Title: QFT2 - Quantum Electrodynamics - Morning Lecture

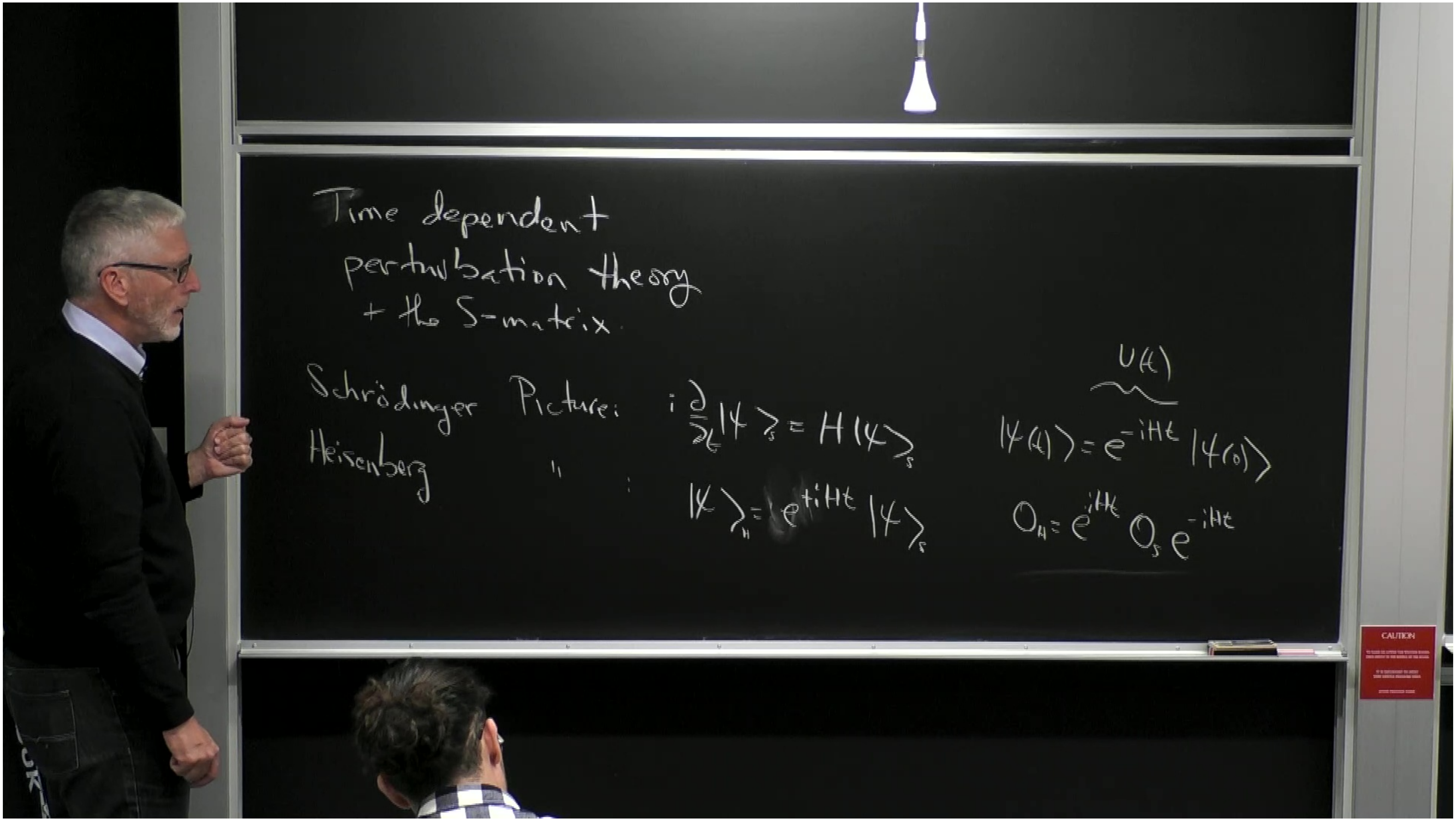
Speakers:

Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

Date: November 08, 2022 - 10:00 AM

URL: https://pirsa.org/22110034

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


Time dependent
perturbation theory
+ the S-matrix.
$U(t)$
Schrödinger Picture: $i\frac{\partial}{\partial t}|\psi\rangle_s = H|\psi\rangle_s$
$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$
Heisenberg " : $|\psi\rangle_H = e^{+iHt}|\psi\rangle_s$
$O_H = e^{iHt} O_s e^{-iHt}$

$$|\psi(t)\rangle_S = \underbrace{e^{\cdots}\, e^{\cdots}\, e^{-iH_0 t_0}}_{U(t,t_0)} |\psi(t_0)\rangle_I$$

$$\frac{\partial U}{\partial t} = i\, e^{iH_0 t} H_0 e^{-iH(t-t_0)} e^{-iH_0 t_0} - i e^{iH_0 t} e^{-iH(t-t_0)} e^{-iH_0 t_0}$$

$$= -i e^{iH_0 t}(H - H_0)e^{-iH(t-t_0)} e^{-iH_0 t_0} = -i V(t)\, U(t,t_0)$$

Time dependent perturbation theory + the S-matrix

$$V(t) = e^{iH_0 t} H_{int}\, e^{-iH_0 t}$$

Schrödinger Picture: $i\frac{\partial}{\partial t}|\psi\rangle_S = H|\psi\rangle_S$ $\quad |\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Heisenberg " : $|\psi\rangle_H = e^{+iHt}|\psi\rangle_S$ $\quad O_H = e^{iHt} O_S e^{-iHt}$

$$\frac{\partial U}{\partial t} = -i V(t) U(t,t_0) \qquad U(t_0,t_0) = I$$

$$U(t,t_0) = I - i \int_{t_0}^{t} dt_1 \, V(t_1) \, U(t_1,t_0)$$

$$= \sum_{n=0}^{\infty} (-i)^n \int_{t_0}^{t} d^4x_1 \int_{t_0}^{t_1} d^4x_2 \cdots \int_{t_0}^{t_{n-1}} d^4x_n \, \mathcal{H}(x_1) \cdots \mathcal{H}(x_n)$$

$$T\left[A(x)\,B(y)\right] = \Theta(x^0-y^0)\,A(x)B(y) + \Theta(y^0-x^0)\,B(y)A(x)$$

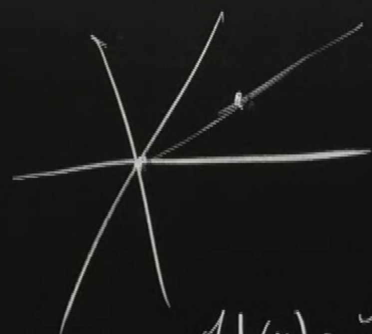
$$\Theta(x) = \begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x<0 \end{cases}$$

$$S = U(\infty,-\infty) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots d^4x_n \, T\left[\mathcal{H}(x_1)\cdots\mathcal{H}(x_n)\right]$$

2 subtleties: 1) How is T defined if we don't agree on whether $x^0 > y^0$?

2) $\partial_\mu T\left[AB\cdots\right] = T\left[\partial_\mu A\, B\cdots\right] + T\left[A\,\partial_\mu B\cdots\right] + \delta^0_\mu\, \delta(x^0-y^0)\, AB\ldots$

i) Spacelike separated obser[illegible] need not agree whether $x^0 > y^0$



$$[\mathcal{H}(x_1), \mathcal{H}(x_2)] = 0 \text{ if } (x_1 - x_2)^2 > 0. \quad (SL)$$

$$\mathcal{H}(x) = \mathcal{H}[A(x,t), A^*(x,t), \partial_\mu A \dots]$$

$$a^*_{p\sigma}|0\rangle = |p\sigma\rangle$$

$$A(x) = \sum_\sigma \int \frac{d^3p}{(2\pi)^3 \sqrt{2\varepsilon_p}} e^{ip\cdot x} a_{p\sigma}$$

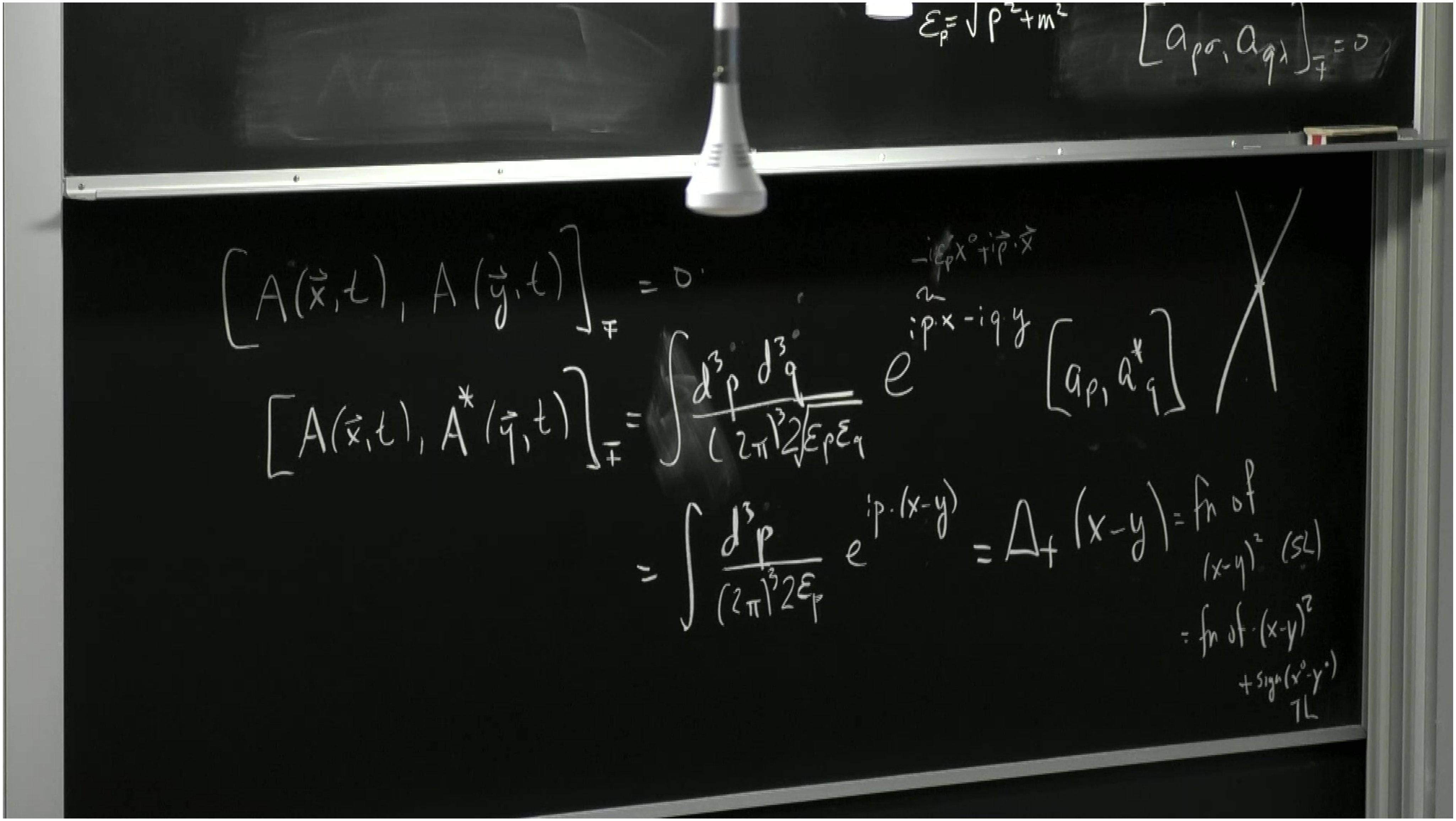
$$\mathcal{E}_p = \sqrt{p^2+m^2} \qquad [a_{p\sigma}, a_{q\lambda}]_\mp = 0$$

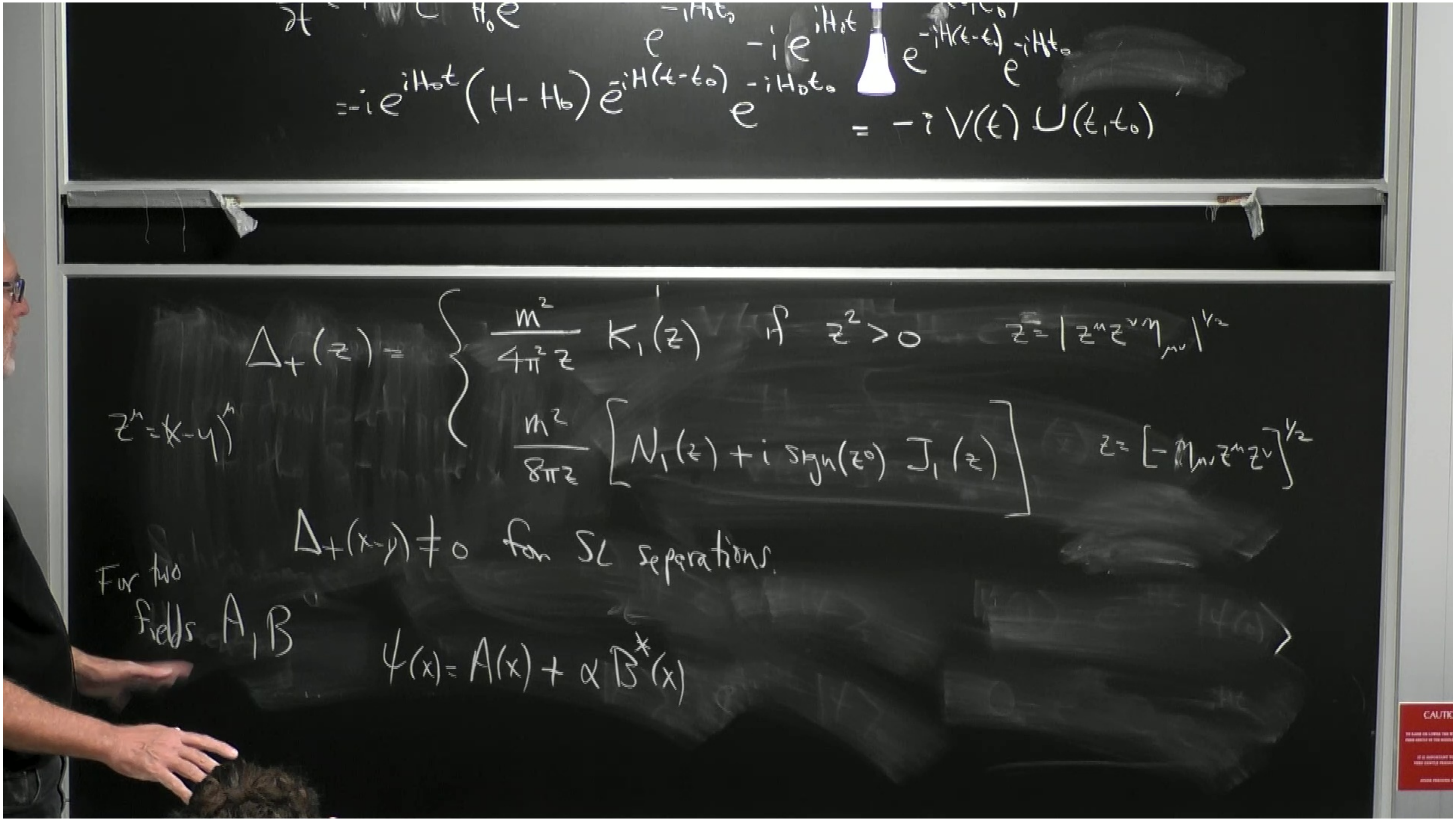
$$[A(\vec{x},t), A(\vec{y},t)]_\mp = 0$$

$$-i\mathcal{E}_p x^0 + i\vec{p}\cdot\vec{x}$$

$$[A(\vec{x},t), A^*(\vec{y},t)]_\mp = \int \frac{d^3p\, d^3q}{(2\pi)^3\, 2\sqrt{\mathcal{E}_p \mathcal{E}_q}}\, e^{ip\cdot x - iq\cdot y}\, [a_p, a^*_q]$$

$$= \int \frac{d^3p}{(2\pi)^3\, 2\mathcal{E}_p}\, e^{ip\cdot(x-y)} = \Delta_+(x-y)$$


$\mathcal{E}_p = \sqrt{p^2+m^2}$
$[a_{p\sigma}, a_{q\lambda}]_\mp = 0$
$[A(\vec{x},t), A(\vec{y},t)]_\mp = 0$
$-i\mathcal{E}_p x^0 + i\vec{p}\cdot\vec{x}$
$[A(\vec{x},t), A^*(\vec{y},t)]_\mp = \int \frac{d^3p\, d^3q}{(2\pi)^3 2\sqrt{\mathcal{E}_p\mathcal{E}_q}} e^{i p\cdot x - i q\cdot y} [a_p, a^*_q]$
$= \int \frac{d^3p}{(2\pi)^3 2\mathcal{E}_p} e^{ip\cdot(x-y)} = \Delta_+(x-y) =$ fn of $(x-y)^2$ (SL)
$=$ fn of $(x-y)^2$ $+ \mathrm{sign}(x^0-y^0)$ TL



$= -i e^{iH_0 t}(H - H_0) e^{-iH(t-t_0)} e^{-iH_0 t_0} = -i V(t) U(t,t_0)$
$\Delta_+(z) = \frac{m^2}{4\pi^2 z} K_1(z)$ if $z^2 > 0$
$z = |z^\mu z^\nu \eta_{\mu\nu}|^{1/2}$
$z^\mu = (x-y)^\mu$
$\frac{m^2}{8\pi z}\left[N_1(z) + i\,\mathrm{sign}(z^0)\,J_1(z)\right]$
$z = \left[-\eta_{\mu\nu} z^\mu z^\nu\right]^{1/2}$
$\Delta_+(x-y) \neq 0$ for SL separations.
For two fields A, B
$\psi(x) = A(x) + \alpha B^*(x)$

$$\Delta_+(z) = \Delta_+(-z) \quad , \text{if } z^2 > 0.$$

$$= \langle \bar{p} x \mid y \rangle$$

$[\psi(\vec{x},t), \psi(\vec{y},t)]_\mp = 0$ if ptcles have same statistics.

$$[\psi(\vec{x},t), \psi^*(\vec{y},t)]_\mp = [A(\vec{x},t), A^*(\vec{y},t)]_\mp \mp |\alpha|^2 [B(\vec{y},t), B^*(\vec{x},t)]_\mp$$

if $\boxed{\begin{matrix} m_A = m_B \\ j_A = j_B \end{matrix}}$

$$= \Delta_+(x-y) \mp |\alpha|^2 \Delta_+(y-x)$$

$= 0$ iff $|\alpha|^2 = 1$ and they are bosons

$$[\psi(x), \psi^*(y)]_- = \Delta_+(x-y) - \Delta_+(y-x) \equiv \Delta(x-y) \neq \text{ for TL sep.}$$

$$= \left(\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m^2}\right) \Delta_+(x-y)$$

$$\left[A(\vec{x},t),\ A(\vec{y},t)\right]_{\mp} = 0$$

$$\left[A_\mu(\vec{x},t),\ A^*_\nu(\vec{y},t)\right]_{\mp} = \sum_{\sigma\lambda} \int \frac{d^3p\, d^3q}{(2\pi)^3 2\sqrt{\mathcal{E}_p \mathcal{E}_q}}\, e^{ip\cdot x - iq\cdot y} \left[a_{p\sigma}, a^*_{q\lambda}\right] u_\mu(p\sigma)\, u^*_\nu(q,\lambda)$$

$-i\mathcal{E}_p x^0 + i\vec{p}\cdot\vec{x}$

$\delta(p-q)$

$\delta_{\sigma\lambda}$

$$= \int \frac{d^3p}{(2\pi)^3 2\mathcal{E}_p}\, e^{ip\cdot(x-y)} = \Delta_+(x-y) = \text{fn of } (x-y)^2 \ (SL)$$

$= \text{fn of } (x-y)^2 + \text{sign}(x^0-y^0)$ TL

$\mathcal{H}(x) = \frac{\lambda}{4!}\varphi^4$

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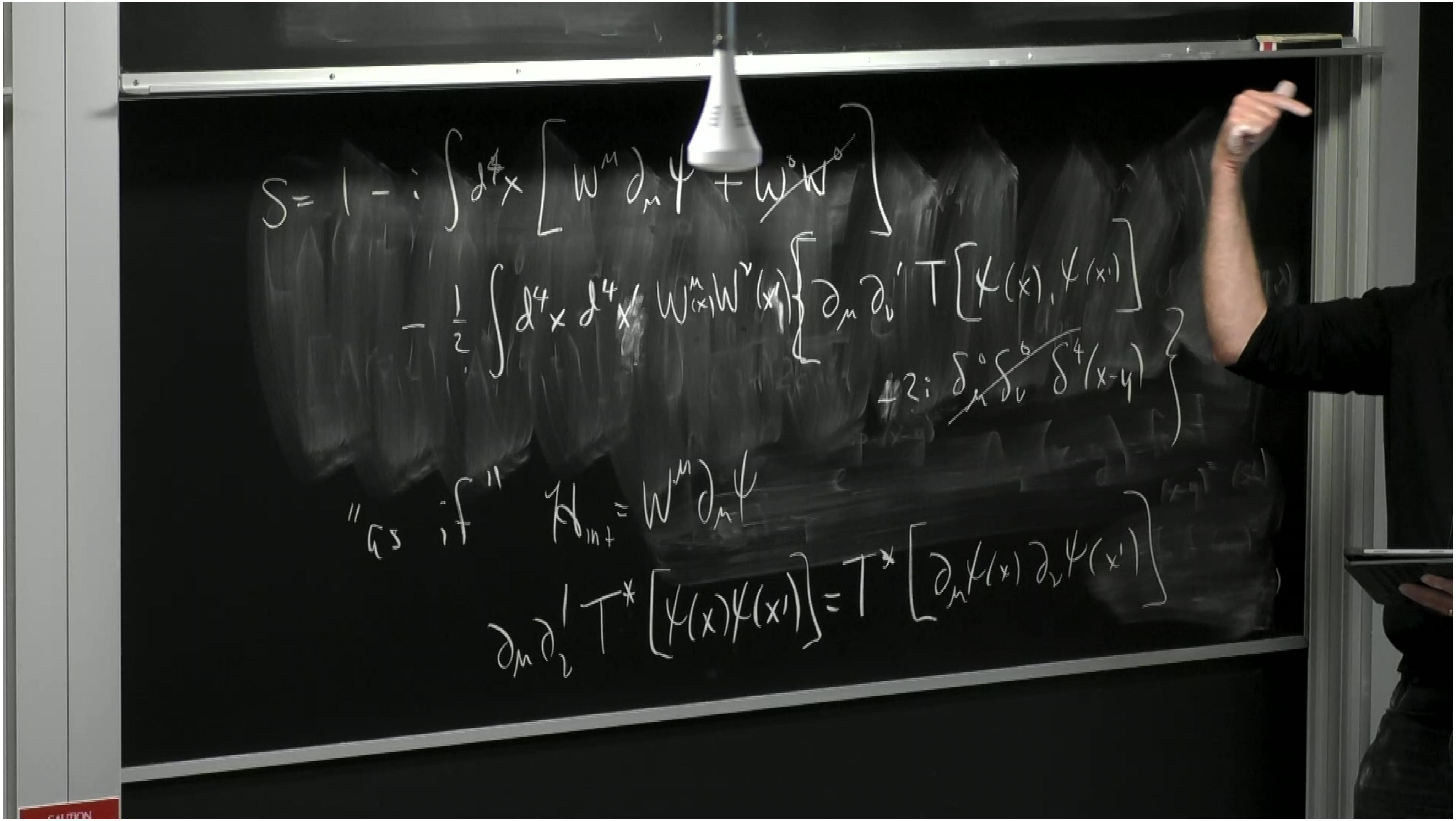
$$S = 1 - i\int d^4x\, W^\mu \partial_\mu \psi^{\nu}\ (+\text{h.c.}) + \frac{(-i)^2}{2}\int d^4x\, d^4y\, W^\mu(x)\, W^\nu(y)\, \underbrace{T\left[\partial_\mu\psi(x)\, \partial_\nu\psi(y)\right]} + \dots$$

Claim: $T\left[\partial_\mu\psi(x)\, \partial_\nu\psi(x')\right]$ is not a tensor.

$$\partial_\mu \partial'_\nu\, T\left[\psi(x)\, \psi(x')\right] = T\left[\partial_\mu\psi(x)\, \partial_\nu\psi(x')\right] + \delta^0_\nu\, \delta(x^0-y^0)\left[\partial_\mu\psi(x), \psi(x')\right]$$

$$+ \delta^0_\mu\, \delta(x^0-y^0)\left[\psi(x), \partial_\nu\psi(x')\right]$$

$$+ \delta^0_\mu\, \delta^0_\nu\, \delta'(x^0-y^0)\left[\psi(x), \psi(x')\right]$$


$S = 1 - i\int d^4x \left[W^\mu \partial_\mu \psi + \cancel{W^0 W^0} \right]$
$-\frac{1}{2}\int d^4x\, d^4x'\, W^\mu(x) W^\nu(x') \left\{ \partial_\mu \partial_\nu' T\left[\psi(x), \psi(x')\right] + \cancel{2i\, \delta_\mu^0 \delta_\nu^0}\, \delta^4(x-y) \right\}$
"as if" $\mathcal{H}_{int} = W^\mu \partial_\mu \psi$
$\partial_\mu \partial_\nu' T^*\left[\psi(x)\psi(x')\right] = T^*\left[\partial_\mu\psi(x)\, \partial_\nu\psi(x')\right]$

$$S = 1 - i\int d^4x \left[W^\mu \partial_\mu \psi + \cancel{W^0 W^0} \right]$$

$$- \frac{1}{2}\int d^4x\, d^4x'\, W^\mu(x)\, W^\nu(x') \left\{ \partial_\mu \partial_\nu' T\left[\psi(x), \psi(x')\right] + \cancel{2i\,\delta_\mu^0 \delta_\nu^0}\, \delta^4(x-y) \right\}$$

"as if" $\mathcal{H}_{int} = W^\mu \partial_\mu \psi$

$$\partial_\mu \partial_\nu' T^*\left[\psi(x)\psi(x')\right] = T^*\left[\partial_\mu \psi(x)\, \partial_\nu \psi(x')\right]$$