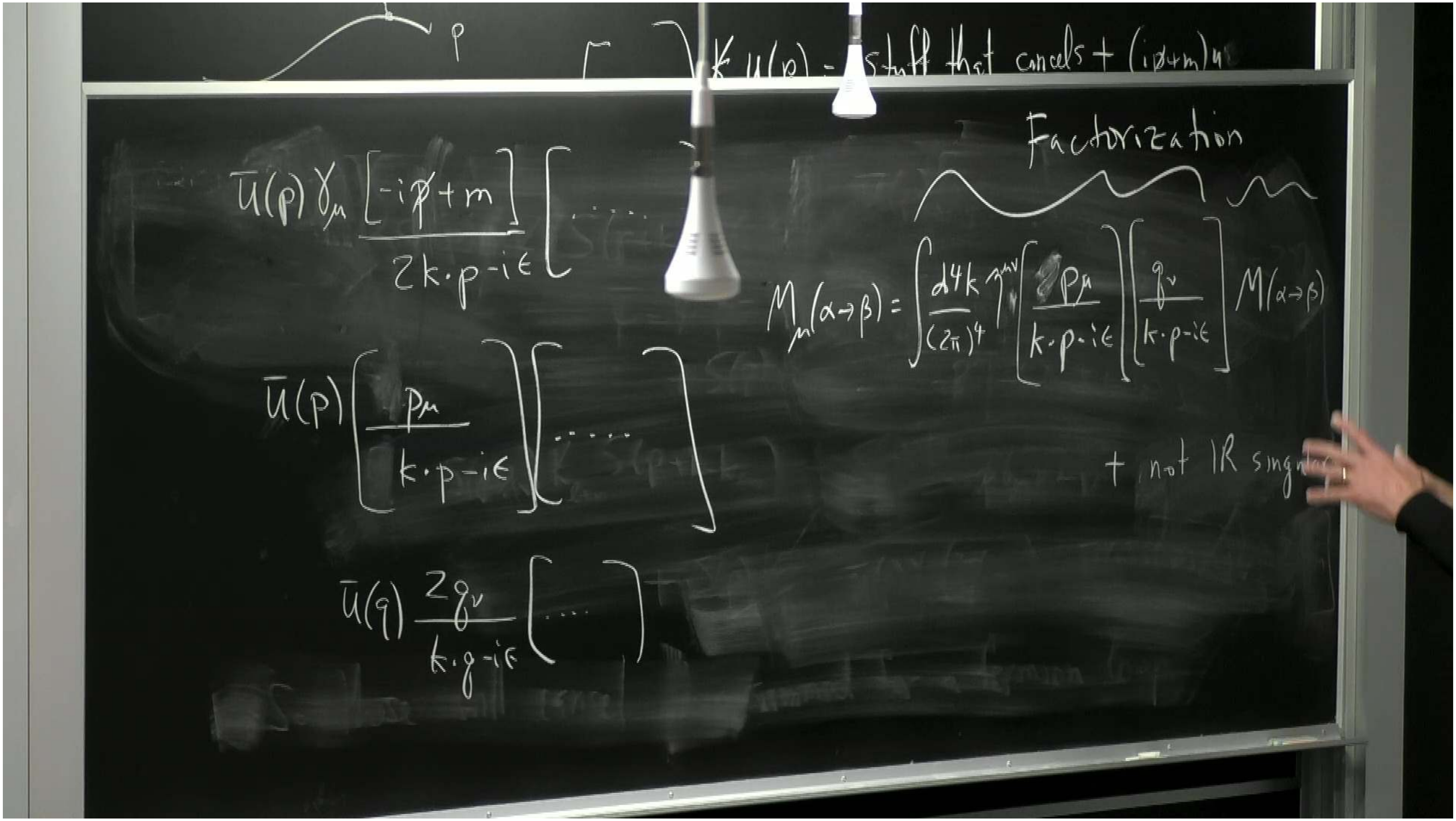
$$\bar{u}(q)\frac{2q_\nu}{k\cdot q - i\epsilon}\left(\;\ldots\;\right)$$

$$\bar{u}(p)\left[i\not{p}+m\right] = 0$$

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu}$$

$$\gamma^\mu \not{p} = p^\lambda \gamma_\mu \gamma_\lambda$$
$$= p^\lambda\left[-\gamma_\lambda\gamma_\mu + 2\eta_{\mu\lambda}\right]$$
$$= -\not{p}\gamma_\mu + 2p_\mu$$

$$\bar{u}(p)\gamma_\mu(-i\not{p}+m) = \bar{u}(p)\left[(i\not{p}+m)\gamma_\mu + 2p_\mu\right]$$


p
stuff that cancels +
Factorization
ū(p) γ_μ [-ip̸+m] / 2k·p-iε [......]
ū(p) [p_μ / k·p-iε] [......]
ū(q) 2q_ν / k·q-iε [....]
M_μ(α→β) = ∫ d4k/(2π)^4 η^μν [p_μ / k·p-iε] [q_ν / k·p-iε] M(α→β)
+ not IR