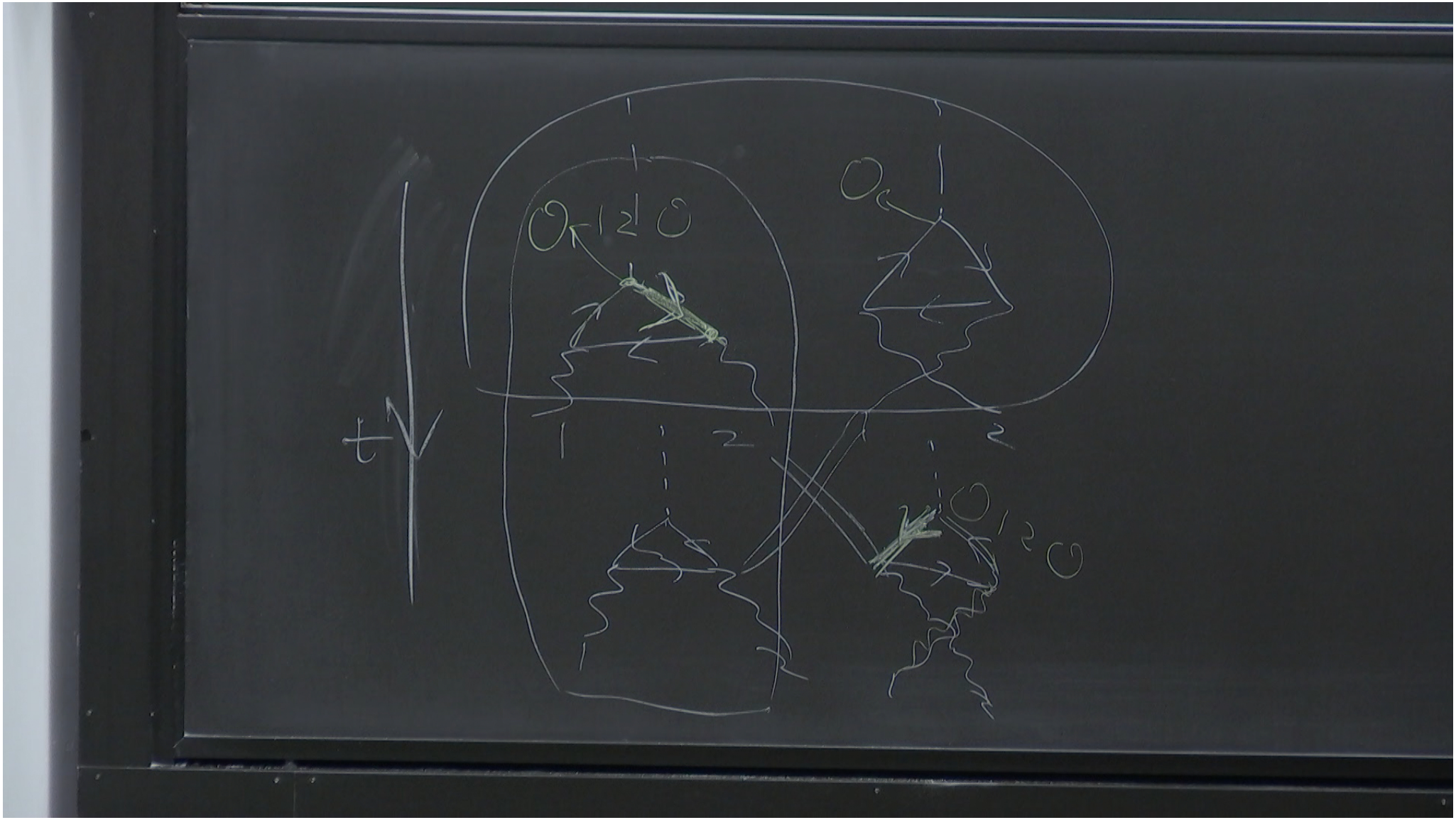
Title: Quantum Field Theory I - Lecture 221108

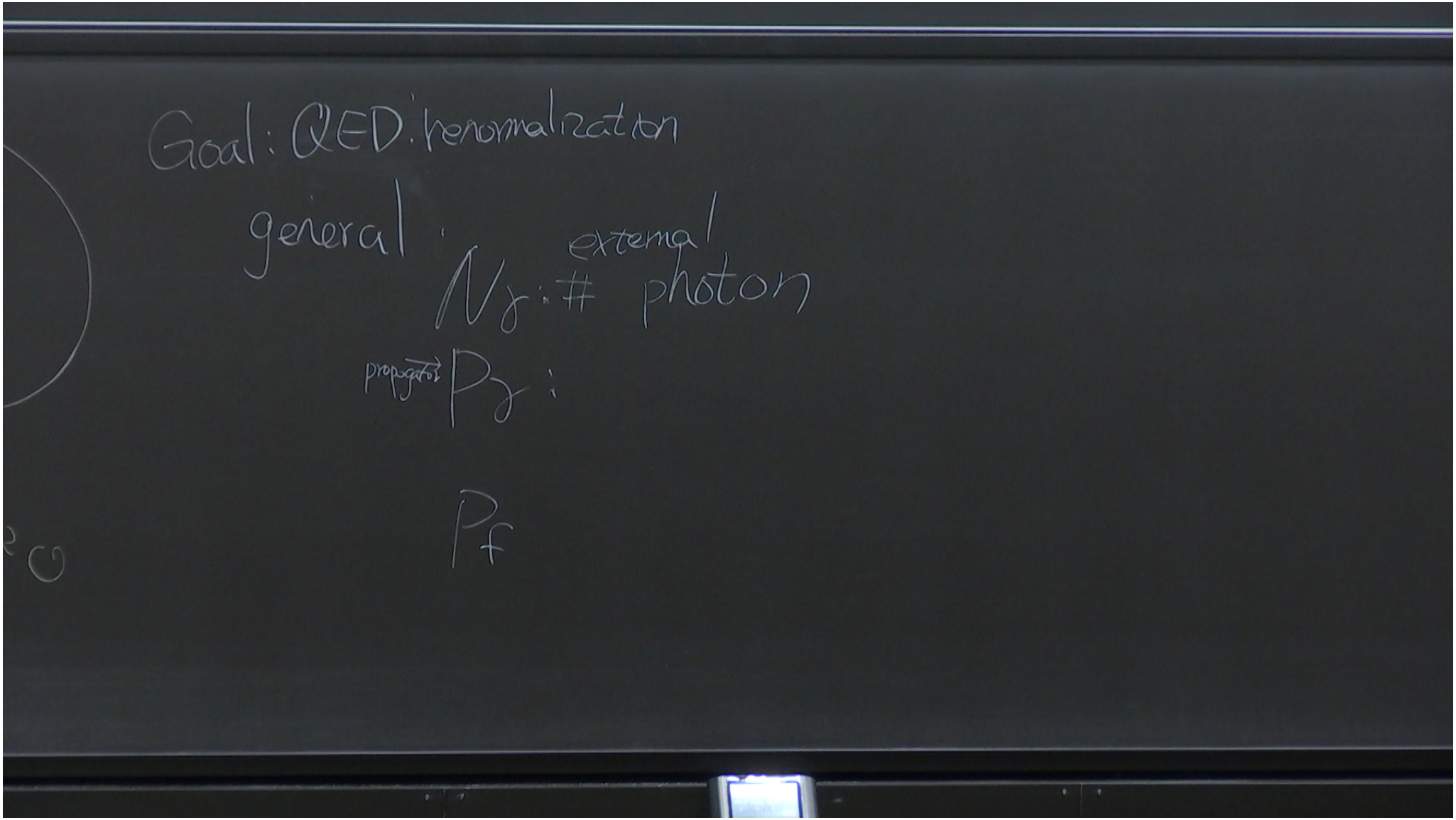
Speakers:

Collection: Quantum Field Theory I (2022/2023)

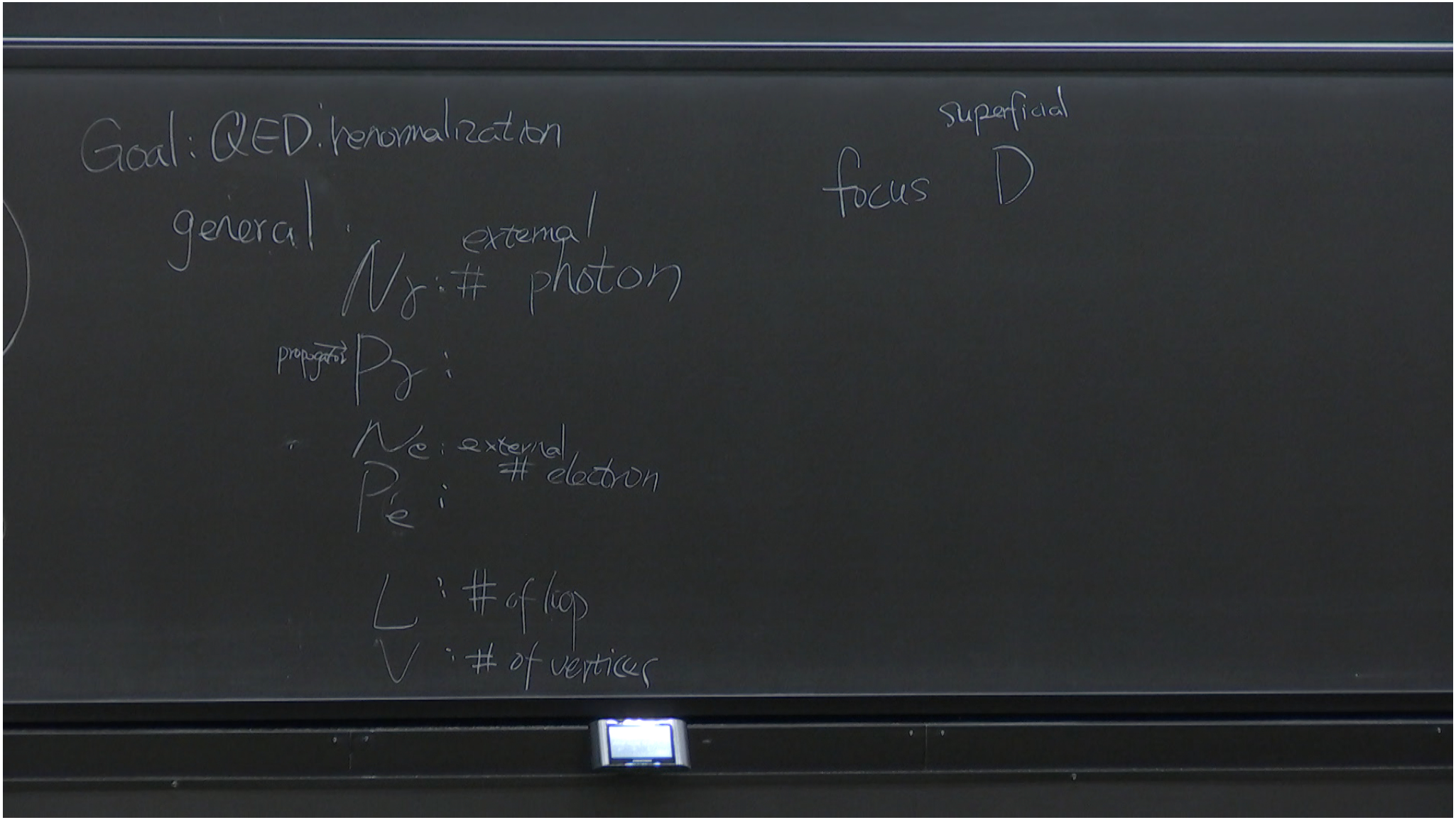
Date: November 08, 2022 - 10:45 AM

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Goal: QED: renormalization
general:
external
N_γ: # photon
propagator P_γ:
P_f


To Robert
neat


Goal: QED: renormalization
general:
external
N_γ: # photon
propagator P_γ:
N_e: external # electron
P_e:
L: # of loop
V: # of vertices
superficial
focus D

superficial

focus D = power of p

$\int d^4p \frac{\text{vertices}}{\text{propagator}}$ Λ^2 $\int d_{pp}$

$\int d^4p\, 1 = p^4$

= power of p numerator

= power of p denominator

$\mu\sim\sim\nu$ $\frac{i\eta^{\mu\nu}}{p^2-m^2+i\epsilon}$

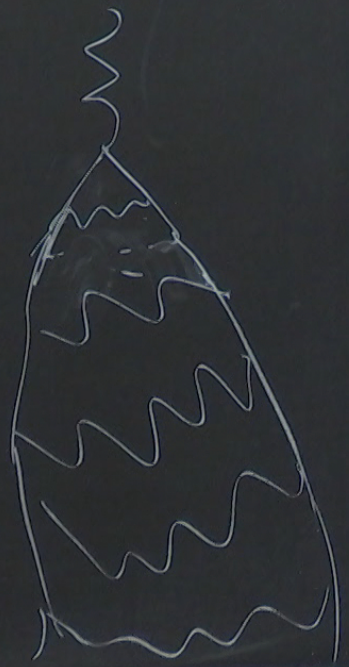
$\rightarrow$ $\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$

$= 4L - 2P_f - P_e$

$\int d^4p \frac{\text{vertices}}{\text{propagator}}$

$\int d^4p\, 1 = p^4$

$\int d^4p\, p$

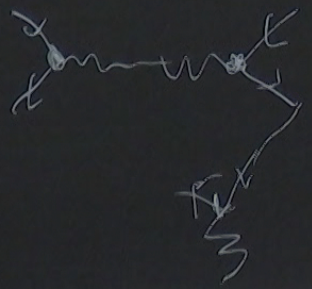


$\gamma: V \times 1 = N_\gamma + 2$

$e: V \times 2 = N_e + 2$

$\gamma: V \times 1 = N_\gamma + 2P_\gamma$

$e: V \times 2 = N_e + 2P_e$

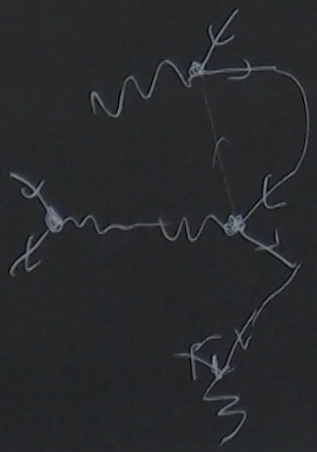


$f: V \times 1 = N_f + 2P_f$

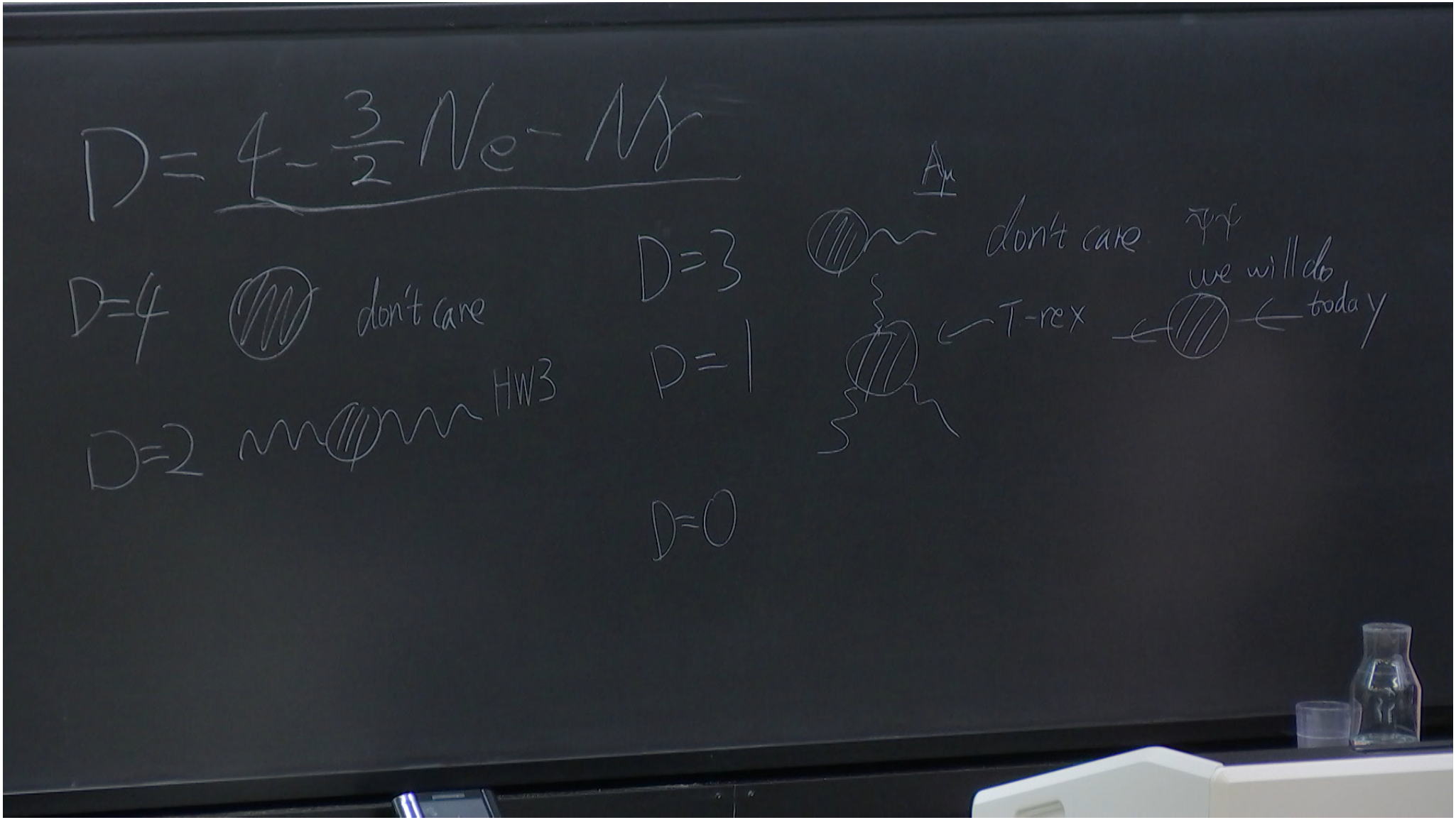
$e: V \times 2 = N_e + 2P_e$

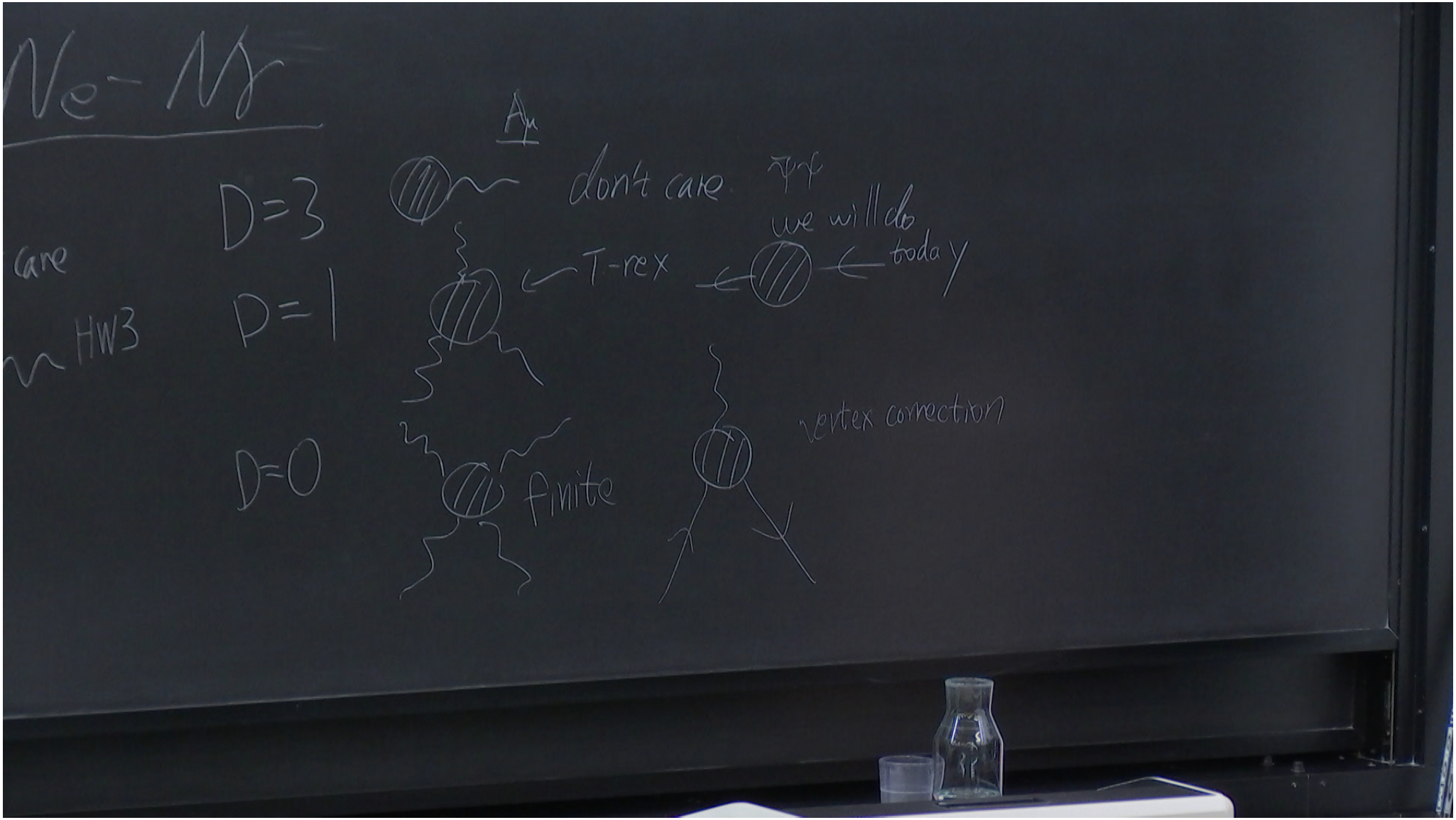
tree

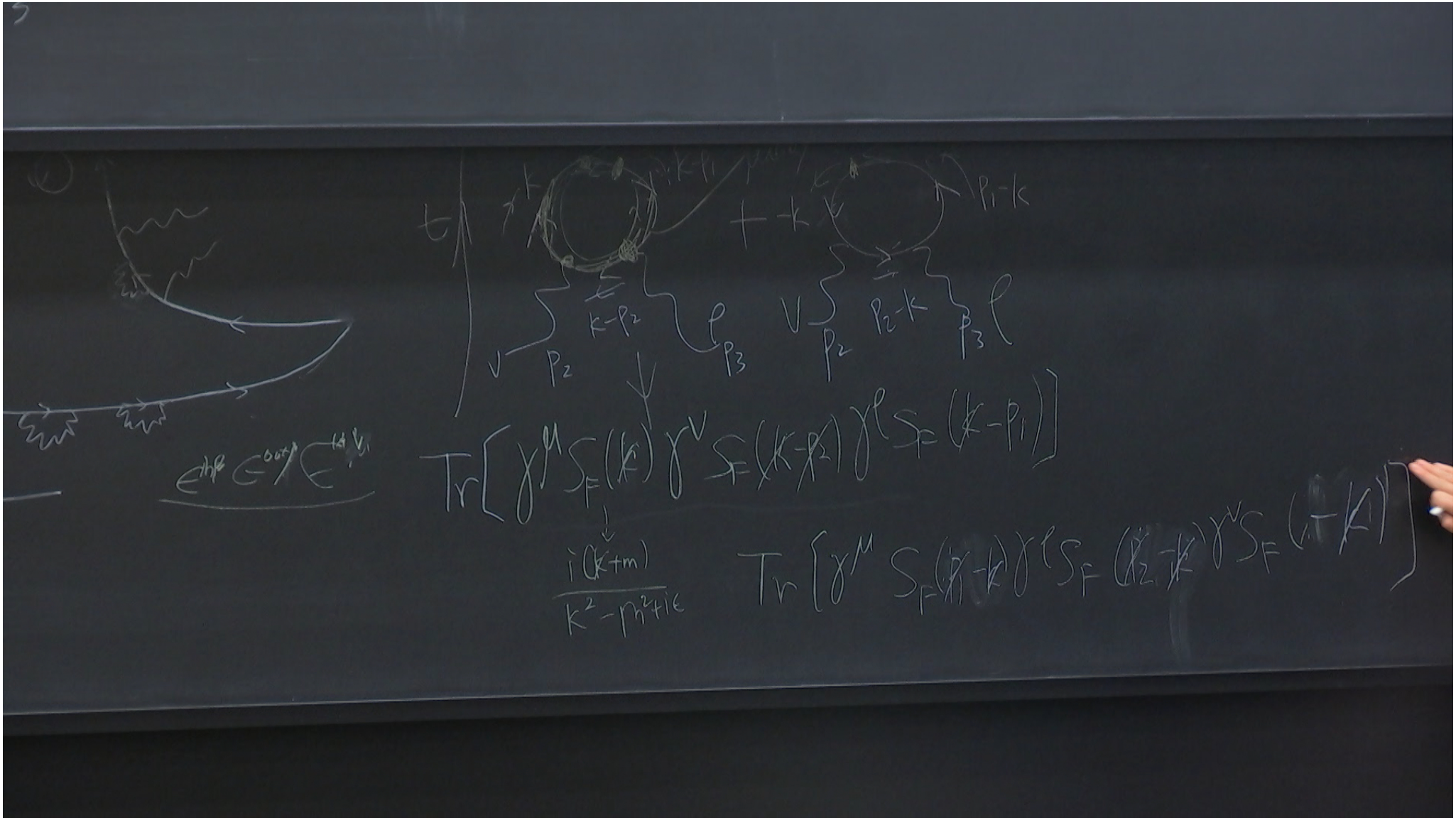
$(P_e + P_f) = V - 1 + L$

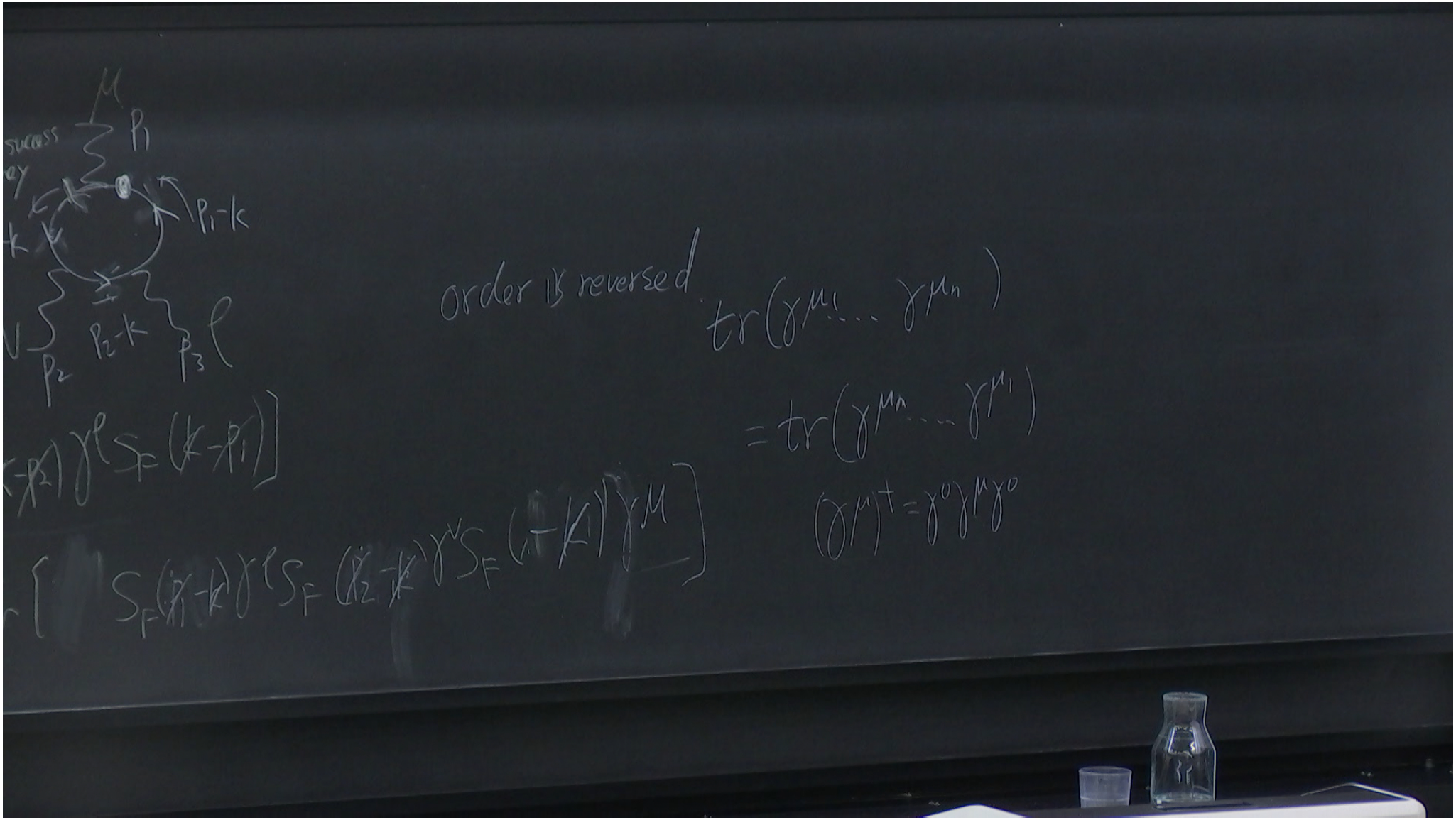


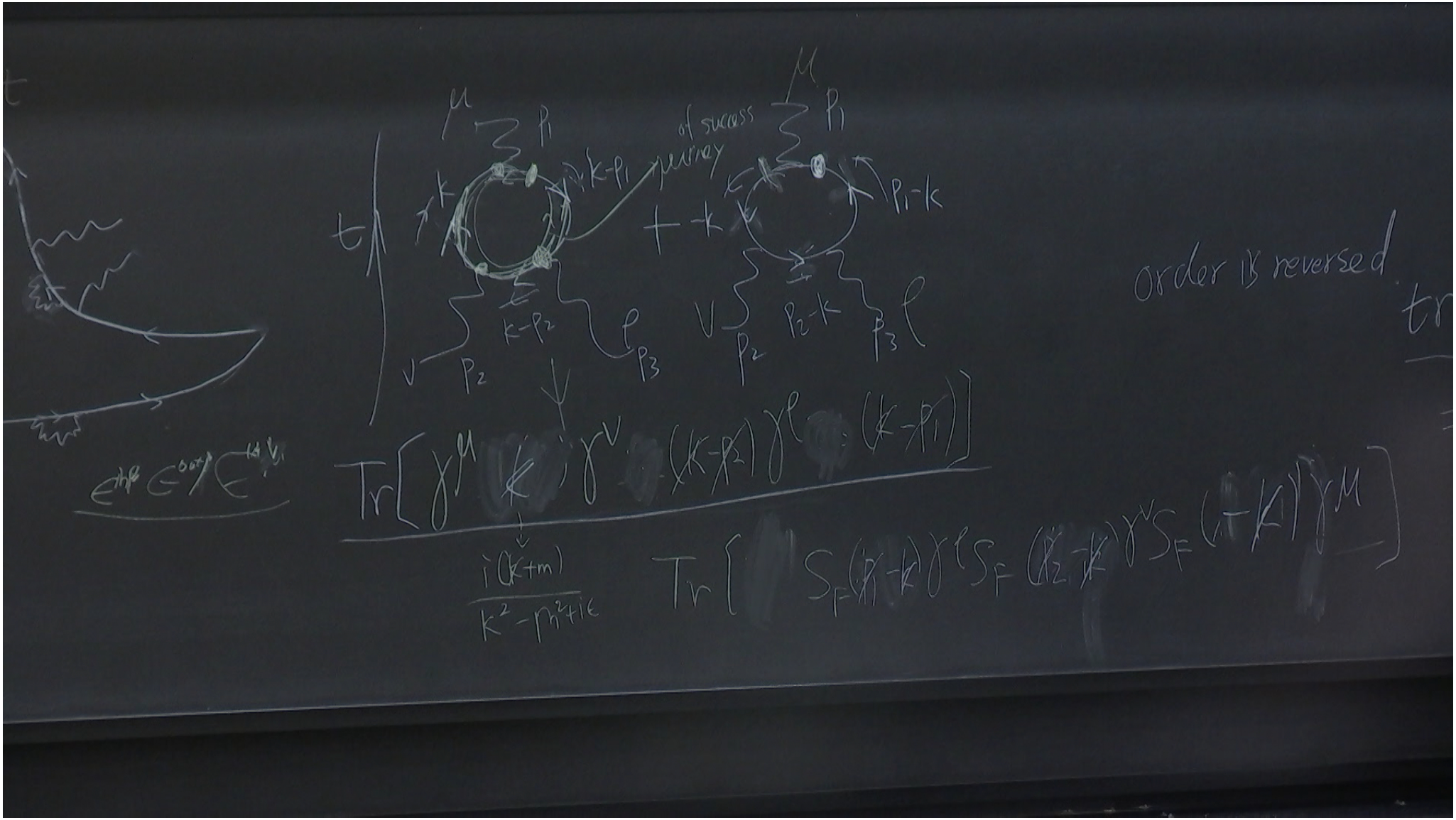
$V - I + L = 1$

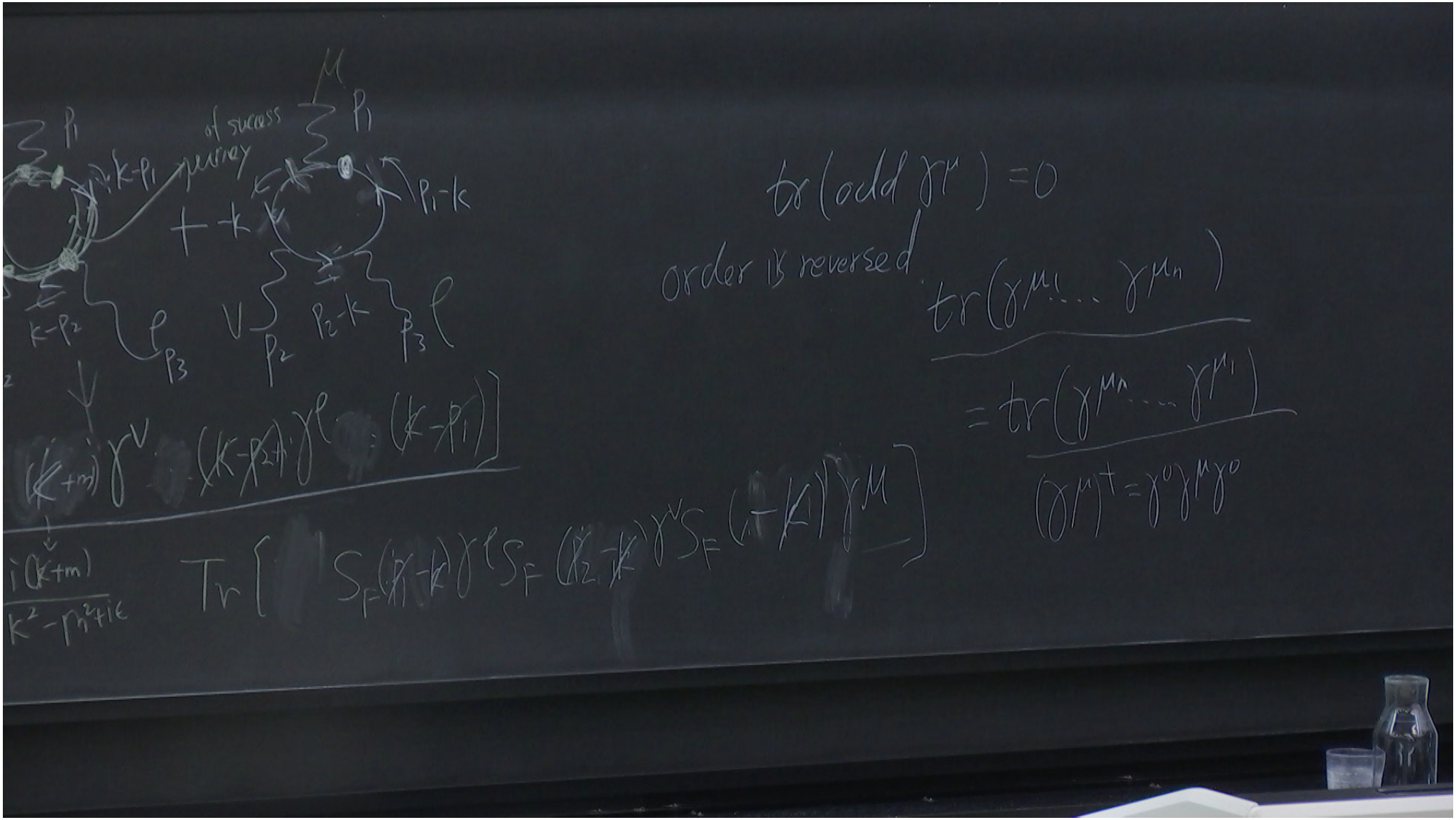

$D = 4 - \frac{3}{2} N_e - N_f$
D=4
don't care
D=2
HW3
D=3
D=1
D=0
Au
don't care
T-rex
we will do today

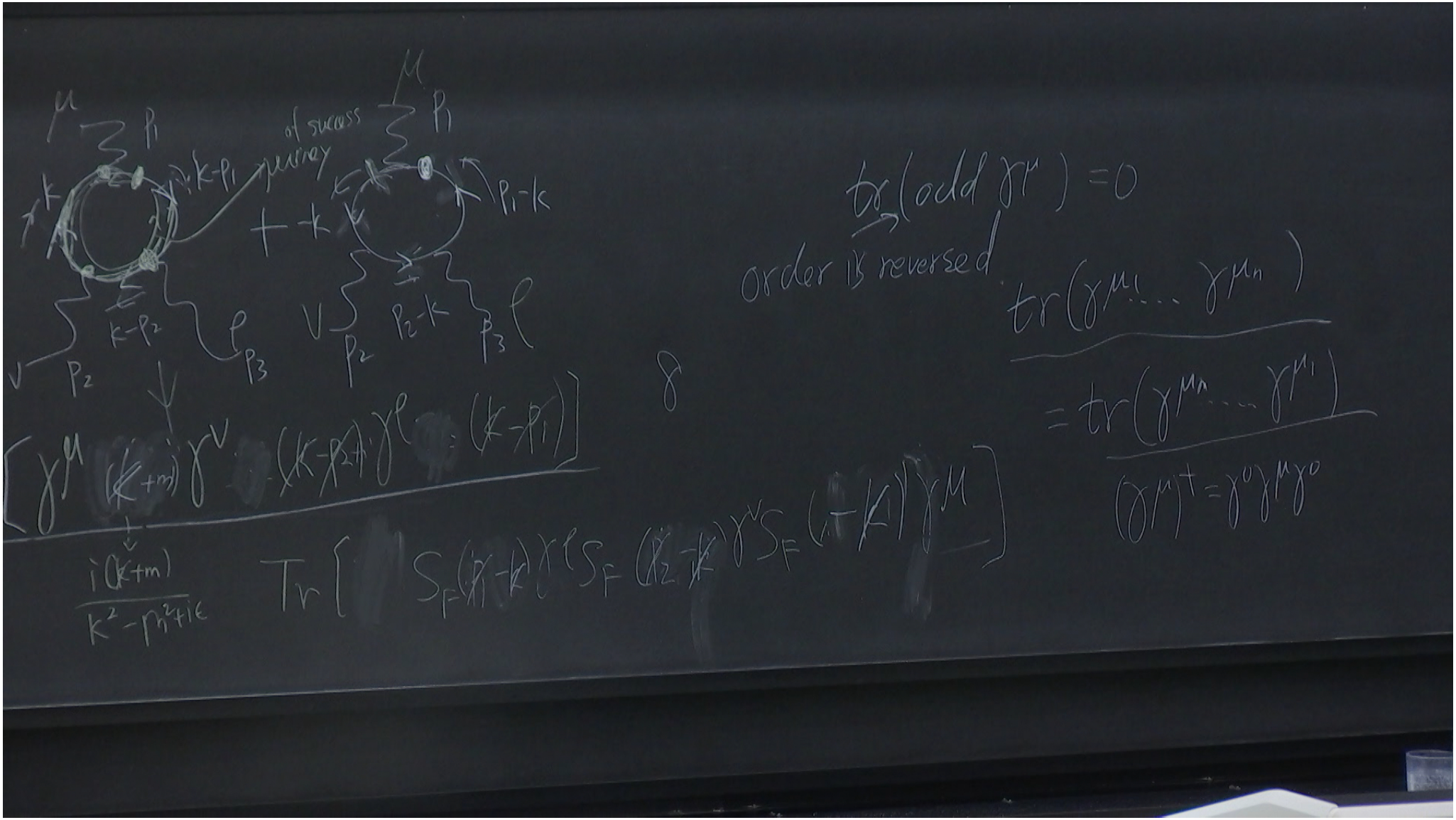

Ne - Nr
D=3
don't care
we will do today
T-rex
D=1
HW3
D=0
finite
vertex correction

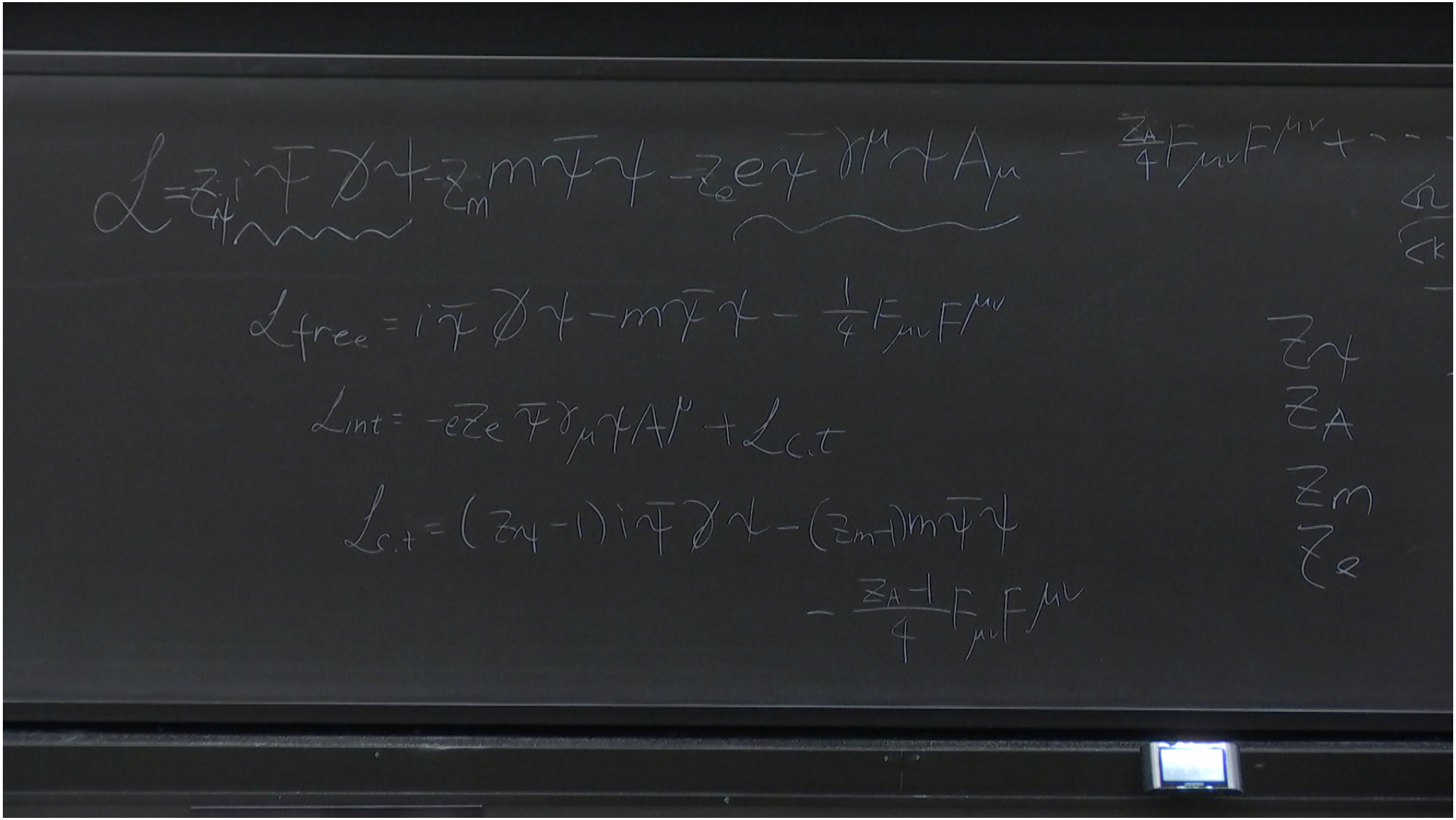

$p_1 - k$
$k - p_2$
$p_2 - k$
p_2
p_3
$\mathrm{Tr}\left[\gamma^\mu S_F(k)\,\gamma^\nu S_F(k-p_2)\,\gamma^\rho S_F(k-p_1)\right]$
$\frac{i(k+m)}{k^2-m^2+i\epsilon}$
$\mathrm{Tr}\left[\gamma^\mu S_F(p_1-k)\,\gamma^\rho S_F(p_2-k)\,\gamma^\nu S_F(p_1-k)\right]$

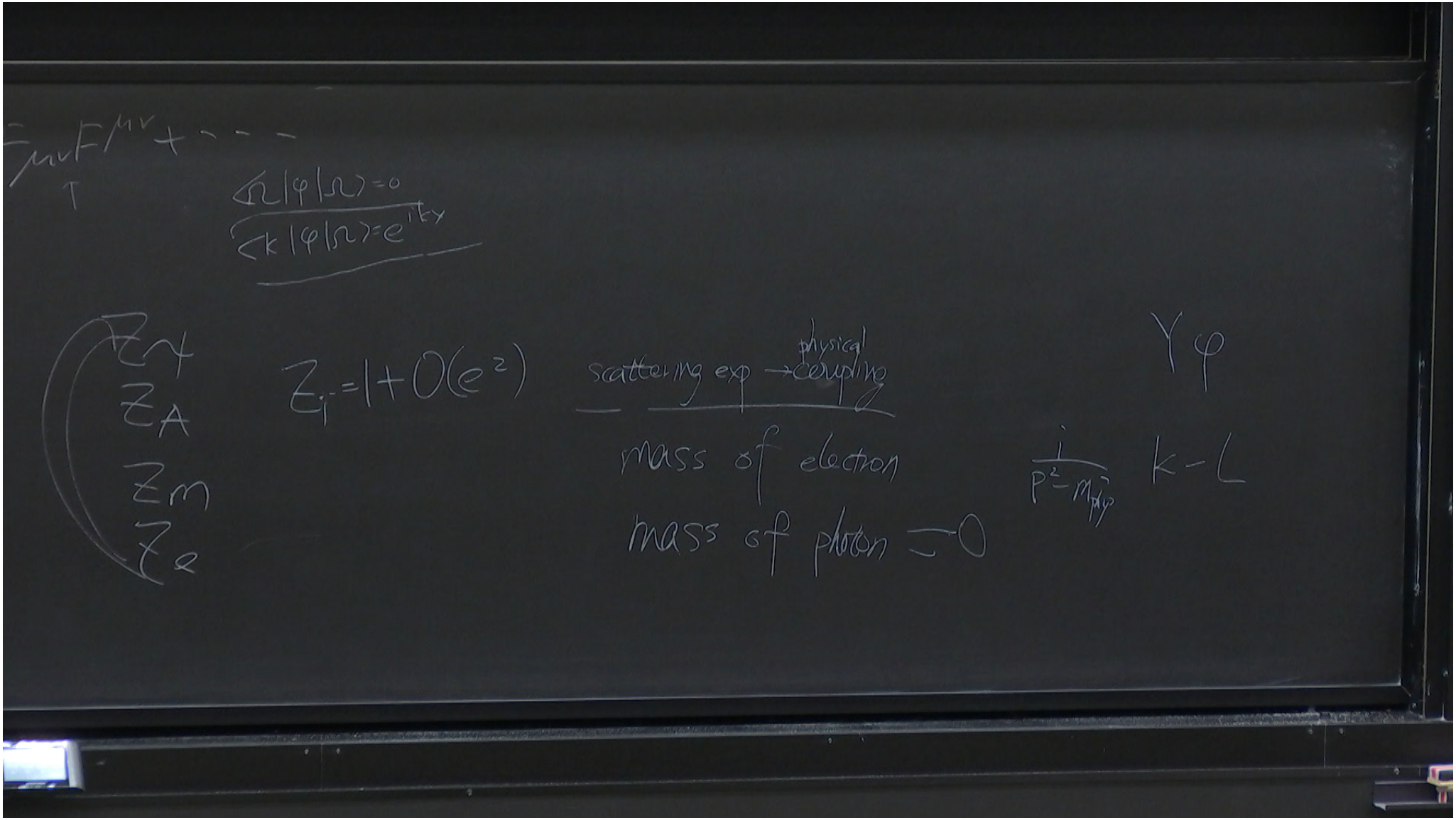

order is reversed
$tr(\gamma^{\mu_1} \dots \gamma^{\mu_n})$
$= tr(\gamma^{\mu_n} \dots \gamma^{\mu_1})$
$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$

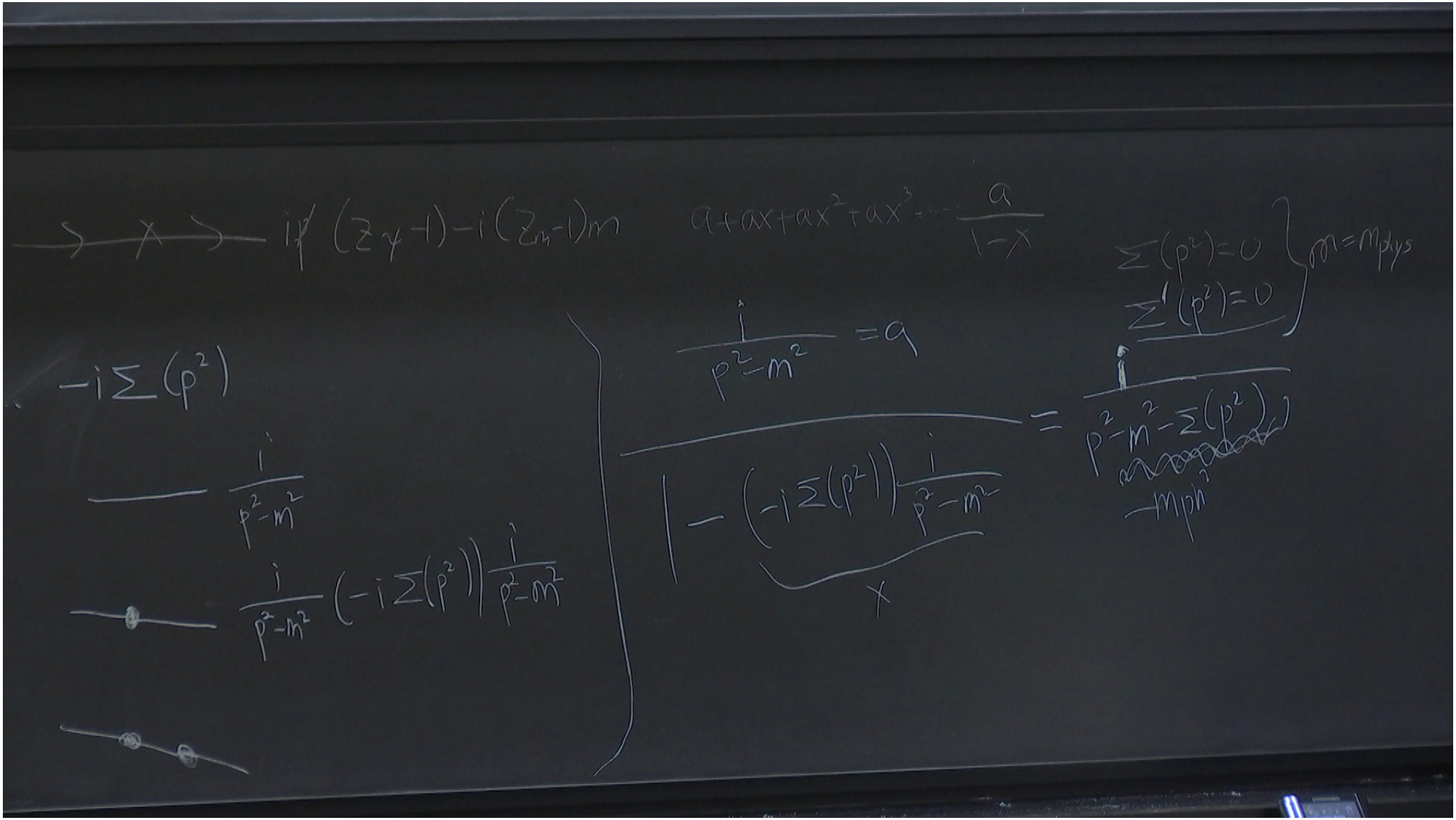

of success
journey
order is reversed

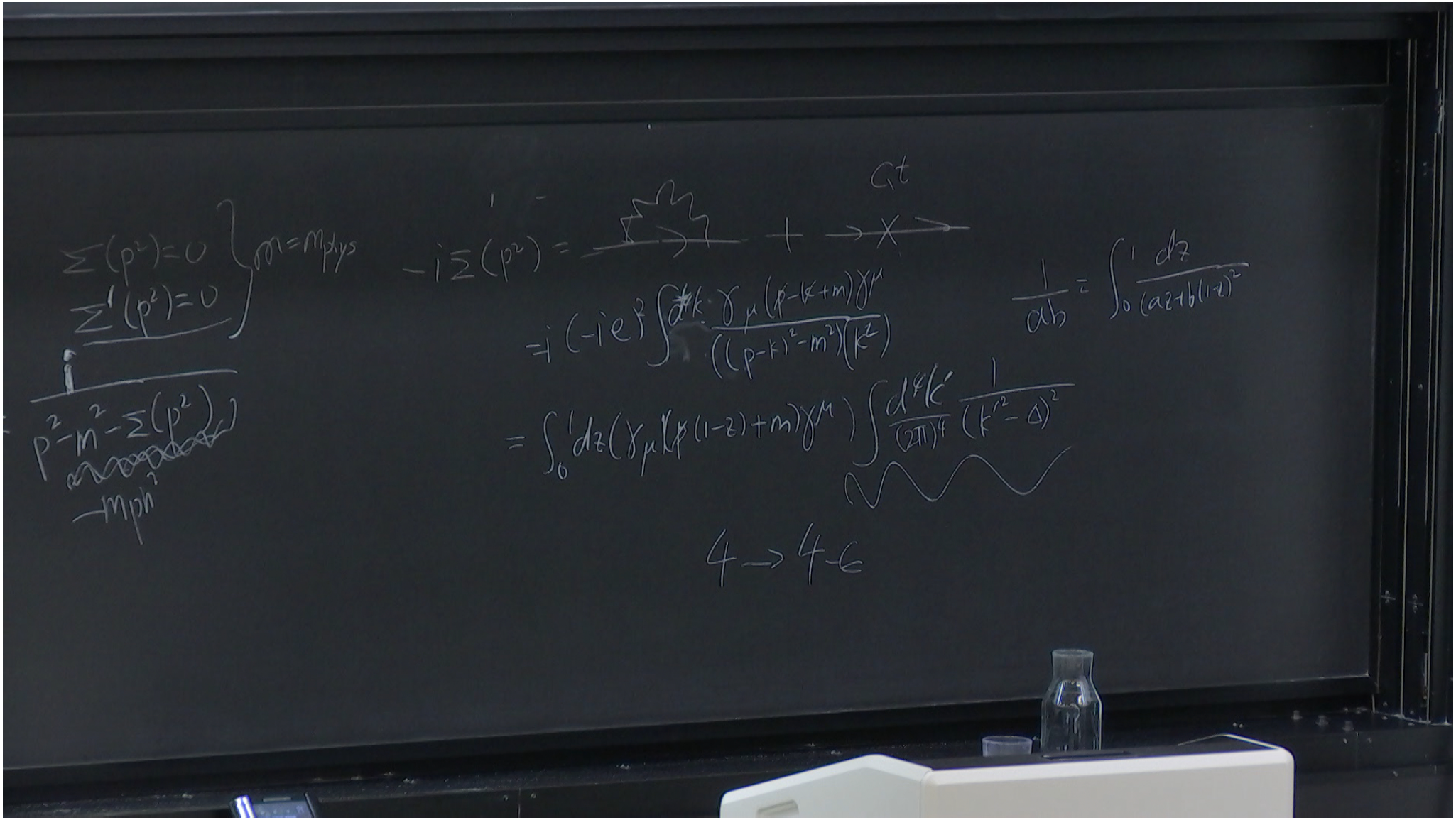

of success
journey
p_1
p_1
$k-p_1$
p_1-k
$k-p_2$
p_3
p_2
p_2-k
p_3
$\frac{i(\not{k}+m)}{k^2-m^2+i\epsilon}$
$\mathrm{Tr}[$
$\mathrm{tr}(\text{odd } \gamma^\mu) = 0$
order is reversed
$\mathrm{tr}(\gamma^{\mu_1} \dots \gamma^{\mu_n})$
$= \mathrm{tr}(\gamma^{\mu_n} \dots \gamma^{\mu_1})$
$(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$


of success
journey
order is reversed


$\mathcal{L} = Z_4 i\bar{\psi}\partial\psi - Z_m m\bar{\psi}\psi - Z_e e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{Z_A}{4}F_{\mu\nu}F^{\mu\nu} + \dots$
$\mathcal{L}_{free} = i\bar{\psi}\partial\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
$\mathcal{L}_{int} = -eZ_e\bar{\psi}\gamma_\mu\psi A^\mu + \mathcal{L}_{c.t}$
$\mathcal{L}_{c.t} = (Z_4 - 1)i\bar{\psi}\partial\psi - (Z_m - 1)m\bar{\psi}\psi$
$- \frac{Z_A - 1}{4}F_{\mu\nu}F^{\mu\nu}$
Z_4
Z_A
Z_m
Z_e


$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\cdots$
T
$\langle\Omega|\varphi|\Omega\rangle=0$
$\langle k|\varphi|\Omega\rangle=e^{ik\cdot x}$
Z_ψ
Z_A
Z_m
Z_e
$Z_i=1+O(e^2)$
scattering exp → physical coupling
mass of electron
mass of photon = 0
$Y\varphi$
$\frac{i}{p^2-m_{\gamma\gamma}^2}$
k - L


$i\not{p}(Z_\psi - 1) - i(Z_m - 1)m$
$a + ax + ax^2 + ax^3 \cdots = \frac{a}{1-x}$
$-i\Sigma(p^2)$
$\frac{i}{p^2 - m^2}$
$\frac{i}{p^2-m^2}(-i\Sigma(p^2))\frac{i}{p^2-m^2}$
$\frac{i}{p^2-m^2} = q$
$q - q(-i\Sigma(p^2))q + q(-i\Sigma(p^2))q(-i\Sigma(p^2))q - \cdots$
$= \frac{i}{p^2 - m^2 - \Sigma(p^2)}$
$\Sigma(p^2) = 0$
$\Sigma'(p^2) = 0$
$m = m_{phys}$
$p^2 - m_{ph}^2$


$\Sigma(p^2)=0$
$\Sigma'(p^2)=0$
$m=m_{phys}$
$\frac{i}{p^2-m^2-\Sigma(p^2)}$
$-m_{ph}^2$
$-i\Sigma(p^2) =$
$+$
Ct
$= i(-ie)^2 \int d^4k \frac{\gamma_\mu (\not{p}-\not{k}+m)\gamma^\mu}{((p-k)^2-m^2)(k^2)}$
$\frac{1}{ab} = \int_0^1 \frac{dz}{(az+b(1-z))^2}$
$= \int_0^1 dz (\gamma_\mu \not{k}(\not{p}(1-z)+m)\gamma^\mu) \int \frac{d^4k'}{(2\pi)^4} \frac{1}{(k'^2-\Delta)^2}$
$4 \to 4-\epsilon$

$$\int \frac{d^4k}{(2\pi)^d} \frac{1}{(k^2-\Delta)^2} = \frac{i}{(4\pi)^{d/2}} \Gamma\left(2-\frac{d}{2}\right)\left(\frac{1}{\Delta}\right)^{2-d/2}$$

$$\Gamma(x) = \frac{1}{x} + \ldots \qquad \Gamma(x)\Big|_{x=\epsilon} = \frac{1}{\epsilon} + \ldots$$

$$1 - \left(2-\frac{d}{2}\right)\log s$$

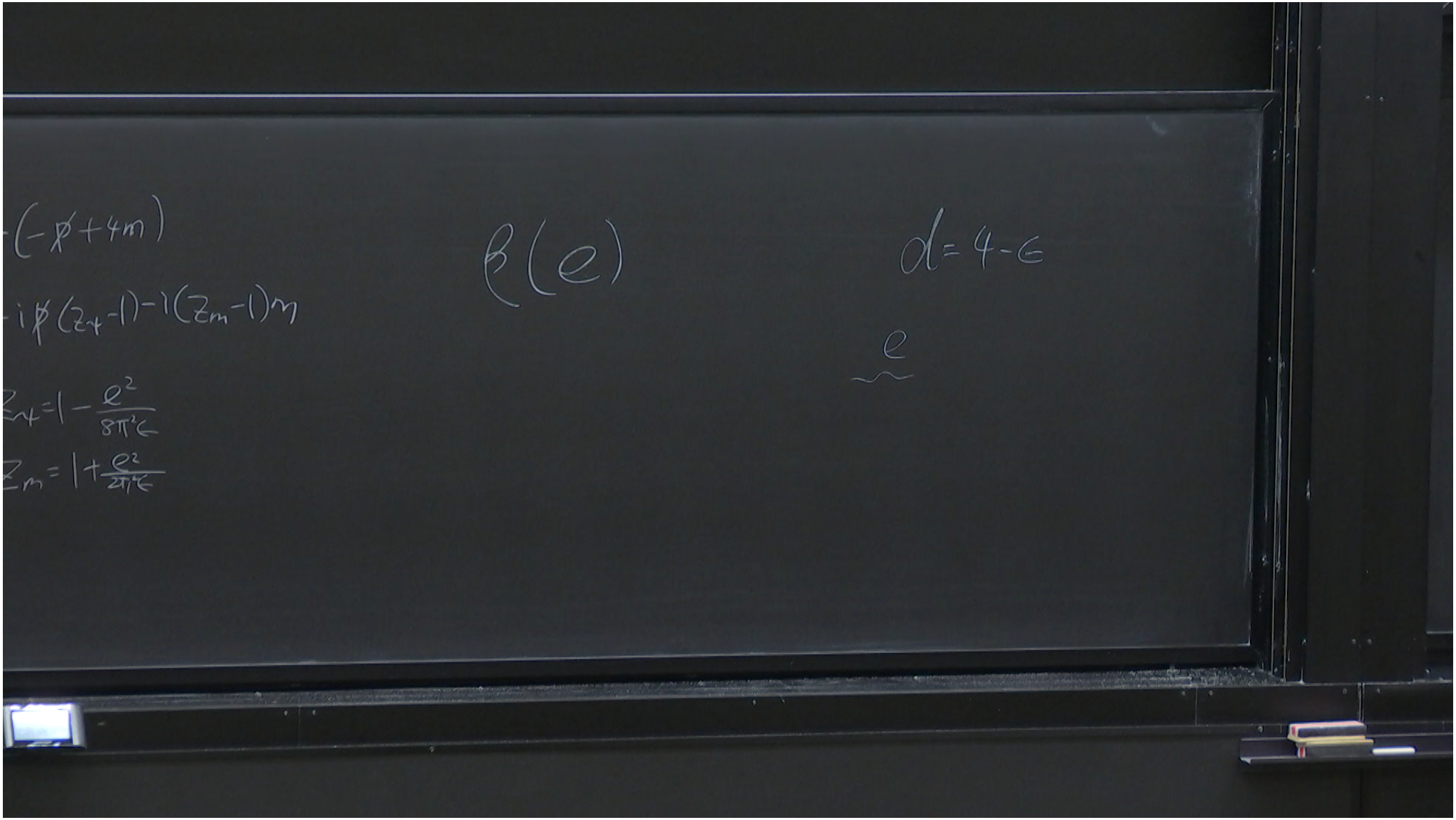
$$-iz(\not{p}) = -i\frac{e^2}{8\pi^2\epsilon}(-\not{p}+4m)$$

$d = 4-\epsilon$

$$+ i\not{p}(z_4 - 1) - i(z_m - 1)m$$

$$z_4 = z_{A4} = 1 - \frac{e^2}{8\pi^2\epsilon}$$

$$z_m = 1 + \frac{e^2}{2\pi^2\epsilon}$$


$-(-\not{p}+4m)$
$-i\not{p}(Z_4-1)-1(Z_m-1)m$
$Z_4=1-\frac{e^2}{8\pi^2\epsilon}$
$Z_m=1+\frac{e^2}{4\pi^2\epsilon}$
$\beta(e)$
$d=4-\epsilon$
e