Title: The Future of Numerical Relativity: Gravitational Memory, BMS Frames, and More

Speakers: Keefe Mitman

Series: Strong Gravity

Date: November 17, 2022 - 1:00 PM

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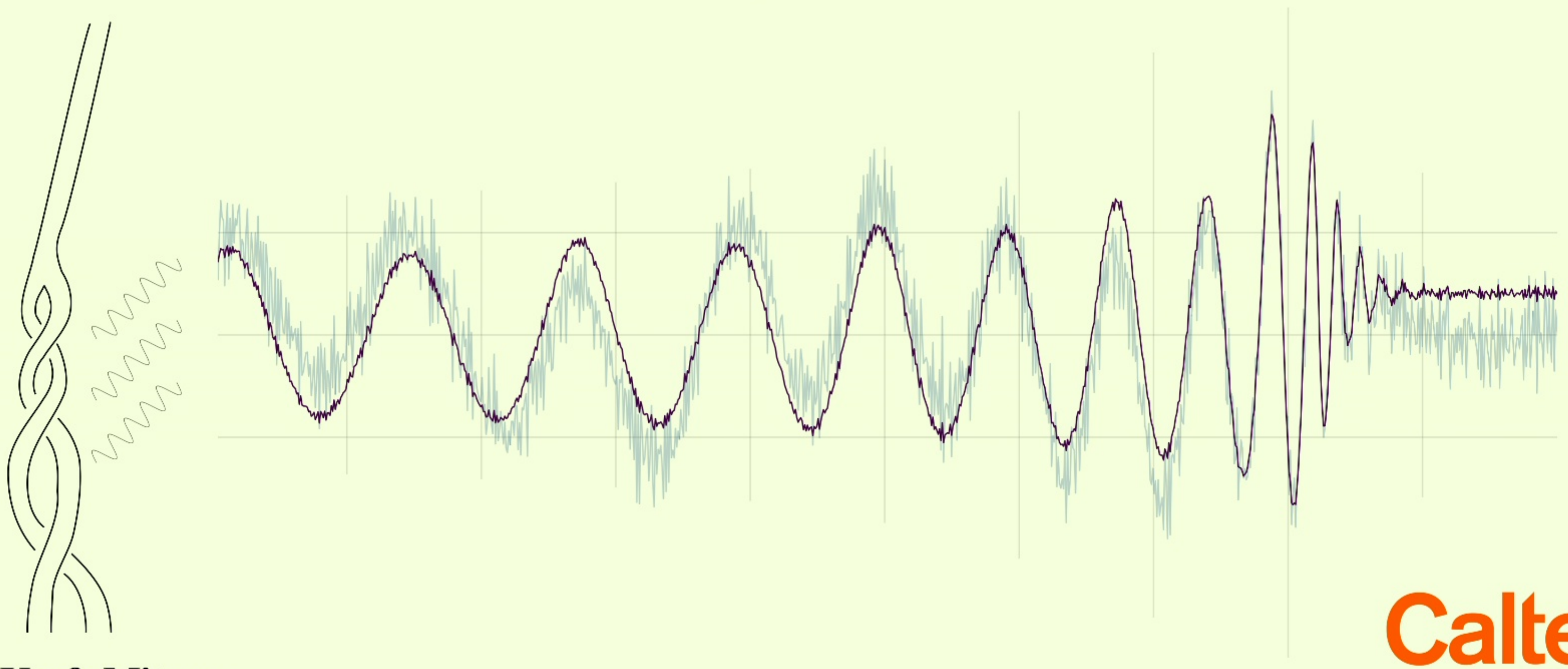
Abstract: As was realized by Bondi, Metzner, van der Burg, and Sachs (BMS), the symmetry group of asymptotic infinity is not the Poincaré group, but an infinite-dimensional group called the BMS group. Because of this, understanding the BMS frame of the gravitational waves produced by numerical relativity is crucial for ensuring that analyses on such waveforms and comparisons with other waveform models are performed properly. Up until now, however, the BMS frame of numerical waveforms has not been thoroughly examined, largely because the necessary tools have not existed. In this talk, I will highlight new methods that have led to improved numerical waveforms; specifically, I will explain what the gravitational memory effect is and how it has recently been resolved in numerical relativity. Following this, I will then illustrate how we fix the BMS frame of numerical waveforms to perform much more accurate comparisons with either quasi-normal mode or post-Newtonian models. Last, I will briefly highlight some exciting results that this work has enabled, such as building memory-containing surrogate models and finding nonlinearities in black hole ringdowns.

Zoom Link: https://pitp.zoom.us/j/96739417230?pwd=Tm00eHhxNzRaOEQvaGNzTE85Z1ZJdz09

The Future of Numerical Relativity:
Gravitational Memory, BMS Frames, and More



Keefe Mitman
Strong Gravity Seminar
Perimeter Institute, November 17, 2022

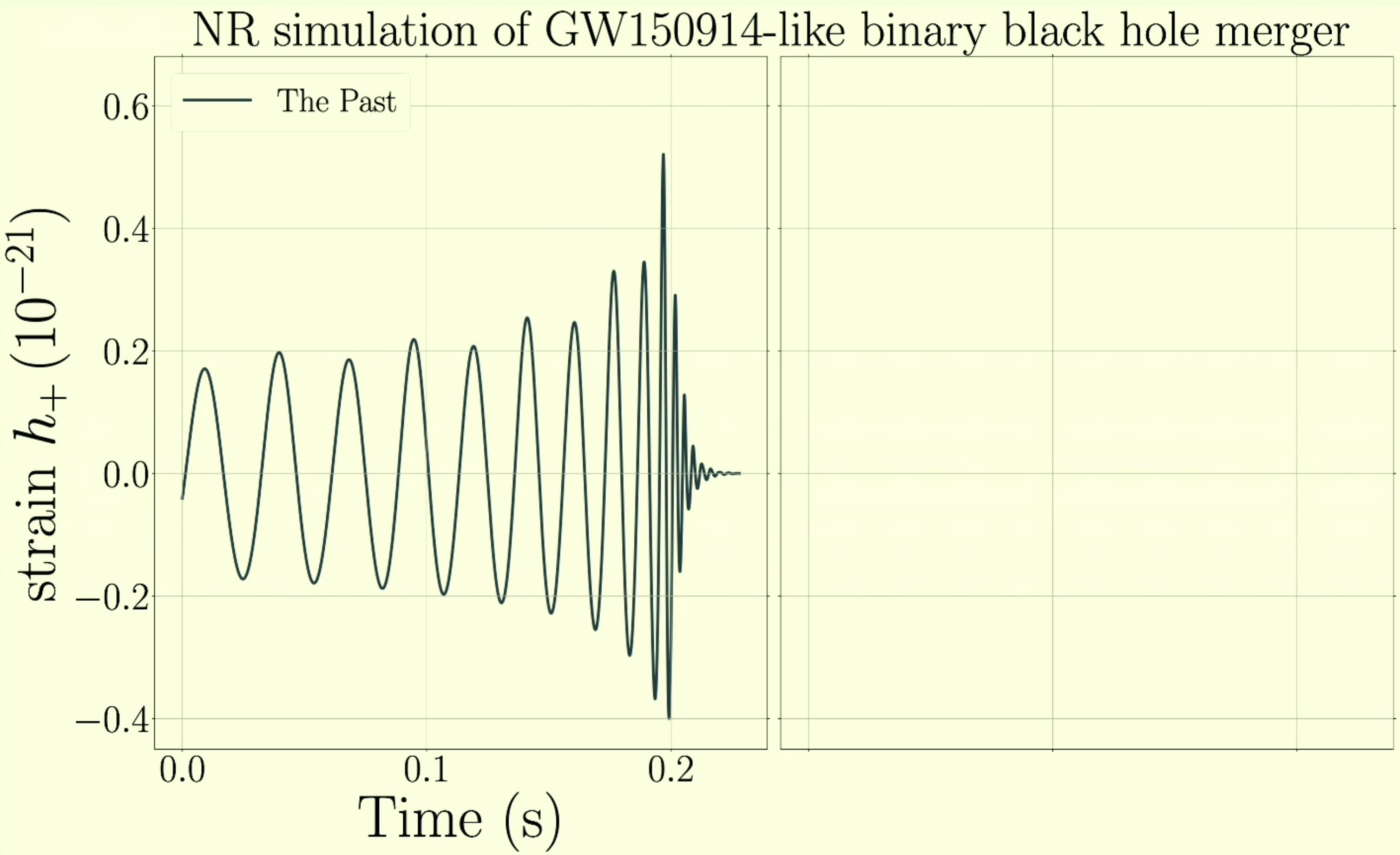
Caltech

SIMULATING eXTREME SPACETIMES
Black holes, neutron stars, and beyond...

Outline

- **Gravitational Memory**
  - Derivation from BMS charges and fluxes
- **Waveform Extraction: Cauchy-characteristic Evolution (CCE)**
  - What CCE is and how it can help replace extrapolation
- **Bondi-van der Burg-Metzner-Sachs (BMS) Frames**
  - Ways to fix the BMS frame
  - Comparing to quasi-normal mode models (QNMs)
  - Comparing to post-Newtonian models (PN)
- **New(er) Developments**
  - Models for 3G detectors
  - Nonlinearities in BH ringdowns

The Past vs. The Future



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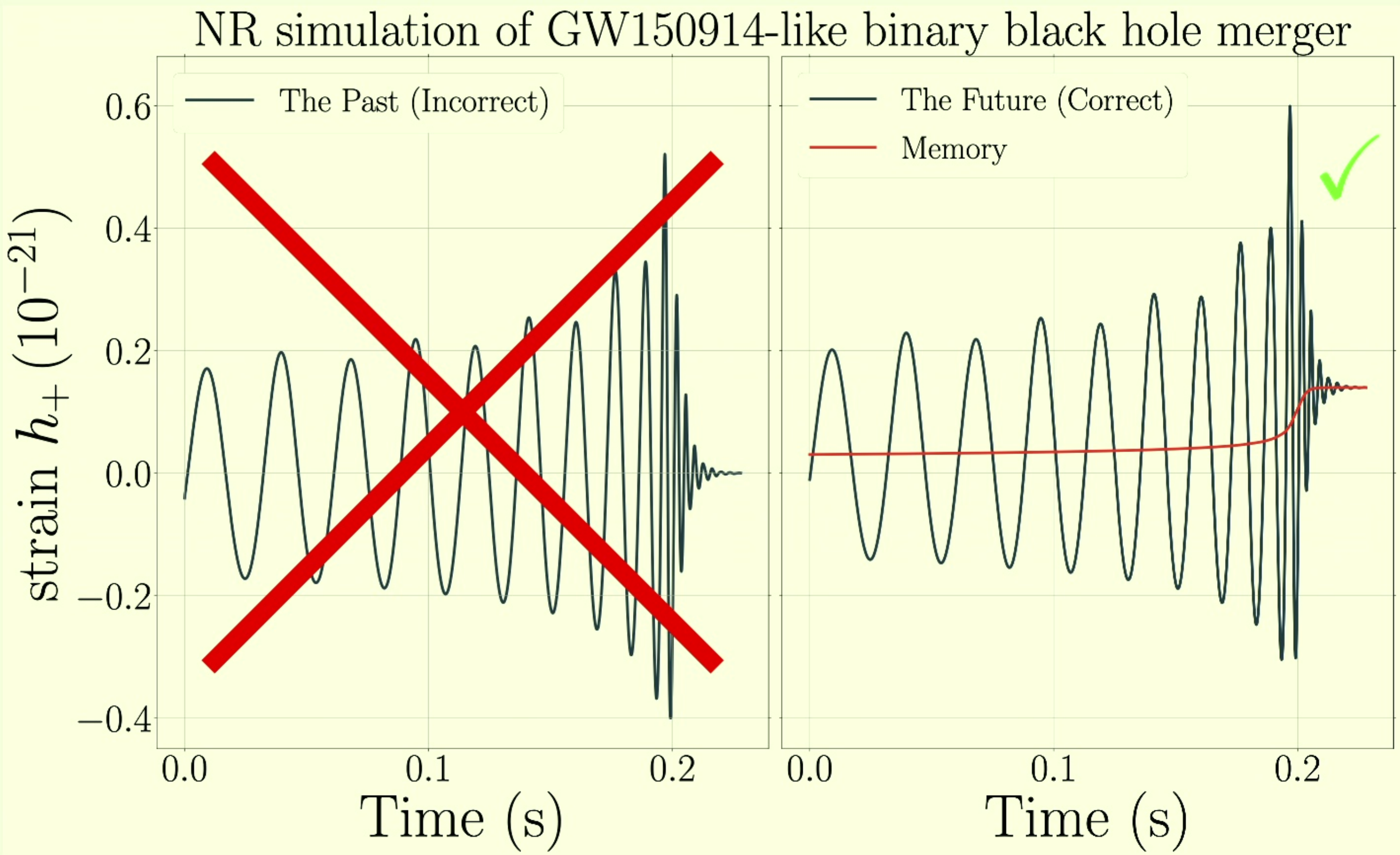


The BMS Group

The Formal Way:

- Find an equivalence class of vector fields $\overrightarrow{\xi}$ satisfying Killing's equation $\mathscr{L}_{\overrightarrow{\xi}} g_{ab} = 0$ (approximately, i.e., w.r.t. fall-off conditions) as one approaches future null infinity $\mathscr{I}^{+}$

- Yields

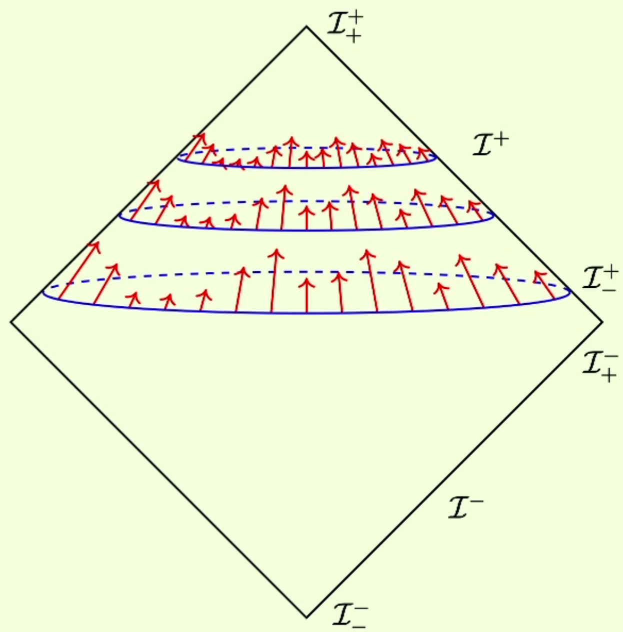
$$\overrightarrow{\xi} = \left[\alpha\left(\theta^{A}\right) + \frac{1}{2} u D_{A} Y^{A}\left(\theta^{B}\right)\right] \partial_{u} + Y^{A}\left(\theta_{B}\right) \partial_{A}$$

where

$Y^{A} = D^{A}\chi + \epsilon^{AB} D_{B}\kappa = \text{boost} + \text{rotation}$

The Easy Way:

- Consider a collection of observers on the celestial two sphere



The BMS Charges and Fluxes

Wald-Zoupas/Dray-Streubel Charges:

$$Q_{\vec{\xi}} = \frac{1}{4\pi}\int d^2\Omega \left[\alpha m + \frac{1}{2}Y^A \hat{N}_A\right]$$

where

$$m \equiv -\operatorname{Re}\left[\psi_2 + \sigma\dot{\bar{\sigma}}\right]$$

$$\hat{N} \equiv -\left(\psi_1 + \sigma\eth\bar{\sigma} + \frac{1}{2}\eth\left(\sigma\bar{\sigma}\right) + u\eth m\right)$$

This choice of charges corresponds to:

- no fluxes in Minkowski space
- real-valued supermomentum

Balance Laws:

For u_0 a non-radiative section of $\mathscr{I}^+$,

$$\operatorname{Re}\left[\bar{\eth}^2\sigma\right] = \operatorname{Re}\left[m + \mathscr{E} + \left(\bar{\eth}^2\sigma - m\right)|^{u_0}\right]$$

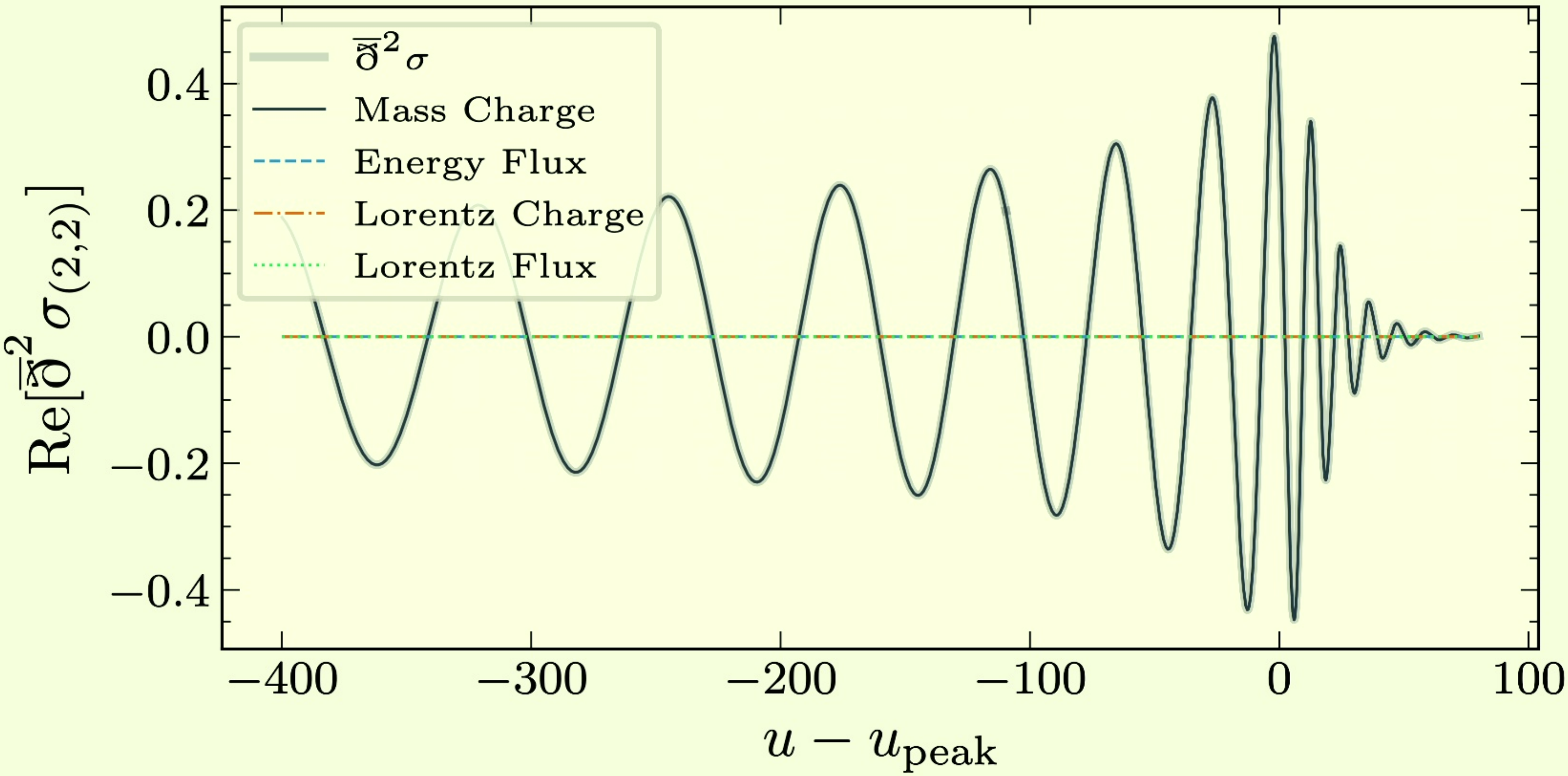
$$\operatorname{Im}\left[\bar{\eth}^2\sigma\right] = -\frac{d}{du}\left(\eth\bar{\eth}\right)^{-1}\operatorname{Im}\left[\bar{\eth}\left(\hat{N} + \mathscr{J}\right)\right]$$

where

$$\mathscr{E} \equiv \int_{u_0}^{u} |\dot{\sigma}|^2 du$$

$$\mathscr{J} \equiv \frac{1}{2}\int_{u_0}^{u}\left(3\dot{\sigma}\eth\bar{\sigma} - 3\sigma\eth\dot{\bar{\sigma}} + \bar{\sigma}\eth\dot{\sigma} - \dot{\bar{\sigma}}\eth\sigma\right) du$$

Numerical Calculation of BMS Charges (Bondi Mass Aspect)



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The Gravitational Memory Effect(s)

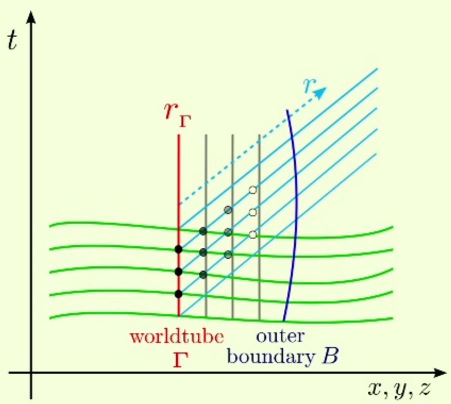
$$\Delta \mathrm{Re}\left[\sigma\right] = \eth^2 \left(\eth^2 \bar{\eth}^2\right)^{-1} \mathrm{Re}\left[\Delta m + \mathscr{E}\right], \quad \int_{-\infty}^{+\infty} \mathrm{Im}\left[\sigma\right] du = -\,\eth^2 \left(\eth^2 \bar{\eth}^2\right)^{-1} \left(\eth \bar{\eth}\right)^{-1} \mathrm{Im}\left[\bar{\eth}\left(\Delta \hat{N} + \mathscr{J}\right)\right]$$

Observable	Parity	Type	Memory
Bondi Mass Aspect	Electric	Ordinary (linear)	Displacement
Energy Flux	Electric	Null (non-linear)	Displacement
Lorentz Aspect	Electric/Magnetic	Ordinary (linear)	Center-of-Mass/Spin
Lorentz Flux	Electric/Magnetic	Null (non-linear)	Center-of-Mass/Spin

Extrapolation vs. Cauchy-characteristic Evolution

Extrapolation:

- Obtain the metric (and derivatives) on a series of finite-radius world-tubes
- Interpolate (radially) between various points on the radially-varying world tubes
- Extrapolate to $r \to \infty$



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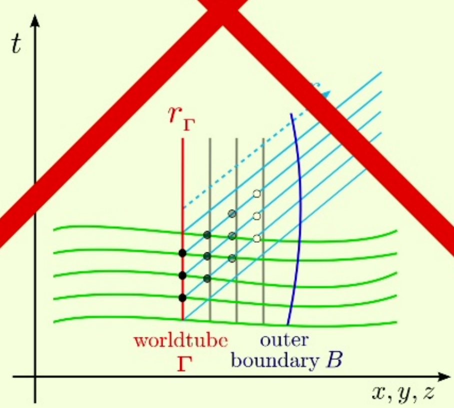
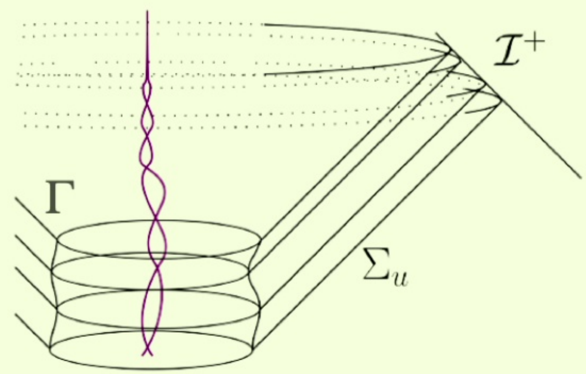
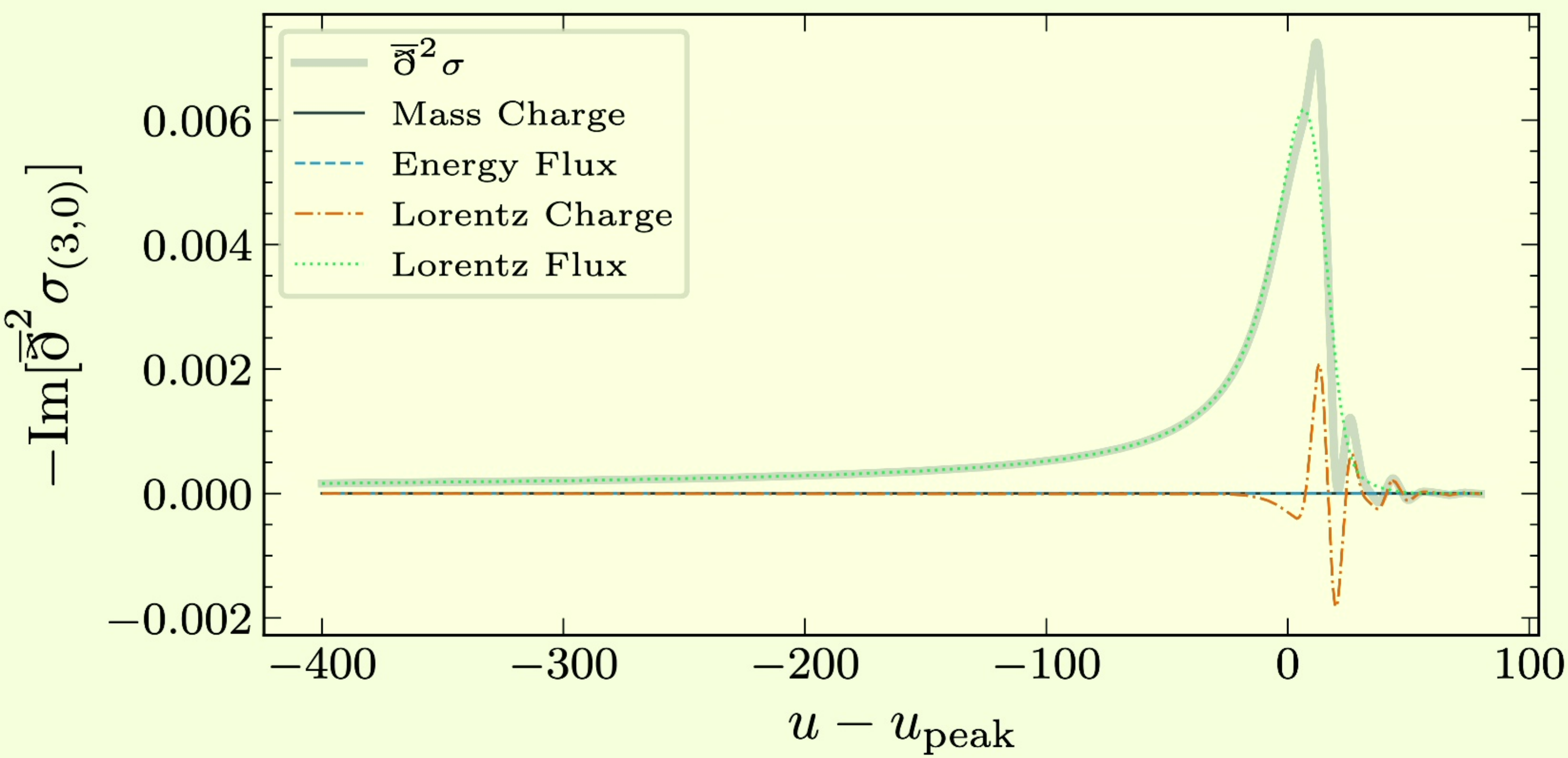


Cauchy-characteristic Evolution

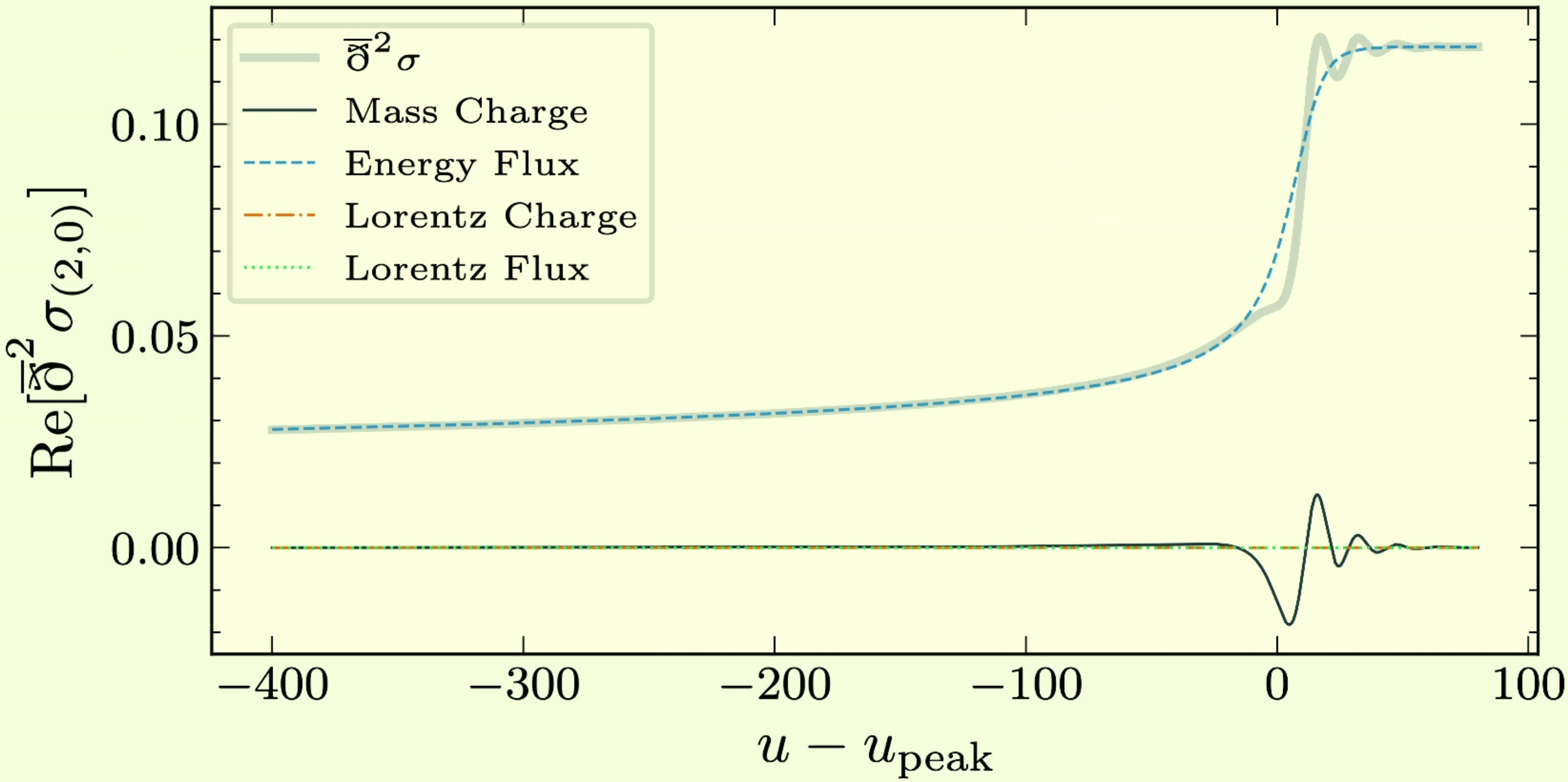
- Obtain the metric (and derivatives) on a finite-radius world-tube Γ
- Initialize the first null hypersurface Σ_u by matching the shear (and derivatives) at Γ
- Evolve Σ_u forward in time



Numerical Calculation of Memory Effects (Lorentz Flux)



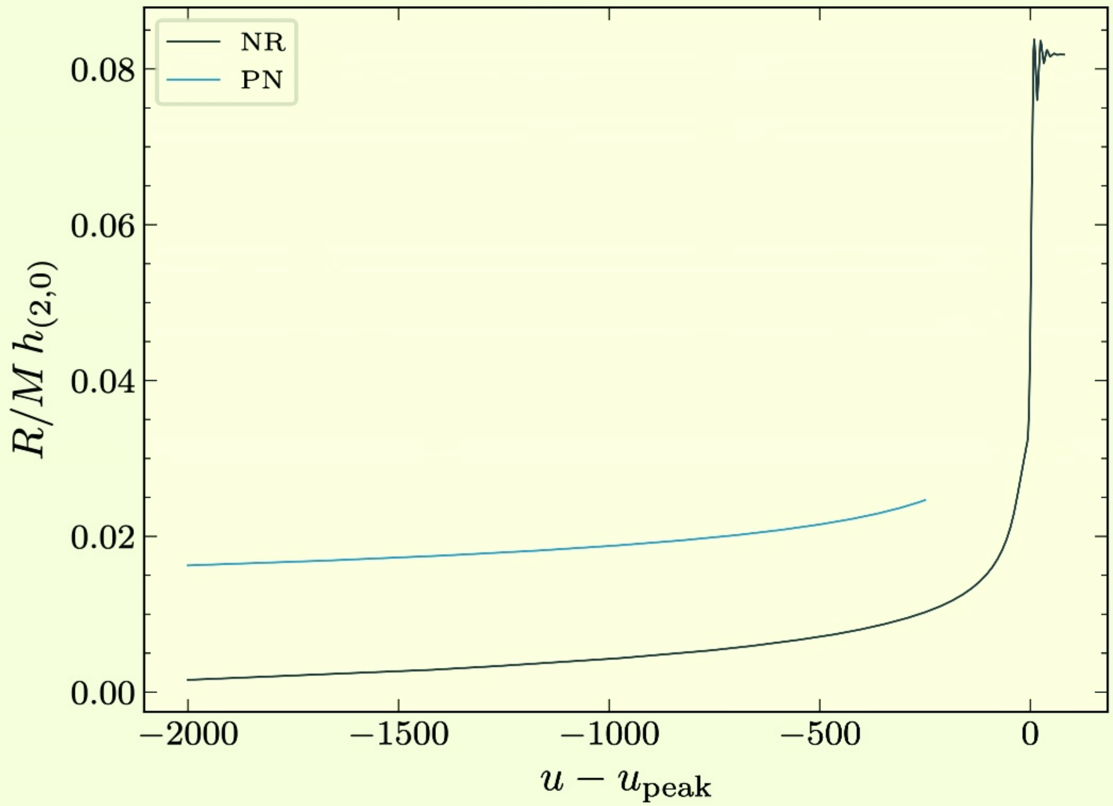
Numerical Calculation of Memory Effects (Energy Flux)



Comparison of Memory Modes in NR vs. PN

What's going wrong?

- CCE requires a robust method for initializing the first null hypersurface
- Current scheme effectively uses the initial data of the Cauchy evolution
- Ideally we would use PN



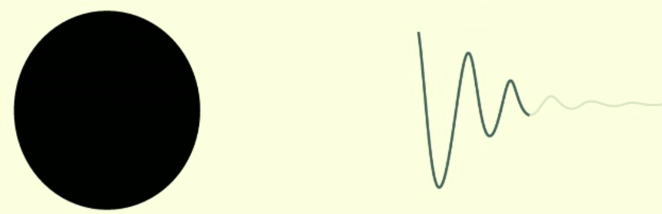
Learning from Quasi-normal Modes

Perturbed BHs emit radiation!

- Can model the strain as

$$h^{\mathrm{QNM}}_{\ell mn\pm} = \mathscr{A}^{\pm}_{\ell mn} e^{-i\omega^{\pm}_{\ell mn}(u-u_0)} \sum_{\ell'} C_{\ell'\ell m}\left(a\omega^{\pm}_{\ell mn}\right)$$

where $\mathrm{Im}\left[\omega\right] < 0$ for stability



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  where $\text{Im}\left[\omega\right] < 0$ for stability
- This implies that as
  $u \to +\infty, \quad h^{\text{QNM}} \to 0$
- But $h(u \to +\infty) \neq 0$ due to memory…

Effect of BMS transformation on Asymptotics:

When acted on by a BMS transformation $\left(\alpha\left(\zeta,\bar{\zeta}\right),\left(a,b,c,d\right)\right)$**...**

- $$\left(u,\zeta\right)\rightarrow\left(u',\zeta'\right)=\left(k\left(u-\alpha\right),\frac{a\zeta+b}{c\zeta+d}\right)$$

- $$\sigma\left(u,\zeta,\bar{\zeta}\right)\rightarrow\sigma'\left(u',\zeta',\bar{\zeta}\right)=\sigma\left(u',\zeta,\bar{\zeta}\right)-\eth^2\alpha\left(\zeta,\bar{\zeta}\right)$$
$$=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\alpha\left(\zeta,\bar{\zeta}\right)\frac{\partial}{\partial u}\right)^n\sigma\left(u,\zeta,\bar{\zeta}\right)-\eth^2\alpha\left(\zeta,\bar{\zeta}\right)$$

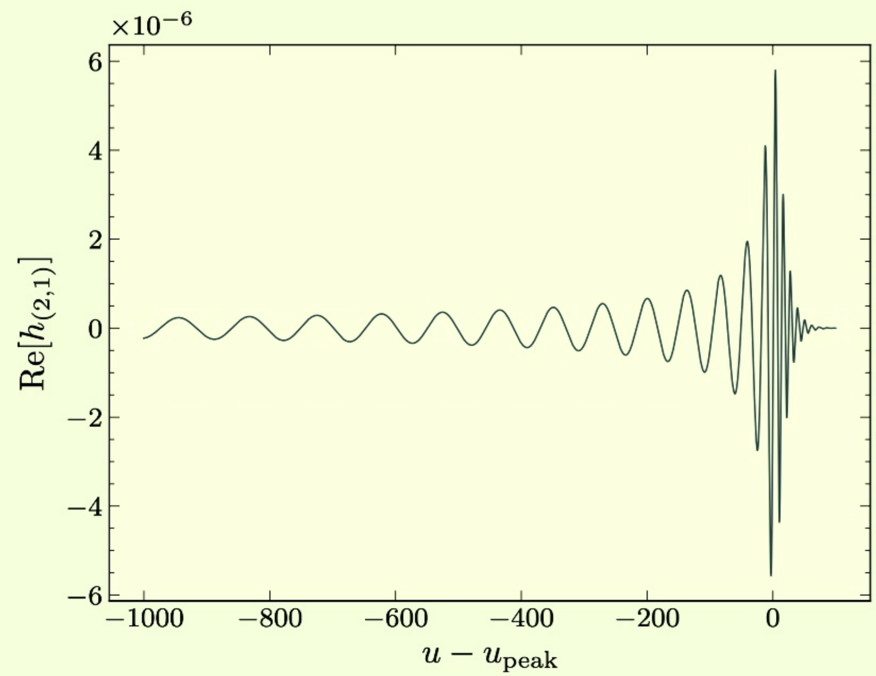
Fixing the Poincaré Frame

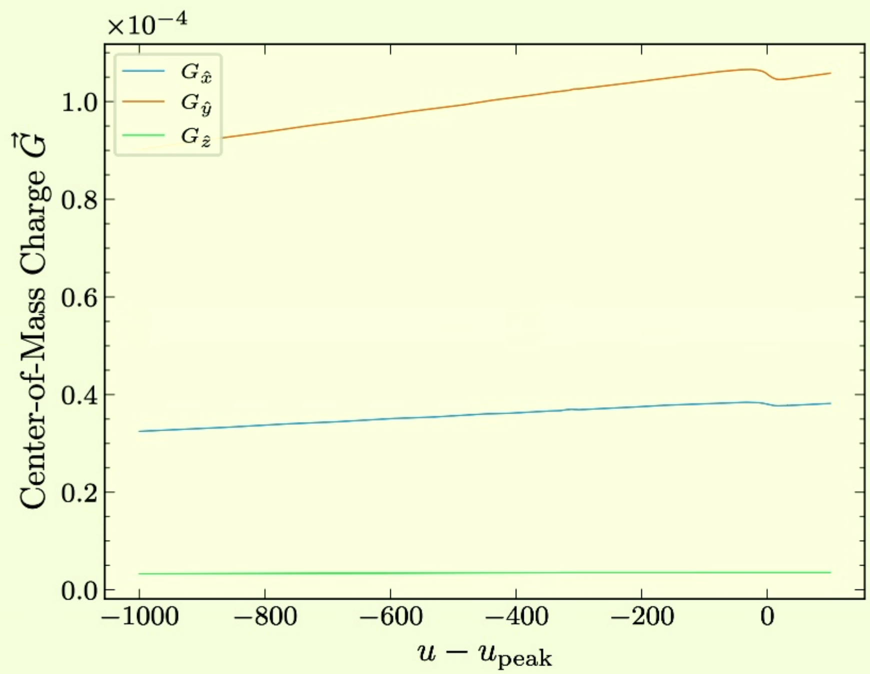
Poincaré Frame:

- translations
- boosts

} → **Center-of-Mass Charge "rest frame"**

- rotations } → **Rotation Charge "Aligned with $\hat{z}$-axis"**





Fixing the BMS Frame with supermomentum

- Use the supermomentum from Dray/Streubel:
  $\Psi_{p,q} = \Psi_2 + \sigma\dot{\bar{\sigma}} + p\left(\eth^2\bar{\sigma}\right) - q\left(\bar{\eth}^2\sigma\right)$
  - $p = q \Rightarrow$ no supermomentum flux in Minkowski space; $p + q = 1 \Rightarrow$ supermomentum is real

- For $p = q = 1/2$, we have the Geroch supermomentum:
  $\Psi_{\mathrm{G}} = \Psi_2 + \sigma\dot{\bar{\sigma}} + \frac{1}{2}\left(\eth^2\bar{\sigma} - \bar{\eth}^2\sigma\right)$
  - This is supertranslation-covariant in non-radiative regimes of $\mathscr{I}$

Fixing the BMS Frame with the Moreschi supermomentum

- Using the Moreschi supermomentum, we have
  $\mathrm{Re}\left[\bar{\eth}^2\sigma\right]' = m' + \Psi_{\mathrm{M}}' = m + \left(\Psi_{\mathrm{M}} - \eth^2\bar{\eth}^2\alpha\right)$

- Define the *superrest frame* as the frame in which $\Psi_{\mathrm{M}} = M_{\mathrm{B}}$

Fixing the BMS Frame with the Moreschi supermomentum

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  $\mathrm{Re}\left[\bar{\eth}^2\sigma\right]' = m' + \Psi_{\mathrm{M}}' = m + \left(\Psi_{\mathrm{M}} - \eth^2\bar{\eth}^2\alpha\right)$

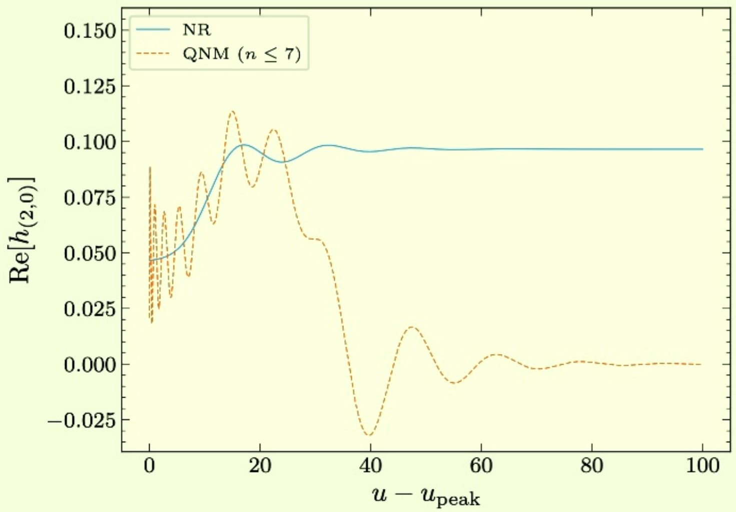
- Define the *superrest frame* as the frame in which $\Psi_{\mathrm{M}} = M_{\mathrm{B}}$
  - For bound systems (like BBHs), this corresponds to there being no "instantaneous" memory

- Can be solved for using the nice section equation:
  $\eth^2\bar{\eth}^2\alpha = \Psi_{\mathrm{M}}\left(u = \alpha, \zeta, \bar{\zeta}\right) + k_{\mathrm{rest}}\left(\alpha, \zeta, \bar{\zeta}\right)^3 M_{\mathrm{B}}(\alpha)$

Fixing the BMS Frame (for QNMs: the remnant BH's *superrest* frame)

BMS Frame:

- supertranslations } → **Moreschi supermomentum**
- translations
- boosts } → **Center-of-Mass Charge "rest frame"**
- rotations } → **Rotation Charge "Aligned with $\hat{z}$-axis"**



Fixing the BMS Frame (for QNMs: the remnant BH's *superrest* frame)

BMS Frame:

- supertranslations → **Moreschi supermomentum**
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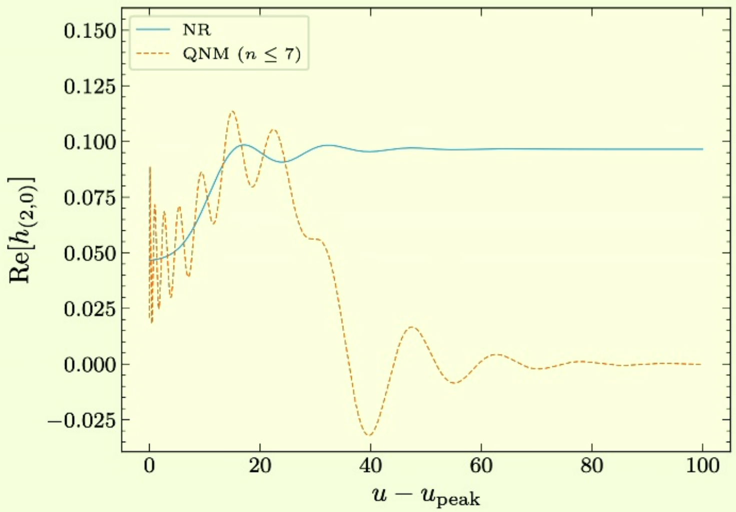
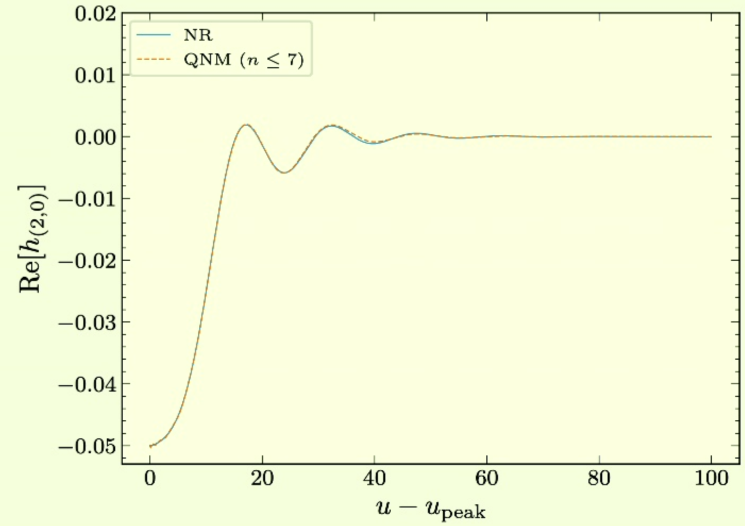


Map NR to superrest frame →



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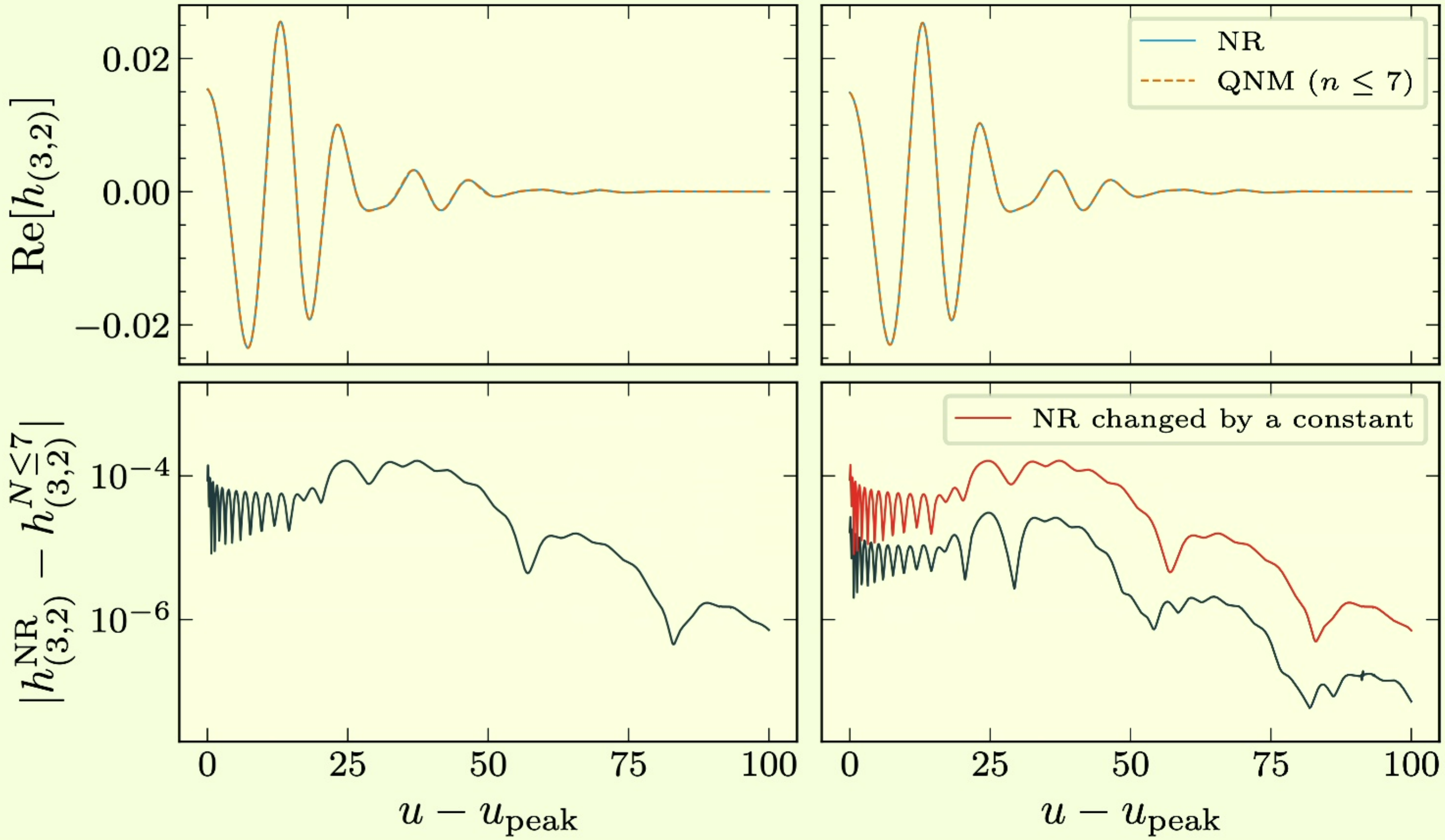
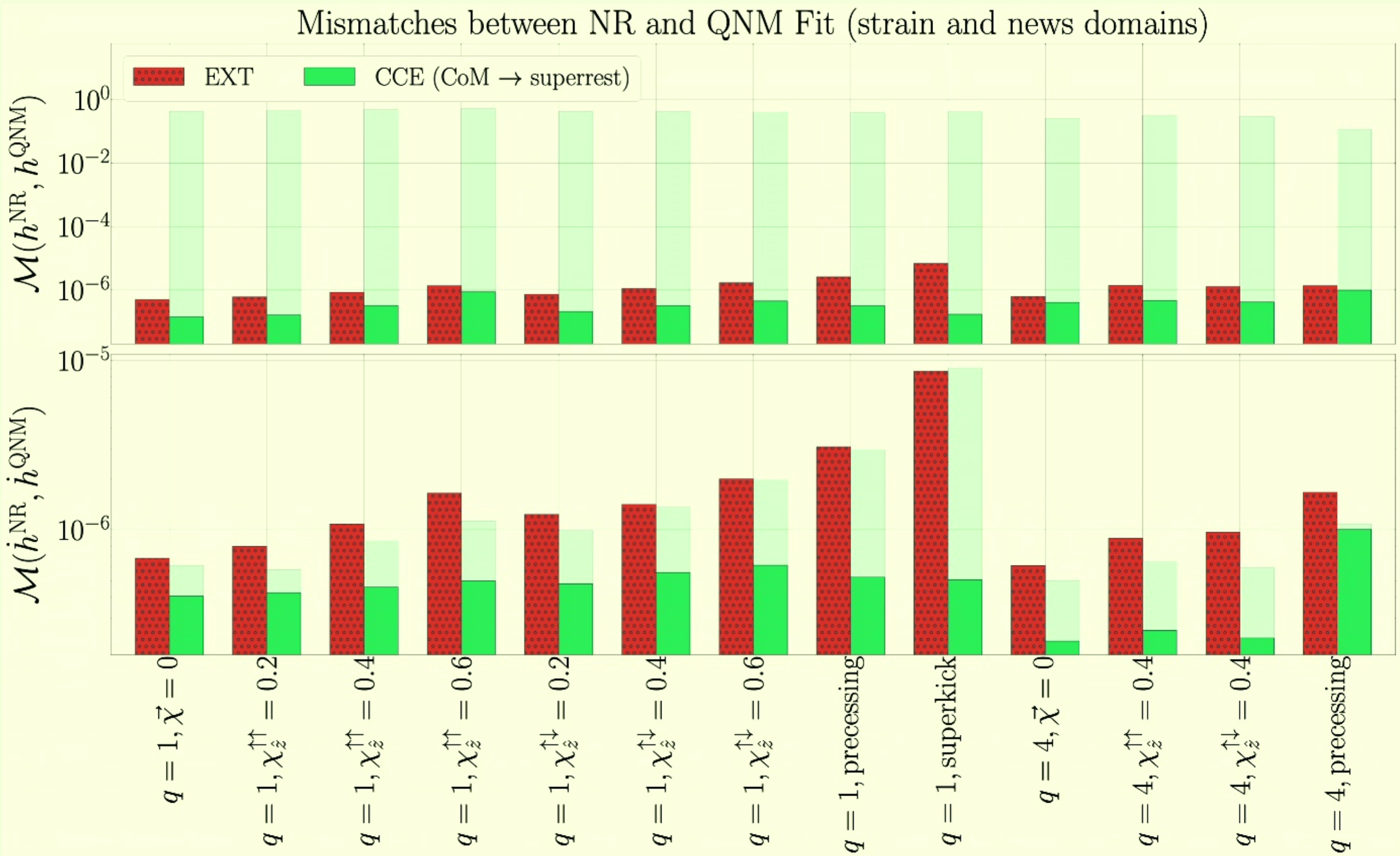


Importance of Fixing the BMS Frame (for QNMs)



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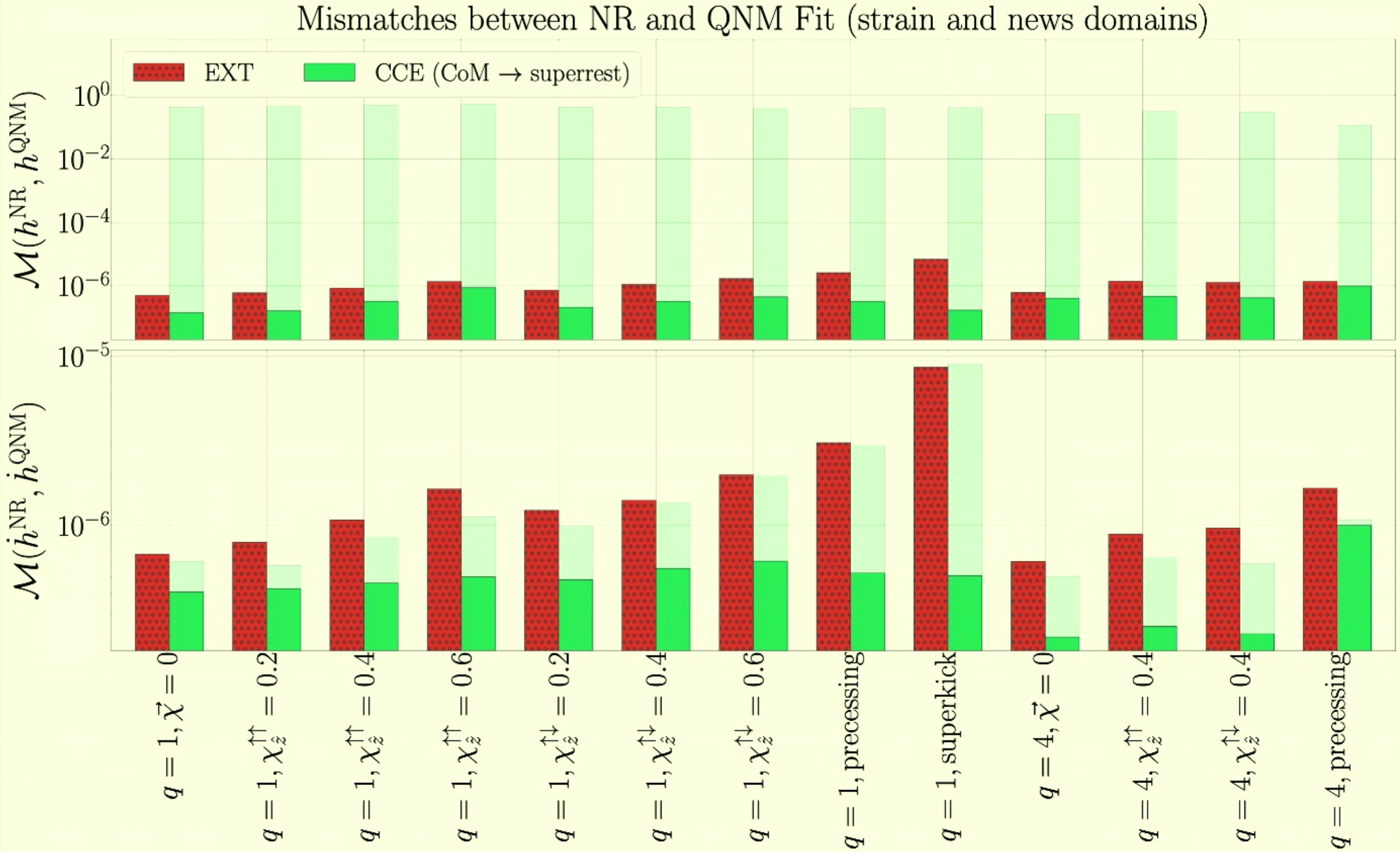


Fixing the BMS Frame (for PN waveforms: the *canonical* frame)

PN waveforms are in the "canonical" BMS frame

$$m\left(u\to-\infty,\zeta,\bar{\zeta}\right)=m_0=\text{constant}$$

$$\sigma\left(u\to-\infty,\zeta,\bar{\zeta}\right)=0$$

$$\hat{N}\left(u\to-\infty,\zeta,\bar{\zeta}\right)=\text{magnetic parity},\ell=1\ \left(\text{only rotational}\right)$$

- Map to this frame by mapping the NR supermomentum to the PN supermomentum
- Need to know the PN Moreschi supermomentum
  - Can compute this "easily" because

$$\Psi_{\mathrm{M}}\left(u,\zeta,\bar{\zeta}\right)=\int_{-\infty}^{u}|\dot{\sigma}|^2du-M_{\mathrm{ADM}}$$

Computing the PN Moreschi supermomentum

$$\mathcal{P}^{(0,0)} = 1 + x\left(-\frac{3}{4} - \frac{\nu}{12}\right) + x^2\left(-\frac{27}{8} + \frac{19\nu}{8} - \frac{\nu^2}{24}\right) + x^3\left(-\frac{675}{64} + \left(\frac{34445}{576} - \frac{205\pi^2}{96}\right)\nu - \frac{155\nu^2}{96} - \frac{35\nu^3}{5184}\right),$$

$$\mathcal{P}^{(0,0)}_{\text{spin}} = x^{3/2}\left(\frac{14S_\ell}{3M^2} + \frac{2\delta\Sigma_\ell}{M^2}\right) + x^2\left(-\frac{16\vec{S}\cdot\vec{S} + 3\vec{\Sigma}\cdot\vec{\Sigma} + 32S_\ell^2 + 9\Sigma_\ell^2}{12M^4} - \frac{4\delta\left(\vec{S}\cdot\vec{\Sigma} + 2S_\ell\Sigma_\ell\right)}{3M^4} + \frac{4\left(\vec{\Sigma}\cdot\vec{\Sigma} + 2\Sigma_\ell^2\right)\nu}{3M^4}\right),$$

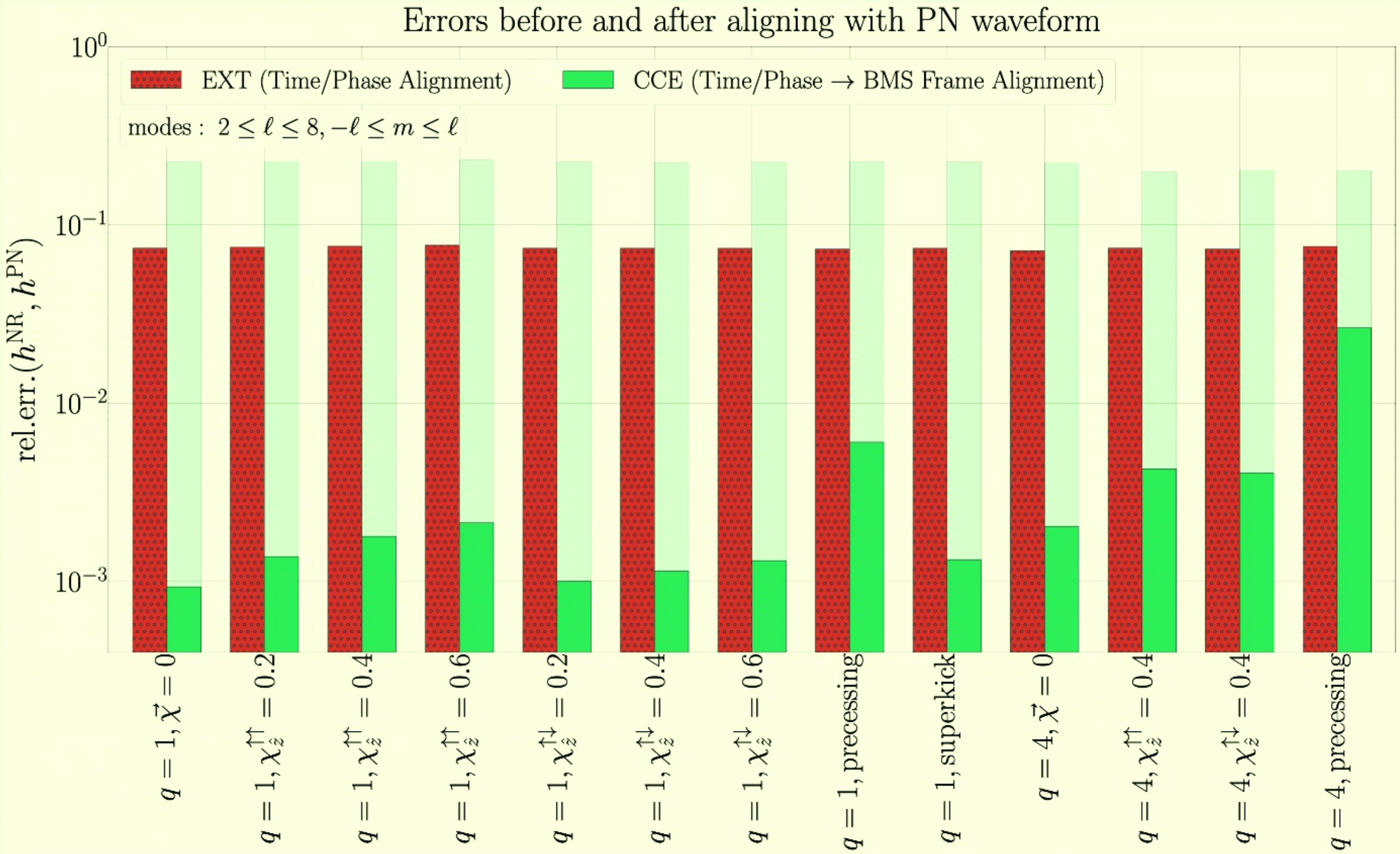
$$\mathcal{P}^{(2,0)} = \frac{2}{7}\sqrt{5}\left\{1 + x\left(-\frac{4075}{4032} + \frac{67\nu}{48}\right) + x^2\left(-\frac{151877213}{67060224} - \frac{123815\nu}{44352} + \frac{205\nu^2}{352}\right) + \pi x^{5/2}\left(-\frac{253}{336} + \frac{253\nu}{84}\right) + x^3\left(-\frac{4397711103307}{532580106240} + \left(\frac{700464542023}{13948526592} - \frac{205\pi^2}{96}\right)\nu + \frac{69527951\nu^2}{166053888} + \frac{1321981\nu^3}{5930496}\right)\right\},$$

$$\mathcal{P}^{(2,0)}_{\text{spin}} = \frac{2}{7}\sqrt{5}\left\{x^{3/2}\left(\frac{16S_\ell}{3M^2} + \frac{419\delta\Sigma_\ell}{160M^2}\right) + x^2\left(-\frac{128\vec{S}\cdot\vec{S} + 24\vec{\Sigma}\cdot\vec{\Sigma} + 256S_\ell^2 + 75\Sigma_\ell^2}{96M^4} - \frac{4\delta\left(\vec{S}\cdot\vec{\Sigma} + 2S_\ell\Sigma_\ell\right)}{3M^4} + \frac{4\left(\vec{\Sigma}\cdot\vec{\Sigma} + 2\Sigma_\ell^2\right)\nu}{3M^4}\right)\right\},$$

$$\mathcal{P}^{(3,1)} = \frac{223}{120\sqrt{21}}\left\{x^3\left(\frac{3872\delta\nu}{223}\right)\right\},$$

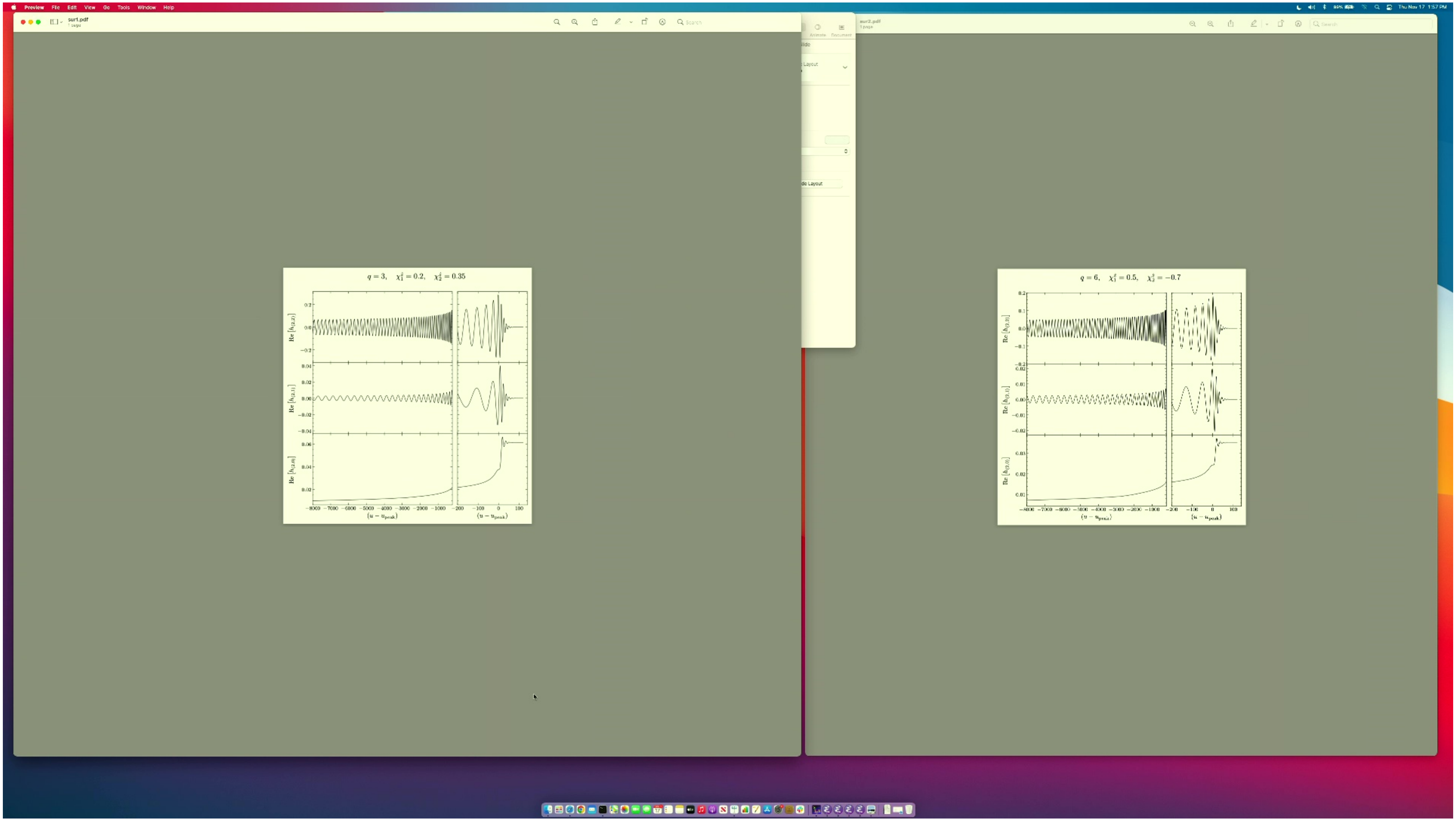
$$\mathcal{P}^{(3,1)}_{\text{spin}} = 0,$$

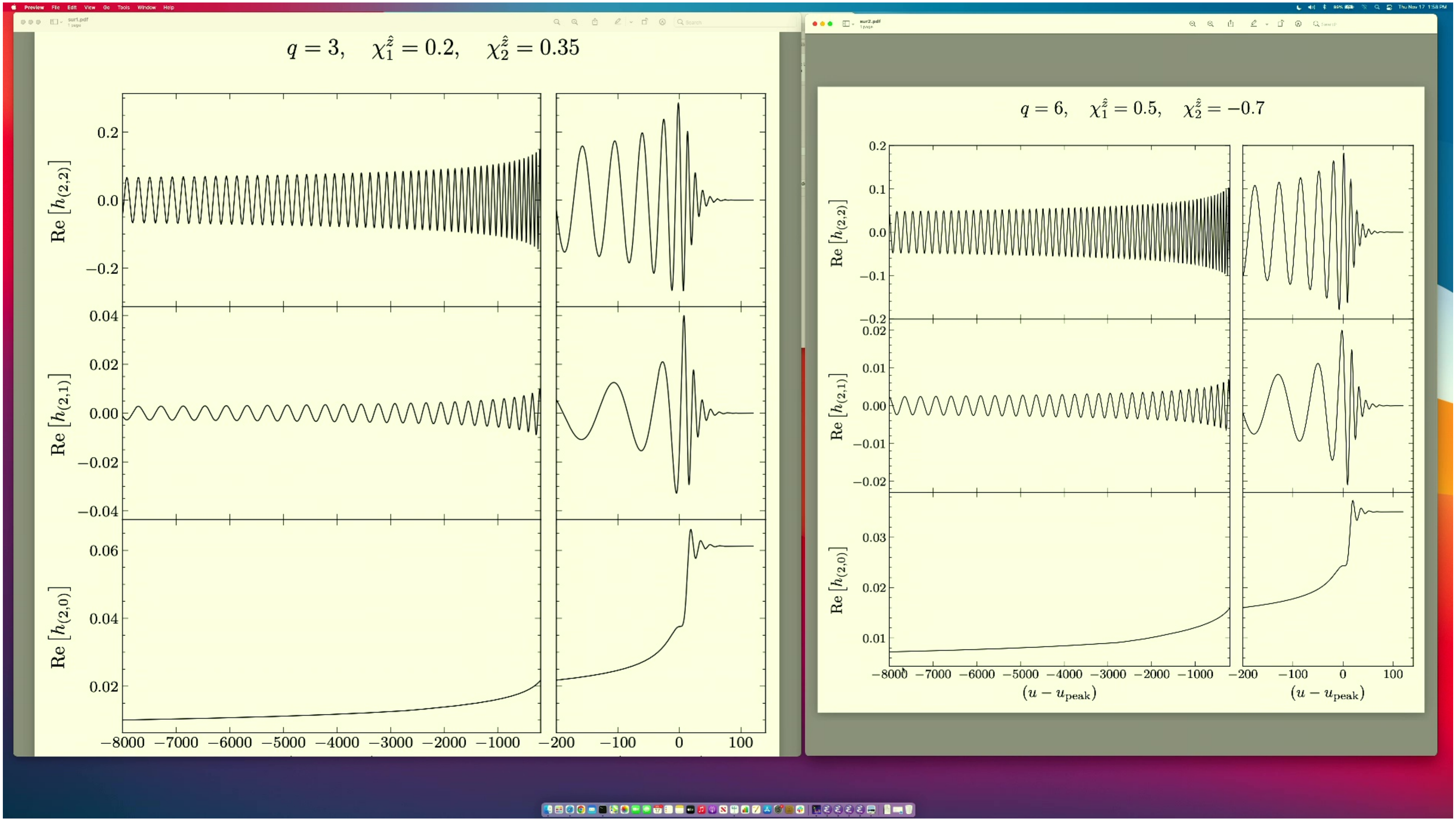
Importance of Fixing the BMS Frame (for PN waveforms)



Constructing CCE surrogate models

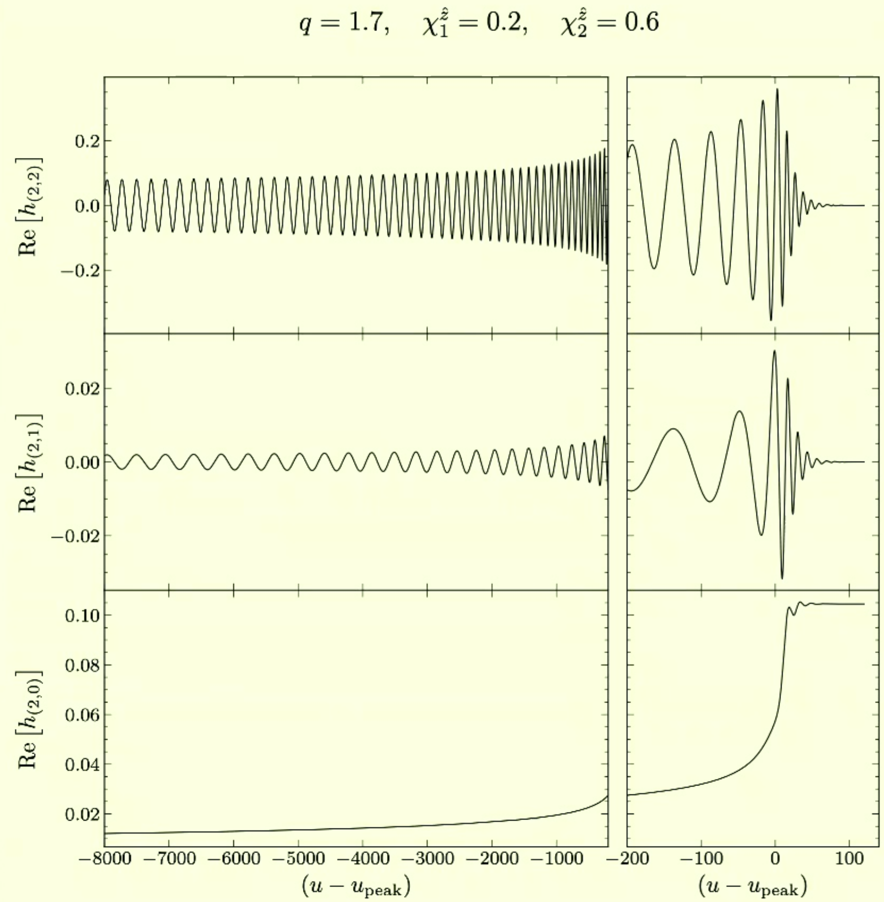
- Build a surrogate for simulations with $q \leq 8$, $|\chi_1^{\hat{z}}| \leq 0.8$, $|\chi_2^{\hat{z}}| \leq 0.8$ using CCE waveforms
- Waveforms are
  - PN from an orbital frequency of $\omega = 2 \times 10^{-4}$ until 20 orbits before merger,
  - A hybridization of NR and PN over a three orbit window,
  - NR for the remaining parts of inspiral, merger, and ringdown
- Alignment of NR and PN prior to hybridization maps NR to the PN BMS frame
- Mismatches between surrogate and training data are $\lesssim 2 \times 10^{-4}$ over $2.25M_\odot \leq M \leq 300M_\odot$
- Better than existing models **and has memory effects!**

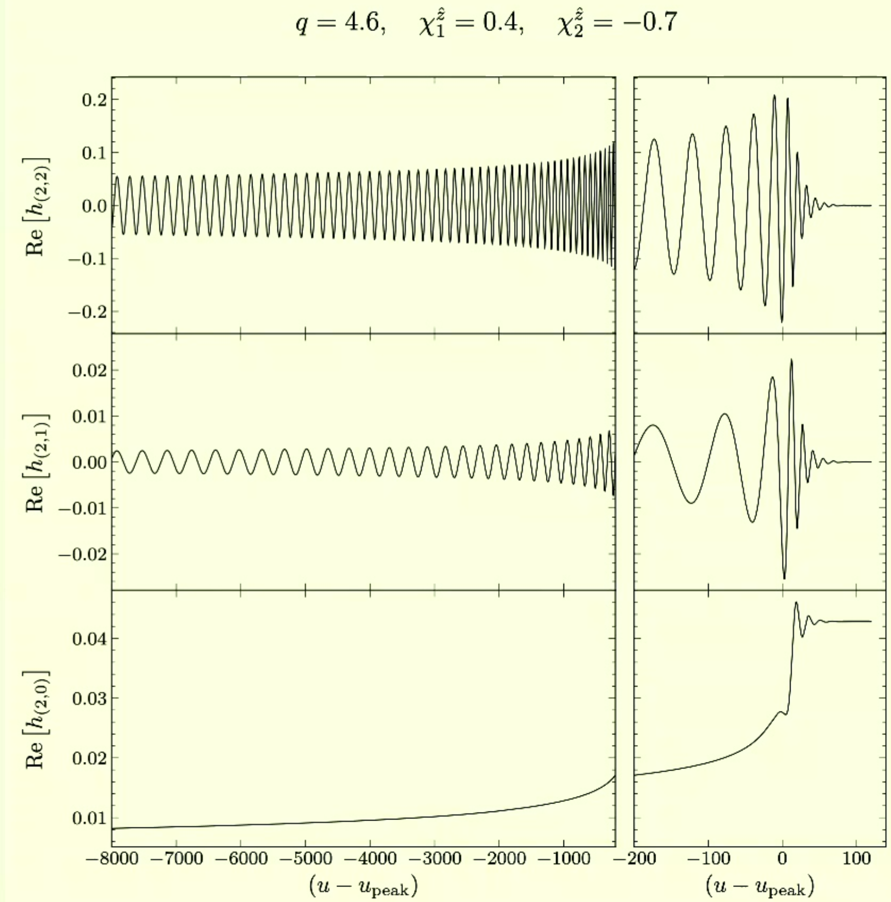

$q = 3, \quad \chi_1^z = 0.2, \quad \chi_2^z = 0.35$
$q = 6, \quad \chi_1^z = 0.5, \quad \chi_2^z = -0.7$



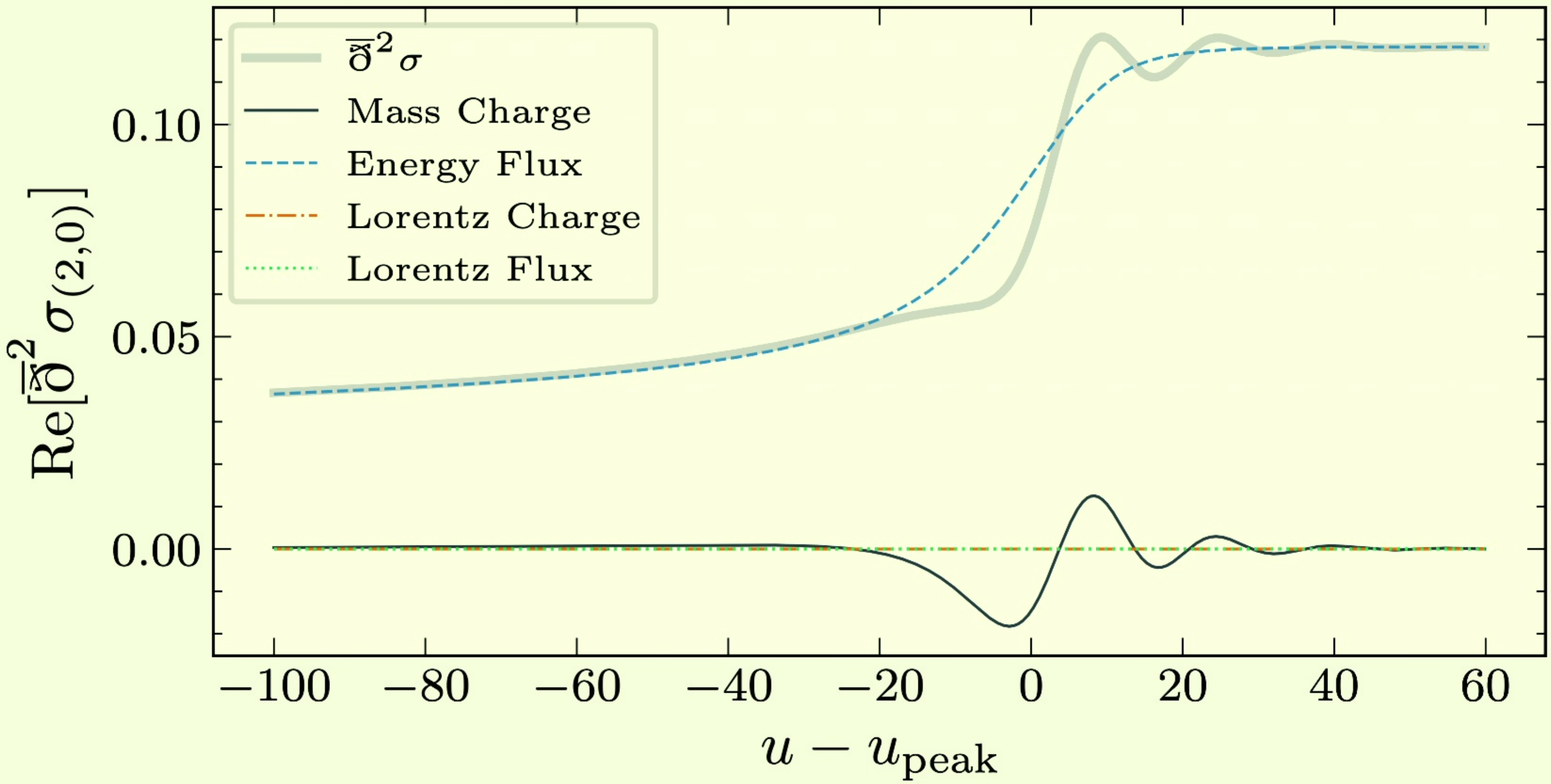
q = 3, χ₁ = 0.2, χ₂ = 0.35
Re [h(2,2)]
Re [h(2,1)]
Re [h(2,0)]
q = 6, χ₁ = 0.5, χ₂ = −0.7
(u − u_peak)
(u − u_peak)

Constructing CCE surrogate models





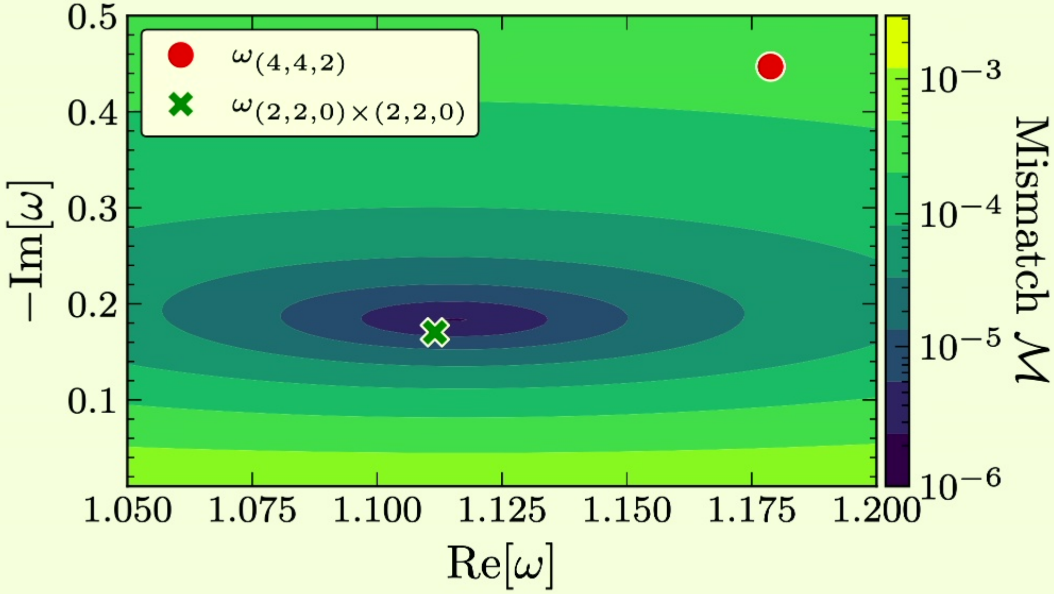
Nonlinearities in BH ringdowns



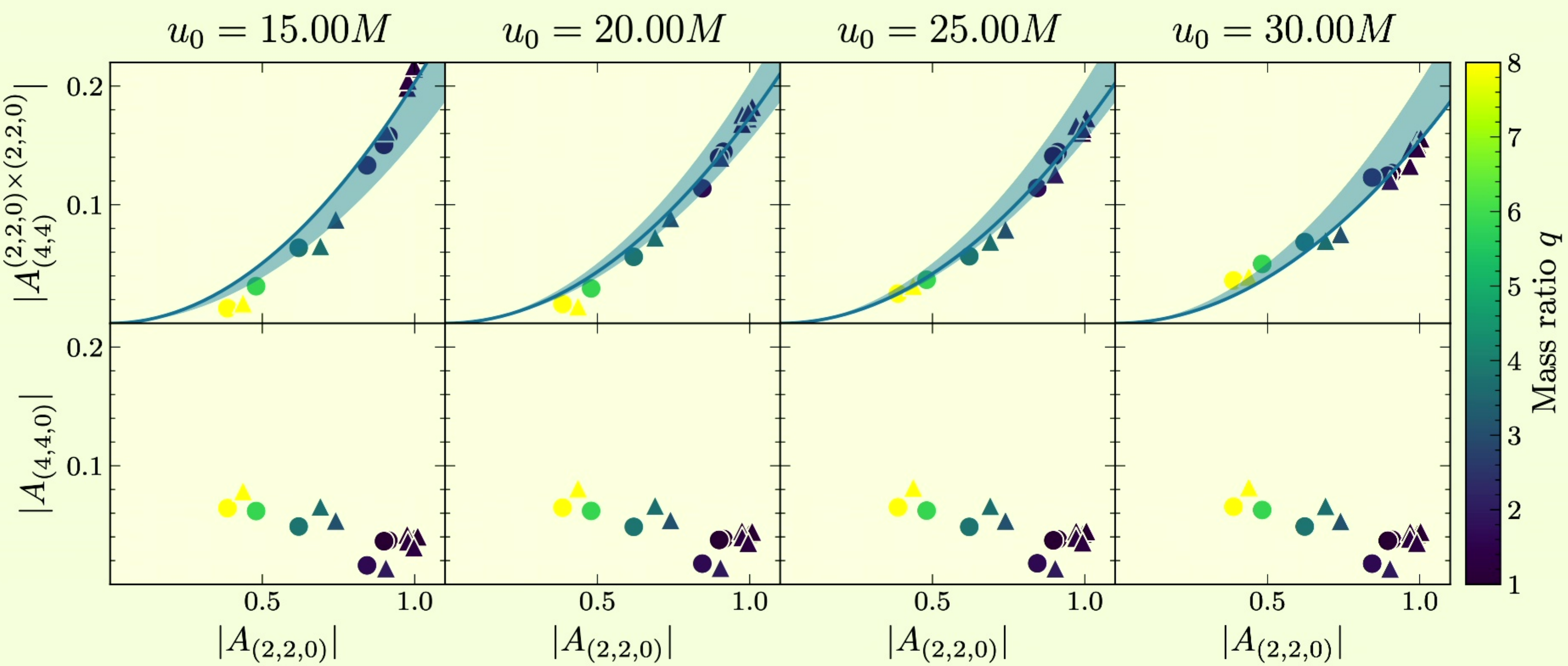
Nonlinearities in BH ringdowns

$$h^{\text{model},L}_{(4,4)} = \sum_{n=0}^{2} A_{(4,4,n)} e^{-i\omega_{(4,4,n)}(u-u_{\text{peak}})}$$

$$h^{\text{model},Q}_{(4,4)} = \sum_{n=0}^{1} A_{(4,4,n)} e^{-i\omega_{(4,4,n)}(u-u_{\text{peak}})} + A^{(2,2)\times(2,2)}_{(4,4)} e^{-i\omega_{(2,2)\times(2,2)}(u-u_{\text{peak}})}$$



Nonlinearities in BH ringdowns



Conclusion

- **BMS Frame Fixing**
  - arXiv:2208.04356, arXiv:2105.02300, arXiv:2110.15922
- **CCE surrogate**
  - In preparation
- **Nonlinearities in BH ringdown**
  - arXiv:2208.07380
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