Title: Words to Describe a Black Hole

Speakers: Ying Lin

Series: Quantum Fields and Strings

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Abstract: We revamp the constructive enumeration of 1/16-BPS states in the maximally supersymmetric Yang-Mills in four dimensions, and search for ones that are not of multi-graviton form. A handful of such states are found for gauge group SU(2) at relatively high energies, resolving a decade-old enigma. Along the way, we clarify various subtleties in the literature, and prove a non-renormalization theorem about the exactness of the cohomological enumeration in perturbation theory. We point out a giant-graviton-like feature in our results, and envision that a deep analysis of our data will elucidate the fundamental properties of black hole microstates.

Zoom link: https://pitp.zoom.us/j/96037678536?pwd=eGdhTWF3UVN1em5uZVpJbWYyM2tzUT09

Words to describe a black hole

Ying-Hsuan Lin (Harvard)

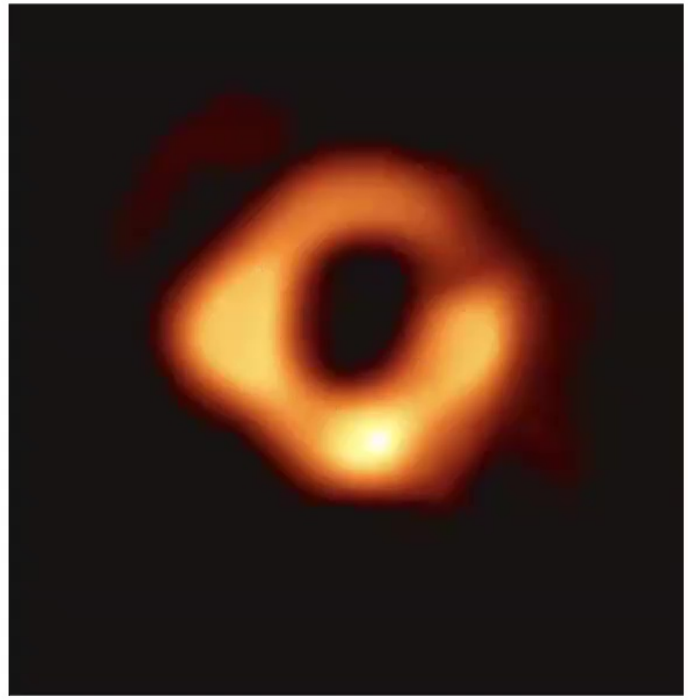
11/29/2022 @ Perimeter Institute QF&S Seminar

2209.06728 with Chi-Ming Chang

M87

Event Horizon Telescope

- Image of a black hole at the center of galaxy.
  - Data taken April 2017.
  - Published April 2019.
- Bright ring is the photon sphere.
- Shadow is the black hole.

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Chi-Ming Chang
(Tsinghua University)
Ying-Hsuan Lin

Why do we like black holes so much?

Index counting

Slide Subtitle

- AdS/CFT provides a more refined setting to count microstates.
  - State-state correspondence means that AdS black hole microstates must be accountable by states in the CFT.
- Semiclassical bulk description valid at strong coupling in CFT, but counting is practically done at weak coupling → study **index**, a deformation-invariant.
- The index counts states with *signs*, whereas the black hole entropy counts states without signs → logically, need not match due to cancellations.
- In practice, there have been numerous successful matches of the asymptotic growth of indices with the Bekenstein-Hawking entropy of AdS black holes.

[Benini-Hristov-Zaffaroni'16'17] + more

Index counting

Slide Subtitle

- The most difficult case:

$$\text{IIB on } AdS_5 \times S^5 \leftrightarrow \mathcal{N} = 4 \ SU(N) \text{ SYM.}$$

(BPS) black hole entropy ↔ (BPS) state counting.

[Gutowski-Reall'04] [Chong-Cvetic-Lu-Pope'05] [Kunduri-Lucietti-Reall'06]

[Kinney-Maldacena-Minwalla-Raju'05]

- Superconformal index can be expressed as an N-dimensional integral.
  - Saddle point analysis revealed O(1) entropy, falling disappointingly short of Bekenstein-Hawking $O(N^2)$ → Attributed to cancellations in the index.

Index counting

Slide Subtitle

- A breakthrough came in 2018.

  [Cabo-Bizet-Cassani-Martelli-Murthy'18] [Choi-Kim-Kim-Nahmgoong'18] [Benini-Milan'18]

  - It was realized that large cancellations were happening not at fixed charges, but between nearby charges → Nature of saddle point analysis.
  - The signed degeneracies oscillate signs rapidly.
  - By turning on complex chemical potentials to cancel the sign oscillation, $O(N^2)$ entropy was obtained!

Gravitons vs black hole microstates

Slide Subtitle

- Not all states in the CFT are dual to single-centered black holes in the bulk.
- There can be gravitons, other black objects, and all sorts of bound states.
- A rough binary classification:
  - **Gas of gravitons:** The energy does not scale with N.
  - **Black objects + other stuff:** The energy does scale with $N^{\#}$.
- CFT states dual to single-gravitons are known. *[Chang-Yin'13]*
  - They are BPS without the need of trace relations, hence no N dependence.

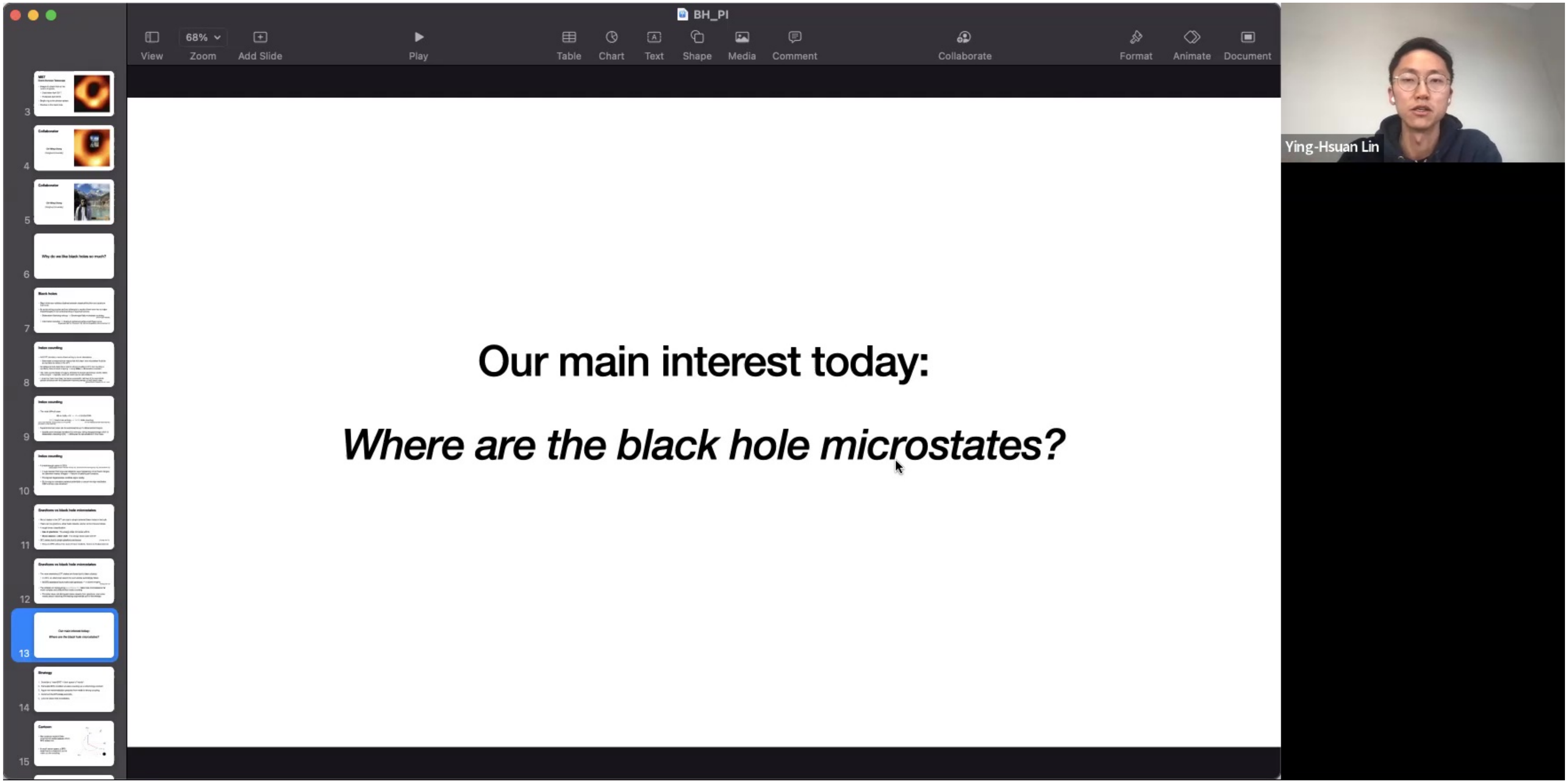
Gravitons vs black hole microstates

Slide Subtitle

- The more interesting CFT states are those dual to black objects.
  - In 2013, an attempted search for such states surprisingly failed.
  - <u>All BPS operators found were multi-gravitons</u> → a bizarre enigma.

[Chang-Yin'13]

- The problem of constructing (candidates for) black hole microstates is far more complex and difficult than index counting.
  - The latter does not distinguish black objects from gravitons, and cares mostly about matching the leading exponential part of the entropy.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Our main interest today:
Where are the black hole microstates?
Ying-Hsuan Lin

Strategy

Slide Subtitle

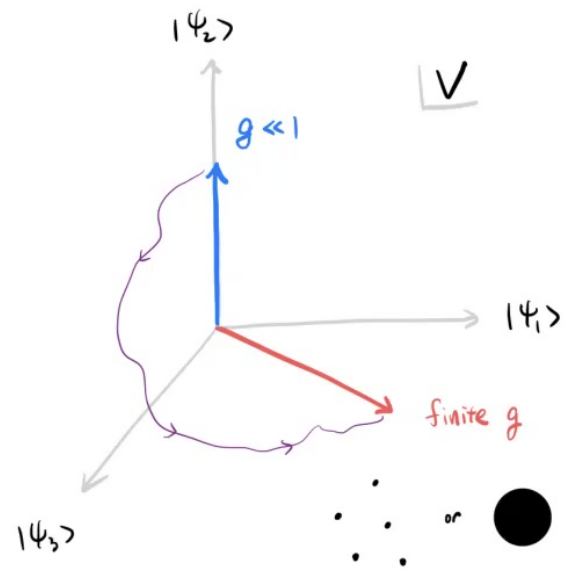
1. Consider a “near-BPS” Hilbert space of “words”.
2. Formulate BPS condition at weak-coupling as a cohomology problem.
3. Argue non-renormalization property from weak to strong coupling.
4. Construct the BPS states explicitly.
5. Look for black hole microstates.

Cartoon

Slide Subtitle

- We construct explicit *finite-dimensional* vector spaces where BPS states live.

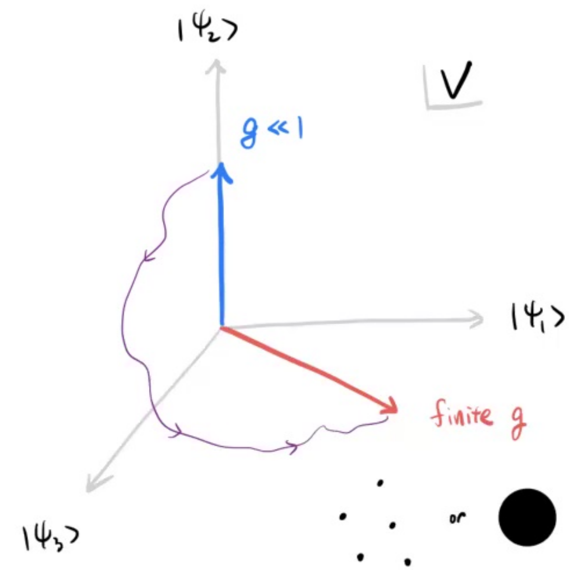
- In each vector space, a BPS state traces a trajectory as we crank up the coupling.

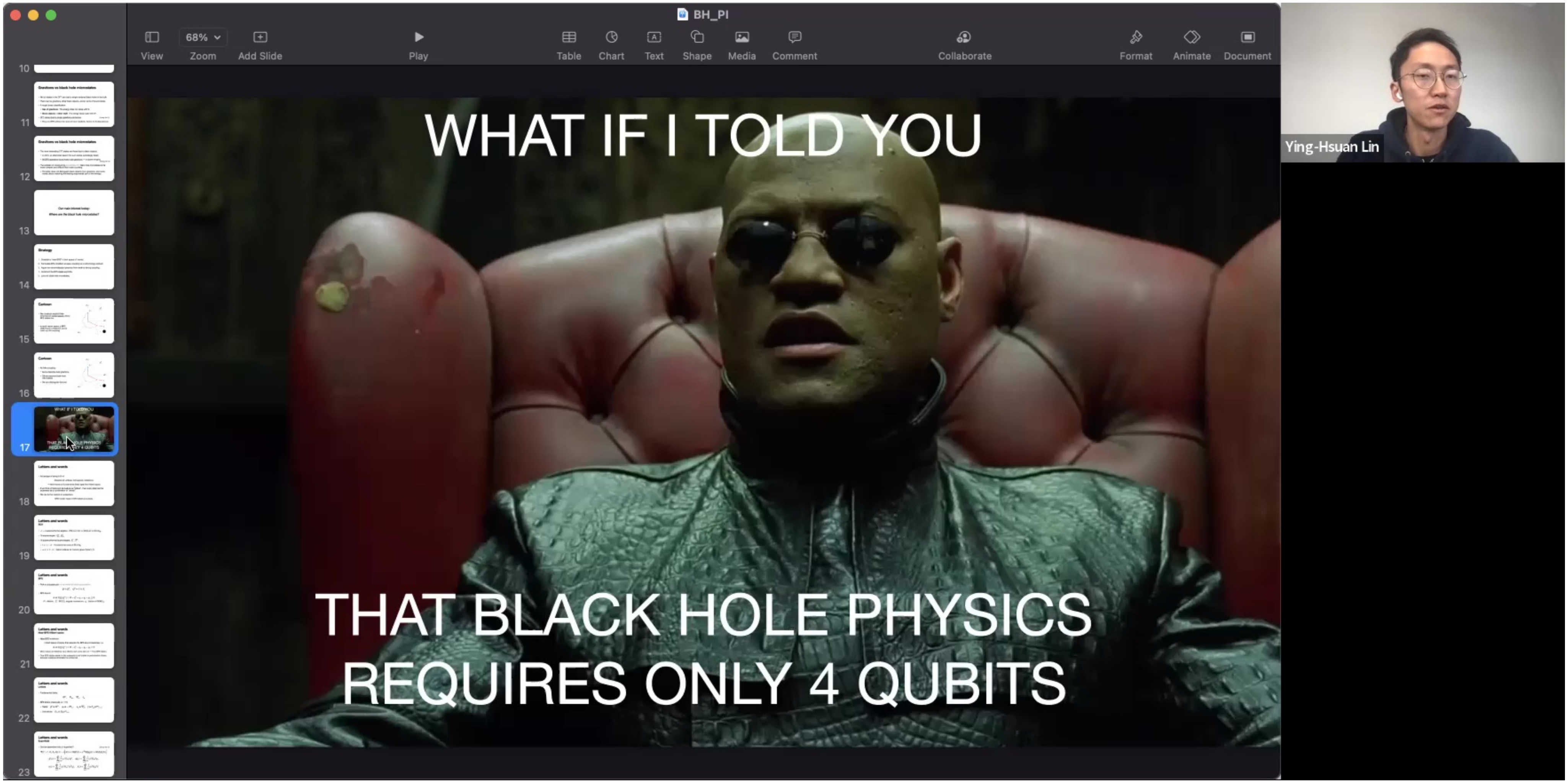


Cartoon

Slide Subtitle

- At finite coupling:
  - Some describe bulk gravitons.
  - Others become black hole microstates.
  - We can distinguish the two!

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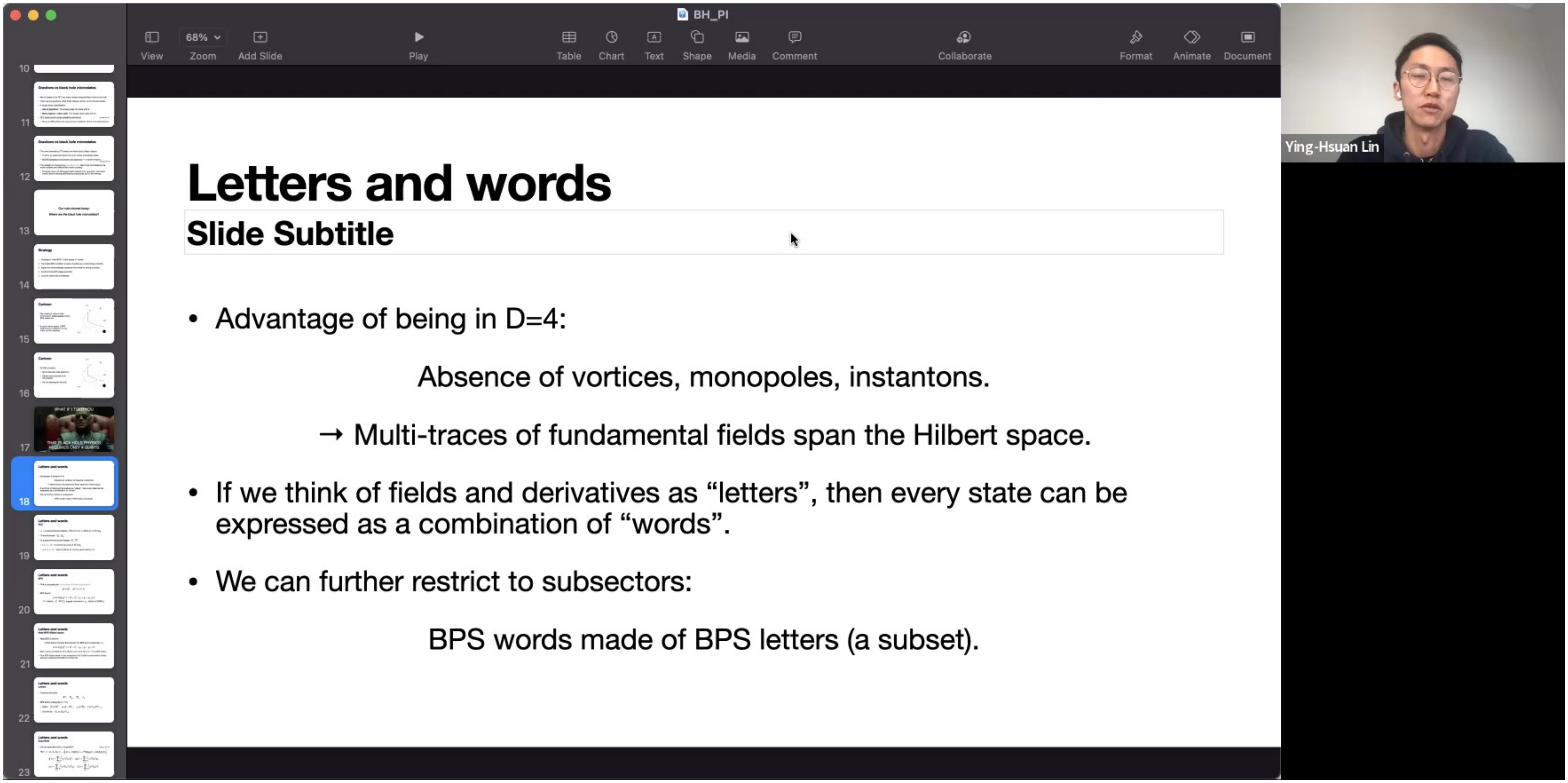
WHAT IF I TOLD YOU
THAT BLACK HOLE PHYSICS
REQUIRES ONLY 4 QUBITS
Ying-Hsuan Lin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Letters and words
Slide Subtitle
• Advantage of being in D=4:
Absence of vortices, monopoles, instantons.
→ Multi-traces of fundamental fields span the Hilbert space.
• If we think of fields and derivatives as “letters”, then every state can be expressed as a combination of “words”.
• We can further restrict to subsectors:
BPS words made of BPS letters (a subset).

Letters and words

SCA

- $\mathcal{N} = 4$ superconformal algebra: $\mathrm{PSU}(2,2\,|\,4) \supset \mathrm{SO}(2,4) \times \mathrm{SU}(4)_{\mathrm{R}}$.
- 16 supercharges: $Q^I_\alpha\,,\ \overline{Q}_{I\dot\alpha}$.
- 16 superconformal supercharges: $S^\alpha_I\,,\ \overline{S}^{I\dot\alpha}$.
    - $I = 1,\cdots,4$: Fundamental index of $\mathrm{SU}(4)_{\mathrm{R}}$.
    - $\alpha, \dot\alpha = \pm\,,\,\dot\pm$: Spinor indices for Lorentz group $\mathrm{Spin}(1,3)$.

Letters and words

BPS

- Pick a conjugate pair (in the sense of radial quantization)

$$Q \equiv Q^4_-, \quad Q^\dagger = S \equiv S_4^-.$$

- BPS bound

$$\Delta \equiv 2\{Q, Q^\dagger\} = D - J_L^3 - q_1 - q_2 - q_3 \geq 0.$$

D : dilation, J_L^3 : $\mathrm{SU}(2)_L$ angular momentum, q_i : Cartan of $\mathrm{SO}(6)_R$.

Letters and words

Near-BPS Hilbert space

- Near-BPS subsector:

  Hilbert space of states that saturate the BPS bound *classically*, i.e.

$$\Delta \equiv 2\{Q, Q^\dagger\} = D - J_L^3 - q_1 - q_2 - q_3 = 0.$$

- Most states are lifted by loop effects, but some are not → True BPS states.
- True BPS states reside in this subsector *to all orders* in perturbation theory, because classical dimension is conserved.

Letters and words

Letters

- Fundamental fields:

$$\Phi^{IJ}, \quad \Psi_{I\alpha}, \quad \overline{\Psi}^{I}_{\dot{\alpha}}, \quad A_{\mu}.$$

- BPS letters (classically $\Delta = 0$):
  - Fields: $\phi^i \equiv \Phi^{4i}, \quad \psi_i \equiv -i\Psi_{i+}, \quad \lambda_{\dot{\alpha}} \equiv \overline{\Psi}^{4}_{\dot{\alpha}}, \quad f \equiv F_{\mu\nu}(\sigma^{\mu\nu})_{++},$
  - Derivatives: $D_{\dot{\alpha}} \equiv D_{\mu}(\sigma^{\mu})_{+\dot{\alpha}}.$

Letters and words

Superfield

- Can be assembled into a "superfield": *[Chang-Yin'13]*

$$\Psi(z^+, z^-, \theta_1, \theta_2, \theta_3) = -\, i \left[\lambda(z) + 2\theta_i \phi^i(z) + \epsilon^{ijk}\theta_i\theta_j\psi_k(z) + 4\theta_1\theta_2\theta_3 f(z) \right].$$

$$\phi^i(z) = \sum_{n=0}^{\infty} \frac{1}{n!}(z^{\dot\alpha}D_{\dot\alpha})^n \phi^i\,, \quad \psi_i(z) = \sum_{n=0}^{\infty} \frac{1}{n!}(z^{\dot\alpha}D_{\dot\alpha})^n \psi_i\,,$$

$$\lambda(z) = \sum_{n=0}^{\infty} \frac{1}{n!}(z^{\dot\alpha}D_{\dot\alpha})^n (z^{\dot\beta}\lambda_{\dot\beta})\,, \quad f(z) = \sum_{n=0}^{\infty} \frac{1}{n!}(z^{\dot\alpha}D_{\dot\alpha})^n f\,.$$

Letters and words

Words

- BPS words are made of BPS letters, e.g.

$$\mathrm{Tr}[\psi_1\phi^2]\,\mathrm{Tr}[\lambda_+D_1D_2f].$$

- In terms of the superfield, e.g.

$$\mathrm{Tr}[(\partial_{\theta_2}\partial_{\theta_3}\Psi)(\partial_{\vec{\theta}}\Psi)^2]\,\mathrm{Tr}[(\partial_{z^+}\Psi)(\partial_{\theta_1}\partial_{z^+}\partial_{z^-}\partial_{\theta_2}\partial_{\theta_3})\Psi].$$

Letters and words

Supercharge

- Classically, the supercharge acts on BPS letters as *[Chang-Yin'13]*

$$\{Q, \Psi(Z)\} = \Psi(Z)^2.$$

- In Lagrangian theories, the action of Q on **fundamental** fields is undeformed non-perturbatively under suitable regularization.

Letters and words

Supercharge

- What about Q acting on **composites**?
  → Classically, the Leibniz rule applies.

- Quantum effects may change the Q action on composites, violating Leibniz.
  e.g. In the presence of axial or Konishi anomaly.
  This issue was pointed out to us by Justin Kulp and Davide Gaiotto.

- We believe that Leibniz holds for Q acting on BPS words in $\mathcal{N} = 4$ SYM.
  Indeed, the Konishi anomaly vanishes. [Eden-Jarczak-Sokatchev-Stanev'05]
  We are trying to formulate a complete argument…

Letters and words
Supercharge

- The supercharge acts on BPS letters by

$$\{Q, \Psi(Z)\} = \Psi(Z)^2,$$

  and on BPS words by the Leibniz rule.

- Where is the coupling dependence?
  - It is dimensionless, so we have absorbed it into the normalization of fields.
  - But $Q^\dagger$ *does* receive corrections at every loop order.

We defined BPS to be annihilated by both Q and $Q^\dagger$.
But $Q^\dagger$ receives quantum corrections.

So how can our result be valid to all loop orders?

Non-renormalization

Slide Subtitle

- Finding the *exact form* of a BPS operator is very difficult beyond one-loop.
- However, determining a finite-dimensional space where it resides is tractable.
- Two ingredients:

Hodge theory

Universal coefficient theorem

Non-renormalization

Slide Subtitle

- *Hodge theory* for the Laplacian $\Delta = \{d, d^\dagger\}$ on differential forms:

Every state with $\Delta = 0$ takes the form $\psi + d\phi$ where $d\psi = 0$.

Harmonic forms ↔ de Rham cohomology

- Replace d by Q, and the exact same arguments go through.
- We learn that:

Computing the Q-cohomology gives the <u>vector space</u> of $\psi + Q\phi$.

Non-renormalization

Slide Subtitle

- Q is one-loop exact, but the Q-cohomology *a priori* may not be.
- In perturbation theory, we can argue that the cohomology is one-loop exact, using the *universal coefficient theorem* for

$$\mathbb{C} \to \mathbb{C}[[g]].$$

  - $\mathbb{C}[[g]]$ is the formal power series ring.
  - Every state or operator takes the form of an infinite series in g.

Non-renormalization

Slide Subtitle

- Let $\mathrm{H}^*(Q;R)$ denote the Q-cohomology with R coefficients.
    - It can in fact be described as a relative Lie superalgebra cohomology.
- Universal coefficient theorem:

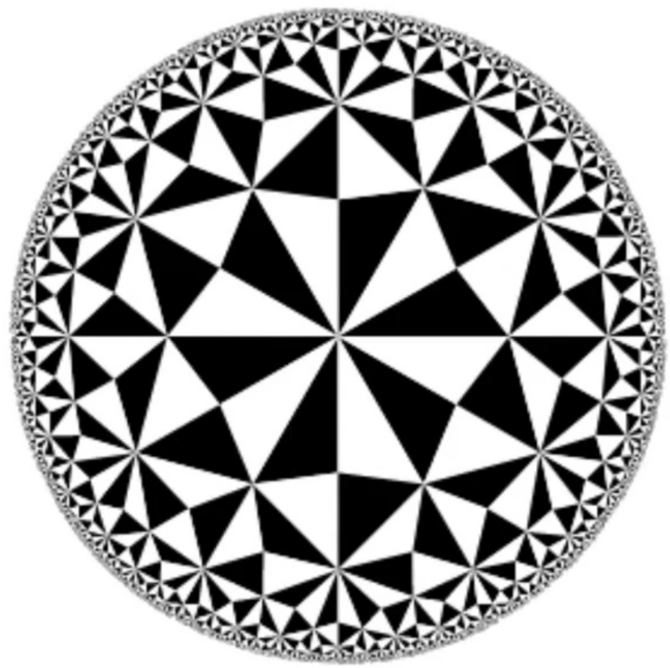
$$\mathrm{H}^*(Q;\mathbb{C}[[g]]) \cong \mathrm{H}^*(Q;\mathbb{C}) \otimes_{\mathbb{C}} \mathbb{C}[[g]].$$

- This is nontrivial, e.g. fails if we replace $\mathbb{C}[[g]]$ by polynomial ring $\mathbb{C}[g]$.

Non-renormalization

Slide Subtitle

- S-duality → Infinitely many weakly coupled points on the conformal manifold.
- + Holomorphy in g
  → Expect non-renormalization non-perturbatively.
  - Conjectured before our perturbative proof.
    [Grant-Grassi-Kim-Minwalla'08]



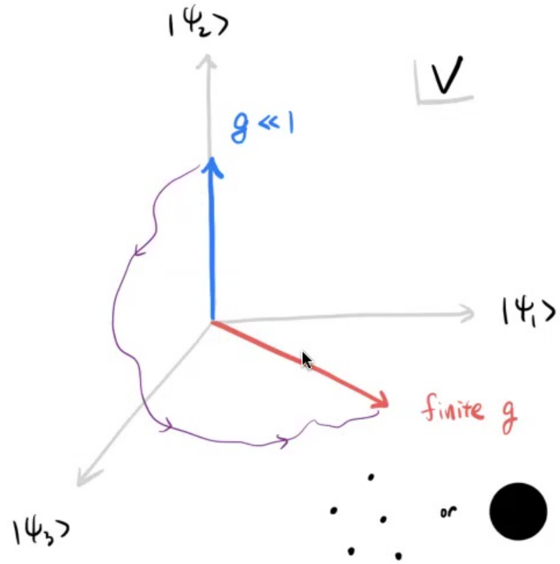
Non-renormalization

Slide Subtitle

- Actual vector depends on $Q^\dagger$ and hence the coupling.
- But the vector *space* does not.
- Construct the vector spaces *explicitly* at one-loop

  → Valid at all loops!

- Some vector spaces contain black hole microstates!



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Gas of gravitons

Slide Subtitle

- Single-gravitons $\leftrightarrow$ Q-cohomology restricted to single-traces. *[Chang-Yin'13]*
  - Explicit representatives

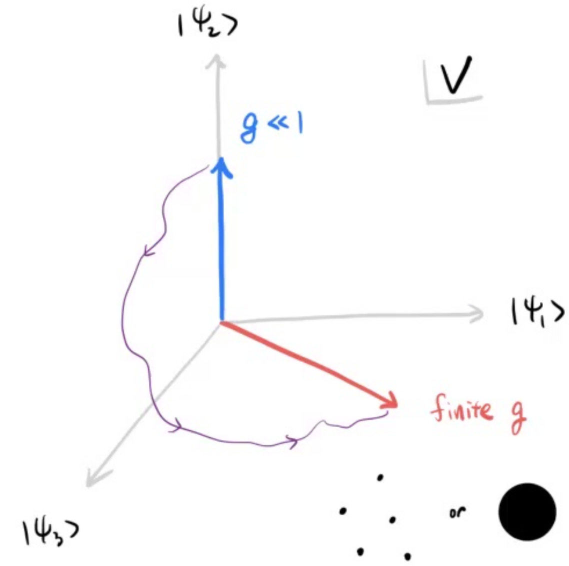
$$\partial_{z^+}^{p_1}\partial_{z^-}^{p_2}\partial_{\theta_1}^{q_1}\partial_{\theta_2}^{q_2}\partial_{\theta_3}^{q_3}\,\mathrm{Tr}\left[(\partial_{z^+}\Psi)^{k_1}(\partial_{z^-}\Psi)^{k_2}(\partial_{\theta_1}\Psi)^{m_1}(\partial_{\theta_2}\Psi)^{m_2}(\partial_{\theta_3}\Psi)^{m_3}\right]\Bigg|_{Z=0}$$

  - Energy and charges do not scale with N.
- Their products give multi-gravitons.
  - Counting at infinite N matches the counting of multi-gravitons in the bulk.

Gas of gravitons

Slide Subtitle

- We explicitly check if the vector space of each Q-cohomology contains a multi-graviton representative.
- The ones that do not represent something interesting in the bulk.



Computation

Slide Subtitle

- Fix charges to work with *finite*-dimensional near-BPS space.
- Choose to work with fields in component form.
    - Alternative: Work with formal traces and impose trace relations
- Find basis of states at fixed charge, then act by Q to find kernel and image.
    - Some fields have Fermi statistics → Data structure requires some design.
    - Finding basis requires determining linear independence, which is too expensive analytically, so needs stable numerical algorithms.

Computation

Slide Subtitle

- The kernel and image of Q provide the explicit construction of the vector space for each cohomology.
  - Dimension of vector space gives counting.
- Check if multi-graviton.

Enumeration results

Slide Subtitle

- The BPS counting data refined by all charges can publicly accessed on

  https://github.com/yinhslin/bps-counting

- To present the data, it is customary to ignore the R-charge fugacities, and track a certain combination of the energy and momentum

$$n = 2(D + J_L^3) \in \mathbb{Z}.$$

- Index-counting folks also prefer $U(N)$ over $SU(N)$.
  - The two are trivially related.

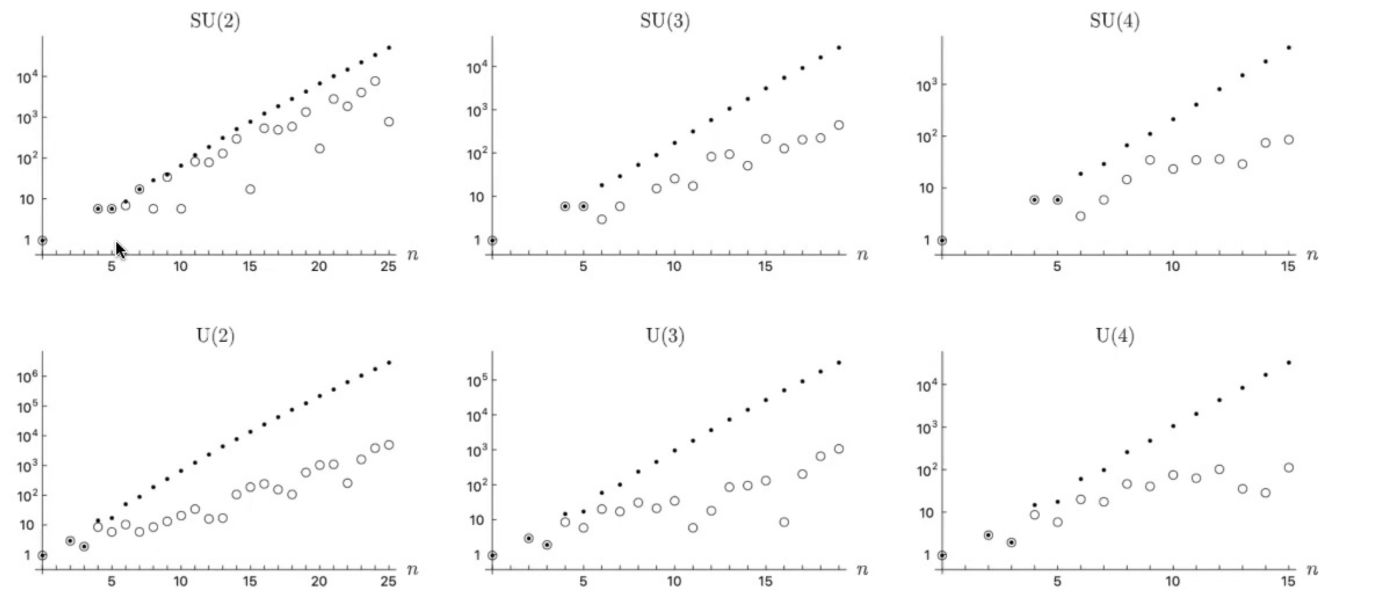
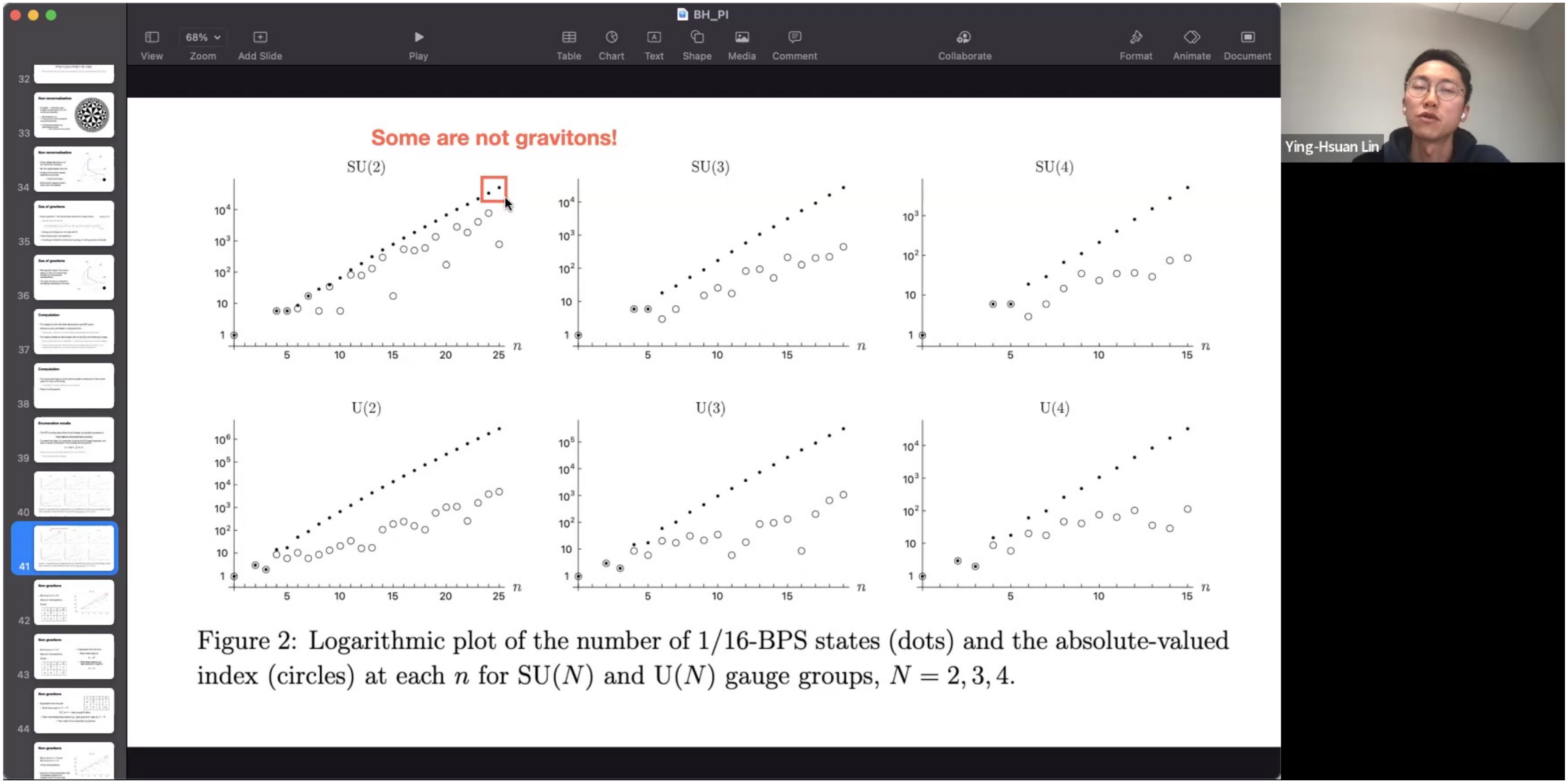


Figure 2: Logarithmic plot of the number of 1/16-BPS states (dots) and the absolute-valued index (circles) at each n for $\mathrm{SU}(N)$ and $\mathrm{U}(N)$ gauge groups, $N = 2, 3, 4$.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Some are not gravitons!
SU(2)
SU(3)
SU(4)
U(2)
U(3)
U(4)
n
Figure 2: Logarithmic plot of the number of 1/16-BPS states (dots) and the absolute-valued index (circles) at each n for SU(N) and U(N) gauge groups, N = 2, 3, 4.
Ying-Hsuan Lin

Non-gravitons

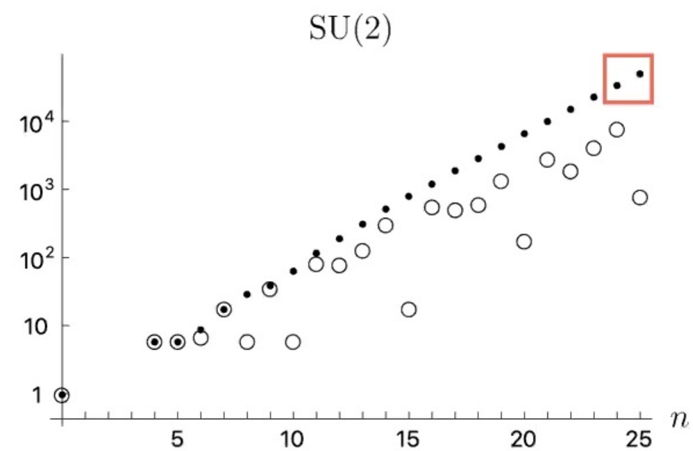
Slide Subtitle

- SU(2) up to $n = 25$:

Most are multi-gravitons…

Except

n	E	J_L^3	J_R^3
24	$\frac{19}{2}$	$\frac{5}{2}$	0
25	10	$\frac{5}{2}$	$\pm\frac{1}{2}$



Non-gravitons

Slide Subtitle

- $\mathrm{SU}(2)$ up to $n = 25$:

  Most are multi-gravitons…

  Except

n	E	J_L^3	J_R^3
24	$\frac{19}{2}$	$\frac{5}{2}$	0
25	10	$\frac{5}{2}$	$\pm\frac{1}{2}$

- Expectation from the bulk:
  - Black holes begin at
    $$E \sim N^2.$$
  - Other large objects, e.g. "giant gravitons" begin at
    $$E \sim N.$$

Non-gravitons

Slide Subtitle

n	E	J_L^3	J_R^3
24	$\frac{19}{2}$	$\frac{5}{2}$	0
25	10	$\frac{5}{2}$	$\pm\frac{1}{2}$

- Expectation from the bulk:
  - Black holes begin at $E \sim N^2$.

    $19/2 \gg 4$ → likely a small N effect.

  - Other intermediate large objects e.g. "giant gravitons" begin at $E \sim N$.

    → They might all be composites of gravitons.

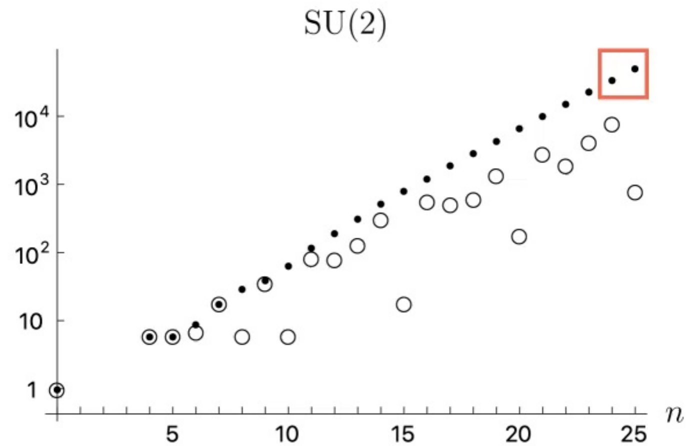
Non-gravitons

Slide Subtitle

- $\mathrm{SU}(3)$ up to $n = 19$ and $\mathrm{SU}(4)$ up to $n = 15$:

  All are multi-gravitons.

- Scarcity of (candidate) black hole microstates explains the negative result 10 years ago.



Non-gravitons

Slide Subtitle

- Fixing the charges to where the $n = 24$ non-graviton was found

  → Q-cohomology is 1-dimensional.

  - All multi-graviton operators are Q-exact.

  - Any Q-closed operator that is not Q-exact is the unique non-graviton state.

Non-gravitons

Slide Subtitle

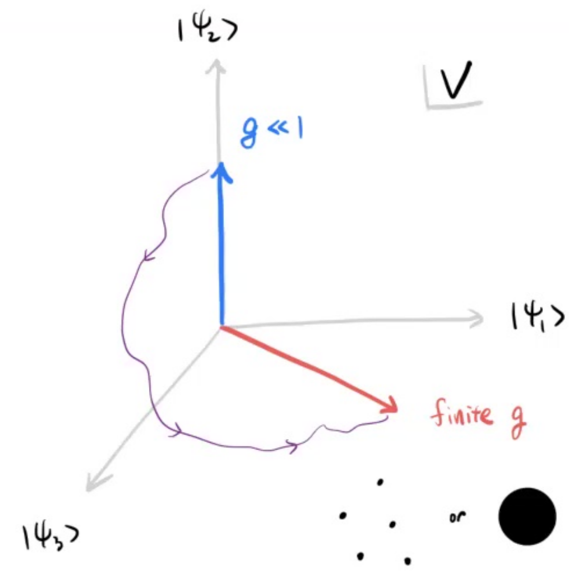
- Any Q-closed operator that is not Q-exact is the *unique* non-graviton state.
- Choi-Kim-Lee-Park used this information to construct a simple representative (a simple basis for our vector space).
- They also pointed out that the $n = 25$ non-graviton states are superconformal descendants of the $n = 24$ non-graviton state.

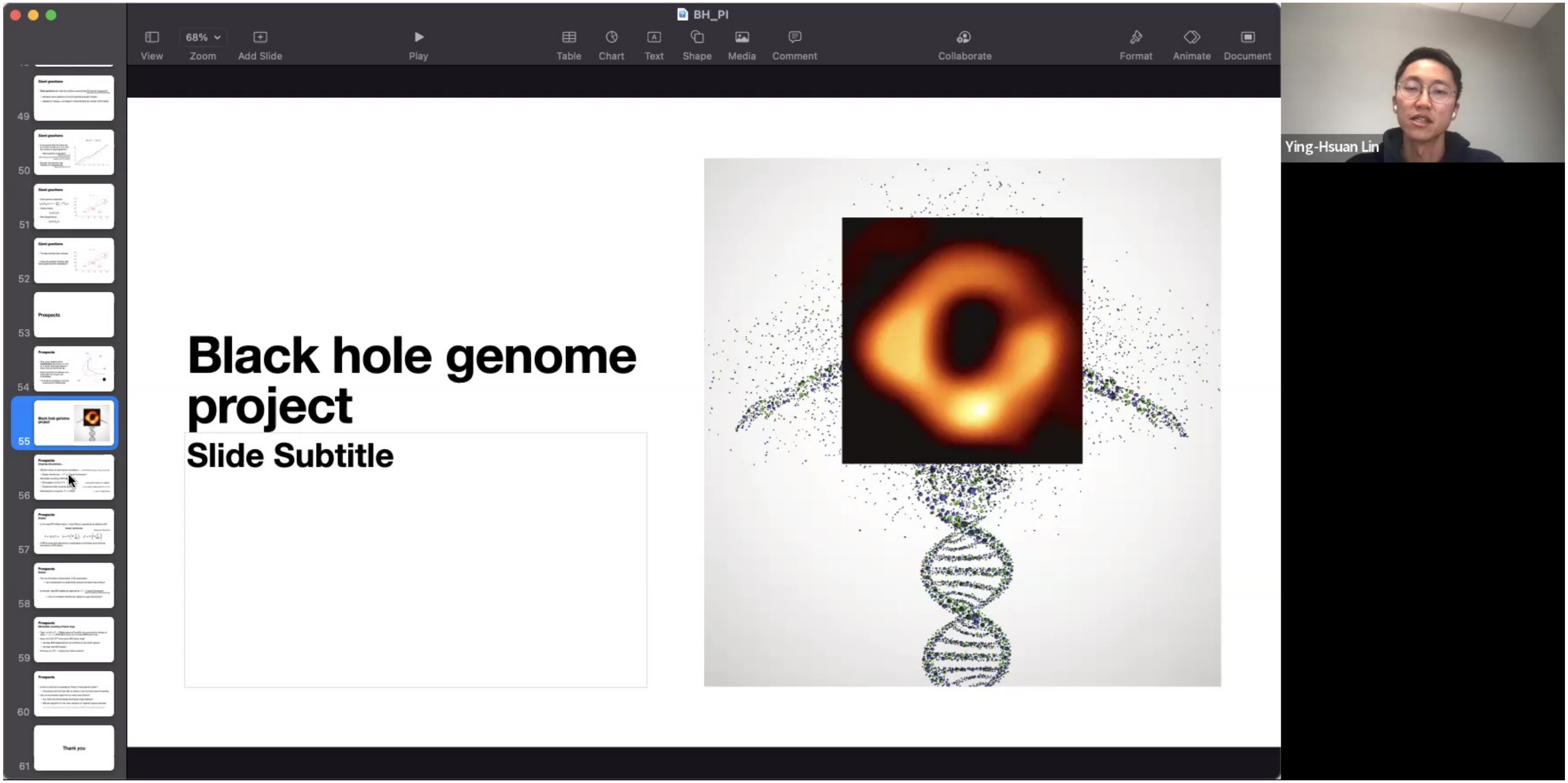
Prospects

Slide Subtitle

- One could imagine some **optimization** procedure to land on a vector that best-mimics a black hole (at fixed finite N).
- Expect gravitons to behave very differently from black hole microstates.

→ It would be *amazing* to actually measure such differences.

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Ying-Hsuan Lin
Prospects
Thank you
Black hole genome project
Slide Subtitle

Prospects

Beisert

- In the near-BPS Hilbert space, 1-loop lifting is captured by an *effective QM*:

<u>Beisert Hamiltonian</u>

[Beisert'03] [Beisert'04]

$$H = \{Q, Q^\dagger\} = \quad Q = \mathrm{Tr}\left(\Psi^2 \frac{\delta}{\delta\Psi}\right), \quad Q^\dagger = \mathrm{Tr}\left(\Psi \frac{\delta^2}{\delta\Psi^2}\right).$$

- $\delta/\delta\Psi$ involves both derivatives of superspace coordinates and functional derivatives of BPS letters.

Prospects

Ongoing discussions…

- Effective theory of near-horizon excitations. *[…with Chi-Ming Chang, Li Feng, Yixiao Tao]*
  - Beisert Hamiltonian → $\mathcal{N} = 2$ super-Schwarzian?
- Microstate counting of BPS black rings.
  - BPS states in D1D5 CFT? *[…with Indranil Halder, Samir Mathur]*
  - Gopakumar-Vafa invariants of CY3? *[…with Indranil Halder, Manki Kim, Xi Yin]*
- Generalization to beyond $\mathcal{N} = 4$ SYM. *[…with Chi-Ming Chang]*

Prospects
Beisert

- One can formulate a bilocal action in 2|3 superspace.

  → Can evaluating the on-shell action produce the black hole entropy?

- In the bulk, near-BPS states are captured by $\mathcal{N} = 2$ super-Schwarzian.
  [Boruch-Heydeman-Illiesu-Turiaci'22]

  → How is the Beisert Hamiltonian related to super-Schwarzian?

Prospects

Beisert

- One can formulate a bilocal action in 2|3 superspace.

  → Can evaluating the on-shell action produce the black hole entropy?

- In the bulk, near-BPS states are captured by $\mathcal{N} = 2$ super-Schwarzian. *[Boruch-Heydeman-Illiesu-Turiaci'22]*

  → How is the Beisert Hamiltonian related to super-Schwarzian?

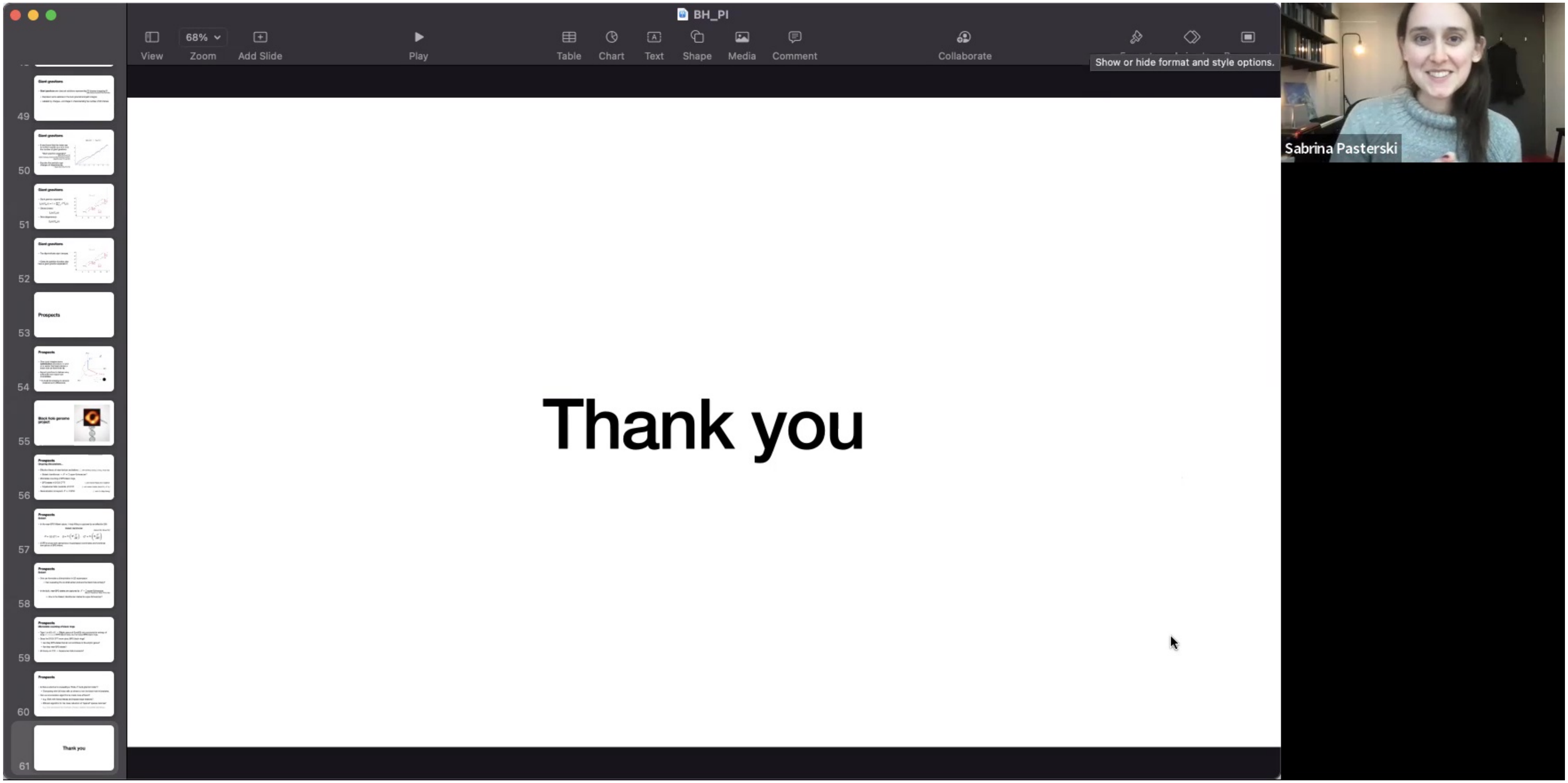
Prospects

Microstate counting of black rings

- Type II on K3 x S^1 → Elliptic genus of Sym(K3) only accounts for entropy of large (3-charged) BPS black holes, but not large BPS black rings.
- Does the D1D5 CFT know about BPS black rings?

  → Are they BPS states that do not contribute to the elliptic genus?

  → Are they near-BPS states?
- M-theory on CY3 → Gopakumar-Vafa invariants?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Thank you
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