Title: Statistical Physics - Lecture 221125

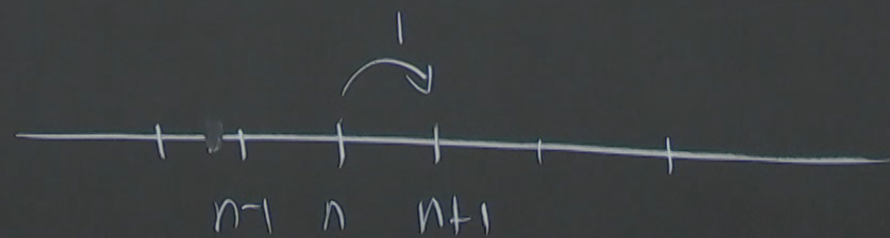
Speakers:

Collection: Statistical Physics (2022/2023)

Date: November 25, 2022 - 9:00 AM

URL: https://pirsa.org/22110017

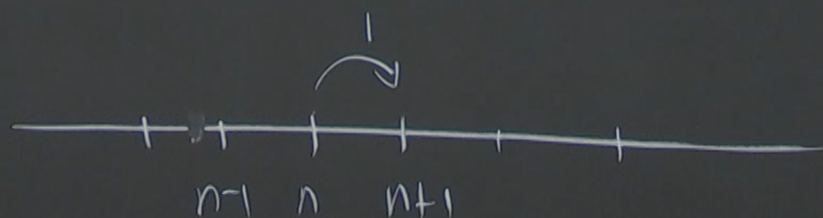
Random Walk on the Line



$$\sum_{n=-\infty}^{\infty}\left[\dot{P}_n(t) = P_{n-1} - 2P_n + P_{n+1}\right] e^{ikn}$$

$$P_k(t) = \sum_{-\infty}^{\infty} P_n(t) e^{ikn}$$

Random Walk on the Line



$$\sum_{n=-\infty}^{\infty}\left[\dot{P}_n(t) = P_{n-1} - 2P_n + P_{n+1}\right] e^{ikn}$$

$$P_k(t) = \sum_{-\infty}^{\infty} P_n(t)\, e^{ikn} \quad ; \quad P_k(t=0) = \cancel{\sum_{-\infty}^{\infty}} \, 1$$

$$\dot{P}_k(t) = \big(\underbrace{e^{ik} - 2 + e^{-ik}}_{2(\cos}$$

assume IC $\quad P_n(t=0) = \delta_{n,0}$

$$P_k(t) = P_k(t=0)\, e^{2(\cos k - 1}$$

$$\boxed{P_k(t) = e^{2(\cos k - 1)t}}$$

$$P_n(t) = I_n(2t)\, e^{-2t}$$

$$\dot{P}_k(t) = (\underbrace{e^{ik} - 2 + e^{-ik}}_{2(\cos k - 1)}) P_k(t) \xrightarrow{k \to 0} \quad P_k(t) \sim e^{-k^2 t}$$

assume IC $\quad P_n(t=0) = \delta_{n,0}$

$$P_k(t) = P_k(t=0)\, e^{2(\cos k - 1)t}$$

$$\boxed{P_k(t) = e^{2(\cos k - 1)t}}$$

$$P_n(t) = I_n(2t)\, e^{-2t}$$

Soln by Laplace transform

$$\tilde{P}_n(s) \equiv \int_0^\infty e^{-st} P_n(t)\, dt$$

$$s\tilde{P}_n(s) - P_n(t=0) = \tilde{P}_{n-1}(s) - 2\tilde{P}_n(s) + \tilde{P}_{n+1}(s)$$

$n \neq 0$ $\quad (s+2)P_n = P_{n-1} + P_{n+1}$

$a = \frac{1}{s+2}$ $\quad P_n = a\left(P_{n-1} + P_{n+1}\right)$

$n = 0$ $\quad (s+2)P_0 - 1 = 2P_1$

try $P_n = A$

$\Rightarrow \lambda =$

$\lambda^2 - \frac{1}{a}\lambda + 1 =$

$\lambda_\pm = \frac{1}{2a} \pm \sqrt{\left(\frac{1}{2a}\right.}$

$\cancel{\lambda_+} > 1 ; \; 0 < \lambda_-$

reject

$\tilde{P}_{n+1}(s)$

try $P_n = A\lambda^n$

$\Rightarrow \lambda = a(1+\lambda^2)$

$\lambda^2 - \frac{1}{a}\lambda + 1 = 0$

$\lambda_{\pm} = \frac{1}{2a} \pm \sqrt{\left(\frac{1}{2a}\right)^2 - 1}$

$\cancel{\lambda_+ > 1}$; $0 < \lambda_- < 1$

reject

$P_n(s) = A\,\lambda_-^{|n|}$

determine A: $(s+2)A - 1 = 2A\lambda_-$

$A = \frac{1}{(s+2-\lambda_-)}$

$P_n(s) = \frac{1}{(s+2-\lambda_-)}\lambda_-^n$

$\underline{s \to 0}$ $(t \to \infty)$

$$\frac{1}{2a} = \frac{s+2}{2} = 1 + s/2$$

$$= 2A\lambda_-$$

$$\lambda_- \cong 1 + s/2 - \sqrt{\cancel{1} + s - \cancel{1}}$$

$$\sim 1 - \sqrt{s}$$

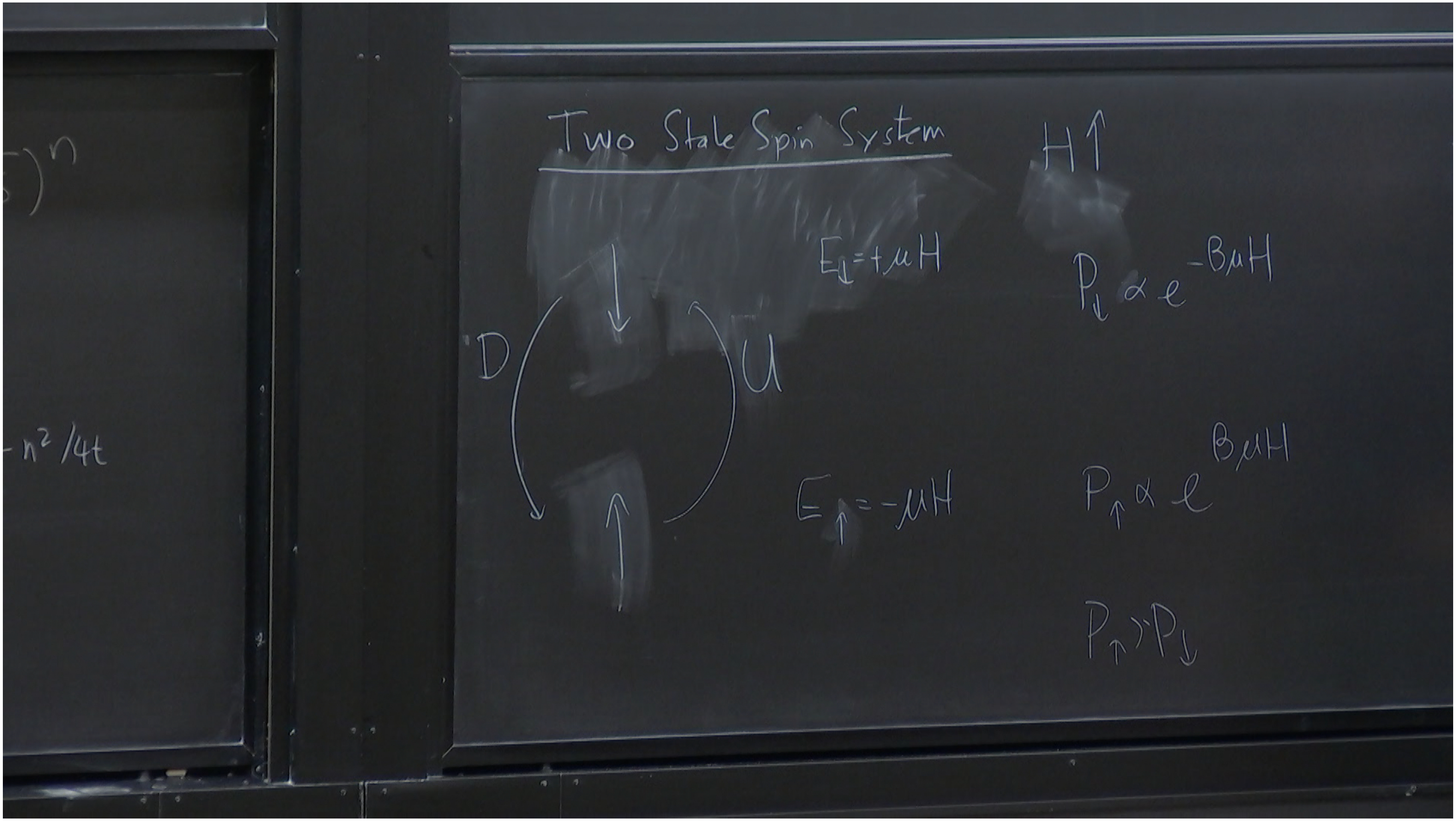
$$(\lambda_-)$$

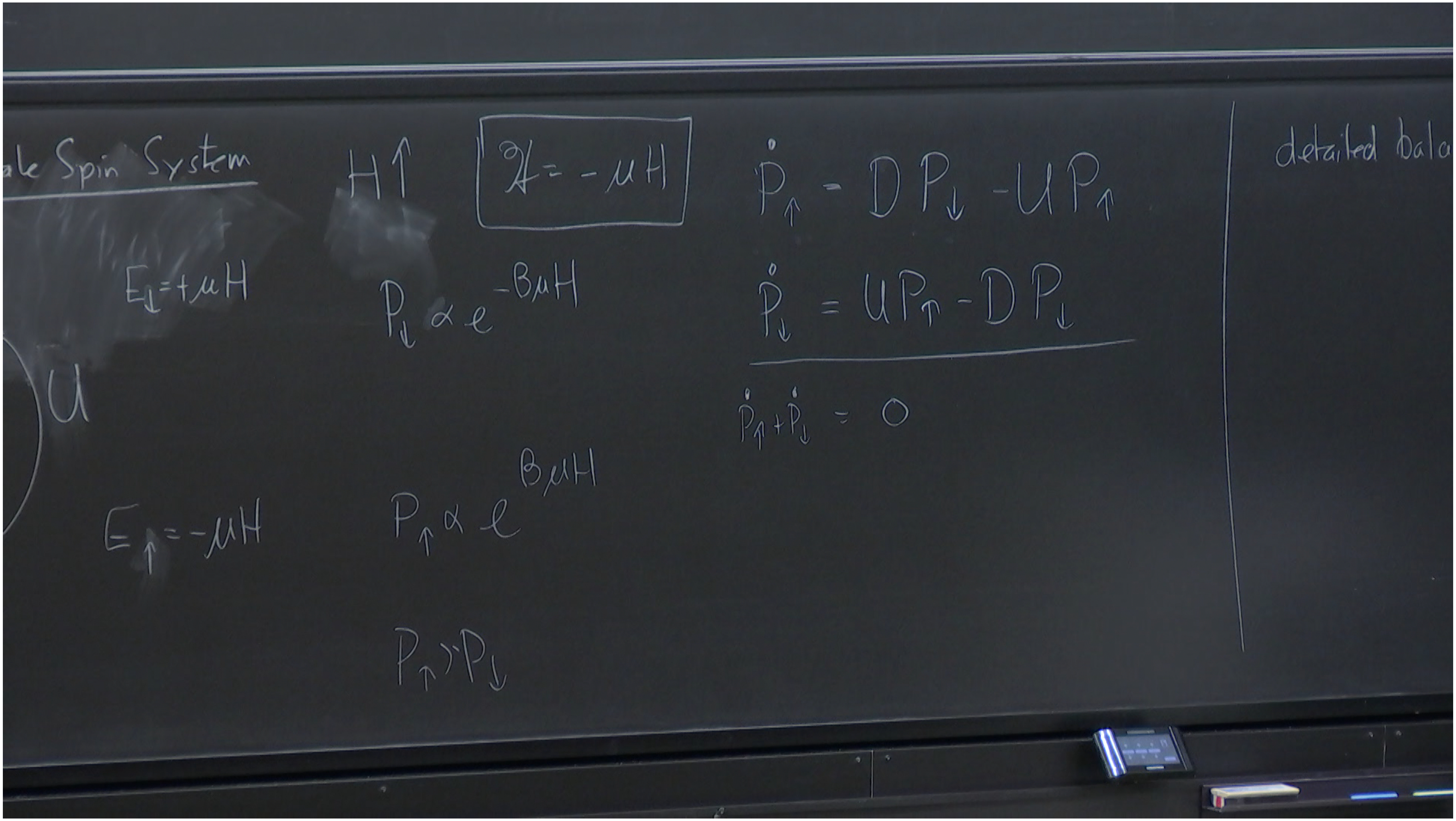
$$A = \frac{1}{s + \cancel{2} - \cancel{2}(1-\sqrt{s})} = \frac{1}{2\sqrt{s}}$$

$$P_n(s) \sim \frac{1}{2\sqrt{s}} (1 - \sqrt{s})^n$$

$$\sim \frac{1}{2\sqrt{s}} e^{-n\sqrt{s}}$$

$$P_n(t) = \frac{1}{\sqrt{4\pi t}} e^{-n^2/4t}$$


Two State Spin System
$H \uparrow$
$E_{\downarrow} = +\mu H$
$P_{\downarrow} \propto e^{-\beta\mu H}$
D
U
$E_{\uparrow} = -\mu H$
$P_{\uparrow} \propto e^{\beta\mu H}$
$P_{\uparrow} \gg P_{\downarrow}$
$)^n$
$-n^2/4t$


ale Spin System
$H \uparrow$
$\boxed{\mathcal{H} = -\mu H}$
$\dot{P}_{\uparrow} = D P_{\downarrow} - U P_{\uparrow}$
$\dot{P}_{\downarrow} = U P_{\uparrow} - D P_{\downarrow}$
$\dot{P}_{\uparrow} + \dot{P}_{\downarrow} = 0$
detailed bala
$E_{\downarrow} = +\mu H$
$P_{\downarrow} \propto e^{-\beta \mu H}$
U
$E_{\uparrow} = -\mu H$
$P_{\uparrow} \propto e^{\beta \mu H}$
$P_{\uparrow} >> P_{\downarrow}$

$-UP_\uparrow$

$-DP_\downarrow$

detailed balance $DP_\downarrow = UP_\uparrow$

$$\frac{D}{U} = \frac{P_\uparrow}{P_\downarrow} = e^{2\beta\mu H}$$

$$S = P_{\uparrow} + P_{\downarrow} = 1 \qquad \dot{S} = 0$$

$$P_{\uparrow} = (S+\Delta)/2 = (1+\Delta)/2$$

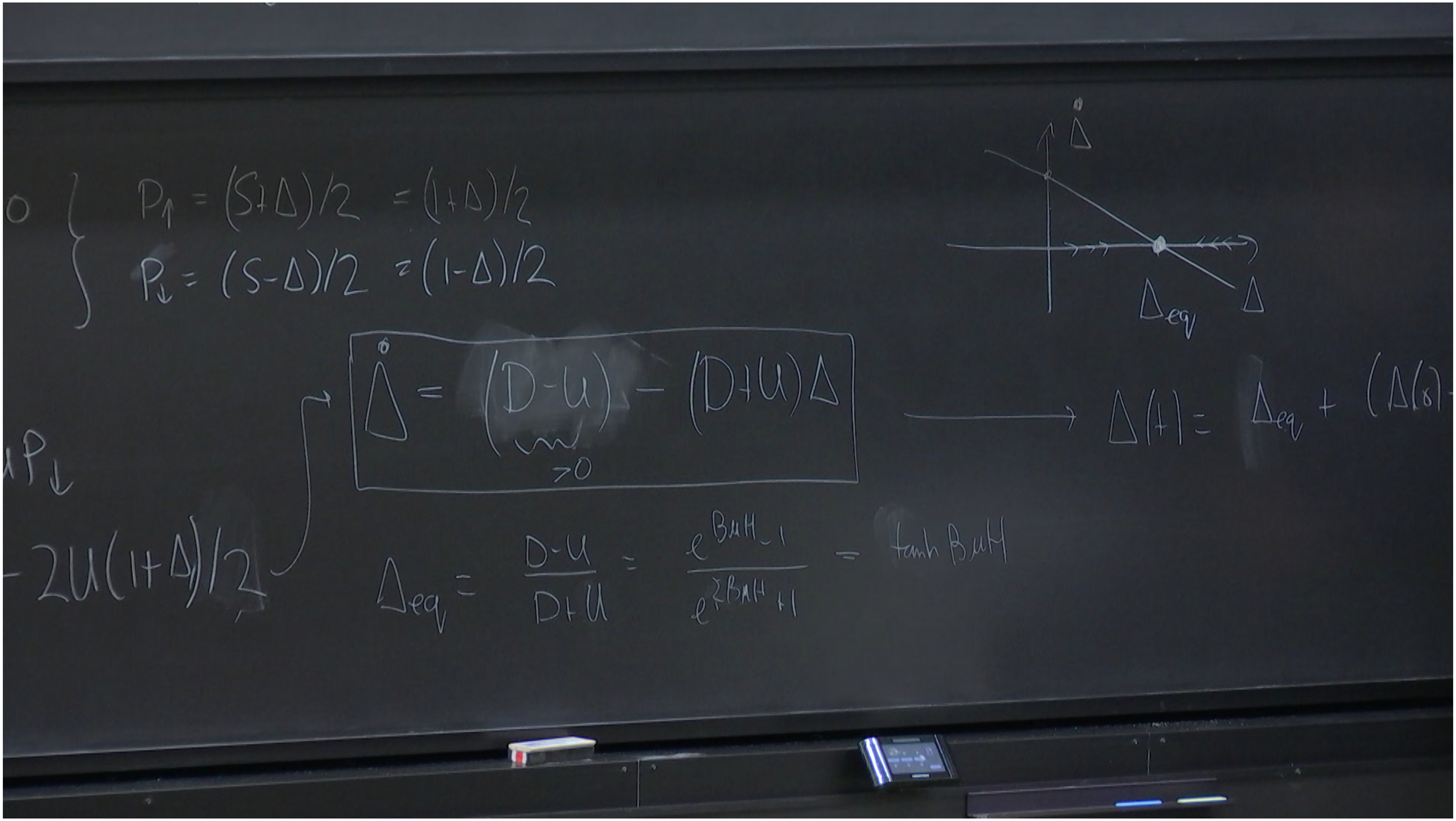
$$P_{\downarrow} = (S-\Delta)/2 = (1-\Delta)/2$$

$$\Delta = P_{\uparrow} - P_{\downarrow}$$

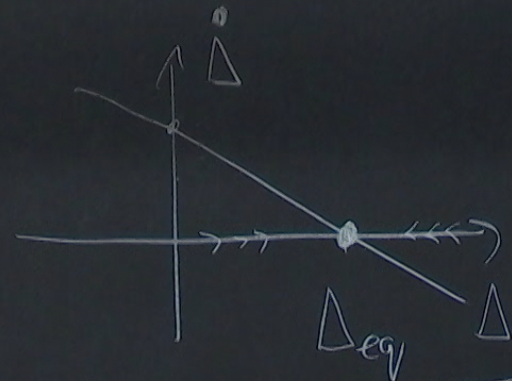
$$\dot{\Delta} = 2DP_{\downarrow} - 2UP_{\downarrow}$$

$$= 2D(1-\Delta)/2 - 2U(1+\Delta)/2$$

$$\boxed{\dot{\Delta} = (D-U) - (D+U)\Delta}$$


$P_\uparrow = (S+\Delta)/2 = (1+\Delta)/2$
$P_\downarrow = (S-\Delta)/2 = (1-\Delta)/2$
$\mu P_\downarrow$
$-2U(1+\Delta)/2$
$\boxed{\dot{\Delta} = \underbrace{(D-U)}_{>0} - (D+U)\Delta}$
$\dot{\Delta}$
Δ_{eq}
Δ
$\Delta(t) = \Delta_{eq} + (\Delta(0)$
$\Delta_{eq} = \frac{D-U}{D+U} = \frac{e^{\beta\mu H}-1}{e^{2\beta\mu H}+1} = \tanh \beta\mu H$

$$\boxed{\underbrace{\ldots - u)}_{>0} - (D+u)\Delta} \longrightarrow \boxed{\Delta(t) = \Delta_{eq} + \left(\Delta(0) - \Delta_{eq}\right) e^{-(D+u)t}}$$



$$\frac{D-u}{D+u} = \frac{e^{2\beta\mu H} - 1}{e^{2\beta\mu H} + 1} = \tanh \beta\mu H$$

Spin Dynamics in the Ising Model

assumption: single spin flip dynamics

$\{s\}' = \{s\}$ with one spin flipped

Master Eqn $$\frac{dP(\{s\},t)}{dt} = \sum_{\{s\}'} W(\{s\}' \to \{s\}) P(\{s\}', t) - \ldots$$

Detailed balance: $$\frac{W(s' \to s)}{W(s \to s')} = \frac{P(s)}{P(s')}$$

$\mathcal{H} = - \ldots$

el

ics

$\{s\}' = \{s\}$ with one spin flipped

$$\sum_{\{s\}'} W(\{s\}' \to \{s\}) P(\{s\}', t) - W(\{s\} \to \{s\}') P(\{s\}, t) = 0$$

in equilibrium

$$\frac{\quad)}{\quad')} = \frac{P(s)}{P(s')}$$

$$\mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j$$

$$\frac{W(s \to s')}{W(s' \to s)}$$

$$\frac{W(s\to s')}{W(s'\to s)} = \frac{P(s')}{P(s)} = \frac{e^{-\beta J \sum s_i s_j}}{e^{+\beta J \sum s_i s_j}} = \frac{1 - s_i \tanh(J\beta \sum s_j)}{1 + s_i \tanh(J\beta \sum_j s_j)}$$

trick for binary variables $s_i = \pm 1$

$$e^{A s_i} = \cosh A s_i + \sinh A s_i$$

$$= \cosh A + s_i \sinh A$$

$$= \cosh A\,(1 + s_i \tanh A)$$

$$\boxed{\Rightarrow W(s_i \to s') = \frac{1}{2}\left[1 - s_i \tanh\left(J\beta \sum s_j\right)\right]}$$

$\sum s_j$ is aligned with s_i $\quad W \to 0$

$\sum s_j$ is antialigned $\quad W \to 1$

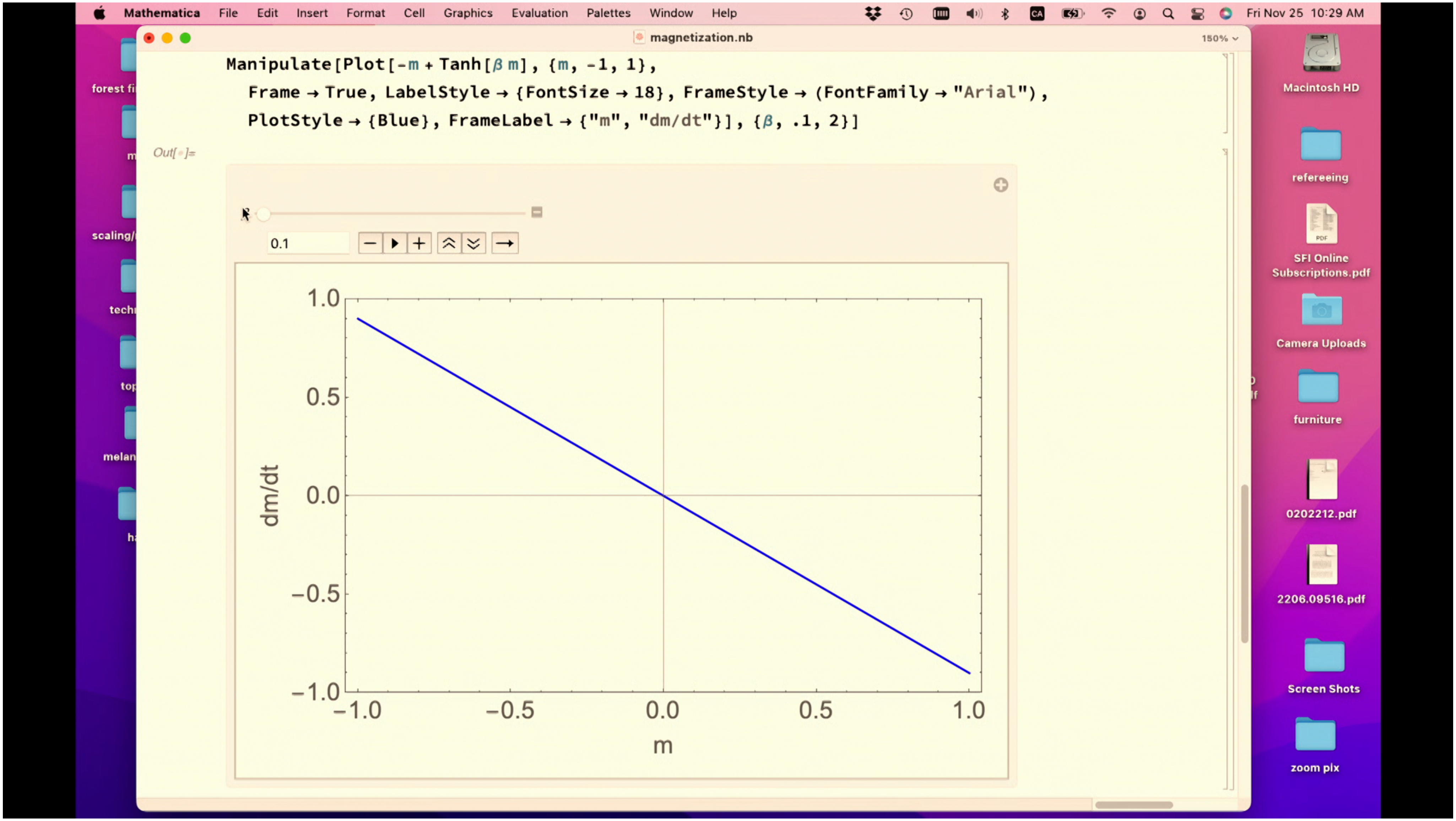
Complete Graph

$$\mathcal{H} = -\frac{1}{N}\sum_{\langle ij\rangle} s_i s_j$$

$$W_i = \frac{1}{2}\Big(1 - s_i \tanh \frac{\beta}{N}\underbrace{\sum_j s_j}_{\approx M}\Big)$$

$$\simeq \frac{1}{2}\big(1 - s_i \tanh \beta m\big)$$

$$\dot{s}_i = -2 s_i W_i$$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0.0
−0.5
−1.0
dm/dt
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−0.5
0.0
0.5
1.0
m
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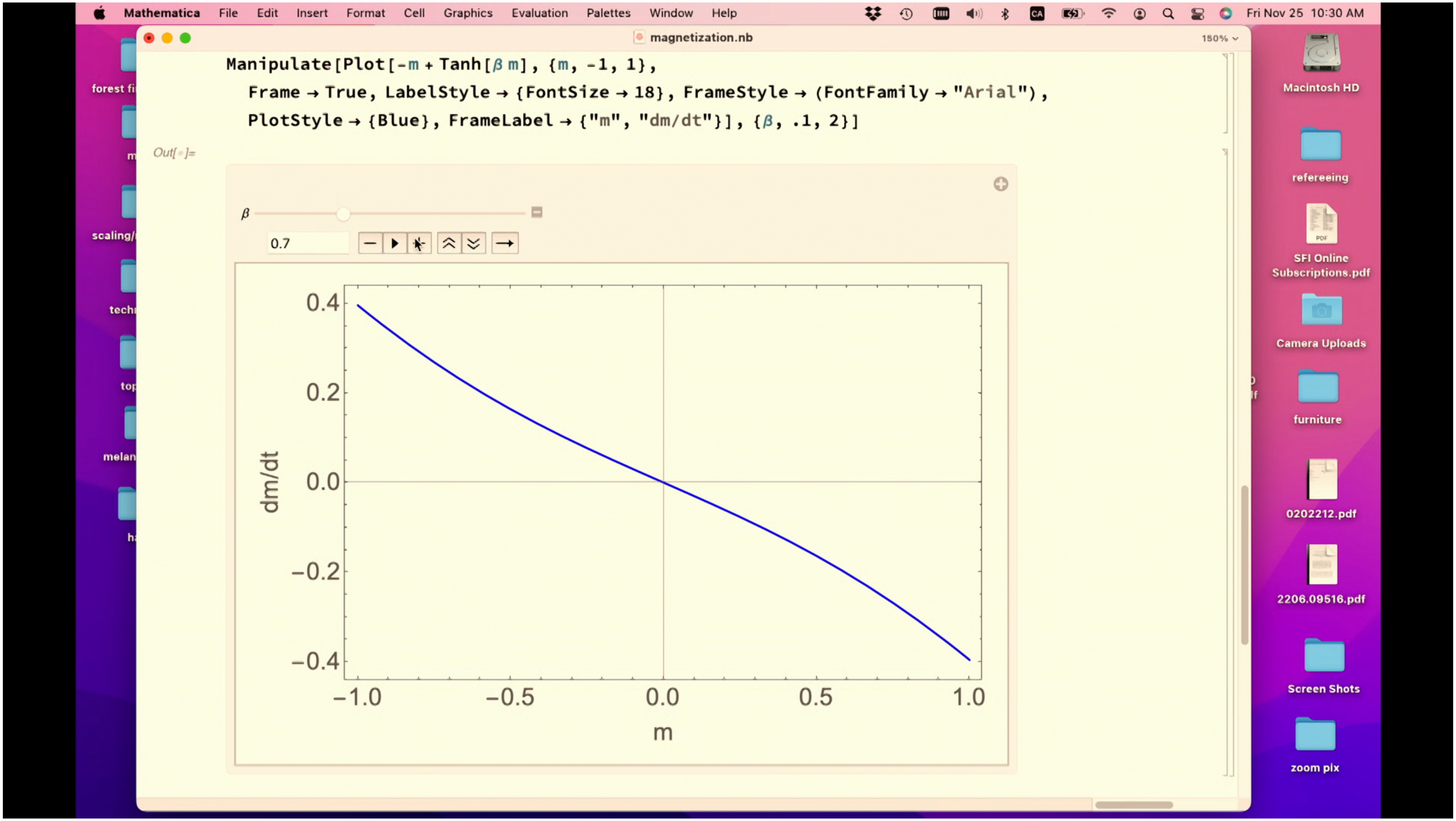
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dm/dt
0.4
0.2
0.0
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−0.4
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m
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Screen Shots
zoom plx

$$\sum_i \left(\dot{S}_i = -S_i + \tanh(\beta m)\right)/N$$

$$\boxed{\dot{m} = -m + \tanh \beta m}$$

$$m = \tanh(\beta J) m$$