Title: The Supersymmetric Index and its Holographic Interpretation

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Series: Quantum Fields and Strings

Date: November 08, 2022 - 2:00 PM

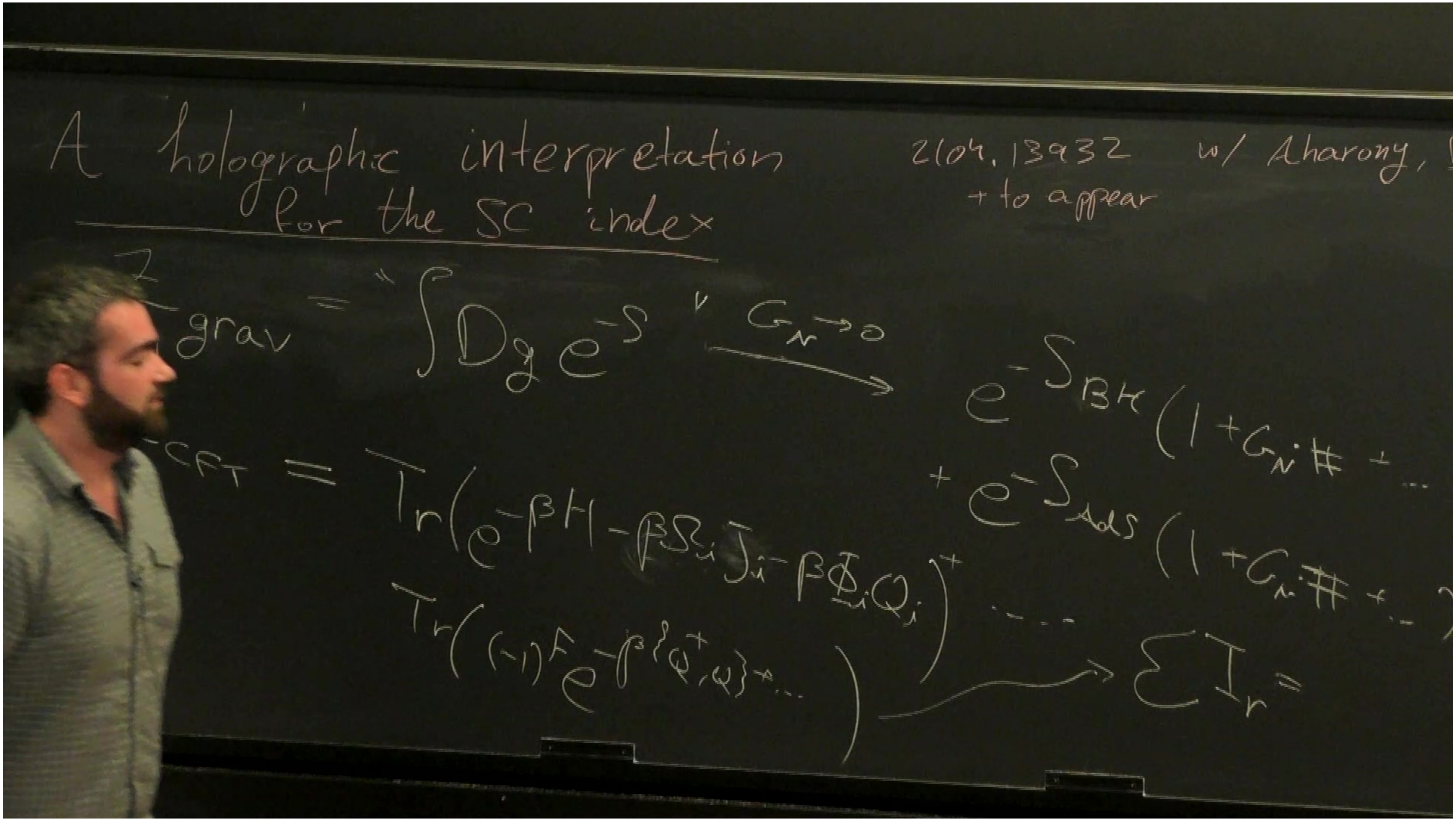
URL: https://pirsa.org/22110020

Abstract: The supersymmetric index of N=4 SU(N) Super Yang-Mills is a well studied quantity. In 2104.13932, using the Bethe Ansatz approach, we analyzed some family of contributions to it. In the large N limit each term in this family has a holographic interpretation - it matches the contribution of a different Euclidean black hole to the partition function of the dual gravitational theory. By taking into account non-perturbative contributions (wrapped D3-branes, similar to Euclidean giant gravitons), we further showed a one to one match between the contributions of the gravitational saddles and this family of contributions to the index, both at the perturbative and non-perturbative levels. I'll end with newer results, concerning the form of these terms at finite N, new solutions to the Bethe Ansatz equations (i.e. additional contributions to the index beyond the ones described in that paper), and some ongoing effort to classify all the solutions to these equations.

Zoom Link: https://pitp.zoom.us/j/95037315617?pwd=ell4WExrSXJ4YUVyaXAzRGJjdjYxUT09

A holographic interpretation for the SC index

2104.13932 w/ Aharony,
+ to appear


A holographic interpretation
for the SC index
2104.13932
+ to appear
w/ Aharony,
Z_grav "=" ∫Dg e^-S "
G_N → 0
e^-S_BH (1 + G_N # + ...
+ e^-S_AdS (1 + G_N # + ...
CFT = Tr(e^-βH - βΩ_i J_i - βΦ_i Q_i) + ...
Tr((-1)^F e^-β{Q†,Q} + ...)
Σ I_r =

A holographic interpretation for the SC index

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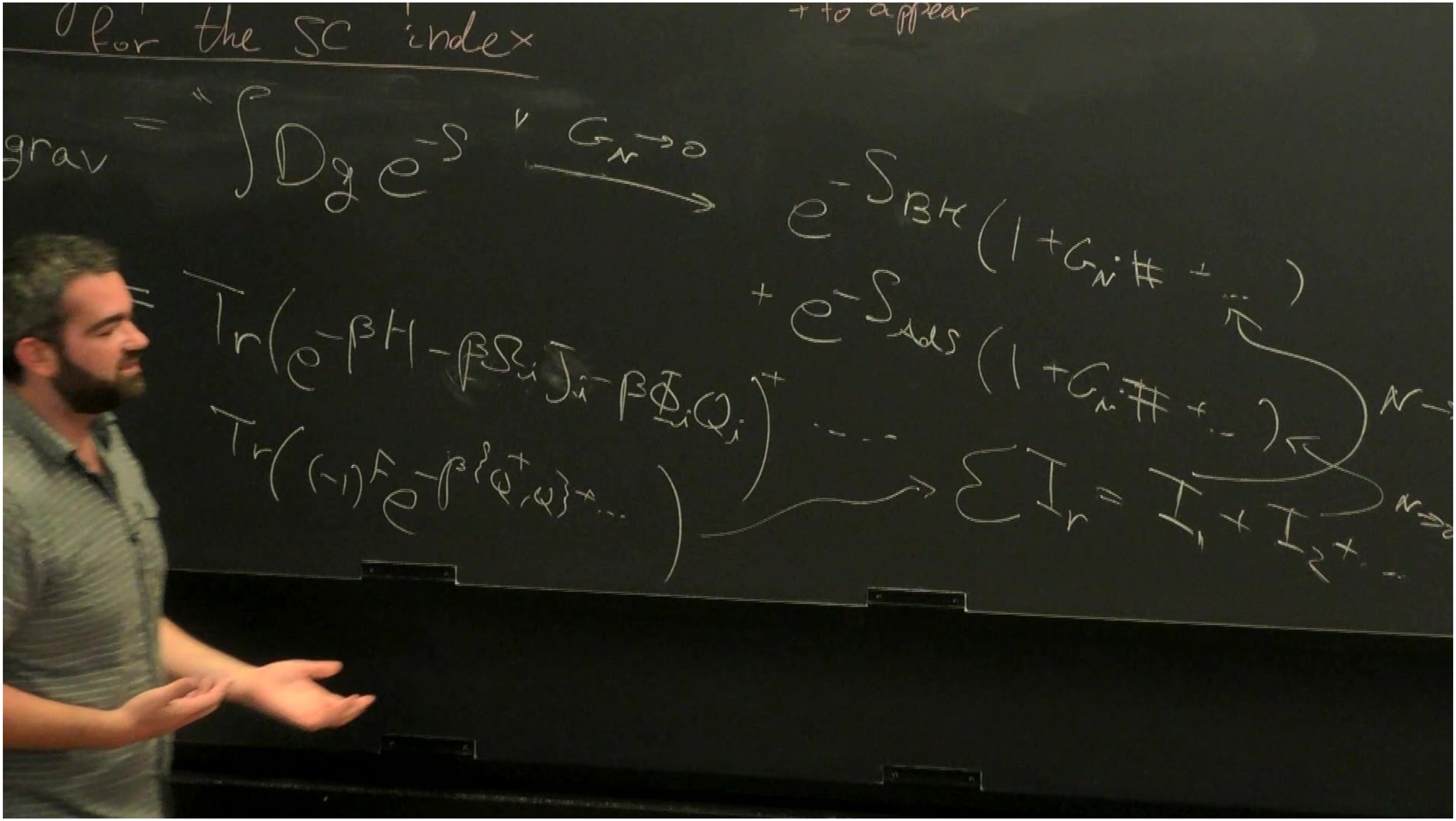
+ to appear

$$Z_{grav} = \text{“}\int Dg\, e^{-S}\text{”} \xrightarrow{G_N \to 0} e^{-S_{BH}}\left(1 + G_N \# + \dots\right) + e^{-S_{AdS}}\left(1 + G_N \# + \dots\right) + \dots$$

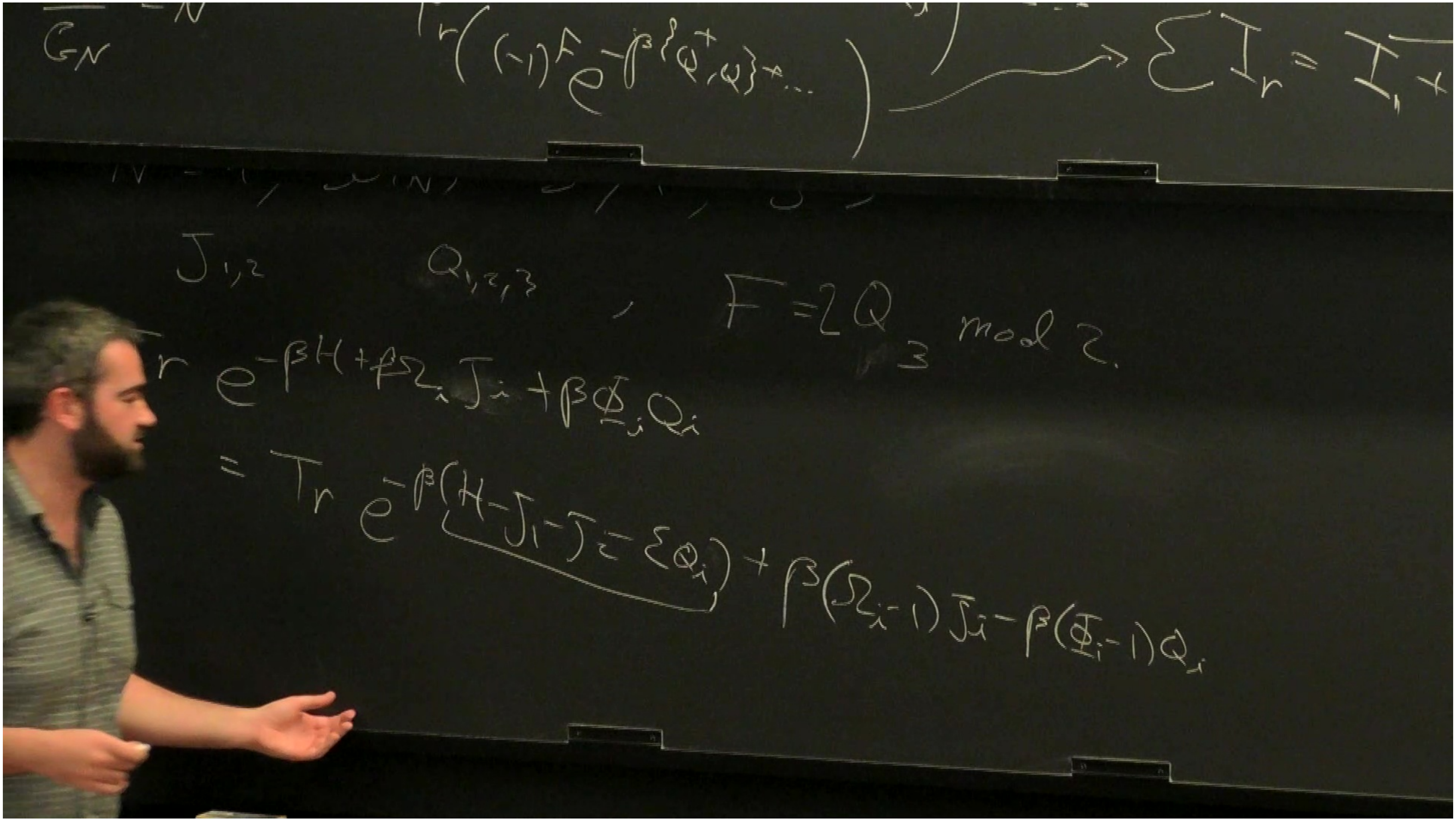
$$\parallel$$

$$Z_{CFT} =$$


for the SC index
+ to appear
grav = " ∫ Dg e^{-S} " G_N → 0 e^{-S_BH} (1 + G_N # + ...)
+ e^{-S_AdS} (1 + G_N # + ...)
= Tr(e^{-βH - βΩ_i J_i - βΦ_i Q_i}) + ...
Tr((-1)^F e^{-β{Q^†, Q} + ...})
Σ I_r = I_1 + I_2 + ...

G_N $\mathrm{Tr}\left((-1)^F e^{-\beta\{\psi^\dagger,\psi\}+\dots} \right) \longrightarrow \sum I_r$

$\underline{e^{-N^2\#}}\,,\ e^{-N^2-N}$


G_N
$\mathrm{Tr}\left((-1)^F e^{-\beta\{Q^\dagger, Q\}+\dots}\right)$
$\sum I_r = I_1 +$
$J_{1,2}$
$Q_{1,2,3}$
$F = 2Q_3$ mod 2.
$\mathrm{Tr}\, e^{-\beta H + \beta\Omega_i J_i + \beta\Phi_i Q_i}$
$= \mathrm{Tr}\, e^{-\beta(H - J_1 - J_2 - \sum Q_i) + \beta(\Omega_i - 1)J_i - \beta(\Phi_i - 1)Q_i}$

Sym , $S^1 \times S^3$

, $F = 2Q_3 \bmod 2.$

$+\beta \Phi_i Q_i$

$J_1 - J_2 - \Sigma Q_i) + \beta(\Omega_i - 1) J_i - \beta(\Phi_i - 1) Q_i$

$= \mathrm{Tr}\, e^{-\beta\{Q^+ Q\}} \ldots$

Sym, $S^1 \times S^3$

$$\beta\left(1 + \sum_i \Omega_i - \sum_a \Phi_a\right) = 2\pi i n$$

$$= \mathrm{Tr}\, e^{-\beta\{Q^\dagger, Q\}} - \dots$$

$$F = 2Q_3 \mod 2.$$

$$+ \beta \Phi_i Q_i$$

$$= \mathrm{Tr}\,(-1)^{2R_3} e^{\left[-\beta\{Q^\dagger, Q\} + \tau\left(J_1 + \frac{1}{2}R_3\right) + \sigma\left(J_1 + \frac{1}{2}R_3\right) + \varphi_{1,2}\left(R_{1,2} - R_3\right)\right]}$$

$$J_1 - J_2 - \sum Q_i) + \beta(\Omega_i - 1) J_i - \beta(\Phi_i - 1) Q_i$$

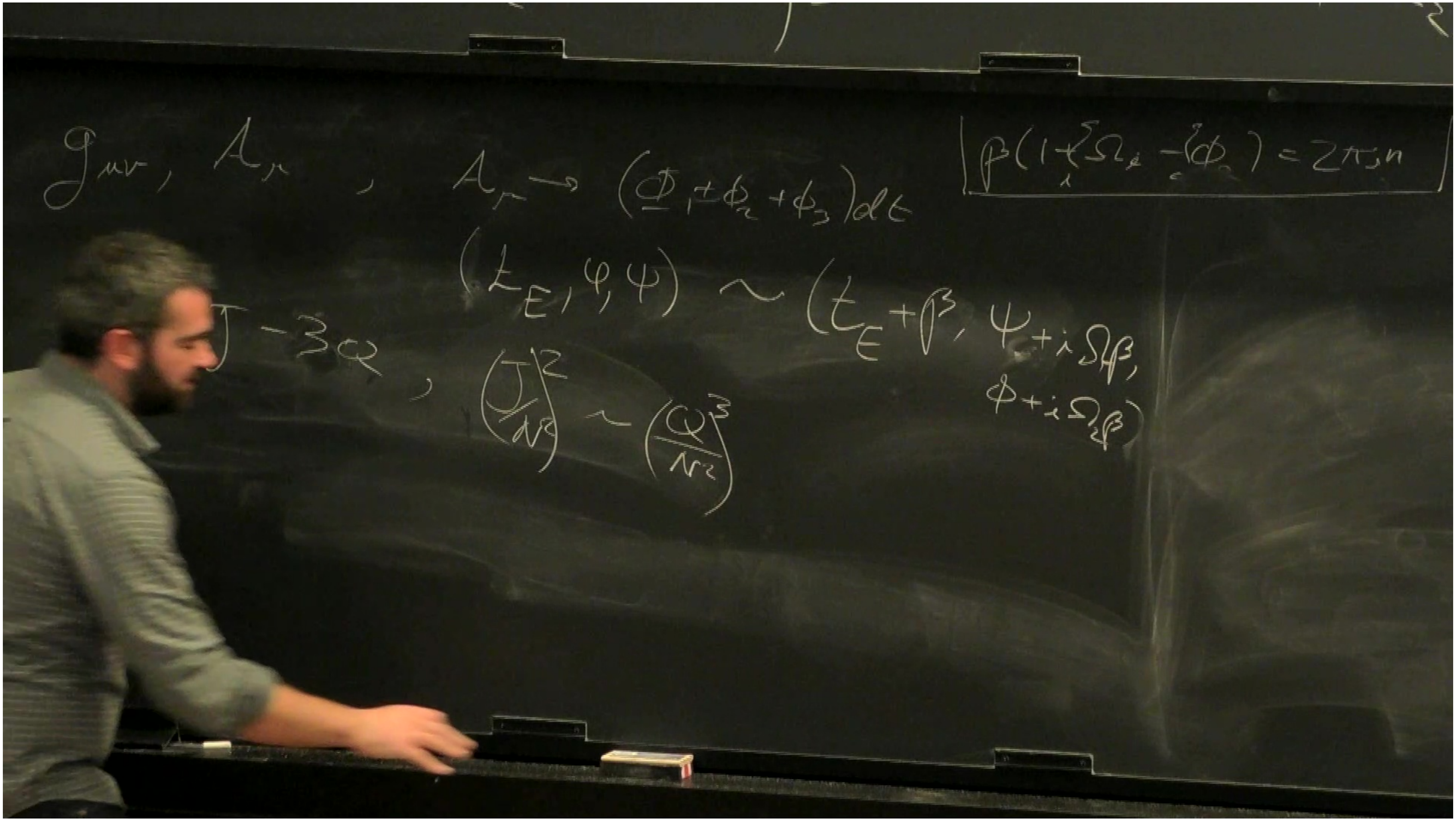
$$\mathrm{Tr}(-1)^F e^{-\beta\{Q^\dagger, Q\} - \underline{\Omega}(J_1 + \frac{1}{2}R_3) + \varphi_i(R_i - R_3)}$$

$$Z(\Omega_i, \Phi_i, \beta) = Z\left(\Omega_i + \frac{4\pi i}{\beta}, \Phi_i, \beta\right)$$

$g_{\mu\nu}$, A_μ , $A_\mu \to (\Phi_1 + \phi_2 + \phi_3)\,dt$

$\beta(1 + \sum_i \Omega_i - \sum_a \Phi_a$

$(t_E, \varphi, \psi) \sim (t_E + \beta,\ \psi + i\Omega_1\beta,\ \phi + i\Omega_2\beta)$


$g_{\mu\nu}$, A_μ, $A_\mu \to (\Phi_1+\Phi_2+\Phi_3)dt$
$\beta(1+\sum_i \Omega_i - \sum_i \Phi_i) = 2\pi i n$
$(t_E, \varphi, \psi) \sim (t_E+\beta, \psi+i\Omega_1\beta, \phi+i\Omega_2\beta)$
$J \sim Q$, $\left(\frac{J}{N^2}\right)^2 \sim \left(\frac{Q}{N^2}\right)^3$

$$g_{\mu\nu}, \quad A_\mu \quad , \quad A_\mu \to (\Phi_1 + \phi_2 + \phi_3)\,dt$$

$$\boxed{\beta\left(1 + \sum_i \Omega_i - \sum_a \Phi_a\right) = 2\pi i n}$$

$$(t_E, \varphi, \psi) \sim (t_E + \beta, \psi + i\Omega_1\beta, \phi + i\Omega_2\beta)$$

$$E = 2J - 3Q \;, \quad \left(\frac{J}{N^2}\right)^2 \sim \left(\frac{Q}{N^2}\right)^3$$

$$g_{\mu\nu}, \quad A_\mu, \quad A_\mu \to (\Phi_1 + \Phi_2 + \Phi_3)\,dt$$

$$(t_E, \varphi, \psi) \sim (t_E + \beta,\ \psi + i\Omega_1\beta,\ \phi + i\Omega_2\beta)$$

$$I = \underbrace{\pi i N^2}\, \frac{\varphi_1 \varphi_2 \varphi_3}{\tau^2}$$

$$Z = e^{\pi i N^2 \frac{\varphi^3}{\tau^2}} + \dots$$

$$Z_{CFT} = \mathrm{Tr}\left(e^{-\beta H - \beta \Omega_i J_i - \beta \Phi_i Q_i}\right) + \ldots \qquad + e^{-S_{AdS}}\left(1 + G_N \# + \ldots\right)$$

$$\frac{1}{G_N} = N^2$$

$$\mathrm{Tr}\left((-1)^F e^{-\beta\{Q^\dagger, Q\} + \ldots}\right) \rightarrow \sum I_r = I_1 \ldots$$

$$T^2 = S^1_\beta \times S^1_t$$

$$\frac{1}{\ } = \frac{\pi i N^2}{\ } \frac{\varphi_1 \varphi_2 \varphi_3}{\tau^2}$$

$$\Omega_1 \rightarrow \Omega_1 + \frac{2\pi i}{\beta}$$

$$\phi + i\Omega(\beta)$$

$$Z = e^{\pi i N^2 \frac{\varphi^3}{\tau^2}}\left(1 + \ldots\right) + e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \ldots$$

$$\mathrm{Tr}\left((-1)^F e^{-\beta\{Q^+,Q\}+\ldots}\right) \rightarrow \sum I_r = I_1 + I_2 + \ldots \quad r\to\infty$$

$$A_\mu \rightarrow (\Phi_1 + \Phi_2 + \Phi_3)\,dt$$

$$(t_E, \varphi, \psi) \sim (t_E + \beta,\ \psi + i\Omega_1\beta + 2\pi,\ \phi + i\Omega_2\beta)$$

$$\Omega_1 \rightarrow \Omega_1 + \frac{2\pi i}{\beta}$$

$$\tau = \frac{\beta(\Omega - 1)}{2\pi i}$$

$$\varphi_i = \frac{\beta(\Phi_i - 1)}{2\pi i}$$

$$\pi i N^2 \frac{\varphi_1\varphi_2\varphi_3}{\tau^2}$$

$$N^2 \frac{\varphi^3}{\tau^2}\left(1 + \ldots\right) + e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \ldots$$

$$\frac{1}{G_N} \sim N \qquad \mathrm{Tr}\left((-1)^F e^{-\beta\{Q^\dagger, Q\}+\dots}\right) \longrightarrow \sum I_r =$$

$$g_{\mu\nu}, \quad \dots, \quad A_\mu \to (\Phi_1+\Phi_2+\Phi_3)\,dt$$

$$(t_E, \varphi, \psi) \sim (t_E+\beta,\ \psi + i\Omega_1\beta + 2\pi,\ \phi + i\Omega_2\beta)$$

$$I = \pi i N^2 \frac{\varphi_1\varphi_2\varphi_3}{\tau^2}$$

$$\Omega_1 \to \Omega_1 + \frac{2\pi i}{\beta}$$

$$Z = e^{\pi i N^2 \frac{\varphi^3}{\tau^2}}\left(1+\dots\right) + e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \dots$$

$$Z(\Omega_i, \Phi_i, \beta) = Z\left(\Omega_i + \frac{4\pi i}{\beta}, \Phi_i, \beta\right)$$

$$\tau \approx \tau + 1$$

$$\int e^{t\{Q,V\} + \frac{1}{G_N} S_{cl}} = e^{N^2(}$$

$$e^{N^2(\dots)}(1 + O(N^0) + $$

A_μ , $\quad A_\mu \to \big(\underbrace{\Phi_1 + \phi_2 + \phi_3}_{\phi}\big)\,dt$

$$\boxed{\beta\left(1+\sum_i \Omega_i - \sum_a \Phi_a\right) = 2\pi i n}$$

$$e^{\int A}$$

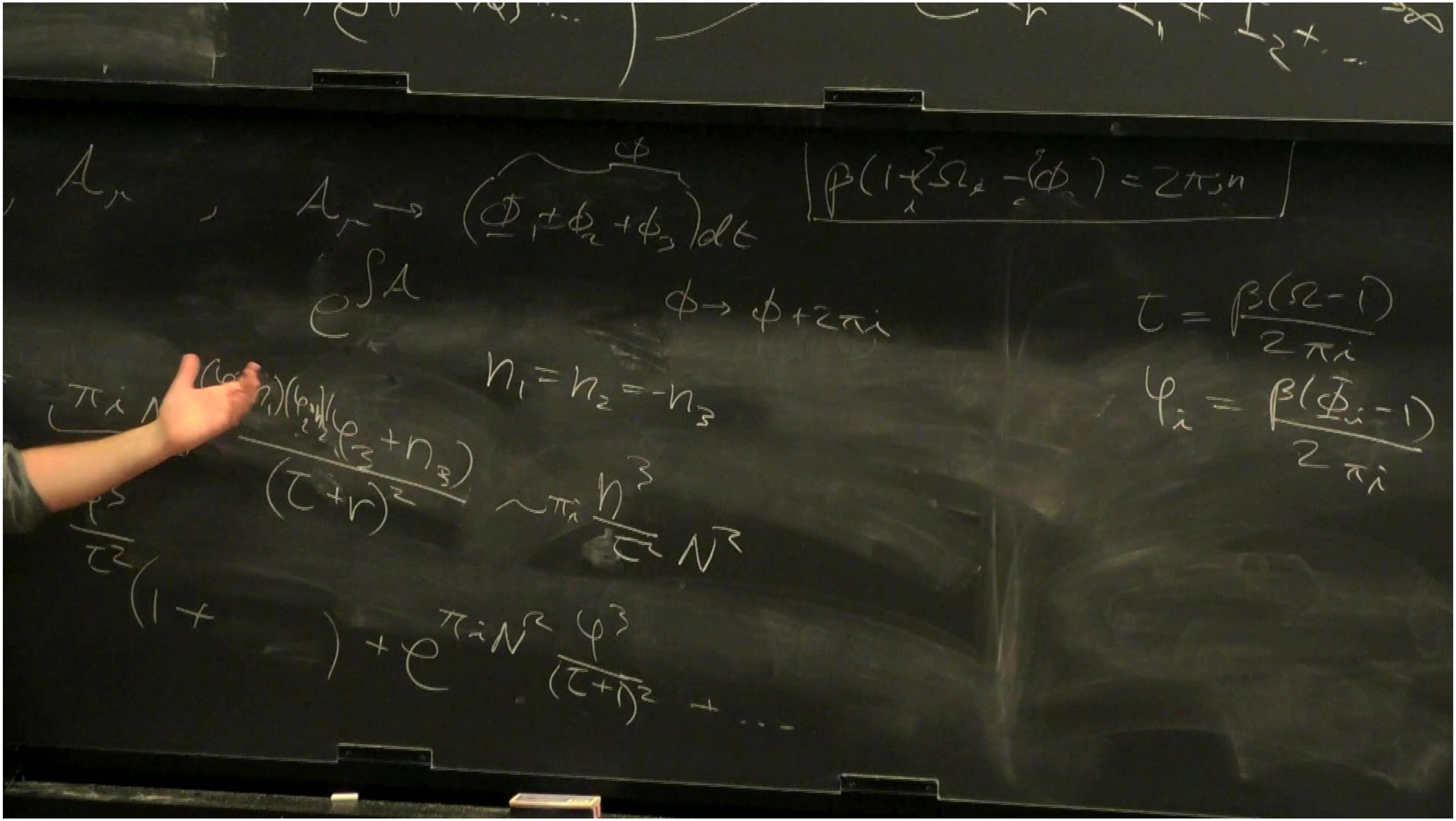
$$\phi \to \phi + 2\pi i$$

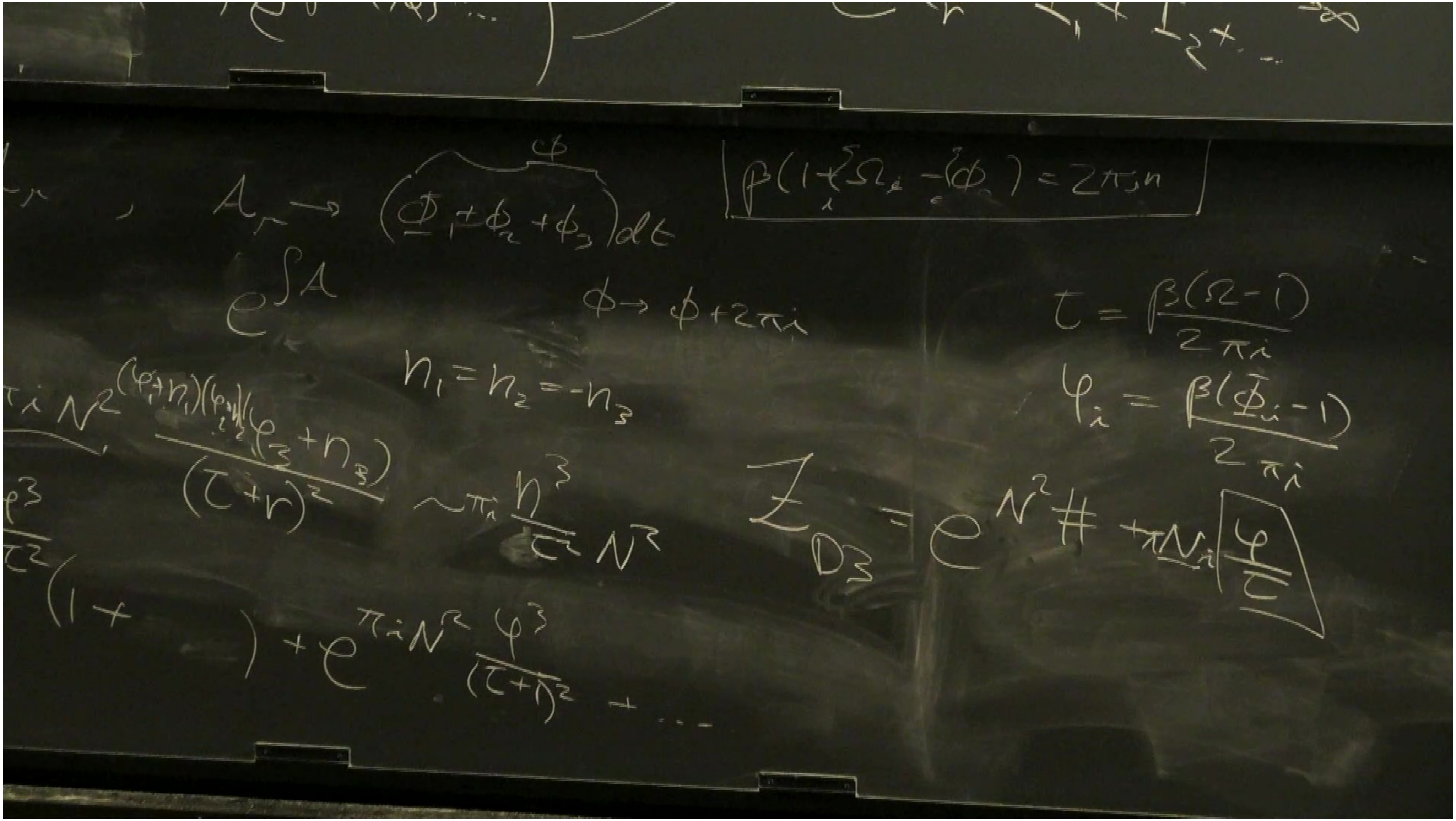
$$\tau = \frac{\beta(\Omega - 1)}{2\pi i}$$

$$\varphi_i = \frac{\beta(\Phi_i - 1)}{2\pi i}$$

$$\pi i N^2 \frac{\varphi_1 \varphi_2 \varphi_3}{\tau^2}$$

$$N^2 \frac{\varphi^3}{\tau^2}\Big(1 + \qquad \Big) + e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \dots$$


$A_\mu \to (\Phi_1+\phi_2+\phi_3)\,dt$
$\beta(1+\sum_i \Omega_i - \sum \Phi_i) = 2\pi i n$
$e^{\int A}$
$\phi \to \phi + 2\pi i$
$\tau = \frac{\beta(\Omega-1)}{2\pi i}$
$n_1 = n_2 = -n_3$
$\varphi_i = \frac{\beta(\Phi_i-1)}{2\pi i}$
$(\tau+r)^2$
$\sim \pi i \frac{n^3}{\tau^2} N^2$
$+ e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \ldots$


$A_\mu \to (\Phi_1 + \phi_2 + \phi_3)\,dt$
$\beta(1 + \sum_i \Omega_i - \sum_i \Phi_i) = 2\pi i n$
$e^{\int A}$
$\phi \to \phi + 2\pi i$
$\tau = \frac{\beta(\Omega - 1)}{2\pi i}$
$n_1 = n_2 = -n_3$
$\varphi_i = \frac{\beta(\Phi_i - 1)}{2\pi i}$
$\sim \pi i \frac{n^3}{\tau^2} N^2$
$Z_{D3} = e^{N^2 \# + \pi i N_i \boxed{\frac{\varphi}{\tau}}}$
$+ e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \ldots$

$$\boxed{\beta(1 + \sum_i \Omega_i - \sum_a \Phi_a) = 2\pi i n}$$

$= 2\pi i$

$$\tau = \frac{\beta(\Omega - 1)}{2\pi i}$$

$$\varphi_i = \frac{\beta(\Phi_i - 1)}{2\pi i}$$

$$Z_{D3} = e^{N^2 \# + \pi N_i \frac{\varphi}{\tau}}$$

$$\mathrm{Tr}(-1)^F e^{-\beta\{Q, Q^\dagger\}}$$

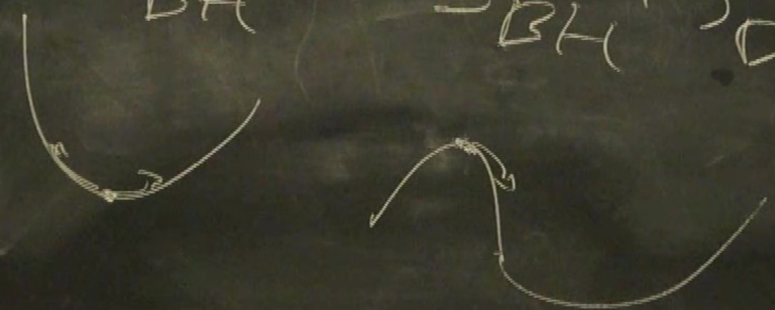
$$Z(\Omega_i, \Phi_i, \beta)$$

$$S_{BH} < S_{BH}$$

$$\mathrm{Tr}(-1)^F e^{-\beta\{Q^\dagger, Q\} - \omega(J_1 + \frac{1}{2}R_3) + \varphi_i(R_i - R}$$

$$Z(\Omega_i, \Phi_i, \beta) = Z\left(\Omega_i + \frac{4\pi i}{\beta}, \Phi_i, \beta\right)$$

$$S_{BH} < S_{BH} + S_{D3}$$



$$\mathrm{Tr}\left((-1)^F e^{-\beta\{Q^\dagger, Q\}} \cdots\right)$$

$$I = \sum_r I_r$$

$$e^{\int A}$$

$$\boxed{\Phi \to \phi + 2\pi i}$$

$$n_1 = n_2 = -n_3$$

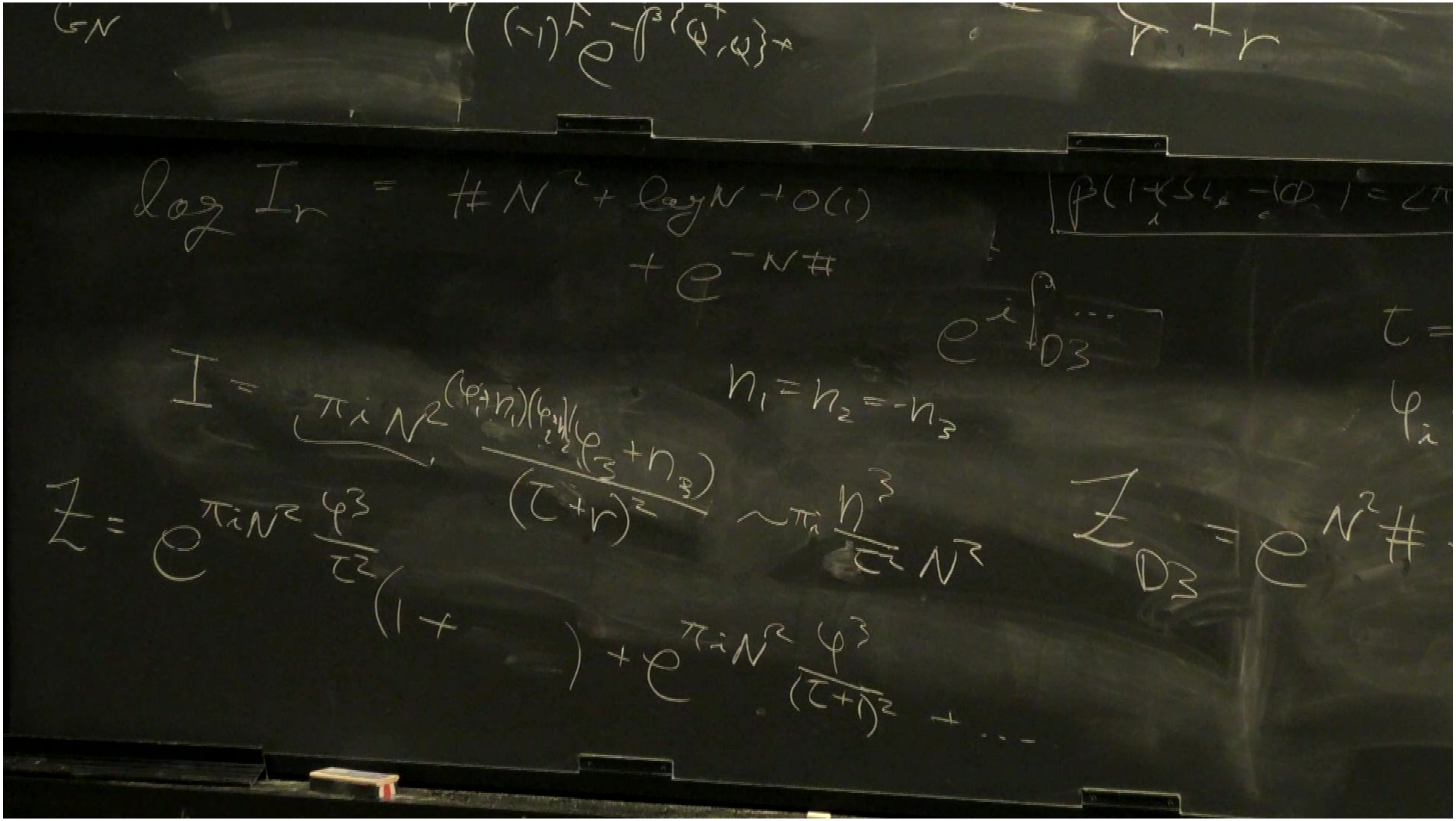
$$\tau = \frac{\beta(\Omega - 1)}{2\pi i}$$

$$\varphi_i = \frac{\beta(\Phi_i - 1)}{2\pi i}$$

$$N^2 \frac{(\varphi_1 + n_1)(\varphi_2 + n_2)(\varphi_3 + n_3)}{(\tau + r)^2} \sim \pi i \frac{n^3}{\tau^2} N^2$$

$$Z_{D3} = e^{N^2 \# + \pi i N \boxed{\frac{\varphi}{\tau}}}$$

$$(1 + \cdots) + e^{\pi i N^2 \frac{\varphi^3}{(\tau+1)^2}} + \cdots$$


log I_r = # N^2 + log N + O(1)
+ e^{-N #}
e^{i ∫_{D3} ...}
n_1 = n_2 = -n_3
I = πi N^2 (φ_1+n_1)(φ_2+n_2)(φ_3+n_3) / (τ+r)^2
~ πi n^3 / τ^2 N^2
Z = e^{πi N^2 φ^3 / τ^2} (1 +) + e^{πi N^2 φ^3 / (τ+1)^2} + ...
Z_{D3} = e^{N^2 #}