Title: Relativity - Lecture 221101

Speakers:

Collection: Relativity (2022/2023)

Date: November 01, 2022 - 9:00 AM

URL: https://pirsa.org/22110024

$-\frac{1}{2} h \eta_{ab}$. Gauge choice $\partial_a \bar{h}^{ab} = 0$ (Lorentz gauge)

$\partial_a A^a = 0$

$G_{ab} = -\frac{1}{2} \Box \bar{h}_{ab}$

$\Box G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$

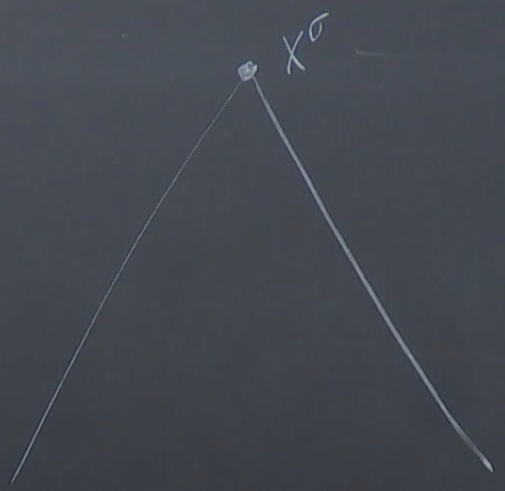
$\Leftrightarrow$

$\Box \bar{h}_{ab} = -16\pi G T_{ab}$

$\partial_a \bar{h}^{a\mu} = 0$ (Lorentz gauge)

$\partial_a A^a = 0$

$\Box G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$



$$\bar{h}_{ab} = h_{ab} - \frac{1}{2} h \eta_{ab}$$

Gauge choice $\partial_a \bar{h}^{ab}$

$$G_{ab} = -\frac{1}{2} \Box \bar{h}_{ab}$$

$$\Box \bar{h}_{ab} = -16\pi G T_{ab}$$

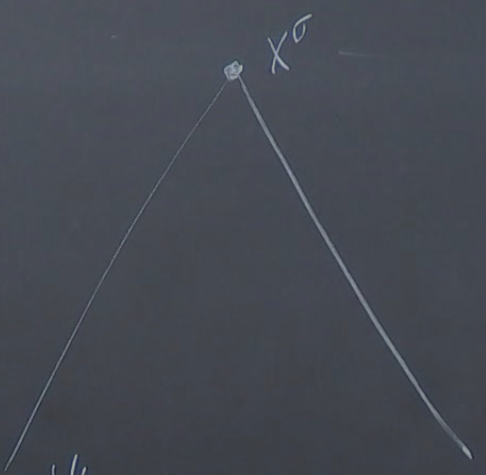
$\Box$

$$\partial_a \bar{h}^{ab} = 0 \quad \text{(Lorentz gauge)}$$

$$\partial_a A^a = 0$$

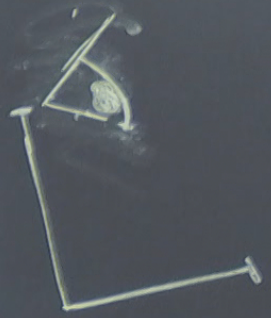
$$\Box G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$$

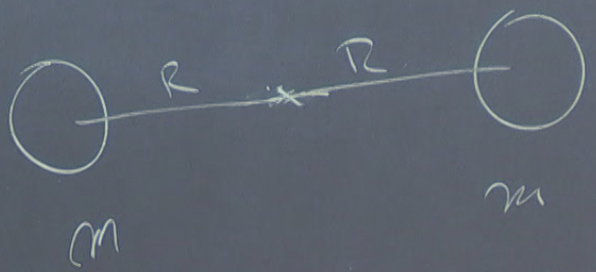
T_{ab} → solution → $\bar{h}_{ab}(x^\sigma) = -16\pi \int G(x^\sigma - y^\sigma) \cdot T_{ab}(y^\sigma)\, d^4y.$



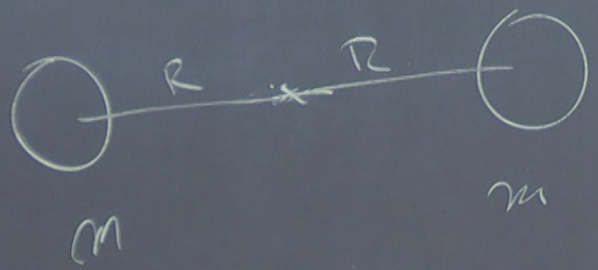
$$G_{\mu\nu} = \frac{1}{2}\left[\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\lambda\right.$$

$G_{\mu\nu} = 8\pi T_{\mu\nu} \longrightarrow h?$



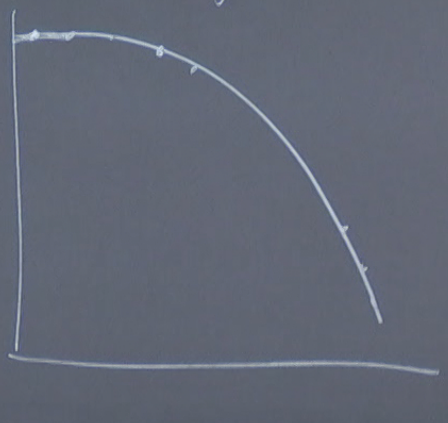


$$-\partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\,\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\,\Box h\,]$$



- Einstein (Robinson)
- Hulse-Taylor Binary pulsar

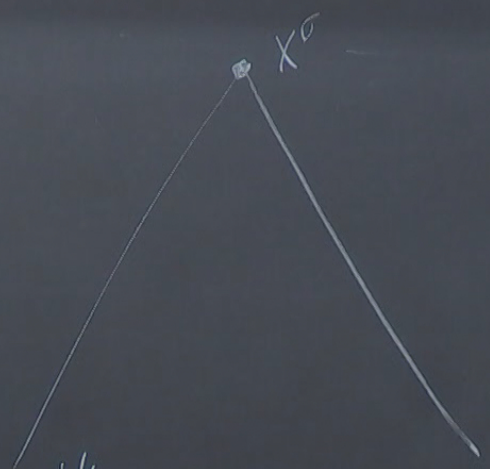
"$\ddot{R}$"



$\bar{h}^{ab} = 0$ (Lorentz gauge)

$\partial_a A^a = 0$

$\Box G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$



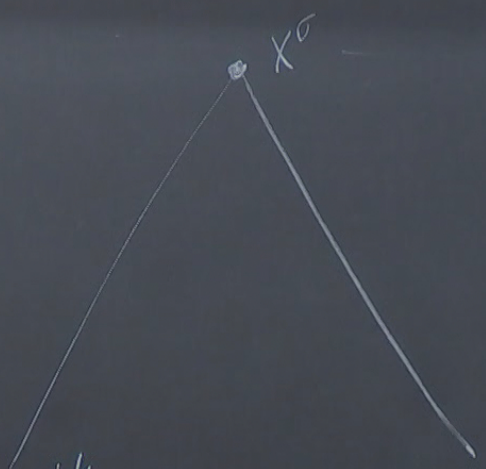
$\rightarrow \bar{h}_{ab}(x^\sigma) = -16\pi \int G(x^\sigma - y^\sigma) \cdot T_{ab}(y^\sigma)\, d^4y.$

$G(x^\sigma - y^\sigma) = -\frac{1}{4\pi |x-y|} \delta(|x-y| - (x^0 - y^0))$

$\bar{h}^{ab} = 0$ (Lorentz gauge)

$\partial_a A^a = 0$

$\Box G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$



$\rightarrow \bar{h}_{ab}(x^\sigma) = -16\pi \int G(x^\sigma - y^\sigma) \cdot T_{ab}(y^\sigma)\, d^4y.$

$G(x^\sigma - y^\sigma) = -\frac{1}{4\pi |\vec{x} - \vec{y}|} \delta(|\vec{x} - \vec{y}| - (x^0 - y^0))$ ⊖

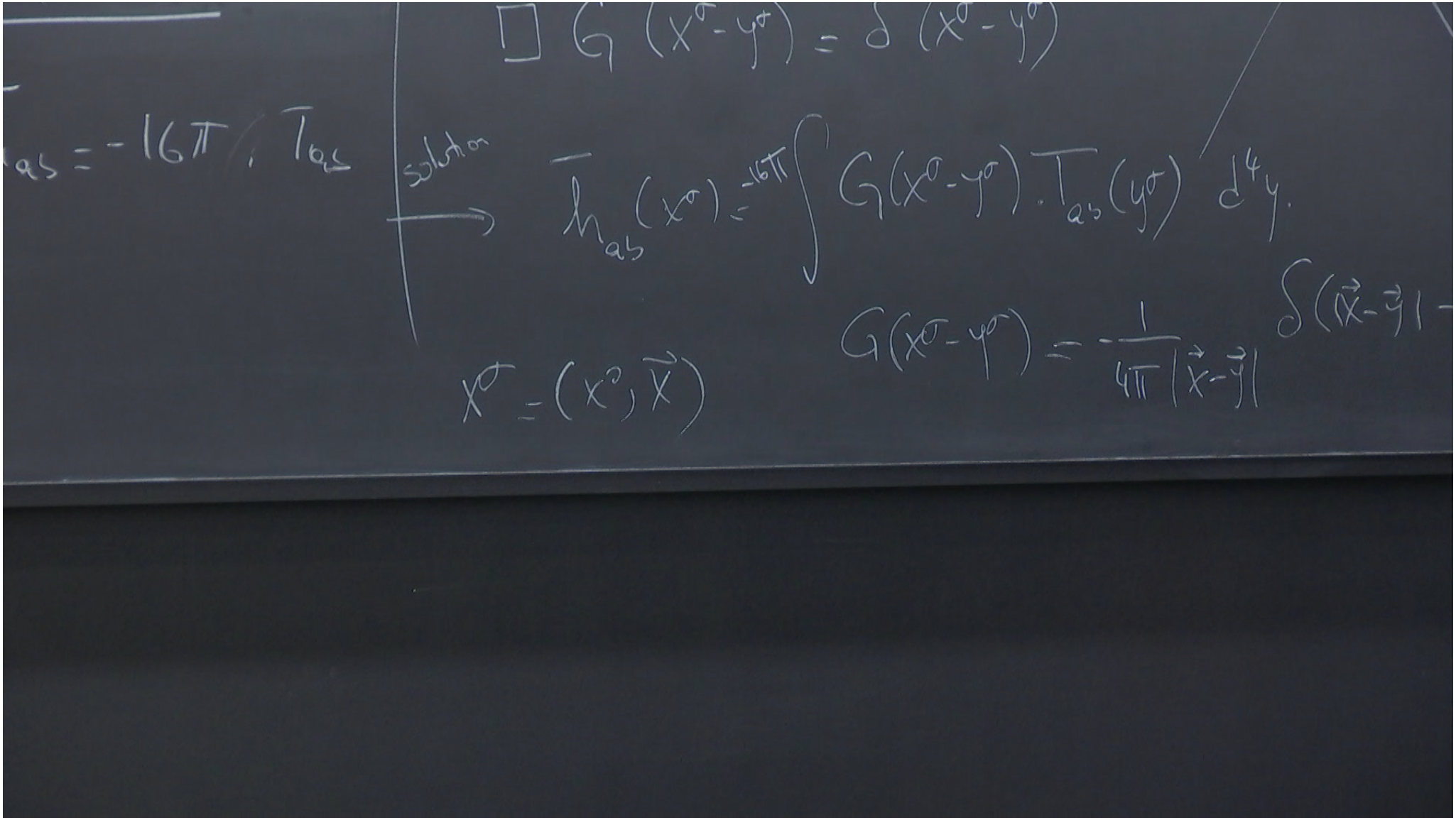
$$\partial_\alpha A^\alpha = 0$$

$$(x^\alpha - y^\alpha) = \delta^{(4)}(x^\alpha - y^\alpha)$$

$$h^{ab}(x^\alpha) = -16\pi \int G(x^\alpha - y^\alpha) \cdot T^{ab}(y^\alpha)\, d^4y.$$

$$G(x^\alpha - y^\alpha) = -\frac{1}{4\pi |\vec{x} - \vec{y}|} \delta(|\vec{x} - \vec{y}| - (x^0 - y^0)) \cdot \Theta(x^0 - y^0)$$




$\Box G(x^\sigma - y^\sigma) = \delta(x^\sigma - y^\sigma)$
$\bar{h}_{ab} = -16\pi\, T_{ab}$
solution
$\bar{h}_{ab}(x^\sigma) = -16\pi \int G(x^\sigma - y^\sigma)\, T_{ab}(y^\sigma)\, d^4y$
$x^\sigma = (x^0, \vec{x})$
$G(x^\sigma - y^\sigma) = -\frac{1}{4\pi|\vec{x}-\vec{y}|}\,\delta(|\vec{x}-\vec{y}|$

$= 0$ (Lorentz gauge)

$\partial_a A^a = 0$

$G(x^\sigma - y^\sigma) = \delta^{(4)}(x^\sigma - y^\sigma)$

$A_{ab}(x^\sigma) = -16\pi \int G(x^\sigma - y^\sigma) \cdot T_{ab}(y^\sigma)\, d^4y.$

$G(x^\sigma - y^\sigma) = -\frac{1}{4\pi |\vec{x} - \vec{y}|}$

$\delta(|\vec{x} - \vec{y}| - (x^0 - y^0)) \cdot \Theta(x^0 - y^0)$

$\Theta = \begin{cases} 1 & x^0 > y^0 \\ 0 & \text{if not} \end{cases}$

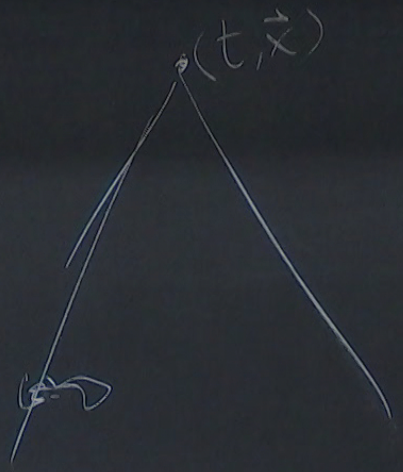
x^0

$x^0, \vec{x})$

$x^\mu = (x^0, \vec{x})$

$$\bar{h}_{ab}(t,\vec{x}) = 4 \int \frac{1}{|x-y|} T_{ab}\left(t - |\vec{x}-\vec{y}|, \vec{y}\right) d^3\vec{y}$$

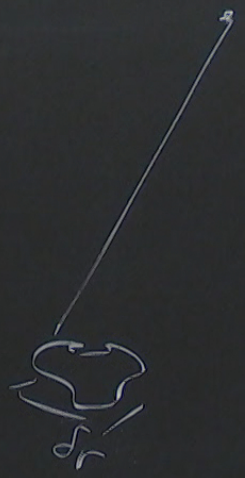
$$t_r = t - |\vec{x}-\vec{y}|$$



$$dt \int d^3y \; e^{i\omega t} \; \frac{T_{ab}\left(t - |\vec{x}-\vec{y}|/y\right)}{|\vec{x}-\vec{y}|} = \frac{4}{\sqrt{2\pi}} \iint dt_r \, d^3y \; e^{i\omega t_r} \; e^{i\omega|\vec{x}-\vec{y}|} \; T_{ab}$$

$$\therefore \; T_{\text{emit across}} \approx \delta r \qquad \delta r \ll \frac{1}{\omega}$$

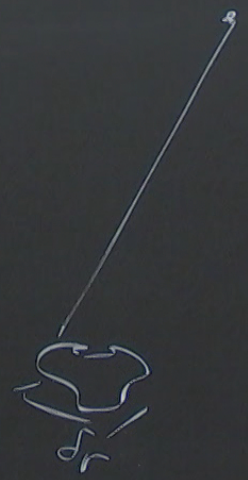
$$T_{\text{charact source}} \approx \frac{1}{\omega}$$



$$e^{i\omega t} \qquad \frac{T_{ab}(t-|\vec{x}-\vec{y}|/c,\vec{y})}{|\vec{x}-\vec{y}|} = \frac{4}{\sqrt{2\pi}}\int\int dt_r\, d^3y\; e^{i\omega t_r}\; e^{i\omega|\vec{x}-\vec{y}|}\; \frac{T_{ab}(t_r,\vec{y})}{|\vec{x}-\vec{y}|}$$

$T_{\text{evolt across}} \approx \delta r \qquad \delta r \ll \frac{1}{\omega_c}$

$T_{\text{charact source}} \approx \frac{1}{\omega_c}$



$$\tilde{h}_{ab}(\omega,\vec{x}) = \frac{1}{\sqrt{2\pi}}\int dt\, e^{i\omega t}\, \bar{h}_{ab}(t,\vec{x}) = \frac{4}{\sqrt{2\pi}}\int dt \int d^3y\, e^{i\omega t}\, \overline{T_{ab}}$$

$$\tilde{h}_{ab}(\omega,\vec{x}) = 4\int d^3y\, e^{i\omega|x-y|}\, \frac{\tilde{T}_{ab}(\omega,\vec{y})}{|x-y|}$$

$\therefore$ $\overline{T}_{\text{exhst across}} \approx$

$\overline{T}_{\text{charact source}} \approx$

$(\omega, \vec{q})$

$\bar{h} \sim e^{i\omega t} F(r)$

$\partial_a \bar{h}^{ab} = 0$

$\partial_t \bar{h}^{ob} + \partial_i \bar{h}^{ib} = 0$

$i\omega \bar{h}^{ob}$

$F(t,r) = T(t) \cdot \bar{F}(r)$

$\sim e^{i\omega t} F(r)$

$$\bar{h}_{ab}(t,\vec{x}) = 4\int \frac{1}{|x-y|}\,\bar{T}_{ab}\left(t-|\vec{x}-\vec{y}|,\vec{y}\right)d^3\vec{y}$$

$$t_r = t-|\vec{x}-\vec{y}|$$

$$\int f\,dy \to \int x(f_y)\,dy$$

$$-\int (\partial f)\,y$$



$$\frac{e^{i\omega r}}{r} \int d^3y\, \tilde{T}_{ij}(\omega, \vec{y})$$

$$\int d^3y \left[\partial_k (T^{kj} y^i) - y^j_J (\partial_k T^{ki}) \right]$$

$$\int d^3y \, T^{0i}(i\omega) y^j$$

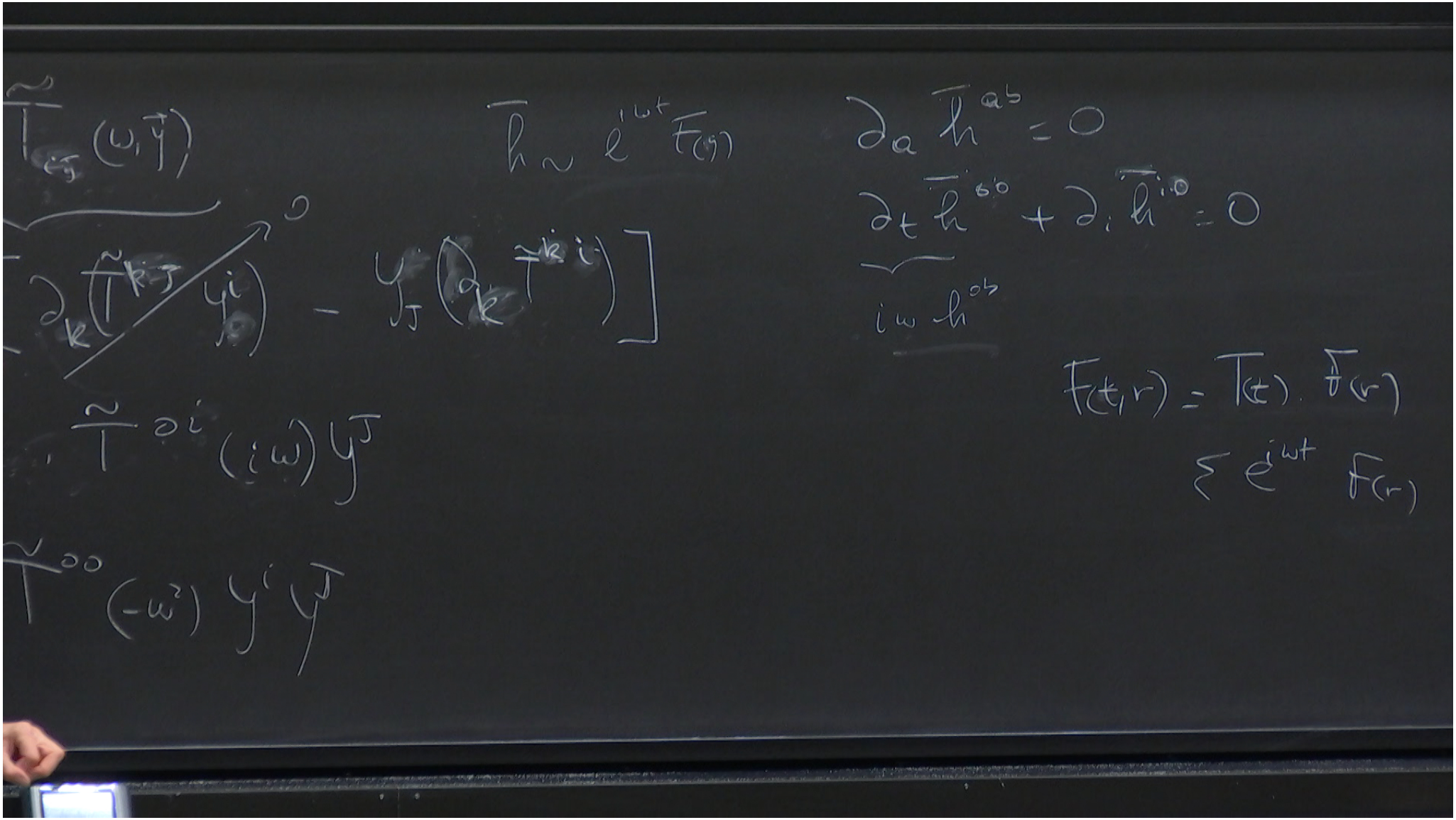
$$\bar{h} \sim e^{i\omega t} F(y)$$

$$\partial_a \bar{h}^{ab} = 0$$

$$\partial_t \bar{h}^{0b} + \partial_i$$

$$\underbrace{\qquad}_{i\omega \bar{h}^{0b}}$$

$$F(t$$


$\partial_a \bar{h}^{ab} = 0$
$\partial_t \bar{h}^{0b} + \partial_i \bar{h}^{ib} = 0$
$i\omega \bar{h}^{0b}$
$h \sim e^{i\omega t} F(y)$
$F(t,r) = T(t) \cdot \vec{F}(r)$
$\sim e^{i\omega t} F(r)$
$\tilde{T}^{0i}(i\omega) y^j$
$\tilde{T}^{00}(-\omega^2) y^i y^j$

$\bar{h} \sim e^{i\omega t} F(r)$

$\partial_a \bar{h}^{a0} = 0$ $\quad$ $\partial_a \bar{h}^{ai} = 0$

$\partial_t \bar{h}^{00} + \partial_i \bar{h}^{i0} = 0$

$\underbrace{\partial_t \bar{h}^{00}}_{i\omega \bar{h}^{00}} + \partial_i \bar{h}^{i0} = 0$

$F(t,r) = T(t) \cdot \bar{F}(r)$

$\sum e^{i\omega t} F(r)$

$\vec{i})^0 - y^i_J (\partial_k T^{kj})]]$

$(i\omega) y^J$

$y^i y^J$

$$\frac{e^{i\omega|x-y|}}{|x-y|} \sim \frac{e^{i\omega r}}{r}$$

$$\tilde{h}_{ij}(\omega,x) = 4\frac{e^{i\omega r}}{r}\int d^3y\ \tilde{T}$$

$$\int d^3y\ [$$

$$\int d^3y\ [$$

$$\Rightarrow \tilde{h}_{ij}(\omega,x) = -2\omega^2\frac{e^{i\omega r}}{r}\tilde{I}_{ij}(\omega)$$

$$+\int d^3y\ \tilde{T}$$

where $\tilde{T}_{ij}(\omega) = \int d^3y\ \tilde{T}^{00}\, y_i y_j$

$\int d^3y\, \tilde{T}_{ij}(\omega, \vec{y})$

$\bar{h} \sim e^{i\omega t} F(y)$

$\boxed{\partial_a \bar{h}^{a0} = 0}$ $\boxed{\partial_a \bar{h}}$

$\partial_t \bar{h}^{b0} + \partial_i \bar{h}^{ib} = 0$

$\underbrace{\quad}_{i\omega \bar{h}^{0b}}$

$\int d^3y \left[\partial_k (\tilde{T}^{kj} y^i) - y^j (\partial_k \tilde{T}^{ki}) \right]$

$F(t, r) = T(t)$

$\sum e^{i\omega t}$

$\int d^3y\, \tilde{T}^{ai}(i\omega) y^j$

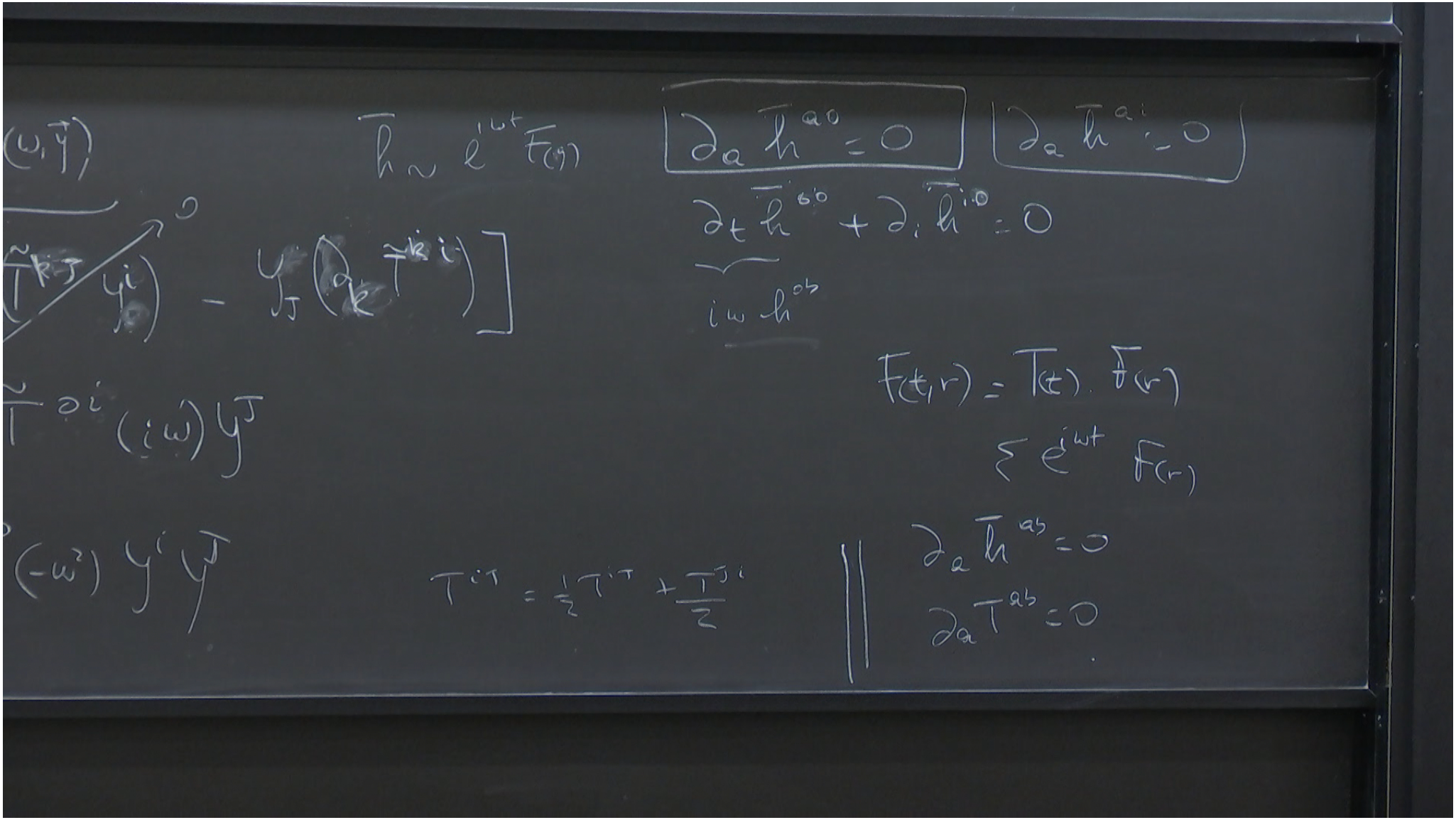
$\int d^3y\, \tilde{T}^{00}(-\omega^2) y^i y^j$

$T^{ij} = \frac{1}{2} T^{ij} + \frac{T^{ji}}{2}$

$$\frac{e}{|x-y|} \approx \frac{e}{r}$$

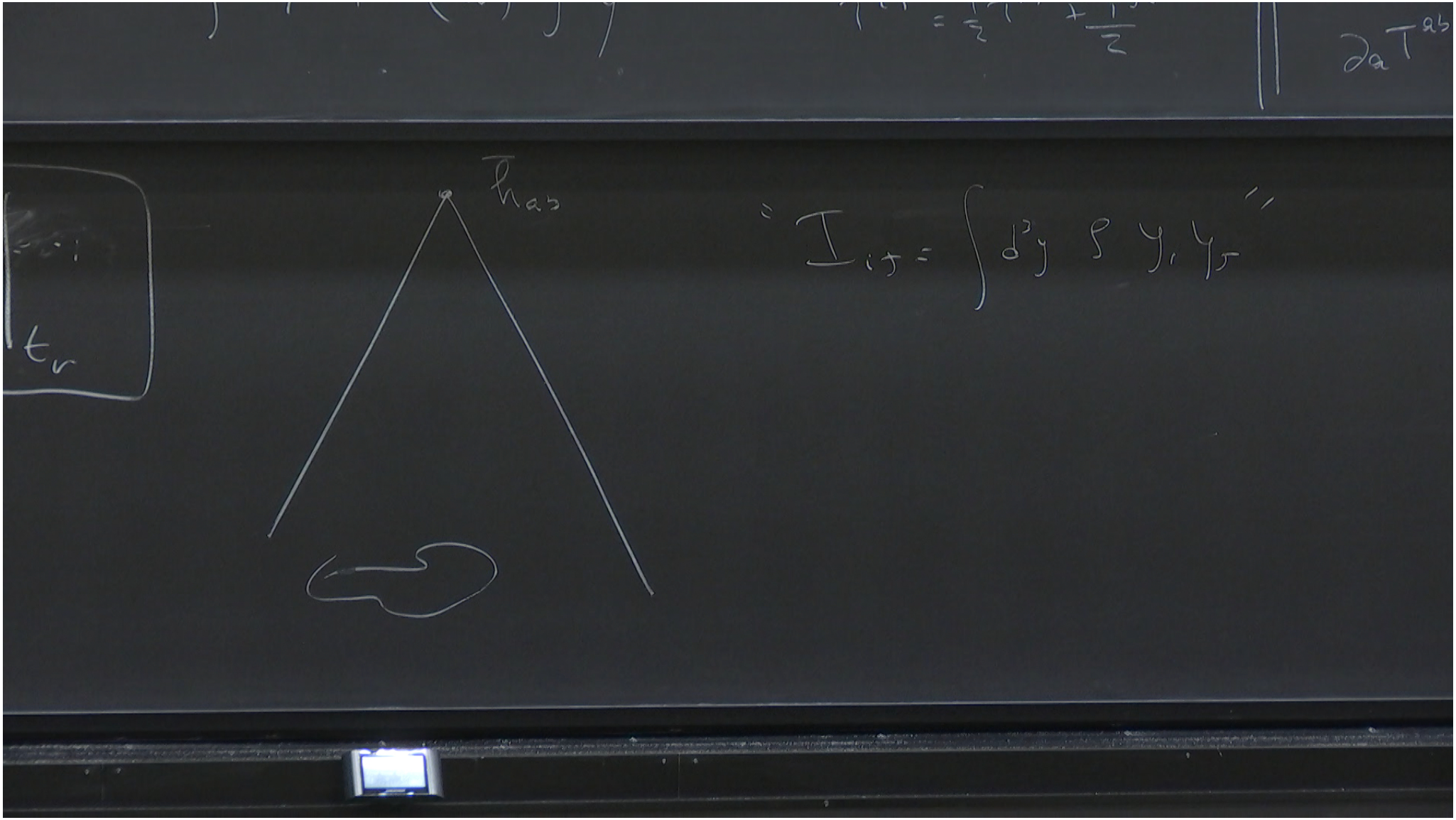
$$\Rightarrow \tilde{h}_{ij}(\omega, x) = -2\omega^2 \frac{e^{i\omega r}}{r} \tilde{I}_{ij}(\omega)$$

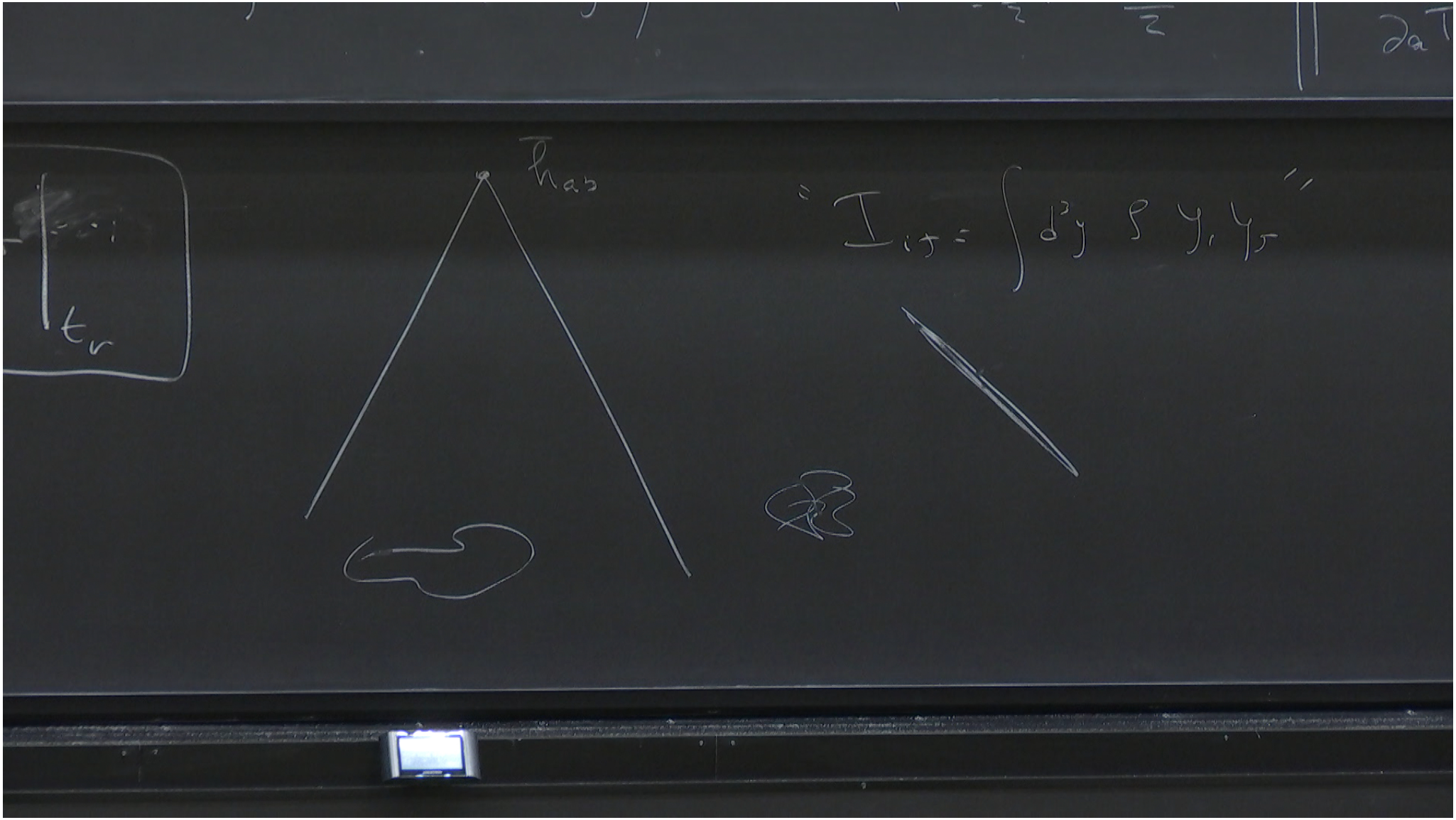
where $\tilde{I}_{ij}(\omega) = \int d^3y \, \tilde{T}^{00}(\omega, y) \, y_i y_j$

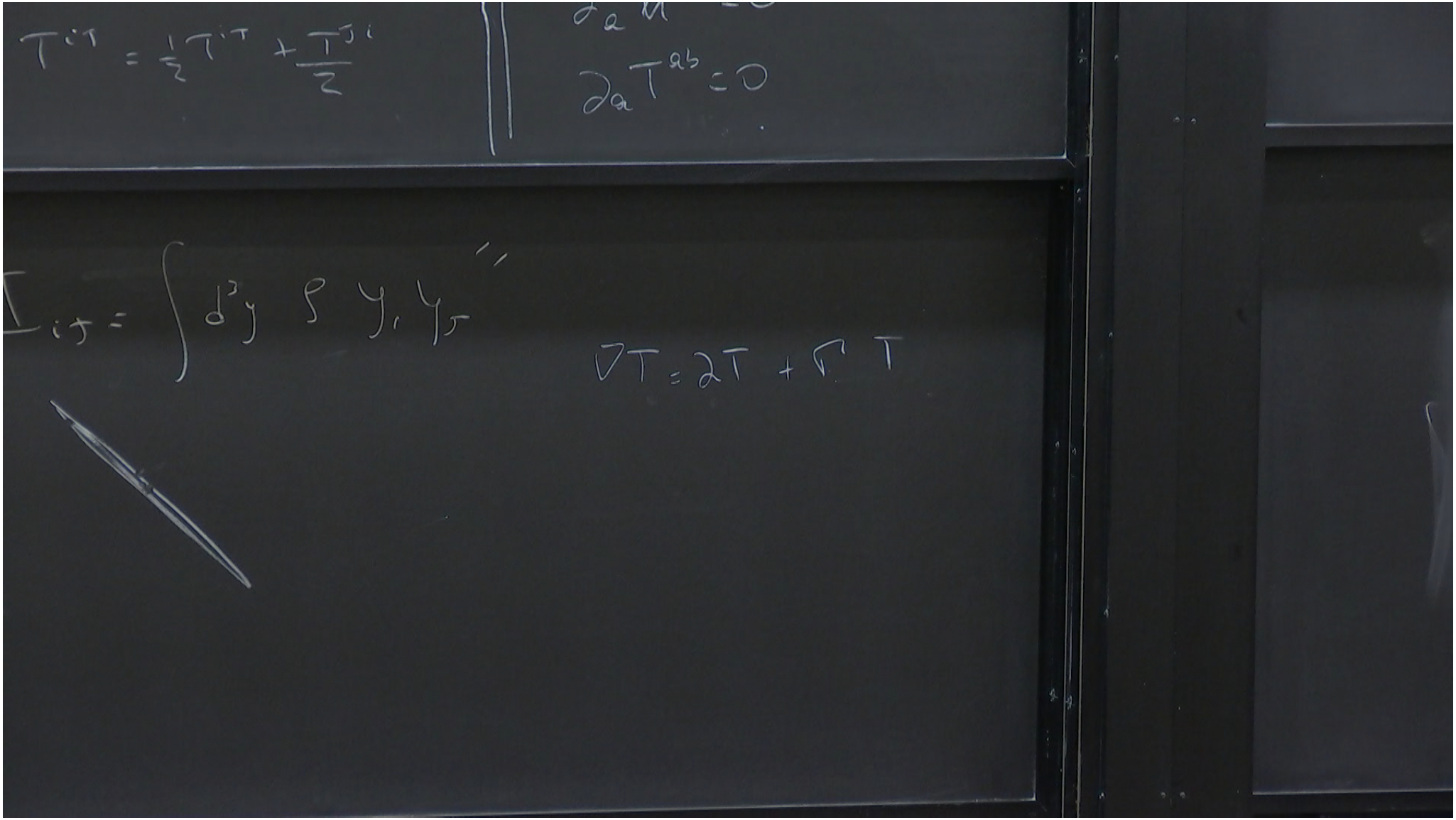

$(\omega, \vec{k})$
$\bar{h} \sim e^{i\omega t} F(r)$
$\partial_a \bar{h}^{a0} = 0$
$\partial_a \bar{h}^{ai} = 0$
$\partial_t \bar{h}^{00} + \partial_i \bar{h}^{i0} = 0$
$i\omega h^{0b}$
$F(t,r) = T(t) \cdot \bar{F}(r)$
$\sum e^{i\omega t} F(r)$
$\partial_a \bar{h}^{ab} = 0$
$\partial_a T^{ab} = 0$
$T^{ij} = \frac{1}{2} T^{ij} + \frac{T^{ji}}{2}$

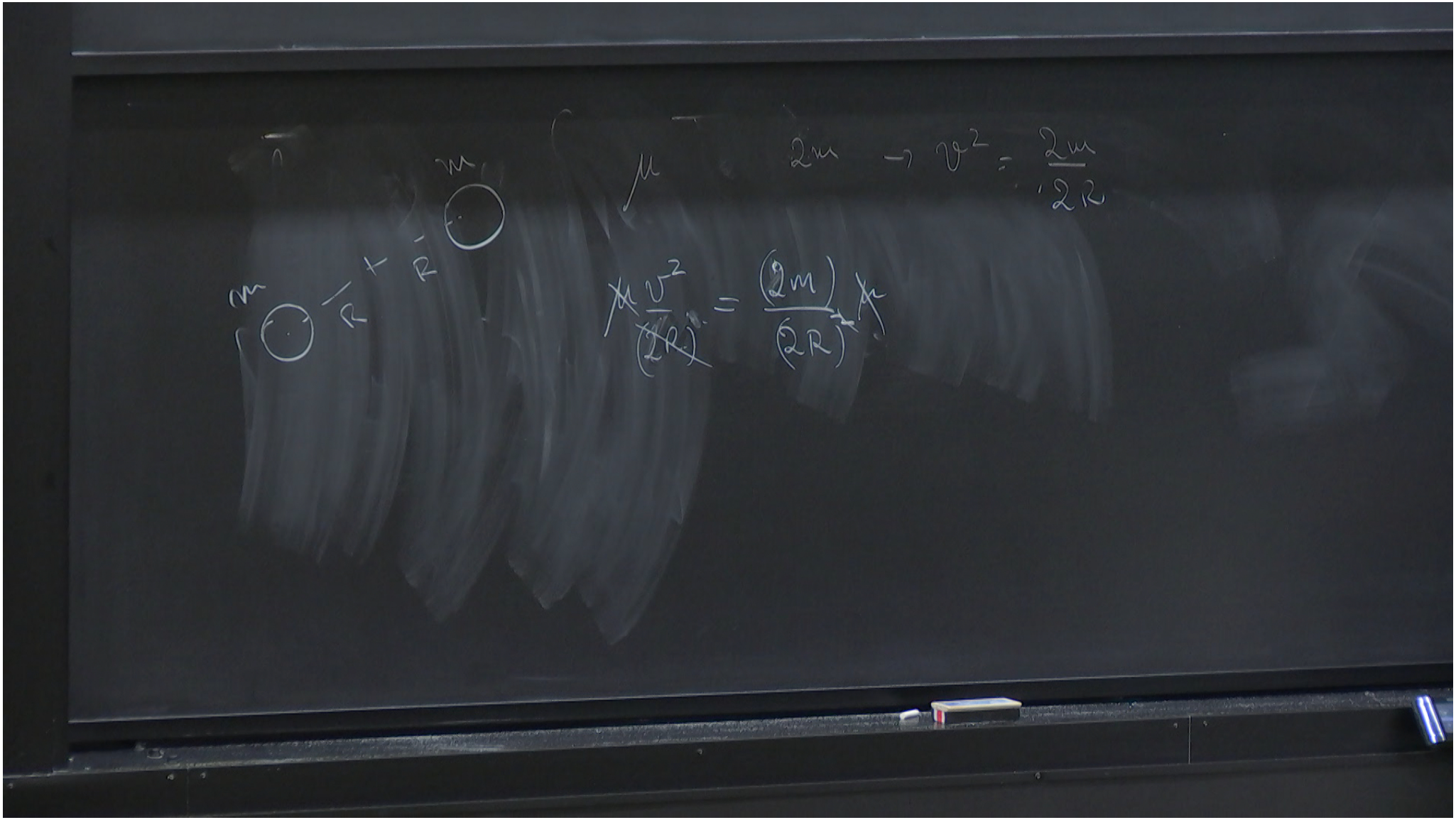
$$\overline{h}_{ij}(t,\vec{x}) = \frac{2}{r} G_N \frac{d^2 I_{ij}}{dt^2}\Big|_{t_r}$$

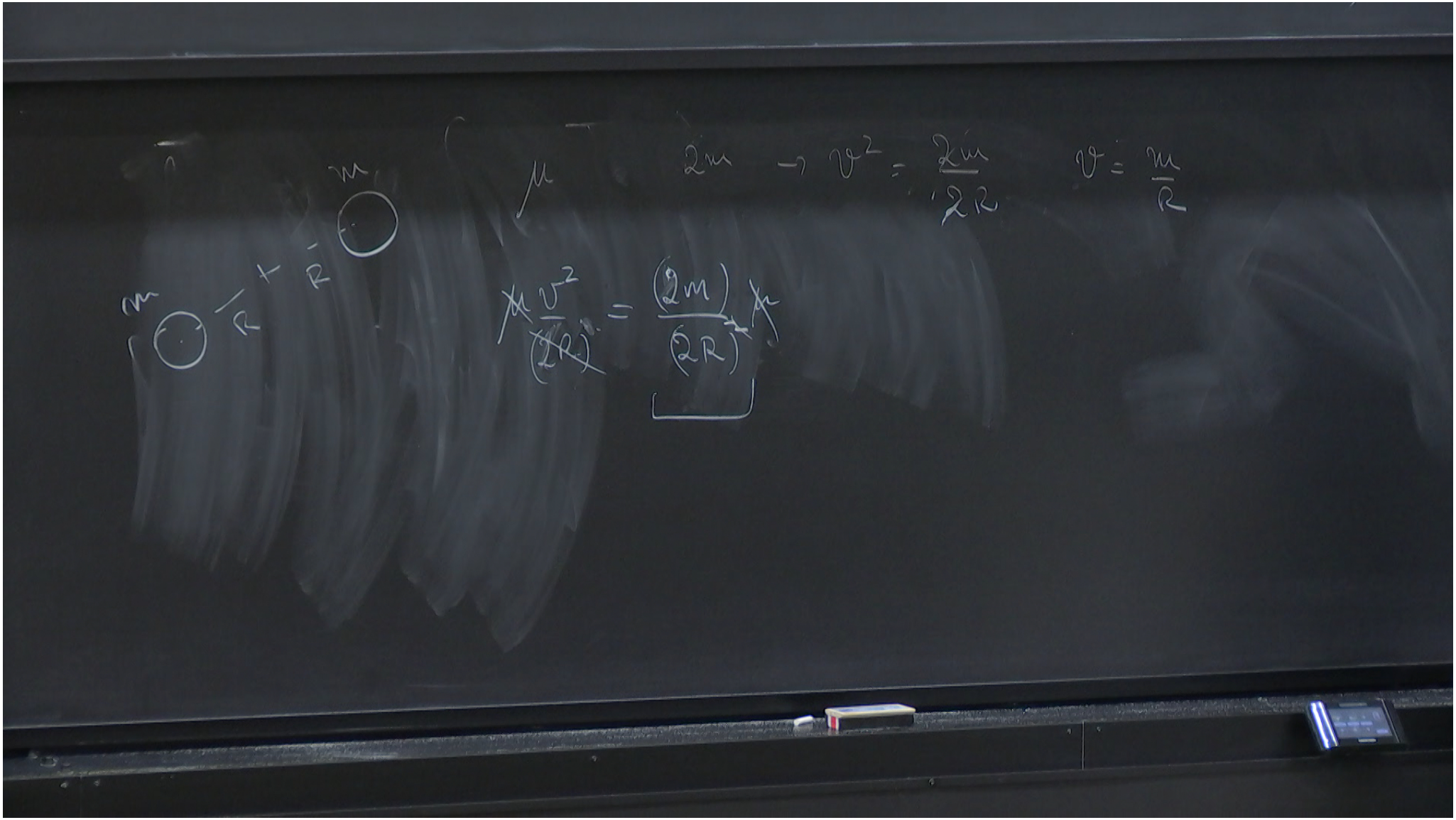


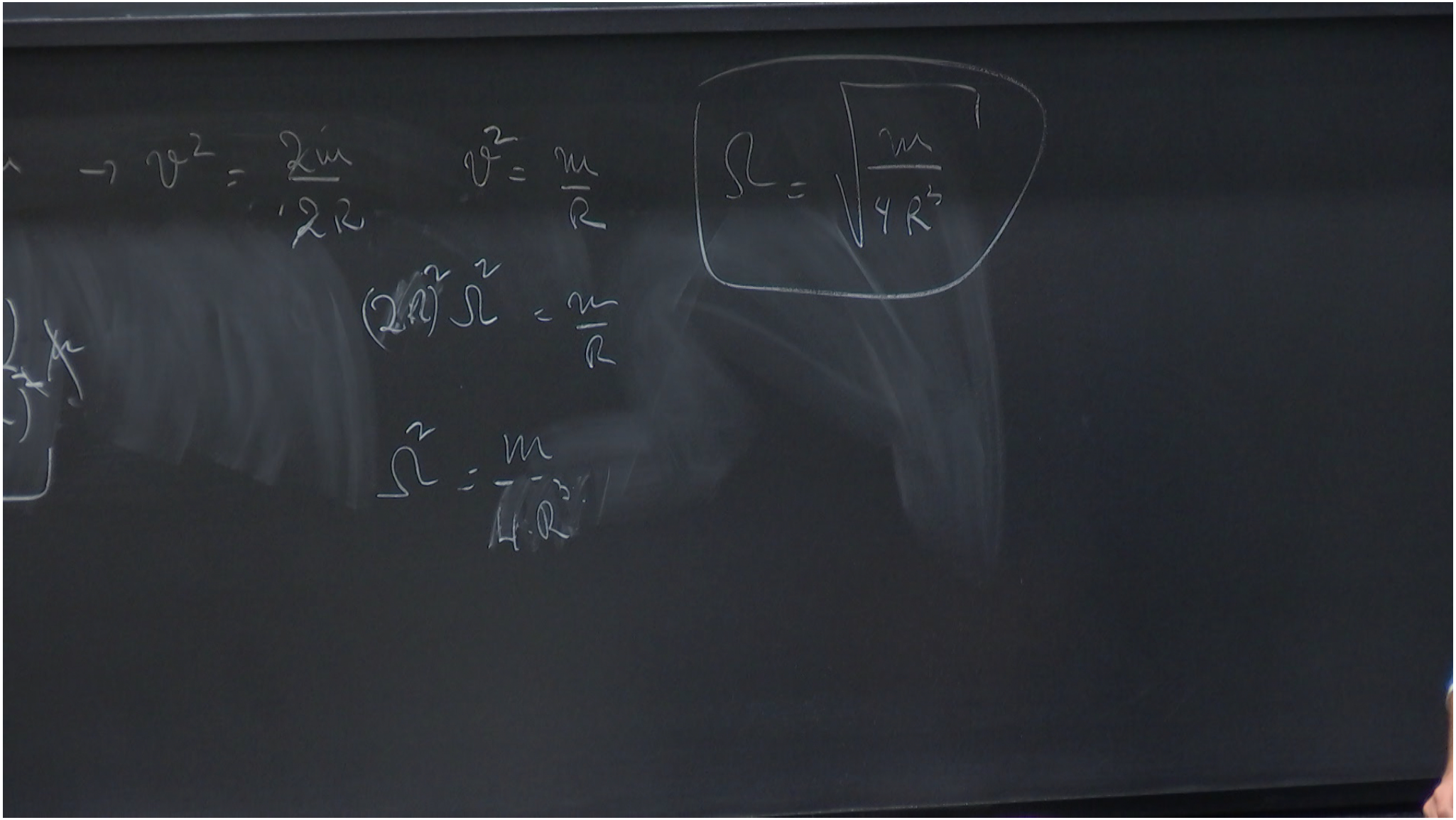

h_{ab}
t_r
$I_{ij} = \int d^3y \, \rho \, y_i y_j$
$\partial_a T^{ab}$

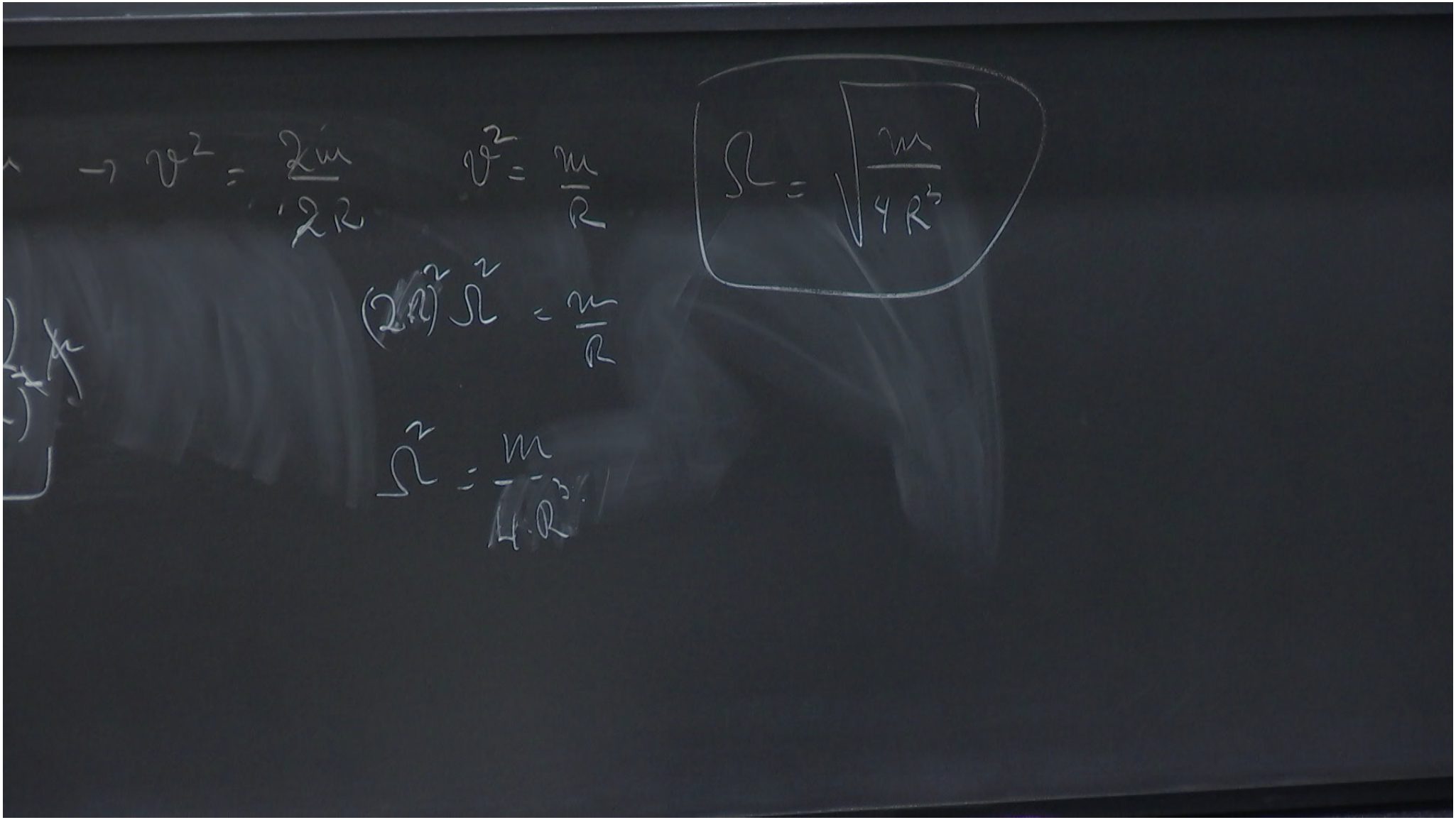

h_{ab}
t_r
$I_{ij} = \int d^3y \, \rho \, y_i y_j$

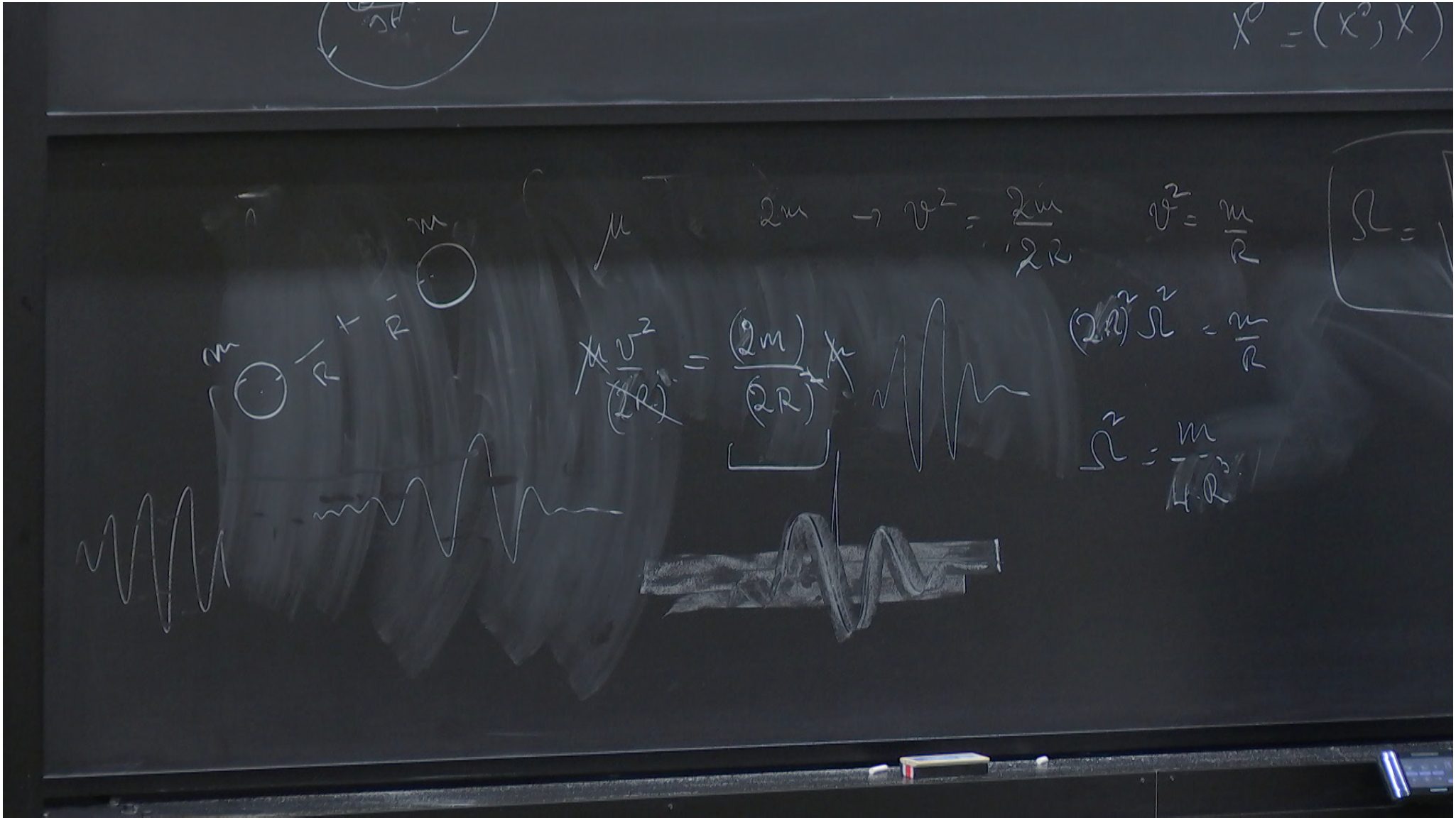

$T^{ij} = \frac{1}{2} T^{ij} + \frac{T^{ji}}{2}$
$\partial_a T^{ab} = 0$
$I_{ij} = \int d^3y \, \rho \, y_i y_j$
$\nabla T = \partial T + \Gamma T$


m
R
R
m
μ
2m
$\rightarrow v^2 = \frac{2m}{2R}$
$\frac{\mu v^2}{(2R)} = \frac{(2m)}{(2R)^2}$


m
m
R
R
μ
2m
$v^2 = \frac{2m}{2R}$
$v = \frac{m}{R}$
$\frac{\mu v^2}{(2R)} = \frac{(2m)}{(2R)^2}$

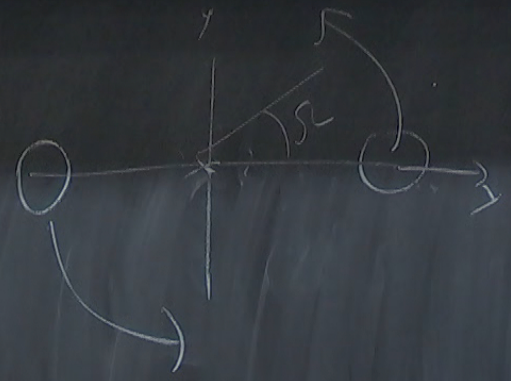

$\rightarrow v^2 = \frac{2m}{2R}$
$v^2 = \frac{m}{R}$
$\Omega = \sqrt{\frac{2m}{4R^3}}$
$(2\pi)^2 \Omega^2 = \frac{m}{R}$
$\Omega^2 = \frac{m}{4R^3}$


$\rightarrow v^2 = \frac{2m}{2R}$
$v^2 = \frac{m}{R}$
$\Omega = \sqrt{\frac{m}{4R^3}}$
$(2R)^2 \Omega^2 = \frac{m}{R}$
$\Omega^2 = \frac{m}{4R^3}$

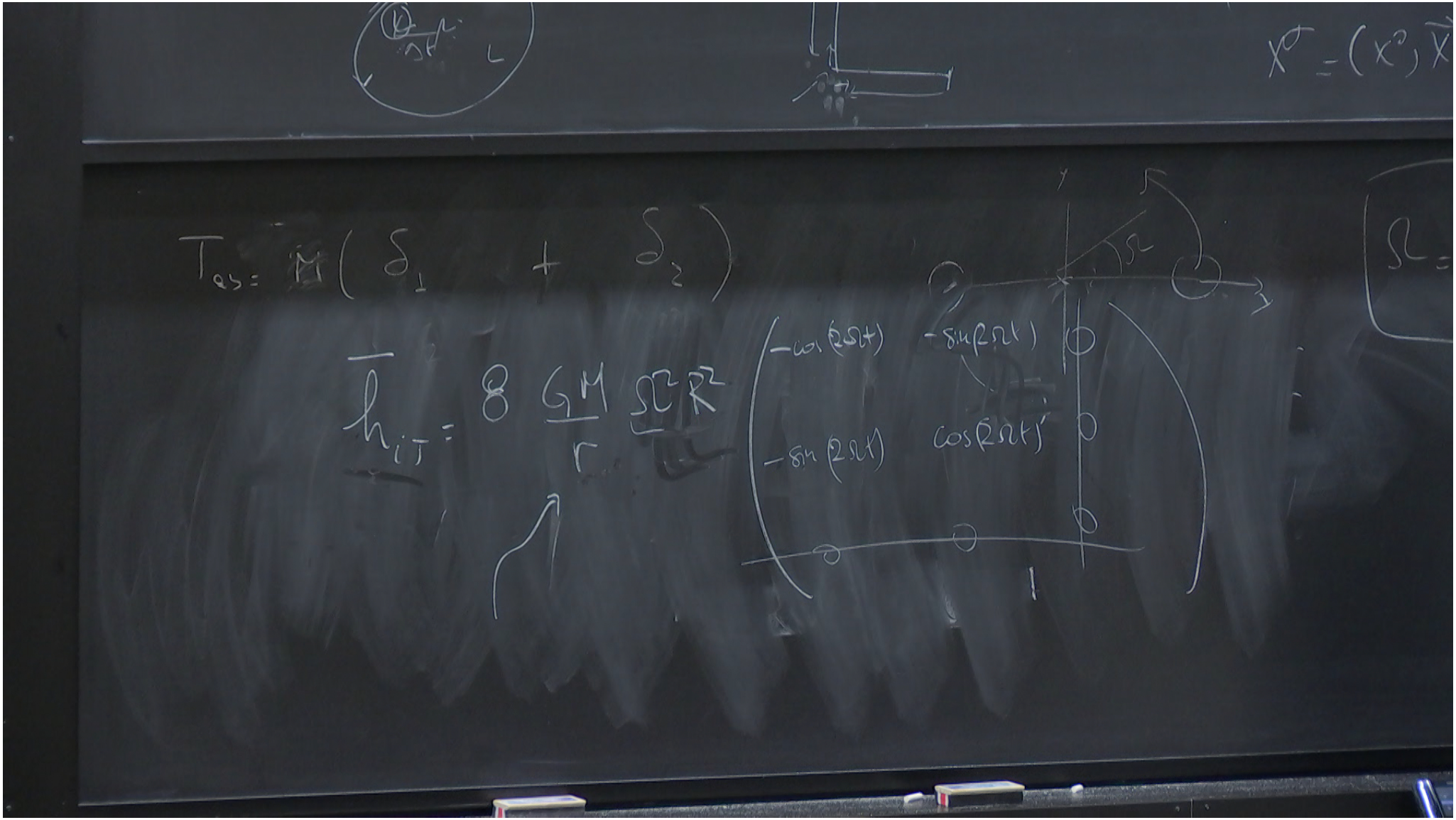

$X^\mu = (x^0, \vec{x})$
$\mu \quad 2m \rightarrow v^2 = \frac{2m}{2R} \quad v^2 = \frac{m}{R}$
$\frac{\mu v^2}{(2R)} = \frac{(2m)}{(2R)^2}\mu$
$(2R)^2 \Omega^2 = \frac{m}{R}$
$\Omega^2 = \frac{m}{4R^3}$

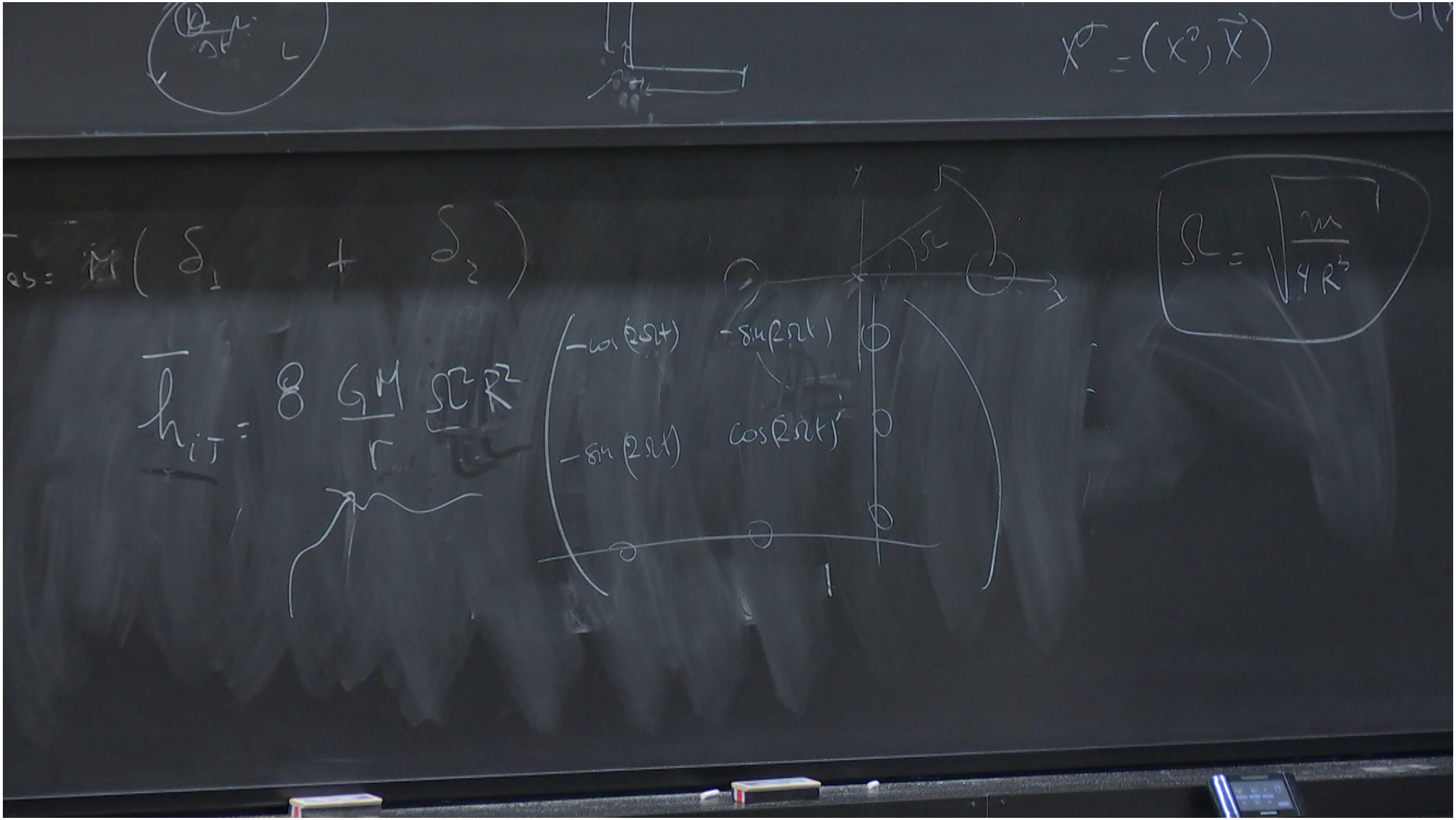
$T_{QS} = M(\delta_1 + \delta_2)$

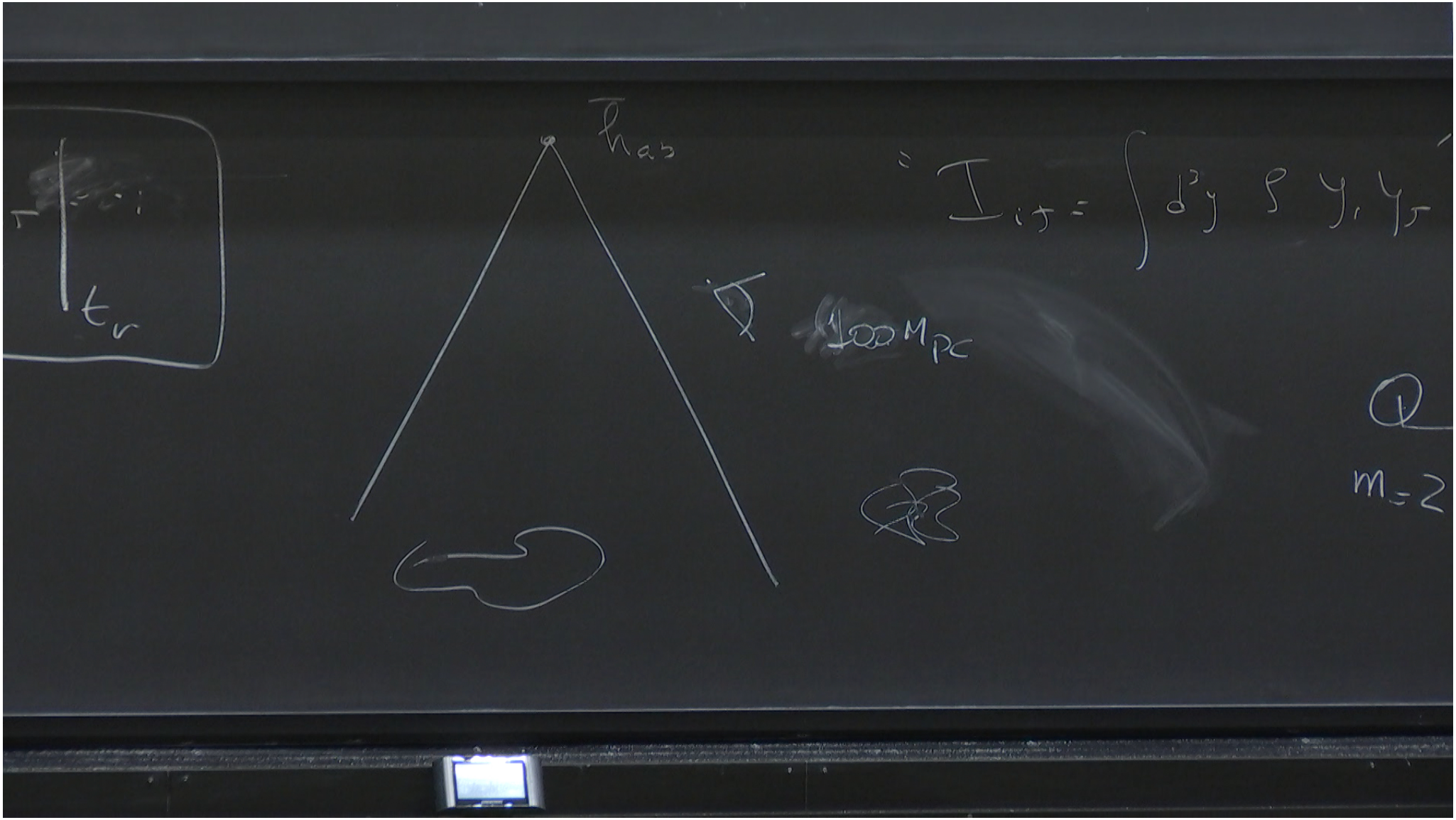
$\bar{h}_{ij} = 8 \frac{GM}{r} [$

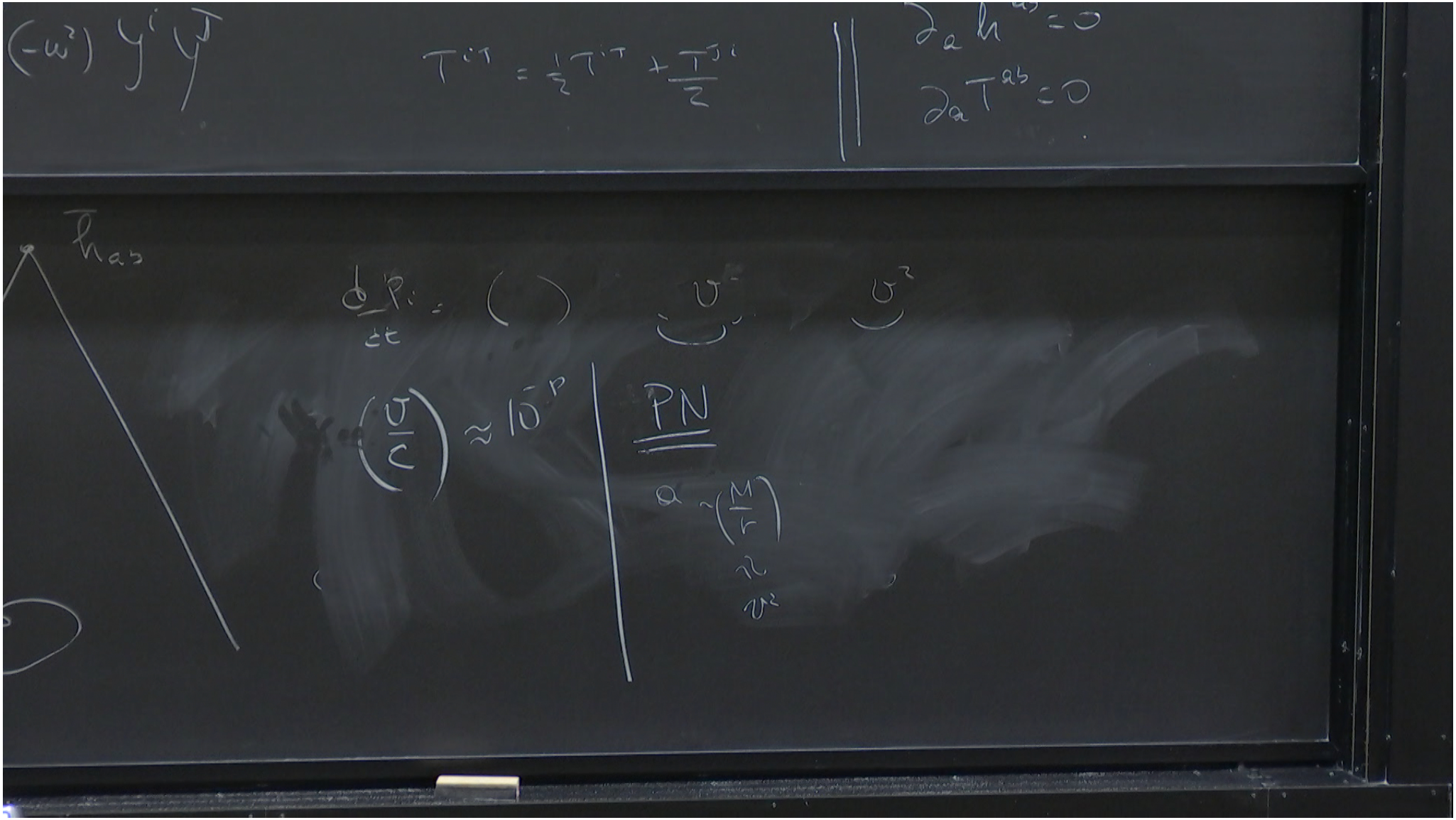


$SL = \sqrt{\frac{}{}}$


$T_{ab} = M(\delta_1 + \delta_2)$
$\bar{h}_{ij} = \frac{8}{r}\frac{GM}{}\frac{\Omega^2 R^2}{}$
$\begin{pmatrix} -\cos(2\Omega t) & -\sin(2\Omega t) & 0 \\ -\sin(2\Omega t) & \cos(2\Omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$x^\mu = (x^0, \vec{x}$


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$\Omega = \sqrt{\frac{2M}{4R^3}}$
$\bar{h}_{ij} = \frac{8GM}{r}\Omega^2 R^2$
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h_{ab}
t_r
$I_{ij} = \int d^3y \; \rho \; y_i y_j$
100 Mpc
m=2


$(-\omega^2)\, y^i y^j$
$T^{ij,TT} = \frac{1}{2} T^{ij,TT} + \frac{T^{ij}}{2}$
$\partial_a h^{ab} = 0$
$\partial_a T^{ab} = 0$
h_{ab}
$\frac{dP_i}{dt} = ( )$
$\left(\frac{v}{c}\right) \approx 10^{-p}$
PN
$a \sim \left(\frac{M}{r}\right)$
$\sim v^2$