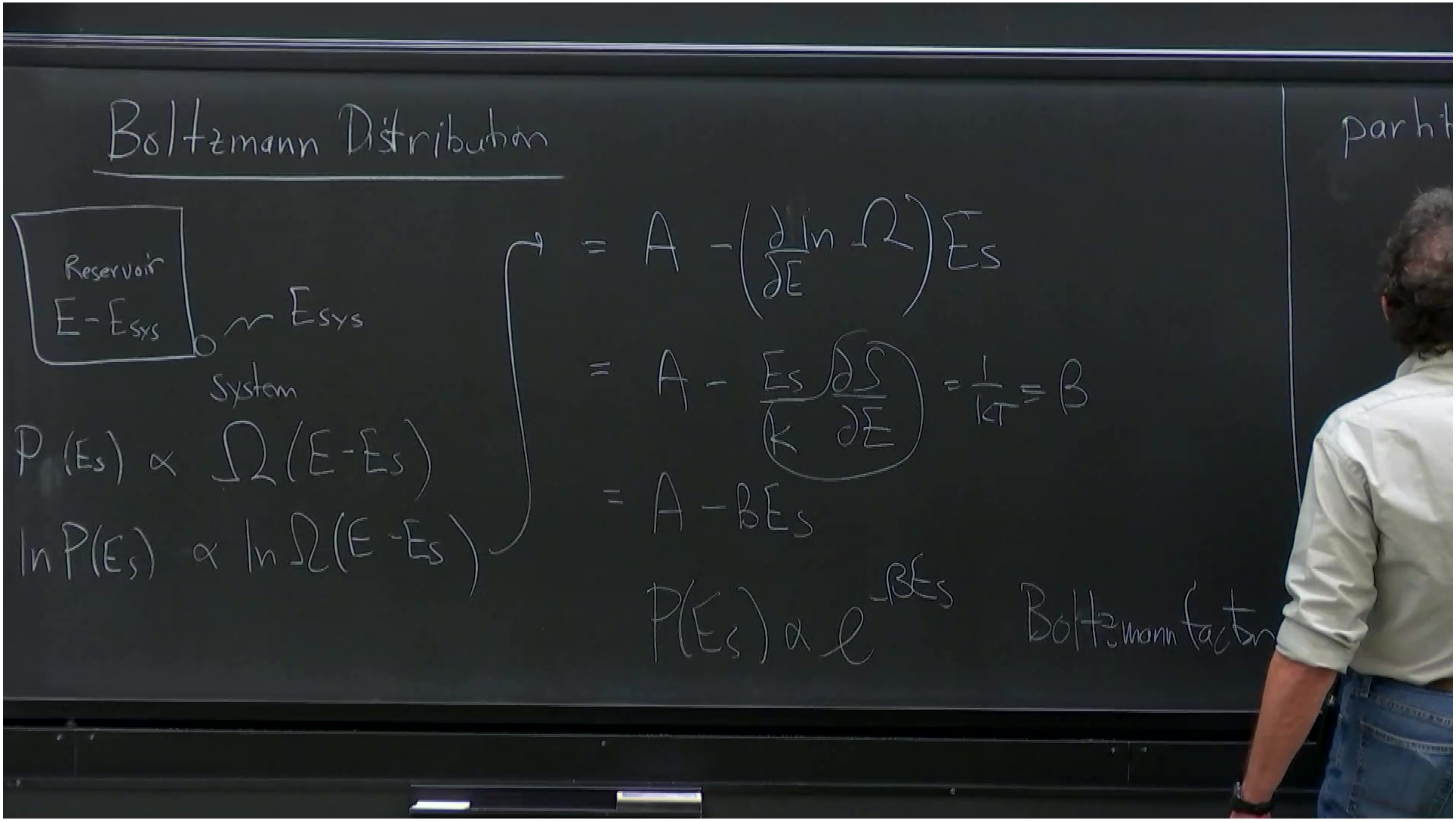
Title: Statistical Physics - Lecture 221115

Speakers:

Collection: Statistical Physics (2022/2023)

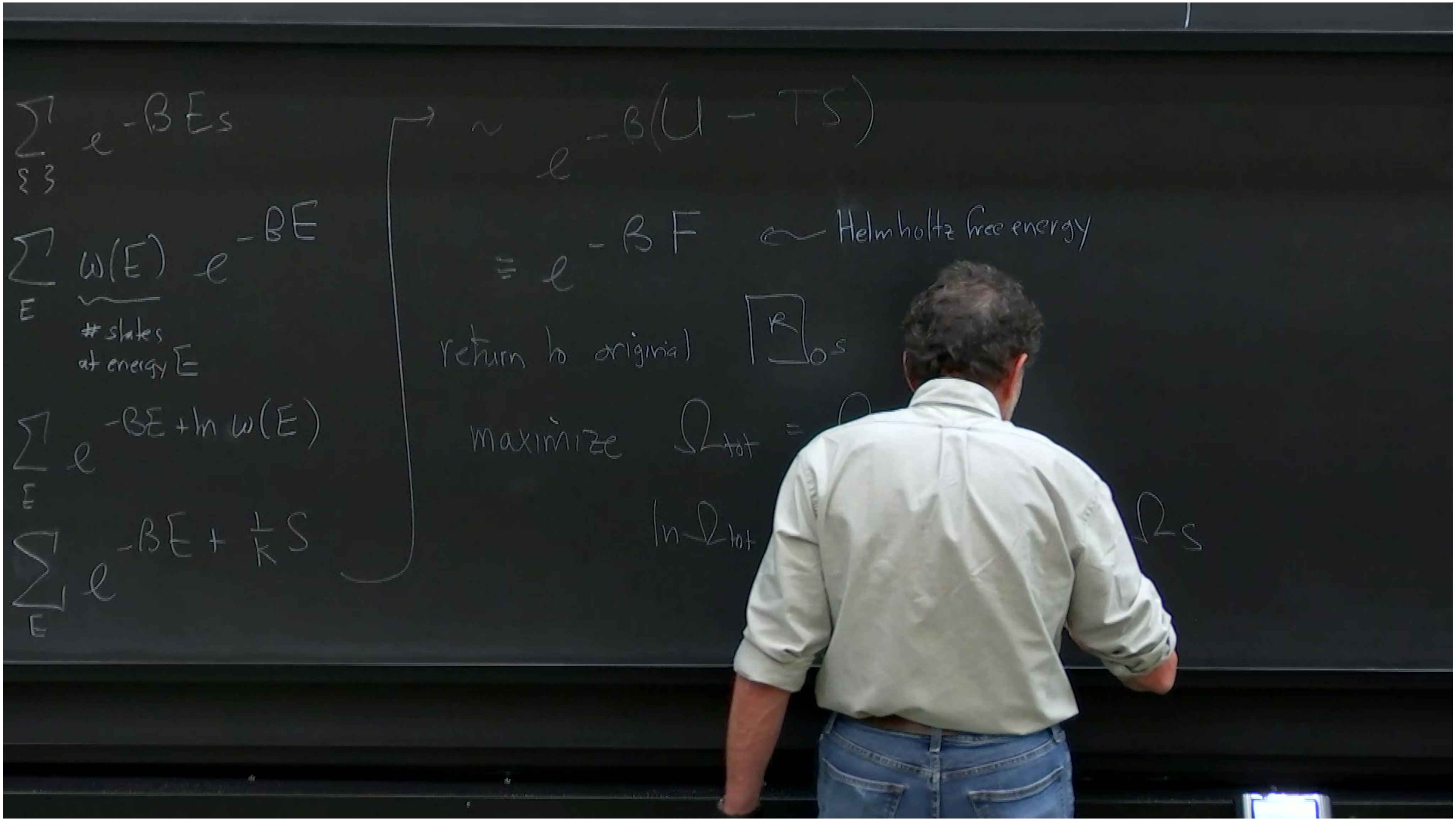
Date: November 15, 2022 - 9:00 AM

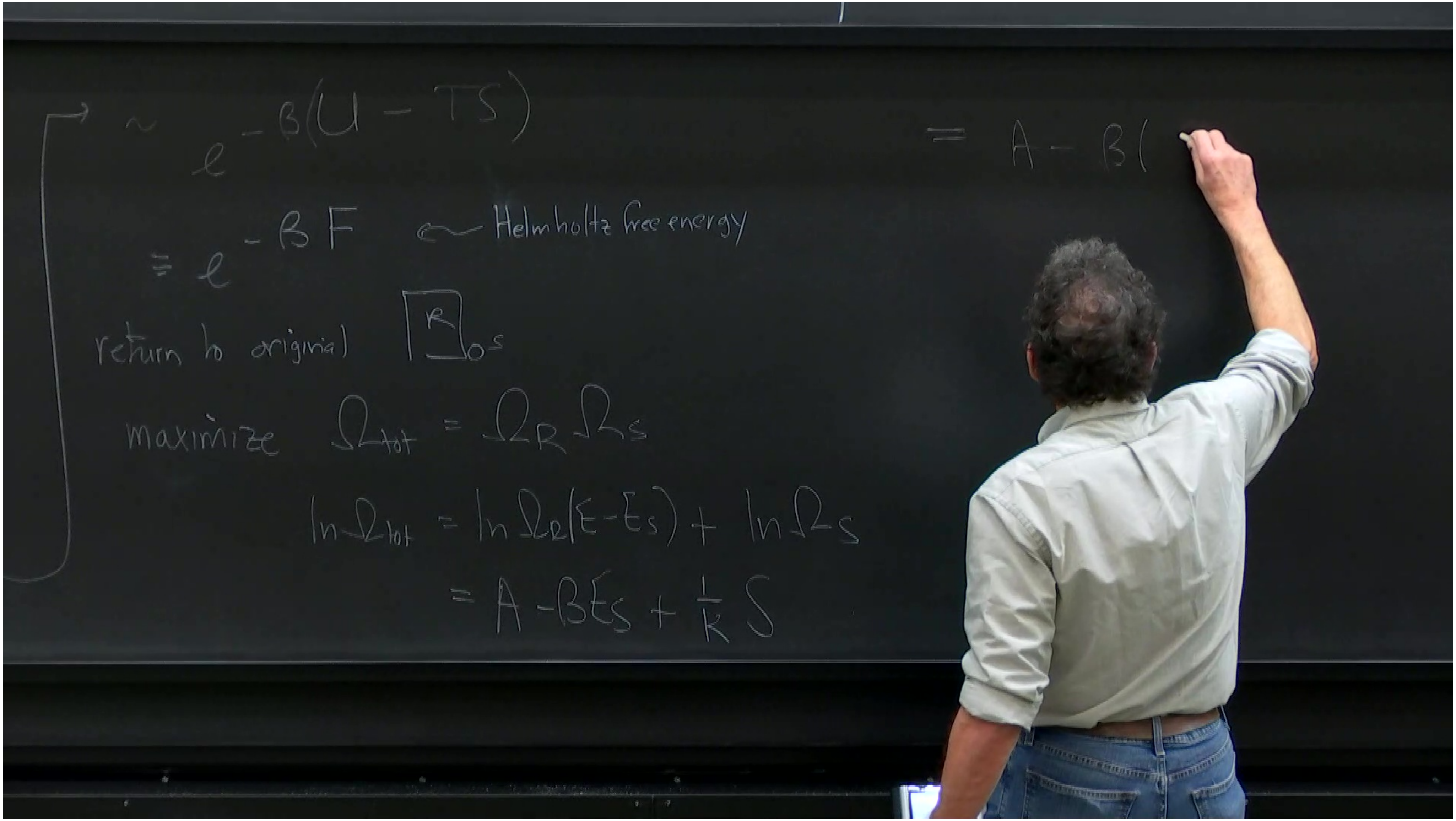
URL: https://pirsa.org/22110012


Boltzmann Distribution
Reservoir
$E - E_{sys}$
E_{sys}
System
$P(E_s) \propto \Omega(E - E_s)$
$\ln P(E_s) \propto \ln \Omega(E - E_s)$
$= A - \left(\frac{\partial}{\partial E} \ln \Omega\right) E_s$
$= A - \frac{E_s}{k} \frac{\partial S}{\partial E} = \frac{1}{kT} = \beta$
$= A - \beta E_s$
$P(E_s) \propto e^{-\beta E_s}$
Boltzmann factor
partit

Partition Function and Free Energy

$$P(E_s) = \frac{e^{-\beta E_s}}{\sum_{\{s\}} e^{-\beta E_s}} \equiv \frac{e^{-\beta E_s}}{Z} \leftarrow \text{partition fn}$$


$\sum_{\{\}} e^{-\beta E_s}$
$\sum_E \underbrace{\omega(E)}_{\text{\# states at energy } E} e^{-\beta E}$
$\sum_E e^{-\beta E + \ln \omega(E)}$
$\sum_E e^{-\beta E + \frac{1}{k} S}$
$\sim e^{-\beta(U - TS)}$
$\equiv e^{-\beta F}$
Helmholtz free energy
return to original $\boxed{R}_{0}\,S$
maximize Ω_{tot} =
$\ln \Omega_{tot}$
Ω_S


~ e^{-β(U - TS)}
= e^{-βF} ← Helmholtz free energy
return to original R S
maximize Ω_tot = Ω_R Ω_s
ln Ω_tot = ln Ω_R(E - E_s) + ln Ω_s
= A - βE_s + (1/k) S
= A - β(

$$\sim e^{-\beta(U-TS)}$$

$$= e^{-\beta F}$$ ← Helmholtz free energy

return to original $\boxed{R}\ \circ\, s$

maximize $\Omega_{tot} = \Omega_R \Omega_s$

$$\ln \Omega_{tot} = \ln \Omega_R(E - E_s) + \ln \Omega_s$$

$$= A - \beta E_s + \frac{1}{k} S$$

$$\ln \Omega_{tot} = A - \beta(E - TS)$$

$$= A - \beta F$$

maximize $\Omega_{tot} \Leftrightarrow$ minimizing

Role of Energy Fluctuations

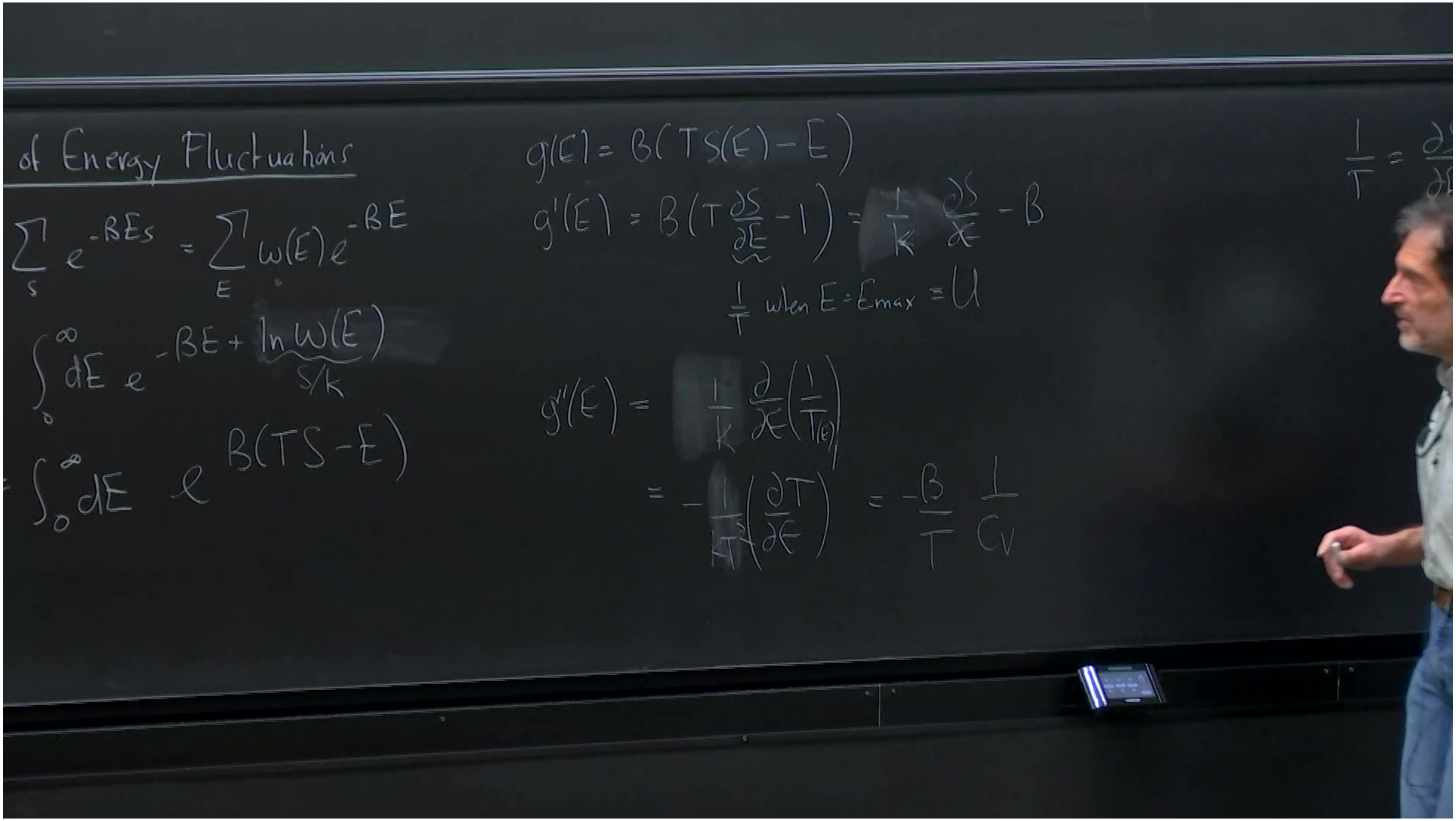
$$Z = \sum_s e^{-\beta E_s} = \sum_E \omega(E)\, e^{-\beta E}$$

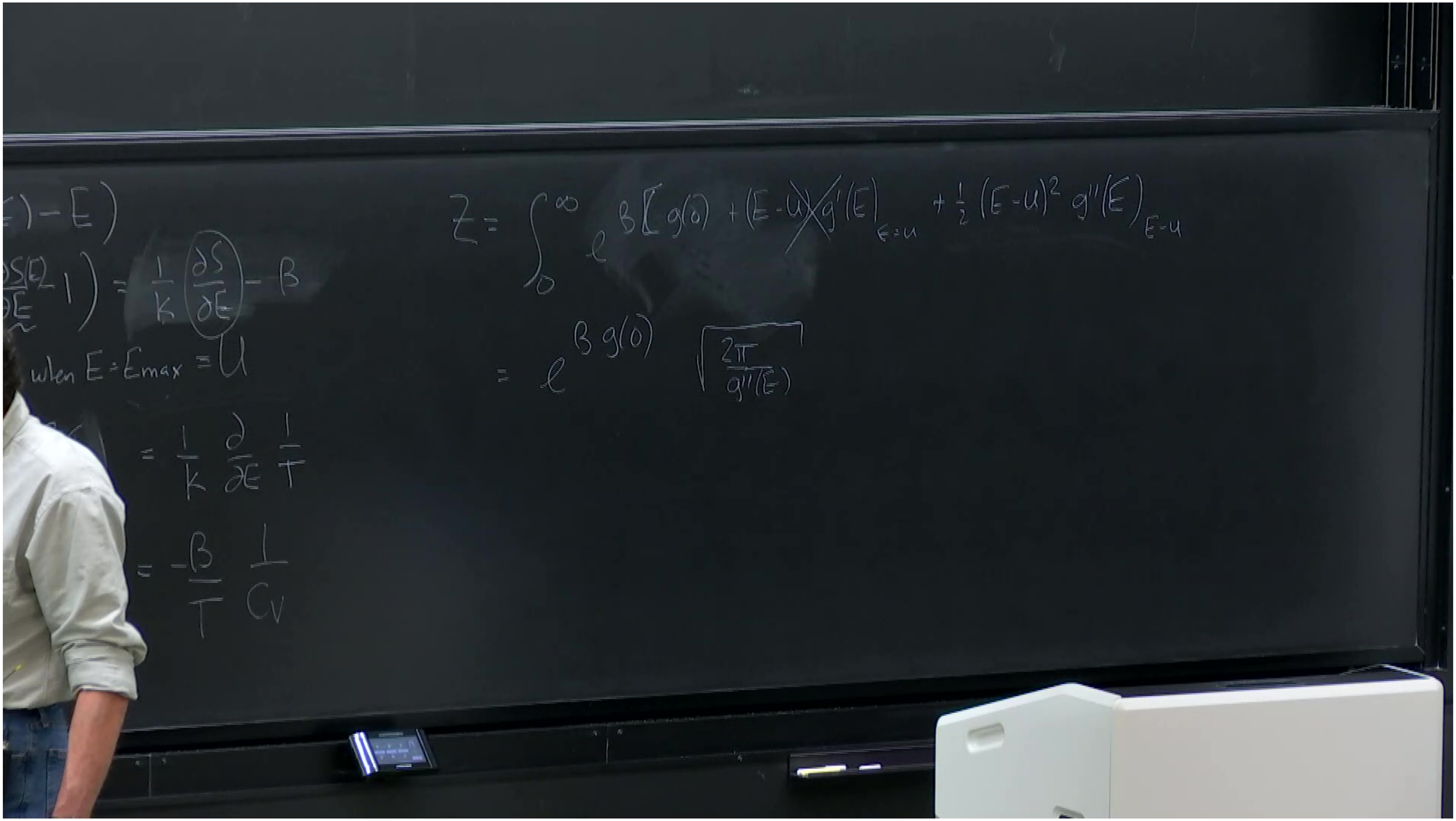
$$= \int_0^\infty dE\, e^{-\beta E + \underbrace{\ln \omega(E)}_{S/k}}$$

$$= \int_0^\infty dE\, e^{\beta(TS - E)}$$

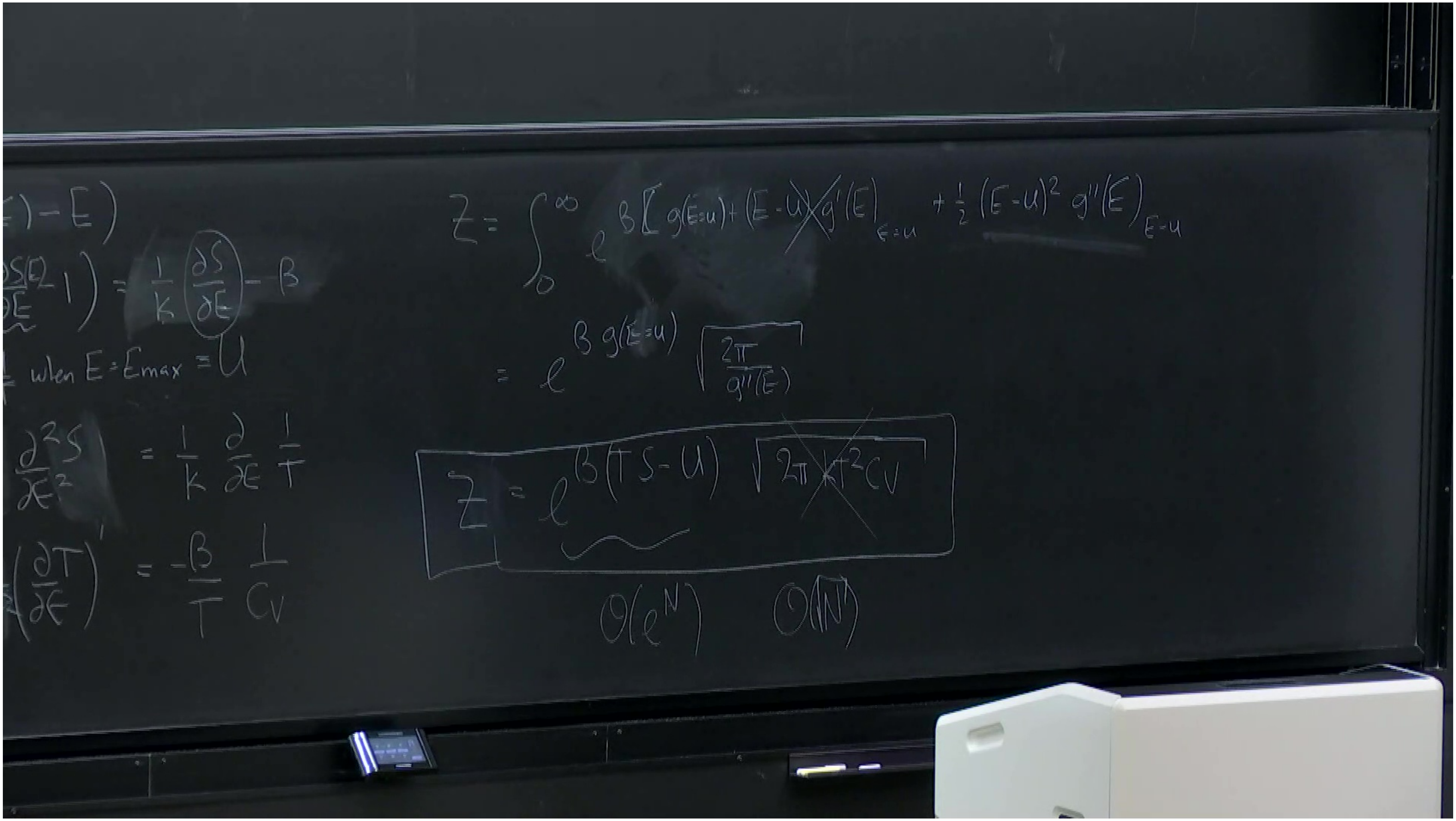
$$g(E) = \beta\left(TS(E) - E\right)$$

$$g'(E) = \beta\left(T \underbrace{\frac{\partial S}{\partial E}}_{\frac{1}{T} \text{ when } E = E_{max} = U} - 1\right) = 0$$


of Energy Fluctuations
$\sum_s e^{-\beta E_s} = \sum_E \omega(E) e^{-\beta E}$
$\int_0^\infty dE\, e^{-\beta E + \underbrace{\ln \omega(E)}_{S/k}}$
$= \int_0^\infty dE\, e^{\beta(TS-E)}$
$g(E) = \beta(TS(E) - E)$
$g'(E) = \beta\left(T\frac{\partial S}{\partial E} - 1\right) = \frac{1}{k}\frac{\partial S}{\partial E} - \beta$
$\frac{1}{T}$ when $E = E_{max} = U$
$g''(E) = \frac{1}{k}\frac{\partial}{\partial E}\left(\frac{1}{T(E)}\right)$
$= -\frac{1}{kT^2}\left(\frac{\partial T}{\partial E}\right) = -\frac{\beta}{T}\frac{1}{C_V}$
$\frac{1}{T} =$



$E) - E)$
$\frac{\partial S(E)}{\partial E} - 1) = \frac{1}{k} \frac{\partial S}{\partial E} - \beta$
when $E = E_{max} = U$
$= \frac{1}{k} \frac{\partial}{\partial E} \frac{1}{T}$
$= -\frac{\beta}{T} \frac{1}{C_V}$
$Z = \int_0^\infty e^{\beta[g(0) + \cancel{(E-U) g'(E)|_{E=U}} + \frac{1}{2}(E-U)^2 g''(E)|_{E=U}}$
$= e^{\beta g(0)} \sqrt{\frac{2\pi}{g''(E)}}$



$E) - E)$
$\frac{\partial S(E)}{\partial E} - 1) = \frac{1}{k}\frac{\partial S}{\partial E} - \beta$
when $E = E_{max} = U$
$\frac{\partial^2 S}{\partial E^2} = \frac{1}{k}\frac{\partial}{\partial E}\frac{1}{T}$
$\left(\frac{\partial T}{\partial E}\right) = -\frac{\beta}{T}\frac{1}{C_V}$
$Z = \int_0^\infty e^{\beta[g(E=u) + \cancel{(E-u) g'(E)|_{E=u}} + \frac{1}{2}(E-u)^2 g''(E)|_{E=u}}$
$= e^{\beta g(E=u)} \sqrt{\frac{2\pi}{g''(E)}}$
$\boxed{Z = e^{\beta(TS-U)} \cancel{\sqrt{2\pi k T^2 C_V}}}$
$O(e^N)$ $O(N)$

Connection of Partition Fn and Thermodynamics

$$S(U,V) \rightarrow U(S,V)$$

$$dU = \text{heat added} + \text{work done}$$

$$= TdS - pdV = \underbrace{\left(\frac{\partial U}{\partial S}\right)_V}_{=\,T} dS + \underbrace{\left(\frac{\partial U}{\partial V}\right)_S}_{=\,-p} dV$$

$\sum_E e$ $\quad = A - \beta E_s + \frac{1}{k} S$

xample: 2-state paramagnet

$\uparrow$ spin up $\qquad \downarrow$ spin down $\qquad H \uparrow$

$E_\uparrow = -\mu H \qquad E_\downarrow = +\mu H$

$P_\uparrow \propto e^{+\beta\mu H} \qquad P_\downarrow = e^{-\beta\mu H}$

$$Z_1 = e^{\beta\mu H} + e^{-\beta\mu H} = 2\cosh\beta\mu H$$

$$P_\uparrow = \frac{e^{\beta\mu H}}{2\cosh\beta\mu H} \qquad P_\downarrow = \frac{e^{-\beta\mu H}}{2\cosh\beta\mu H}$$

$$\langle m\rangle = \frac{\mu e^{\beta\mu H}}{2\cosh\beta\mu H} - \frac{\mu e^{-\beta\mu H}}{2\cosh\beta\mu H}$$

$$\boxed{\langle m\rangle = \mu\tanh\beta\mu H}$$

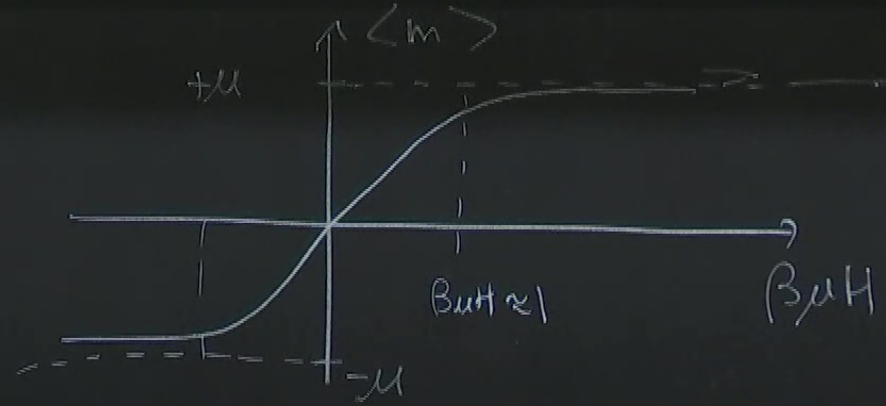
$$= A - \beta E_s + \frac{1}{k} S$$

$$Z_1 = e^{\beta\mu H} + e^{-\beta\mu H} = 2\cosh\beta\mu H$$

$$P_\uparrow = \frac{e^{\beta\mu H}}{2\cosh\beta\mu H} \qquad P_\downarrow = \frac{e^{-\beta\mu H}}{2\cosh\beta\mu H}$$

$$\langle m \rangle = \frac{\mu e^{\beta\mu H}}{2\cosh\beta\mu H} - \frac{\mu e^{-\beta\mu H}}{2\cosh\beta\mu H}$$

$$\boxed{\langle m \rangle = \mu \tanh\beta\mu H}$$



$$Z_1 = 2\cosh \beta\mu H$$

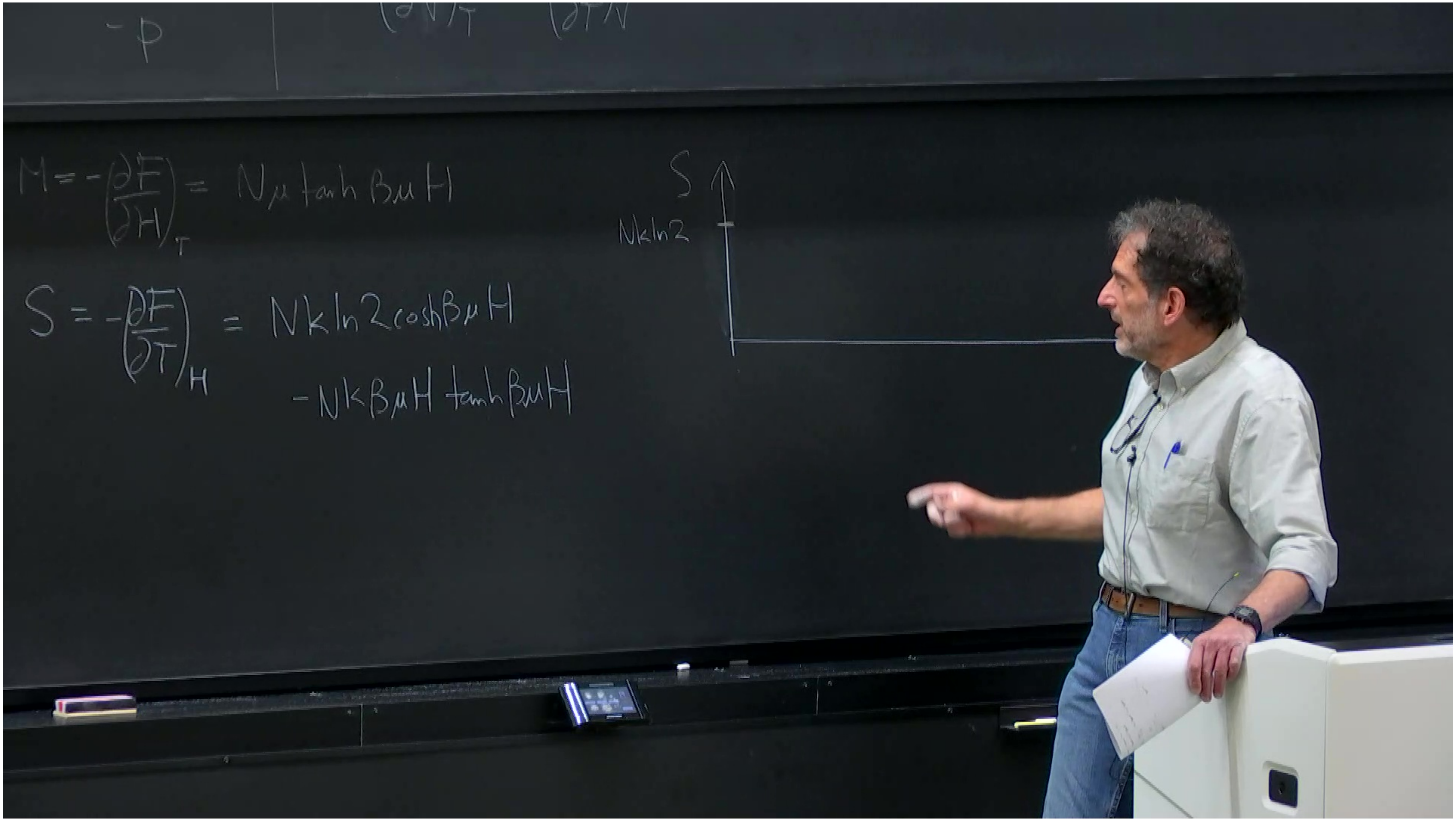
$$Z_N = Z_1^{\,N}$$

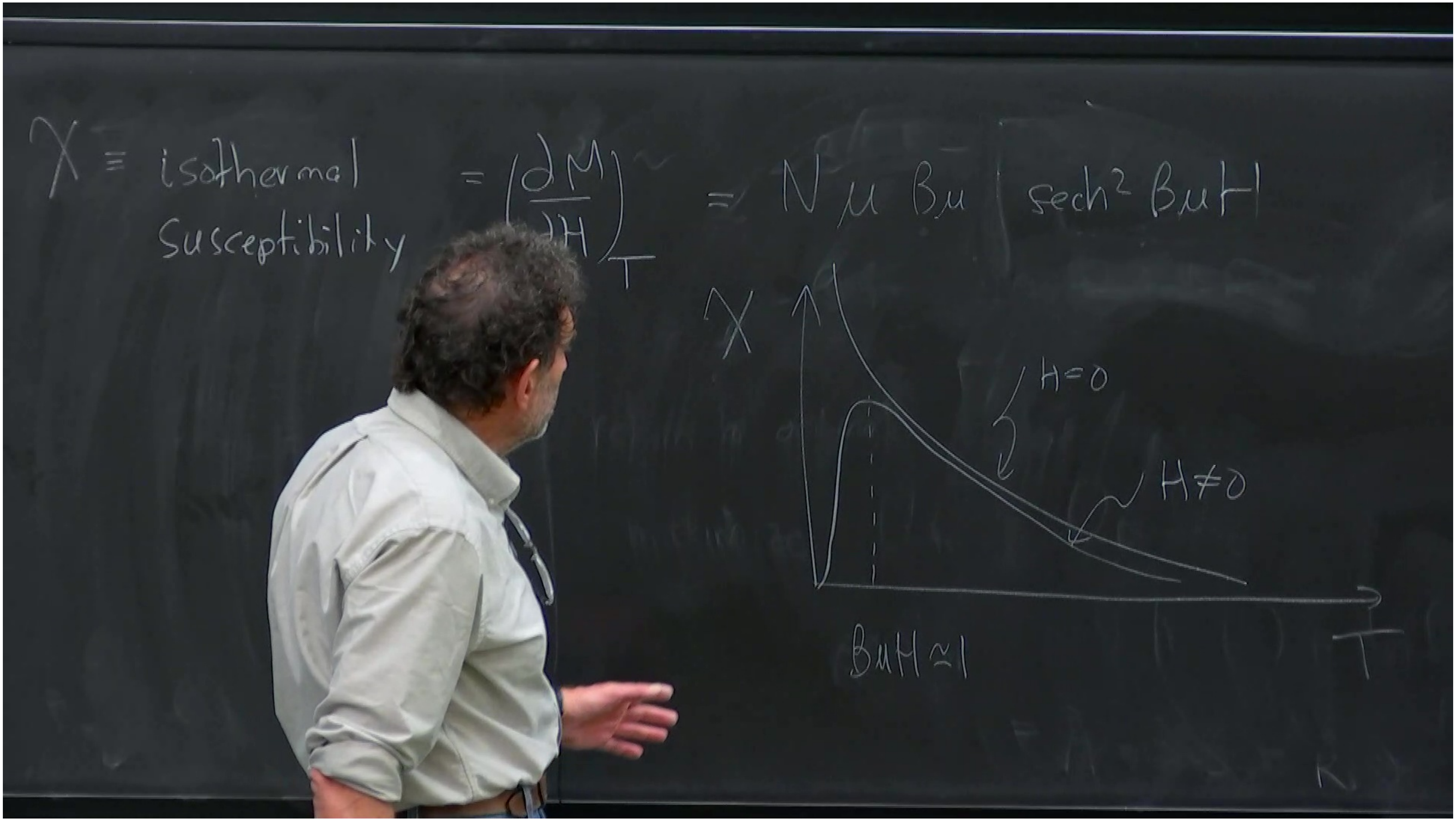
$$F = -kTN \ln 2\cosh \beta\mu H$$

$$dF = -S\,dT - M\,dH$$

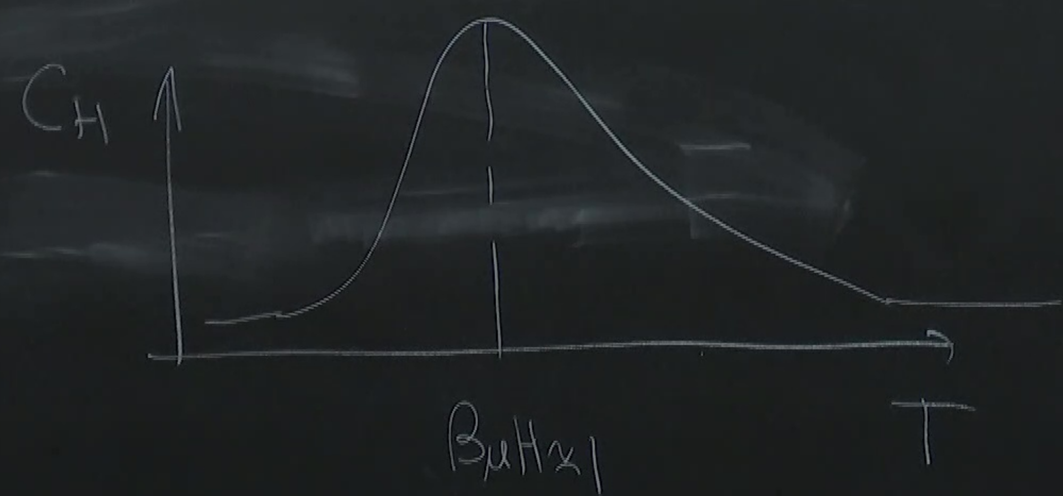
$$M = -\left(\frac{\partial F}{\partial H}\right)_T = N\mu \tanh \beta\mu H$$

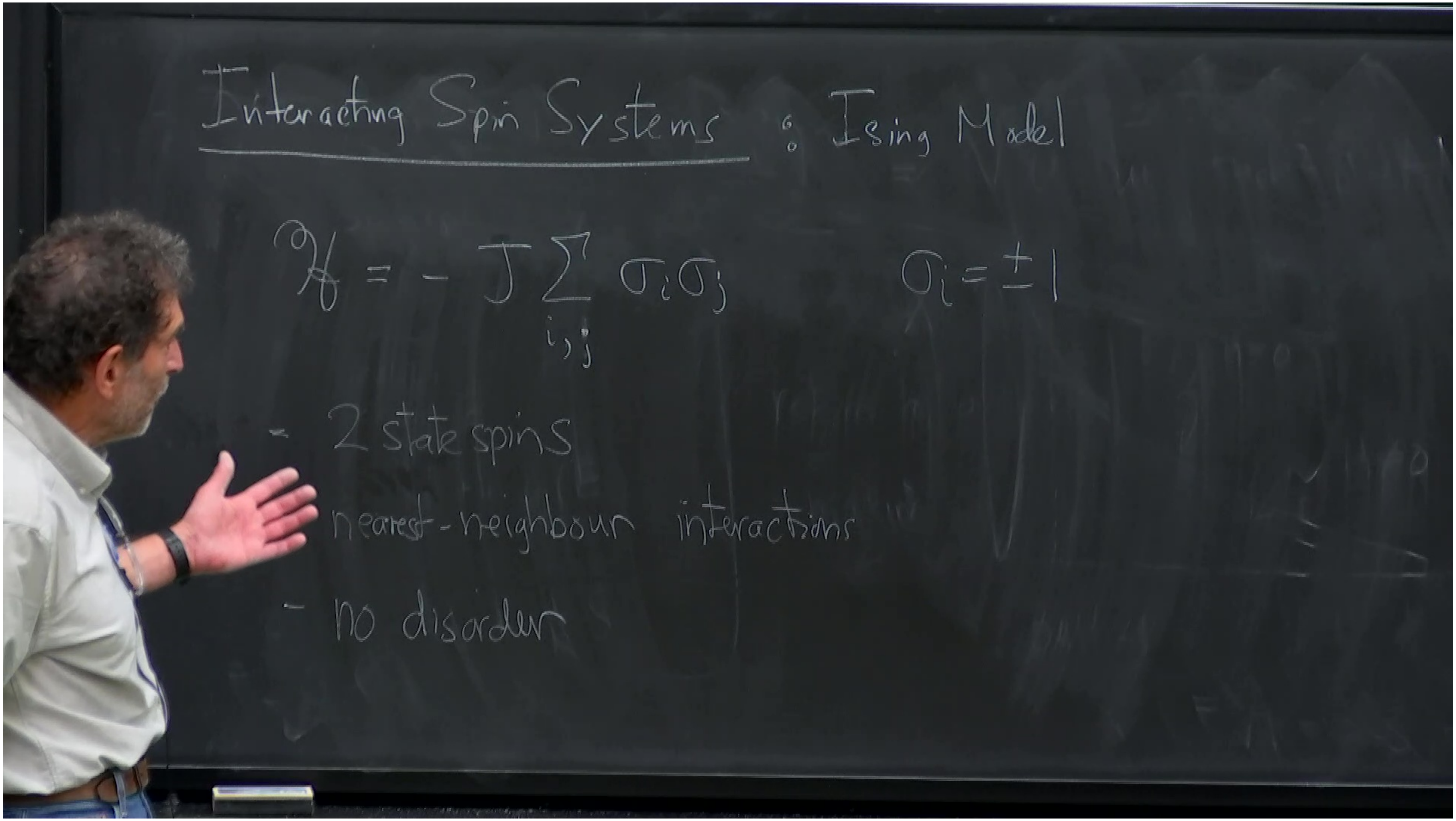
$$S = -\left(\frac{\partial F}{\partial T}\right)_H = Nk \ln 2\cos \beta\mu H - Nk\beta\mu H \tanh \beta\mu H$$

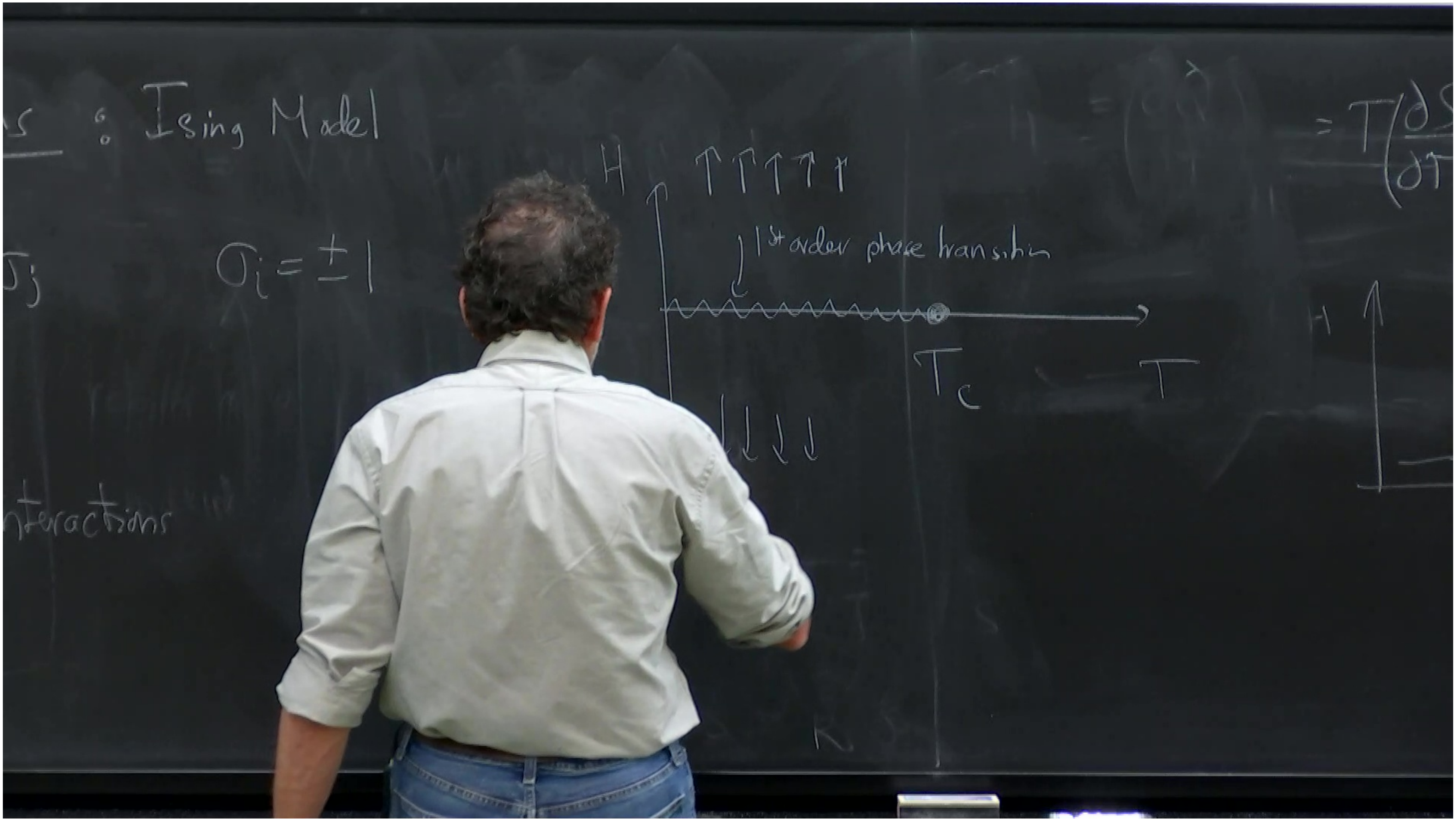

-P
$M = -\left(\frac{\partial F}{\partial H}\right)_T = N\mu \tanh \beta\mu H$
$S = -\left(\frac{\partial F}{\partial T}\right)_H = Nk \ln 2\cosh \beta\mu H$
$- Nk\beta\mu H \tanh \beta\mu H$
S
$Nk\ln 2$


χ ≡ isothermal susceptibility
= (∂M/∂H)_T
= N μ βμ sech² βμH
χ
H=0
H≠0
βμH≃1
T

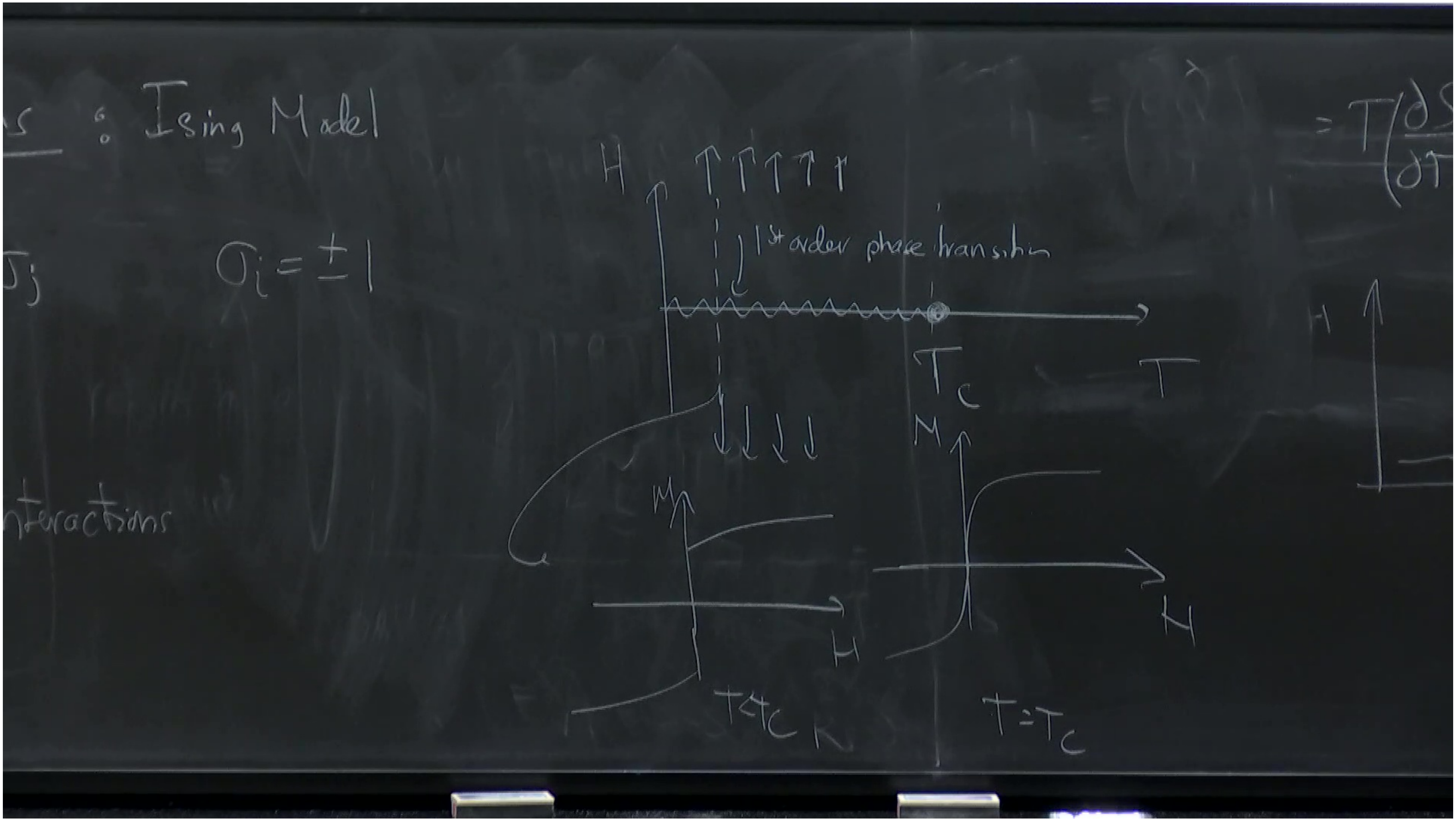
$$C_H = \left(\frac{\partial Q}{\partial T}\right)_H = T\left(\frac{\partial S}{\partial T}\right)_H = Nk(\beta\mu H)^2 \, \mathrm{sech}^2 \beta\mu H$$




Interacting Spin Systems : Ising Model
$\mathcal{H} = -J\sum_{i,j}\sigma_i\sigma_j$
$\sigma_i = \pm 1$
- 2 state spins
- nearest-neighbour interactions
- no disorder



Ising Model
$\sigma_i = \pm 1$
interactions
H
1st order phase transition
T_c
T


Ising Model
σi = ±1
nteractions
H
1st order phase transition
T
Tc
M
H
T<Tc
T=Tc