Title: Quantum Field Theory II - Lecture 221130

Speakers:

Collection: Quantum Field Theory II (2022/2023)

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SU(2) $A_\mu(x) = A_\mu^a \cdot t_a$ $t_a = \frac{1}{2}\sigma_a$ 3 generators of the Lie Alg. of SU(2) $[t_a, t_b] = i f_{abc} t_c$

↳ Adj Repr.

$f_{abc} = \epsilon_{abc}$ for SU(2)

2×2 traceless Hermitean matrix

Gauge transf.

$A_\mu(x) \to A_\mu(x) + D_\mu \alpha(x)$ $\alpha(x) = \alpha^a(x)\, t_a$

↳ 3 infinit. gauge transf

$D_\mu \alpha(x) = \partial_\mu \alpha(x) - i\,[A_\mu(x), \alpha(x)]$ D_μ cov. derivative

SU(2)

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$g(x) \in SU(2)$ $A_\mu(x) \longrightarrow g(x) \cdot [A_\mu(x) - i\partial_\mu]\, g^{-1}(x)$

$a = \frac{1}{2}\sigma_a$ 3 generators

f the Lie Alg. of $SU(2)$

natrix

$= \alpha^a(x)\, t_a$

↳ 3 infinit. gauge transf

v. derivation

$$\boxed{)+i\partial_\mu]g^{-1}(x) = A'_\mu(x)}$$

$$[t_a, t_b] = i f_{abc}\, t_c$$

$f_{abc} = \epsilon_{abc}$ for $SU(2)$

reduces to

$$g(x) = 1 + i\alpha(x) + \cdots$$

$U(1)$ maxwell

$$g(x) = e^{i\alpha(x)}$$

$$A_\mu \to e^{i\alpha(x)}(A_\mu + i\partial_\mu)e^{-i\alpha(x)}$$

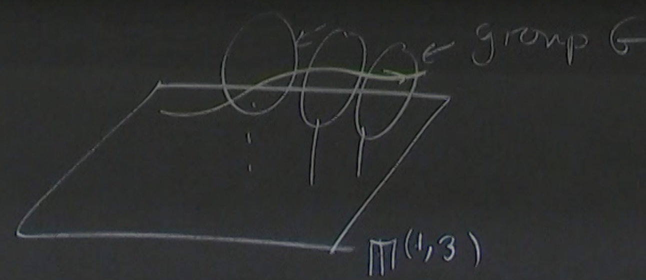
$$= A_\mu + \partial_\mu \alpha$$

$g(x) \in SU(2)$ $\quad$ $A_\mu(x) \longrightarrow g(x)\cdot[A_\mu($

$$A(x) = A_\mu(x)\cdot dx^\mu$$

$\uparrow$ 2×2 matrices

connexion on a fiber bundle



$$A(x) \longrightarrow g(x)\left(A(x) + i\, dx_\mu \frac{\partial}{\partial x^\mu}\right) g^{-1}(x) = A_g(x)$$

compact notation

$$\to g(x)\cdot\left[A_\mu(x) + i\partial_\mu\right]g^{-1}(x) = A'_\mu(x)$$

$$\int D[A]\, \exp\left(i S[A]\right)$$

Formal Functional Integral

$$S[A] = -\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}\left[F_{\mu\nu}\cdot F^{\mu\nu}\right]$$

$$S[A_g] = S[A]$$ gauge invariance

$$\boxed{g(x)\cdot\left[A_\mu(x) + i\partial_\mu\right] g^{-1}(x) = A'_\mu(x)}$$

$$\int D[A] \exp\left(iS[A]\right)$$

Formal Functional Integral

$$D[A] = \prod_x \prod_\mu \prod_a dA^a_\mu(x)$$

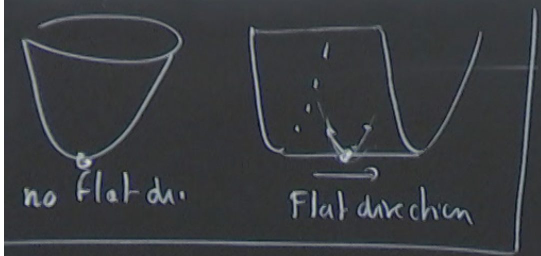
$$D[A_g] = D[A] \quad \text{for any } g$$

$$S[A] = -\frac{1}{2g^2}\int d^4x \, \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu}\right]$$

$$S[A_g] = S[A] \quad \text{gauge invariance}$$

"Flat Directions" $\Rightarrow$ propagator is ill defined

to solve it: Choosing a gauge <u>fixing condition</u>



$$\int D[A] \stackrel{?}{=} \int D[A]\, \delta[F[A]]$$

$$\delta[F[A]] = \prod_x \prod_a \delta[F^a(x)] = \prod_x \prod_x \delta[\partial^\nu A^a_\nu(x)]$$

Dirac δ-function

hoosing a transverse slices

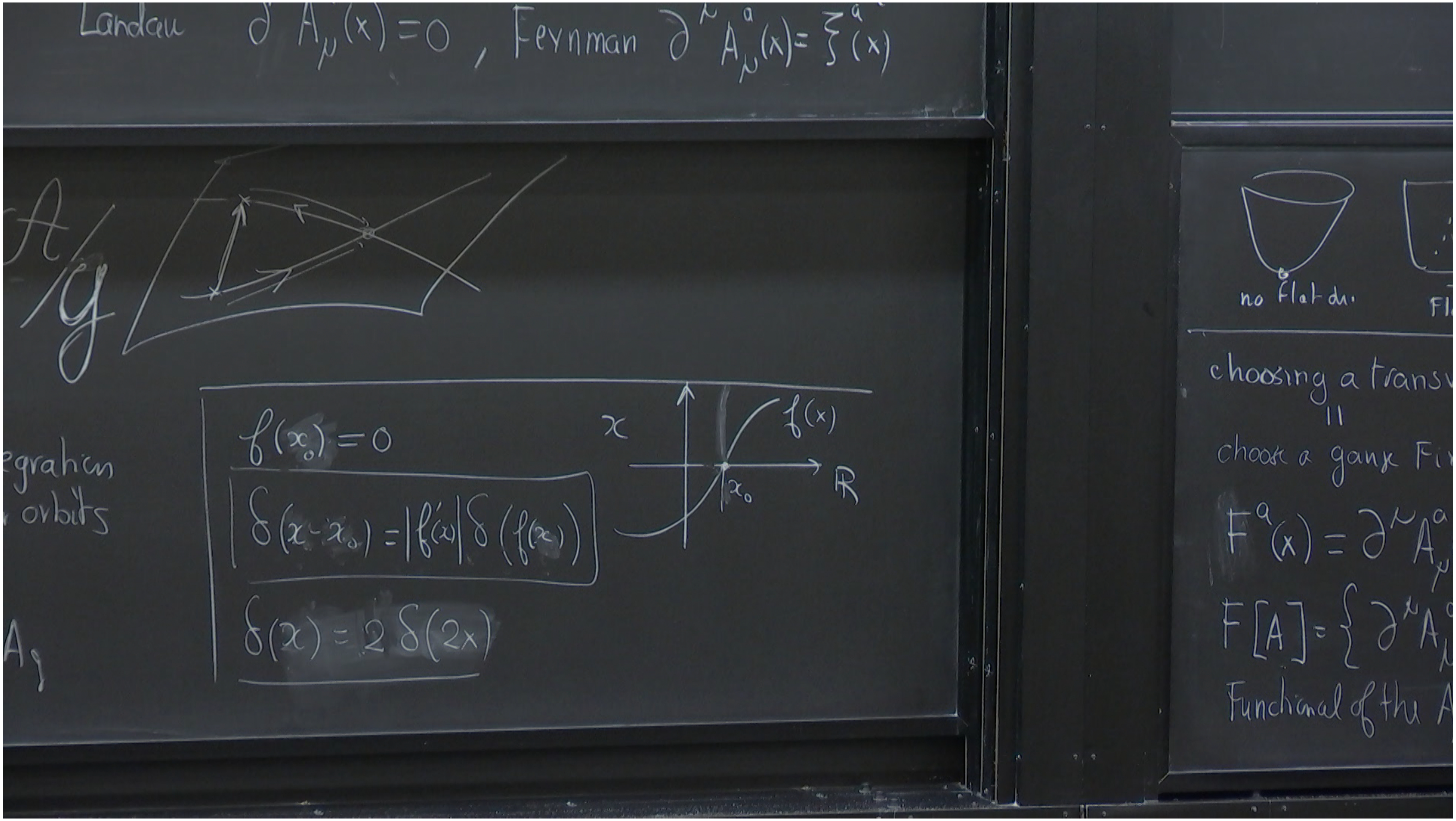
||

hoose a gauge Fixing condition

$$F^a(x) = \partial^\nu A^a_\nu(x) = 0$$

$$F[A] = \{ \partial^\nu A^a_\nu(x) : a, x \}$$

Functional of the $A^a_\nu(x)$ $\longrightarrow$ $F^a(x) := \partial^\nu A^a_\nu(x)$


Landau $\partial^\mu A_\mu(x)=0$, Feynman $\partial^\mu A^a_\mu(x)=\xi^a(x)$
$\mathcal{A}/\mathcal{G}$
$f(x_0)=0$
$\delta(x-x_0)=|f'(x)|\,\delta(f(x))$
$\delta(x)=2\,\delta(2x)$
x
$f(x)$
x_0
$\mathbb{R}$
egration
orbits
no flat dir.
choosing a transv
choose a gauge Fix
$F^a(x)=\partial^\mu A^a_\mu$
$F[A]=\{\partial^\mu A_\mu$
Functional of the A

Functional of the $A^a_\nu(x) \longrightarrow F^a(x) := \partial^\nu A^a_\nu(x)$

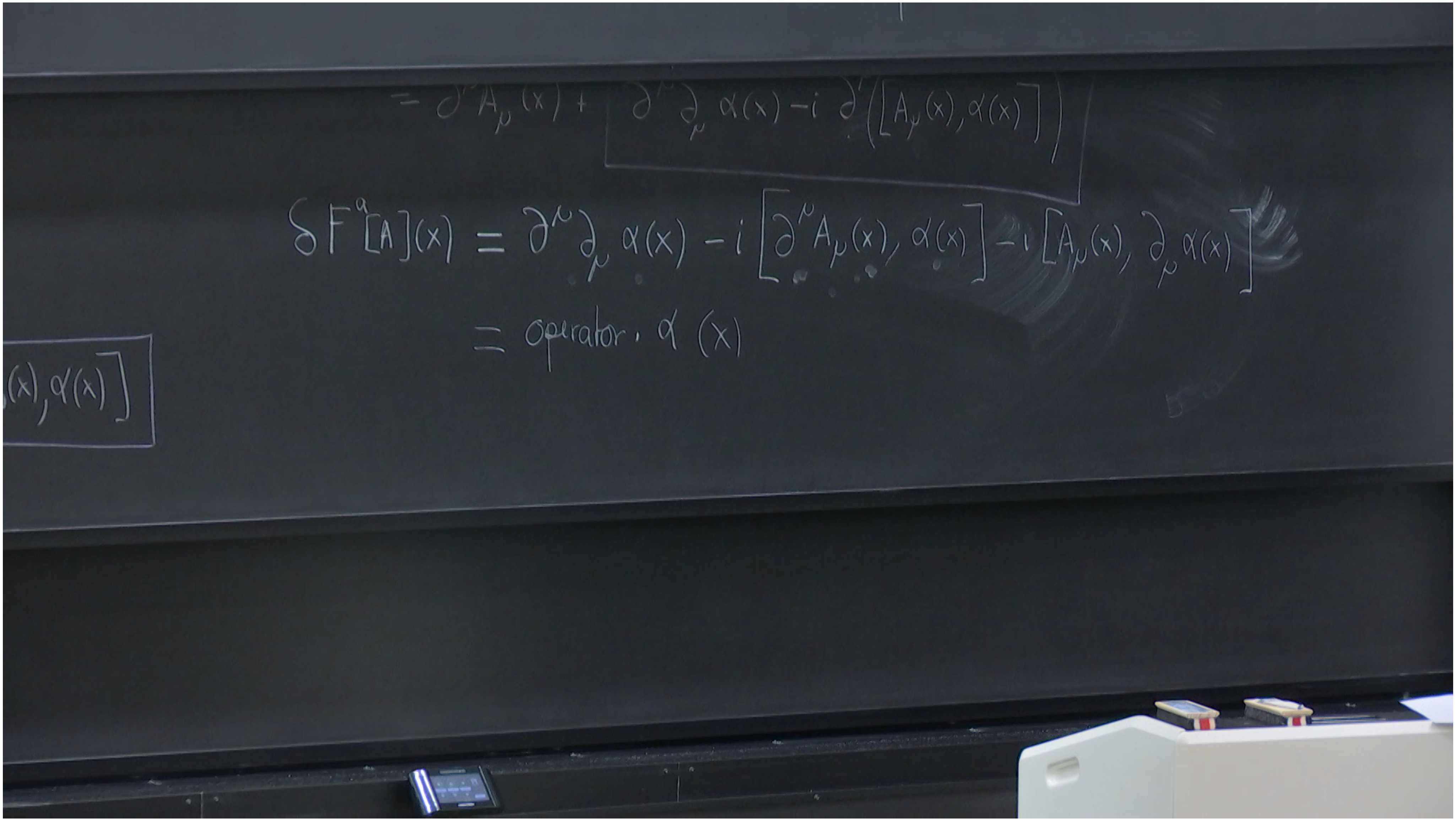
What is this " $F[A]$ "

How does gauge Fixing $F^a[A](x) = \partial^\mu A^a_\mu(x)$

changes when I perform a gauge transformation?

$$g(x) = \mathbb{1} + i\,\alpha(x) + \cdots$$

$$A_\mu(x) \longrightarrow A_\mu(x) + \delta A_\mu(x) = A_\mu(x) + D_\mu \alpha(x) + \cdots$$


$= \partial^\nu A_\nu(x) + \partial^\mu \partial_\mu \alpha(x) - i\, \partial^\nu([A_\nu(x), \alpha(x)])$
$\delta F^a[A](x) = \partial^\mu \partial_\mu \alpha(x) - i\left[\partial^\mu A_\mu(x), \alpha(x)\right] - i\left[A_\mu(x), \partial_\mu \alpha(x)\right]$
$= \text{operator} \cdot \alpha(x)$
$(x), \alpha(x)]$

$$= \partial^\mu A^a_\mu(x) + \boxed{\partial^\mu \partial_\mu \alpha(x) - i\,\partial^\mu\left(\left[A_\mu(x), \alpha(x)\right]\right)}$$

$$\delta F^a[A](x) = \partial^\mu \partial_\mu \alpha(x) - i\left[\left(\partial^\mu A_\mu(x)\right), \alpha(x)\right] - i\left[A_\mu(x), \left(\partial^\mu \alpha(x)\right)\right]$$

$$= \underset{\text{differential}}{\text{operator}} \cdot \alpha(x)$$

$$\boxed{\ldots(x), \alpha(x)]}$$