

Title: Statistical Physics - Lecture 221123

Speakers:

Collection: Statistical Physics (2022/2023)

Date: November 23, 2022 - 9:00 AM

URL: <https://pirsa.org/22110016>

Renormalization Group

Percolation

$p < p_c$ finite clusters

$p > p_c$ ∞ clusters, complementary
finite clusters

$p = p_c$ scale invariance
clusters of all sizes
power law size distribution
 s^{-2}

Spin system

$T > T_c$ small islands of order in
sea of disorder

$T < T_c$ small islands of $\downarrow$ in a sea of $\uparrow$

$T = T_c$ scale invariance
clusters of all sizes
power law size distribution

Spatial Rescaling

Percolation

start at $p < p_c \rightarrow p = 0$

$p > p_c \rightarrow p = 1$

$p = p_c \rightarrow \text{invariant}$

Spin System

start at $T > T_c \rightarrow T = \infty$

$T < T_c \rightarrow T = 0$

$T = T_c \rightarrow \text{invariant}$

Percolation

start at $p < p_c \rightarrow p = 0$

$p > p_c \rightarrow p = 1$

$p = p_c \rightarrow$ invariant

$$\xi(p) \sim (p - p_c)^{-\nu}$$

$$P(p) \sim (p - p_c)^{\beta}$$

$$\langle S^2 \rangle \sim (p - p_c)^{-\gamma}$$

Spin System

start at $T > T_c \rightarrow T = \infty$

$T < T_c \rightarrow T = 0$

$T = T_c \rightarrow$ invariant

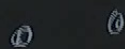
$$\xi(T) \sim (T - T_c)^{-\nu}$$

$$M(T) \sim (T_c - T)^{\beta}$$

$$\chi(T) \sim (T - T_c)^{-\gamma}$$

$$\langle S \rangle = \sum_s S N_s = \text{const. in percolation}$$

$$p(1-p)^2$$



Rescale Factor b

$$\Sigma(\delta p') = \Sigma(\delta p)/b$$

$$\Sigma \sim (\delta p)^{-2} \quad \Sigma' \sim (\delta p')^{-2}$$

$$\left(\frac{dp'}{dp}\right)^{+2} = \frac{\Sigma'}{\Sigma} = b$$

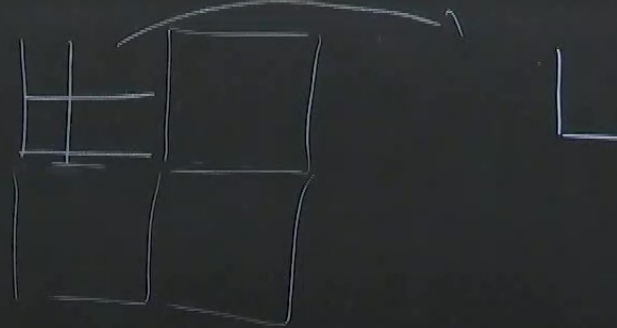
$$\frac{dp'}{dp} = b^{+1/2}$$

RG relation to
calculate critical exponent

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Percolation

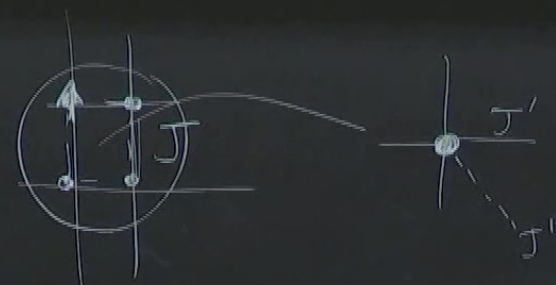


Percolation

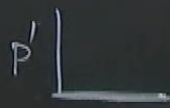
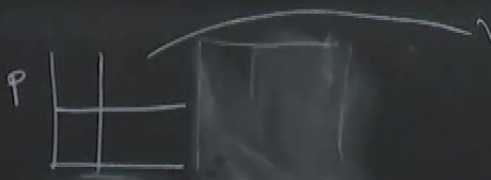


percolating configs

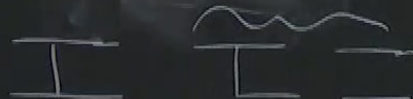
Spin System



Percolation

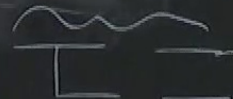


percolating configs



5 bonds

1



4 bonds

5



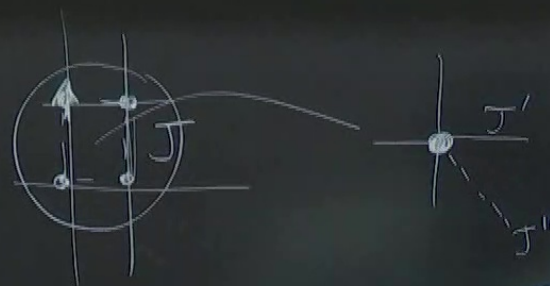
3 bonds

8 configs



2 configs

Spin System



$$p' = p^5 + 2$$

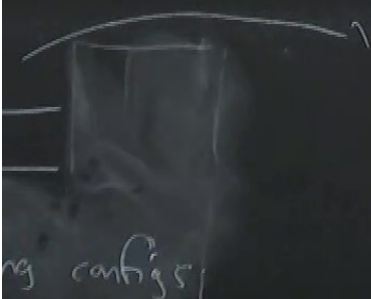
$$2p^2(1-p)^3$$

$$S^2 \sim (p-p_c)^{-\alpha}$$

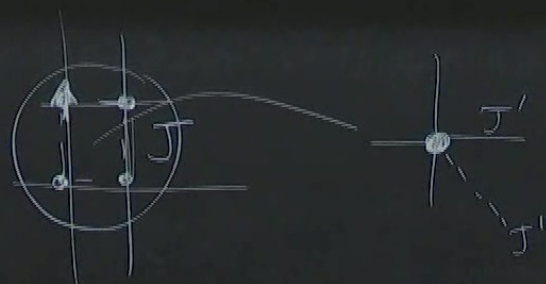
$$\chi(T) \sim (T-T_c)^0$$

ation

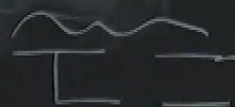
Spin System



p'



ing configs



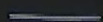
4 bonds

5



3 bonds

8 configs



2 configs

$$p' = p^5 + 5p^4(1-p) + 8p^3(1-p)^2 + 2p^2(1-p)^3$$

fixed pt at $p' = 1/2$!

$$\left. \frac{dp'}{dp} \right|_{p^* = 1/2} = 13/8$$

$$\nu = \frac{\ln b}{\ln \left. \frac{dp'}{dp} \right|_{p^*}} \approx 1.43$$

Rescale Factor b

$$\xi(\delta p') = \xi(\delta p) / b$$

$$\xi \sim (\delta p)^{-\nu} \quad \xi' \sim (\delta p')^{-\nu}$$

$$\left(\frac{d\xi}{d\delta p} \right)^{-\nu} = \frac{d\xi'}{d\delta p'} = b$$

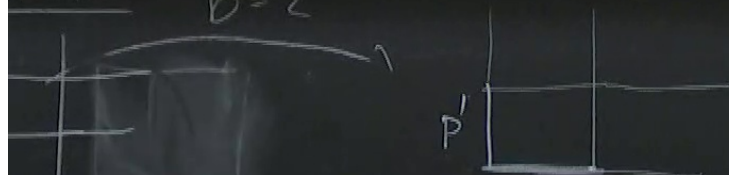
$$\frac{d\delta p'}{d\delta p} = b^{+1/\nu}$$

RG relation to
Calculate critical exponent

$$\langle S^2 \rangle \sim (p - p_c)^{-\alpha}$$

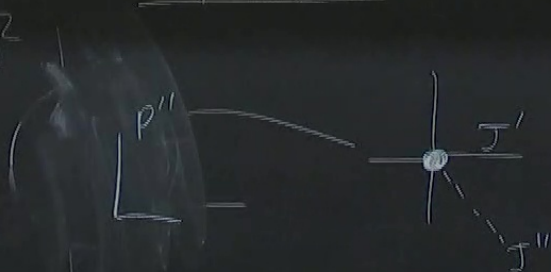
$$\chi(T) \sim (T - T_c)^{-\beta}$$

lotion $b=2$



$b=2$

Spin System



ing configs



4 bonds

5



3 bonds

8 configs



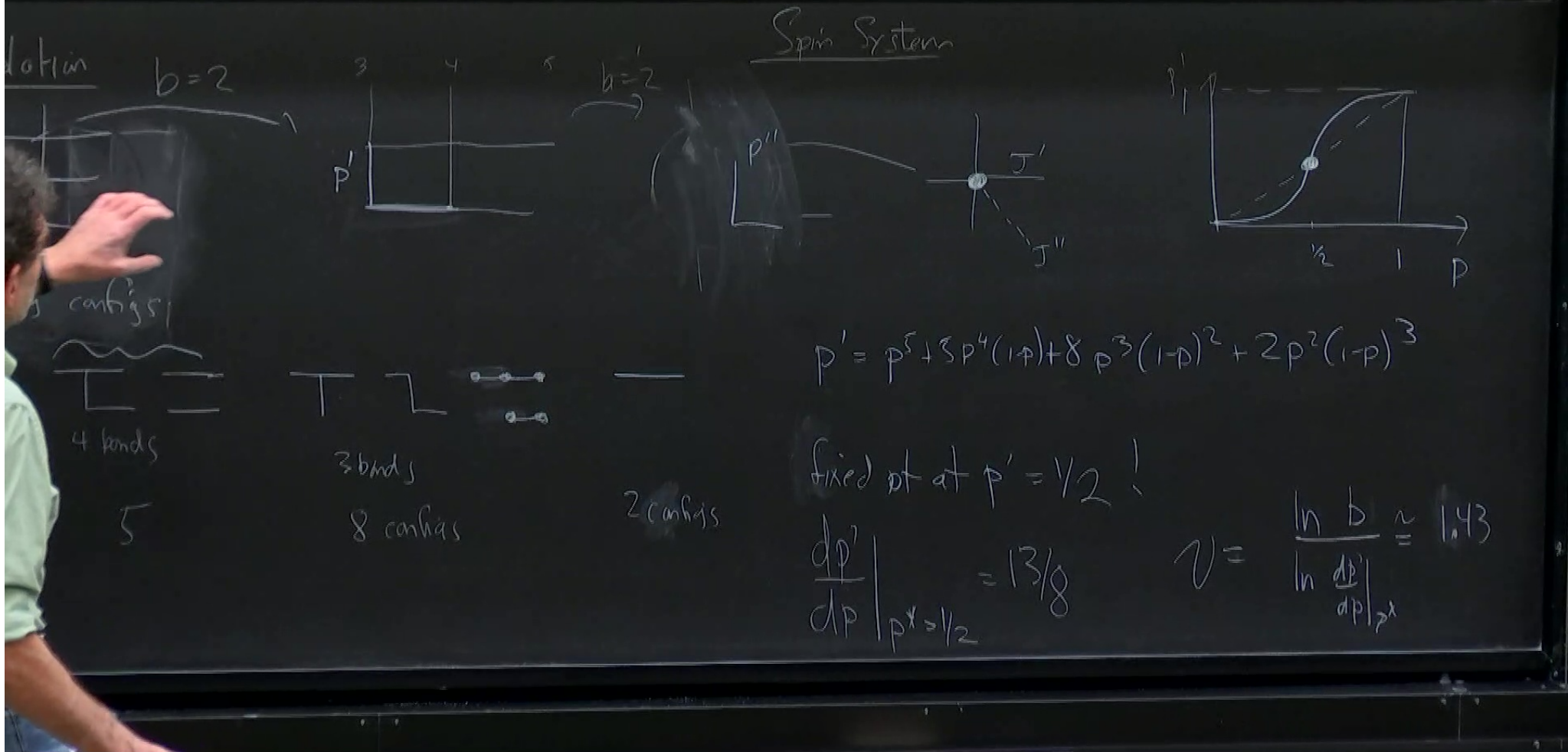
2 configs

$$p' = p^5 + 3p^4(1-p) + 8p^3(1-p)^2 + 2p^2(1-p)^3$$

fixed pt at $p' = 1/2$!

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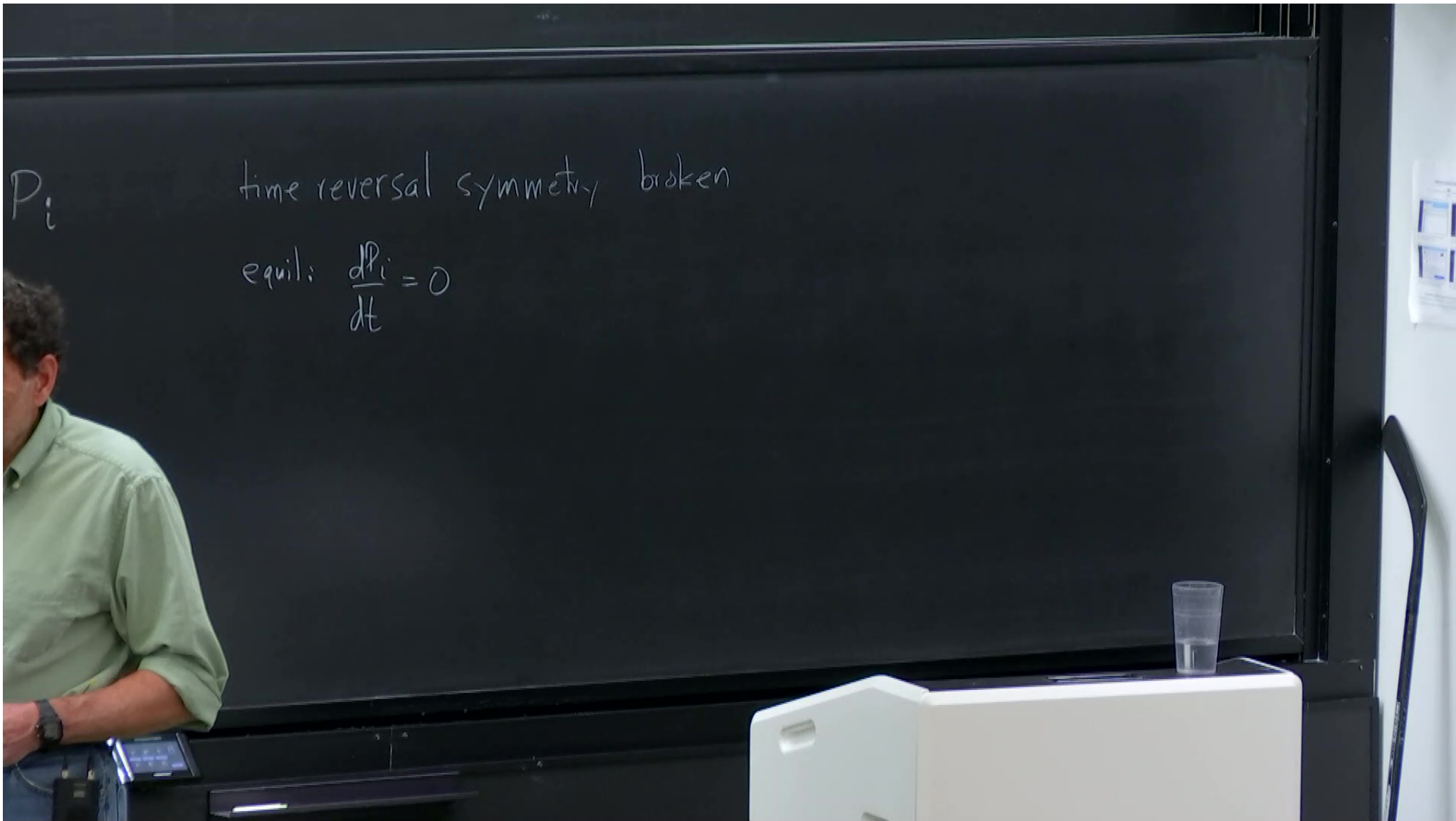


Master Eqn

$$\frac{dP_i}{dt} = \sum_j W_{j \rightarrow i} P_j - \sum_j W_{i \rightarrow j} P_i$$

inflow outflow

i, P_i
 j
microstate



P_i

time reversal symmetry broken

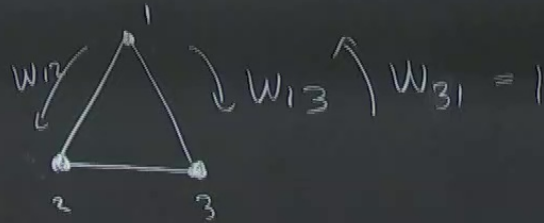
equil: $\frac{dP_i}{dt} = 0 \rightarrow$ total inflow = total outflow at each site

$$\sum_j W_{j \rightarrow i} P_j = \sum_j W_{i \rightarrow j} P_i$$

detailed
balance

$$W_{j \rightarrow i} P_j = W_{i \rightarrow j} P_i$$

Example



$$\tilde{P}_i(s) = \int_0^{\infty} P_i(t) e^{-st} dt$$

$$\int_0^{\infty} \begin{bmatrix} \dot{P}_1 = P_2 + P_3 - 2P_1 \\ \dot{P}_2 = P_3 + P_1 - 2P_2 \\ \dot{P}_3 = P_1 + P_2 - 2P_3 \end{bmatrix} e^{-st} dt$$

$$\text{I.C.: } P_i(t=0) = \delta_{i,j_0}$$

$$\int_0^{\infty} \dot{P}_2 e^{-st} dt$$

$31 = 1$

$$\tilde{P}_i(s) = \int_0^{\infty} P_i(t) e^{-st} dt$$

C.: P_i

$$\begin{aligned} \int_0^{\infty} \dot{P}_2 e^{-st} dt &= P_2 \left(e^{-st} \right) \Big|_0^{\infty} + \int_0^{\infty} P_2 s e^{-st} dt \\ &= 0 - P_2(t=0) + s P_2(s) \end{aligned}$$

$$31 = 1$$

$$\tilde{P}_i(s) = \int_0^{\infty} P_i(t) e^{-st} dt$$

$$C.: P_i(t=0) = \delta_{i,1}$$

$$\int_0^{\infty} \dot{P}_2 e^{-st} dt = P_2 \left(e^{-st} \right) \Big|_0^{\infty} + \int_0^{\infty} P_2 s e^{-st} dt$$

$$= 0 - P_2(t=0) + s P_2(s)$$

$$\rightarrow s P_1 - 1 = P_2 + P_3 - 2 P_1$$

$$s P_2 = P_3 + P_1 - 2 P_2$$

$$s P_3 = P_1 + P_2 - 2 P_3$$

$$31 = 1$$

$$C.: P_i(t=0) = \delta_{i,1}$$

$$\rightarrow \begin{cases} sP_1 - 1 = P_2 + P_3 - 2P_1 \\ sP_2 = P_3 + P_1 - 2P_2 \\ sP_3 = P_1 + P_2 - 2P_3 \end{cases}$$

$$P_2 = P_3 = \frac{1}{3s(s+3)}$$

$$P_1 =$$

$$P_2 = P_3 = \frac{1}{3s(s+3)} = \frac{1}{3s} - \frac{1}{(s+3)3} \rightarrow P_2(t) = P_3(t) = \frac{1}{3}(1 - e^{-3t})$$

$$P_1 = \frac{s+1}{3s(s+3)} = \frac{1}{3} - \frac{1}{3s} + \frac{1}{3(s+3)}$$

$$P_1(t) = \frac{2}{3} + \frac{1}{3}e^{-3t}$$

$$\dot{P} = W P \quad \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

$$W = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$\text{equilibrium } \dot{P} = 0 \rightarrow WP = 0$$

equilibrium

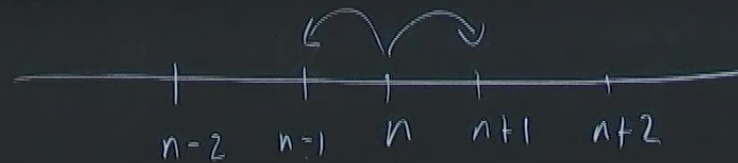
equilibrium $\dot{P}=0 \rightarrow WP=0$

equilibrium corresponds to eigenvector
with eigenvalue 0

if W_{ij} all same, equiprobable state is equilibrium

$$\det W = 0$$

Example 2



$$\int_0^{\infty} dt \left\{ \begin{array}{l} \dot{P}_n = P_{n-1} - 2P_n + P_{n+1} \\ P_n(t=0) = \delta_{n,0} \end{array} \right. e^{-st}$$