Title: Session 2 - Viktoriia Voloshyna

Speakers:

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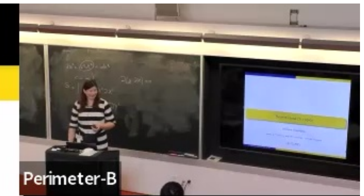
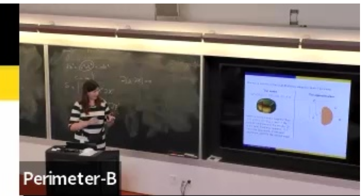


Recent research topics

Viktoriia Voloshyna

Imath, CPT (France) and KNU (Ukraine) → PI UW (Canada)

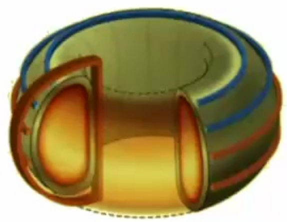
24/10/2022



Numerical solution of the Grad-Shafranov equation (Gloria Faccanoni)

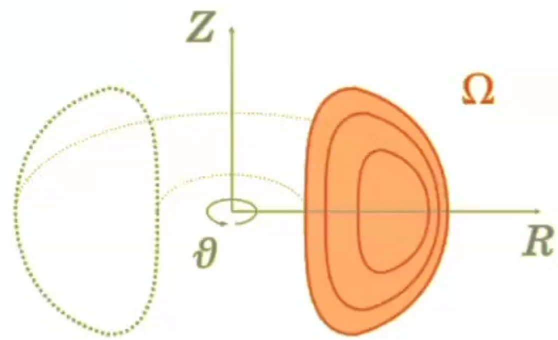
The model

$$-\Delta^*\psi(R,Z) = f(R,\psi(R,Z)) \text{ in } \Omega,$$



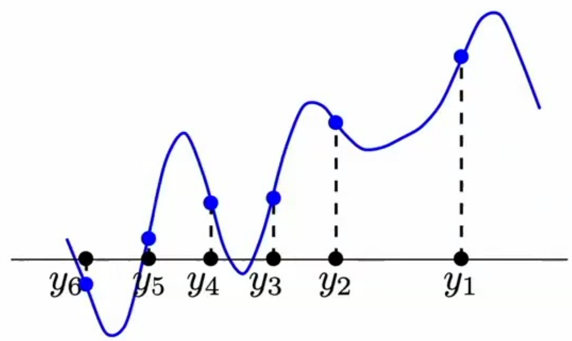
where ψ is the poloidal magnetic flux, $f(R,\psi(R,Z)) = R\mu_0 J$, with J the toroidal component of the density, Δ^* is the Grad-Shafranov operator, R is the radial coordinate, Z the axial coordinate, and θ is the toroidal angle.

The approximation

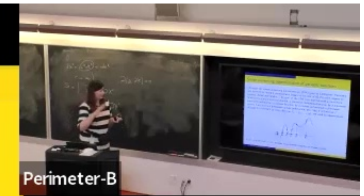


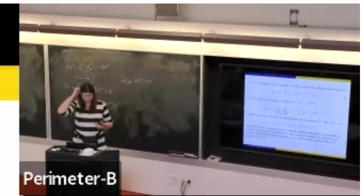
Shape preserving approximation of periodic functions

Although the shape preserving approximation (SPA) goes to Chebyshev, Bernstein and Lorentz its modern development began at the end of 60-th in the papers by Lorentz, Zeller and Shisha. The goal of the SPA is to approximate a monotone function by a monotone polynomial, a piecewise monotone function by a piecewise monotone polynomial, a convex function by a convex polynomial, a piecewise convex function by a piecewise convex polynomial, etc. Say, if a function f changes its convexity at the collection $Y_3 = \{y_1, ..., y_6\}$, we wish to approximate



it by a polynomial, that changes its convexity exactly at these points.





$f \in C^{(r)}$, the space of $2\pi-$periodic, r times continuously differentiable functions.

$$E_n(f) \leq \frac{c(k,r)}{n^r}\omega_k\left(f^{(r)}, \frac{1}{n}\right), \quad n \geq 1. \tag{1}$$

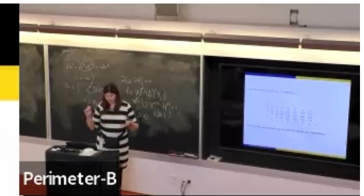
k-th modulus of continuity of a function $g \in C \quad (= C^{(0)})$,

$$\omega_k(g,t) = \sup_{h\in[0,t]} \|\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}g(\cdot + jh)\|, \quad t \geq 0,$$

provides more accurate scale of smoothness than the spaces W^r. Say, for $f \in W^r$,

$$\|f^{(r)}\| \leq 1 \quad \Longleftrightarrow \quad \omega_r\left(f,t\right) \leq t^r.$$

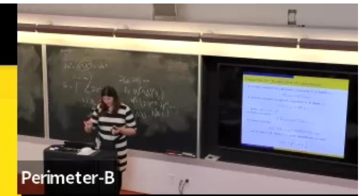
We proved that, in opposite to the classical Jackson inequality, the inequality in cannot be improved in the sense that the number N cannot be replaced by a number independent of the collection Y_s for the coconvex approximation for functions from W^r.



A is the set of all pairs (k, r) marked by $\oplus$

r	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\iddots$
3	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\cdots$
2	$\oplus$	$\oplus$	$\oplus$	$\oplus$	CE	CE	CE	$\cdots$
1	$\oplus$	$\oplus$	$\oplus$	CE	CE	CE	CE	$\cdots$
0		$\oplus$	$\oplus$	$\oplus$	CE	CE	CE	$\cdots$
	0	1	2	3	4	5	6	k

These results are obtained jointly with Dzyubenko and Yushchenko.



Inequalities for the derivative of a polynomial

- Bernstein inequality for trigonometric polynomials T_n of degree $\leq n$:

$$\|T_n'\|_{C(\mathbb{R})} \leq n\|T_n\|_{C(\mathbb{R})};$$

- Bernstein inequality for algebraic polynomials P_n of degree $\leq n$:

$$\|\varphi P_n'\|_{C[-1,1]} \leq n\|P_n\|_{C[-1,1]},$$

where $\varphi(x) = \sqrt{1-x^2}$.

- Markov inequality:

$$\|P_n'\|_{C[-1,1]} \leq n^2\|P_n\|_{C[-1,1]};$$

- Dzyadyk inequality:

$$\|\varphi_n^{1-s} P_n'\|_{C[-1,1]} \leq c(s)n\|\varphi_n^{-s} P_n\|_{C[-1,1]},$$

valid for each $s \in \mathbb{R}$, where $c(s) = \text{const}$, depends only on s and

$$\varphi_n(x) := \sqrt{1 - x^2 + \frac{1}{n^2}}.$$

For each natural $s \geq 1$ and $n \geq 2s$ Halan and Shevchuk recently found an exact constant in Dzyadyk inequality

$$\|\varphi_n^{1-s} P_n'\|_{C[-1,1]} \leq c(s,n) n \|\varphi_n^{-s} P_n\|_{C[-1,1]}, \qquad (2)$$

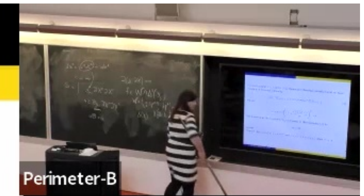
namely

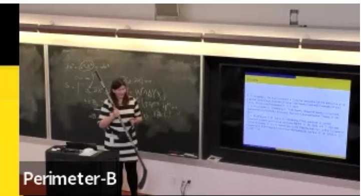
$$c(s,n) = \left(1 + s\frac{\sqrt{1+n^2}-1}{n}\right)^2 - s.$$

We proved that this constant $c(s,n)$ is exact in the inequality (2) for

$$s \leq n < 2s$$

as well.
Remark that $c(s,n) \leq 1 + s + s^2$ and $c(s,n) \to 1 + s + s^2$, when $n \to \infty$.





Papers

1) V. Voloshyna, On exact constant in Dzyadyk inequality for the derivative of an algebraic polynomial, Bulletin of Taras Shevchenko National University of Kyiv. Series: Physics and Mathematics, V.1, 2022.

2) G. Dzyubenko, V. Voloshyna, L. Yushchenko, Negative results in coconvex approximation of periodic functions, Journal of Approximation Theory, V. 267, 2021.

3) V. Koshmanenko, O. Satur, V. Voloshyna, Point spectrum in conflict dynamical systems with fractal partition, MFAT, V. 25, 2019, n.4, P. 324-338.

4) Koshmanenko, V. D., V. Voloshyna, Limit Distributions for Conflict Dynamical System with Point Spectra, Ukrainian Mathematical Journal, V. 70, 2019, n. 12, P.1861-1872.