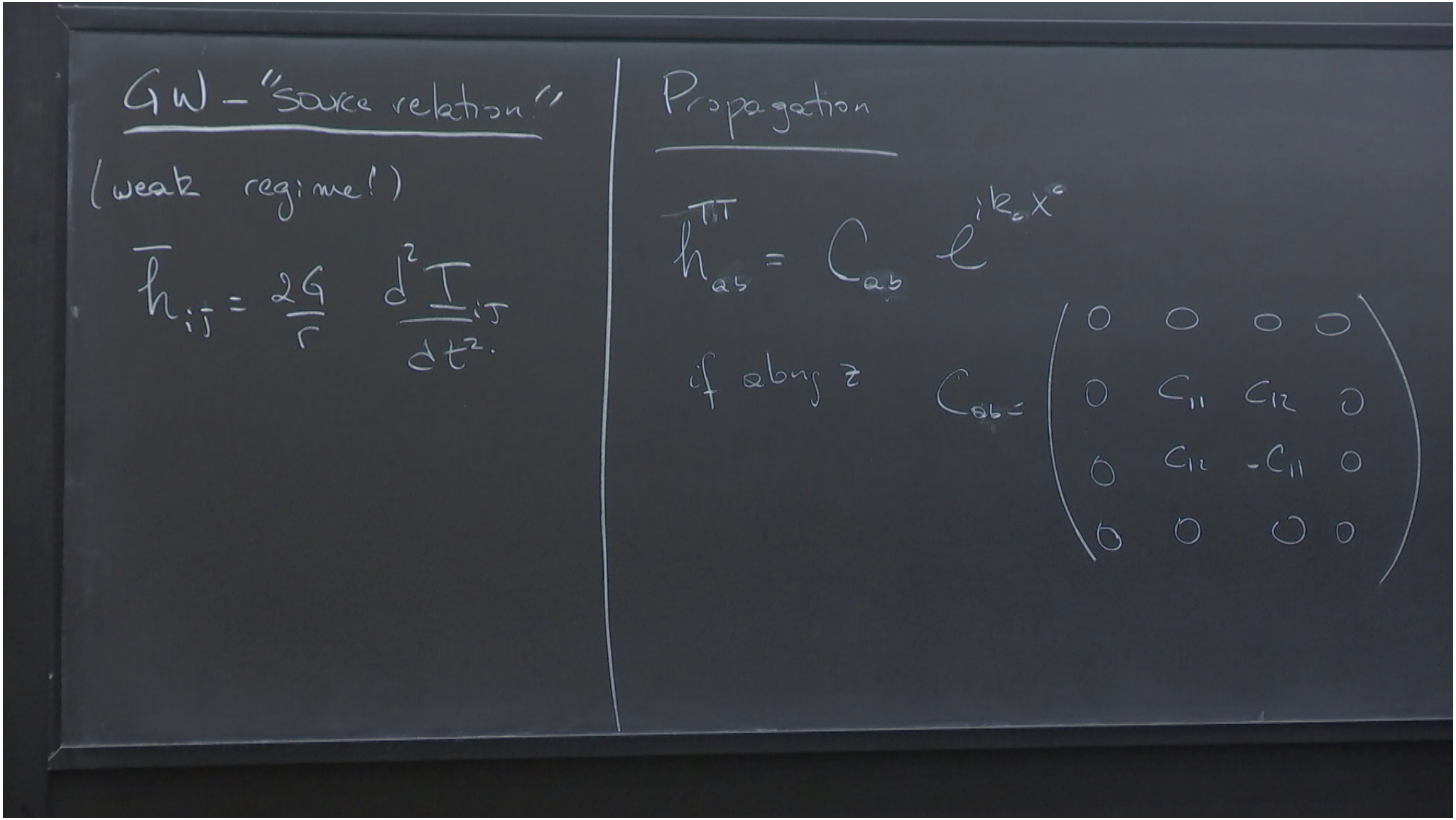
Title: Relativity - Lecture 221102

Speakers:

Collection: Relativity (2022/2023)

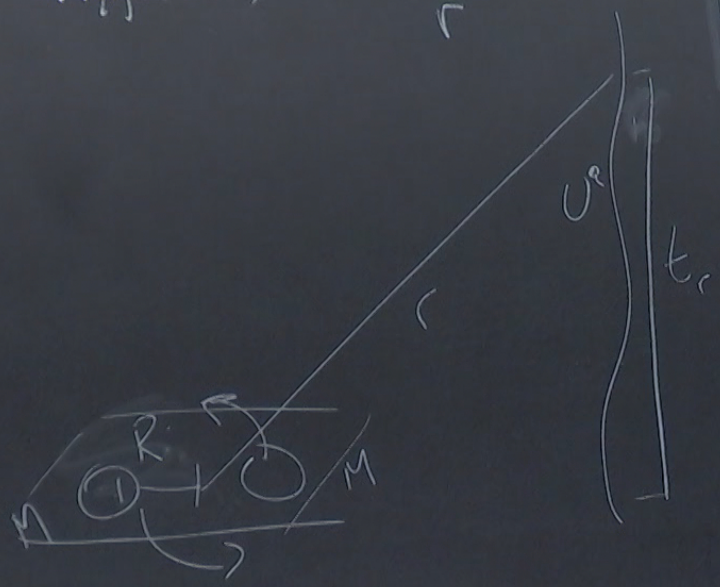
Date: November 02, 2022 - 9:00 AM

URL: https://pirsa.org/22110025


GW - "source relation"
(weak regime!)
$\bar{h}_{ij} = \frac{2G}{r} \frac{d^2 \underline{I}_{ij}}{dt^2}$
Propagation
$h^{TT}_{ab} = C_{ab} e^{ik_c x^c}$
if $a,b = 2$
$C_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

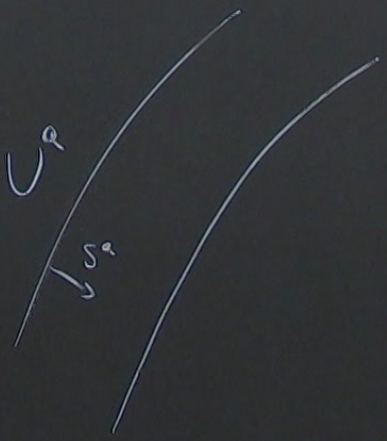
Binary ("adiabatic" regime)

$$\bar{h}_{ij}^{TT}(t,\vec{x}) = \frac{8GM}{r}\,\Omega^2 R^2 \begin{pmatrix} -\cos(2\Omega t_r) & -\sin(2\Omega t_r) & 0 \\ \sin(2\Omega t_r) & \cos(2\Omega t_r) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



→ $U^\mu = (1, \vec{0})$

→ $\tau \sim x^0 \sim t$



$U^\nu U^\rho S^\sigma$

$\rightarrow U^\rho = (1, \vec{0}) + \text{correct}$ $\sim h$

$\rightarrow \tau \sim x^0 \sim t$

S^σ

but $R_{\mu\rho\sigma\sigma} = \frac{1}{2}\left[\partial_\rho^2 h^{TT}_{\mu\sigma} + \partial_\sigma \partial_\mu h^{TT}_{\rho\rho} - \partial_\sigma \partial_\rho h^{TT}_{\mu\rho} - \partial_\mu \partial_\rho h^{TT}_{\sigma\rho}\right]$

U^ρ

S^ρ

$$\frac{D^2 S^\mu}{d\tau^2} = R^\mu{}_{\nu\rho\sigma} U^\nu U^\rho S^\sigma$$

$$\frac{\partial^2 S^\mu}{\partial t^2} = R^\mu{}_{0 0 0 \sigma} S^\sigma$$

but

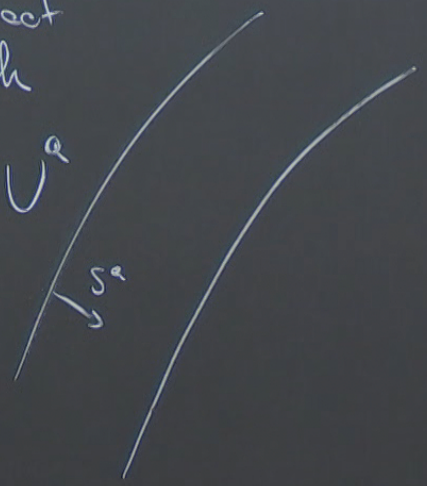
$\hookrightarrow$

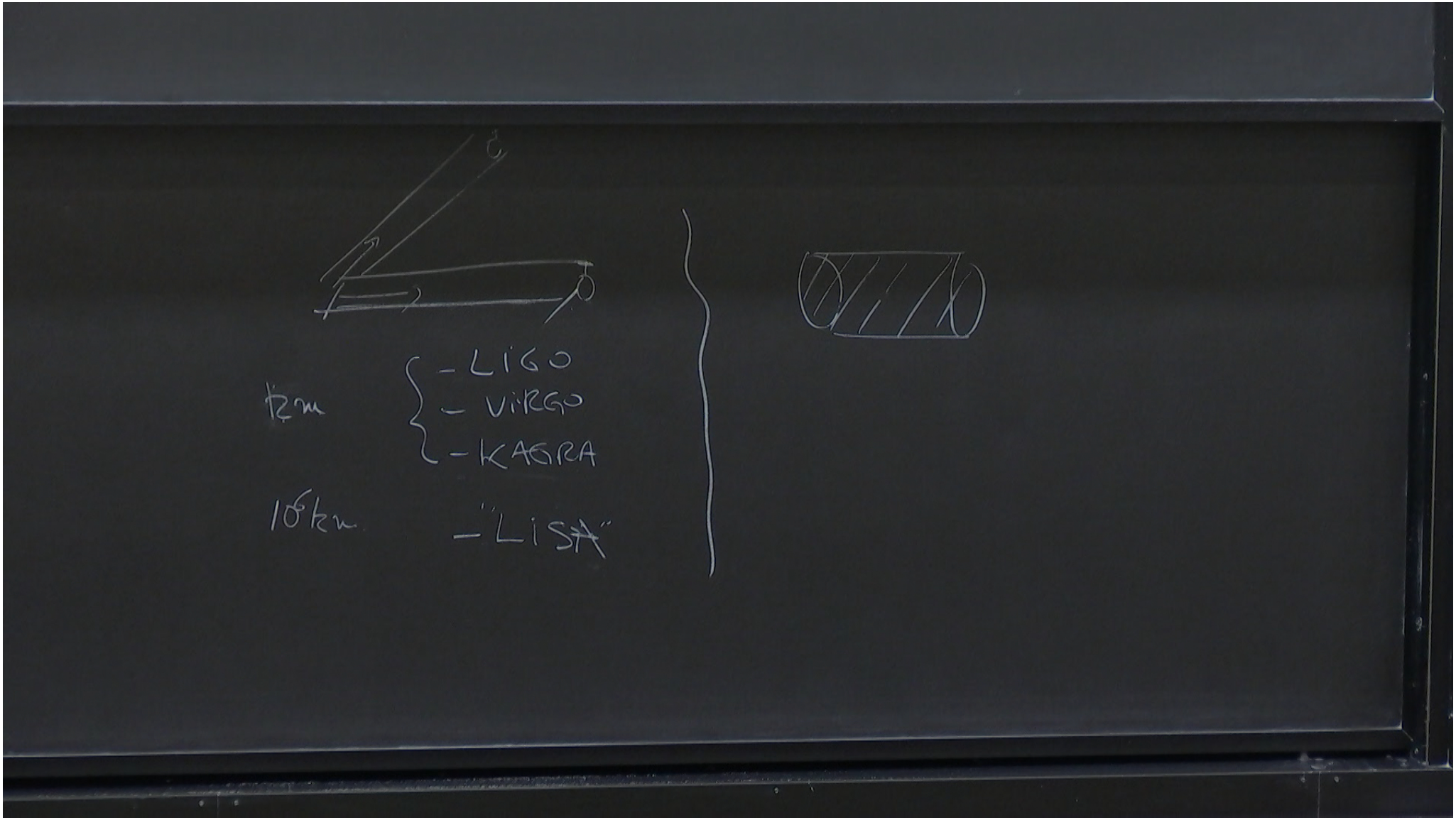
$$\boxed{\frac{\partial^2 S^\mu}{\partial t^2} = \frac{1}{2} S^\sigma \frac{\partial^2}{\partial t^2} h^{TT}{}^\mu{}_\sigma}$$

$\rightarrow U^\mu = (1, \vec{0}) + \text{correct}$ $\sim h$

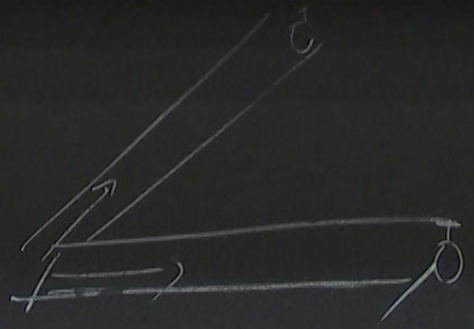
$\rightarrow \tau \sim x^0 \sim t$

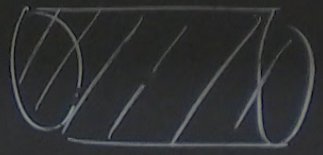
but $R_{\mu 0 0 \sigma} = \frac{1}{2}\left[\partial_0^2 h^{TT}_{\mu\sigma} + \partial_\sigma \partial_\mu h^{TT}_{00} - \partial_0 \partial_0 h^{TT}_{\mu\sigma} - \partial_\mu \partial_\sigma h^{TT}_{00}\right]$




- LIGO
- VIRGO
- KAGRA
- "LISA"

$h \sim \frac{\Delta L}{L}$





4km { - LIGO
- VIRGO
- KAGRA

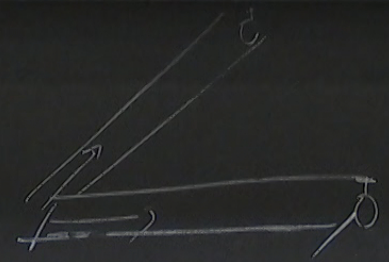
10km - "LISA"

{ 2 projects from China

$\mu\nu\sigma = \frac{1}{2}\left[\partial_\sigma h_{\mu\nu} + \partial_\nu \partial_\mu h'_{\sigma 0} - \partial_\sigma \partial_0 h^{TT}_{\mu 0} - \partial_\mu \partial_0 h^{TT}_{\nu\sigma}\right]$

$\eta \to \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ $(t, \vec{x})$

$h \sim \frac{\Delta L}{L}$

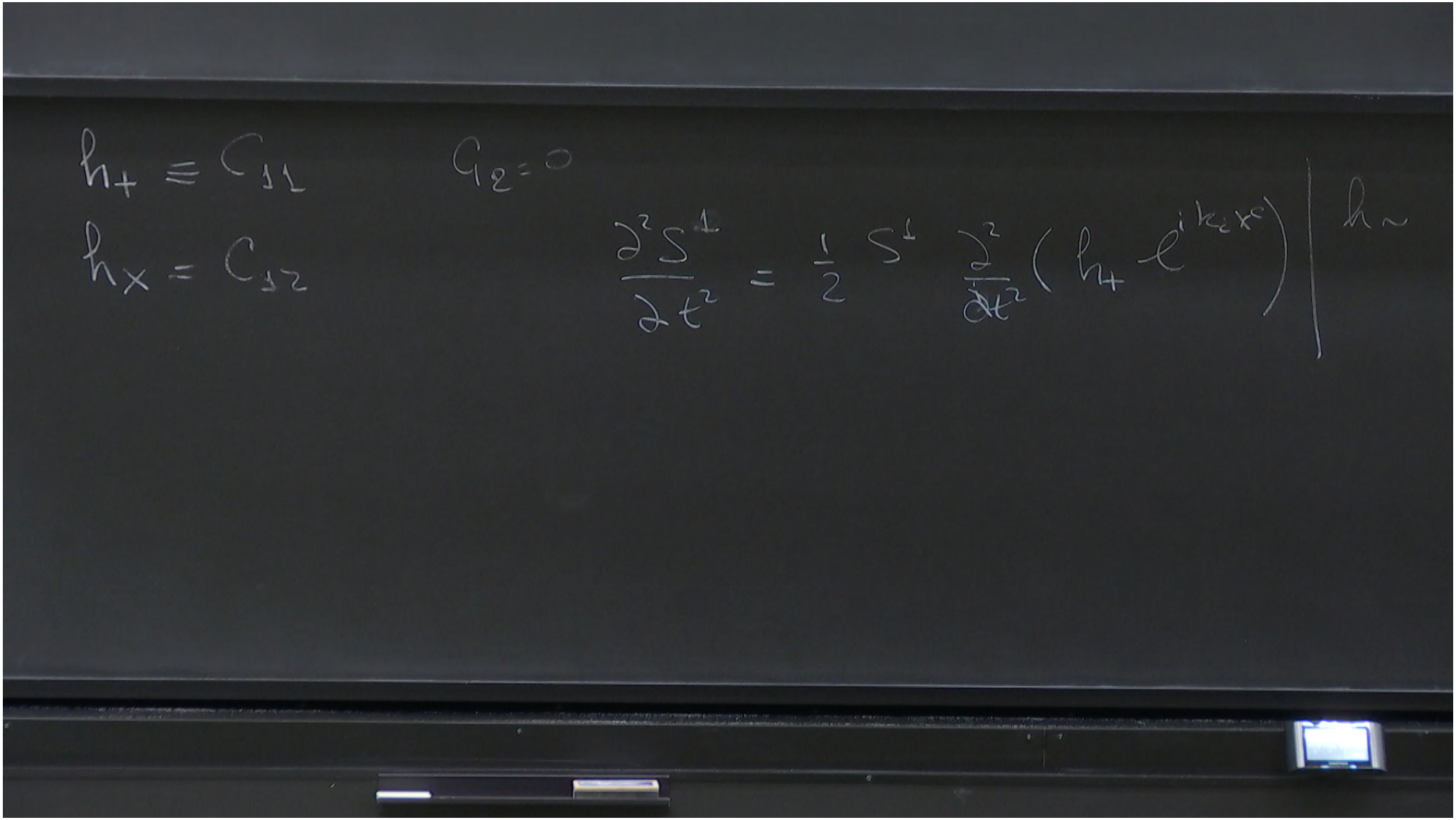


- LIGO
- VIRGO
- KAGRA

~ 2 km

$\sim 10^6$ km — "LISA"




$h_+ \equiv C_{11}$ $C_{12} = 0$
$h_\times = C_{12}$
$\frac{\partial^2 S^1}{\partial t^2} = \frac{1}{2} S^1 \frac{\partial^2}{\partial t^2}\left(h_+ e^{i k_a x^a}\right)\Big| h_\sim$

$$\frac{d^2 S^\mu}{dt^2} = R^\mu{}_{00\sigma}\, S^\sigma \qquad \text{but } R_{\mu 00\sigma} = \frac{1}{2}\frac{\partial^2}{\partial t^2} h^{TT}{}_{\mu\sigma}$$

$$\hookrightarrow \quad \boxed{\frac{d^2 S^\mu}{dt^2} = \frac{1}{2} S^\sigma \frac{\partial^2}{\partial t^2} h^{TT}{}_{\mu\sigma}}$$

$h_+ \equiv C_{11} \qquad C_{12} = 0$

$h_\times = C_{12}$

$$\frac{d^2 S^1}{dt^2} = \frac{1}{2} S^1 \frac{\partial^2}{\partial t^2}\left(h_+ e^{i k_z x^0}\right) \Big| \quad h_\sim$$

GW – "source relation"

(weak regime!)

$$\bar{h}_{ij} = \frac{2G}{r} \frac{d^2 \underline{\underline{I}}_{ij}}{dt^2}$$

Propagation

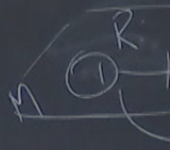
$$h^{TT}_{ab} = C_{ab}\, e^{ik_c x^c}$$

if along z $\quad C_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$k = (\omega, 0, 0, \omega)$ $\quad x^c = (t, \vec{x})$

Binary

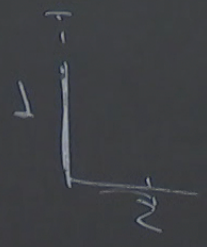
$\bar{h}_{ij}(t$



$h_+ \equiv C_{11}$ $\quad (C_{12}=0)$

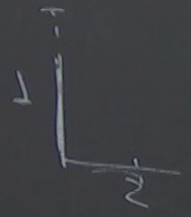
$h_\times = C_{12}$

$$\frac{\partial^2 S^{1/2}}{\partial t^2} = \frac{1}{2} S^{1/2} \frac{\partial^2}{\partial t^2}\left(h_+ e^{i k_z x^0}\right)$$



$$S^{1,2}_{(t)} = \left(1 \pm h_+ e^{i k_z x^0}\right) S^{1/2}$$

$\frac{\partial t}{\partial t}$



$$S^{1,2}_{(t)} = \left(1 \pm \frac{h_x}{2} e^{i k_c x^c}\right) S^{1,2}(0)$$

2km

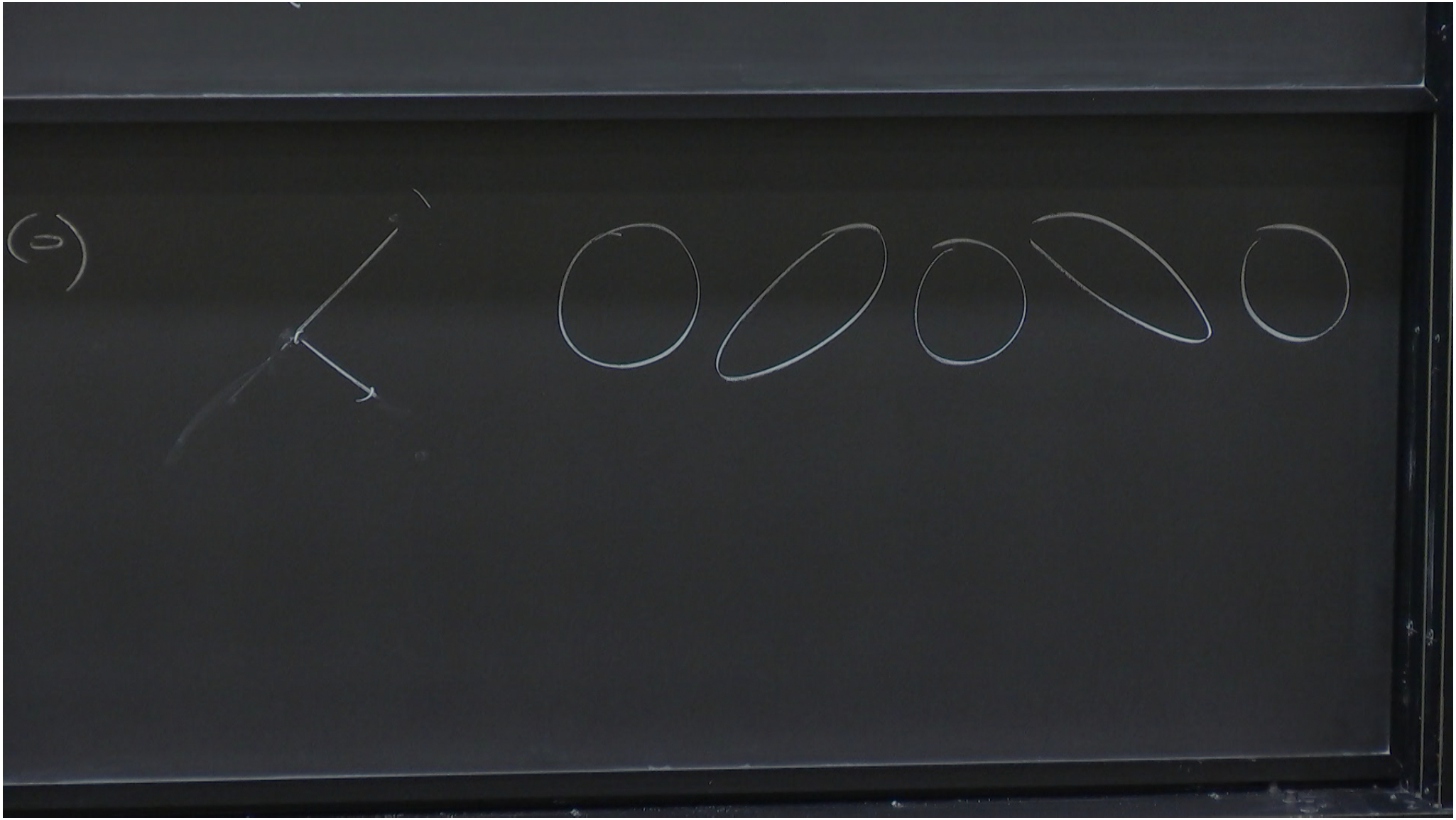
10km



$C_{11} = 0 \quad (h_x = 0)$

$C_{12} \neq 0 \quad (h_x \neq 0)$

$$S^{1,2}_{(t)} = S^{1,2}(0) + \frac{1}{2} h_x e^{i k_c x^c} S^{2,2}(0)$$



2 projects from China

L

h_x

h_+

$T_{0\mu} \to E = T_{00}$

$\partial_\alpha T^{\alpha 0} = 0 \qquad p^i = T_{0i}$

$\partial_t T^{00} + \partial_i T^{i0} = 0$

$\boxed{\partial_t E + \partial_i p^i = 0}$

$g = \eta + h + P$

$k = (\omega, 0, 0, \omega)$

$x^\mu = (t, \vec{x})$

$G_{ab}(\eta + h + h^{(2)})$

$G_{ab}^{(0)}(\eta) = 0$

$\hookrightarrow G_{ab}^{(1)}(h) = 0$

$\underline{P}$, $\underline{h^2}$

$G_{ab}(\eta + \varepsilon h + \varepsilon^2 P)$

$G^{(0)}_{ab}(\eta) = 0$

$\hookrightarrow G^{(1)}_{ab}(h) = 0$

$G^{(2)}_{ab}(P) = t_{ab}(h^2)$

P, h^2

$\Gamma\,\Gamma$

$(\varepsilon h)\,(\varepsilon h) = \varepsilon^2 h^2$

$g = \eta + h + P$

$k = (\omega, 0, 0, \omega)$ $\quad x^\mu = (t, \vec{x})$

$G_{ab}(\eta + \varepsilon h + \varepsilon^2 P)$

$G^{(0)}_{ab}(\eta) = 0$

$\hookrightarrow G^{(1)}_{ab}(h) = 0$

$G^{(2)}_{ab}(P) = t_{ab}(h^2)$

P, h^2

$\Gamma\,\Gamma$

$(\varepsilon h)\,(\varepsilon h) = \varepsilon^2 h^2$

$G_{ab}(g) =$

$\Delta E = -\iint T_{i0}\, dS^i$

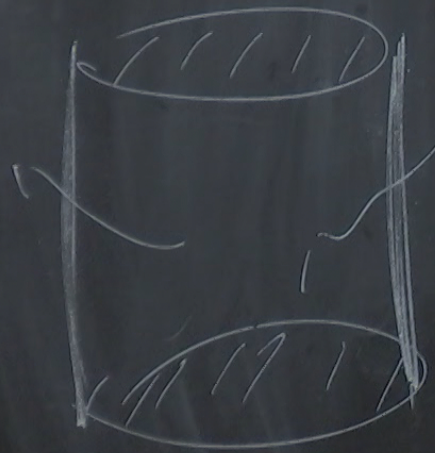
$\int dt \int d^3x\, \partial_i T^{i0}$

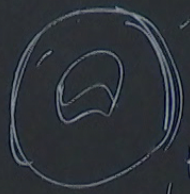
$\int dt \int dS_i\, T^i$

$T_{es} \to$

P

$\partial_\alpha T^{\alpha\nu} = 0$



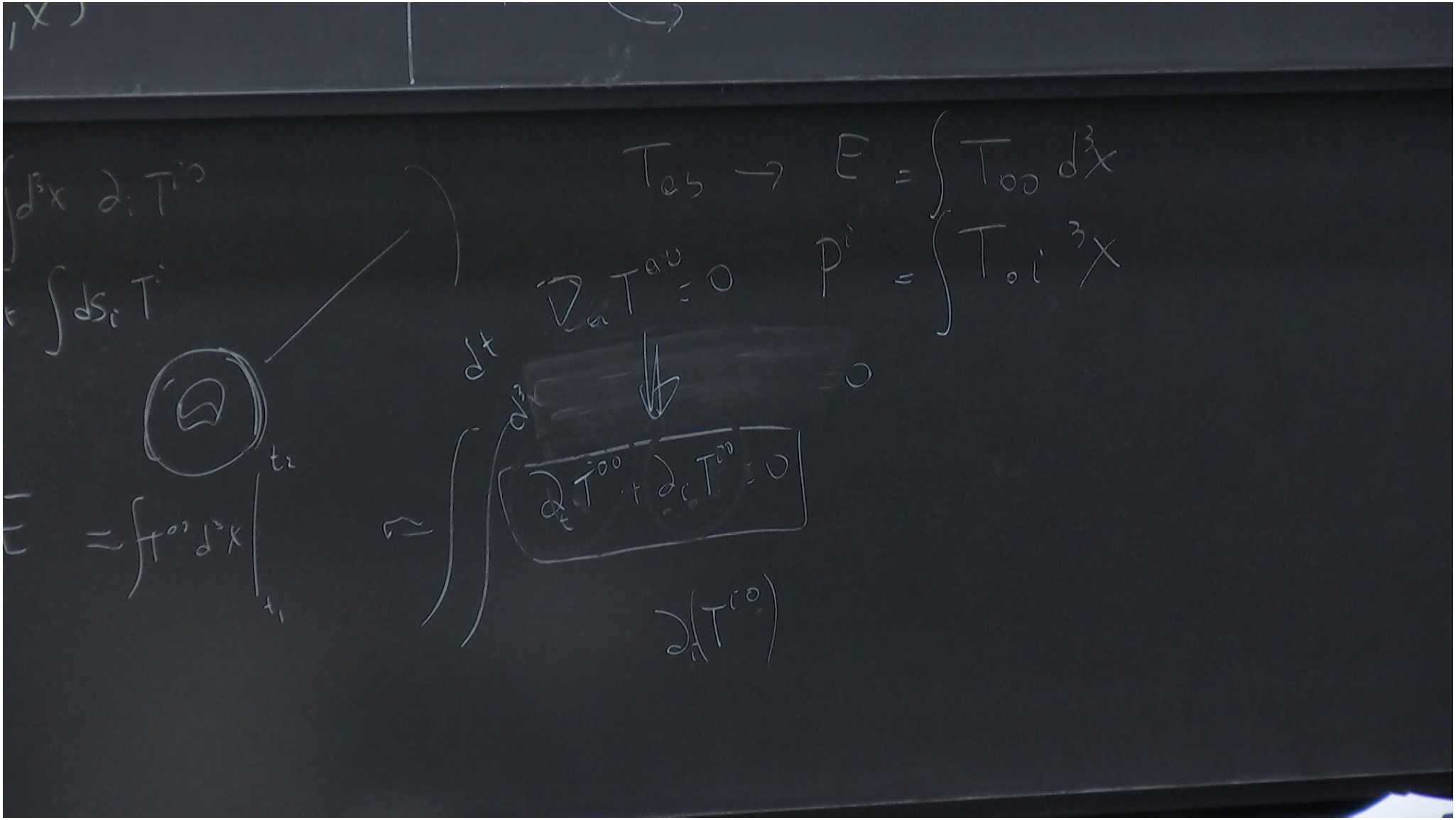


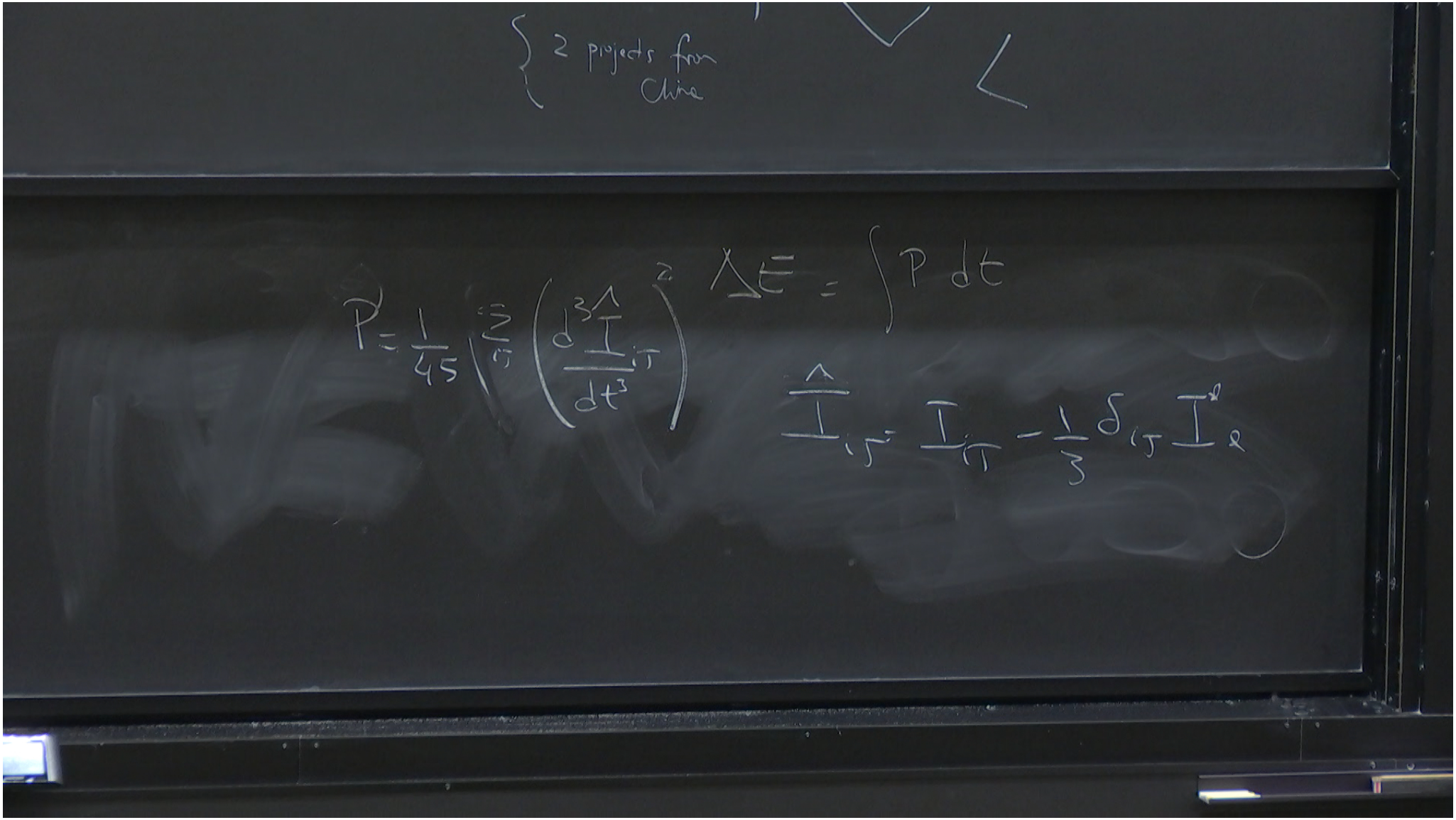
$\Delta E = \int T^{00} d^3x \Big|_{t_1}^{t_2}$

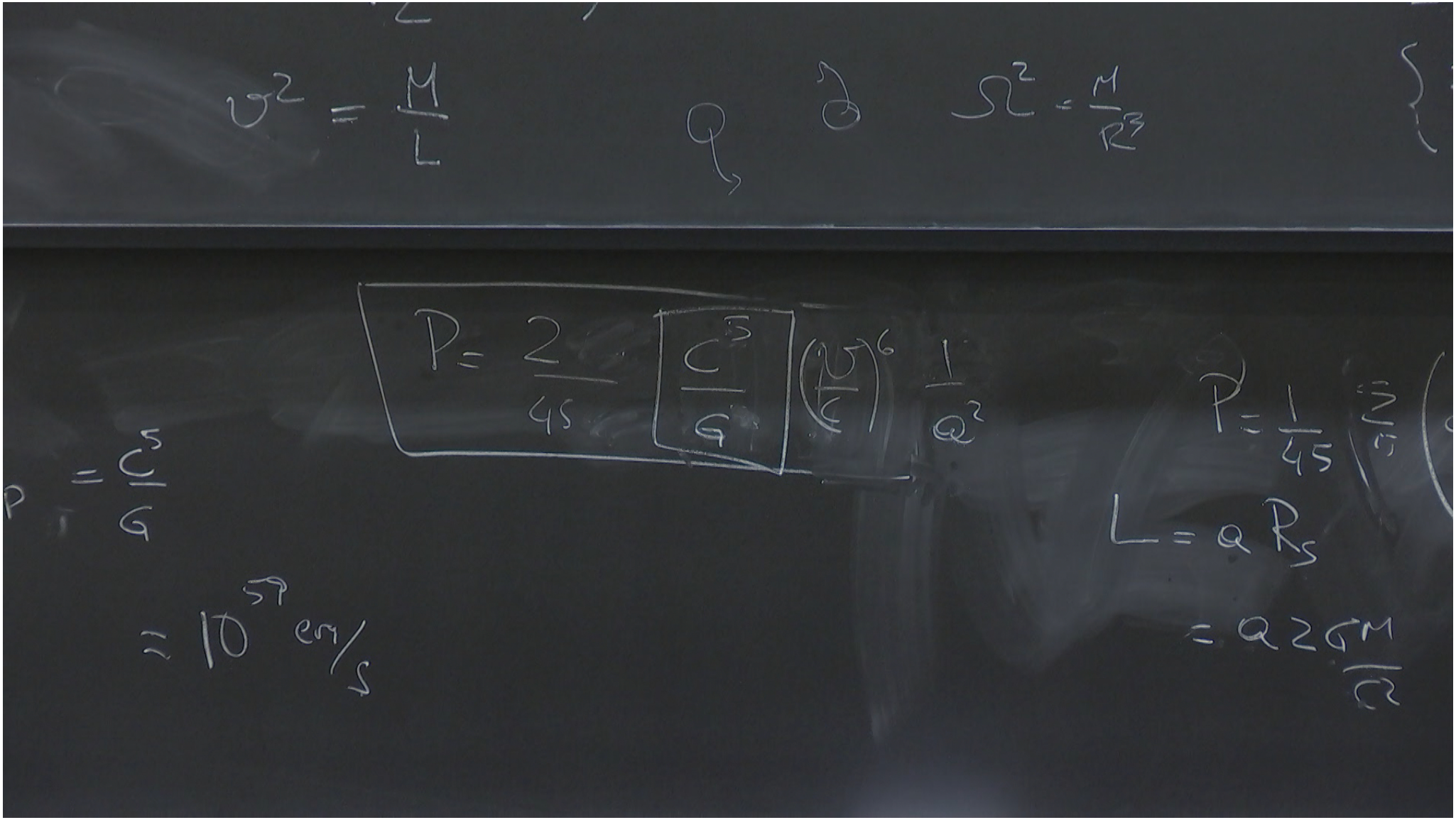
$\simeq \int dt \int d^3x$

$\partial_t T^{00} + \partial_i T^{i0} = 0$

$\partial_i (T^{i0})$


$\int d^3x\, \partial_i T^{i0}$
$+ \int dS_i\, T^i$
$T_{\alpha 5} \to E = \int T_{00}\, d^3x$
$D_\alpha T^{\alpha\mu} = 0$
$P^i = \int T_{0i}\, d^3x$
dt
$= 0$
$E = \int T^{00} d^3x \Big|_{t_1}^{t_2}$
$\partial_t T^{00} + \partial_i T^{i0} = 0$
$\partial_i (T^{i0})$


2 projects from China
$P = \frac{1}{45} \sum_{ij} \left(\frac{d^3 \hat{I}_{ij}}{dt^3} \right)^2$
$\Delta E = \int P \, dt$
$\hat{I}_{ij} = I_{ij} - \frac{1}{3} \delta_{ij} I_{kk}$


$v^2 = \frac{M}{L}$
$\Omega^2 = \frac{M}{R^3}$
$P = \frac{2}{45} \frac{c^5}{G} \left(\frac{v}{c}\right)^6 \frac{1}{Q^2}$
$P = \frac{c^5}{G}$
$\approx 10^{59}$ erg/s
$P = \frac{1}{45}$
$L = 9 R_S$
$= 9 \frac{2GM}{c}$

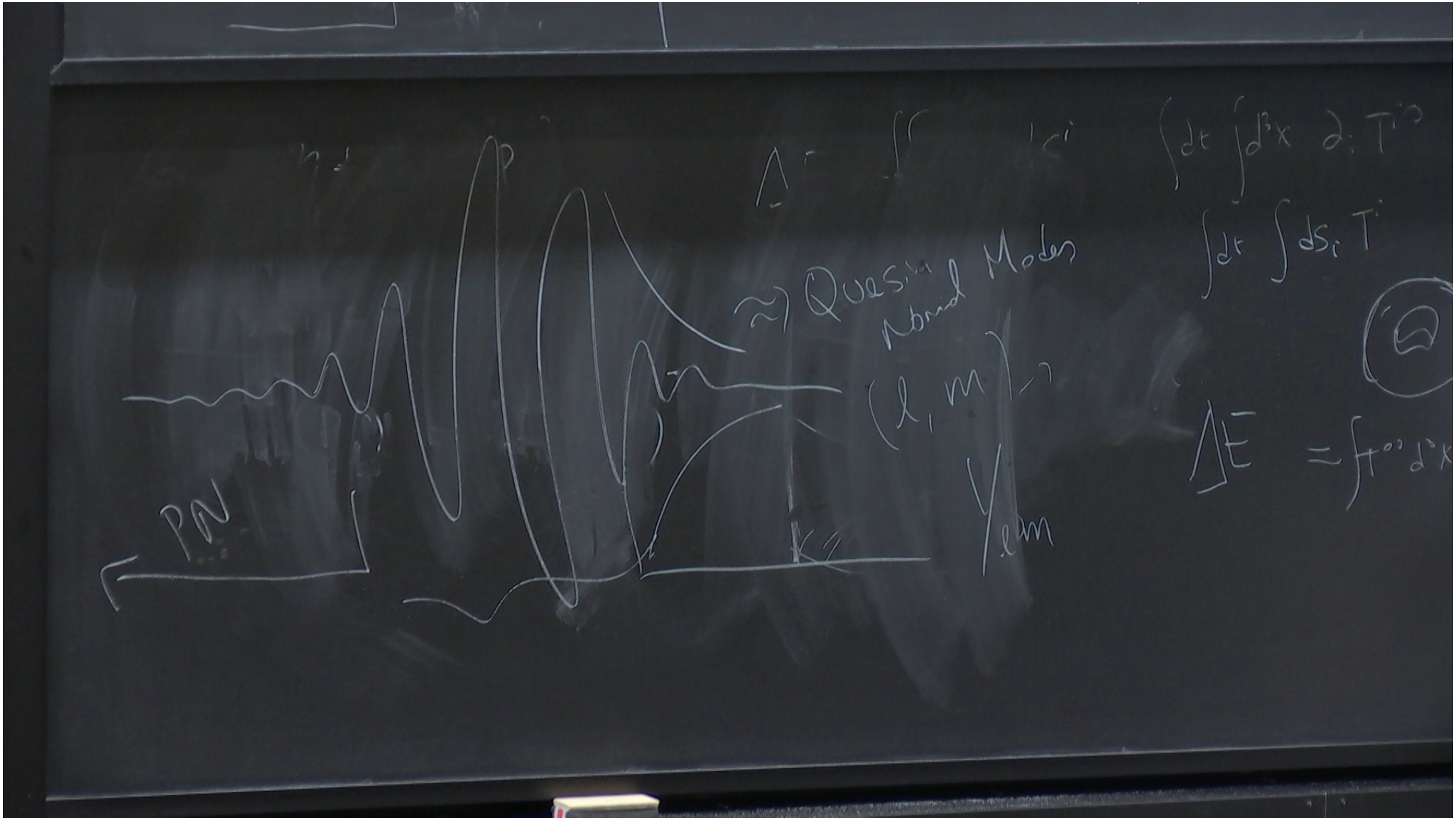
$v^2 = \frac{M}{L}$ q ∂ Ω^2 =

$t_{planck} = \left(\frac{\hbar G}{c^5}\right)^{1/2}$

$t_{planck} = \left(\frac{\hbar c^5}{G}\right)^{1/2}$ ~ $\frac{t_p}{t_p} \equiv L_p = \frac{c^5}{G}$

$= 10^{59}$ cm/s

$$P = \frac{2}{45}\left[\frac{c^3}{G}\right]\left(\frac{v}{c}\right)^6 \frac{1}{Q^2}$$


PN
≈ Quasi-Normal Modes
(l, m)
Y_lm
ΔE = ∫ dt ∫ d³x ∂_i T^0
∫ dt ∫ dS_i T^i