Title: Statistical Physics - Lecture 221116

Speakers:

Collection: Statistical Physics (2022/2023)

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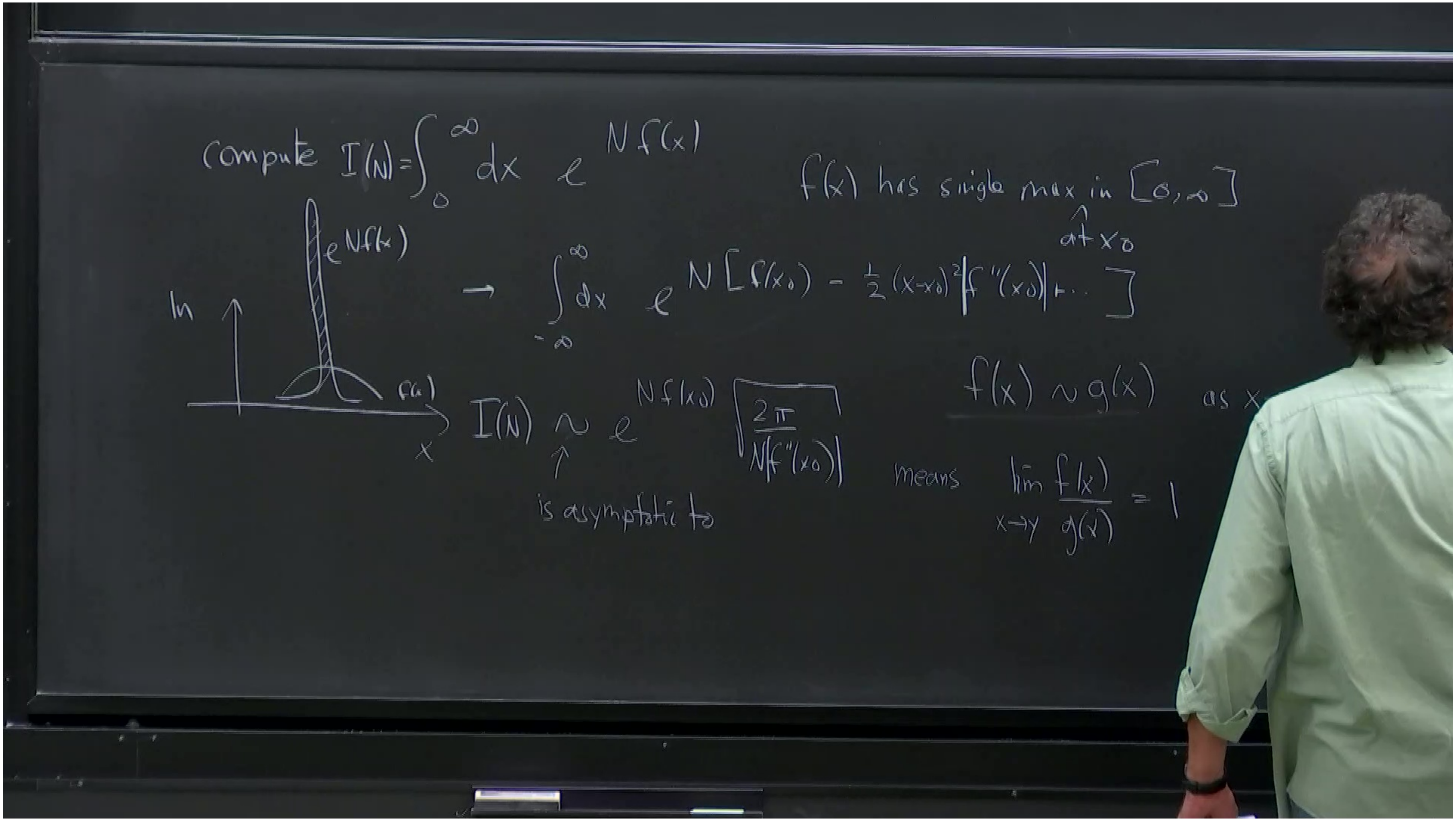
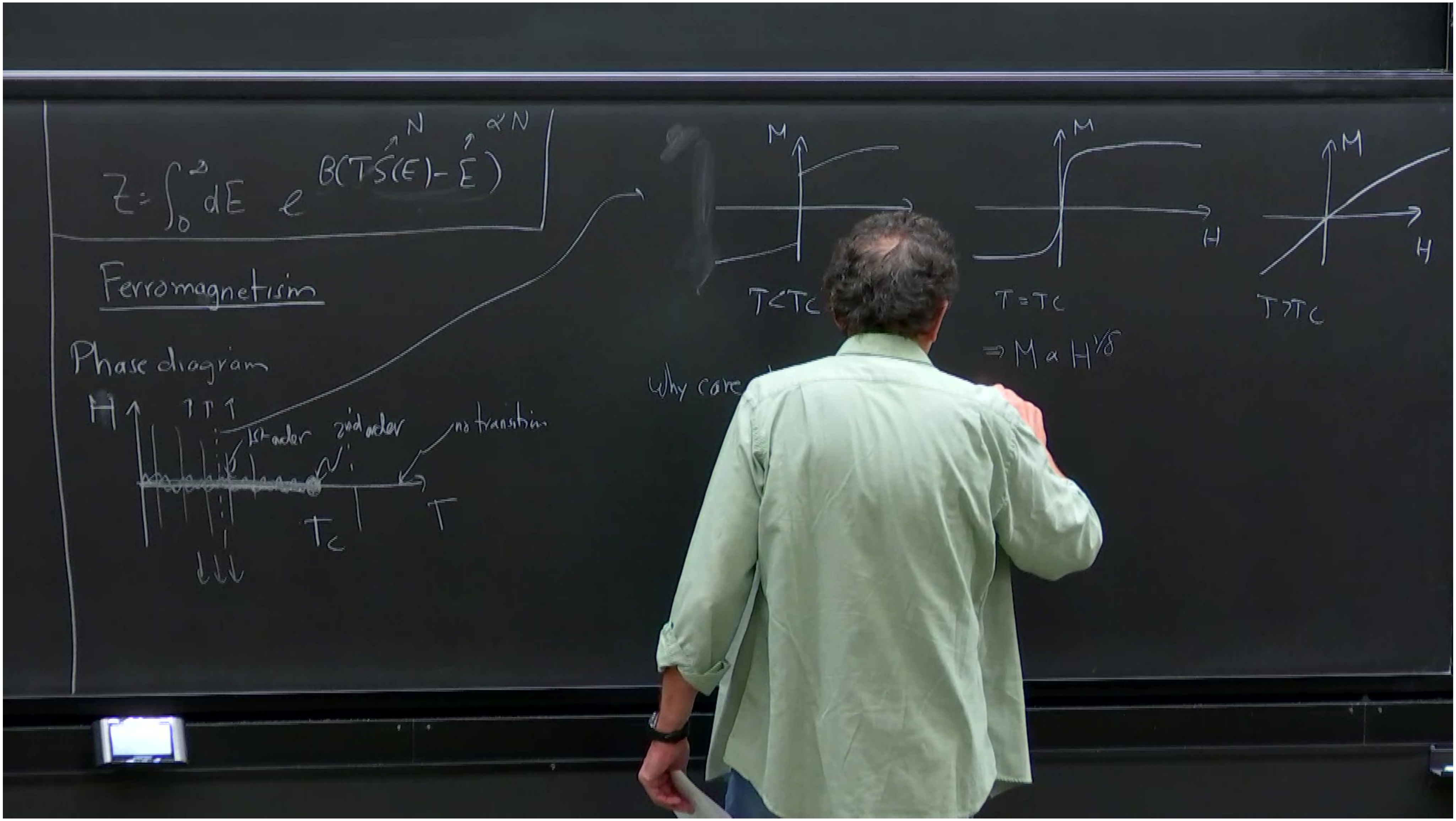


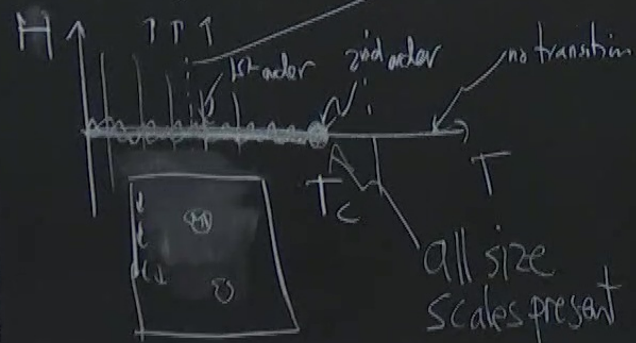
Compute $I(N) = \int_0^\infty dx \; e^{Nf(x)}$
$f(x)$ has single max in $[0, \infty]$
at x_0
$e^{Nf(x)}$
ln
$f(x)$
x
$\to \int_{-\infty}^{\infty} dx \; e^{N\left[f(x_0) - \frac{1}{2}(x-x_0)^2 |f''(x_0)| + \dots\right]}$
$I(N) \sim e^{Nf(x_0)} \sqrt{\frac{2\pi}{N|f''(x_0)|}}$
is asymptotic to
$f(x) \sim g(x)$ as x
means $\lim_{x \to y} \frac{f(x)}{g(x)} = 1$

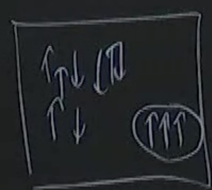

$Z = \int_0^\infty dE \; e^{\beta(T\hat{S}(E) - \hat{E})}$
N
αN
Ferromagnetism
Phase diagram
H
1st order
2nd order
no transition
T_c
T
M
H
$T<T_c$
$T=T_c$
$T>T_c$
$\Rightarrow M \propto H^{1/\delta}$
Why care

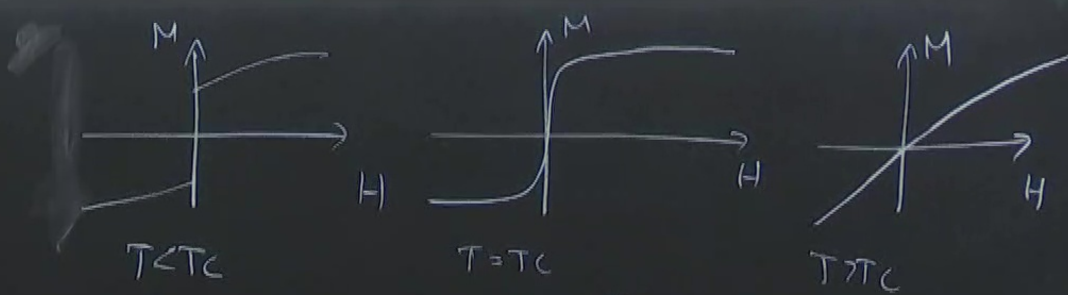
$$Z = \int_0^\infty dE \, e^{\beta(T\overset{N}{S}(E) - \overset{\propto N}{E})}$$

Ferromagnetism

Phase diagram

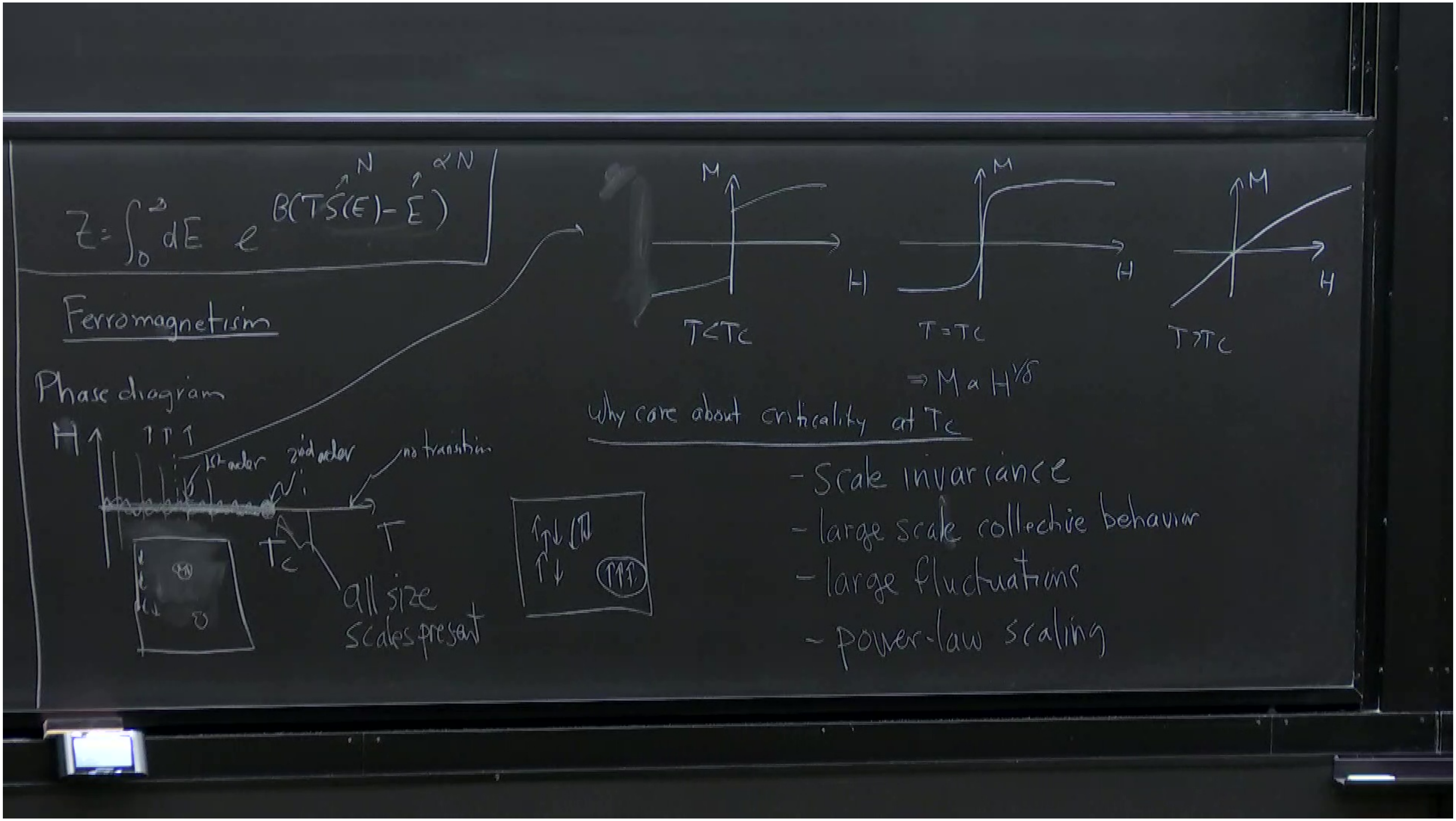






$\Rightarrow M \propto H^{1/\delta}$

Why care about criticality at T_c


$Z = \int_0^\infty dE \, e^{\beta(TS(E) - E)}$
$\propto N$
N
Ferromagnetism
Phase diagram
H
T
T_c
1st order
2nd order
no transition
all size scales present
M
H
$T < T_c$
$T = T_c$
$T > T_c$
$\Rightarrow M \propto H^{1/\delta}$
Why care about criticality at T_c
- Scale invariance
- large scale collective behavior
- large fluctuations
- power-law scaling

Critical Behaviours

$M \sim (T_c - T)^{\boxed{\beta}} \quad T < T_c$

$\quad = 0 \quad T > T_c$

$C_H \sim (T_c - T)^{\alpha}$

$\chi \sim (T - T_c)^{-\gamma}$

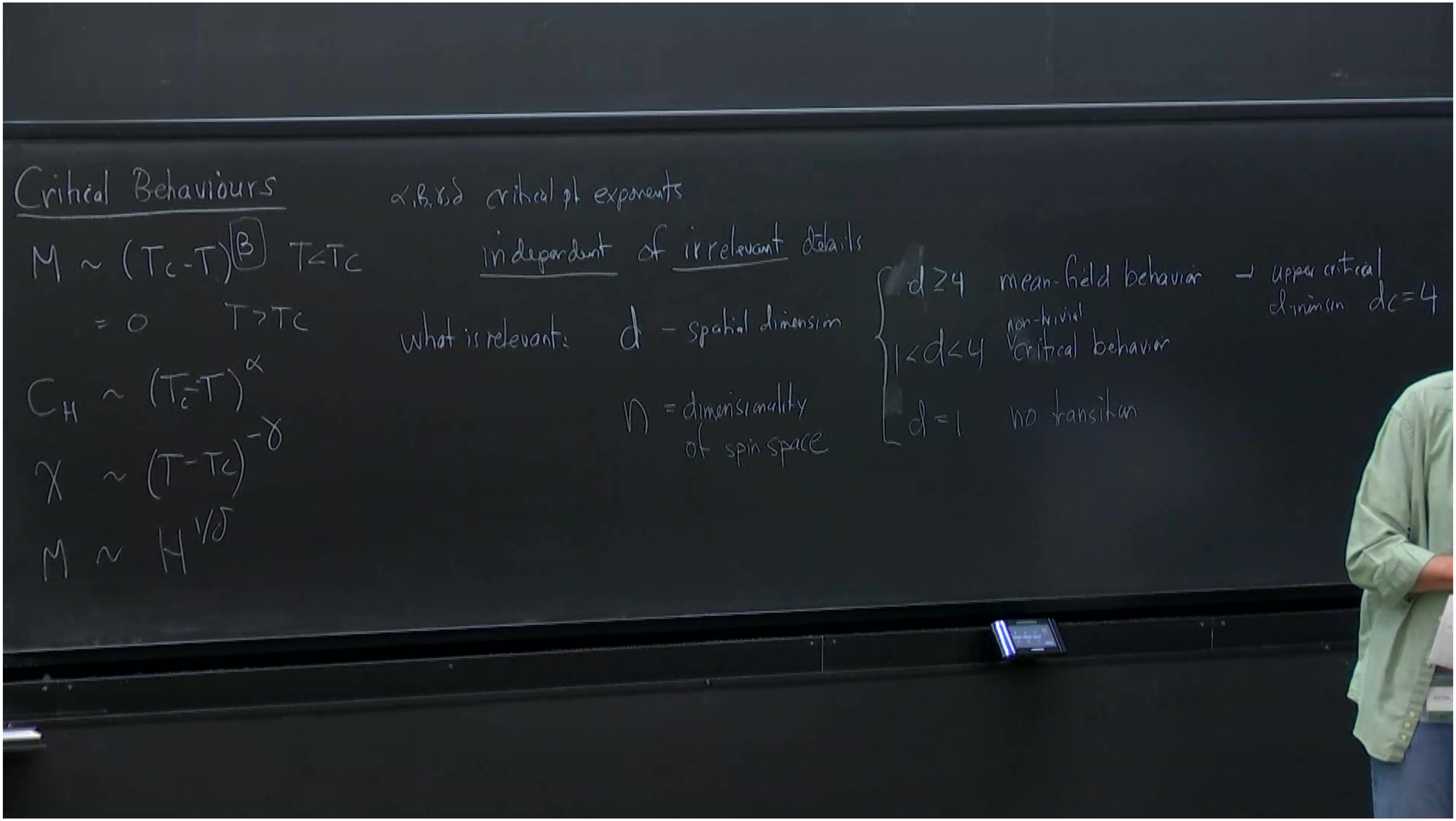
$M \sim H^{1/\delta}$

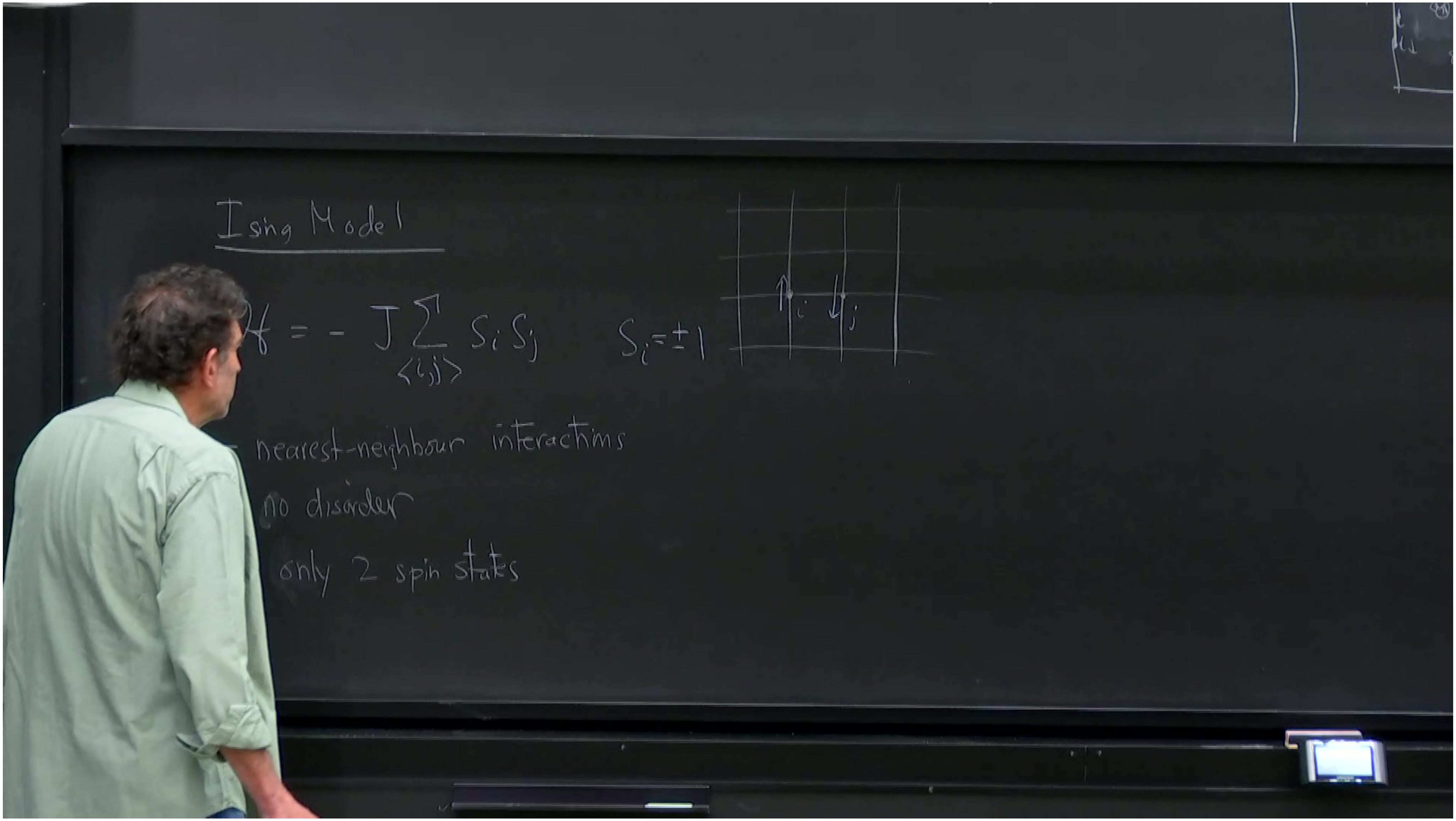
$\alpha, \beta, \gamma, \delta$ critical pt exponents

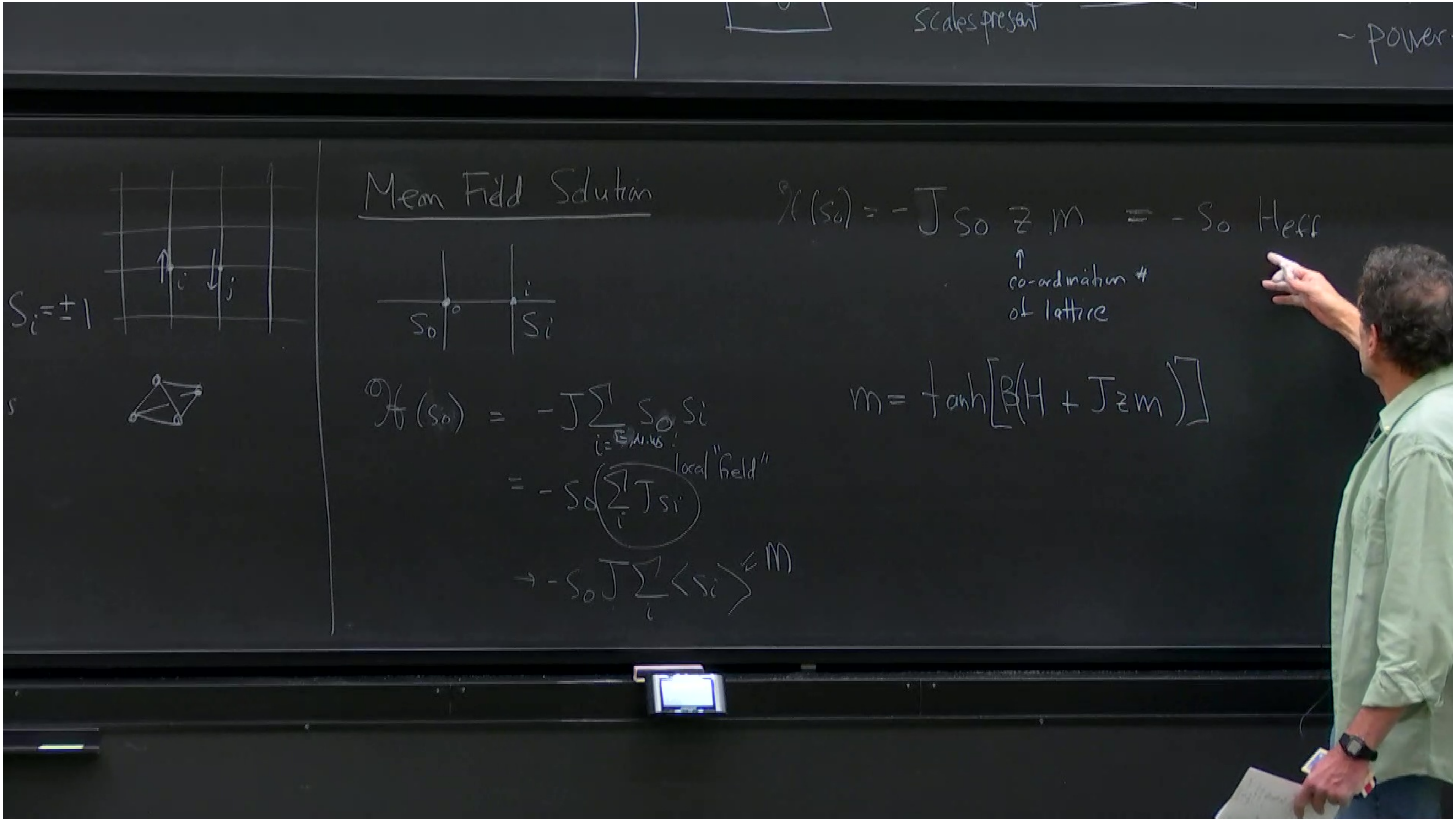
independent of irrelevant details

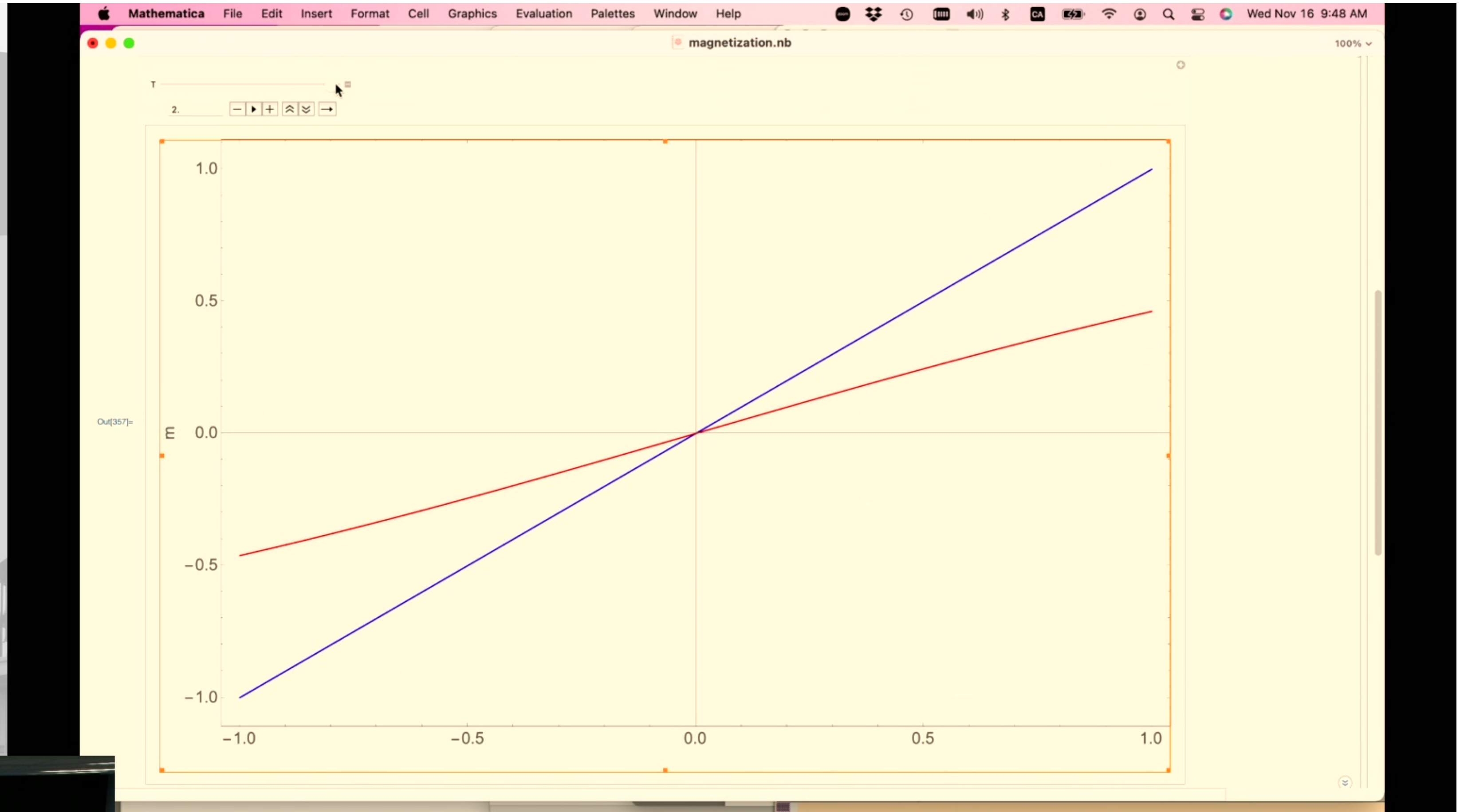
what is relevant: d - spatial dimension

$d \geq 4$ mean-field behavior


Critical Behaviours
M ~ (Tc - T)^β T<Tc
= 0 T>Tc
C_H ~ (Tc - T)^α
χ ~ (T - Tc)^-γ
M ~ H^1/δ
α, β, γ, δ critical pt exponents
independent of irrelevant details
what is relevant: d - spatial dimension
n = dimensionality of spin space
d ≥ 4 mean-field behavior → upper critical dimension dc = 4
1 < d < 4 non-trivial critical behavior
d = 1 no transition


Ising Model
$\mathcal{H} = -J \sum_{\langle i,j \rangle} S_i S_j$
$S_i = \pm 1$
nearest-neighbour interactions
no disorder
only 2 spin states


scales present
~ power
$S_i = \pm 1$
Mean Field Solution
S_0
S_i
$\mathcal{H}(S_0) = -J \sum_i S_0 S_i$
$= -S_0 \left(\sum_i J S_i \right)$
local "field"
$\rightarrow -S_0 J \sum_i \langle S_i \rangle = m$
$\mathcal{H}(S_0) = -J S_0 z m = -S_0 H_{eff}$
co-ordination #
of lattice
$m = \tanh[\beta(H + Jzm)]$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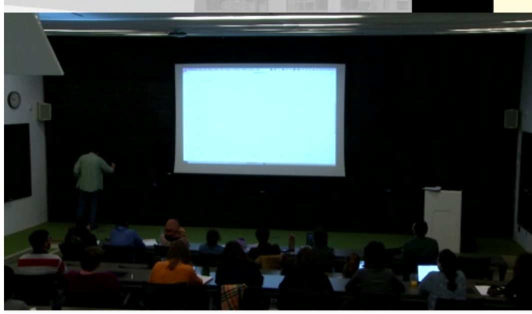
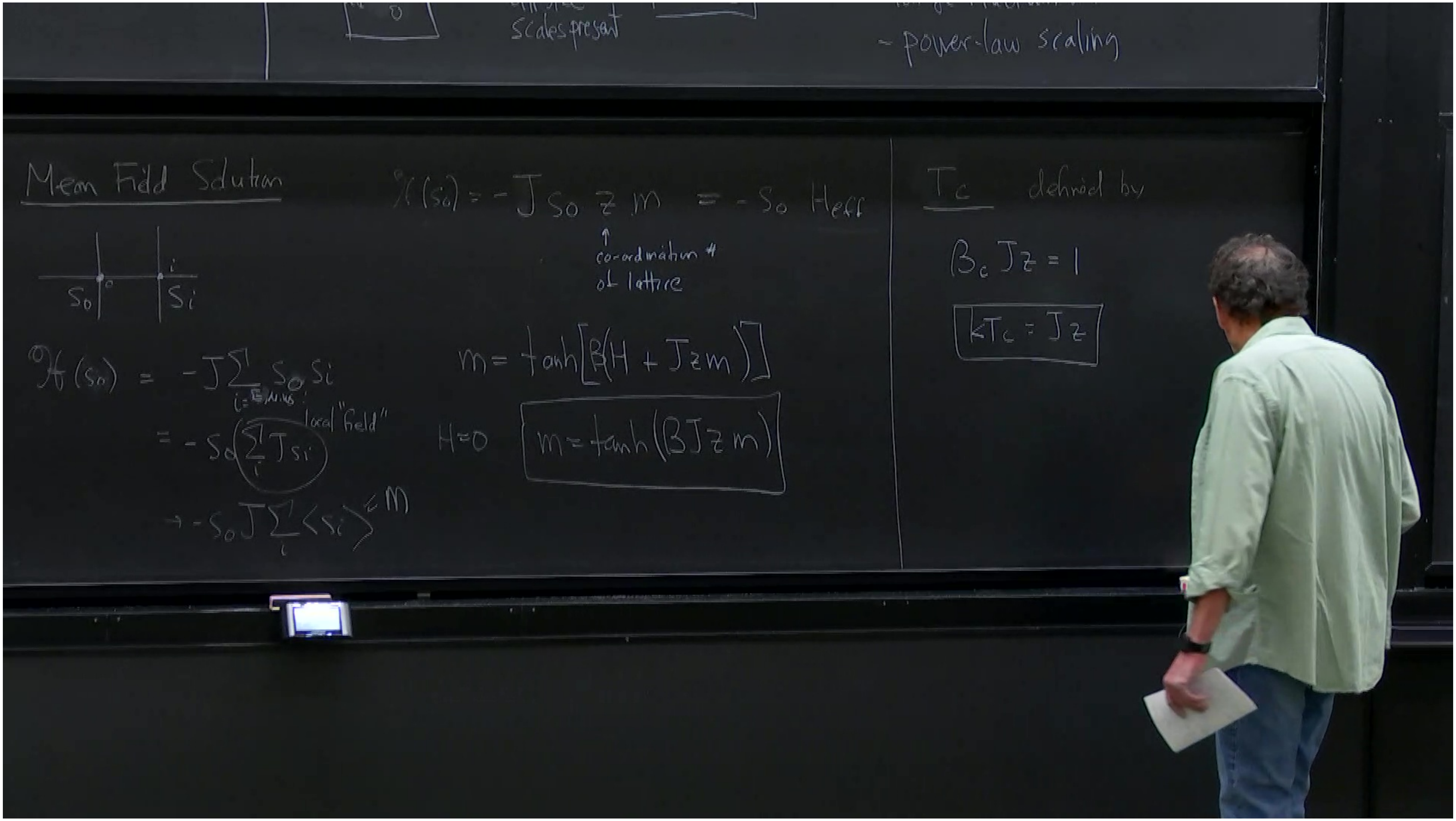
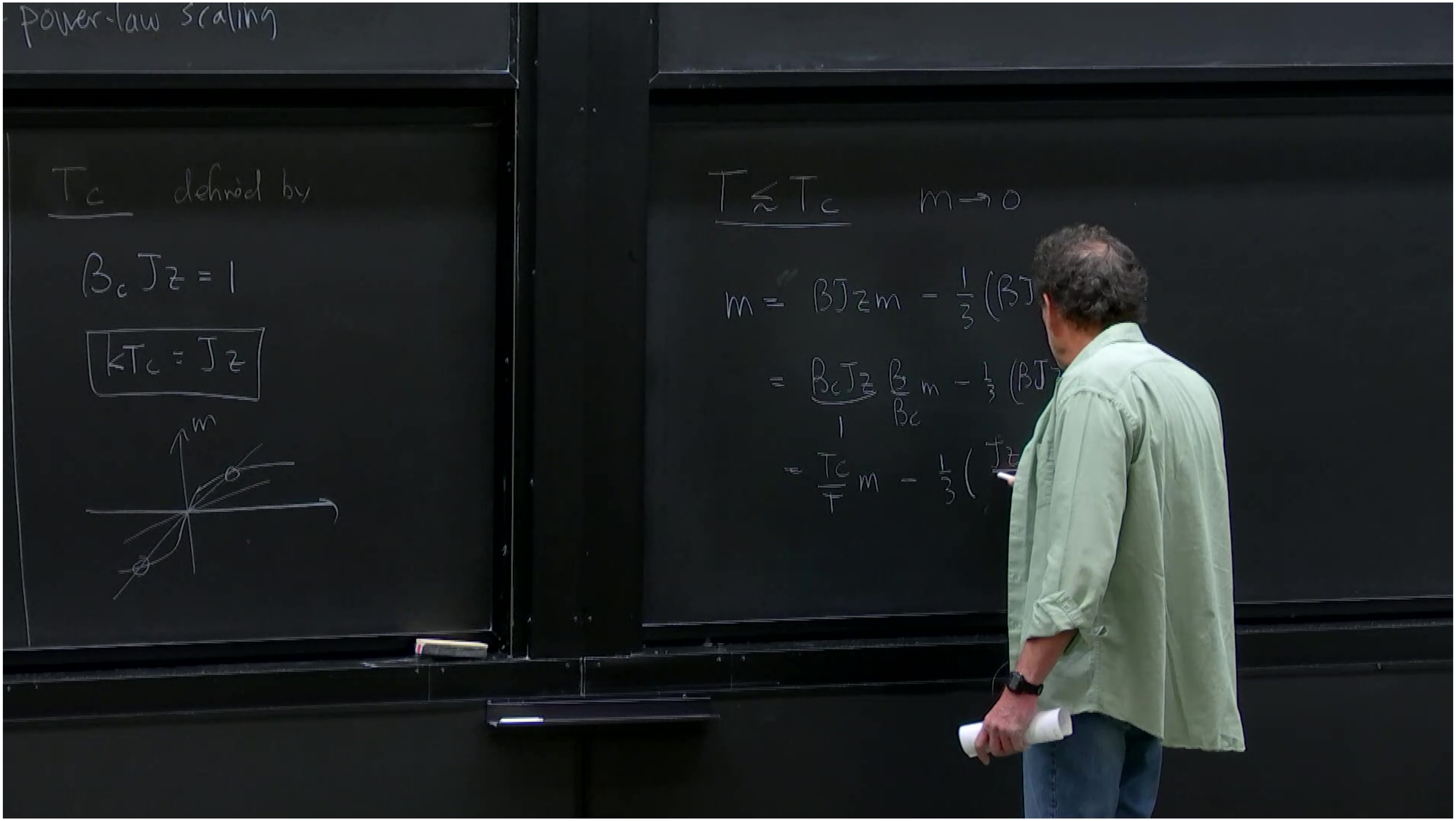
magnetization.nb
100%
T
2.
Out[357]=
m
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0




scales present
- power-law scaling
Mean Field Solution
H(s0) = -J s0 z m = -s0 Heff
co-ordination #
of lattice
s0
si
H(s0) = -J Σ s0 si
= -s0 Σ J si
local "field"
→ -s0 J Σ <si> = m
m = tanh[β(H + Jzm)]
H=0
m = tanh(βJzm)
Tc defined by
βc Jz = 1
kTc = Jz


power-law scaling
Tc defined by
βc J z = 1
kTc = J z
m
T ≲ Tc
m → 0
m = βJzm − 1/3 (βJ
= βcJz β/βc m − 1/3 (βJ
1
= Tc/T m − 1/3 (

$T \lesssim T_c \qquad m \to 0$

$$m = \beta J z m - \frac{1}{3}(\beta J z)^3 + \cdots$$

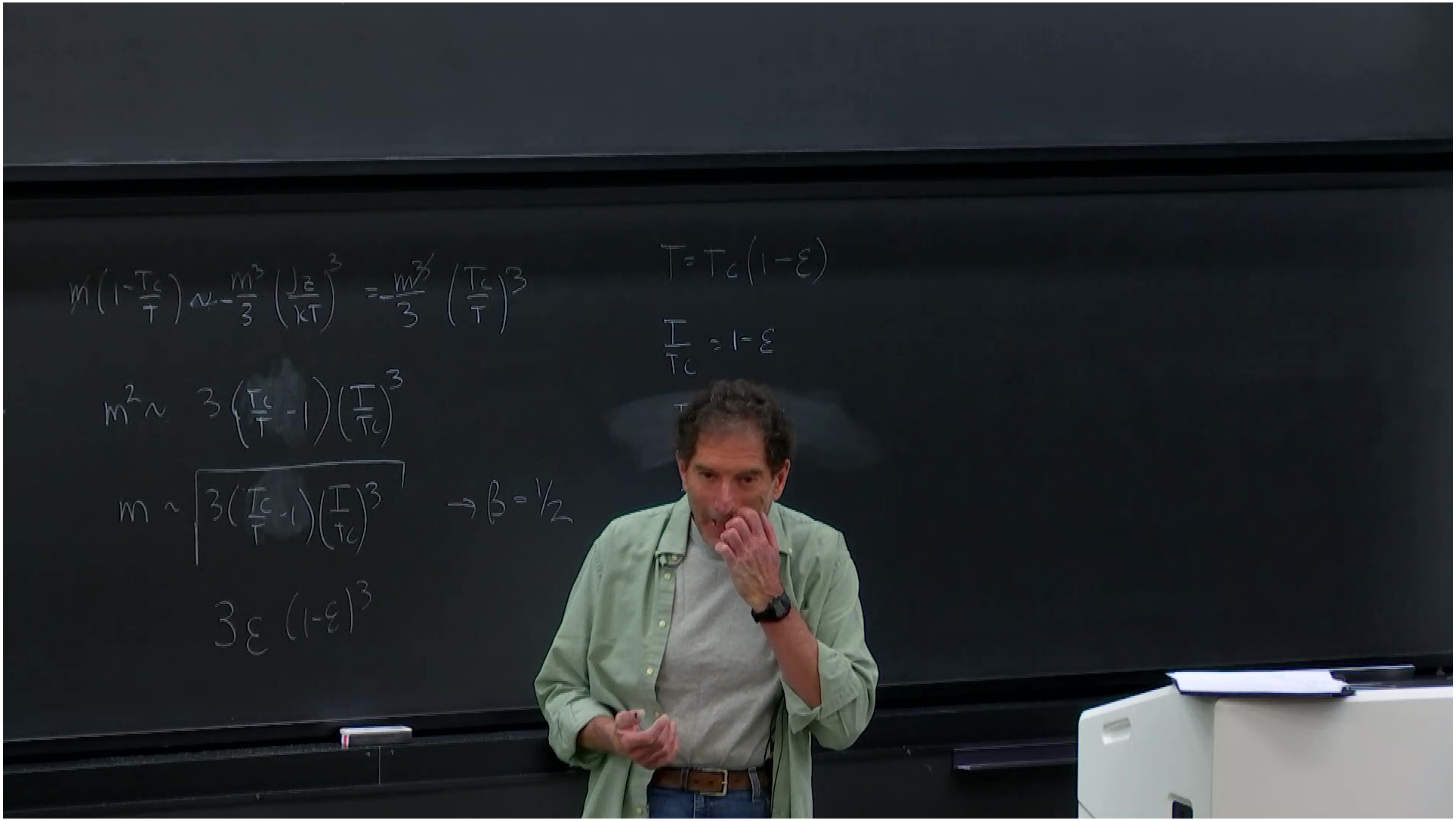
$$= \underbrace{\beta_c J z}_{1} \frac{\beta}{\beta_c} m - \frac{m^3}{3}(\beta J z)^3 + \cdots$$

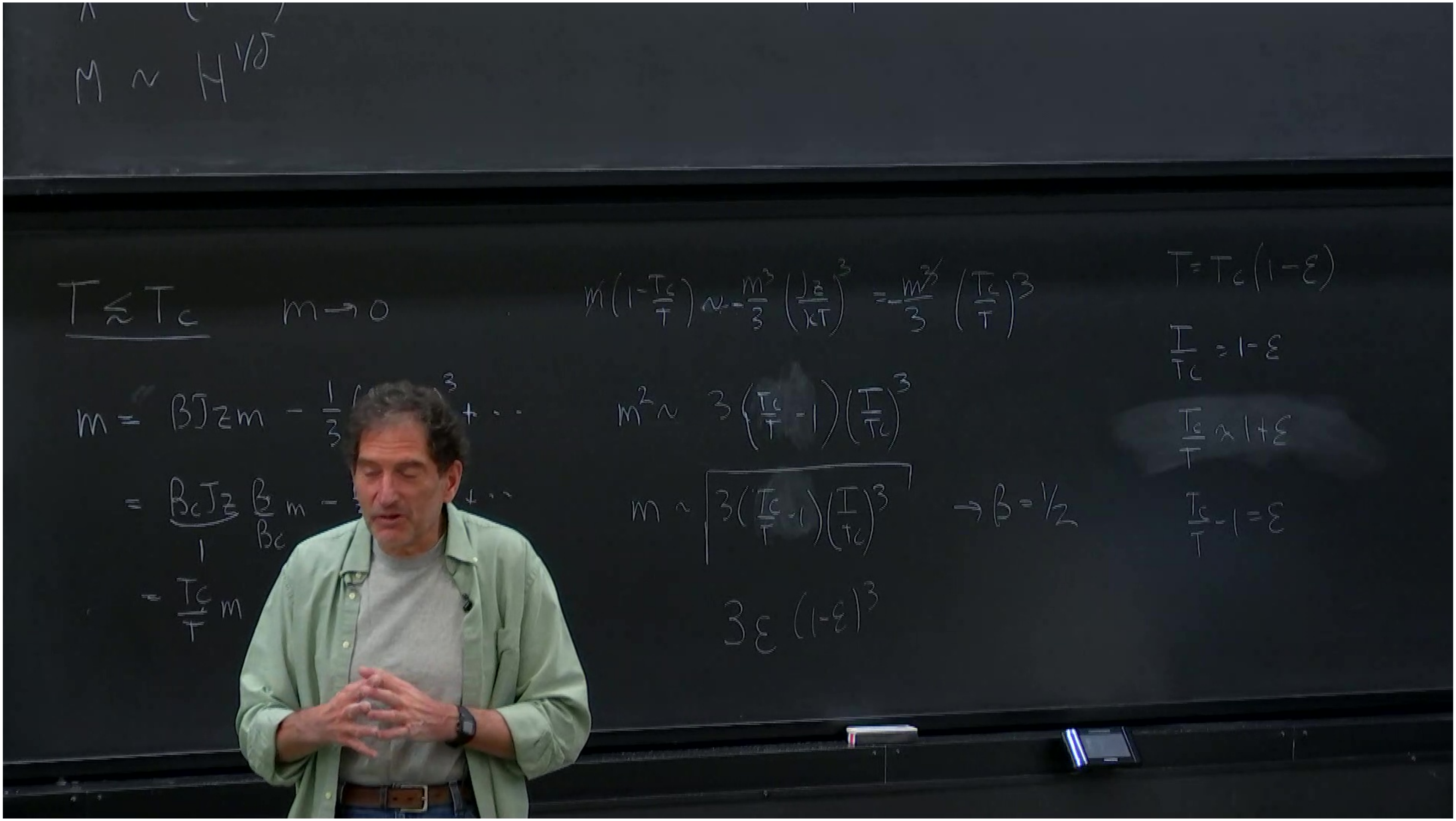
$$= \frac{T_c}{T} m - \frac{m^3}{3}\left(\frac{Jz}{kT}\right)^3 + \cdots$$

$$\cancel{m}\left(1 - \frac{T_c}{T}\right) \sim -\frac{m^3}{3}\left(\frac{Jz}{kT}\right)^3 = -\frac{m^{\cancel{3}}}{3}\left(\frac{T_c}{T}\right)^3$$

$$m^2 \sim 3\left(\frac{T_c}{T} - 1\right)\left(\frac{T}{T_c}\right)^3$$

$$m \sim \sqrt{3\left(\frac{T_c}{T} - 1\right)\left(\frac{T}{T_c}\right)^3} \to \beta = 1/2$$


$m\left(1-\frac{T_c}{T}\right) \approx -\frac{m^3}{3}\left(\frac{Jz}{kT}\right)^3 = -\frac{m^3}{3}\left(\frac{T_c}{T}\right)^3$
$T = T_c(1-\varepsilon)$
$\frac{T}{T_c} = 1-\varepsilon$
$m^2 \sim 3\left(\frac{T_c}{T}-1\right)\left(\frac{T}{T_c}\right)^3$
$m \sim \sqrt{3\left(\frac{T_c}{T}-1\right)\left(\frac{T}{T_c}\right)^3} \rightarrow \beta = 1/2$
$3\varepsilon(1-\varepsilon)^3$


M ~ H^{1/δ}
T ≲ T_c   m → 0
m(1 − T_c/T) ≈ −m^3/3 (Jz/kT)^3 = −m^3/3 (T_c/T)^3
T = T_c(1 − ε)
T/T_c = 1 − ε
m = βJzm − 1/3 ( )^3 + ...
m^2 ~ 3(T_c/T − 1)(T/T_c)^3
T_c/T ≈ 1 + ε
= β_cJz β/β_c m − ... + ...
m ~ √(3(T_c/T − 1)(T/T_c)^3) → β = 1/2
T_c/T − 1 = ε
= T_c/T m
3ε(1 − ε)^3

$\to -S_0 J \sum_i \langle s_i \rangle$

Susceptibility $\chi_T = \left(\frac{\partial M}{\partial H}\right)_T$ isothermal susceptibility

$T \to T_c \Rightarrow m$ small

$$m = \tanh(\beta H + \beta J z m)$$

$$\approx \beta H + \beta J z m$$

$$m(1 - \beta J z) = \beta H$$

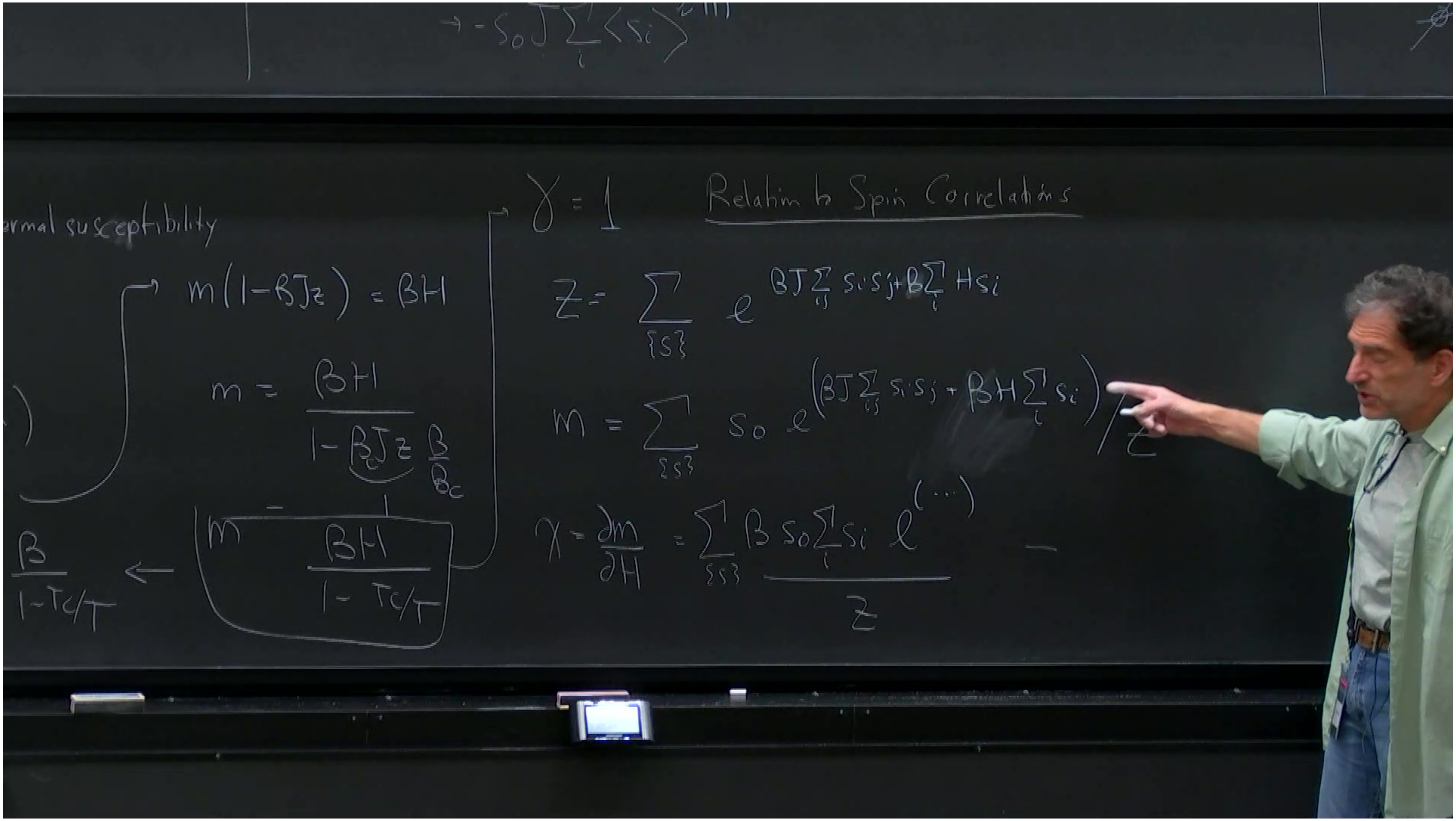
$$m = \frac{\beta H}{1 - \beta_c J z \frac{\beta}{\beta_c}}$$

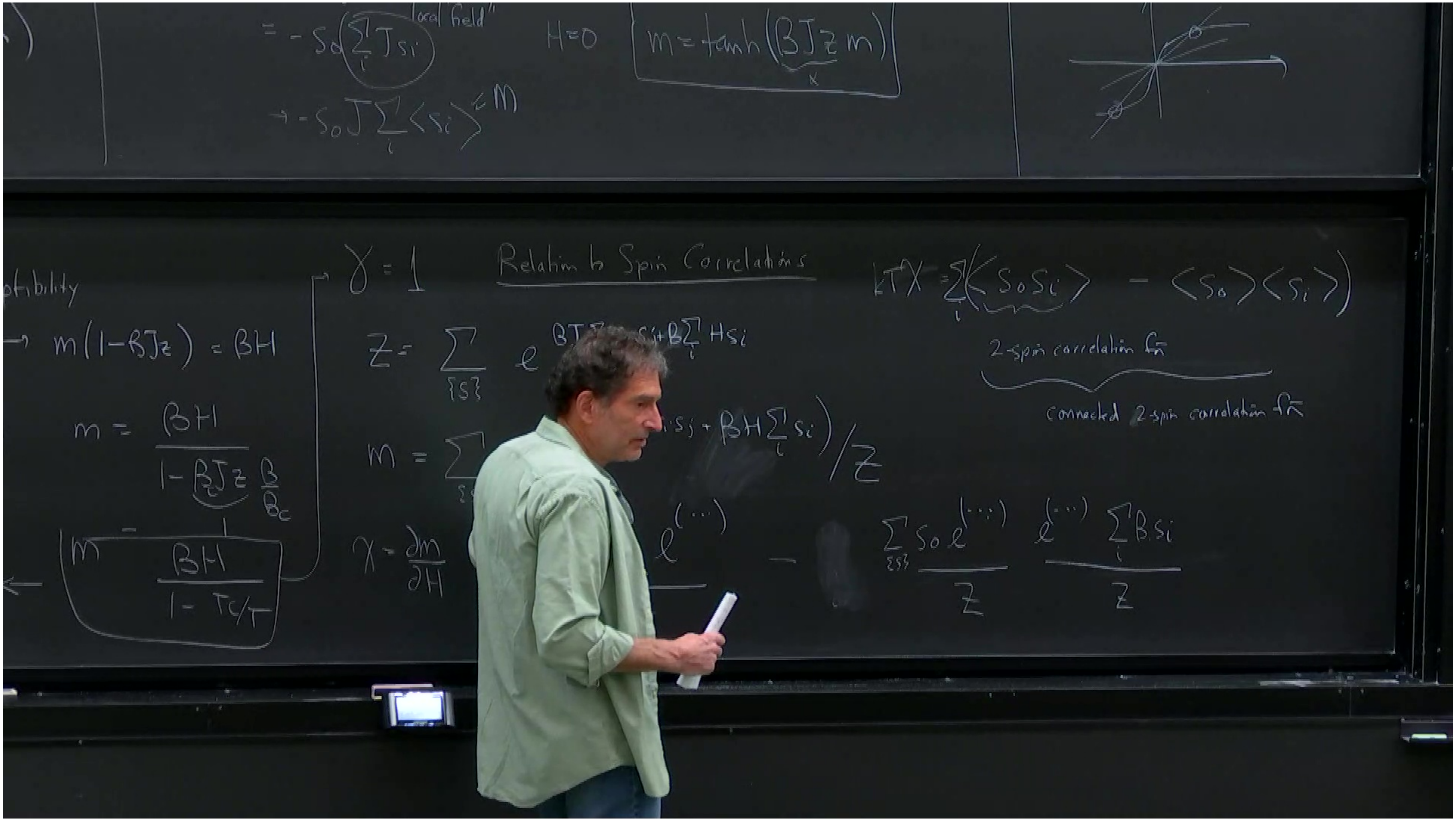
$$\boxed{m = \frac{\beta H}{1 - T_c/T}}$$

$$\chi = \frac{\beta}{1 - T_c/T} \leftarrow$$

$$\to \gamma = 1$$

Relation to


→ −s0 J Σi ⟨si⟩
ermal susceptibility
m(1−βJz) = βH
m = βH / (1−βJz β/βc)
m = βH / (1−Tc/T)
β / (1−Tc/T) ←
γ = 1
Relation to Spin Correlations
Z = Σ{s} e^(βJ Σij si sj + β Σi H si)
m = Σ{s} s0 e^(βJ Σij si sj + βH Σi si) / Z
χ = ∂m/∂H = Σ{s} β s0 Σi si e^(...) / Z


"local field"
H=0
m = tanh(βJz m)
γ = 1
Relation to Spin Correlations
kTχ = Σ (<S₀Sᵢ> − <S₀><Sᵢ>)
2-spin correlation fn
connected 2-spin correlation fn
m(1−βJz) = βH
m = βH / (1 − βJz β/βc)
m = βH / (1 − Tc/T)
χ = ∂m/∂H