Title: QFT2 - Quantum Electrodynamics - Morning Lecture

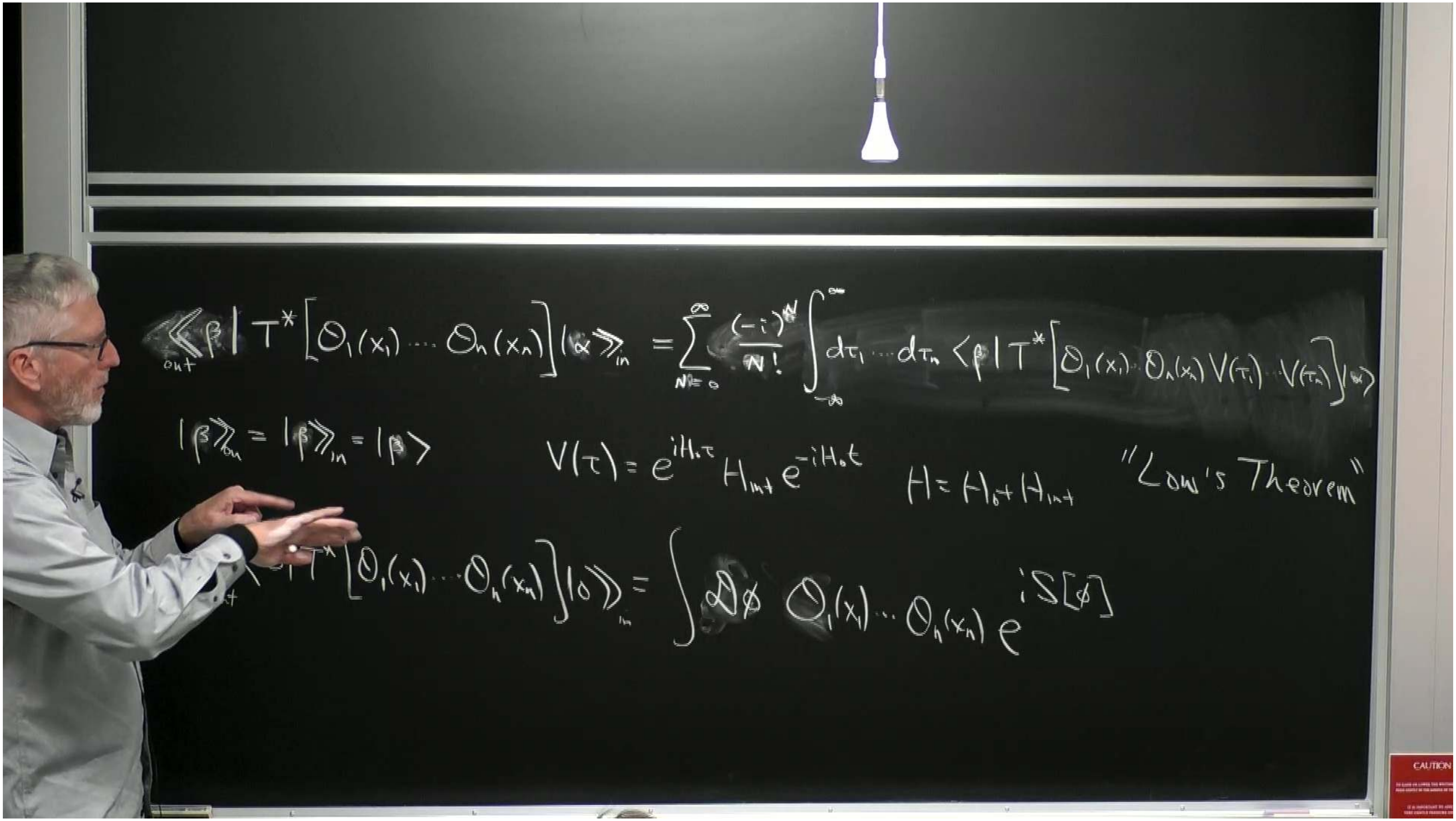
Speakers:

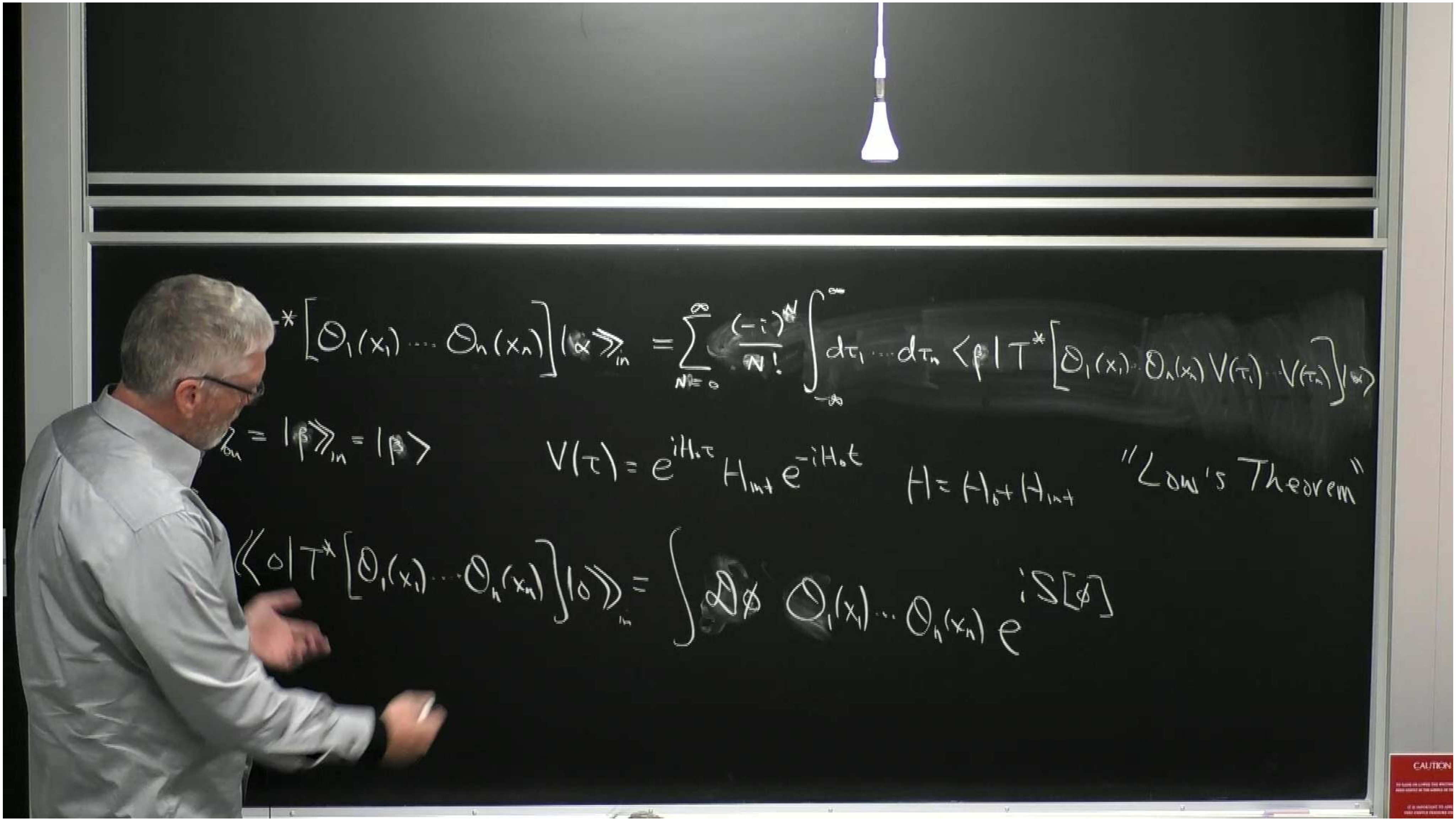
Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

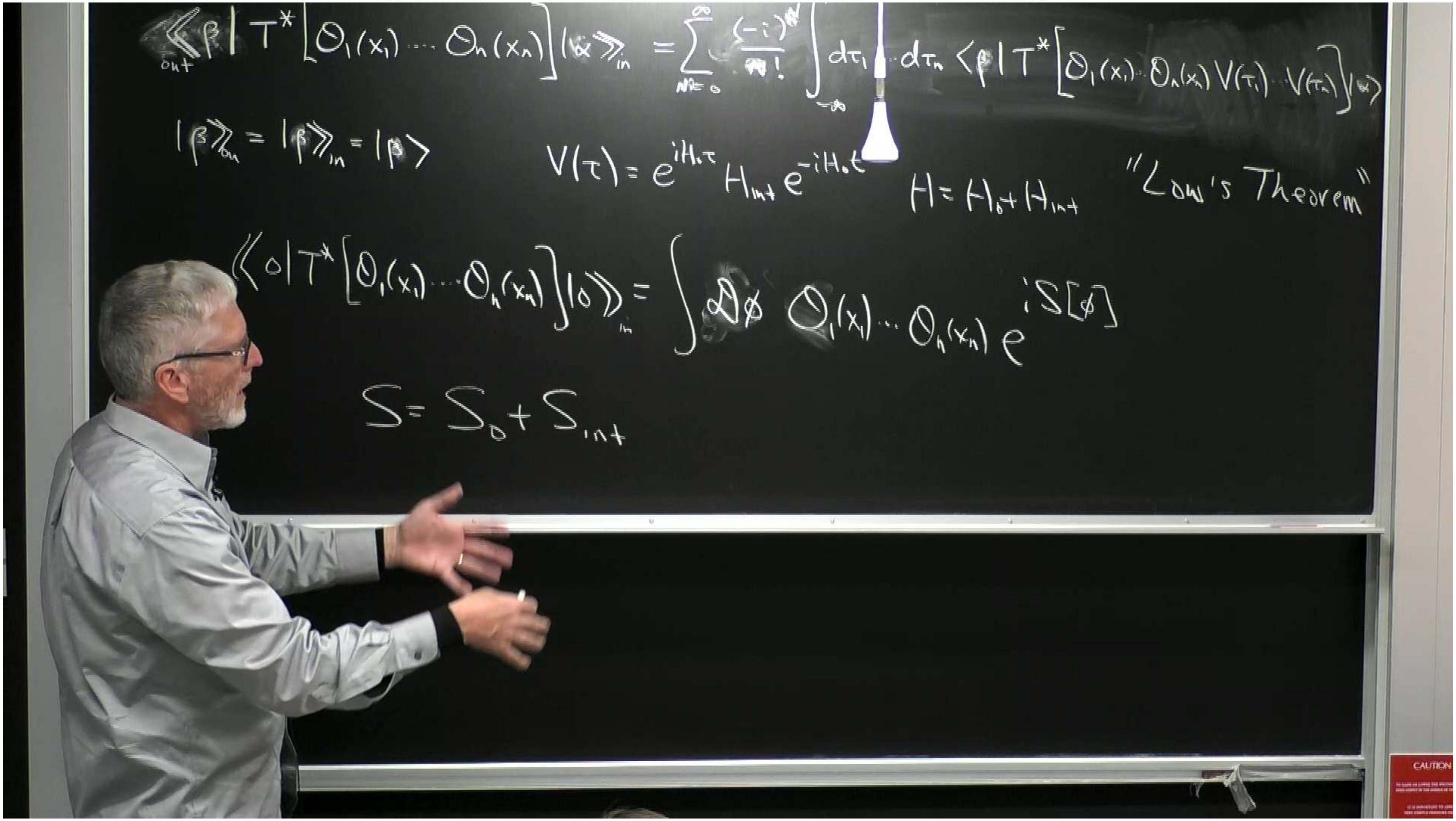
Date: November 22, 2022 - 10:00 AM

URL: https://pirsa.org/22110036

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


${}_{out}\langle\langle\beta| T^*[O_1(x_1)\cdots O_n(x_n)]|\alpha\rangle\rangle_{in} = \sum_{N=0}^{\infty}\frac{(-i)^N}{N!}\int_{-\infty}^{\infty} d\tau_1\cdots d\tau_N \langle\beta|T^*[O_1(x_1)\cdots O_n(x_n)V(\tau_1)\cdots V(\tau_N)]|\alpha\rangle$
$|\beta\rangle\rangle_{out} = |\beta\rangle\rangle_{in} = |\beta\rangle$
$V(\tau) = e^{iH_0\tau} H_{int} e^{-iH_0 t}$
$H = H_0 + H_{int}$
"Low's Theorem"
$T^*[O_1(x_1)\cdots O_n(x_n)]|0\rangle\rangle_{in} = \int \mathcal{D}\phi\, O_1(x_1)\cdots O_n(x_n)\, e^{iS[\phi]}$


$T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)]|\alpha\rangle\rangle_{in} = \sum_{N=0}^{\infty}\frac{(-i)^N}{N!}\int_{-\infty}^{\infty} d\tau_1\cdots d\tau_N \langle\beta|T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)V(\tau_1)\cdots V(\tau_N)]|\alpha\rangle$
$= |\beta\rangle\rangle_{in} = |\beta\rangle$
$V(\tau) = e^{iH_0\tau} H_{int} e^{-iH_0 t}$
$H = H_0 + H_{int}$
"Low's Theorem"
$\langle\langle 0|T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)]|0\rangle\rangle_{in} = \int \mathcal{D}\phi\, \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\, e^{iS[\phi]}$
CAUTION


$_{out}\langle\langle\beta| T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)]|\alpha\rangle\rangle_{in} = \sum_{N=0}^{\infty}\frac{(-i)^N}{N!}\int_{-\infty}^{\infty} d\tau_1\cdots d\tau_n \langle\beta| T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)V(\tau_1)\cdots V(\tau_n)]|\alpha\rangle$
$|\beta\rangle\rangle_{out} = |\beta\rangle\rangle_{in} = |\beta\rangle$
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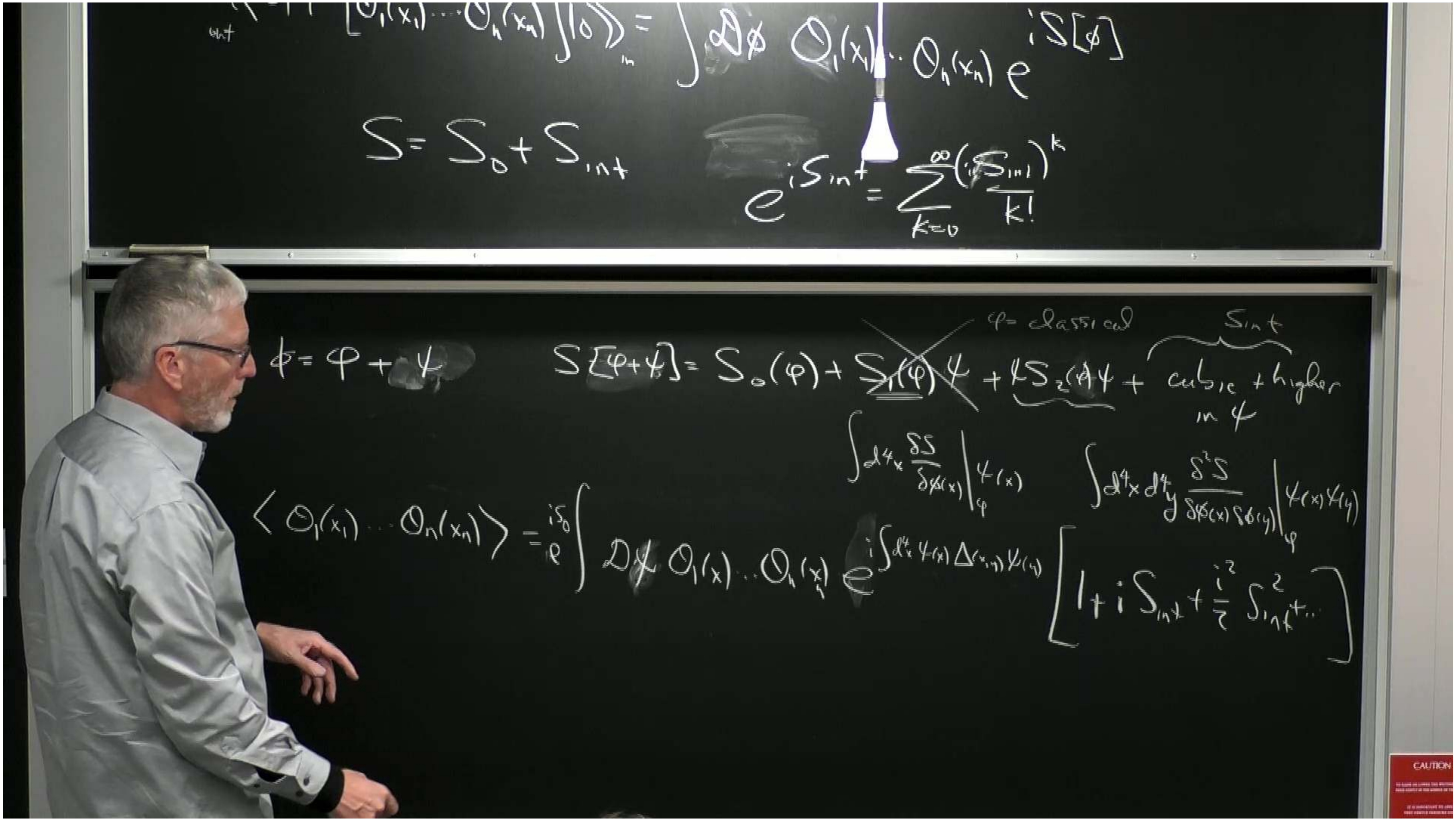
"Low's Theorem"
$\langle\langle 0| T^*[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)]|0\rangle\rangle_{in} = \int \mathcal{D}\phi\, \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\, e^{iS[\phi]}$
$S = S_0 + S_{int}$
CAUTION


S = S_0 + S_int
φ = classical
S_int
cubic + higher in ψ

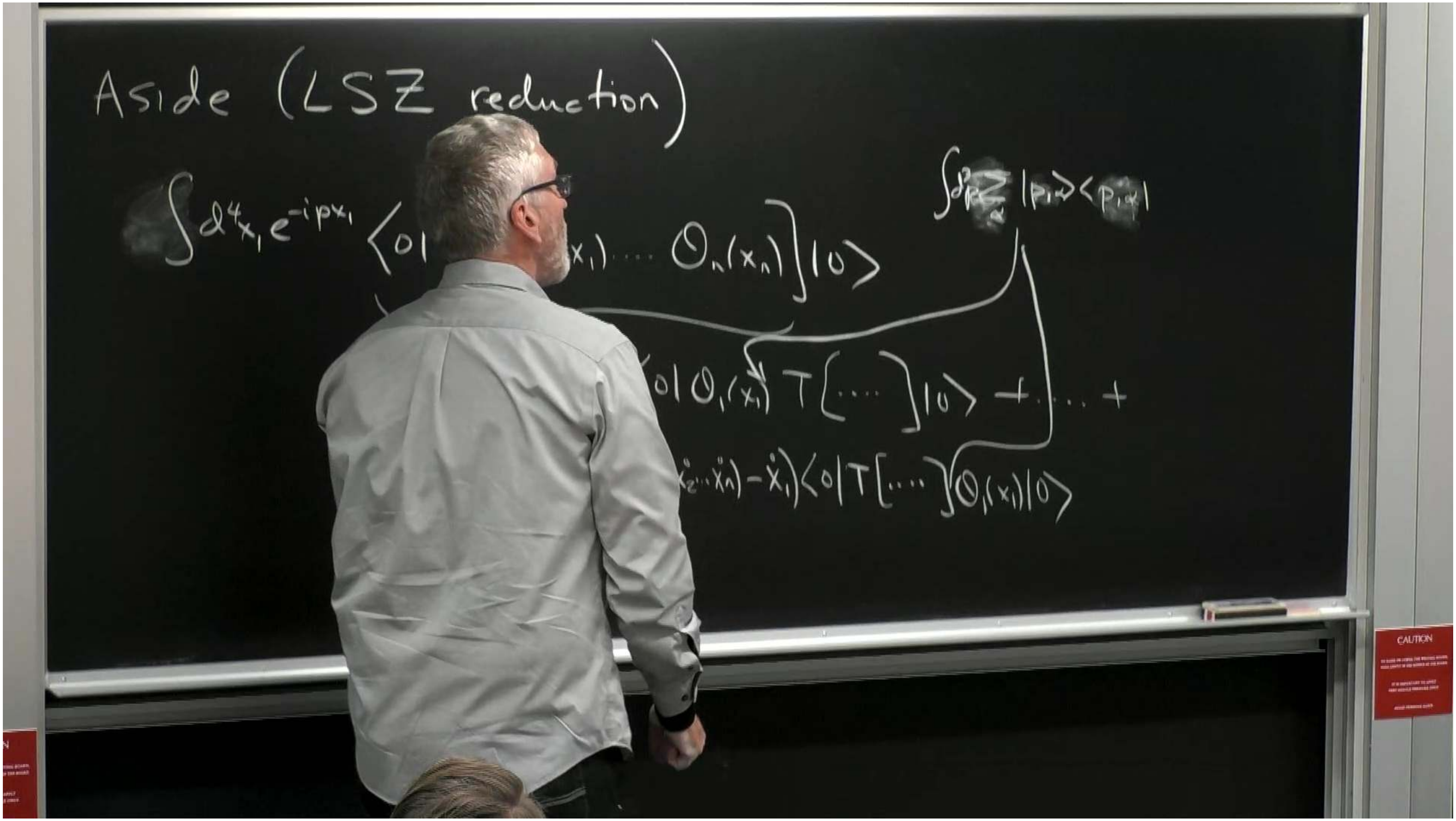
Aside (LSZ reduction)

$$\sum_N |N\rangle\langle N|$$

$$\int d^4x_1\, e^{-ipx_1} \underbrace{\langle 0|T\left[\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\right]|0\rangle}$$

$$\Theta\left(x_1^0-\max(x_2^0\ldots x_n^0)\right)\langle 0|\mathcal{O}_1(x_1)\,T\left[\cdots\right]|0\rangle + \ldots +$$

$$+\,\Theta\left(\min(x_2^0\ldots x_n^0)-x_1^0\right)\langle 0|T\left[\cdots\right]\mathcal{O}_1(x_1)|0\rangle$$


Aside (LSZ reduction)

singular part of the answer

$$\propto \sum_{\alpha} \frac{1}{p^2+m_\alpha^2-i\epsilon} \langle 0|\mathcal{O}_1(0)|\vec{p},\alpha\rangle\langle\vec{p},\alpha|T(\dots)|0\rangle$$

$$\langle H | \psi_e^*(x)\, \psi_p^*(y) | 0 \rangle$$

Singular part of the answer

$$\propto \sum_\alpha \frac{1}{p^2+m_\alpha^2-i\epsilon} \langle 0|\mathcal{O}_1(0)|\vec{p},\alpha\rangle\langle \vec{p},\alpha|T(\dots)|0\rangle$$

$$\langle \phi_1(x_1) \cdots \phi_n(x_n) \rangle = \int \mathcal{D}\phi \; \phi_1(x_1) \cdots \phi_n(x_n) \, e^{iS[\phi]}$$

Generating functional:

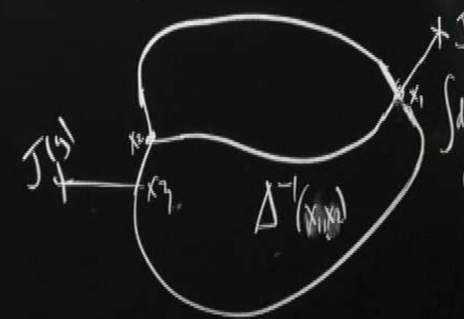
$$Z[J(x)] = \int \mathcal{D}\phi \, e^{iS[\phi] + i\int d^4x \, J(x)\phi(x)}$$

$$(-i)^n \frac{\delta^n Z}{\delta J(x_1) \cdots \delta J(x_n)}\bigg|_{J=0} = \int \mathcal{D}\phi \; \phi(x_1) \cdots \phi(x_n) \, e^{iS[\phi] + i\int J\phi} = \langle \phi(x_1) \cdots \phi(x_n) \rangle_J$$

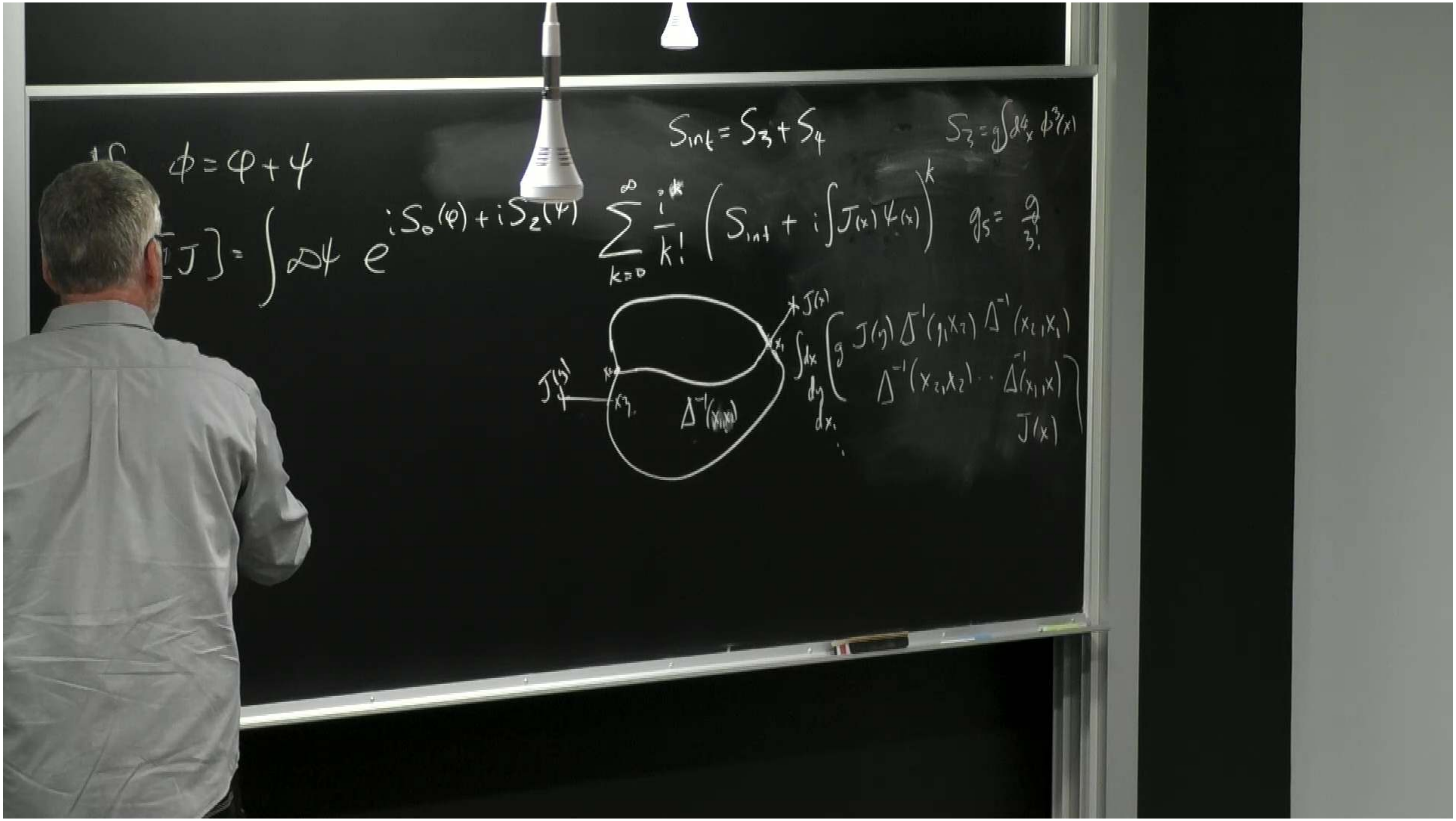
If $\phi = \varphi + \psi$

$$S_{int} = S_3 + S_4 \qquad S_3 = g\int d^4x\, \phi^3(x)$$

$$Z[J] = \int \mathcal{D}\psi\, e^{iS_0(\varphi) + iS_2(\psi)} \sum_{k=0}^{\infty} \frac{i^k}{k!}\left(S_{int} + i\int J(x)\psi(x)\right)^k$$



$$\int dx\, dy\, dx_1 \ldots \left(g\, J(y)\, \Delta^{-1}(y,x_2)\, \Delta^{-1}(x_2,x_1)\, \Delta^{-1}(x_2,x_2) \ldots \Delta^{-1}(x_1,x)\, J(x)\right)$$

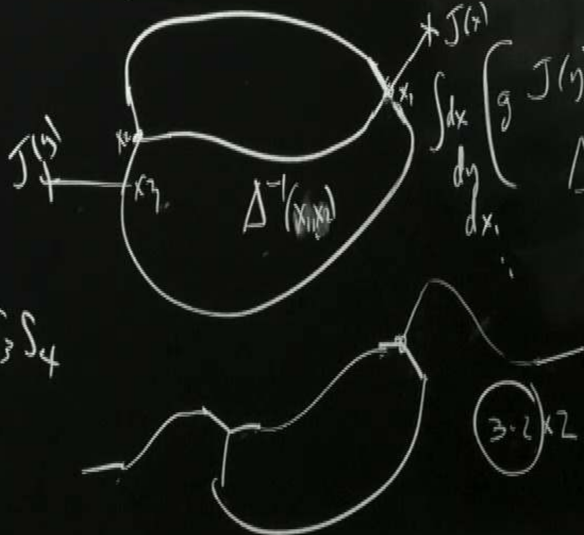


Sint = S3 + S4
S3 = g ∫ d4x φ3(x)
φ = φ + ψ
gs = g/3!

If $\phi = \varphi + \psi$

$S_{int} = S_3 + S_4$ $\qquad$ $S_3 = g\int d^4x\, \phi^3(x)$

$$Z[J] = \int \mathcal{D}\psi\, e^{iS_0(\varphi) + iS_2(\psi)} \sum_{k=0}^{\infty} \frac{i^k}{k!}\left(S_{int} + i\int J(x)\psi(x)\right)^k \qquad g_s = \frac{g}{3!}$$

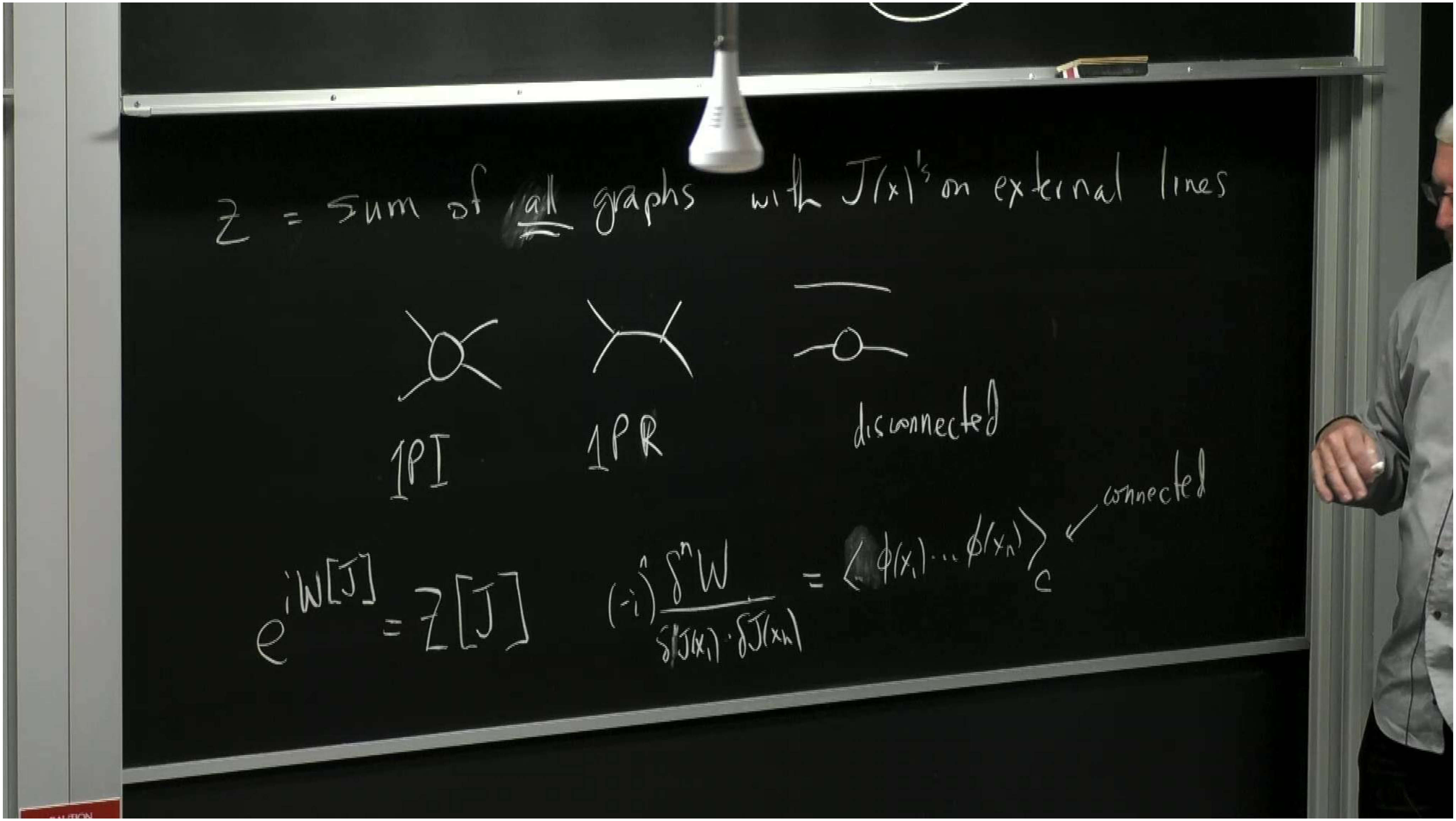


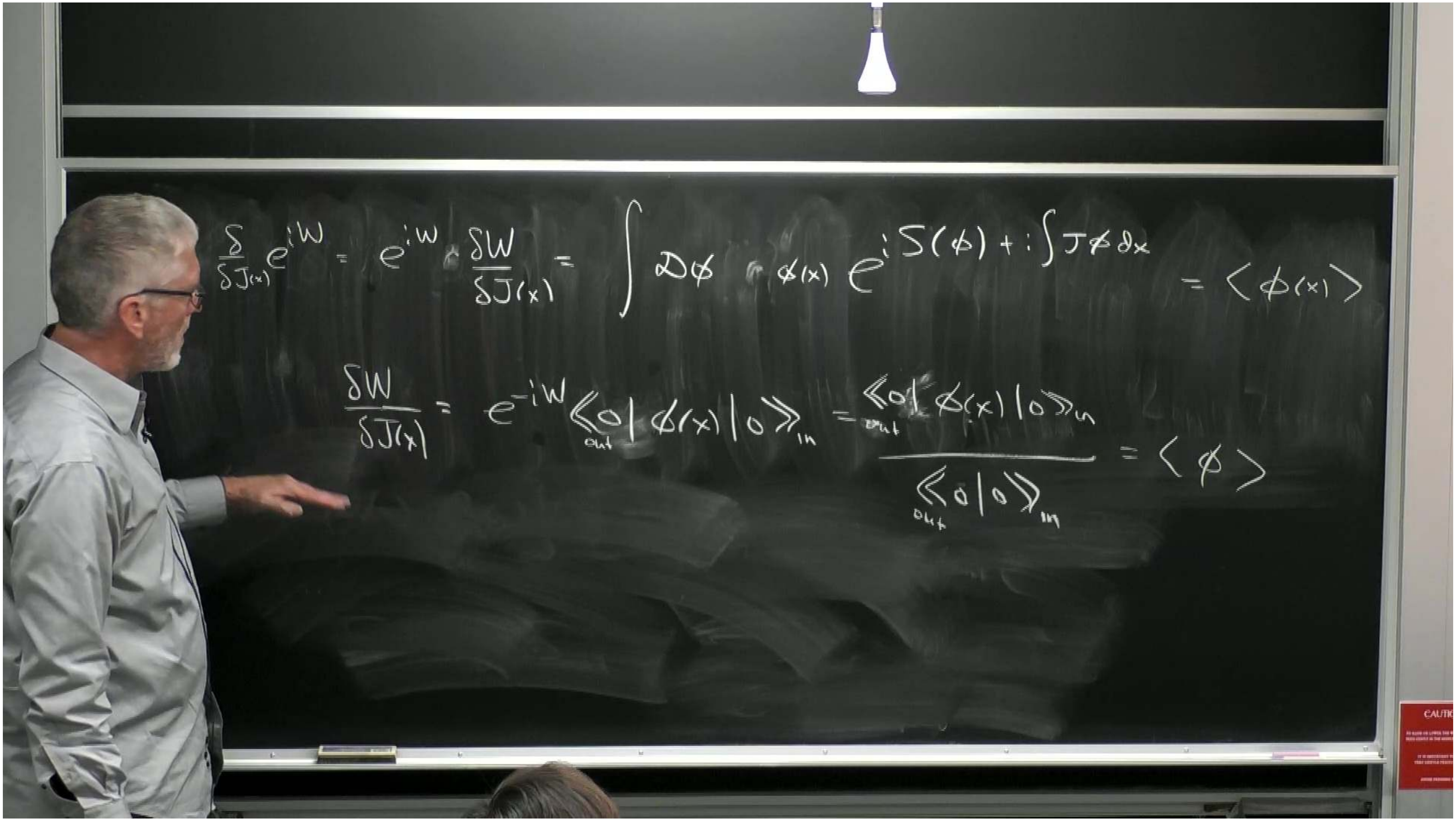
$$\int dx\, dy\, dx_1 \cdots \left(g\, J(y)\, \Delta^{-1}(y,x_2)\, \Delta^{-1}(x_2,x_1)\, \Delta^{-1}(x_2,x_2) \cdots \Delta^{-1}(x_1,x)\, J(x) \right)$$

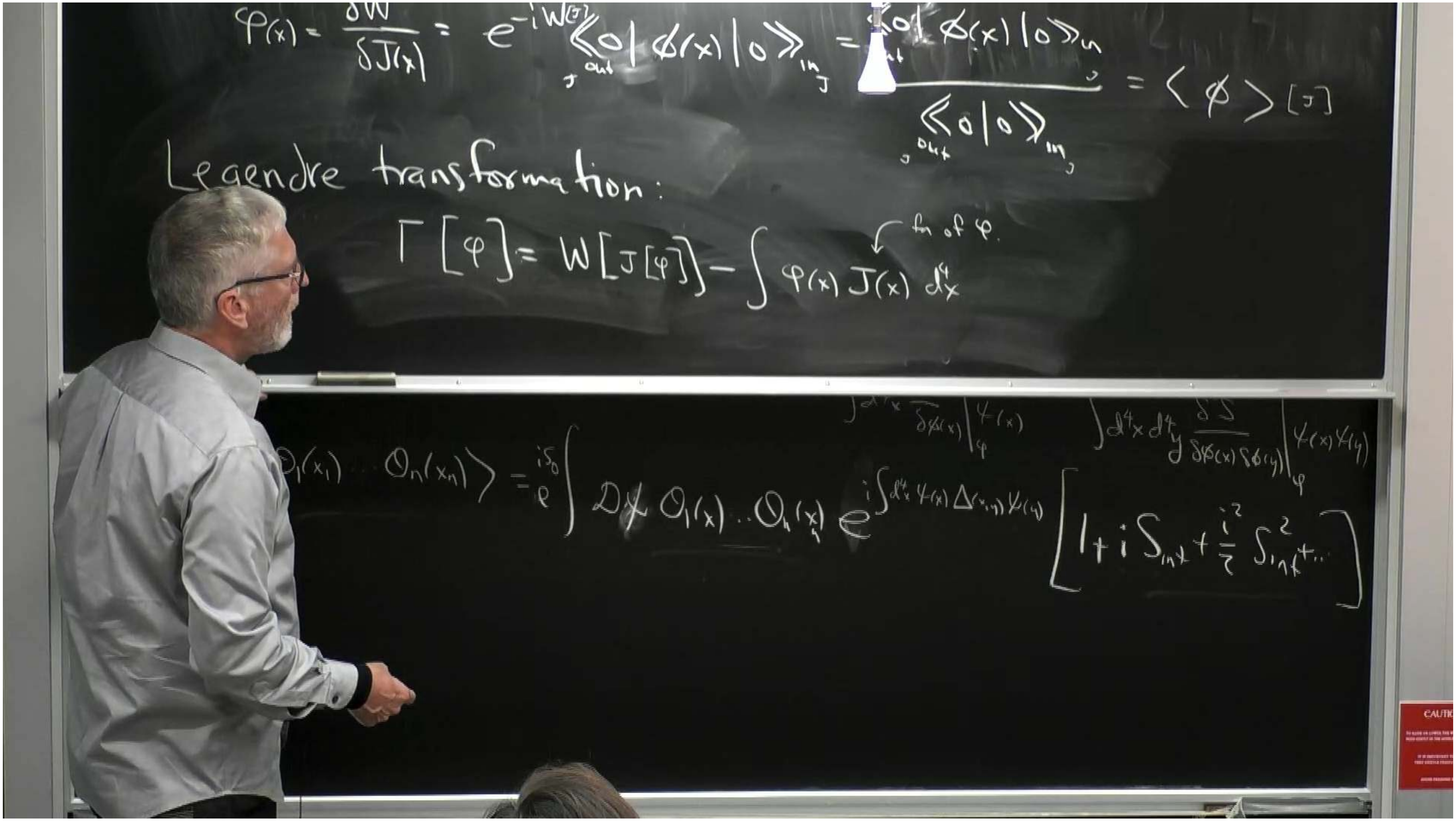
$S_{int} = S_3 + S_4$

$$S_{int}^2 = (S_3+S_4)^2 = S_3^2 + S_4^2 + 2S_3S_4$$

$3\cdot 2 \times 2$


Z = sum of all graphs with J(x)'s on external lines
1PI
1PR
disconnected
connected
$e^{iW[J]} = Z[J]$
$(-i)^n \frac{\delta^n W}{\delta J(x_1) \cdots \delta J(x_n)} = \langle \phi(x_1) \ldots \phi(x_n) \rangle_c$


$\frac{\delta}{\delta J(x)} e^{iW} = e^{iW} \frac{\delta W}{\delta J(x)} = \int \mathcal{D}\phi \; \phi(x) \, e^{iS(\phi) + i\int J\phi \, dx} = \langle \phi(x) \rangle$
$\frac{\delta W}{\delta J(x)} = e^{-iW} {}_{out}\langle\langle 0 | \phi(x) | 0 \rangle\rangle_{in} = \frac{{}_{out}\langle\langle 0 | \phi(x) | 0 \rangle\rangle_{in}}{{}_{out}\langle\langle 0 | 0 \rangle\rangle_{in}} = \langle \phi \rangle$


φ(x) = δW/δJ(x) = e^{-iW[J]} ⟪0| φ(x) |0⟫ = ⟪0| φ(x) |0⟫ / ⟪0|0⟫ = ⟨φ⟩[J]
Legendre transformation:
Γ[φ] = W[J[φ]] − ∫ φ(x) J(x) d⁴x
fn of φ.
O₁(x₁) ... Oₙ(xₙ)⟩ = e^{iS₀} ∫ Dφ O₁(x) .. Oₙ(x) e^{i∫d⁴x φ(x) Δ(x,y) φ(y)} [1 + i S_int + i²/2 S_int² + ...]

$$\int \varphi(x) J(x)\, d^4x$$

$$\frac{\delta\Gamma}{\delta\varphi(y)} = \cancel{\int d^4x\, \frac{\delta W}{\delta J(x)} \frac{\delta J(x)}{\delta\varphi(y)}} - J(y) - \cancel{\int d^4x\, \varphi(x) \frac{\delta J(x)}{\delta\varphi(y)}}$$

$$= -J(y)$$

Compare: $\frac{\delta S}{\delta\varphi(x)} + J(x) = 0$ classical eq.

$$\Gamma[\varphi] = W[J[\varphi]] - \int \varphi(x) \underbrace{J(x)}_{\text{fn of } \varphi} d^4x$$

$$\frac{\delta \Gamma}{\delta \varphi(y)} = \cancel{\int d^4x \frac{\delta W}{\delta J(x)} \frac{\delta J(x)}{\delta \varphi(y)}} - J(y) - \cancel{\int d^4x\, \varphi(x) \frac{\delta J(x)}{\delta \varphi(y)}}$$

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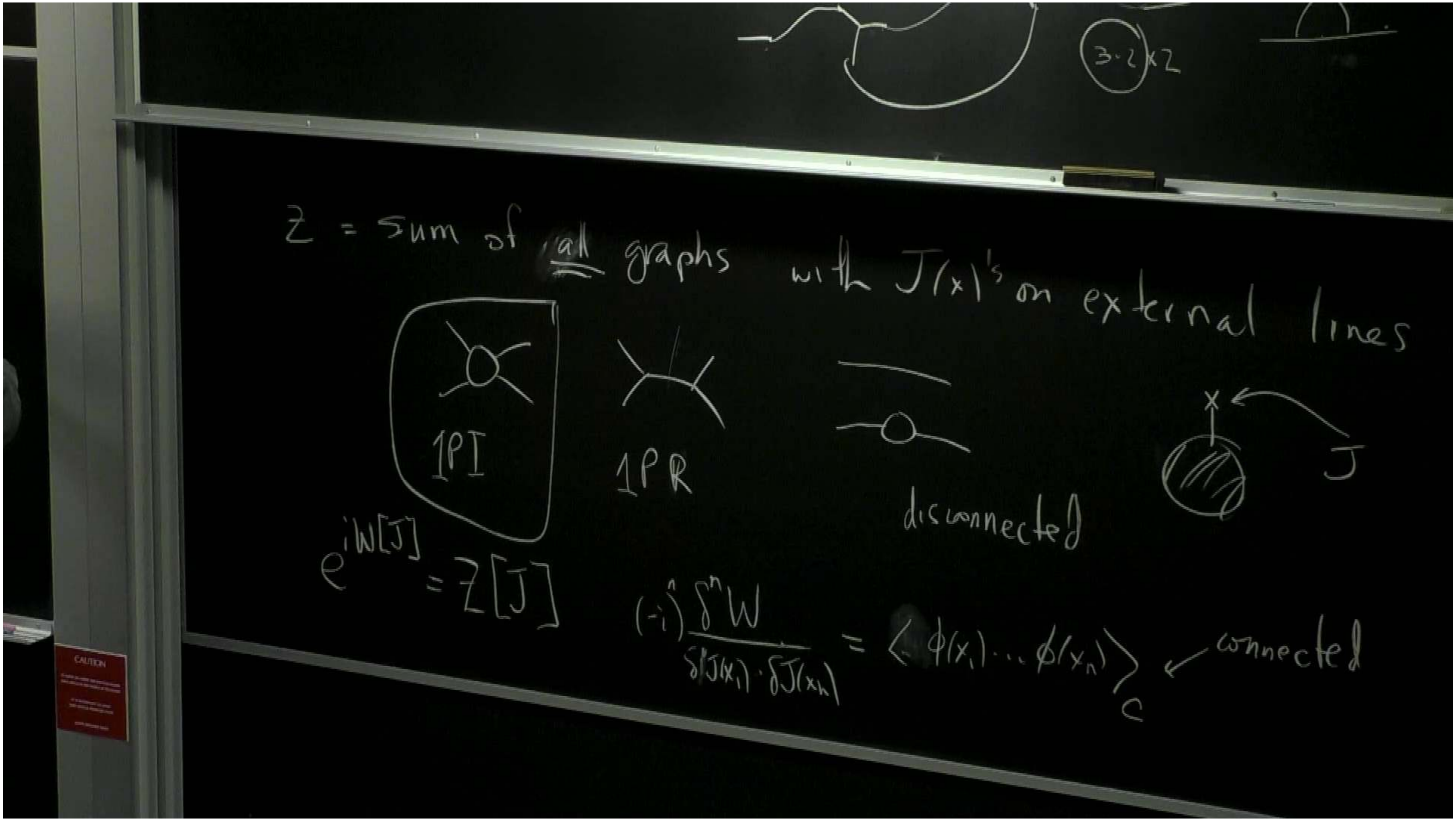
Compare: $\frac{\delta S}{\delta \varphi(x)} + J(x) = 0$ classical eq.

Static

$$-\Gamma[\varphi(\vec{x})] = \min_{\psi} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \quad \text{subject to } \langle \psi | \hat{\varphi} | \psi \rangle = \varphi(\vec{x})$$



3·2 x 2
Z = sum of all graphs with J(x)'s on external lines
1PI
1PR
disconnected
x
J
$e^{iW[J]} = Z[J]$
$(-i)^n \frac{\delta^n W}{\delta J(x_1) \cdots \delta J(x_n)} = \langle \phi(x_1) \cdots \phi(x_n) \rangle_c$
connected

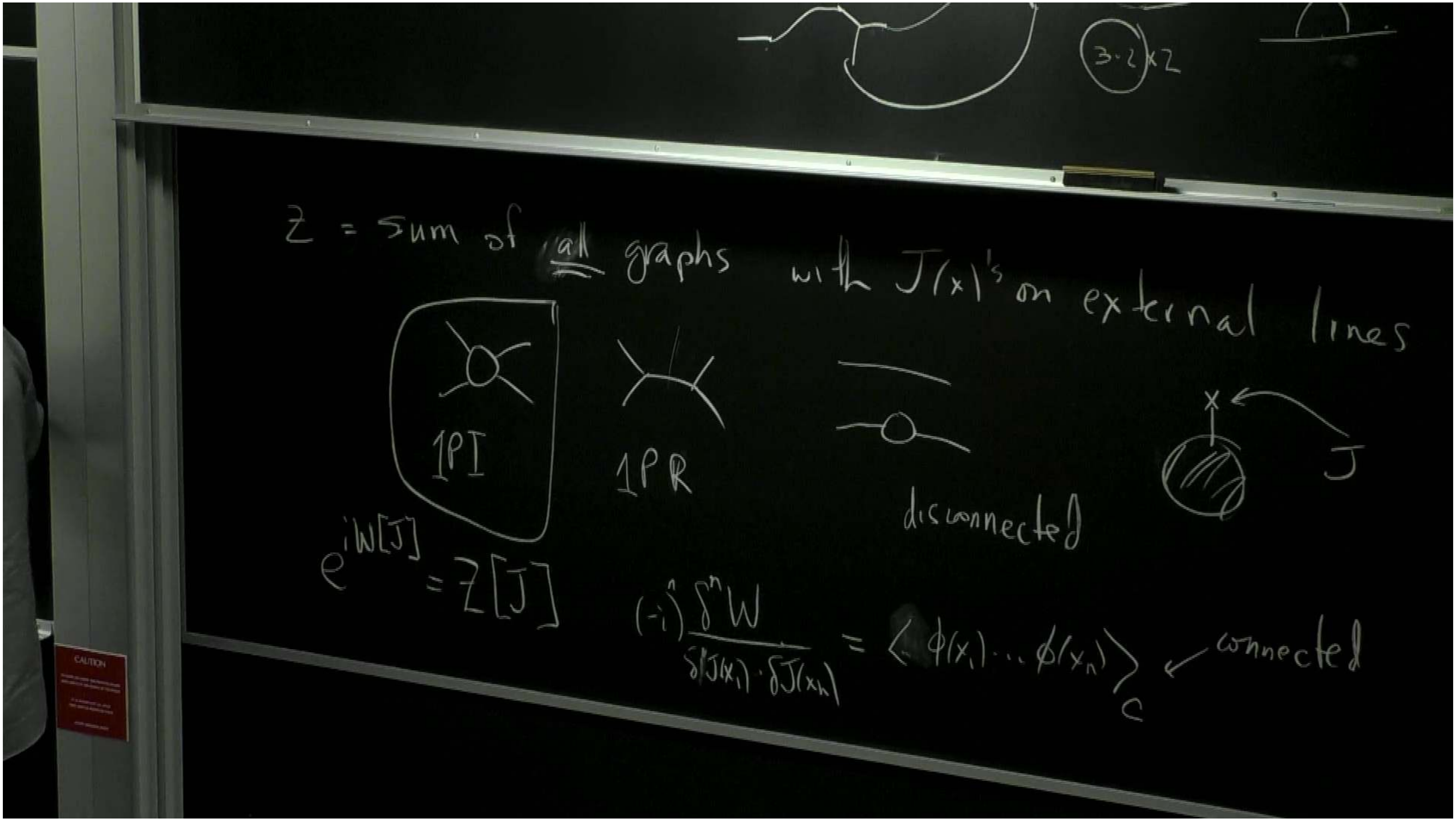
$$e^{iW(J)} = \int \mathcal{D}\phi \; e^{iS(\phi) + i\int \phi(x) J(x)\, d^4x}$$

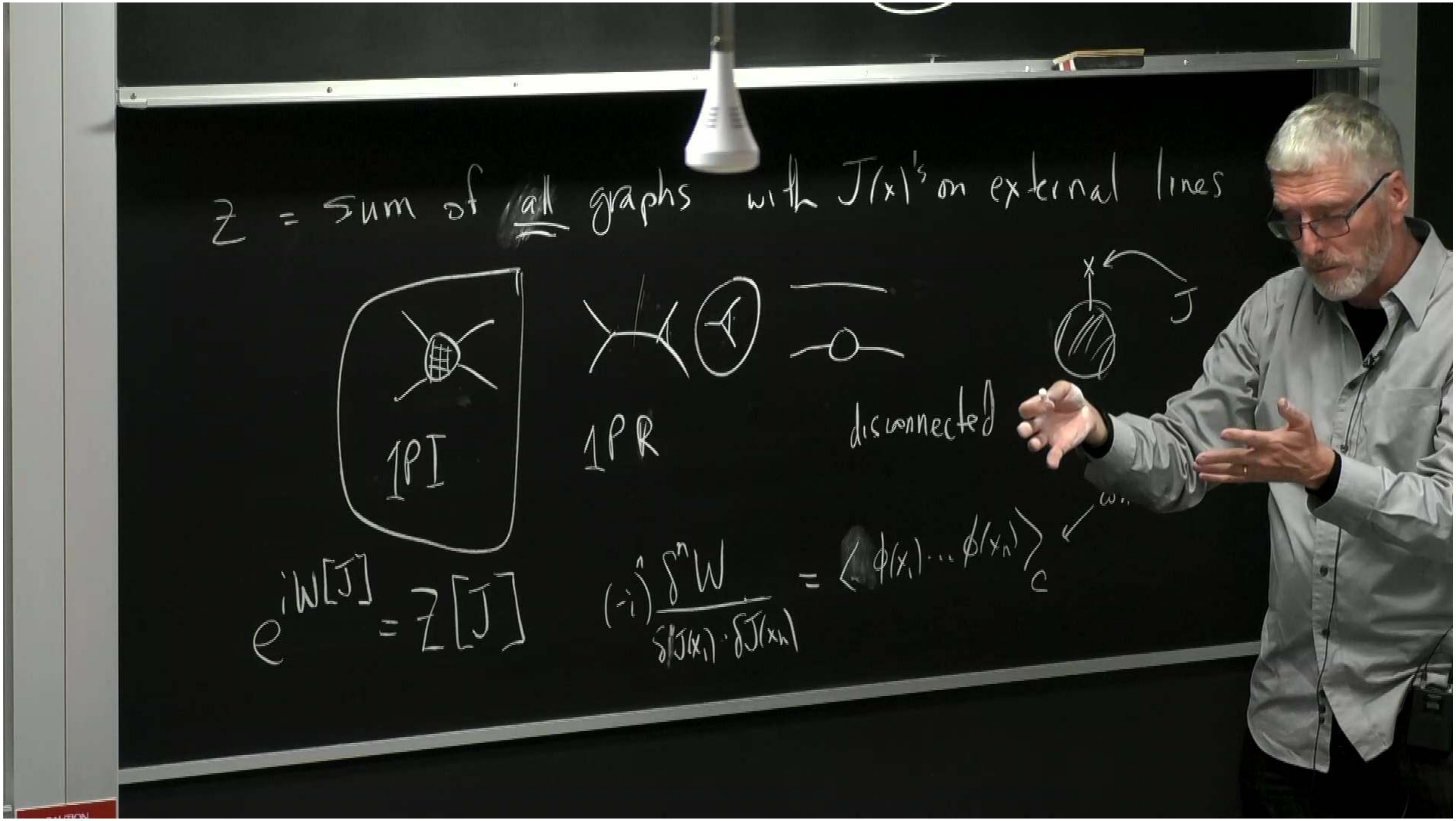
$$e^{i\Gamma[\varphi]} = e^{iW[J] - i\int \varphi J\, d^4x} = \int \mathcal{D}\psi \; e^{iS[\varphi+\psi] + i\int \psi J\, d^4x}$$

$J[\varphi]$

$$\boxed{J(\varphi) = -\frac{\delta \Gamma}{\delta \varphi}[\varphi]}$$

Claim: Γ = sum of all 1PI graphs.


3·2 x2
Z = sum of all graphs with J(x)'s on external lines
1PI
1PR
disconnected
J
e^{iW[J]} = Z[J]
(-i)^n δ^n W / δJ(x1)···δJ(xn) = ⟨φ(x1)···φ(xn)⟩_c
connected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$e^{iW[J]} = Z[J]$
$(-i)^n \frac{\delta^n W}{\delta J(x_1)\cdots\delta J(x_n)} = \langle \phi(x_1)\ldots\phi(x_n)\rangle_c$

$$\Gamma[\varphi] = W[J[\varphi]] - \int \varphi(x) \overset{\text{fn of } \varphi}{J(x)}\, d^4x$$

$$\frac{\delta \Gamma}{\delta \varphi(y)} = \cancel{\int d^4x \frac{\delta W}{\delta J(x)} \frac{\delta J(x)}{\delta \varphi(y)}} - J(y) - \cancel{\int d^4x\, \varphi(x) \frac{\delta J(x)}{\delta \varphi(y)}}$$

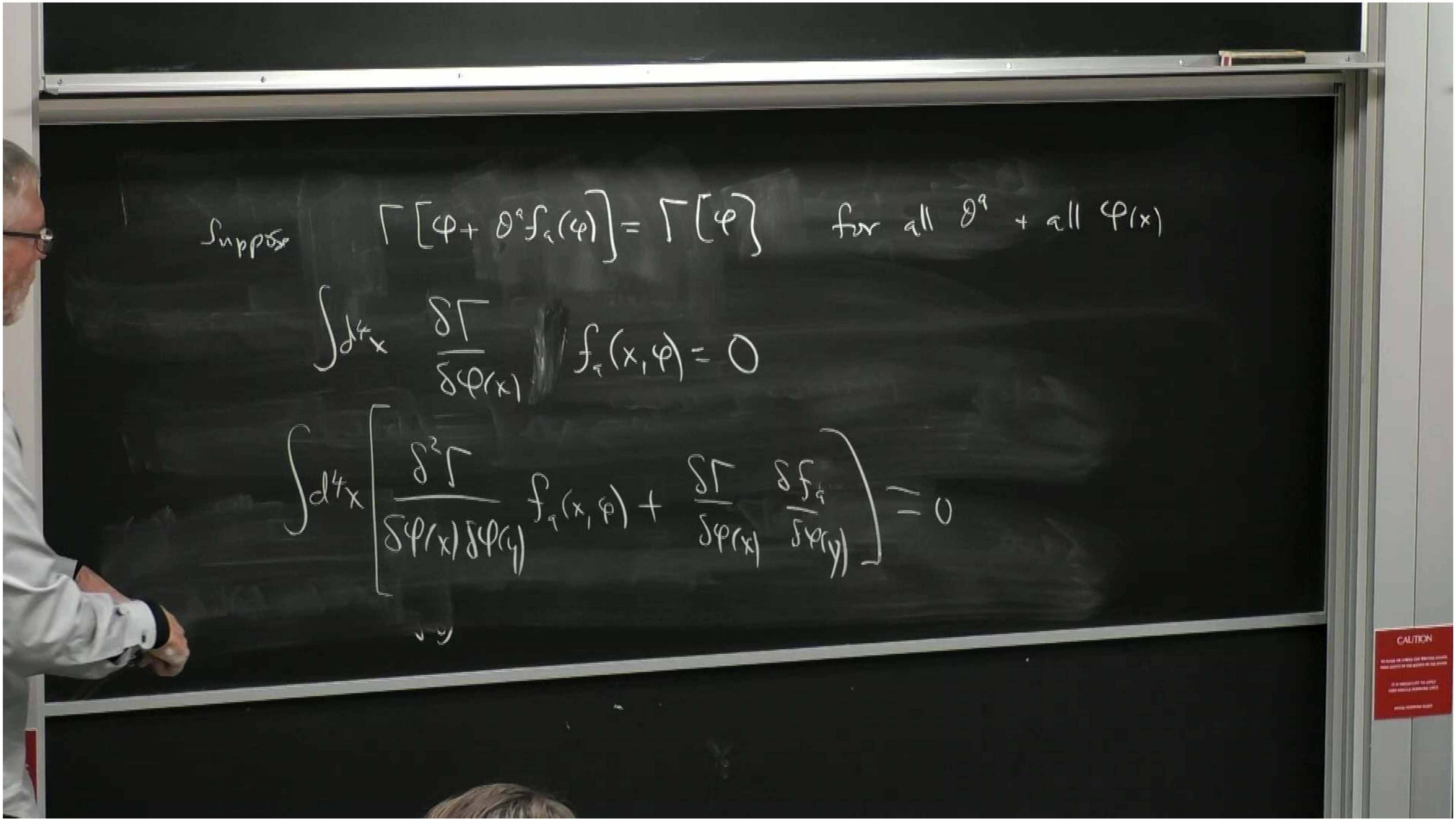
$$\frac{\delta \Gamma}{\delta \varphi(y)} = -J(y)$$

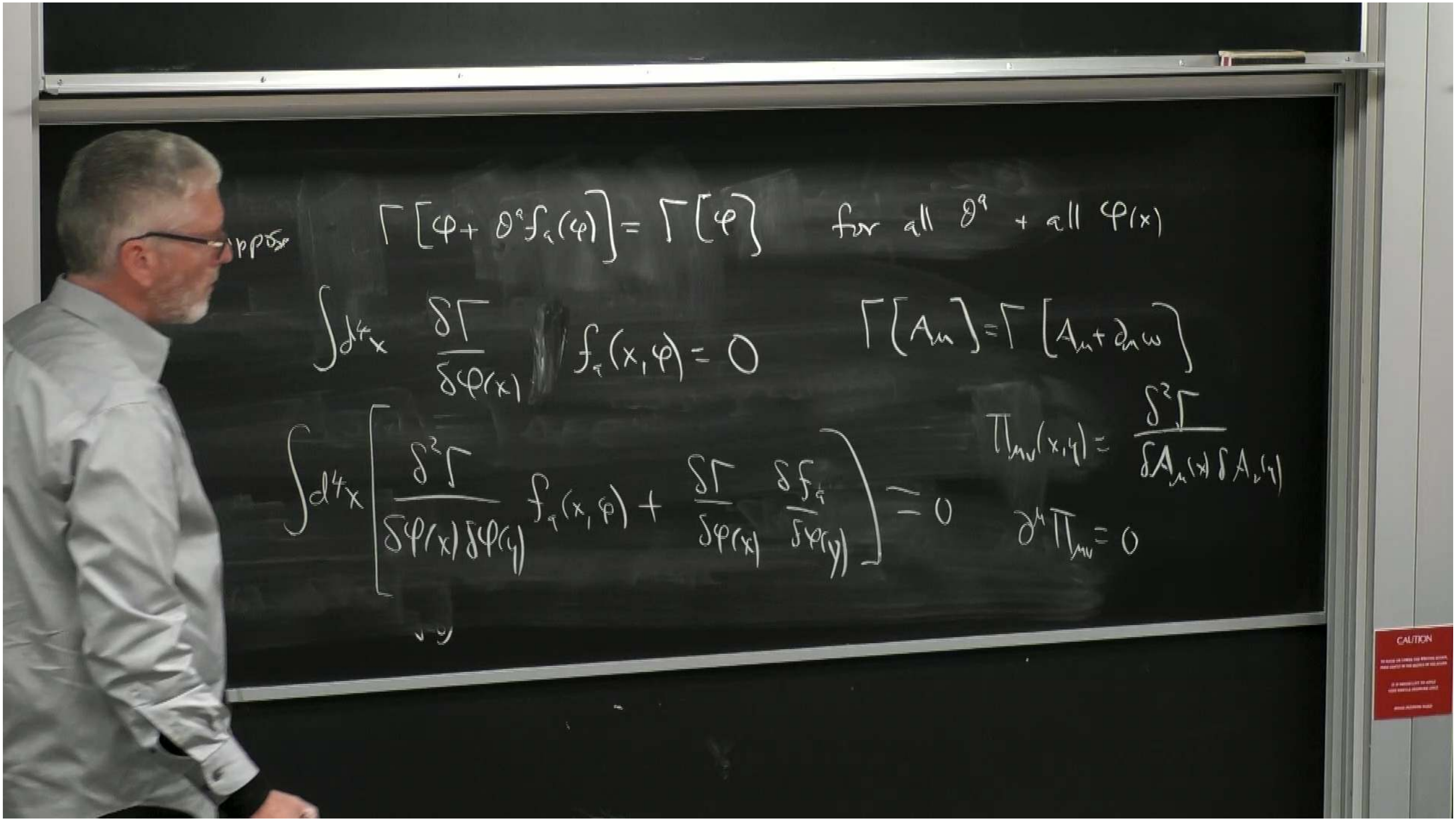
Compare: $\frac{\delta S}{\delta \varphi(x)} + J(x) = 0$ classical eq.

Static

$$-\Gamma[\varphi(\vec{x})] = \min_{\psi} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} \quad \text{subject to } \langle \psi | \phi | \psi \rangle = \varphi(\vec{x})$$

$$(-i)^n \frac{\delta^n \Gamma}{\delta \varphi(x_1) \cdots \delta \varphi(x_n)} \bigg|_{\varphi_0} = \langle \phi(x_1) \cdots \phi(x_n) \rangle_{1PI}$$


Suppose $\Gamma[\varphi + \theta^a f_a(\varphi)] = \Gamma[\varphi]$ for all θ^a + all $\varphi(x)$
$\int d^4x \, \frac{\delta \Gamma}{\delta \varphi(x)} f_a(x,\varphi) = 0$
$\int d^4x \left[\frac{\delta^2 \Gamma}{\delta \varphi(x) \delta \varphi(y)} f_a(x,\varphi) + \frac{\delta \Gamma}{\delta \varphi(x)} \frac{\delta f_a}{\delta \varphi(y)} \right] = 0$



$\Gamma[\varphi + \theta^a f_a(\varphi)] = \Gamma[\varphi]$ for all θ^a + all $\varphi(x)$
$\int d^4x \, \frac{\delta\Gamma}{\delta\varphi(x)} f_a(x,\varphi) = 0$
$\Gamma[A_\mu] = \Gamma[A_\mu + \partial_\mu \omega]$
$\Pi_{\mu\nu}(x,y) = \frac{\delta^2\Gamma}{\delta A_\mu(x)\,\delta A_\nu(y)}$
$\int d^4x \left[\frac{\delta^2\Gamma}{\delta\varphi(x)\,\delta\varphi(y)} f_a(x,\varphi) + \frac{\delta\Gamma}{\delta\varphi(x)} \frac{\delta f_a}{\delta\varphi(y)} \right] = 0$
$\partial^\mu \Pi_{\mu\nu} = 0$

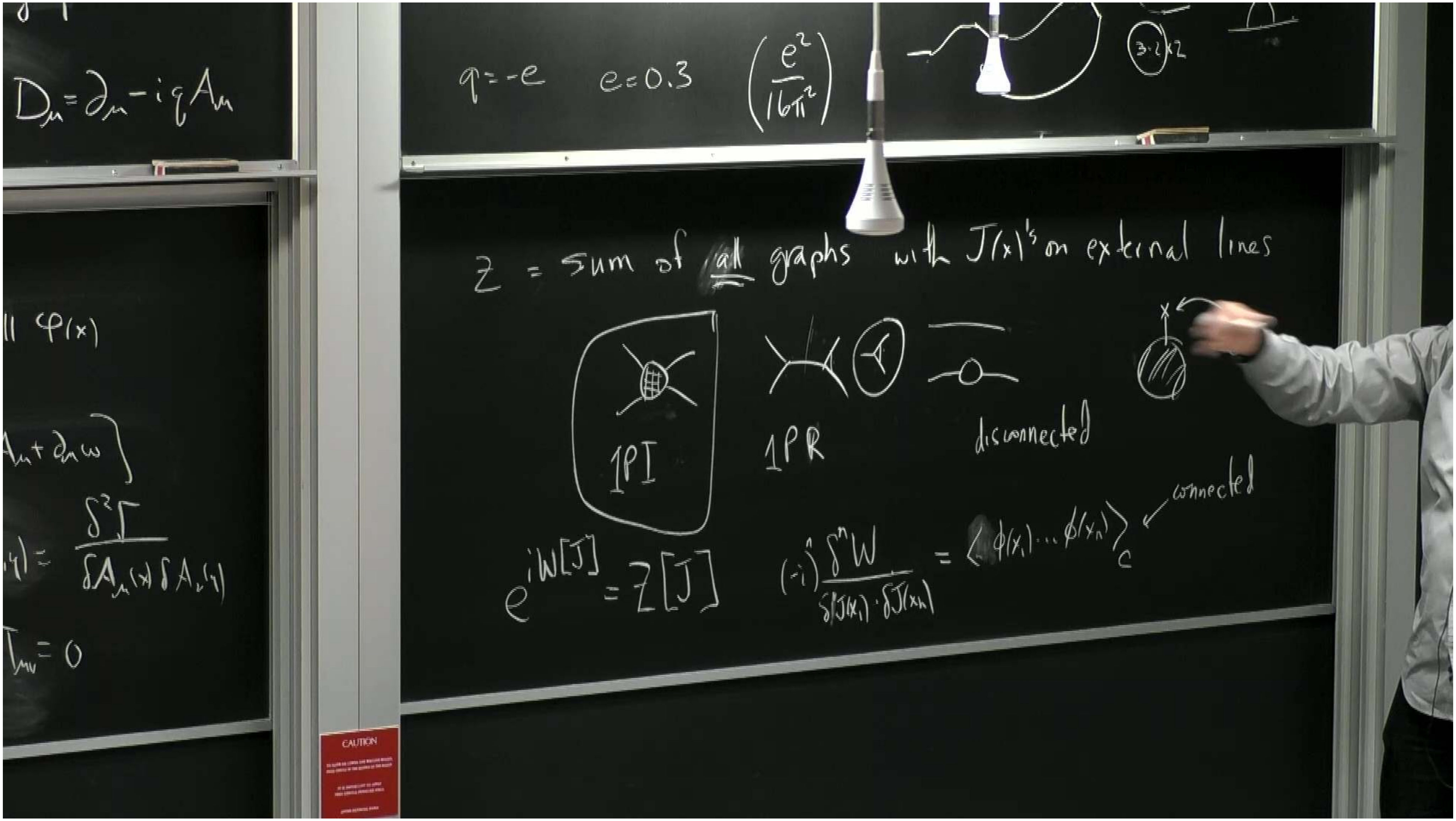
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi + iq\bar{\psi}\not{A}\psi \qquad D_\mu = \partial_\mu - iqA_\mu$$

Suppose $\Gamma[\varphi + \theta^a f_a(\varphi)] = \Gamma[\varphi]$ for all θ^a + all $\varphi(x)$

$$\int d^4x \, \frac{\delta \Gamma}{\delta \varphi(x)} f_a(x,\varphi) = 0 \qquad \Gamma[A_\mu] = \Gamma[A_\mu + \partial_\mu \omega]$$

$$\int d^4x \left[\frac{\delta^2 \Gamma}{\delta \varphi(x)\,\delta \varphi(y)} f_a(x,\varphi) + \frac{\delta \Gamma}{\delta \varphi(x)} \frac{\delta f_a}{\delta \varphi(y)} \right] = 0 \qquad \Pi_{\mu\nu}(x,y) = \frac{\delta^2 \Gamma}{\delta A_\mu(x)\,\delta A_\nu(y)}$$

$$\partial^\mu \Pi_{\mu\nu} = 0$$


$D_\mu = \partial_\mu - iqA_\mu$
$q = -e$
$e = 0.3$
$\left(\frac{e^2}{16\pi^2}\right)$
Z = sum of all graphs with J(x)'s on external lines
1PI
1PR
disconnected
connected
$e^{iW[J]} = Z[J]$
$(-i)^n \frac{\delta^n W}{\delta J(x_1) \cdots \delta J(x_n)} = \langle \phi(x_1) \cdots \phi(x_n) \rangle_c$