Title: Locality bounds on quantum dynamics with measurements

Speakers:

Series: Quantum Matter

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Abstract: In non-relativistic systems, the Lieb-Robinson Theorem imposes an emergent speed limit (independent of the relativistic limit set by c), establishing locality under unitary quantum dynamics and constraining the time needed to perform useful quantum tasks. We have extended the Lieb-Robinson Theorem to quantum dynamics with measurements. In contrast to the general expectation that measurements can arbitrarily violate spatial locality, we find at most an (M+1)-fold enhancement to the speed of quantum information, provided the outcomes of M local measurements are known; this holds even when classical communication is instantaneous. Our bound is asymptotically optimal, and saturated by existing measurement-based protocols (the "quantum repeater"). Our bound tightly constrain the resource requirements for quantum computation, error correction, teleportation, generating entangled resource states (Bell, GHZ, W, and spin-squeezed states), and preparing SPT states from short-range entangled states.

Zoom Link: https://pitp.zoom.us/j/95640053536?pwd=Z05oWlFRSEFTZWFRK2dwcHdsWlBBdz09

Locality bounds on quantum dynamics with measurement

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November 8, 2022

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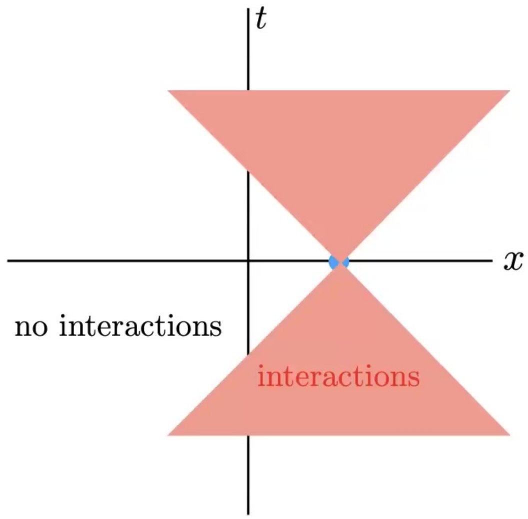
- Introduction to the Lieb-Robinson Theorem
- Adding Measurements
- Stinespring Formalism
- Applications
- Summary

1 In special relativity, spacelike separated points can't communicate!



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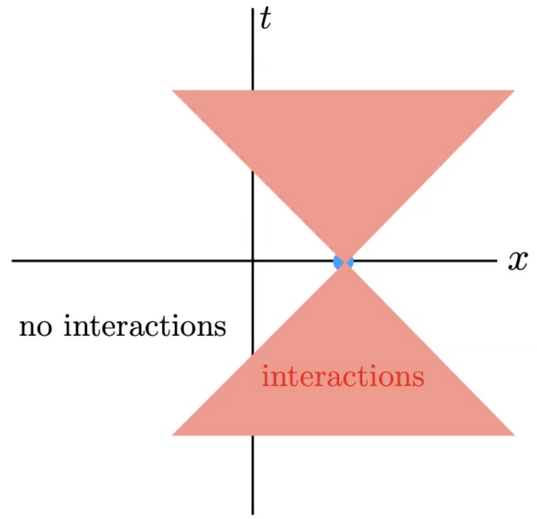


In relativistic quantum field theory:

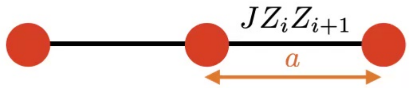
$$[\mathcal{O}_1(\mathbf{x}_1, t_1), \mathcal{O}_2(\mathbf{x}_2, t_2)] = 0, \text{ if } (\mathbf{x}_1 - \mathbf{x}_2)^2 > c^2(t_1 - t_2)^2$$

2

Lieb-Robinson Theorem: *emergent locality* in many-body quantum systems with spatially local interactions:

[Lieb, Robinson; (1972)]; [Hastings, Koma; math-ph/0507008]

$$\|[A_0(t), B_L]\| \lesssim \left(\frac{vt}{L}\right)^L, \qquad v \sim \frac{Ja}{\hbar}$$



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$$\|[A_0(t), B_L]\| \lesssim \left(\frac{vt}{L}\right)^L, \qquad v \sim \frac{Ja}{\hbar}$$

Why? Defining $\mathcal{L} = \mathrm{i}[H, \cdot]$,

$$A_0(t) = A_0 + \mathcal{L}tA_0 + \frac{(\mathcal{L}t)^2}{2!}A_0 + \cdots + \underbrace{\frac{(\mathcal{L}t)^L}{L!}A_0 + \cdots}_{\text{first terms that survive commutator!}}$$

we must have a sequence of

$$\mathcal{A} = [H_{L-1,L}, \cdots, [H_{1,2}, \cdots, [H_{0,1}, A_0]]] \subset A_0(t)$$

to have $[\mathcal{A}, B_L] \neq 0$.

3 Lieb-Robinson bounds constrain almost anything useful! For example, to prepare an entangled state (e.g. Bell pair)

$$|\mathrm{Bell}\rangle = \left[\alpha\frac{|00\rangle_{0L}+|11\rangle_{0L}}{\sqrt{2}} + \beta\frac{|01\rangle_{0L}+|10\rangle_{0L}}{\sqrt{2}}\right]\otimes|\psi\rangle_{1,\ldots,L-1}.$$

out of $|\psi(\alpha,\beta)\rangle = (\alpha|0\rangle+\beta|1\rangle)_0\otimes|0\cdots0\rangle_{1\ldots L}$, wait $t > L/v$.

This is because

$$\mathrm{e}^{\mathrm{i}Ht}X_0X_L\mathrm{e}^{-\mathrm{i}Ht}Z_0|\psi(\alpha,\beta)\rangle = |\psi(-\beta,\alpha)\rangle,$$
$$Z_0\mathrm{e}^{\mathrm{i}Ht}X_0X_L\mathrm{e}^{-\mathrm{i}Ht}|\psi(\alpha,\beta)\rangle = |\psi(\beta,-\alpha)\rangle = -|\psi(-\beta,\alpha)\rangle,$$
$$\|[(X_0X_L)(t), Z_0]\| = 2.$$

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Also bounds: correlation functions, finite correlation length in gapped ground state, classical/quantum simulatability, time to prepare topological state, etc.

4

The past few years have seen huge progress on extending the Lieb-Robinson theorem:

- dissipative (Lindblad) dynamics
- locality with power-law interactions [Chen, Lucas; 1907.07637]; [Kuwahara, Saito; 1910.14477], [Tran *et al*; 2103.15828]

$$H(t) = \sum_{i,j\in\mathbb{Z}^d} J_{ij}^{\alpha\beta}(t)\frac{X_i^\alpha X_j^\beta}{|i-j|^\alpha}, \quad |J_{ij}^{\alpha\beta}(t)| \le 1$$

where the light cone is $t \gtrsim r^{\min(1,\alpha-2d)}$.

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- Bose-Hubbard model (and related): [Yin, Lucas; 2106.09726]

$$H = \sum_{i,j\in\mathbb{Z}} J_{ij}(t) b_i^\dagger b_j + \sum_{i\in\mathbb{Z}} f\left(b_i^\dagger b_i\right)$$

  where (in 1d, boson density $\bar{n}$) $t \gtrsim r/\bar{n}$

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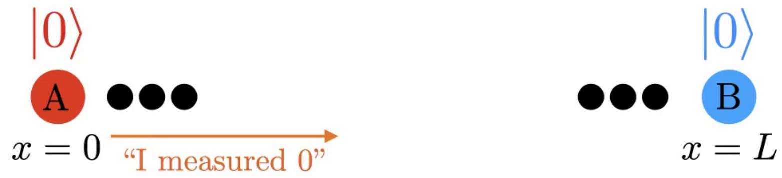
Does wave function collapse break relativity?

Assuming $\alpha = 1, \beta = 0$, A/B cannot confirm their outcomes agree until classical communication (at c) established.



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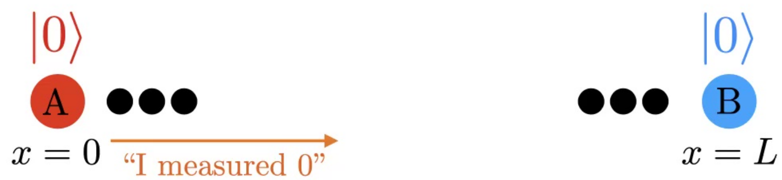

But in a non-relativistic world ($c = \infty$), the measurement M breaks locality!

$$\mathsf{M}X_0|\mathrm{Bell}\rangle = X_L\mathsf{M}|\mathrm{Bell}\rangle.$$

6

While Lieb-Robinson bounded $|\text{Bell}\rangle$ preparation time from unentangled qubits, measurement ruins locality in $|\text{Bell}\rangle$. (Why it's a useful resource!)

The main question for today: **Can we salvage the Lieb-Robinson bound for dynamics with measurements?**

The answer: **Starting with unentangled qubits, if M local measurement outcomes used to perform error correction, one qubit can move distance L in time t if**

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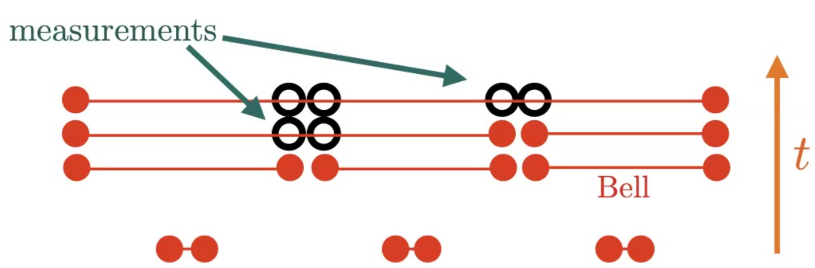
This theorem is optimal, holds for circuits or continuous time dynamics, incorporates the possibility of "feedback", and is saturated by simple protocols.

[Friedman, Yin, Hong, Lucas; 2206.09929]

7 "Quantum repeater" is the optimal protocol that saturates $(M+1)vt \gtrsim L$. Illustrate it with teleportation protocol:

$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle)_0 \otimes |0\cdots 0\rangle_{1\cdots L}.$$
$$|\psi(T)\rangle = |\phi\rangle_{0\cdots N-1} \otimes (\alpha|0\rangle + \beta|1\rangle)_L \,.$$

$|\phi\rangle$ depends on measurement outcomes.



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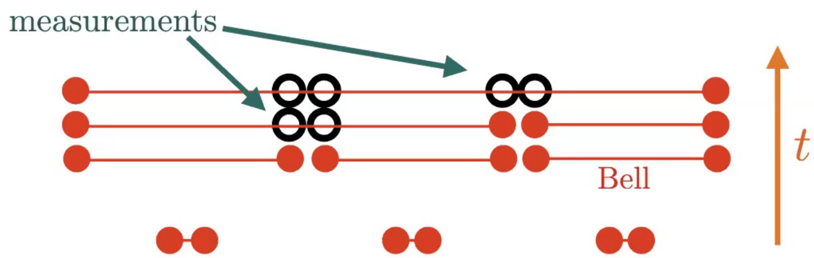

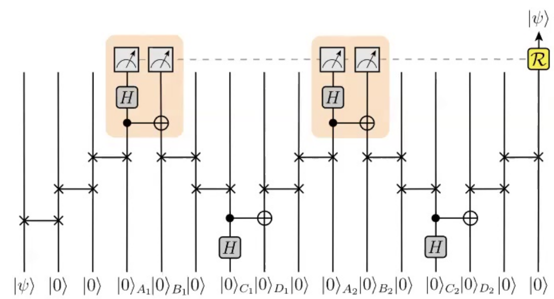

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Measurement collapses $|\psi\rangle$. X_0 transform independent of $|\psi\rangle$?

Avoid question. Look for a **unitary realization of measurement**:

$$[\text{measure } Z]\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{\text{physical}} \otimes \underbrace{|\tilde{0}\rangle}_{\text{measurement outcome}}$$

$$\to \alpha|0\rangle|\tilde{0}\rangle + \beta|1\rangle|\tilde{1}\rangle.$$

The auxiliary **Stinespring qubit** recorded whether I got $Z|0\rangle = +|0\rangle$ or $Z|1\rangle = -|1\rangle$ in my measurement outcome.

Entangled qubit with knowledge of measurement outcome.

9

The unitary M, which measures Z, is a CNOT gate between physical/Stinespring qubits. It acts on operators as:

$$\mathsf{M}^\dagger X \mathsf{M} = X\widetilde{X}, \quad \mathsf{M}^\dagger Y \mathsf{M} = Y\widetilde{X}, \quad \mathsf{M}^\dagger Z \mathsf{M} = Z, \quad \mathsf{M}^\dagger \widetilde{Z} \mathsf{M} = Z\widetilde{Z}$$

Once a measurement happens, the Stinespring qubit records outcome – **never touch again**. Each measurement has its own Stinespring qubit!

In Schrödinger picture, protocol P acts as $P|\psi\rangle$.
In Heisenberg picture, P acts as $P^\dagger O P$.

Since a physical process has

$$\langle\psi_0|\langle\widetilde{0}|[P^\dagger O P]|\psi_0\rangle|\widetilde{0}\rangle,$$

operators with $\widetilde{X}$ are "death". Quantum information is destroyed. For expectation values to be non-zero, we want to only see $\widetilde{Z}$. Measurement outcomes are a classical resource.

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10 Let's see Stinespring in action. Suppose

$$|\psi(0)\rangle = \frac{\alpha(|00\rangle + |11\rangle) + \beta(|01\rangle + |10\rangle)}{\sqrt{2}} \otimes |\widetilde{0}\rangle.$$

I measure Z_2 and error correct to obtain:

$$|\psi\rangle = \mathsf{RM}|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |2\text{+SS}\rangle.$$

$$\mathsf{M} = \frac{1 + Z_2}{2} + \frac{1 - Z_2}{2}\widetilde{X}, \quad \mathsf{R} = \frac{1 + \widetilde{Z}}{2} + \frac{1 - \widetilde{Z}}{2}X_1$$

The final **logical operators** X_1 and Z_1 evolve into:

$$\mathsf{M}^\dagger \mathsf{R}^\dagger X_1 \mathsf{RM} = \mathsf{M}^\dagger X_1 \mathsf{M} = X_1,$$

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X_1 is logical for both $|\psi\rangle$ and $|\psi(0)\rangle$.
Z_1 is logical for $|\psi\rangle$, but $Z_1 Z_2$ is logical for $|\psi(0)\rangle$.

11 No cloning theorem implies **useful quantum task must be done in "one shot".**

To protect a logical bit without QEC, logical operator can have no $\widetilde{X}$ or $\widetilde{Z}$. Thus, trace out Stinespring. Measurement is Lindblad-like dissipation! (Does nothing "interesting"...)

Error correction needed for measurement to be helpful.

In "measurement-induced phase transitions", one finds **post-selection problem**: a task may be achievable, but only if all measurements have correct outcome. In the thermodynamic limit, e^{-Lt} probability a quantum phase discoverable.

[Friedman, Hart, Nandkishore; 2210.07256]

A practical measurement-assisted phase of matter will be a quantum error correcting code?

12 To understand our theorem, consider Heisenberg dynamics of logical Paulis. Schematically, logical X is

$$X_{\mathrm{L}} \to U^\dagger \mathsf{M}^\dagger \mathsf{R}^\dagger X_f \underbrace{\mathsf{R}}_{\text{QEC}} \underbrace{\mathsf{M}}_{\text{meas.}} \underbrace{U}_{\text{unitary}}$$

$$\sim U^\dagger \mathsf{M}^\dagger X_f \underbrace{\widetilde{Z}_j}_{\text{meas. outcome}} \underbrace{\mathsf{M}}_{\text{meas.}} \underbrace{U}_{\text{unitary}}$$

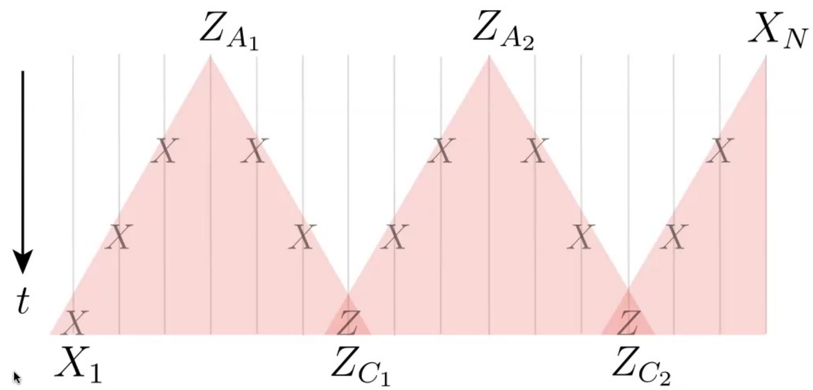
$$\sim U^\dagger X_f \widetilde{Z}_j \underbrace{Z_j}_{\text{physical}} \underbrace{U}_{\text{unitary}}$$

$$\sim X_i \cdot \widetilde{Z}_j (\text{stabilizer for } |\psi(0)\rangle)$$

Crucial points:

- Without QEC, only unitary dynamics can grow operator.
- With QEC and measurement, seed physical operators at measurement locations, creating non-locality.
- **Measurement outcomes must be used to error correct.** If not, anything "quantum" destroyed.

13 In the quantum repeater example before:



One measurement/finite depth circuit can disrupt qubit far away:

$$X_N \xrightarrow{\text{“QEC”}} X_N \widetilde{Z}_1 \xrightarrow{\text{meas.}} X_N \widetilde{Z}_1 Z_1$$

To generate useful quantum resource, push non-commutativity of (X_N, Z_N) to (X_1, Z_1), requiring “intersecting light cones”!

14

To re-cap, we've proved that

$$(M+1)vt \geq L$$

if M measurement outcomes/QEC used to transmit one qubit distance L in time t.

It is possible to make $M \sim L$ so $t \sim 1$. This requires $\mathrm{O}(L)$ initial qubits to be in finely tuned state, so QEC works. **Measurement is not a free resource for preparing quantum states.**

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Sometimes, "feed forward" (changing U based on intermediate M) can be helpful. This does not help to saturate the Lieb-Robinson bound.

15 Can we send Q qubits for the price of 1? No!

$$\left(1 + \frac{M'}{Q}\right) T \gtrsim L.$$

M' = local Pauli measurement *outcomes* used for QEC.

Need $(M+1) \gtrsim L/T$ **for each qubit.**

If false (for Clifford dynamics), a clever logical X/Z pair, formed out of products of $(X_1, Z_1, \ldots, Z_Q)$, can be transmitted with $M + 1 < L/T$ measurements. Violates 1-qubit theorem.

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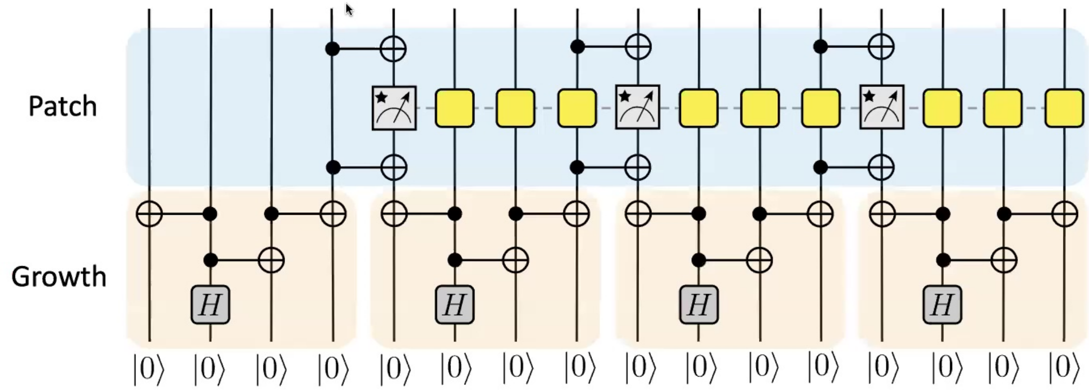
We have proved this in general, for $M' < L$, *except* for non-Clifford dynamics with measurement locations dependent on prior measurements, and $Q > 1$. A technical challenge only?

16 A corollary of our theorem shows that the GHZ state

$$|\mathrm{GHZ}\rangle = \alpha|0\cdots0\rangle + \beta|1\cdots1\rangle$$

cannot be prepared from $(\alpha|0\rangle + \beta|1\rangle) \otimes |0\cdots0\rangle$ unless $(M+1)vt \gtrsim L$.



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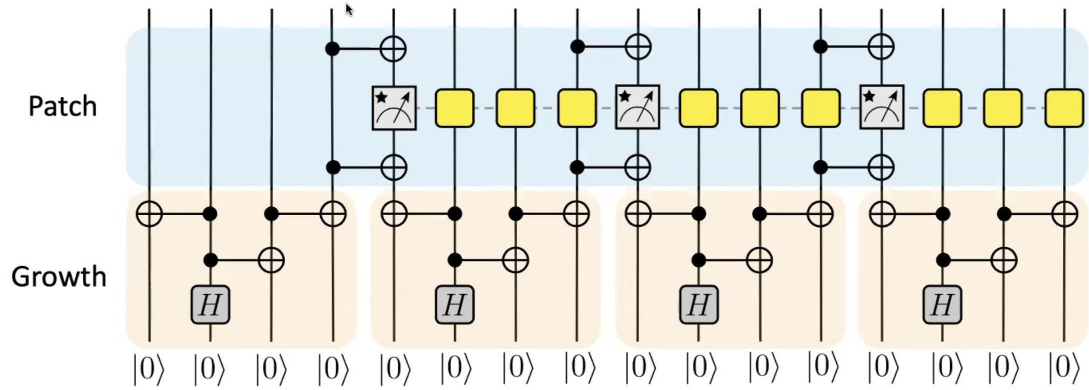
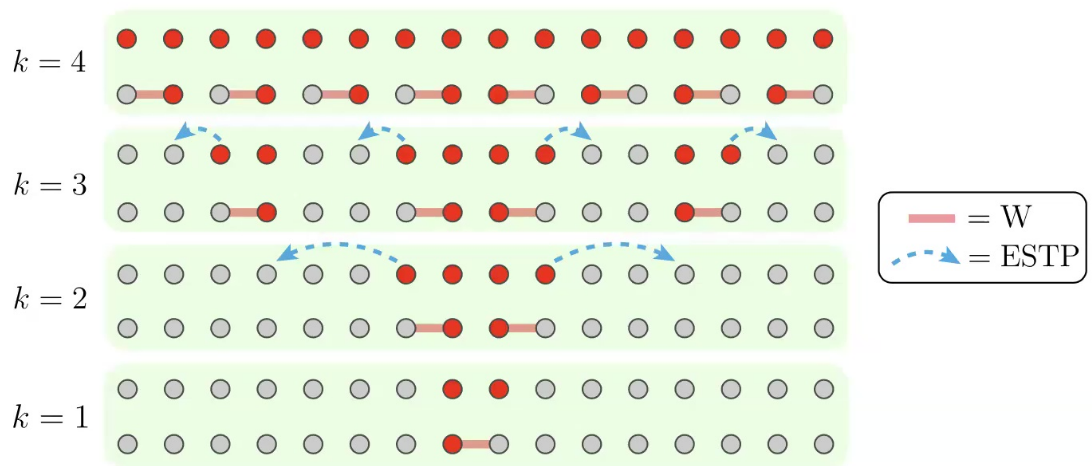


Also cannot transform "efficiently" between $|\mathrm{GHZ}\rangle$ and $|\mathrm{Bell}\rangle$ using any combination of measurement + unitary dynamics.

17 Our theorem also applies for preparing

$$|\mathrm{W}\rangle = \alpha|0\cdots 0\rangle + \beta\frac{|10\cdots 0\rangle + |01\cdots 0\rangle + |00\cdots 1\rangle}{\sqrt{N}},$$

but in this case we do not know if $(M+1)vt = L$ can be *saturated*. (We have $T \sim \log L$ and $M \sim L\log L$.)



18 A spin squeezed state (made out of L spin-$\frac{1}{2}$ particles on 1d chain) obeys (WLOG)

$$\langle J^\alpha \rangle = \delta_z^\alpha, \qquad \min_\theta \frac{\langle (J_x \cos\theta + J_y \sin\theta)^2 \rangle}{\langle J^z \rangle^2} = \frac{\xi^2}{L}.$$

We give tight bounds on preparing spin squeezed states in local 1d chains: if we want

$$\xi^2 \sim \frac{f}{L}, \quad f \geq 1$$

then we need

$$(M+1)vt \gtrsim \frac{L}{f}.$$

Saturated (with $M=0$) by preparing f independent max-squeezed regions.

19 $\mathbb{Z}_2 \times \mathbb{Z}_2$ SPT phases can protect a qubit against extensive measurement. We'd like to prepare with finite t unitary.

These phases can be used as a resource for teleportation and MBQC. [Stephen *et al*; 1611.08053]

$(M+1)vt \gtrsim L$, thus no SPT phase (nor anything else) can be used to teleport without extensive measurements/decoding.

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We generalized the Lieb-Robinson Theorem to dynamics with measurement. For **unentangled initial states**, sending one qubit requires M local measurement outcomes, where

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This **optimal bound** saturated by quantum repeater.

This constrains the preparation of (in 1d):

- GHZ state
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and also implies limits on how easy it might be to decode a qubit out of an SPT phase.

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Open: Can these bounds suggest optimal solutions to quantum hardware design? When can measurements and feedback actually help implement gates in a real quantum computer?

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