Title: QFT2 - Quantum Electrodynamics - Morning Lecture

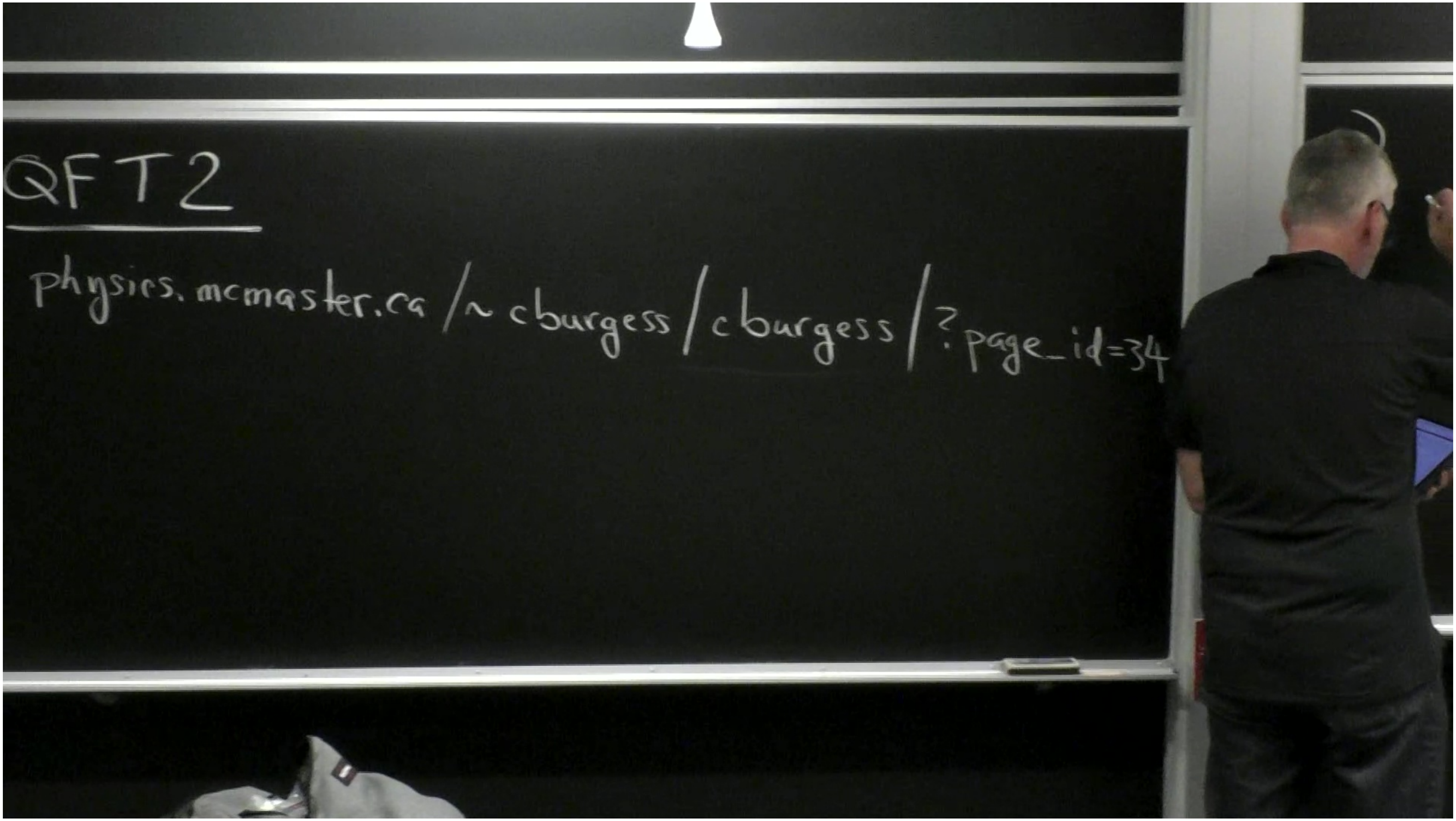
Speakers:

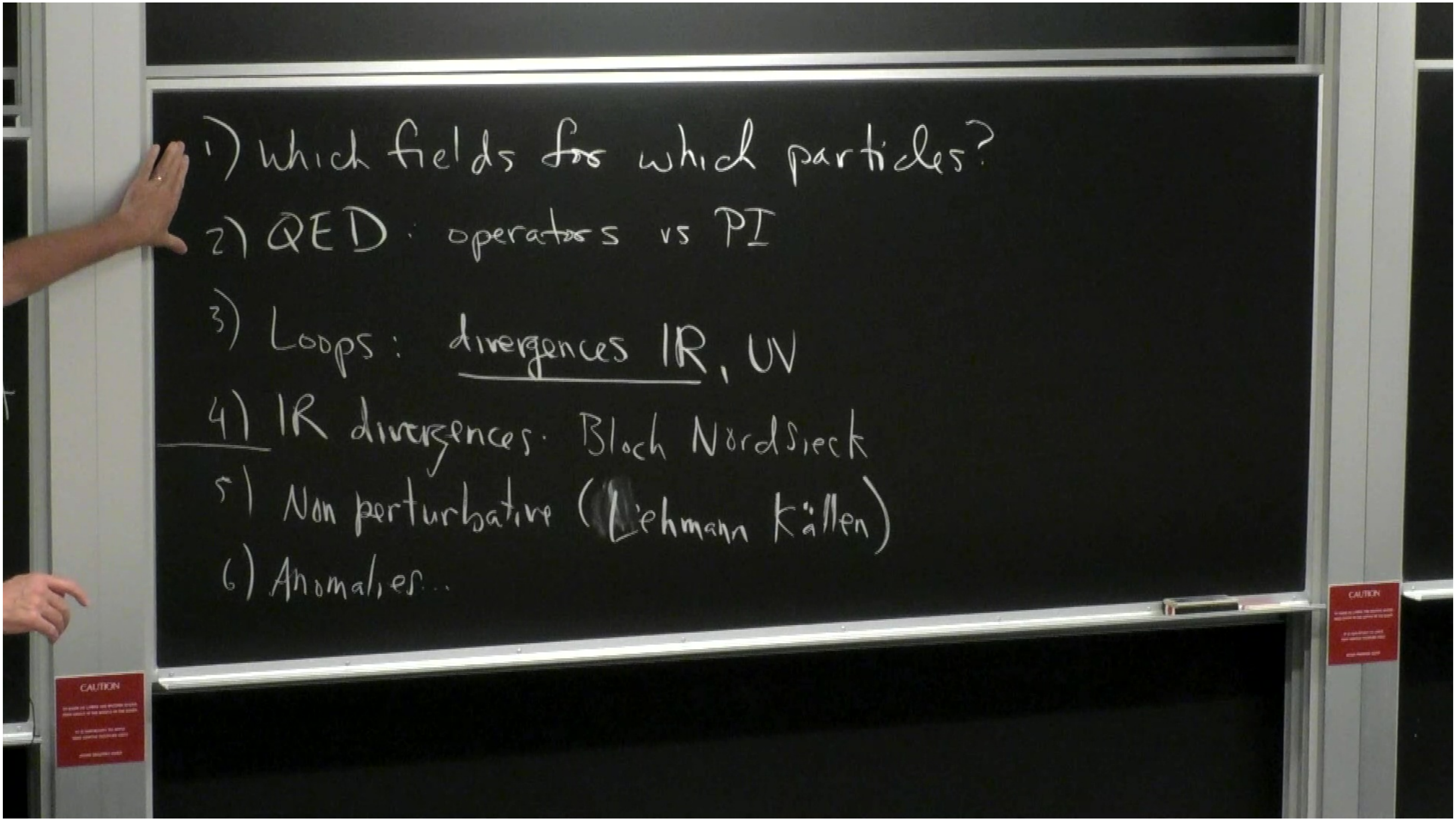
Collection: Special Topics in Physics - QFT2: Quantum Electrodynamics (Cliff Burgess)

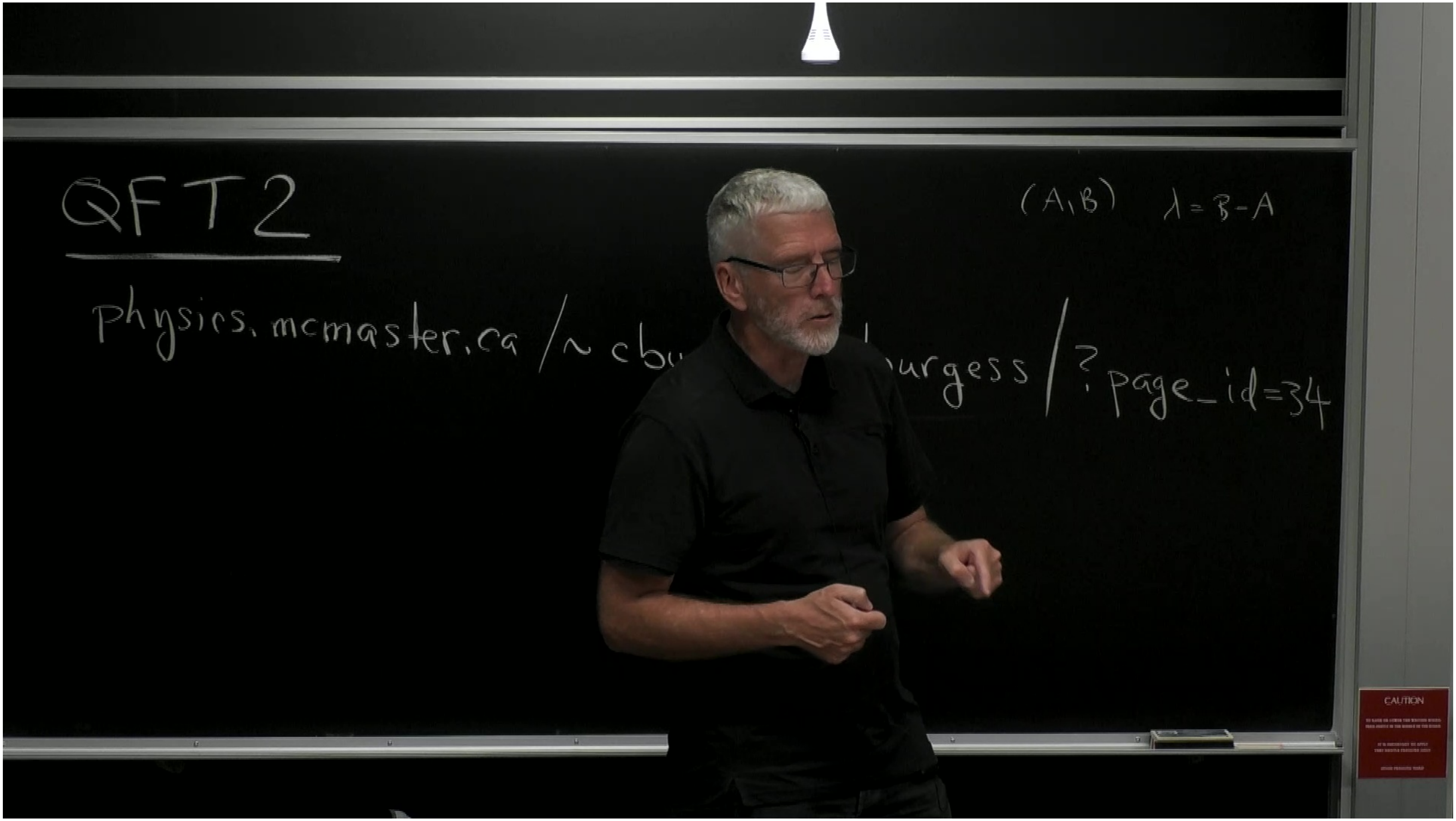
Date: November 01, 2022 - 10:00 AM

URL: https://pirsa.org/22110033

Abstract: This course uses quantum electrodynamics (QED) as a vehicle for covering several more advanced topics within quantum field theory, and so is aimed at graduate students that already have had an introductory course on quantum field theory. Among the topics hoped to be covered are: gauge invariance for massless spin-1 particles from special relativity and quantum mechanics; Ward identities; photon scattering and loops; UV and IR divergences and why they are handled differently; effective theories and the renormalization group; anomalies.


QFT 2
physics.mcmaster.ca/~cburgess/cburgess/?page_id=34


1) Which fields for which particles?
2) QED: operators vs PI
3) Loops: divergences IR, UV
4) IR divergences. Bloch Nordsieck
5) Non perturbative (Lehmann Källen)
6) Anomalies..


QFT2
(A,B) λ = B − A
physics.mcmaster.ca/~cbu
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CAUTION

1) Which fields for which particles?

2) QED: operators vs PI

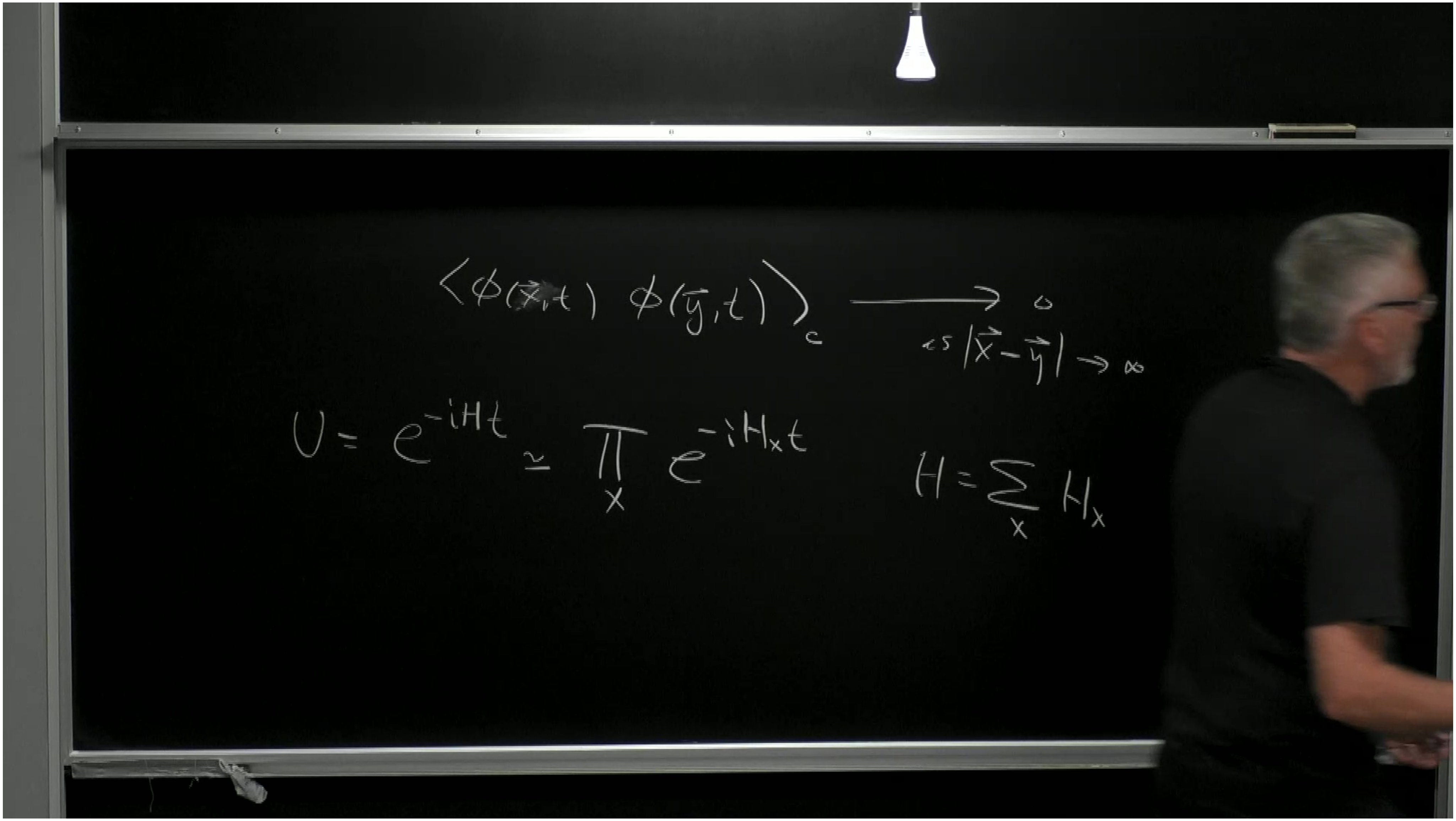
$\boxed{\text{– ward identities}}$

3) Loops: divergences IR, UV

4) IR divergences: Bloch Nordsieck

5) Non perturbative (Lehmann Källen)

6) Anomalies...


$\langle \phi(\vec{x},t)\ \phi(\vec{y},t) \rangle_c \longrightarrow 0$
as $|\vec{x}-\vec{y}| \to \infty$
$U = e^{-iHt} \simeq \prod_x e^{-iH_x t}$
$H = \sum_x H_x$

6) Anomalies...

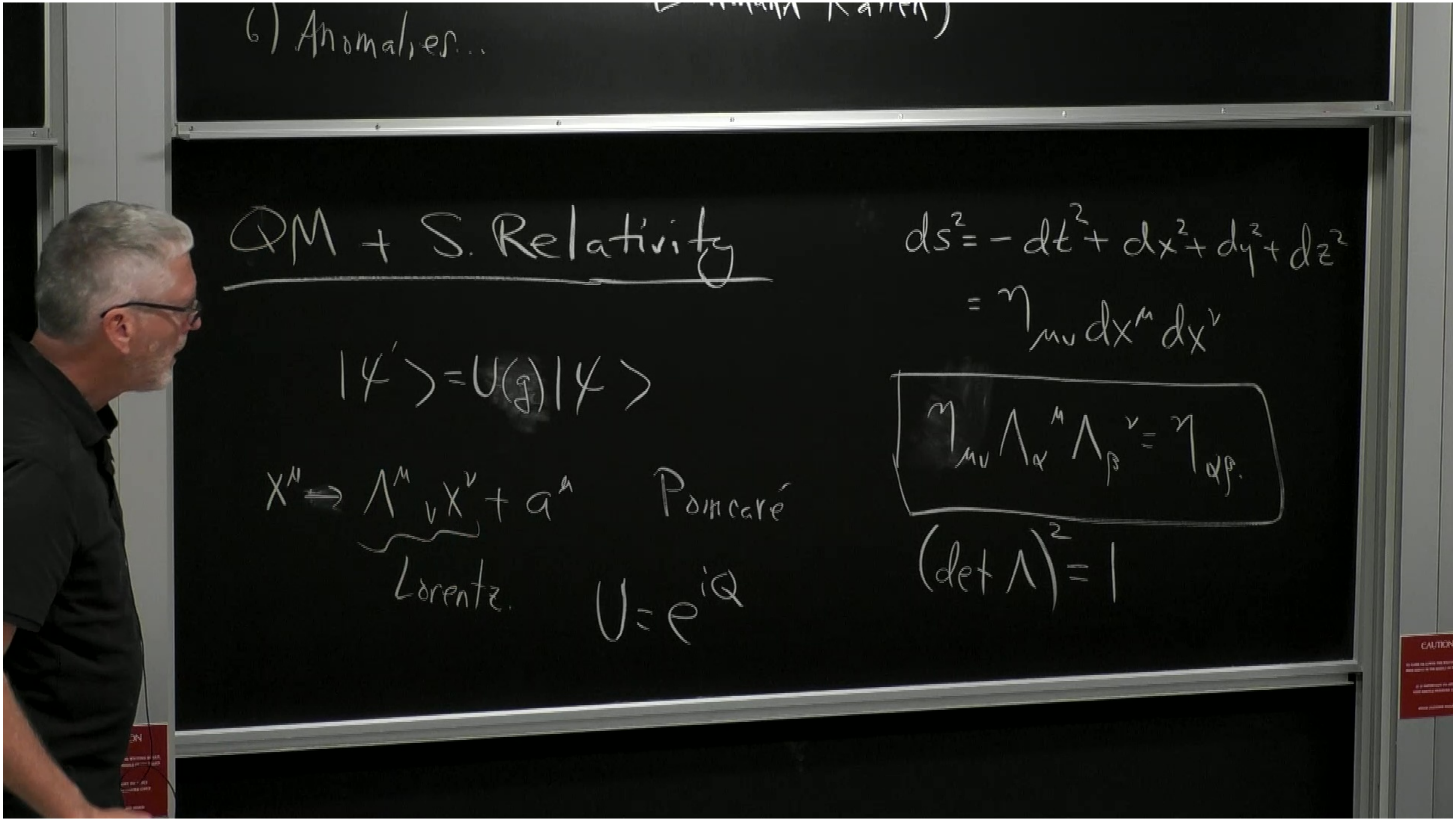
QM + S. Relativity

$$|\psi'\rangle = U(g)|\psi\rangle$$

$$X^\mu \Rightarrow \underbrace{\Lambda^\mu{}_\nu X^\nu}_{\text{Lorentz.}} + a^\mu \qquad \text{Poincaré}$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$
$$= \eta_{\mu\nu} dx^\mu dx^\nu$$

$$p^2$$


6) Anomalies...
QM + S. Relativity
$|\psi'\rangle = U(g)|\psi\rangle$
$x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu$
Poincaré
Lorentz
$U = e^{iQ}$
$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
$= \eta_{\mu\nu} dx^\mu dx^\nu$
$\eta_{\mu\nu} \Lambda_\alpha{}^\mu \Lambda_\beta{}^\nu = \eta_{\alpha\beta}$
$(\det \Lambda)^2 = 1$

$$\eta_{00} = -1 = \Lambda_0{}^{\mu}\Lambda_0{}^{\nu}\eta_{\mu\nu}$$

$$= -(\Lambda_0{}^0)^2 + \sum_i (\Lambda_0{}^i)^2$$

$$(\Lambda_0{}^0)^2 = 1 + \text{positive}$$

$$P_\mu{}^\nu = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad T_\mu{}^\nu = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\Lambda_\mu{}^\nu = \delta_\mu{}^\nu + \omega_\mu{}^\nu$$

$$\eta_{\mu\nu} \Lambda_\alpha{}^\mu \Lambda_\beta{}^\nu = \eta_{\alpha\beta} \longrightarrow \omega_{\mu\nu} = -\omega_{\nu\mu}$$

$$\omega_{\mu\nu} = \eta_{\nu\alpha} \omega_\mu{}^\alpha$$

$$U(\Lambda_1, a_1) U(\Lambda_2, a_2) = U(\Lambda_3, a_3)$$

$$\Lambda_3 = (\Lambda_2 \Lambda_1) \qquad a_3 = (\Lambda_2 a_1) + a_2$$

$$\boxed{\Lambda_\mu{}^\nu = \delta_\mu{}^\nu + \omega_\mu{}^\nu}$$

$$\eta_{\mu\nu}\Lambda_\alpha{}^\mu\Lambda_\beta{}^\nu = \eta_{\alpha\beta} \longrightarrow \omega_{\mu\nu} = -\omega_{\nu\mu}$$

$$\omega_{\mu\nu} = \eta_{\nu\alpha}\omega_\mu{}^\alpha$$

$$U(1+\omega, a) = 1 + \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu} + i a^\mu P_\mu + \dots$$

$$U(\Lambda_1, a_1)\,U(\Lambda_2, a_2) = U(\Lambda_3, a_3)$$

$$\Lambda_3 = (\Lambda_2\Lambda_1) \qquad a_3 = (\Lambda_2 a_1) + a_2$$

$$U(\Lambda,a)\,U(1+\omega,\epsilon)\,U^{-1}(\Lambda,a) = U(\Lambda,a)\left[1 + \frac{i}{2}\omega_{\mu\nu}J^{\mu\nu} + i\epsilon_\mu P^\mu + \cdots\right]U^{-1}(\Lambda,a)$$

$$\to \quad U(\Lambda,a)\,J^{\mu\nu}\,U^{-1}(\Lambda,a) = \Lambda_\alpha{}^\mu\Lambda_\beta{}^\nu\left(J^{\alpha\beta} - P^\alpha a^\beta + P^\beta a^\alpha\right)$$

$$U(\Lambda,a)\,P^\mu\,U^{-1}(\Lambda,a) = \Lambda_\alpha{}^\mu P^\alpha$$

$$\left[J^{\mu\nu}, J^{\lambda\rho}\right] = -i\,J^{\mu\lambda}\eta^{\nu\rho} \pm \text{perm.} \qquad \left[J^{\mu\nu}, P^\lambda\right] = i\eta^{\mu\nu}P^\lambda - i\eta^{\mu\lambda}P^\nu$$

$$\boxed{\Lambda_\mu{}^\nu = \delta_\mu{}^\nu + \omega_\mu{}^\nu}$$

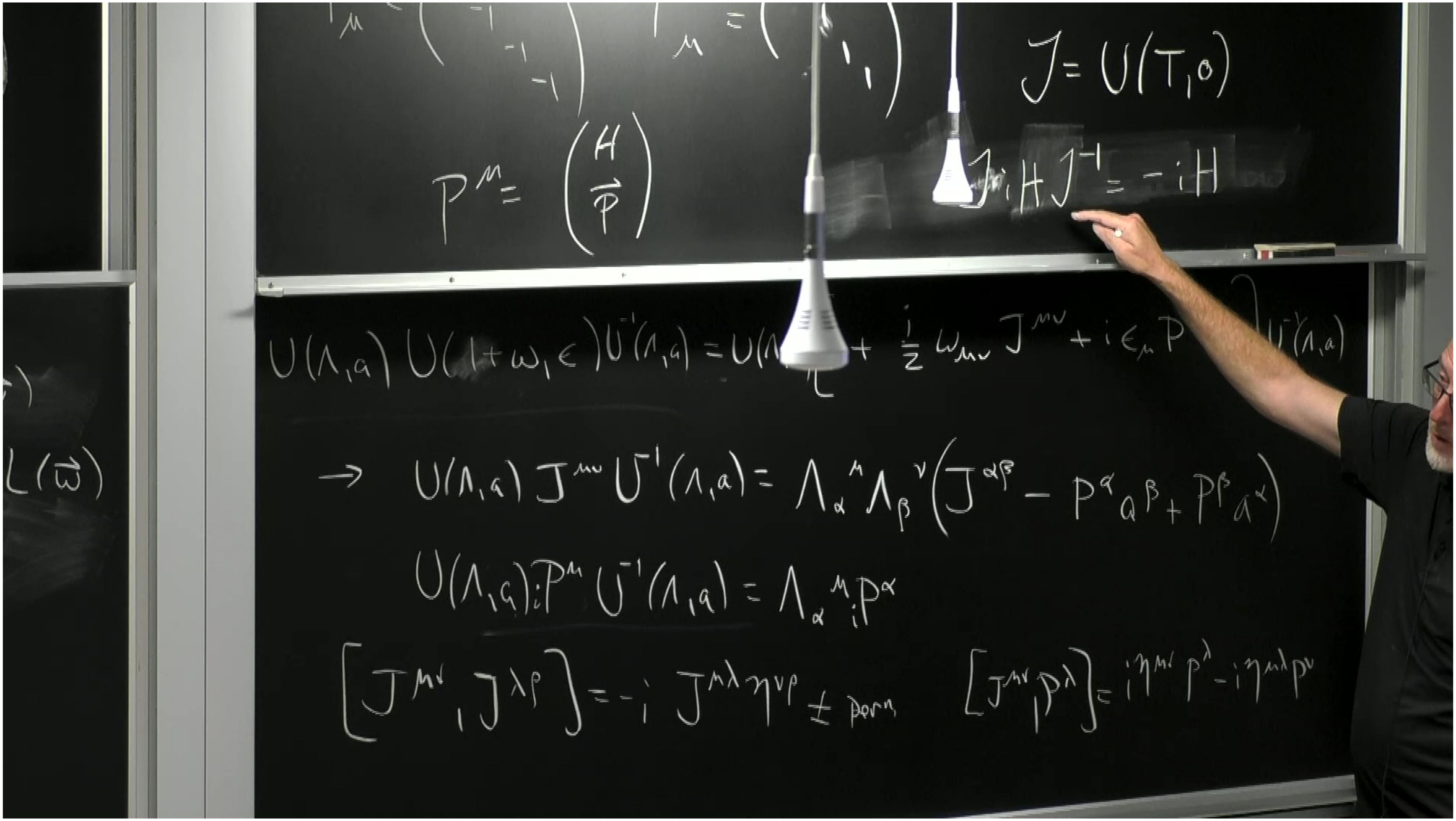
$$\eta_{\mu\nu}\Lambda_\alpha{}^\mu\Lambda_\beta{}^\nu = \eta_{\alpha\beta} \longrightarrow \omega_{\mu\nu} = -\omega_{\nu\mu}$$

$$\omega_{\mu\nu} = \eta_{\nu\alpha}\,\omega_\mu{}^\alpha$$

$$L(a\vec{v} + b\vec{w}) = a^* L(\vec{v}) + b^* L(\vec{w})$$

$$\underline{U(\Lambda_1, a_1)\,U(\Lambda_2, a_2) = U(\Lambda_3, a_3)}$$

$$\Lambda_3 = (\Lambda_2\Lambda_1) \qquad a_3 = (\Lambda_2 a_1) + a_2$$

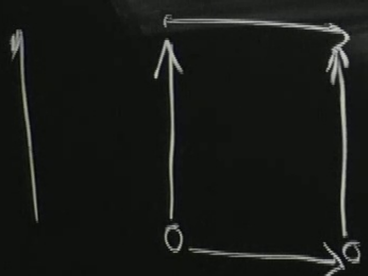

$J = U(T,0)$
$J\, iH\, J^{-1} = -iH$
$P^\mu = \begin{pmatrix} H \\ \vec{P} \end{pmatrix}$
$U(\Lambda,a)\, U(1+\omega,\epsilon)\, U^{-1}(\Lambda,a) = U(\Lambda \quad + \frac{i}{2}\omega_{\mu\nu} J^{\mu\nu} + i\epsilon_\mu P \quad) U^{-1}(\Lambda,a)$
$\rightarrow \quad U(\Lambda,a)\, J^{\mu\nu}\, U^{-1}(\Lambda,a) = \Lambda_\alpha{}^\mu \Lambda_\beta{}^\nu \left(J^{\alpha\beta} - P^\alpha a^\beta + P^\beta a^\alpha\right)$
$U(\Lambda,a)\, P^\mu\, U^{-1}(\Lambda,a) = \Lambda_\alpha{}^\mu P^\alpha$
$[J^{\mu\nu}, J^{\lambda\rho}] = -i J^{\mu\lambda}\eta^{\nu\rho} \pm \text{perm}$
$[J^{\mu\nu}, P^\lambda] = i\eta^{\mu\nu} P^\lambda - i\eta^{\mu\lambda} P^\nu$

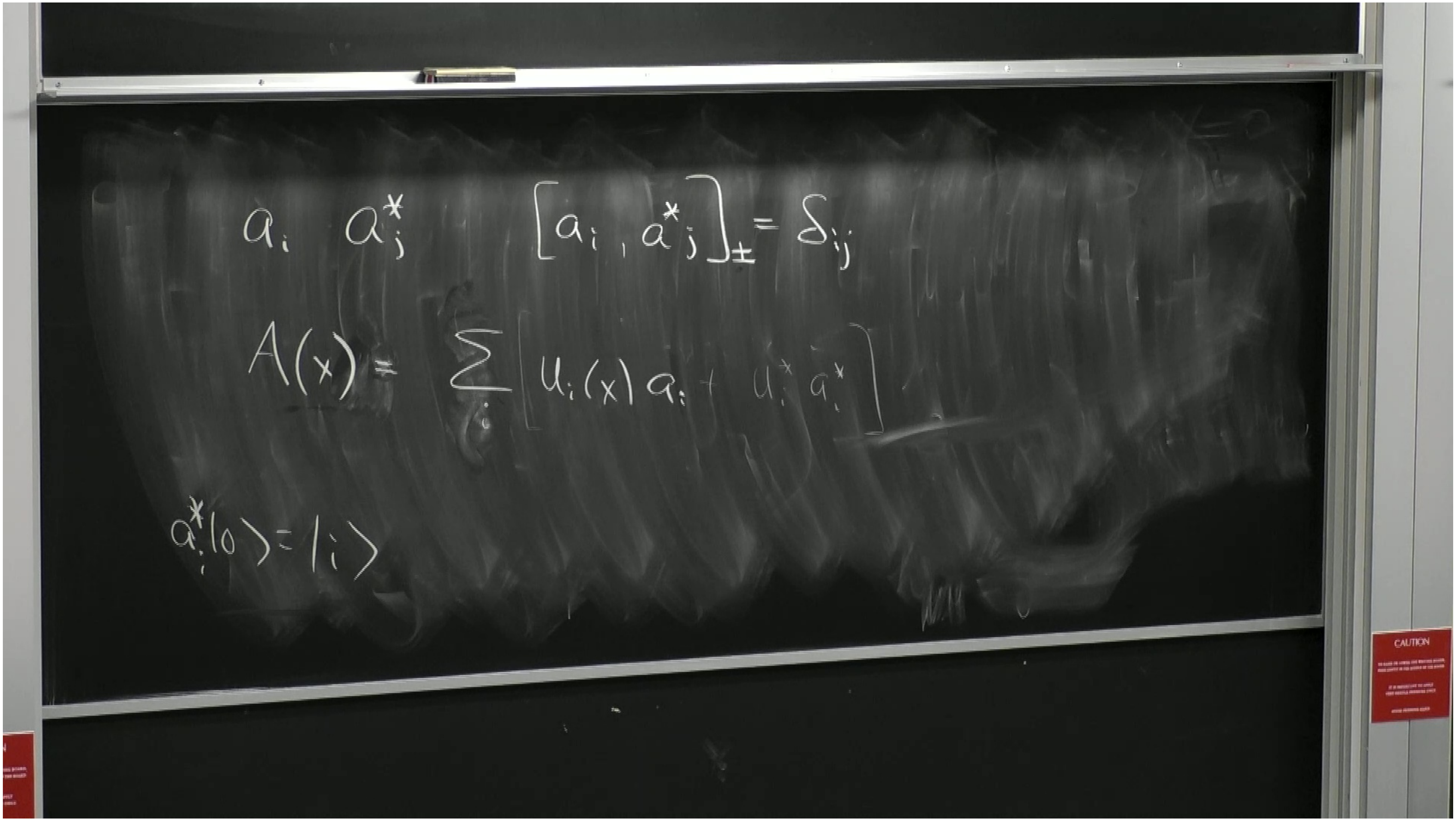
$$H = \int d^3x \, \mathcal{H}(x)$$

$$H = \sum_x H_x$$

$$U = e^{-iHt} = \prod_x U_x$$

$$|\alpha_{in}\rangle = U\,|\beta_{in}\rangle \qquad |\alpha_{out}\rangle = U\,|\beta_{out}\rangle$$





$a_i \quad a_j^* \qquad [a_i , a_j^*]_\pm = \delta_{ij}$
$A(x) = \sum_i \left[u_i(x)\, a_i + u_i^*\, a_i^* \right]$
$a_i^* |0\rangle = |i\rangle$

$$a_i \quad a_j^* \qquad [a_p, a_q^*]_\pm = \delta^3(p-q)$$

$$A(x,t) = \int \frac{d^3p}{(2\pi)^{3/2}}\, a_p\, e^{i\vec{p}\cdot\vec{x} - i\omega t} \qquad [A(\vec{x},t), A^*(\vec{y},t)] = \delta^3(x-y)$$

$$a_i^*|0\rangle = |i\rangle$$

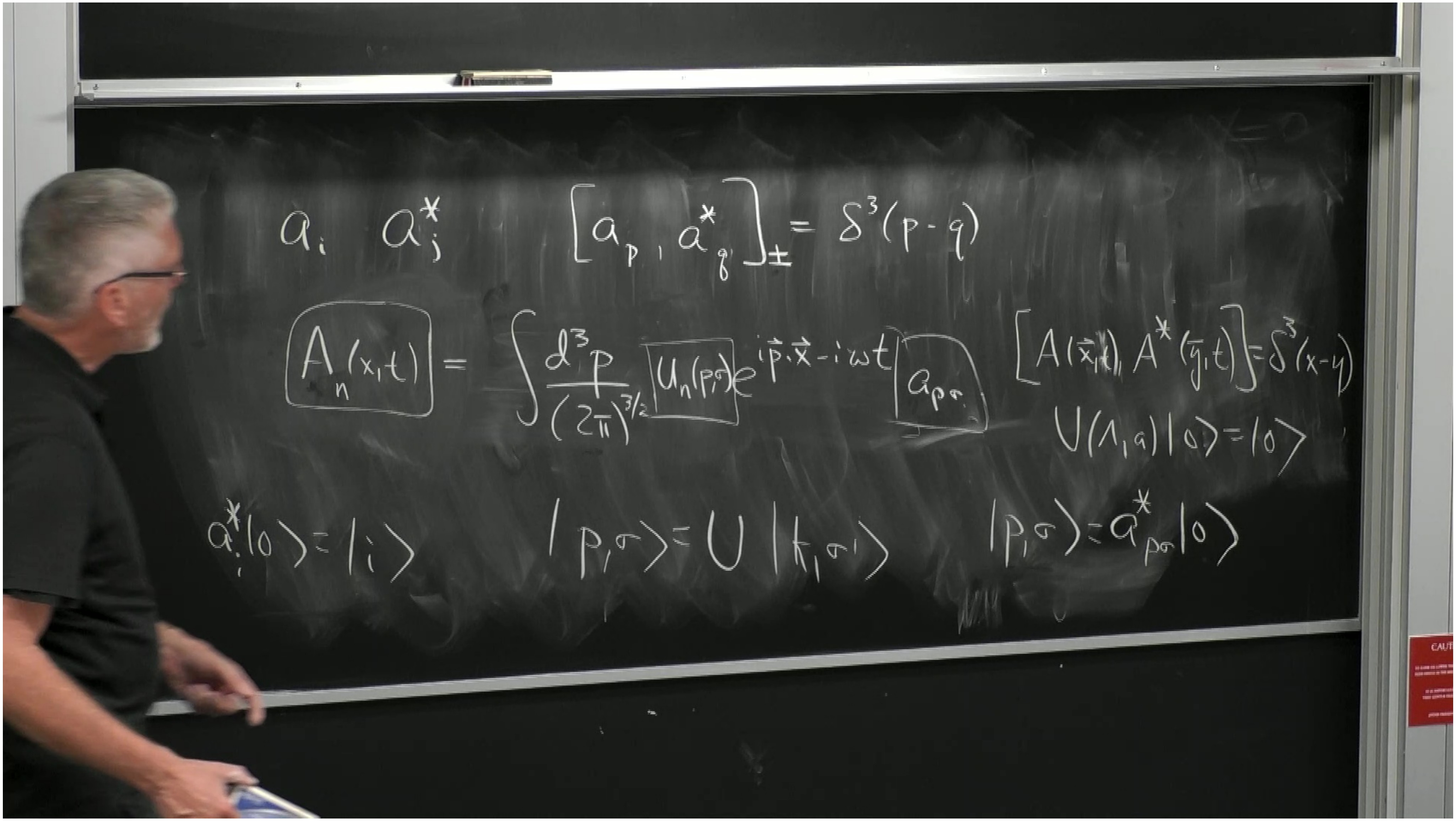
$a_i \quad a_j^*$

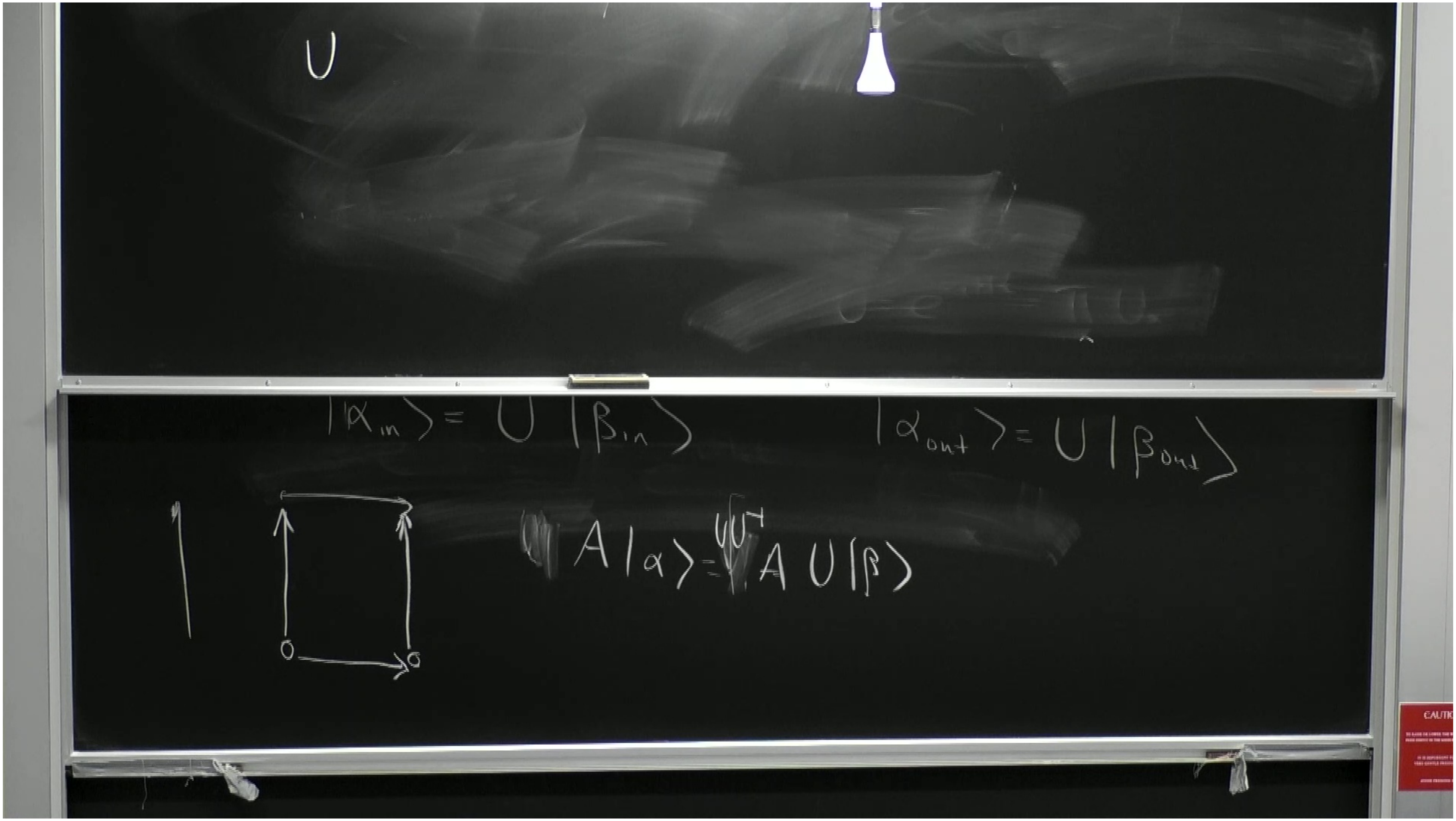
$$[a_p, a_q^*]_\pm = \delta^3(p-q)$$

$$\boxed{A_n(x,t)} = \int \frac{d^3p}{(2\pi)^{3/2}} \boxed{u_n(p,\sigma)} e^{i\vec{p}\cdot\vec{x} - i\omega t} \boxed{a_{p\sigma}}$$

$$[A(\vec{x},t), A^*(\vec{y},t)] = \delta^3(x-y)$$

$$a_i^* |0\rangle = |i\rangle$$


$a_i \quad a_j^*$
$[a_p, a_q^*]_\pm = \delta^3(p-q)$
$\boxed{A_n(x,t)} = \int \frac{d^3p}{(2\pi)^{3/2}} \boxed{u_n(p\sigma)} e^{i\vec{p}\cdot\vec{x} - i\omega t} \boxed{a_{p\sigma}}$
$[A(\vec{x},t), A^*(\vec{y},t)] = \delta^3(x-y)$
$U(\Lambda,a)|0\rangle = |0\rangle$
$a_i^*|0\rangle = |i\rangle$
$|p,\sigma\rangle = U|k,\sigma\rangle$
$|p,\sigma\rangle = a_{p\sigma}^*|0\rangle$

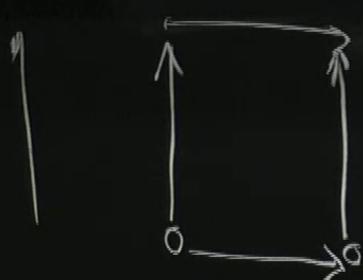

U
|α_in⟩ = U |β_in⟩
|α_out⟩ = U |β_out⟩
A |α⟩ = U U† A U |β⟩

$$U(\Lambda, 0) = U(\Lambda)$$

$$U(\Lambda_1)\, A_m\, U^{-1}(\Lambda_1) = D_{mn}(\Lambda_1^{-1})\, A_n$$

$$U(\Lambda_2)U(\Lambda_1)A_m U^{-1}(\Lambda_1)\, U^{-1}(\Lambda_2) = D_{mn}(\Lambda_1^{-1})\, \underbrace{U(\Lambda_2)\, A_n\, U^{-1}(\Lambda_2)}_{D_{np}(\Lambda_2^{-1})\, A_p}$$

$$= \left[D(\Lambda_1^{-1})\, D(\Lambda_2^{-1})\right]_{mp} A_p$$



$$A|\alpha\rangle = U^{-1} A\, U|\beta\rangle$$

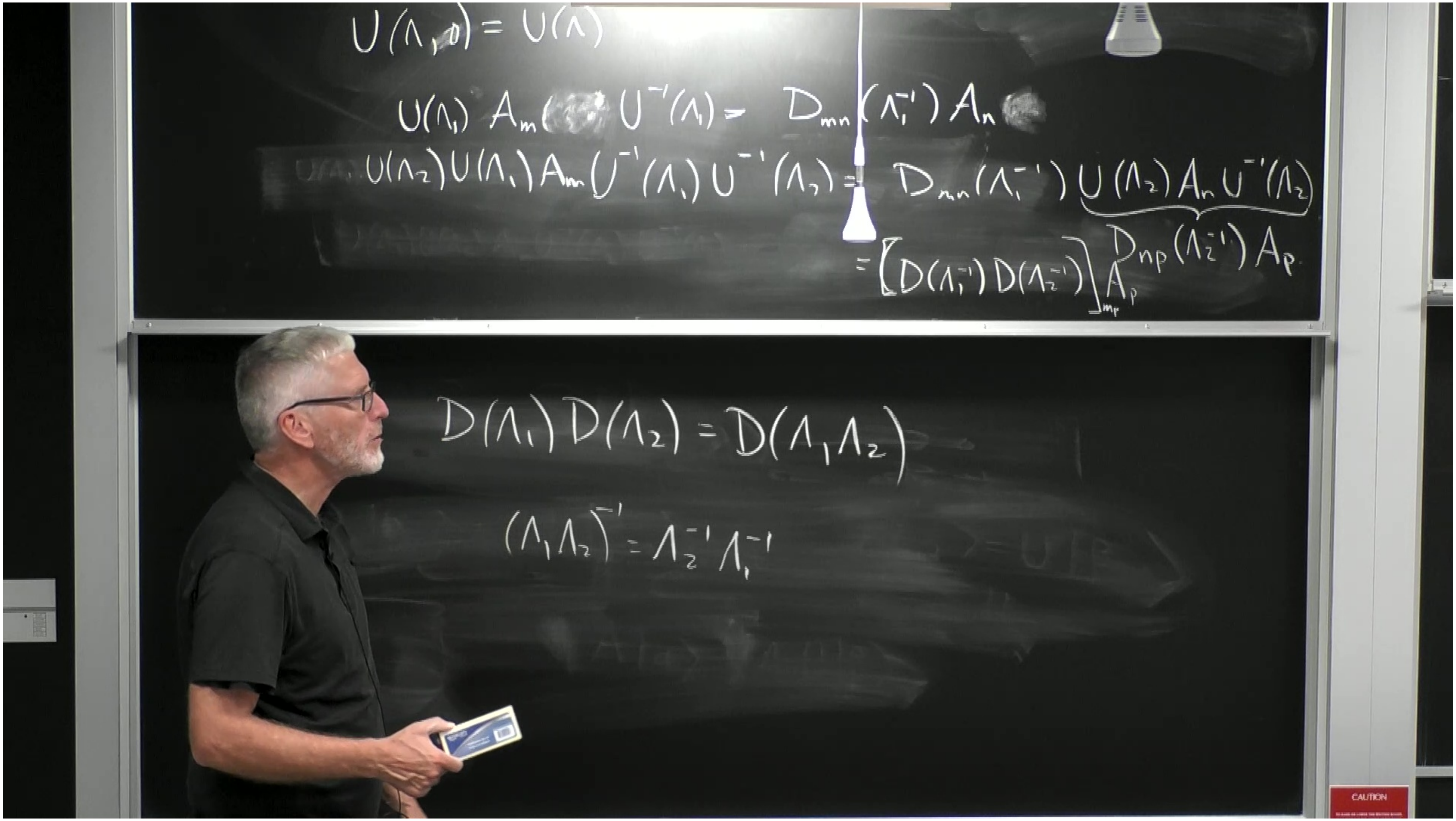
$$U(\Lambda, 0) = U(\Lambda)$$

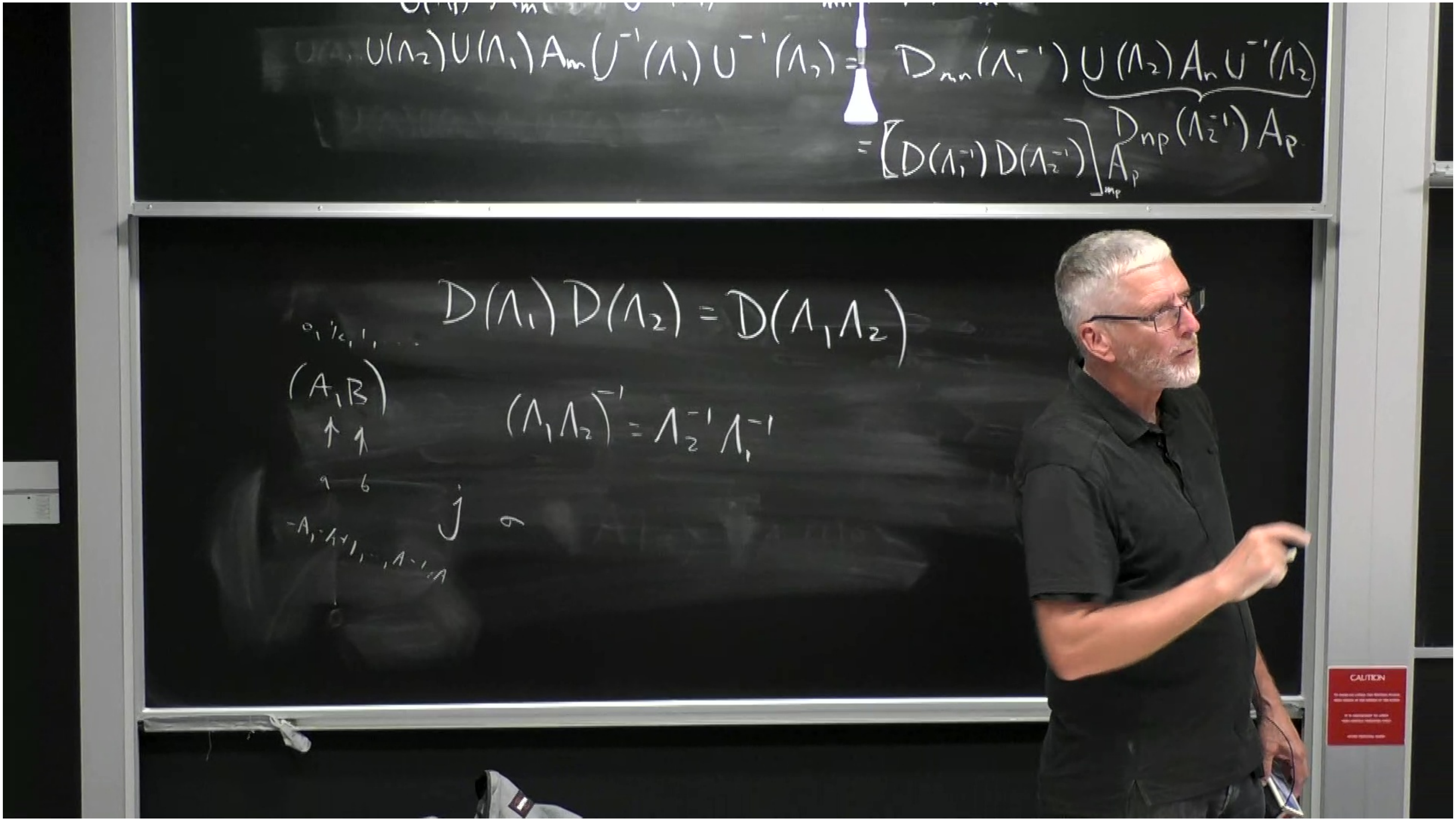
$$U(\Lambda_1)\, A_m\, U^{-1}(\Lambda_1) = D_{mn}(\Lambda_1^{-1})\, A_n$$

$$U(\Lambda_2)U(\Lambda_1)A_m U^{-1}(\Lambda_1)\, U^{-1}(\Lambda_2) = D_{mn}(\Lambda_1^{-1})\, \underbrace{U(\Lambda_2)\, A_n\, U^{-1}(\Lambda_2)}_{D_{np}(\Lambda_2^{-1})\, A_p}$$

$$= \left[D(\Lambda_1^{-1})\, D(\Lambda_2^{-1})\right]_{mp} A_p$$

$$D(\Lambda_1)\, D(\Lambda_2) = D(\Lambda_1 \Lambda_2)$$


$U(\Lambda, 0) = U(\Lambda)$
$U(\Lambda_1) A_m U^{-1}(\Lambda_1) = D_{mn}(\Lambda_1^{-1}) A_n$
$U(\Lambda_2) U(\Lambda_1) A_m U^{-1}(\Lambda_1) U^{-1}(\Lambda_2) = D_{mn}(\Lambda_1^{-1}) \underbrace{U(\Lambda_2) A_n U^{-1}(\Lambda_2)}_{D_{np}(\Lambda_2^{-1}) A_p}$
$= [D(\Lambda_1^{-1}) D(\Lambda_2^{-1})]_{mp} A_p$
$D(\Lambda_1) D(\Lambda_2) = D(\Lambda_1 \Lambda_2)$
$(\Lambda_1 \Lambda_2)^{-1} = \Lambda_2^{-1} \Lambda_1^{-1}$


$U(\Lambda_2)U(\Lambda_1)A_m U^{-1}(\Lambda_1)U^{-1}(\Lambda_2) = D_{mn}(\Lambda_1^{-1})\, U(\Lambda_2)A_n U^{-1}(\Lambda_2)$
$D_{np}(\Lambda_2^{-1})A_p$
$= [D(\Lambda_1^{-1})D(\Lambda_2^{-1})]_{mp} A_p$
$D(\Lambda_1)D(\Lambda_2) = D(\Lambda_1\Lambda_2)$
(A, B)
$(\Lambda_1\Lambda_2)^{-1} = \Lambda_2^{-1}\Lambda_1^{-1}$
j
CAUTION