Title: QFT2 - Quantum Electrodynamics - Morning Lecture

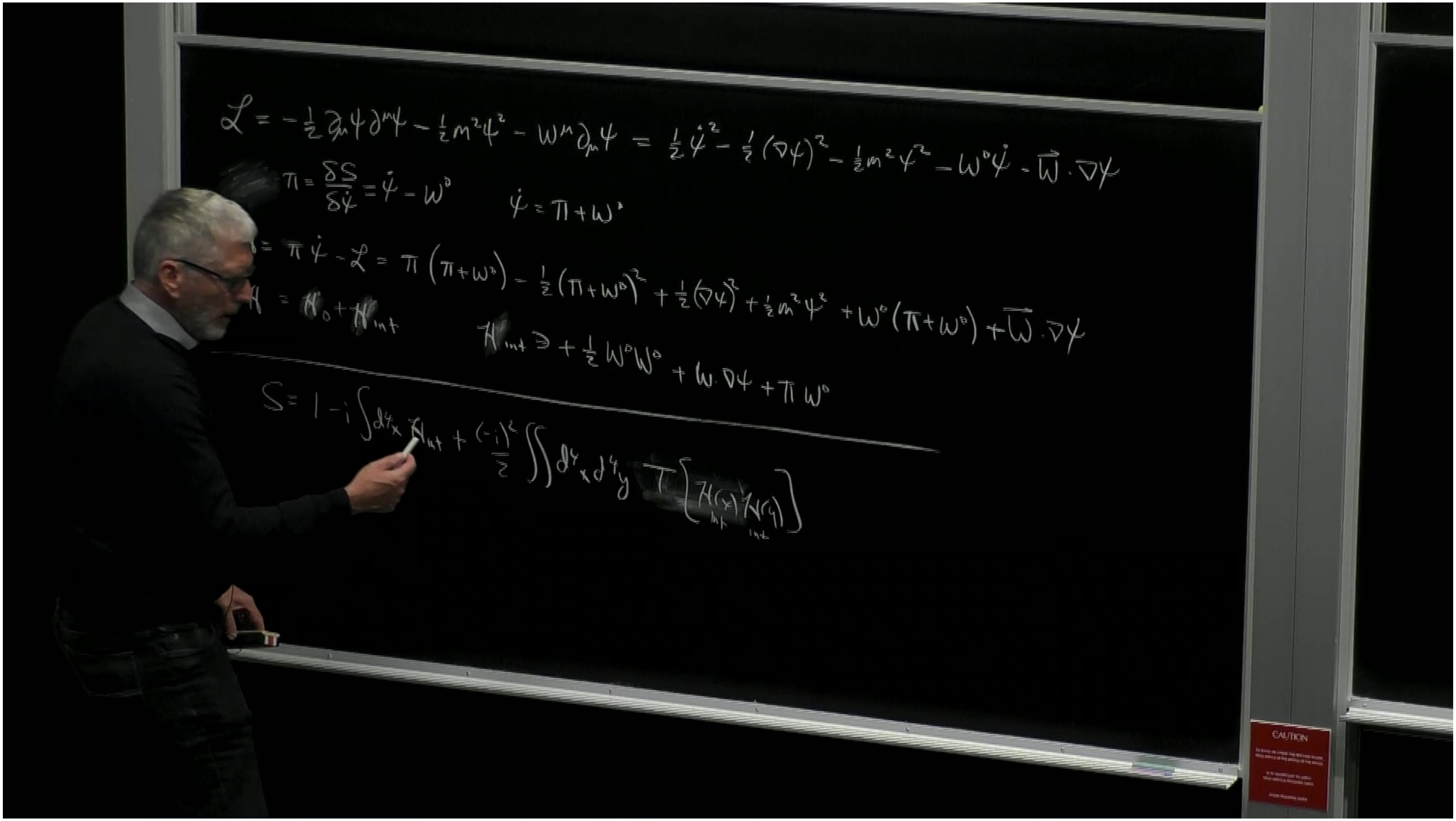
Speakers:

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

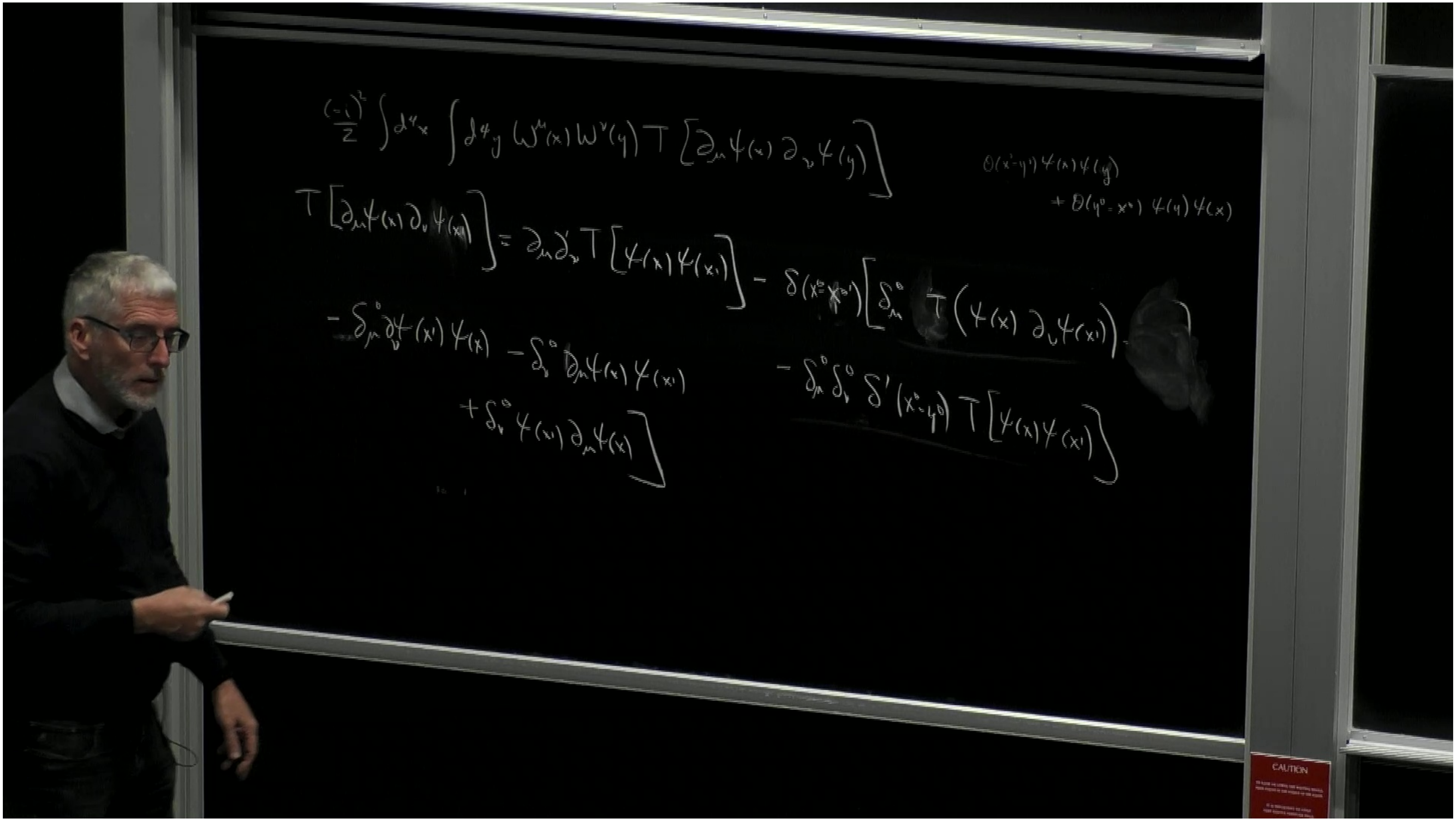
Date: November 15, 2022 - 10:00 AM

URL: https://pirsa.org/22110035

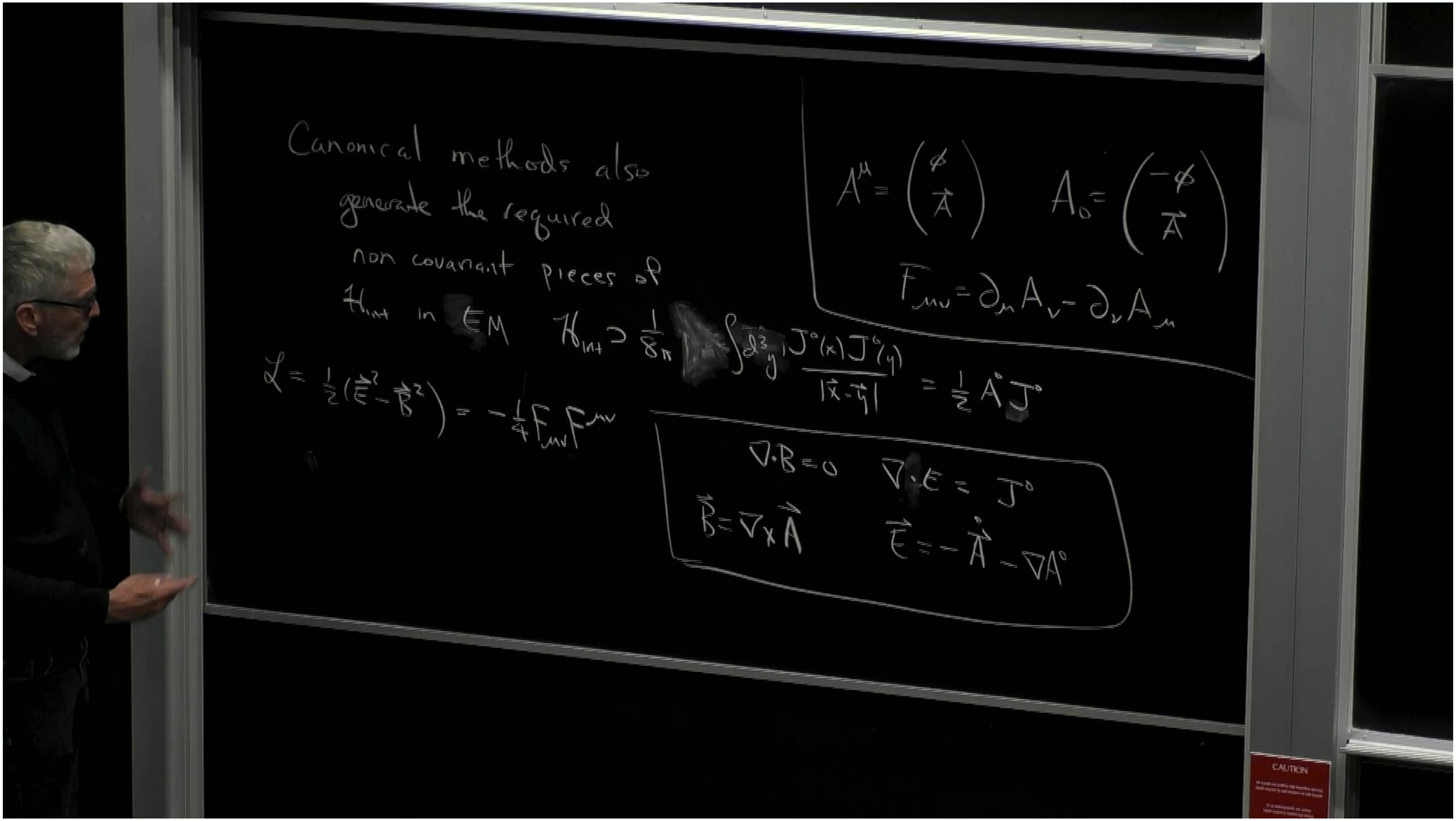
Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.



$\mathcal{L} = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{2}m^2\psi^2 - W^\mu\partial_\mu\psi = \frac{1}{2}\dot\psi^2 - \frac{1}{2}(\nabla\psi)^2 - \frac{1}{2}m^2\psi^2 - W^0\dot\psi - \vec{W}\cdot\nabla\psi$
$\pi = \frac{\delta S}{\delta\dot\psi} = \dot\psi - W^0$
$\dot\psi = \pi + W^0$
$= \pi\dot\psi - \mathcal{L} = \pi(\pi + W^0) - \frac{1}{2}(\pi + W^0)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{1}{2}m^2\psi^2 + W^0(\pi + W^0) + \vec{W}\cdot\nabla\psi$
$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}$
$\mathcal{H}_{int} \ni +\frac{1}{2}W^0W^0 + W\cdot\nabla\psi + \pi W^0$
$S = 1 - i\int d^4x\, \mathcal{H}_{int} + \frac{(-i)^2}{2}\iint d^4x\, d^4y\, T\left[\mathcal{H}_{int}(x)\mathcal{H}_{int}(y)\right]$
CAUTION



$\frac{(-i)^2}{2}\int d^4x \int d^4y\, W^\mu(x) W^\nu(y)\, T\left[\partial_\mu \psi(x)\, \partial_\nu \psi(y)\right]$
$\theta(x^0-y^0)\,\psi(x)\psi(y)$
$+\,\theta(y^0-x^0)\,\psi(y)\psi(x)$
$T\left[\partial_\mu\psi(x)\partial_\nu\psi(x')\right] = \partial_\mu\partial'_\nu T\left[\psi(x)\psi(x')\right] - \delta(x^0-x'^0)\left[\delta_\mu^0\, T\left(\psi(x)\,\partial_\nu\psi(x')\right)\right.$
$-\,\delta_\mu^0\,\partial'_\nu\psi(x')\psi(x) - \delta_\nu^0\,\partial_\mu\psi(x)\psi(x')$
$\left.+\,\delta_\nu^0\,\psi(x')\,\partial_\mu\psi(x)\right]$
$-\,\delta_\mu^0\delta_\nu^0\,\delta'(x^0-y^0)\, T\left[\psi(x)\psi(x')\right]$
CAUTION


Canonical methods also generate the required non covariant pieces of $\mathcal{H}_{int}$ in EM
$\mathcal{H}_{int} \supset \frac{1}{8\pi} \int d^3y \, \frac{J^0(x) J^0(y)}{|\vec{x}-\vec{y}|} = \frac{1}{2} A^0 J^0$
$\mathcal{L} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$A^\mu = \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix}$ $A_\mu = \begin{pmatrix} -\phi \\ \vec{A} \end{pmatrix}$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\nabla \cdot B = 0$ $\nabla \cdot E = J^0$
$\vec{B} = \nabla \times \vec{A}$ $\vec{E} = -\dot{\vec{A}} - \nabla A^0$
CAUTION

$$\vec{\pi} = \frac{\delta S}{\delta \dot{\vec{A}}} = -\vec{E}$$

$$\boxed{\pi_0 = \frac{\delta S}{\delta \dot{A}^0} = 0}$$

Constraint: cannot solve for $\dot{A}_0 = f(\ldots\ldots)$

"Constrained Hamiltonian systems"

Dirac (1950's CJM)

Henneaux + Teitelboim

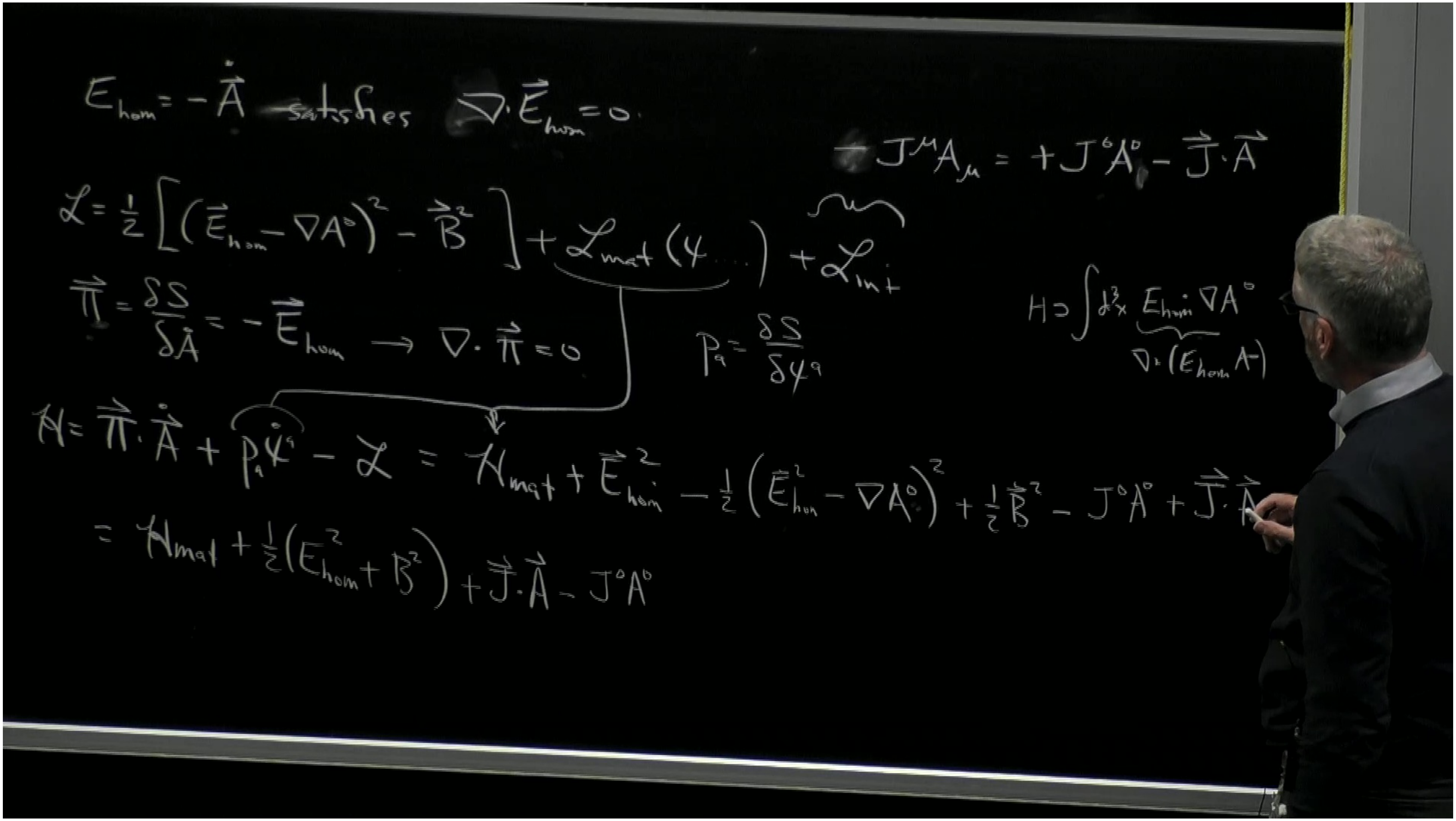
Strategy: treat A^0 as an explicit function of the other degrees of freedom

$$-\nabla^2 A^0 = J^0 \longrightarrow A^0(x,t) = \frac{1}{4\pi}\int d^3y \, \frac{J^0(y,t)}{|\vec{x}-\vec{y}|}$$

$E_{hom} = -\dot{\vec{A}}$ satisfies $\nabla \cdot \vec{E}_{hom} = 0$

$$\mathcal{L} = \frac{1}{2}\left[(\vec{E}_{hom} - \nabla A^0)^2 - \vec{B}^2\right] + \mathcal{L}_{mat}(\psi \ldots) + \underbrace{\mathcal{L}_{int}}_{+J}$$

π


$E_{hom} = -\dot{\vec{A}}$ ~~satisfies~~ $\nabla\cdot\vec{E}_{hom} = 0$
$-J^{\mu}A_{\mu} = +J^{0}A^{0} - \vec{J}\cdot\vec{A}$
$\mathcal{L} = \frac{1}{2}\left[(\vec{E}_{hom} - \nabla A^{0})^{2} - \vec{B}^{2}\right] + \mathcal{L}_{mat}(\psi \dots) + \mathcal{L}_{int}$
$\vec{\pi} = \frac{\delta S}{\delta \dot{\vec{A}}} = -\vec{E}_{hom} \rightarrow \nabla\cdot\vec{\pi} = 0$
$p_q = \frac{\delta S}{\delta \dot{\psi}^{q}}$
$H \supset \int d^3x\, \underbrace{E_{hom}\cdot\nabla A^{0}}_{\nabla\cdot(E_{hom}A^{0})}$
$\mathcal{H} = \vec{\pi}\cdot\dot{\vec{A}} + p_q\dot{\psi}^{q} - \mathcal{L} = \mathcal{H}_{mat} + \vec{E}_{hom}^{2} - \frac{1}{2}(\vec{E}_{hom} - \nabla A^{0})^{2} + \frac{1}{2}\vec{B}^{2} - J^{0}A^{0} + \vec{J}\cdot\vec{A}$
$= \mathcal{H}_{mat} + \frac{1}{2}(E_{hom}^{2} + B^{2}) + \vec{J}\cdot\vec{A} - J^{0}A^{0}$

$E_{hom} = -\dot{\vec{A}}$ satisfies $\nabla\cdot\vec{E}_{hom} = 0$.

$$-J^{\mu}A_{\mu} = +J^0 A^0 - \vec{J}\cdot\vec{A}$$

$$\mathcal{L} = \frac{1}{2}\left[(\vec{E}_{hom} - \nabla A^0)^2 - \vec{B}^2\right] + \mathcal{L}_{mat}(\psi \ldots) + \mathcal{L}_{int}$$

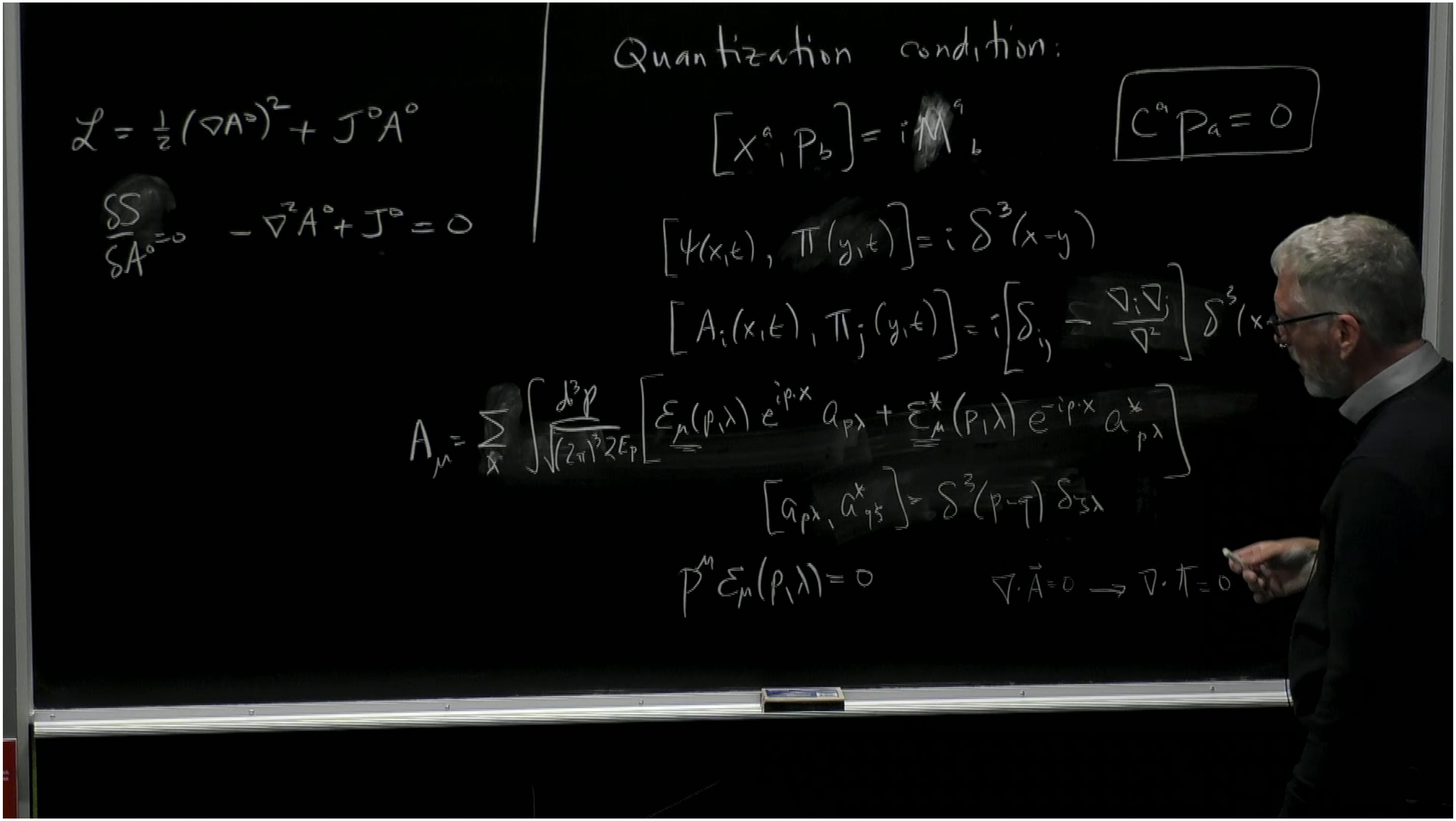
$$\vec{\pi} = \frac{\delta S}{\delta \dot{\vec{A}}} = -\vec{E}_{hom} \to \nabla\cdot\vec{\pi} = 0$$

$$p_a = \frac{\delta S}{\delta \dot{\psi}^a}$$

$$\cancel{H \supset \int d^3x\, \underbrace{E_{hom}\cdot\nabla A^0}_{\nabla\cdot(E_{hom}A^0)}}$$

$$\mathcal{H} = \vec{\pi}\cdot\dot{\vec{A}} + p_a\dot{\psi}^a - \mathcal{L} = \mathcal{H}_{mat} + \vec{E}_{hom}^2 - \frac{1}{2}(\vec{E}_{hom} - \nabla A^0)^2 + \frac{1}{2}\vec{B}^2 - J^0A^0 + \vec{J}\cdot\vec{A}$$

$$= \mathcal{H}_{mat} + \frac{1}{2}(E_{hom}^2 + B^2) + \vec{J}\cdot\vec{A} - J^0A^0 + \underbrace{\frac{1}{2}A^0\nabla^2A^0}_{-\frac{1}{2}A^0J^0}$$


$\mathcal{L} = \frac{1}{2}(\nabla A^0)^2 + J^0 A^0$
$\frac{\delta S}{\delta A^0} = 0 \quad -\nabla^2 A^0 + J^0 = 0$
Quantization condition:
$[X^a, P_b] = i M^a{}_b$
$\boxed{C^a P_a = 0}$
$[\psi(x,t), \pi(y,t)] = i\delta^3(x-y)$
$[A_i(x,t), \pi_j(y,t)] = i\left[\delta_{ij} - \frac{\nabla_i \nabla_j}{\nabla^2}\right]\delta^3(x-$
$A_\mu = \sum_\lambda \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\left[\varepsilon_\mu(p,\lambda)\, e^{ip\cdot x} a_{p\lambda} + \varepsilon^*_\mu(p,\lambda)\, e^{-ip\cdot x} a^*_{p\lambda}\right]$
$[a_{p\lambda}, a^*_{q\xi}] = \delta^3(p-q)\,\delta_{\xi\lambda}$
$p^\mu \varepsilon_\mu(p,\lambda) = 0$
$\nabla\cdot\vec{A} = 0 \longrightarrow \nabla\cdot\pi = 0$

$$\mathcal{L} = \frac{1}{2}(\nabla A^0)^2 + J^0 A^0$$

$$\frac{\delta S}{\delta A^0} = 0 \quad -\nabla^2 A^0 + J^0 = 0$$

Quantization condition:

$$[X^a, P_b] = i M^a{}_b \qquad \boxed{C^a P_a = 0}$$

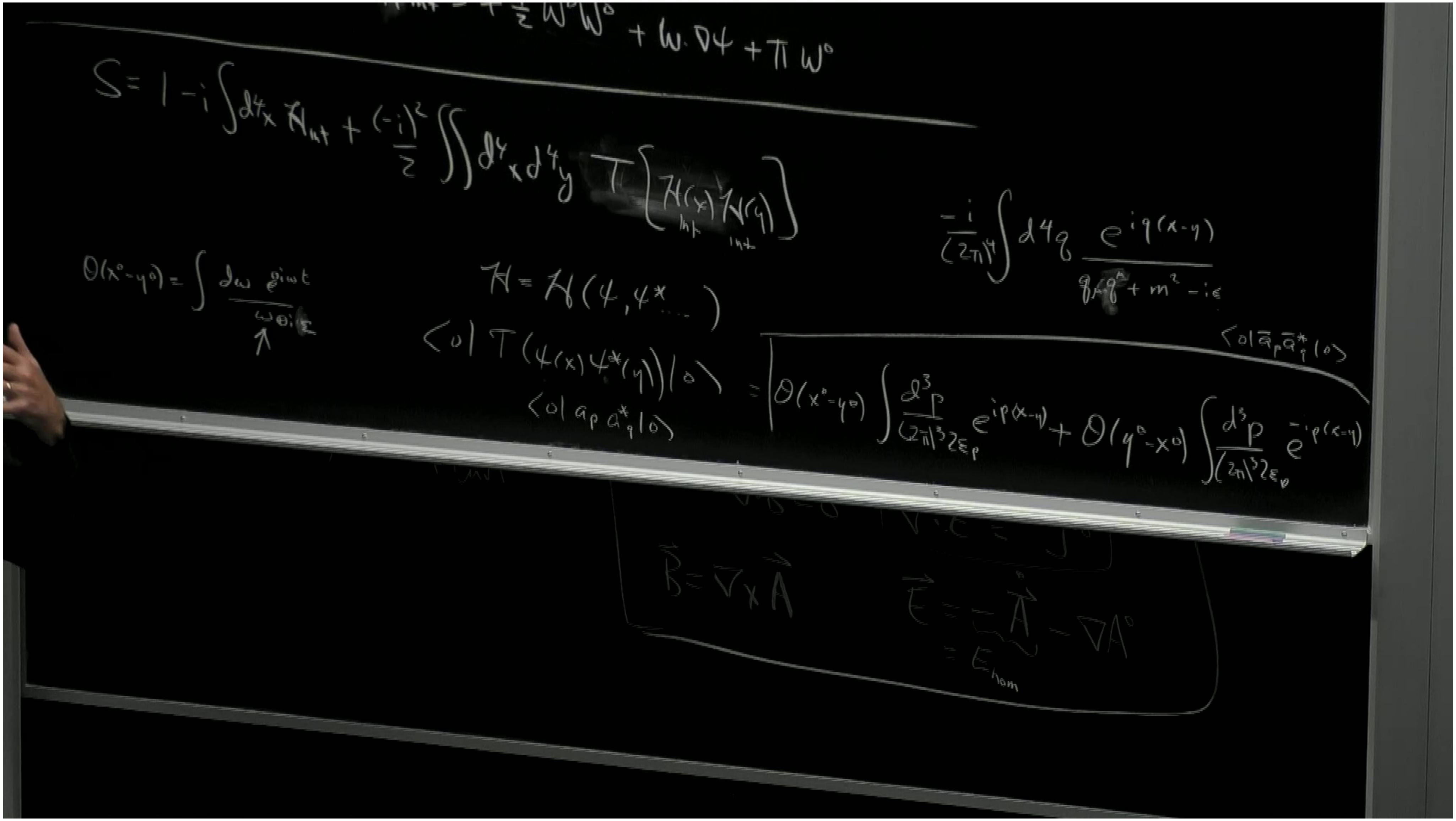
$$[\psi(x,t), \pi(y,t)] = i\,\delta^3(x-y)$$

$$[A_i(x,t), \pi_j(y,t)] = i\left[\delta_{ij} - \frac{\nabla_i \nabla_j}{\nabla^2}\right]\delta^3(x-y)$$

$$A_\mu = \sum_\lambda \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_p}}\left[\varepsilon_\mu(p,\lambda)\, e^{ip\cdot x} a_{p\lambda} + \varepsilon^*_\mu(p,\lambda)\, e^{-ip\cdot x} a^*_{p\lambda}\right]$$

$$[a_{p\lambda}, a^*_{q\xi}] = \delta^3(p-q)\,\delta_{\xi\lambda}$$

$$p^\mu \varepsilon_\mu(p,\lambda) = 0 \qquad \nabla\cdot\vec{A} = 0 \Rightarrow \nabla\cdot\pi = 0$$


$+ W \cdot D\psi + \pi W^0$
$S = 1 - i \int d^4x \, \mathcal{H}_{int} + \frac{(-i)^2}{2} \iint d^4x \, d^4y \, T\left[\mathcal{H}_{int}(x) \mathcal{H}_{int}(y)\right]$
$\frac{-i}{(2\pi)^4} \int d^4q \, \frac{e^{iq(x-y)}}{q_\mu q^\mu + m^2 - i\epsilon}$
$\Theta(x^0 - y^0) = \int d\omega \, \frac{e^{i\omega t}}{\omega \pm i\epsilon}$
$\mathcal{H} = \mathcal{H}(\psi, \psi^* \ldots)$
$\langle 0 | \bar{a}_p \bar{a}^*_q | 0 \rangle$
$\langle 0 | T(\psi(x) \psi^*(y)) | 0 \rangle = \Theta(x^0 - y^0) \int \frac{d^3p}{(2\pi)^3 2E_p} e^{ip(x-y)} + \Theta(y^0 - x^0) \int \frac{d^3p}{(2\pi)^3 2E_p} e^{-ip(x-y)}$
$\langle 0 | a_p a^*_q | 0 \rangle$
$\vec{B} = \nabla \times \vec{A}$
$\vec{E} = -\dot{\vec{A}} - \nabla A^0$
$= E_{hom}$

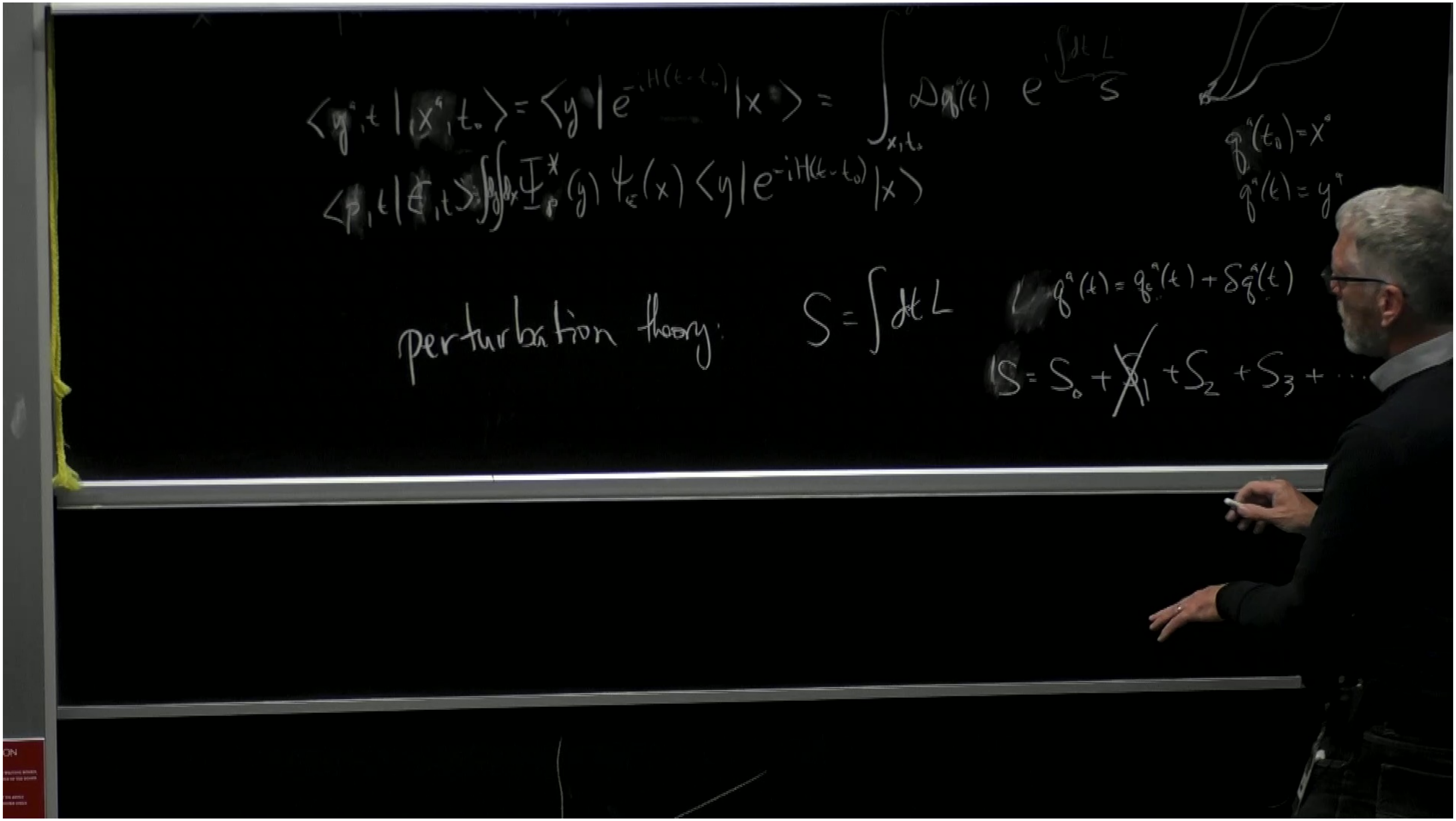
Path integrals: (Point: they allow calculations to be done explicitly using only $\mathcal{L}$ for which Lor. Inv. is manifest)

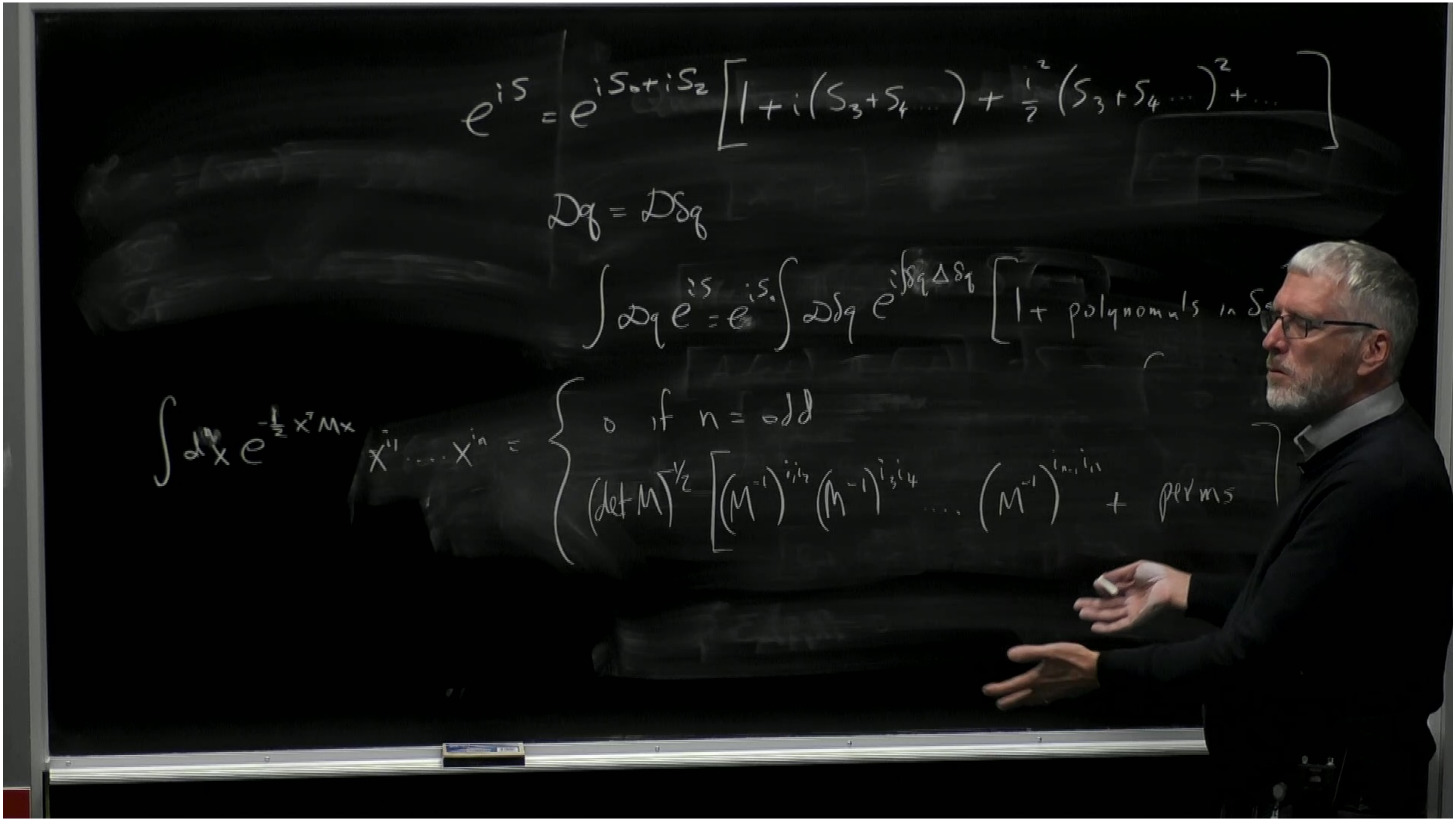
$x^a(t) \qquad p_a(t) \qquad H(p,x)$

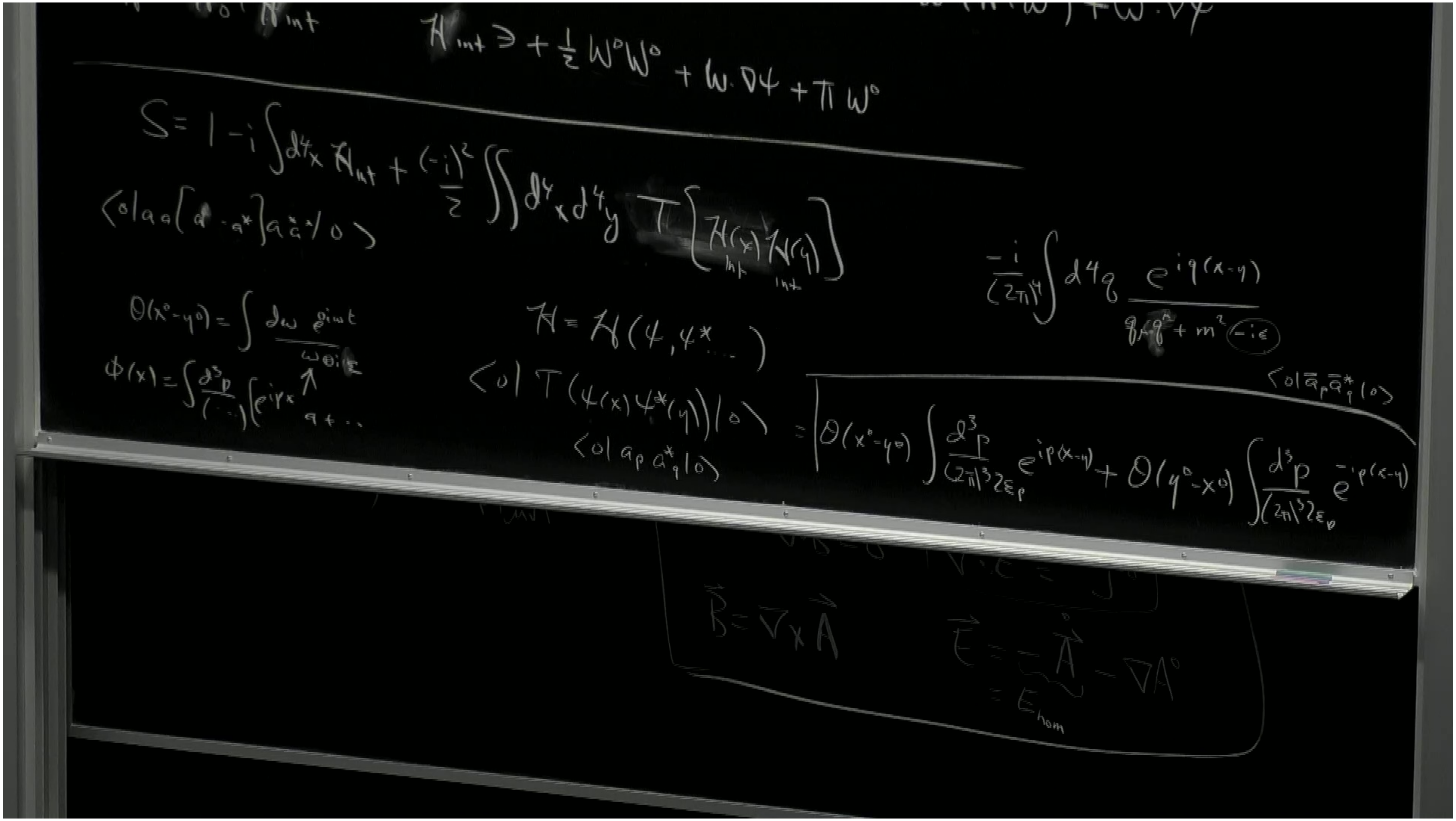
$$\langle y^a,t|x^a,t_0\rangle = \langle y|e^{-iH(t-t_0)}|x\rangle = \int_{x,t_0}^{y,t} \mathcal{D}q^a(t)\; e^{i\int dt\, L}$$

$$\langle p,t|E,t_0\rangle = \int dy\,dx\; \Psi_p^*(y)\,\psi_E(x)\,\langle y|e^{-iH(t-t_0)}|x\rangle$$

$q^a(t_0)=x^a$


$\langle y^a, t \mid x^a, t_0 \rangle = \langle y \mid e^{-iH(t-t_0)} \mid x \rangle = \int_{x, t_0} \mathcal{D}q^a(t)\, e^{i \overbrace{\int dt\, L}^{S}}$
$q^a(t_0) = x^a$
$q^a(t) = y^a$
$\langle p, t \mid E, t_0 \rangle = \int dy\, dx\, \Phi^*_p(y)\, \psi_E(x) \langle y \mid e^{-iH(t-t_0)} \mid x \rangle$
perturbation theory: $S = \int dt\, L$
$q^a(t) = q_c^a(t) + \delta q^a(t)$
$S = S_0 + \cancel{S_1} + S_2 + S_3 + \cdots$


$e^{iS} = e^{iS_0 + iS_2}\left[1 + i(S_3 + S_4 \dots) + \frac{i^2}{2}(S_3 + S_4 \dots)^2 + \dots\right]$
$\mathcal{D}q = \mathcal{D}\delta q$
$\int \mathcal{D}q\, e^{iS} = e^{iS_0}\int \mathcal{D}\delta q\, e^{i\delta q \Delta \delta q}\left[1 + \text{polynomials in } \delta q\right.$
$\int d^n x\, e^{-\frac{1}{2}x^T M x}\, x^{i_1} \dots x^{i_n} = \begin{cases} 0 \text{ if } n = \text{odd} \\ (\det M)^{-1/2}\left[(M^{-1})^{i_1 i_2}(M^{-1})^{i_3 i_4} \dots (M^{-1})^{i_{n-1} i_n} + \text{perms}\right] \end{cases}$


H_int ∋ + ½ W⁰W⁰ + W·∇φ + π W⁰
S = 1 − i ∫d⁴x H_int + (−i)²/2 ∬ d⁴x d⁴y T[H_int(x) H_int(y)]
Θ(x⁰−y⁰) = ∫ dω e^{iωt}/(ω ± iε)
φ(x) = ∫ d³p/(…) [e^{ipx} a + …]
H = H(φ, φ*, …)
⟨0| T(φ(x) φ*(y)) |0⟩
⟨0| a_p a*_q |0⟩
= Θ(x⁰−y⁰) ∫ d³p/((2π)³ 2ε_p) e^{ip(x−y)} + Θ(y⁰−x⁰) ∫ d³p/((2π)³ 2ε_p) e^{−ip(x−y)}
−i/(2π)⁴ ∫ d⁴q e^{iq(x−y)}/(q_μ q^μ + m² − iε)
⟨0| ā_p ā*_q |0⟩
B = ∇×A
E = −Ȧ − ∇A⁰ = E_hom

$$e^{iS} = e^{iS_0 + iS_2}\left[1 + i(S_3 + S_4 \dots) + \frac{i^2}{2}(S_3 + S_4 \dots)^2 + \dots\right]$$

$$\mathcal{D}q = \mathcal{D}\delta q$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2$$
$$= +\frac{1}{2}\phi(\Box + m^2)\phi$$

$$\int \mathcal{D}q\, e^{iS} = e^{iS_0}\int \mathcal{D}\delta q\, e^{i\int \delta q \Delta \delta q}\left[1 + \text{polynomials in } \delta q\right]$$

$$\int d^n x\, e^{-\frac{1}{2}x^T M x}\, x^{i_1} \dots x^{i_n} = \begin{cases} 0 \text{ if } n = \text{odd} \\ (\det M)^{-1/2}\left[(M^{-1})^{i_1 i_2}(M^{-1})^{i_3 i_4} \dots (M^{-1})^{i_{n-1} i_n} + \text{perms}\right] \end{cases}$$

e.g. $q^n(t) \to \phi^n(\vec{x}, t)$

$$\phi = \varphi + \delta\phi$$

$$\mathcal{L} = \underbrace{\mathcal{L}_0 + \cancel{\mathcal{L}_1} + \mathcal{L}_2}\; + \underbrace{\mathcal{L}_3 + \dots}_{1 + \dots}$$

$$\langle \varphi(\cdot), t \,|\, T[\hat{\phi}(z_1,t_1) \cdots \hat{\phi}(z_n,t_n)] \,|\, \psi(\cdot), t_0 \rangle = \int \mathcal{D}\phi \; e^{i\int d^4x\, \mathcal{L}} \; \phi(z_1,t_1) \dots \phi(z_n,t_n)$$

$$\hat{\phi}(x)\,|\varphi(\cdot)\rangle = \varphi(x)\,|\varphi(\cdot)\rangle$$

$$e^{iS} = e^{iS_0 + iS_2}\left[1 + i(S_3 + S_4 \dots) + \frac{i^2}{2}(S_3 + S_4 \dots)^2 + \dots\right]$$

$$\mathcal{D}q = \mathcal{D}\delta q$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$
$$= +\frac{1}{2}\phi(\Box - m^2)\phi$$

$$\int \mathcal{D}q\, e^{iS} = e^{iS_0}\int \mathcal{D}\delta q\, e^{i\delta q\,\Delta\,\delta q}\left[1 + \text{polynomials in } \delta q\right]$$

$$\int d^n x\, e^{-\frac{1}{2}x^T M x}\, x^{i_1}\dots x^{i_n} = \begin{cases} 0 & \text{if } n = \text{odd} \\ (\det M)^{-1/2}\left[(M^{-1})^{i_1 i_2}(M^{-1})^{i_3 i_4}\dots(M^{-1})^{i_{n-1} i_n} + \text{perms}\right] \end{cases}$$

$$\int \mathcal{D}\phi\, e^{-\frac{i}{2}\int d^4x\, \phi\Delta\phi}\, \phi(x,t)\,\phi(y,\tau) = (\det\Delta)^{-1/2}\, G(x,y)$$

$$\Delta u_n = \lambda_n u_n$$

$$\det\Delta = \prod_n \lambda_n$$

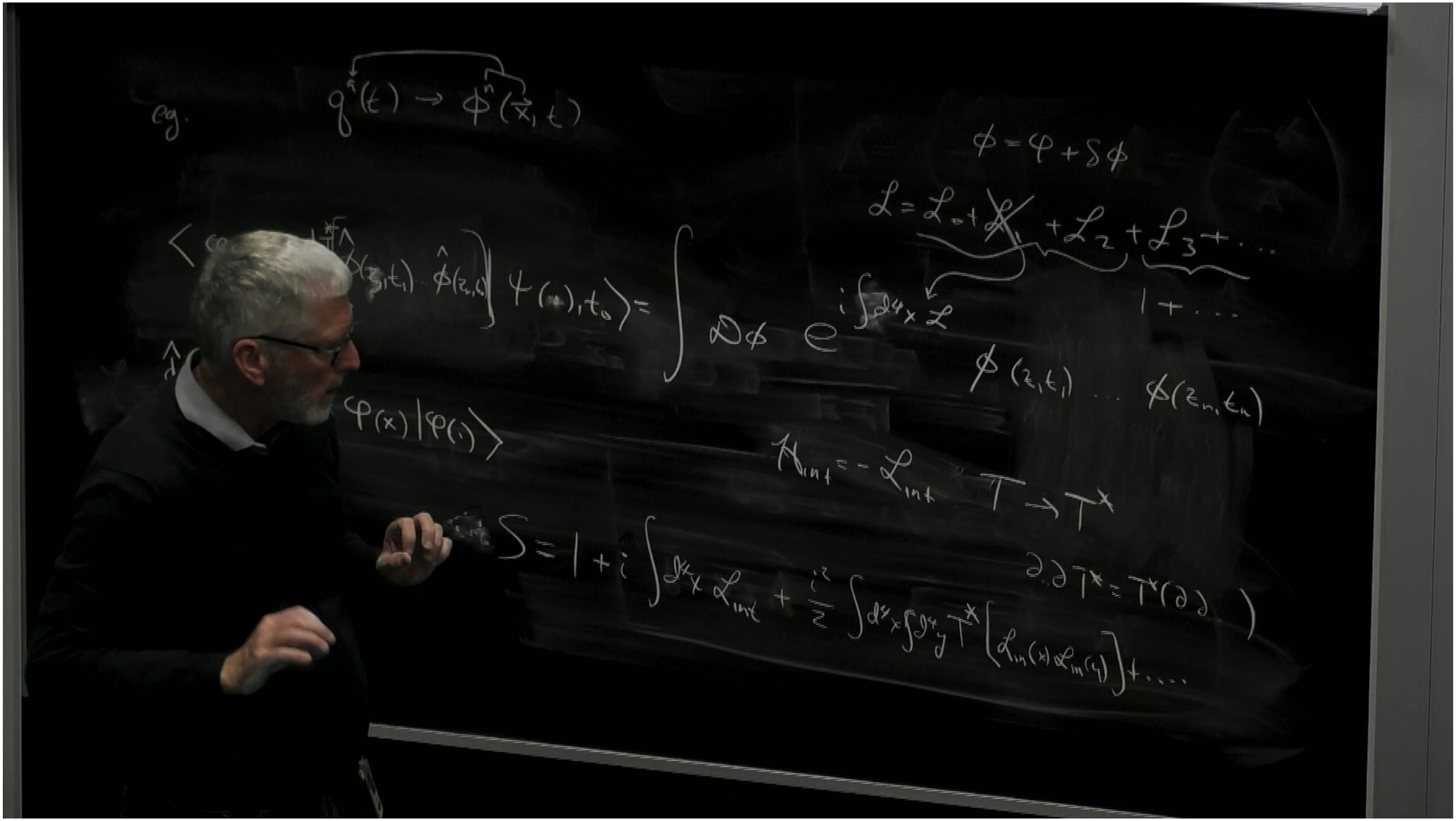
e.g. $q^n(t) \to \phi^n(\vec{x}, t)$

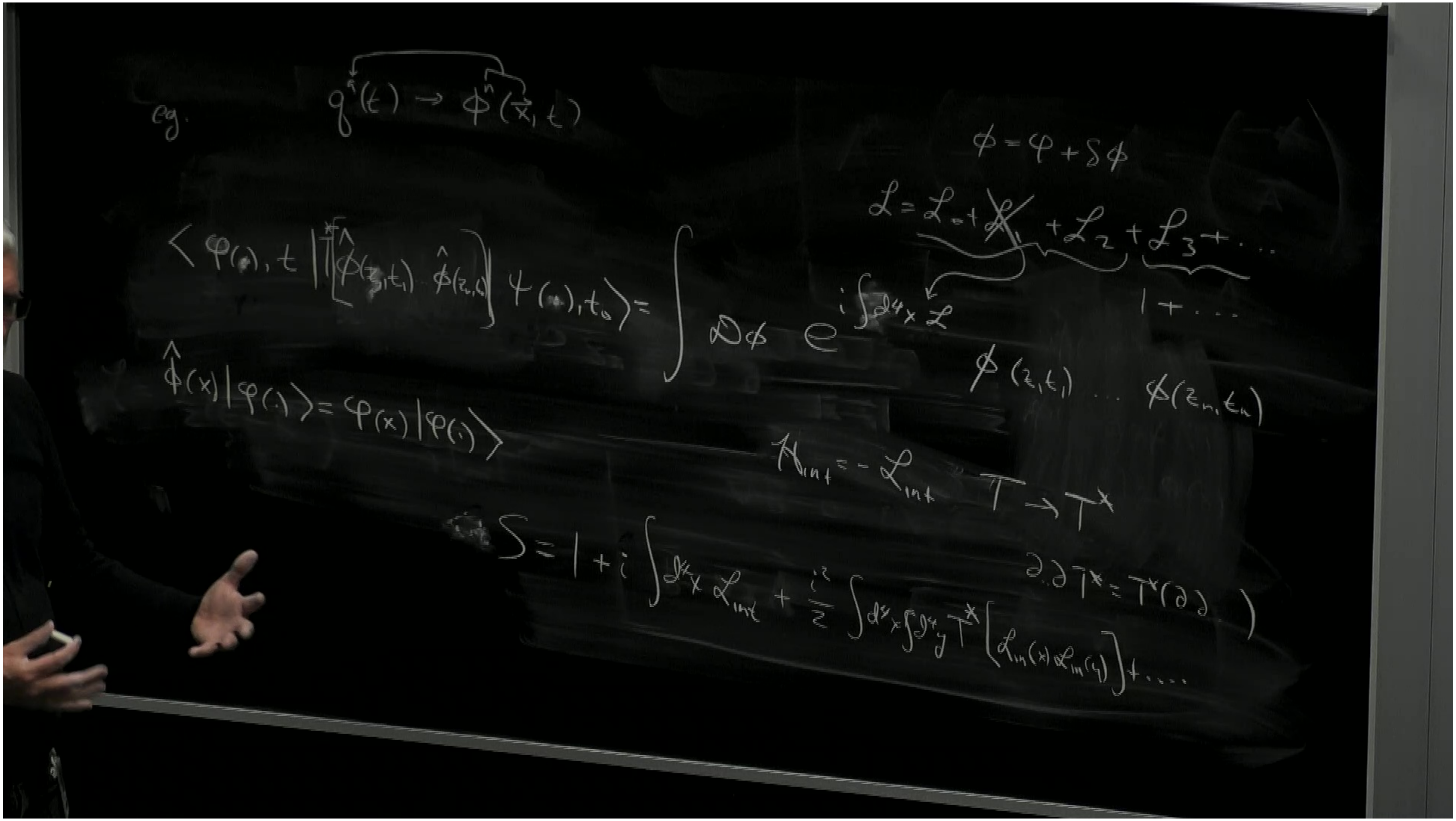
$\phi = \varphi + \delta\phi$

$\mathcal{L} = \mathcal{L}_0 + \cancel{\mathcal{L}_1} + \mathcal{L}_2 + \underbrace{\mathcal{L}_3 + \dots}_{1 + \dots}$

$$\langle \varphi(\cdot), t | T[\hat{\phi}(z_1,t_1) \dots \hat{\phi}(z_n,t_n)] | \psi(\cdot), t_0 \rangle = \int \mathcal{D}\phi \; e^{i\int d^4x \, \mathcal{L}} \; \phi(z_1,t_1) \dots \phi(z_n,t_n)$$

$$\hat{\phi}(x) | \varphi(\cdot) \rangle = \varphi(x) | \varphi(\cdot) \rangle$$


e.g.
$q^n(t) \rightarrow \phi^n(\vec{x}, t)$
$\phi = \varphi + \delta\phi$
$\mathcal{L} = \mathcal{L}_0 + \cancel{\mathcal{L}_1} + \mathcal{L}_2 + \mathcal{L}_3 + \ldots$
$= \int \mathcal{D}\phi \; e^{i\int d^4x \mathcal{L}}$
$\phi(z_1, t_1) \ldots \phi(z_n, t_n)$
$\mathcal{H}_{int} = -\mathcal{L}_{int}$
$T \rightarrow T^*$
$S = 1 + i\int d^4x \, \mathcal{L}_{int} + \frac{i^2}{2}\int d^4x \int d^4y \, T^*\left[\mathcal{L}_{int}(x)\mathcal{L}_{int}(y)\right] + \ldots$


eg. $q^n(t) \to \phi^n(\vec{x}, t)$
$\phi = \varphi + \delta\phi$
$\mathcal{L} = \mathcal{L}_0 + \cancel{\mathcal{L}_1} + \mathcal{L}_2 + \mathcal{L}_3 + \ldots$
$1 + \ldots$
$\langle \varphi(\cdot), t | T^*[\hat{\phi}(z_1, t_1) \ldots \hat{\phi}(z_n, t_n)] | \psi(\cdot), t_0 \rangle = \int \mathcal{D}\phi \, e^{i \int d^4x \mathcal{L}} \phi(z_1, t_1) \ldots \phi(z_n, t_n)$
$\hat{\phi}(x) | \varphi(\cdot) \rangle = \varphi(x) | \varphi(\cdot) \rangle$
$\mathcal{H}_{int} = -\mathcal{L}_{int}$
$T \to T^*$
$\partial \partial T^* = T^*(\partial \partial \ldots)$
$S = 1 + i \int d^4x \, \mathcal{L}_{int} + \frac{i^2}{2} \int d^4x \int d^4y \, T^*[\mathcal{L}_{int}(x) \mathcal{L}_{int}(y)] + \ldots$