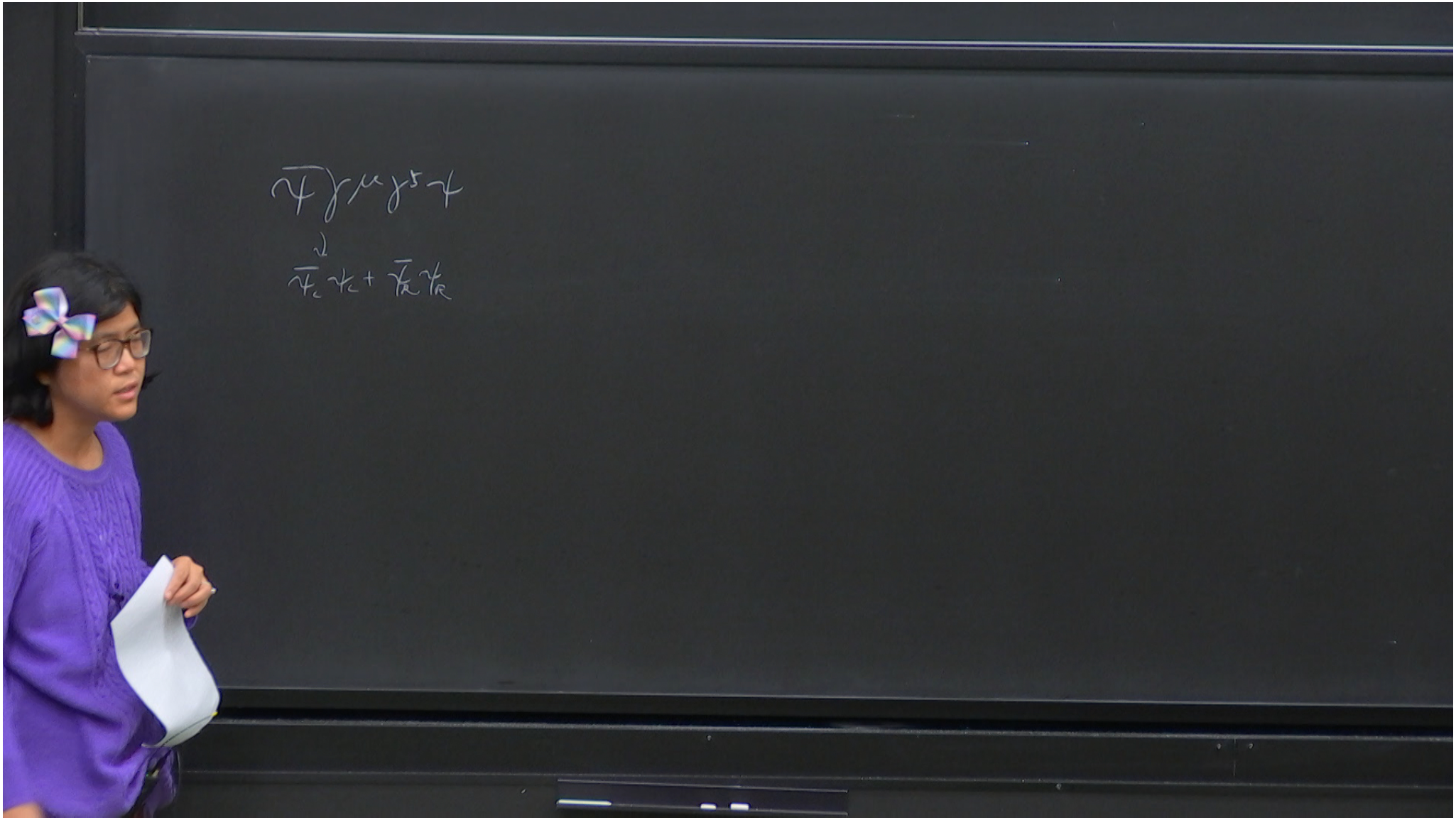
Title: Quantum Field Theory I - Lecture 221101

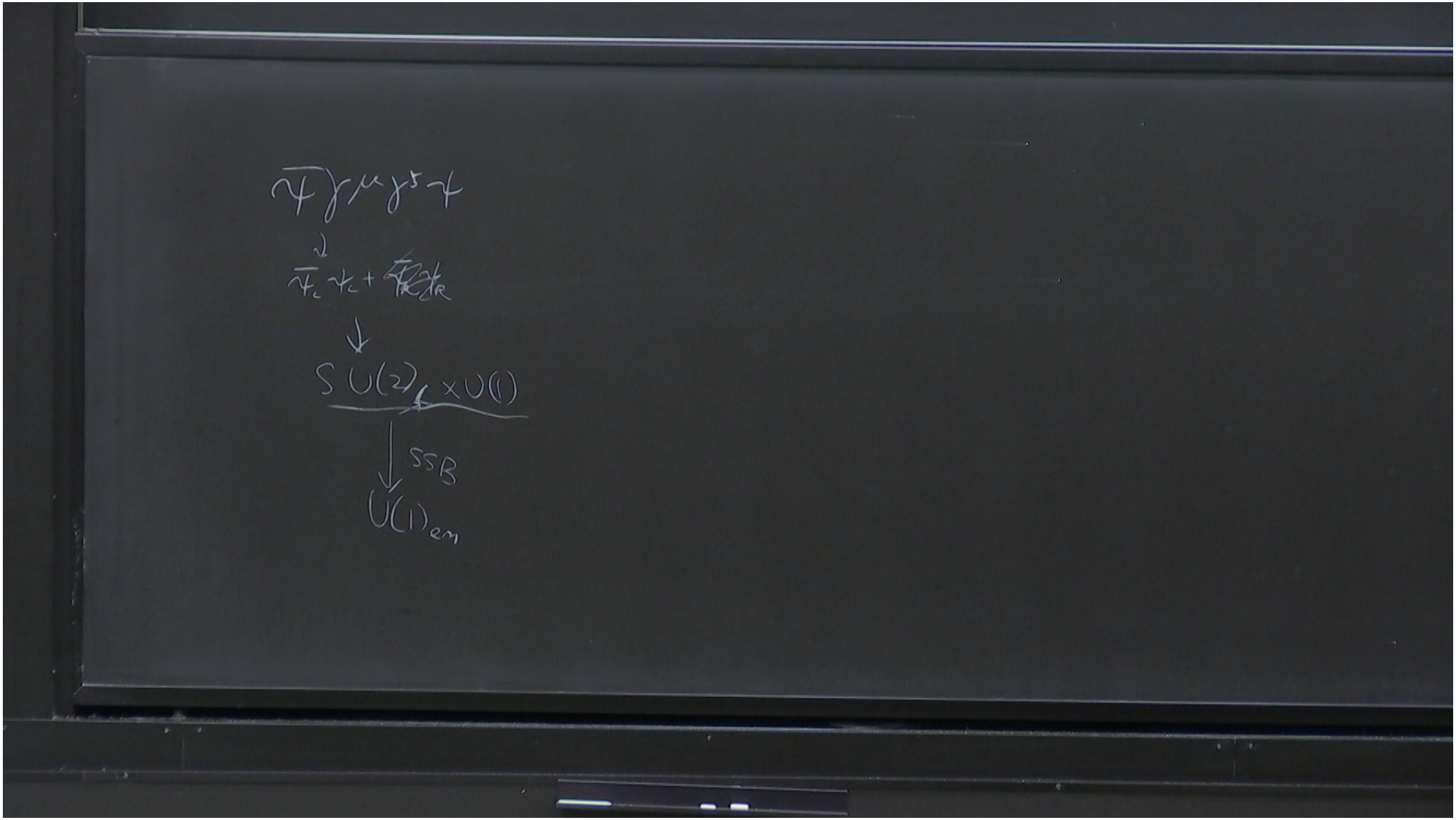
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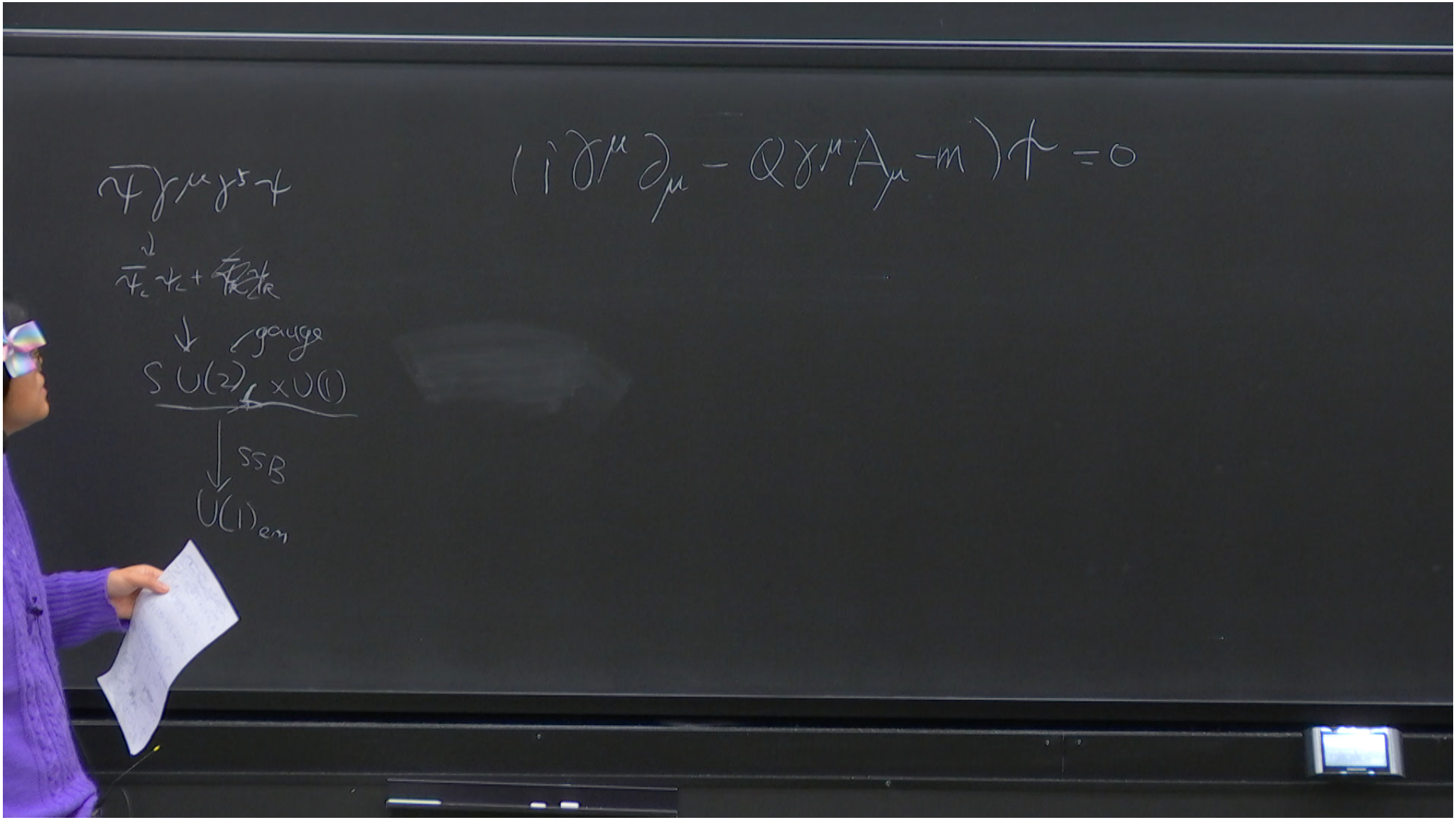
Collection: Quantum Field Theory I (2022/2023)

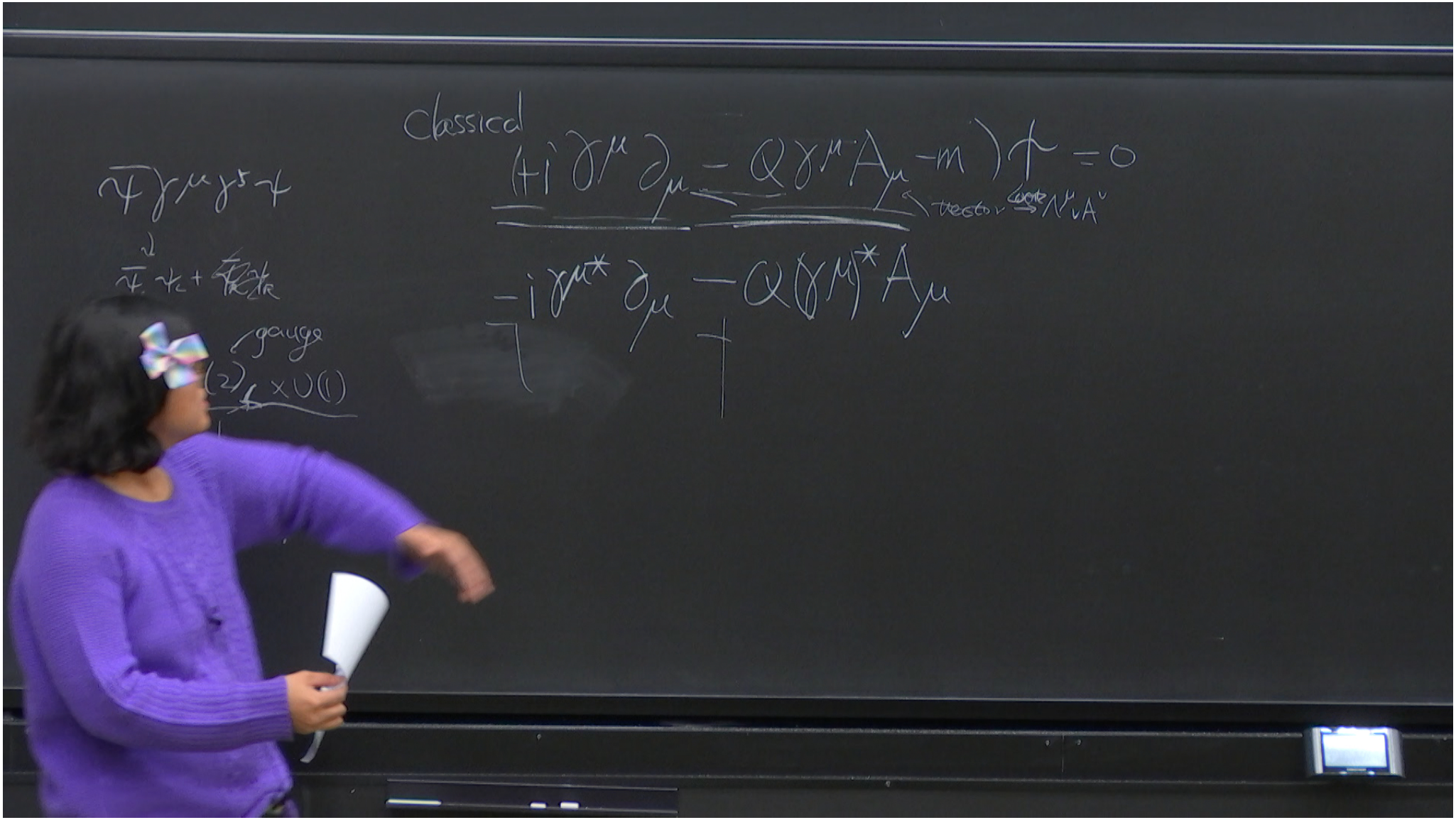
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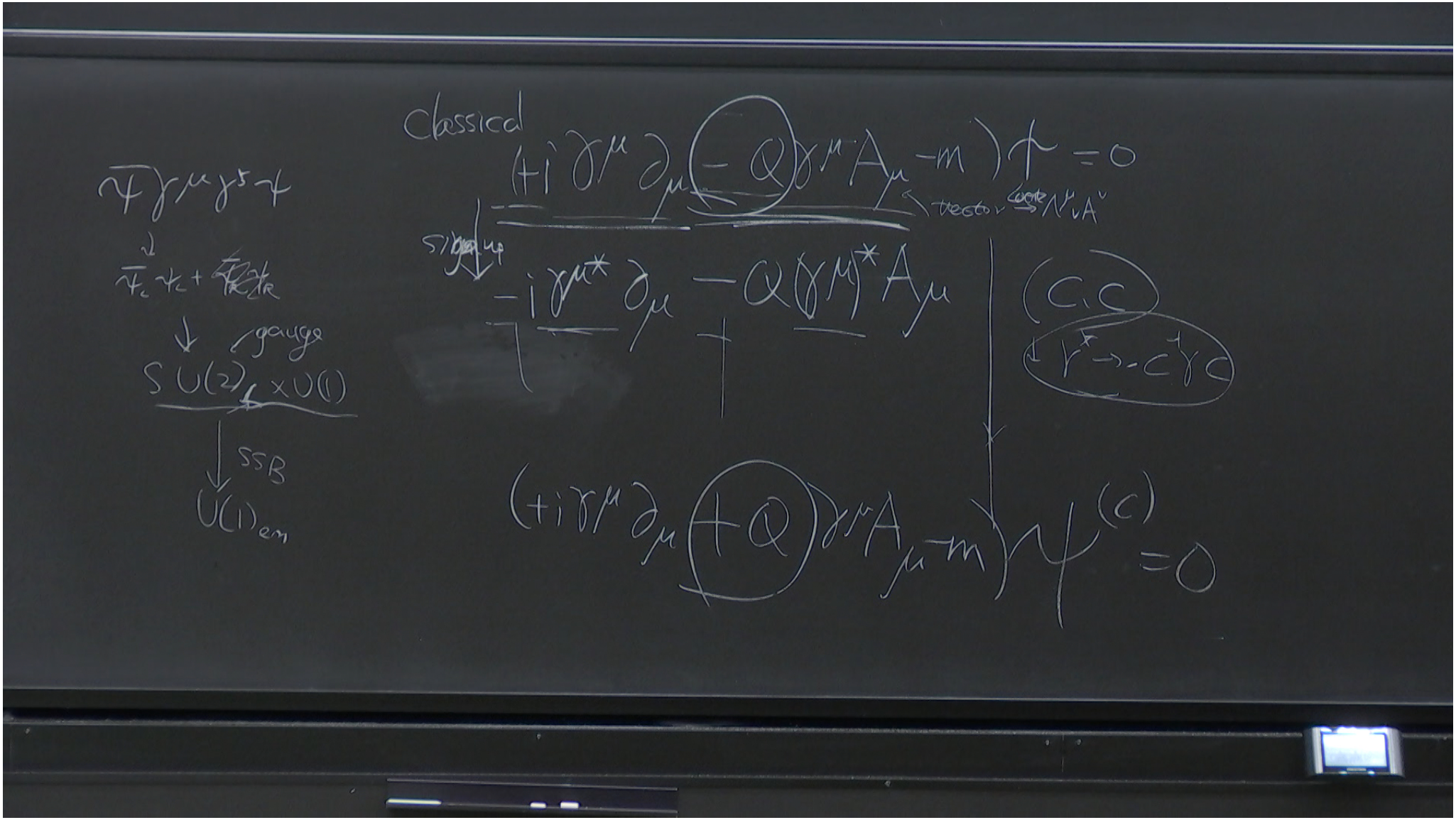
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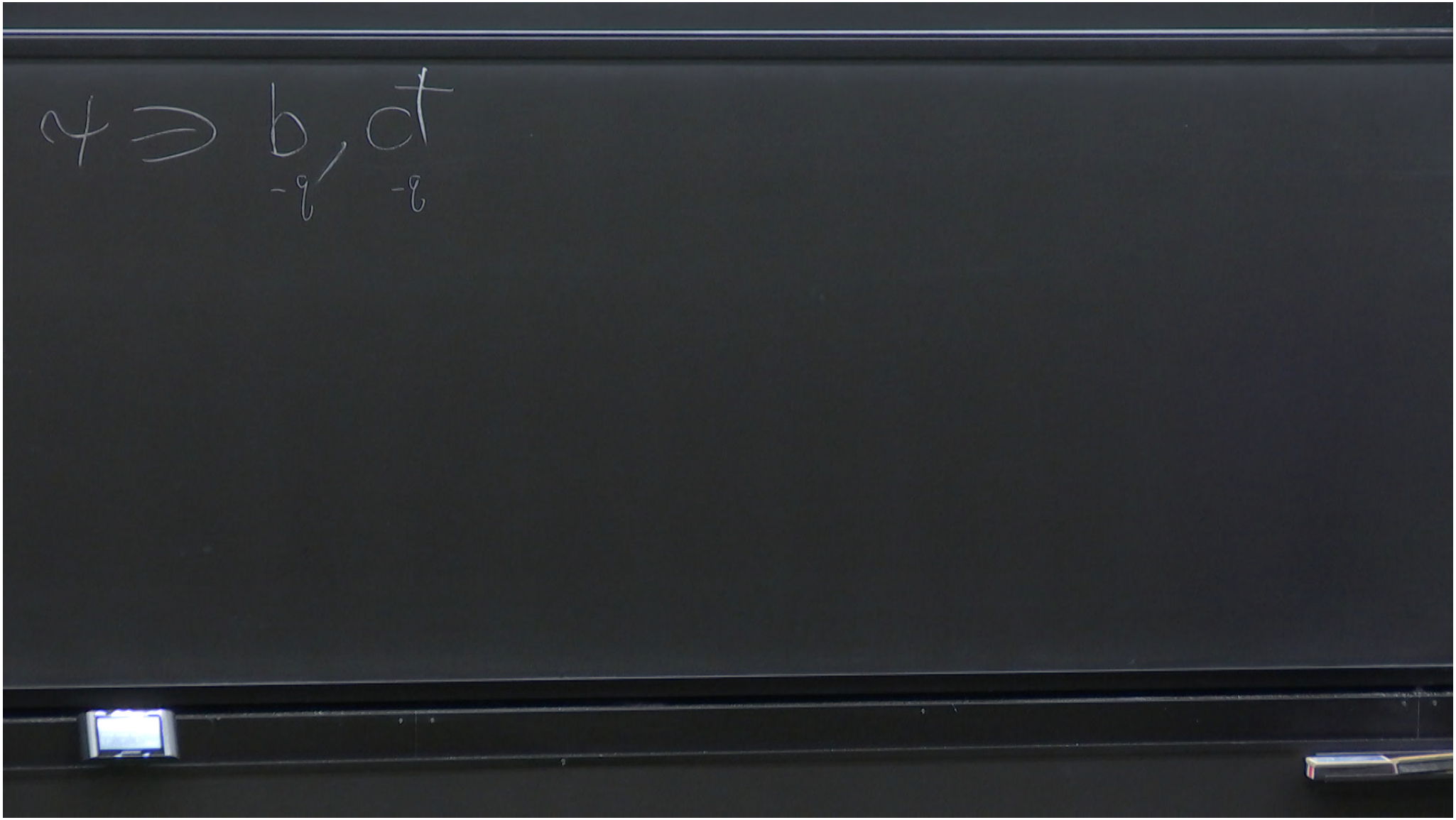

$\bar{\psi}\gamma^\mu\gamma^5\psi$
$\bar{\psi}_L\gamma_L + \bar{\psi}_R\gamma_R$


$\bar{\psi}\gamma^\mu\gamma^5\psi$
$\bar{\psi}_L\psi_L + \bar{\psi}_R\psi_R$
$SU(2)_L \times U(1)$
SSB
$U(1)_{em}$


$\bar{\psi}\gamma^\mu\gamma^5\psi$
$\bar{\psi}_L\psi_L + \bar{\psi}_R\psi_R$
gauge
$SU(2)_L \times U(1)$
SSB
$U(1)_{em}$
$(i\gamma^\mu\partial_\mu - Q\gamma^\mu A_\mu - m)\psi = 0$


classical
$(i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu - m)\psi = 0$
$-i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu$
$\bar{\psi}\gamma^\mu\gamma^5\psi$
gauge
$SU(2) \times U(1)$


classical
gauge
SU(2)L × U(1)
SSB
U(1)em
vector
scalar
C.C


b, ct
-q -q

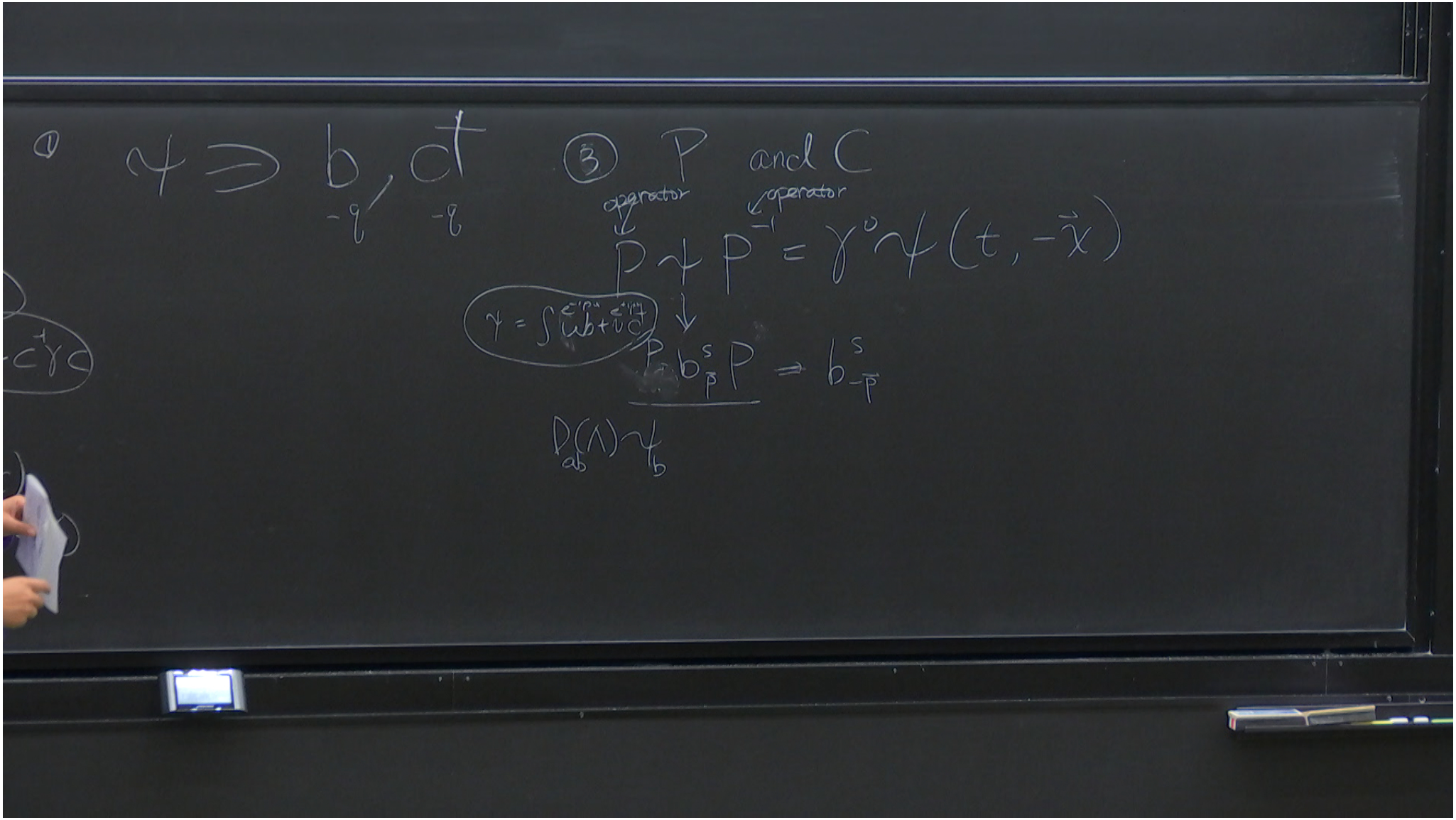
$\psi \supset b, c^\dagger$
$-q \quad -q$

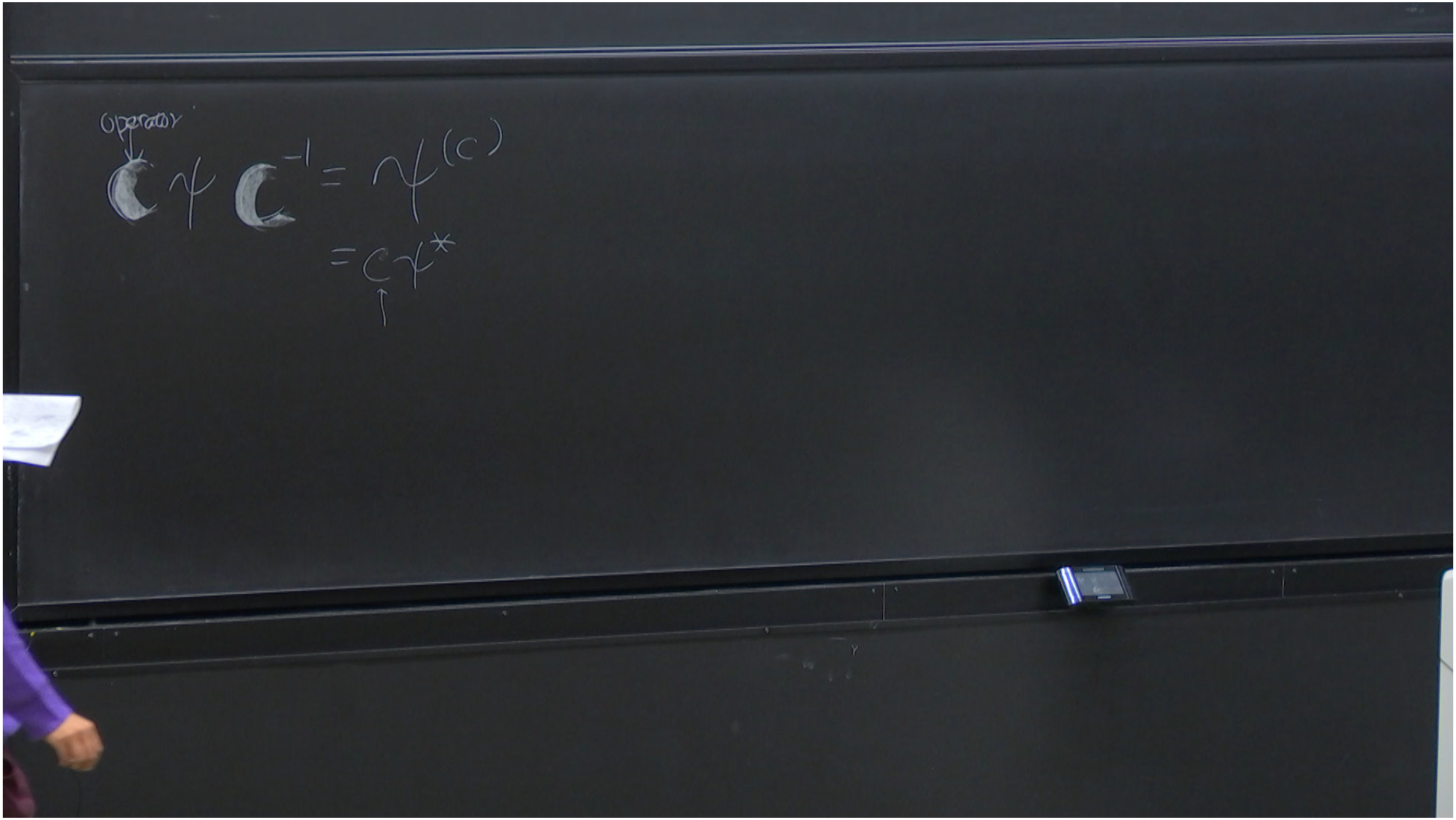
③ P and C

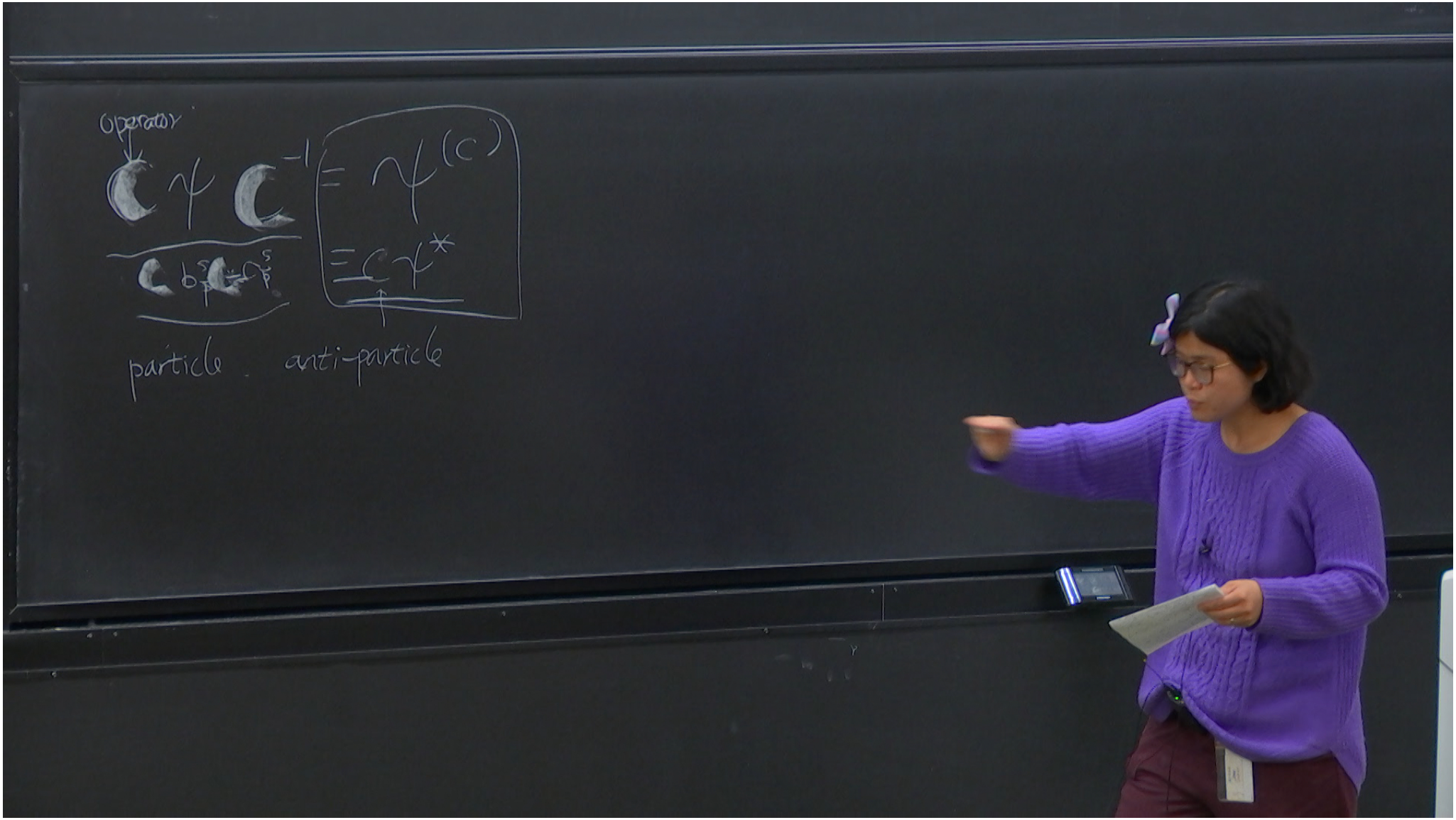
$$P\psi P^{-1} = \gamma^0 \psi(t, -\vec{x})$$

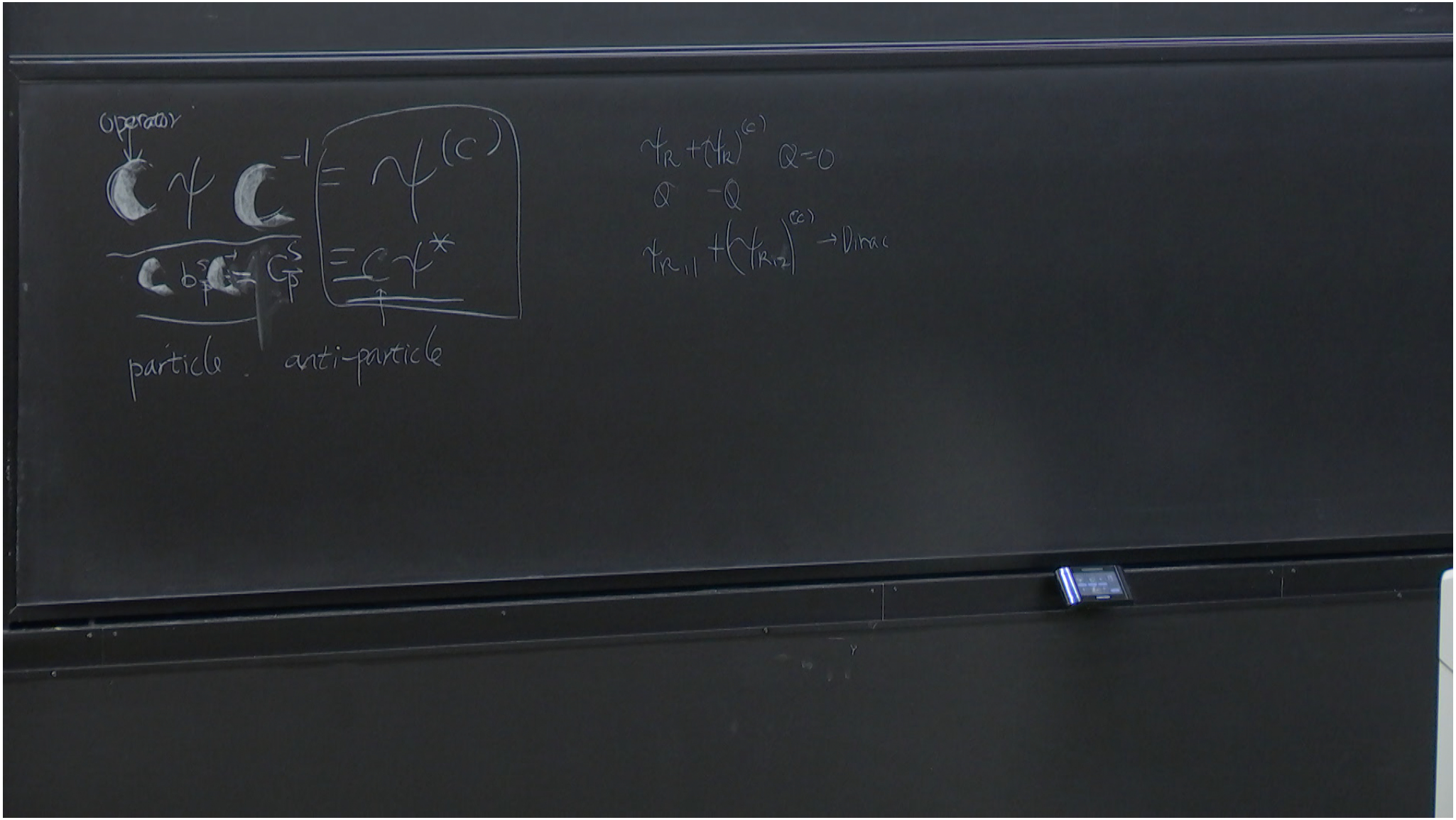
$$\downarrow$$

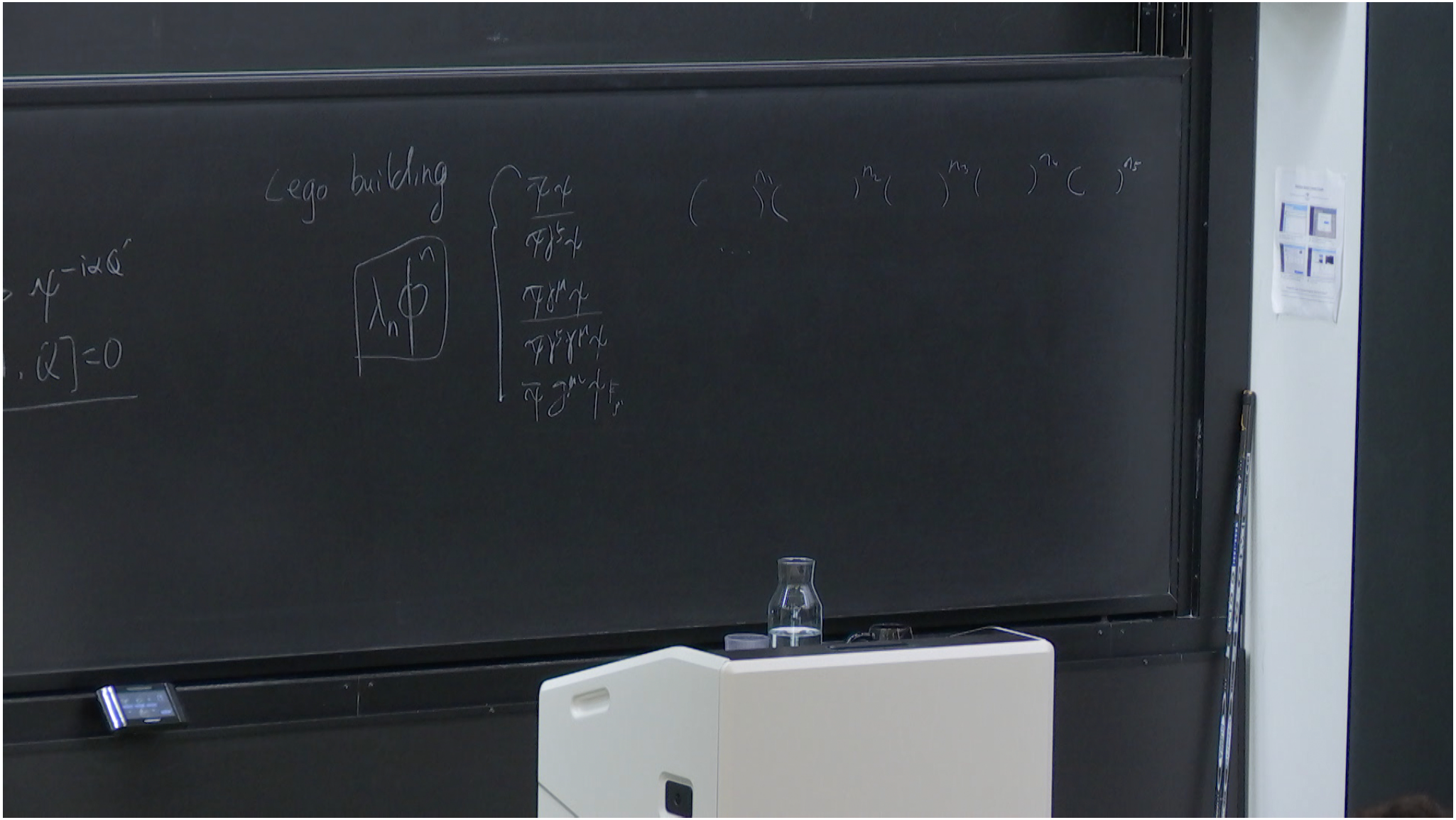
$$P b^s_{\vec{p}} P^{-1} \to b^s_{-\vec{p}}$$

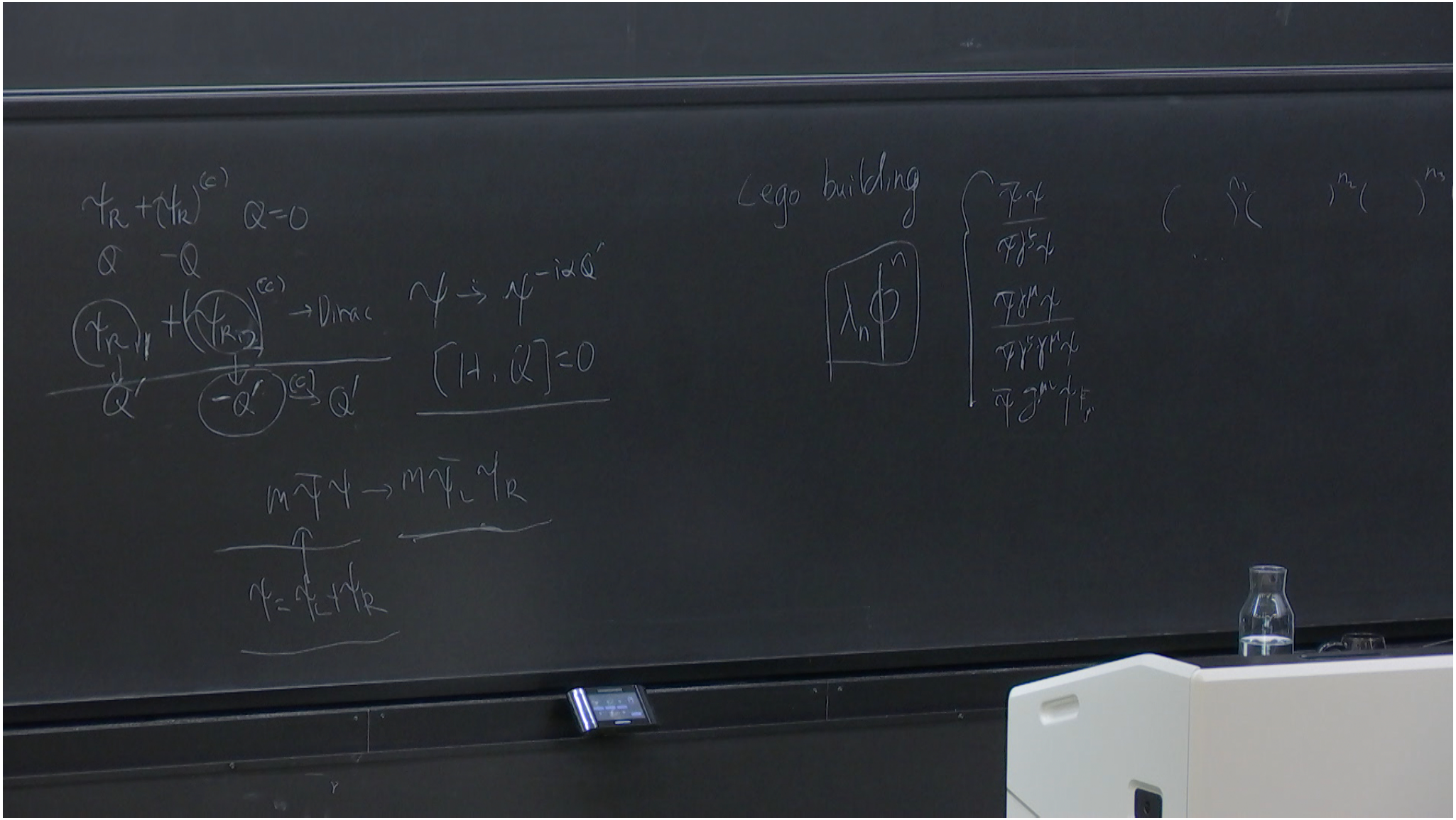

① ψ ⊃ b, c†
-q -q
③ P and C
operator
operator
P ψ P⁻¹ = γ⁰ ψ (t, -x⃗)
ψ = ∫ ub + ic
P b_p^s P = b_-p^s
D_ab(Λ) ψ_b

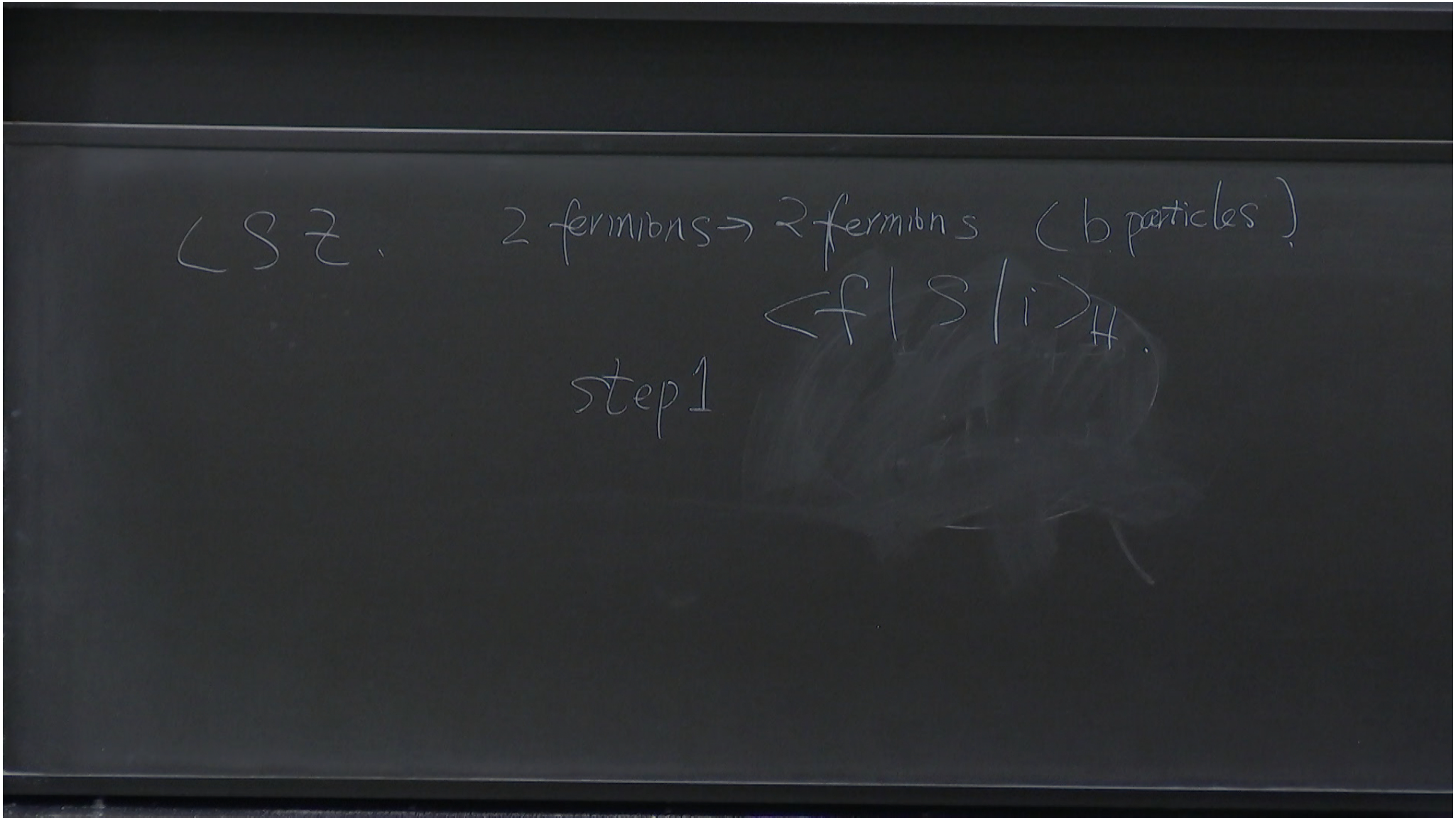

Operator
$C\psi C^{-1} = \psi^{(C)}$
$= C\psi^*$


Operator
particle
anti-particle


Operator
particle
anti-particle
Dirac


Lego building


Lego building
$\lambda_n \phi^n$
$\psi \to \psi^{-i\alpha Q'}$
$[H, Q] = 0$
$m\bar{\psi}\psi \to m\bar{\psi}_L\psi_R$
$\psi = \psi_L + \psi_R$
Dirac


LSZ. 2 fermions → 2 fermions (b particles)
$\langle f|S|i\rangle_H$.
step 1

2 fermions → 2 fermions (b particles)

$$\langle f|S|i\rangle_H$$

step 1 $\quad S|i\rangle_H = b_1^+(-\infty)\, b_2^+(-\infty)|\Omega\rangle \qquad b_1 \equiv b_{\vec{k}}^{s_1}$

$$|f\rangle_H = b_3^+(+\infty)\, b_4^+(+\infty)|\Omega\rangle$$

$$\langle f|S|i\rangle_H .$$

$$= \langle \Omega | b_4(+\infty)\, b_3(+\infty)\, b_1^\dagger(-\infty)\, b_2^\dagger(-\infty) | \Omega \rangle$$

$b_1 \equiv b_E^{S_1}$

$\rangle$

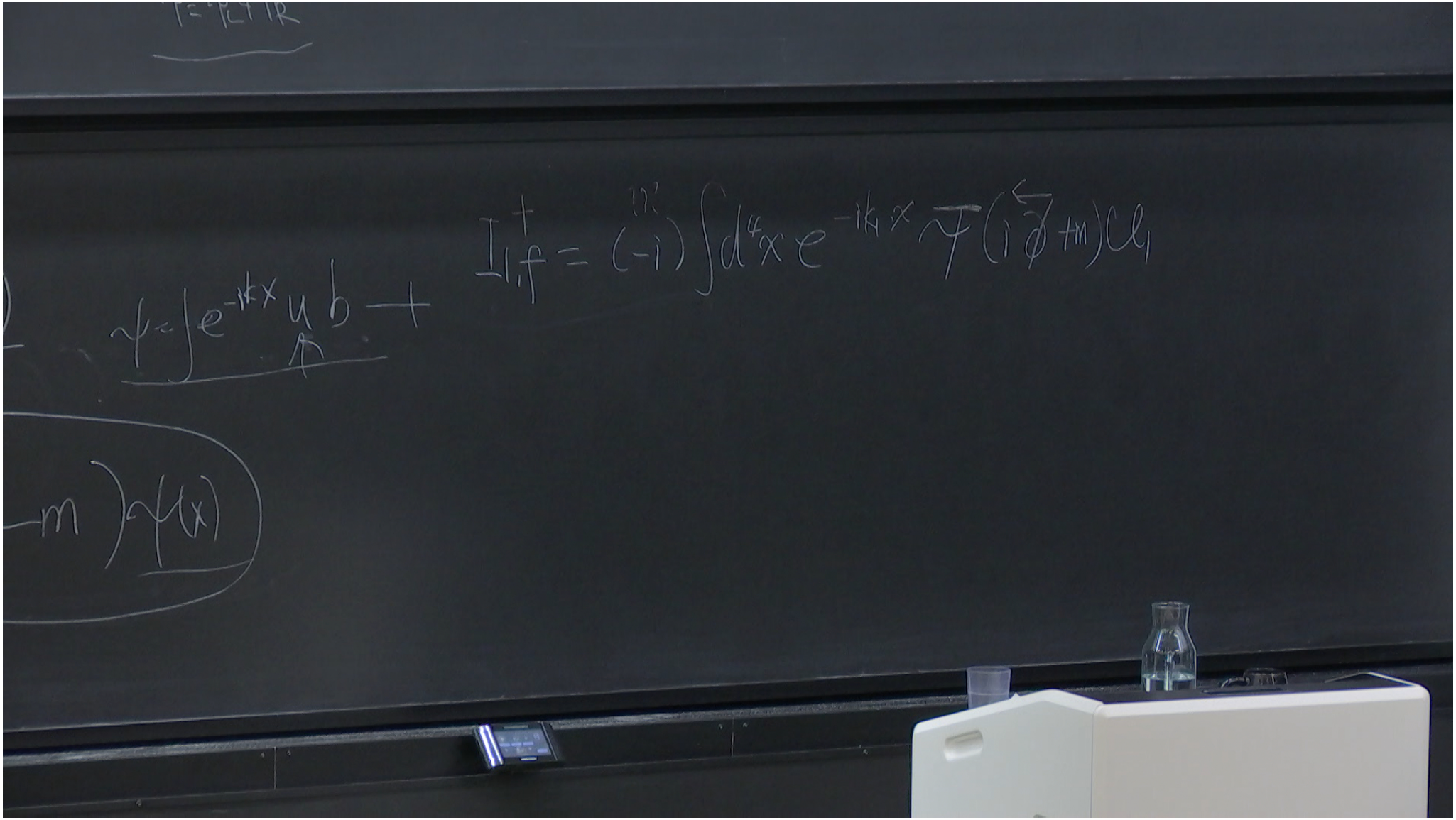
$$I_1 \equiv a_1(+\infty) - a_1(-\infty)$$
$$= i \int d^4x \, e^{+ik_i x} (\partial^2 + m^2) \phi(x)$$

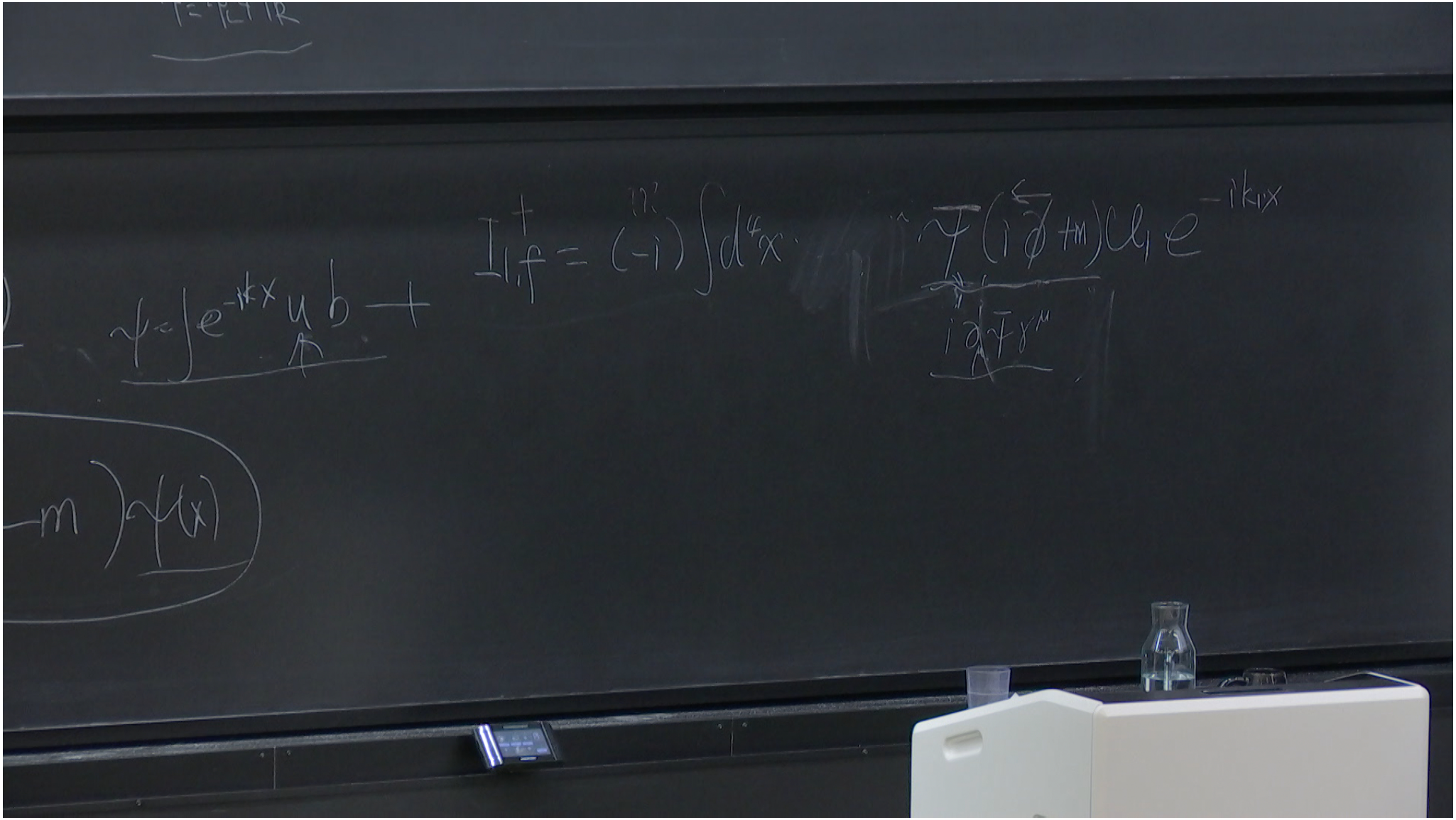
$$I_1 \equiv a_1(+\infty) - a_1(-\infty)$$

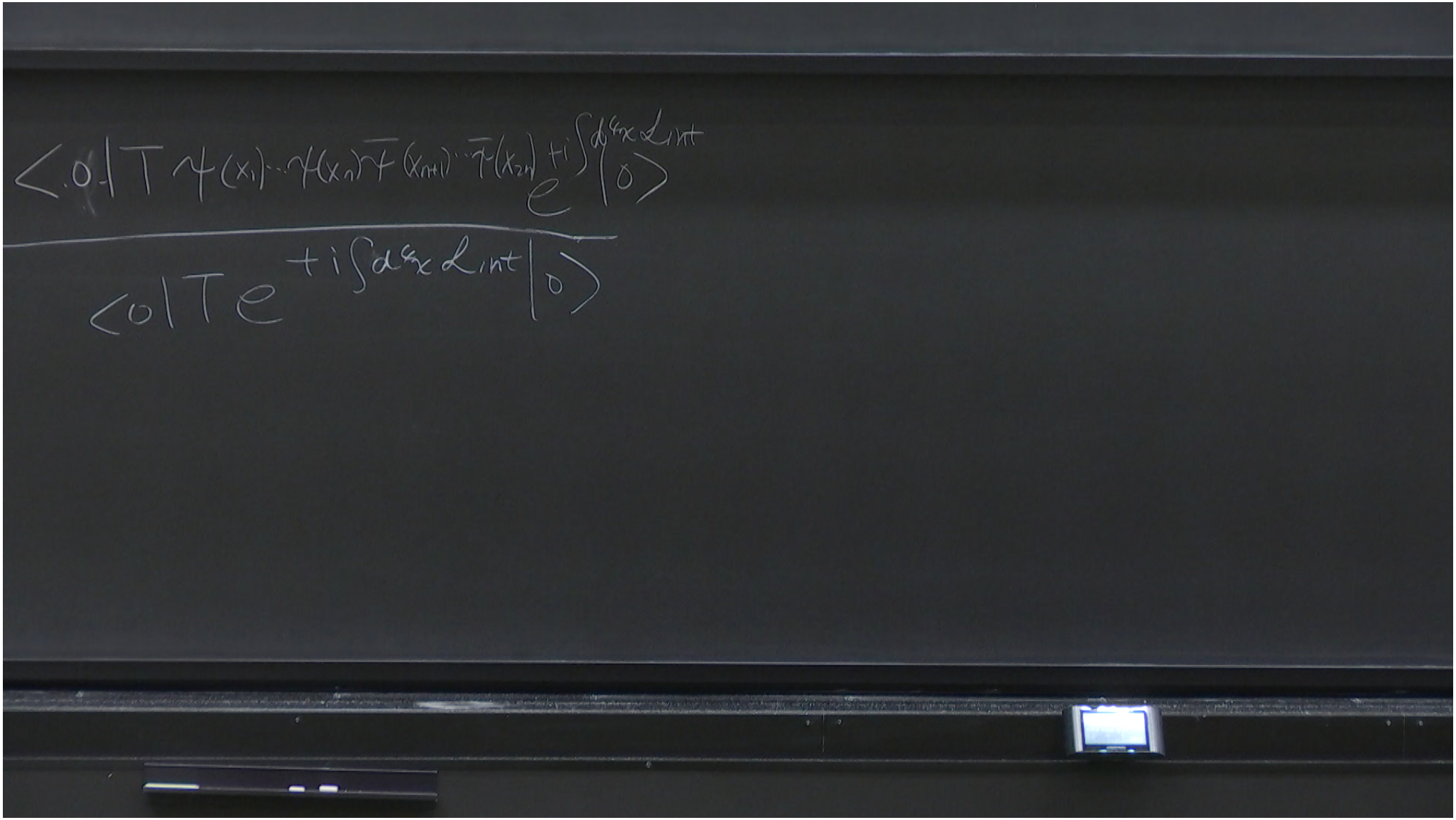
$$= i\int d^4x \underbrace{e^{+ik_1\cdot x}}_{①} \underbrace{(\partial^2 + m^2)}_{②} \underbrace{\phi(x)}_{③}$$

$$I_{1,f} = b_1(+\infty) - b_1(-\infty)$$

$$= -i\int d^4x\, e^{+ik_1\cdot x}\, (i\not{\partial} - m)\psi(x)$$


$\psi = \int e^{-ikx} u\, b -$
$I_{1,f}^{+} = (-i)^{n} \int d^4x\, e^{-ikx} \bar{\psi} (i \overleftarrow{\partial} + m) \mathcal{U}_1$
$-m)\psi(x)$


$\psi \sim e^{-ikx} u b +$
$I_{i,f}^{+} = (-i)^{n} \int d^4x$
$\bar{\mathcal{T}}(i\overleftarrow{\partial} + m) \mathcal{U}_i e^{-ik_i x}$
$i\bar{\mathcal{T}}\gamma^{\mu}$
$-m)\psi(x)$

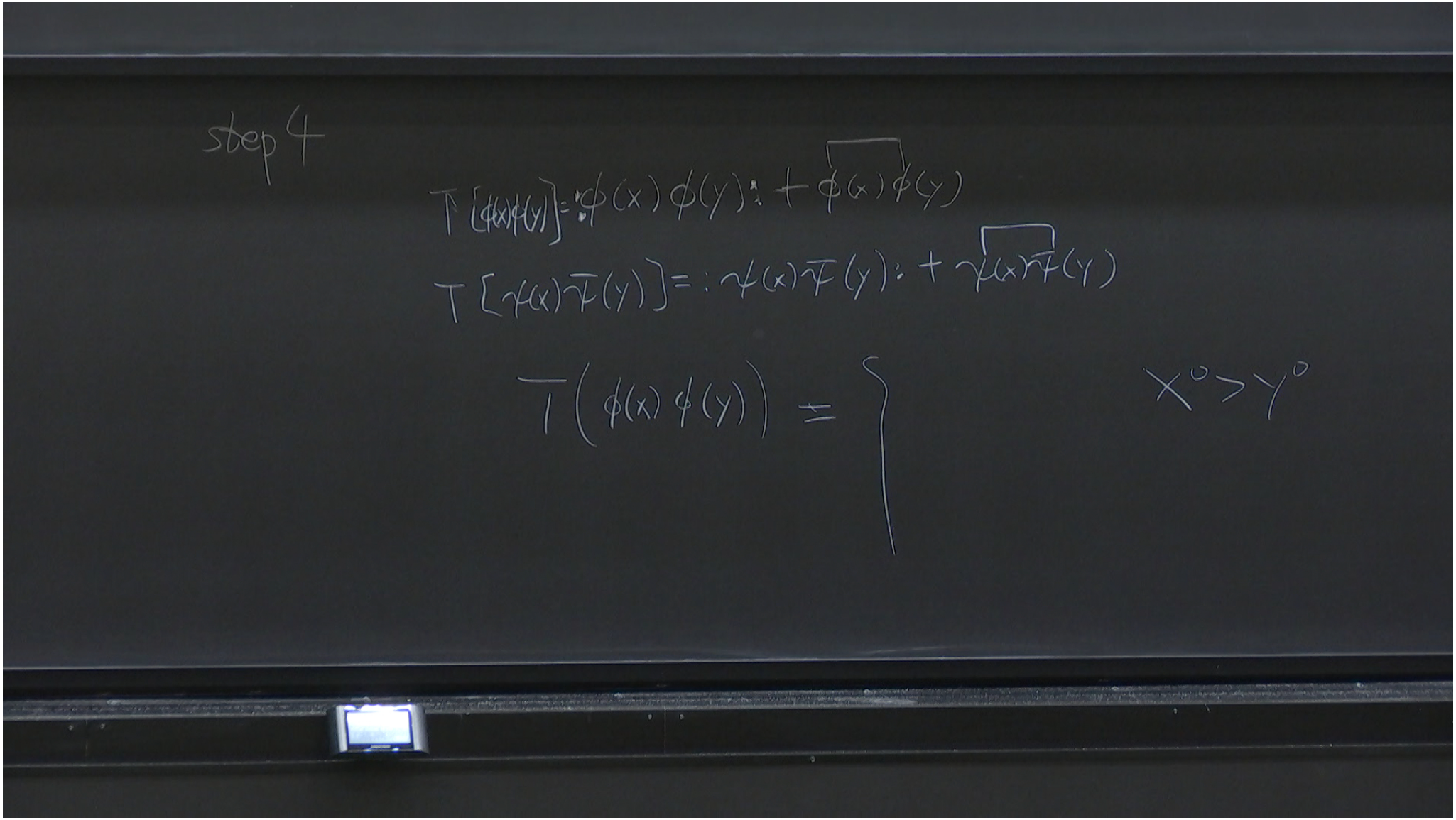

$\langle 0|T\psi(x_1)\cdots\psi(x_n)\bar{\psi}(x_{n+1})\cdots\bar{\psi}(x_{2n})e^{+i\int d^4x\mathcal{L}_{int}}|0\rangle$
$\langle 0|Te^{+i\int d^4x\mathcal{L}_{int}}|0\rangle$

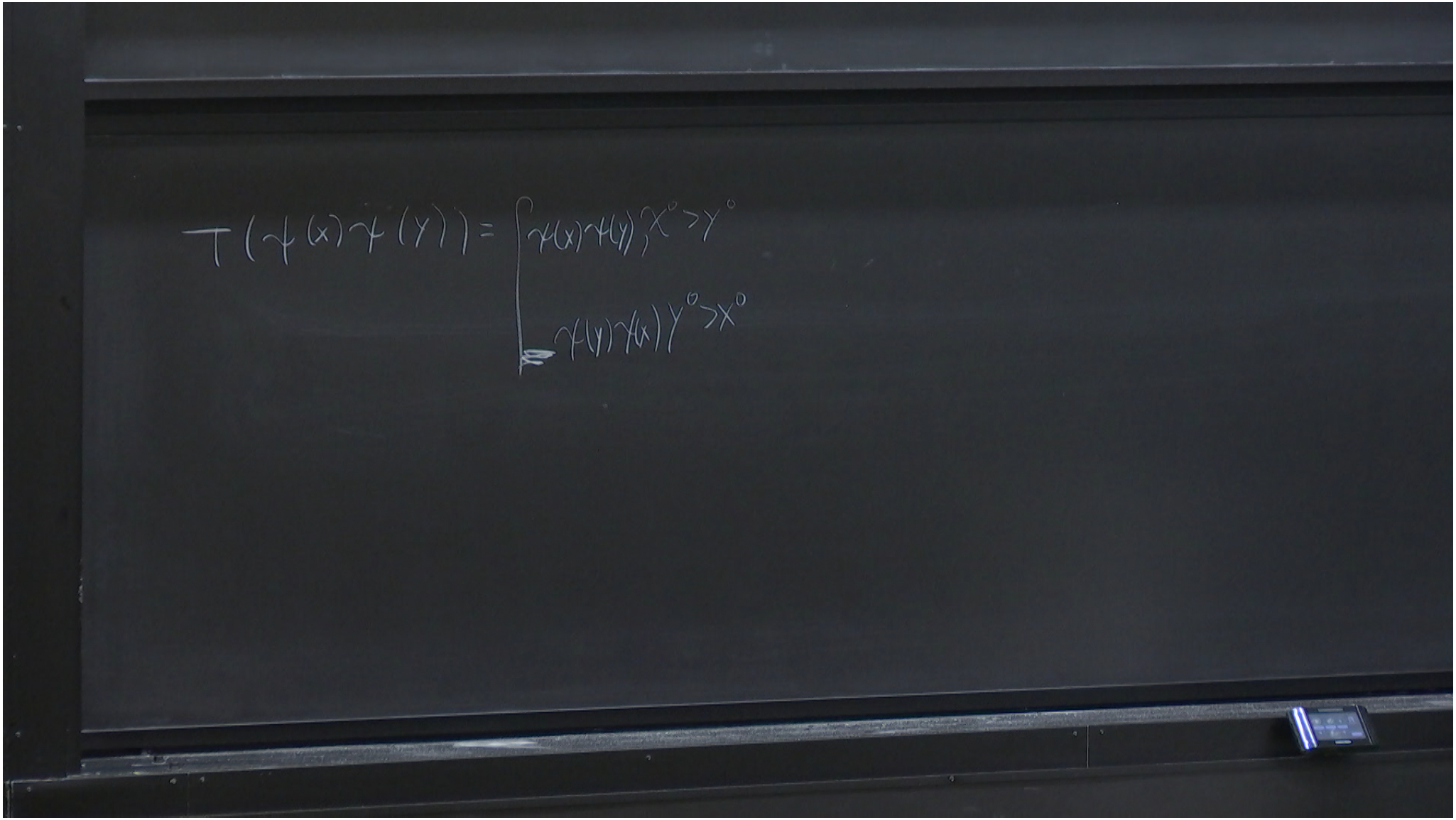
step 4

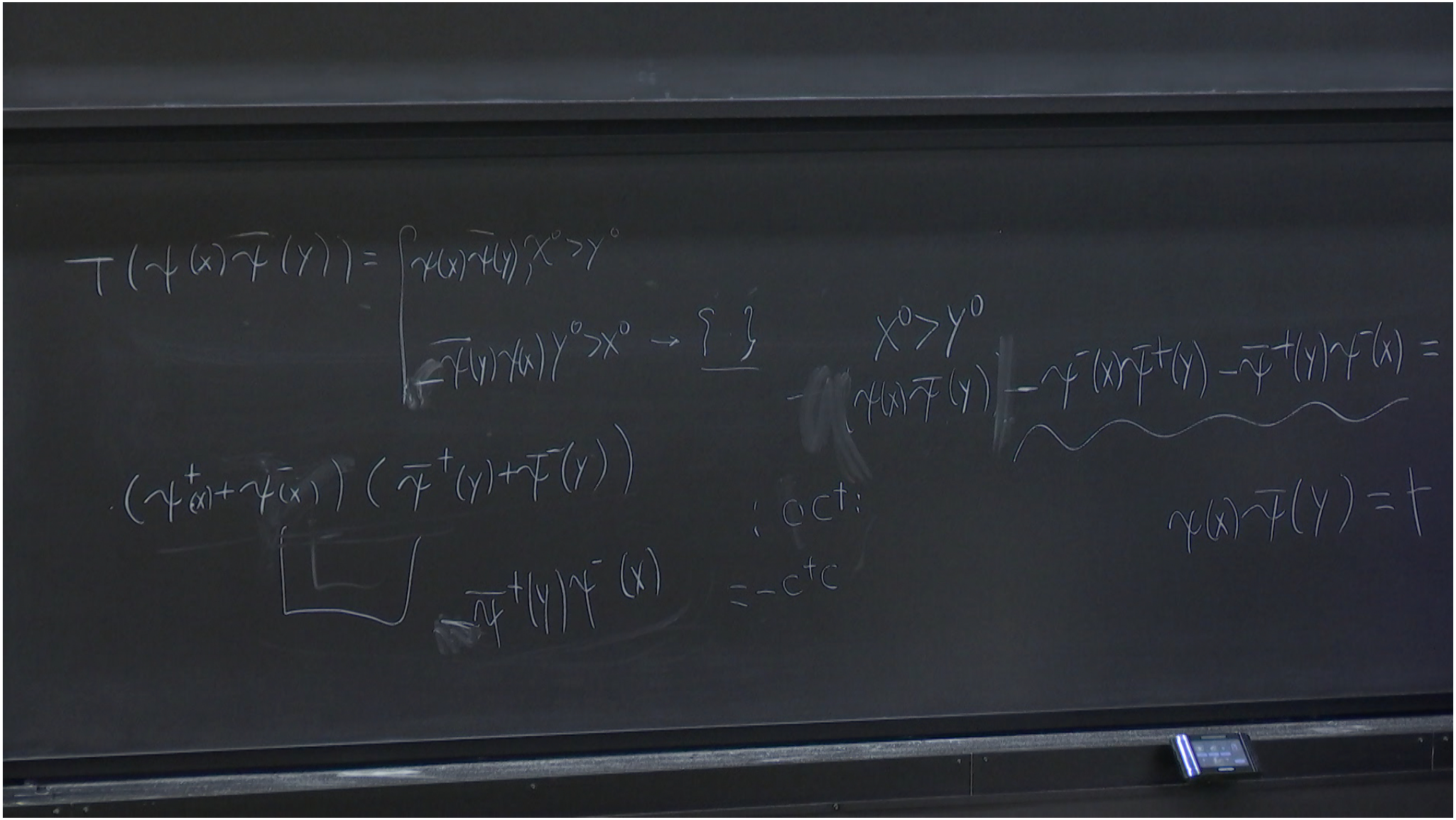
$$T[\phi(x)\phi(y)] = :\phi(x)\phi(y): + \overbrace{\phi(x)\phi(y)}$$

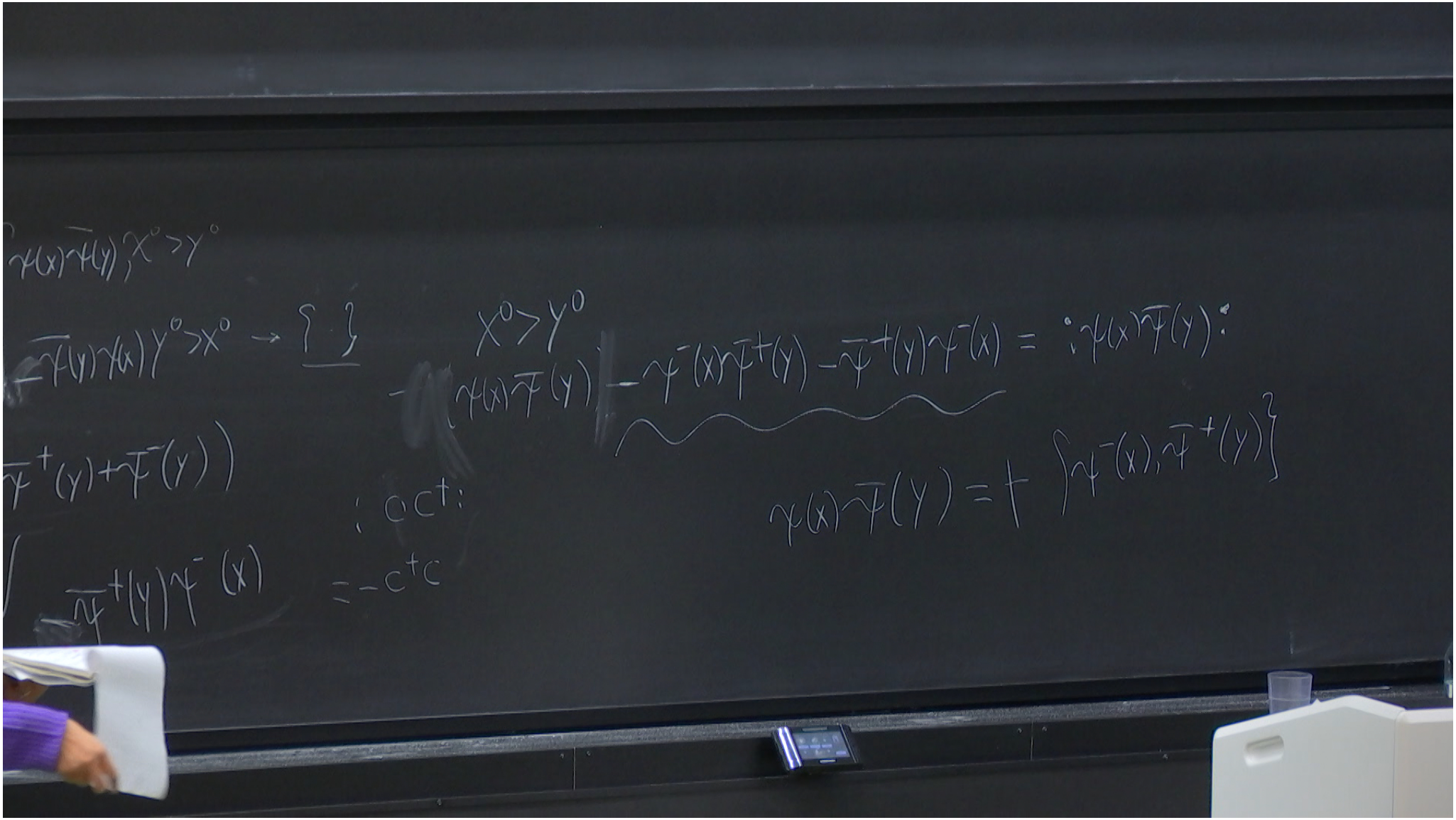
$$T[\chi(x)\bar{\chi}(y)] = :\chi(x)\bar{\chi}(y): + \overbrace{\chi(x)\bar{\chi}(y)}$$

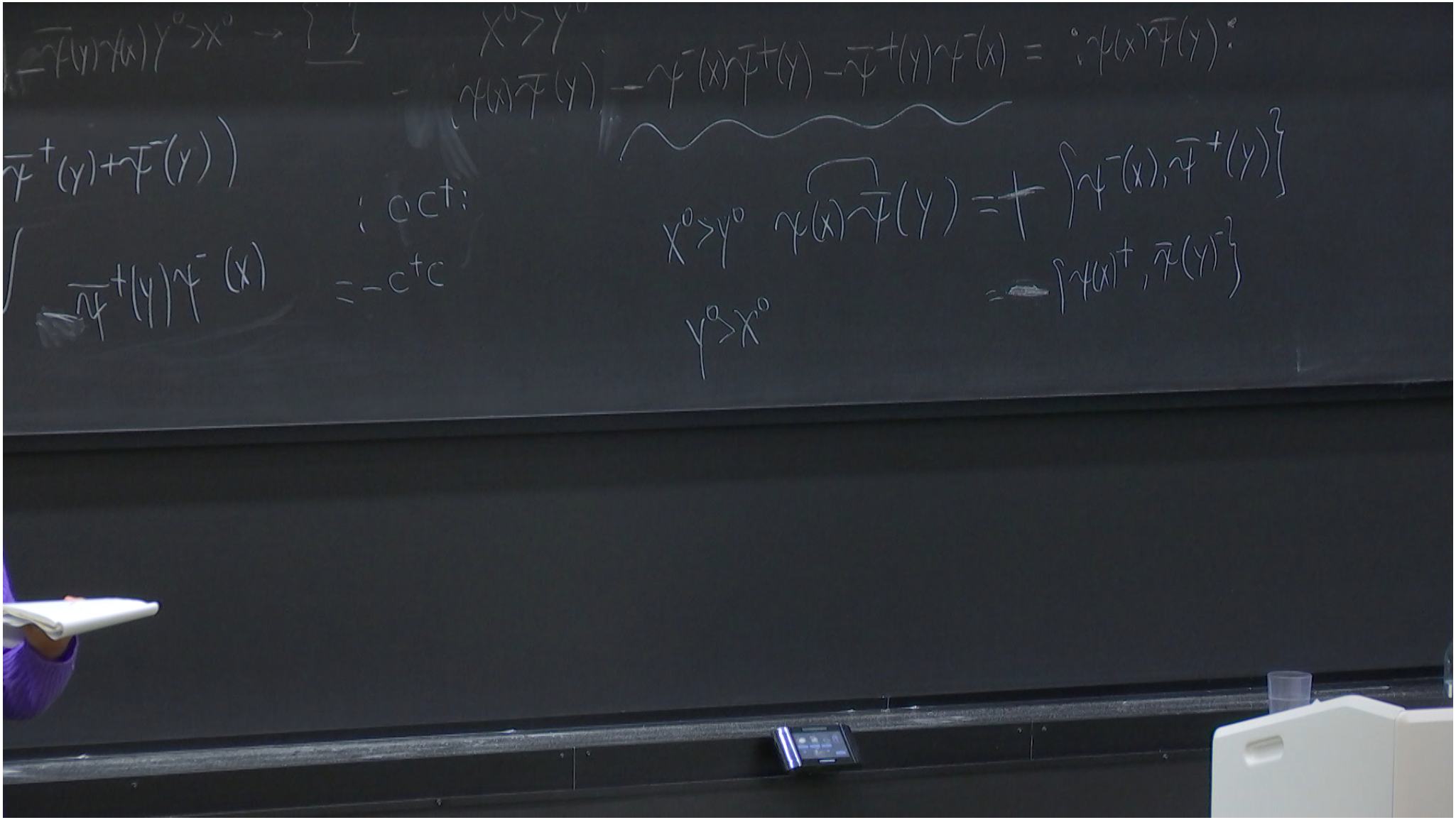



step 4
$T[\phi(x)\phi(y)] =: \phi(x)\phi(y) : + \overbrace{\phi(x)\phi(y)}$
$T[\psi(x)\bar{\psi}(y)] =: \psi(x)\bar{\psi}(y) : + \overbrace{\psi(x)\bar{\psi}(y)}$
$T(\phi(x)\phi(y)) = \begin{cases} \\ \\ \end{cases}$
$x^0 > y^0$


$T(\tau(x)\tau(y)) = \begin{cases} \tau(x)\tau(y), & x^0 > y^0 \\ \tau(y)\tau(x), & y^0 > x^0 \end{cases}$


$T(\psi(x)\bar{\psi}(y)) = \begin{cases} \psi(x)\bar{\psi}(y); x^0 > y^0 \\ -\bar{\psi}(y)\psi(x); y^0 > x^0 \end{cases}$
$x^0 > y^0$
$\psi(x)\bar{\psi}(y) - \psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^+(y)\psi^-(x) =$
$(\psi^+(x) + \psi^-(x))(\bar{\psi}^+(y) + \bar{\psi}^-(y))$
$: c c^\dagger :$
$\psi(x)\bar{\psi}(y) = 1$
$\bar{\psi}^+(y)\psi^-(x)$
$= -c^\dagger c$


$\psi(x)\bar{\psi}(y)$; $x^0 > y^0$
$\bar{\psi}(y)\psi(x)$ $y^0 > x^0$ → { }
$\bar{\psi}^+(y) + \bar{\psi}^-(y)$)
$\bar{\psi}^+(y)\psi^-(x)$
$x^0 > y^0$
$(\psi(x)\bar{\psi}(y)$ $-\psi^-(x)\bar{\psi}^+(y) - \bar{\psi}^+(y)\psi^-(x) = :\psi(x)\bar{\psi}(y):$
: oct:
$= -c^+c$
$\psi(x)\bar{\psi}(y) = +\{\psi^-(x), \bar{\psi}^+(y)\}$



$$\psi(x)^- = \int_p u\, b\, e^{-ipx}$$

$$\bar{\psi}(y)^+ = \int dV_q\, \bar{u}\, b^+ e^{+iq\cdot y}$$

$$= \{\psi(x)^-, \bar{\psi}(y)^+\} = \int dV_p\, u\bar{u}\, e^{-ip\cdot(x-y)}$$

$$= \int dV_p\, (\not{p}+m)\, e^{-ip\cdot(x-y)}$$

$$S_F(x-y) = (i\not{\partial}_x + m)($$

$$S_F(x-y) = (i\partial_x + m)\left(\Theta(x^0-y^0)\int d\tilde{V}_p\, e^{-ip\cdot(x-y)} + \Theta(y^0-x^0)\int d\tilde{V}_p\, e^{ip\cdot(y-x)}\right)$$

$$S_F(x-y) = (i\not{\partial}_x + m)\Big(\Theta(x^0-y^0)\int dV_p\, e^{-ip\cdot(x-y)} + \Theta(y^0-x^0)\int dV_p\, e^{ip\cdot(y-x)}\Big)$$

$$= (i\not{\partial}_x + m)\,\Delta_F(x-y)$$

$$S_F(x-y) = (i\not{\partial}_x + m)\Big(\Theta(x^0-y^0)\int dV_p\, e^{-ip\cdot(x-y)} + \Theta(y^0-x^0)\int dV_p\, e^{ip\cdot(y-x)}\Big)$$

$$= (i\not{\partial}_x + m)\,\Delta_F(x-y)$$

$$= \int \frac{d^4p}{(2\pi)^4}\,\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}\,e^{-ip(x-y)}$$

$$(\not\partial_x + m)\left(\theta(x^0-y^0)\int dV_p\, e^{-ip\cdot(x-y)} + \theta(y^0-x^0)\int dV_p\, e^{ip\cdot(y-x)}\right)$$

$$(\not\partial_x + m)\,\Delta_F(x-y)$$

$$\int \frac{d^4p}{(2\pi)^4}\,\frac{i(\not p + m)}{p^2 - m^2 + i\epsilon}\,e^{-ip\cdot(x-y)}$$

$$(\partial^2 + m^2)\,\Delta_F(x-y) = -i\,\delta(x-y)$$

$$(i\not\partial - m)\,S_F(x-y) = i\,\delta(x-y)$$