Title: Holographic scattering from quantum error-correction

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Series: Colloquium

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Abstract: We revisit the problem of how interactions emerge in quantum gravity. Namely, we show that bulk scattering of multiple particles in the AdS space requires multipartite entanglement on the boundary. This statement can be proven by two totally different methods, 1) general relativity and 2) quantum cryptographic argument. Furthermore, we argue that interactions among particles in the scattering event emerge from the mechanism of entanglement-assisted quantum error-correcting codes (EAQECCs) which utilize pre-existing multipartite entanglement in CFT. We also propose a concrete protocol to implement a certain class of multi-partite unitary interactions by using transversal logical operators of quantum codes. This talk is based on a (very) recent work with Alex May and Jonathan Sorce.

Zoom Link: https://pitp.zoom.us/j/91349028320?pwd=TGF2Q2ZNdTZtZGxkQ0NiMURLdW5Zdz09

Holographic Scattering from Quantum error-correction

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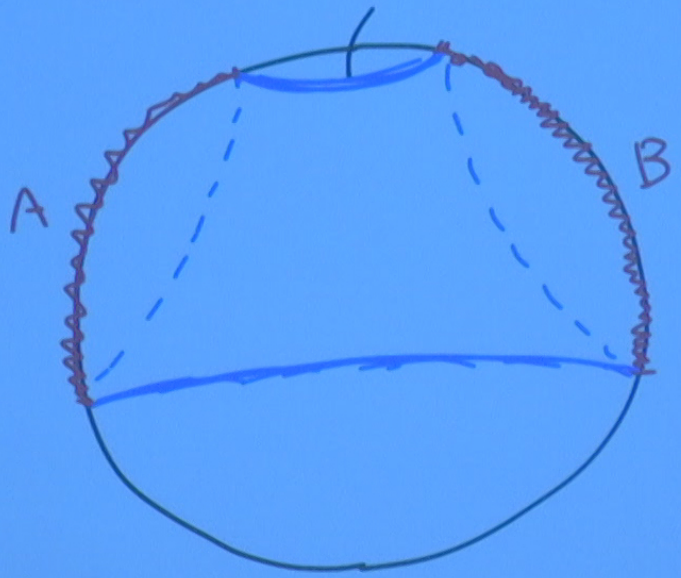
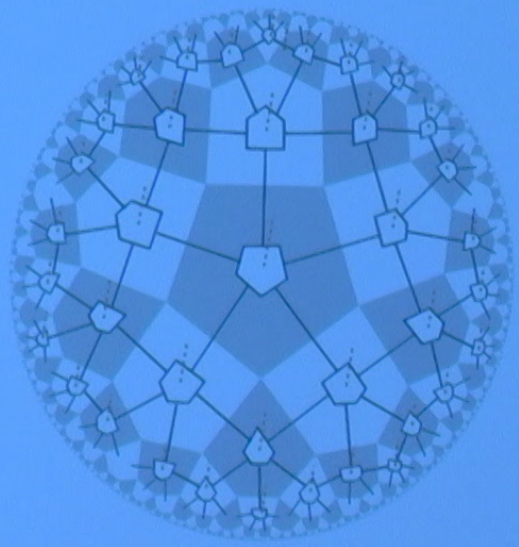
w/ Alex May (Stanford)
Jonathan Sorce (MIT)

Spacetime from entanglement



Entanglement ← connected wedge
(like wormhole)

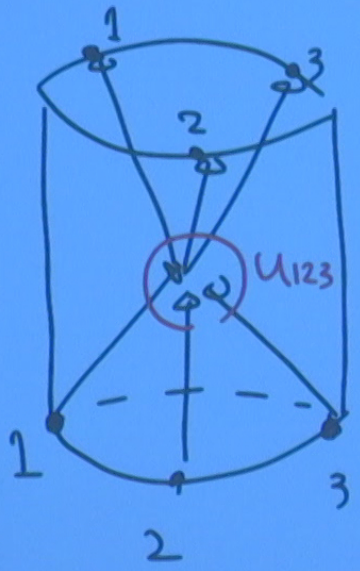
Ryu-Takayanagi formula



Tensor-network

Quantum error-correction

Holographic Scattering?



U_{123} : Arbitrary unitary op.

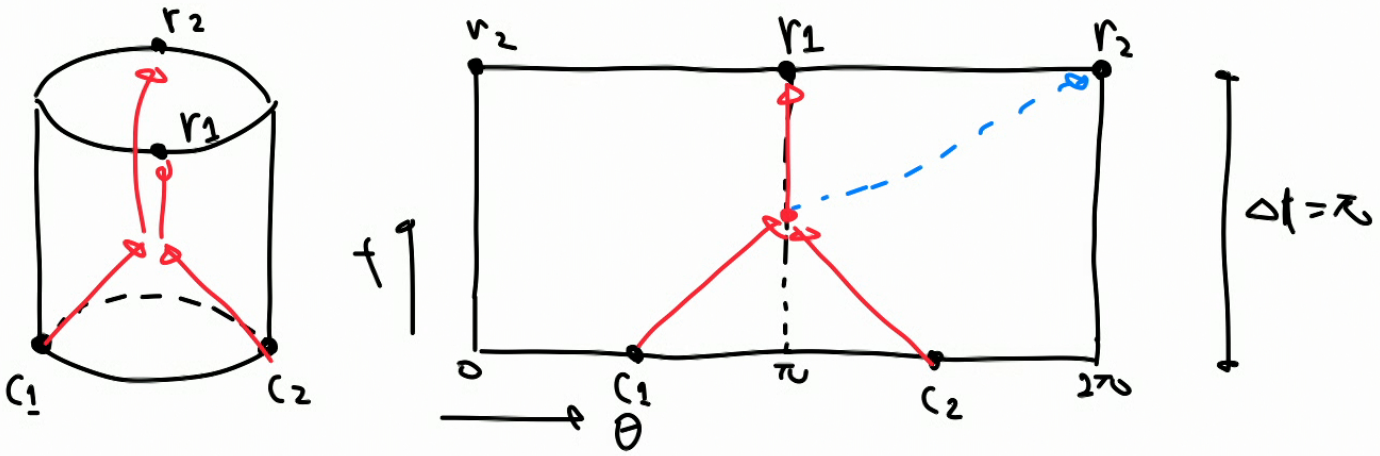
How is U_{123} realized on boundary QM?

Important work by Alex May (2019)

Holographic Scattering

Entanglement

2 particle Scattering Puzzle

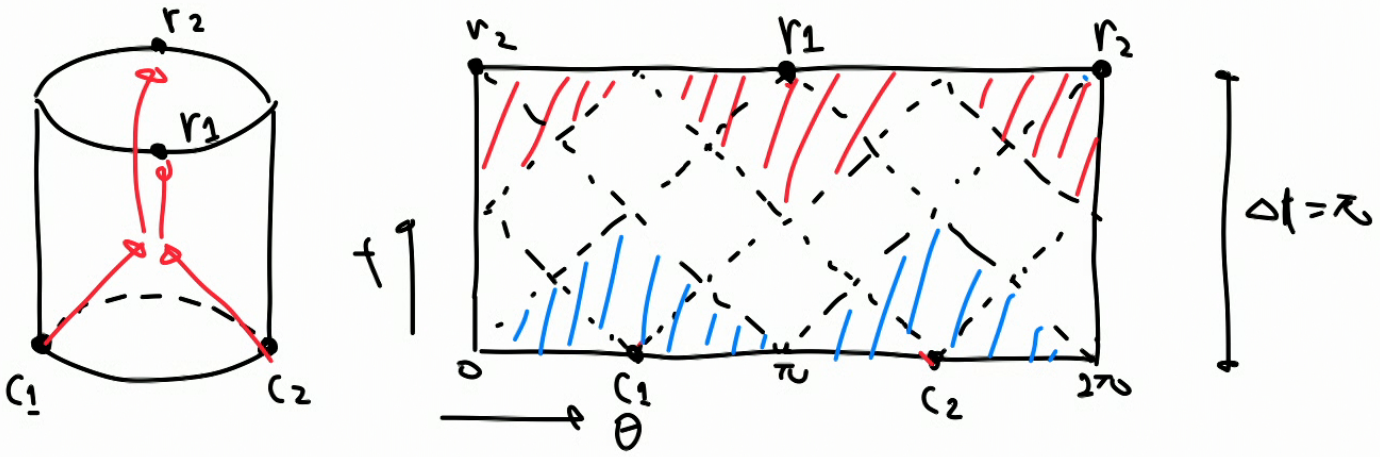


bulk $\Delta t = \pi$ boundary $\Delta t > \pi$

Not enough time for boundary scattering !!

(After Maj, 2019)

Future and Past light cones

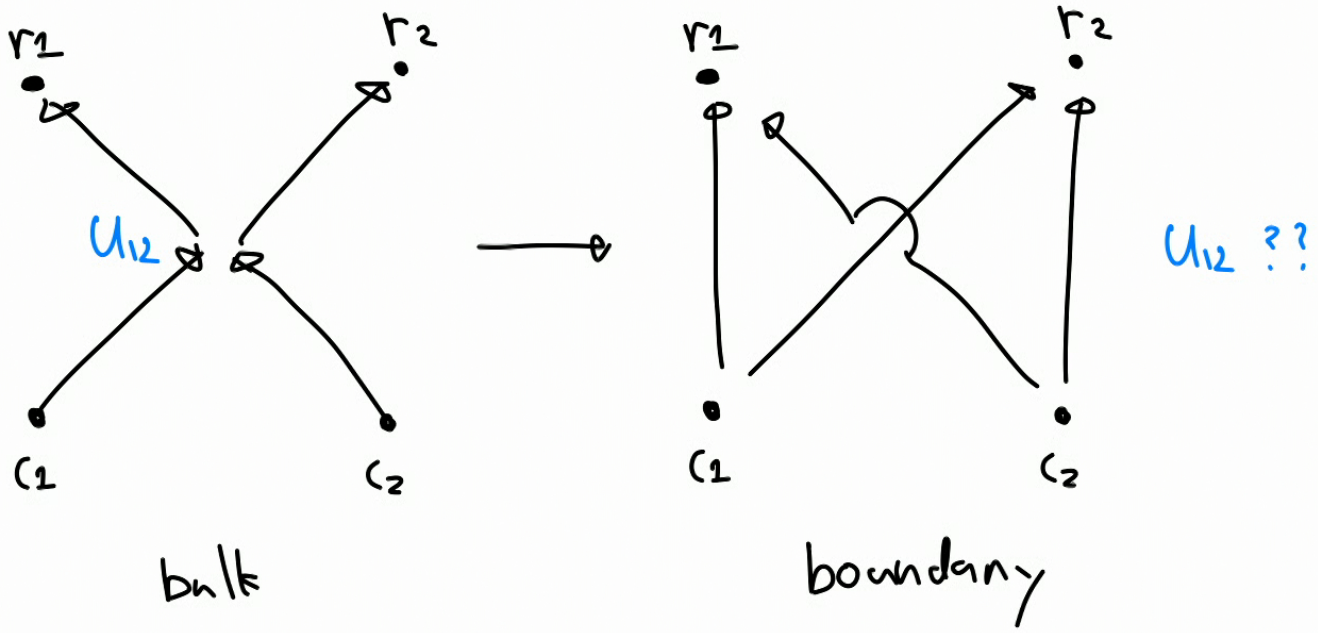


$J_+(C_1) \cap J_+(C_2)$ Future of C_1, C_2.

$J_-(r_1) \cap J_-(r_2)$ Past of r_1, r_2.

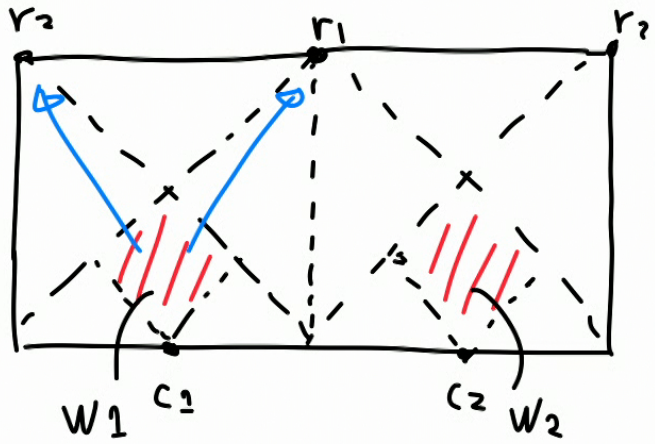
No overlap !! (No direct boundary scattering)

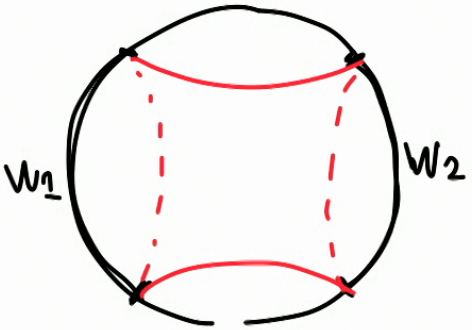
Boundary causal graph



Resolution : Entanglement as resource

[Alex May 2019]





$W_1 = J_+(C_1) \cap J_-(r_1) \cap J_-(r_2)$

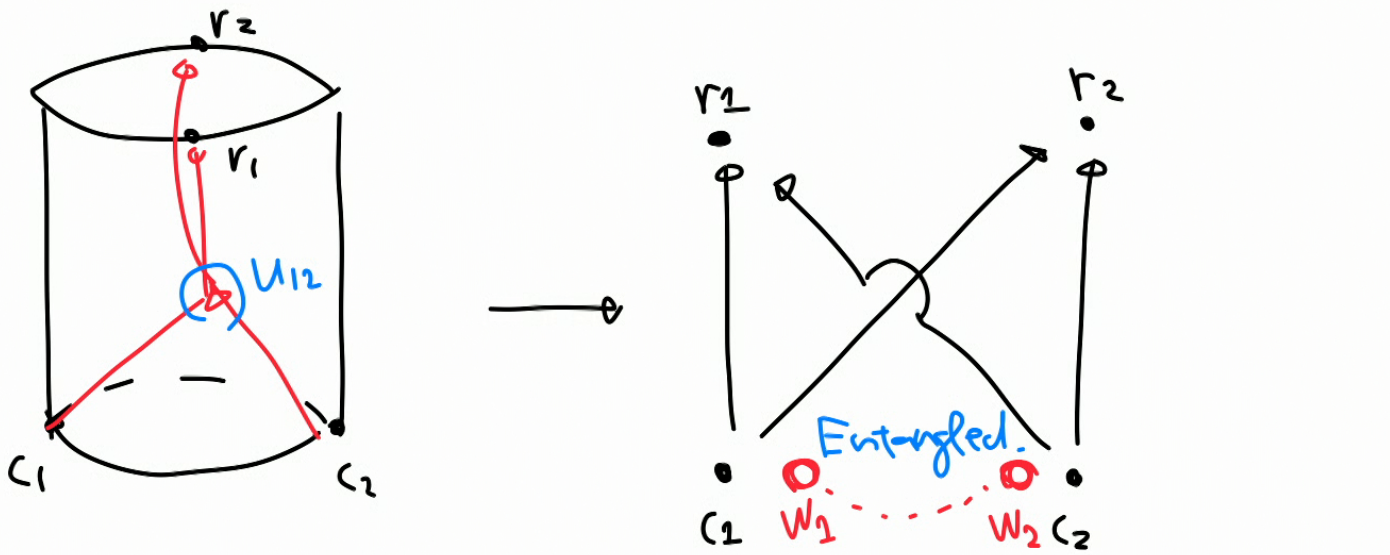
W_1 can signal to r_1, r_2

$I(W_1, W_2) = O(\frac{1}{G_N})$

W_1 & W_2 entangled !

(via Ryu-Takayanagi formula)

With entanglement ...



bulk

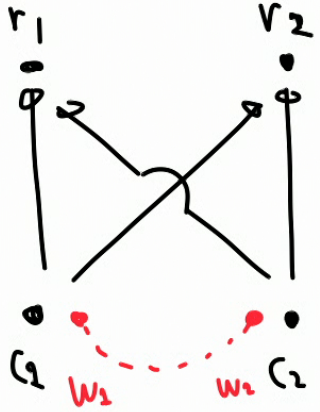
boundary

Idea

Somehow use $W_1 - W_2$ entanglement to implement U_{12}

Entanglement-Assisted Quantum Computation

Some U_{12} can be implemented efficiently !! (@I)

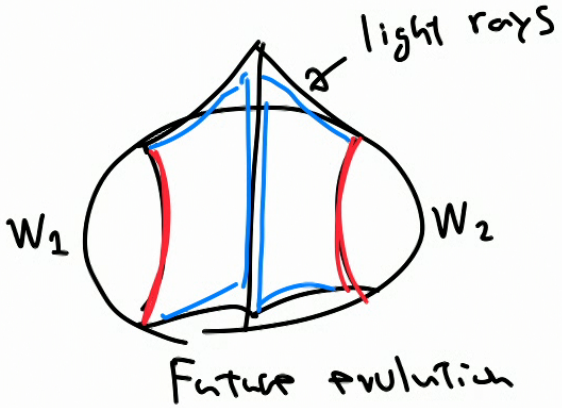


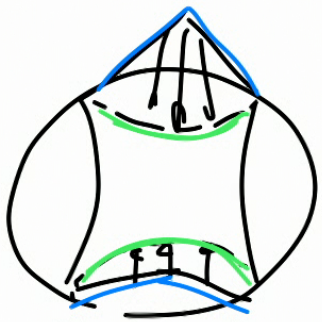
- Teleportation "BBq protocol". (1984)
  Quantum cryptography
- Arbitrary U_{12}: Port-based teleportation
  (Hiroshima-Ishizaka 2008)
  (Beigi-König 2012)

- But ... For n qubit, exp(n) EPRs needed in general.

Scattering implies entanglement

- [Thm] If $C_1, C_2 \longrightarrow r_1, r_2$ scattering is possible on bulk, then W_1 & W_2's wedges are connected.
- [idea] Use the focusing conjecture.
- [May, Pennington, Sorce 2020]
  [New result] more precise proof. & works for $N \geq 3$ particles

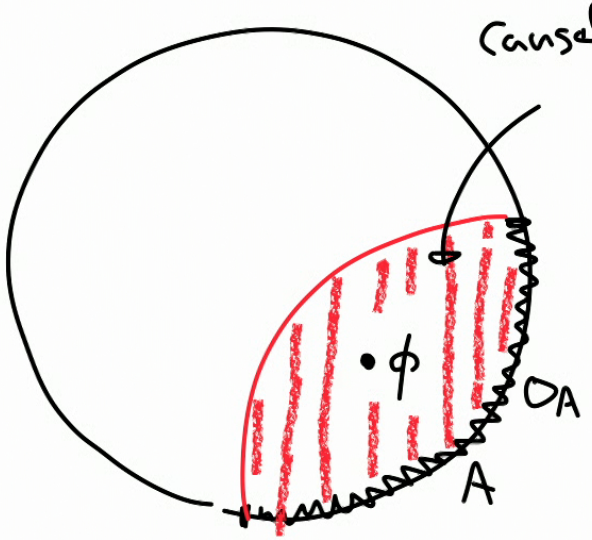




Scattering from

quantum error-correction?

Bulk operator from boundary



Causal wedge reconstruction

$\phi \longrightarrow O_A$

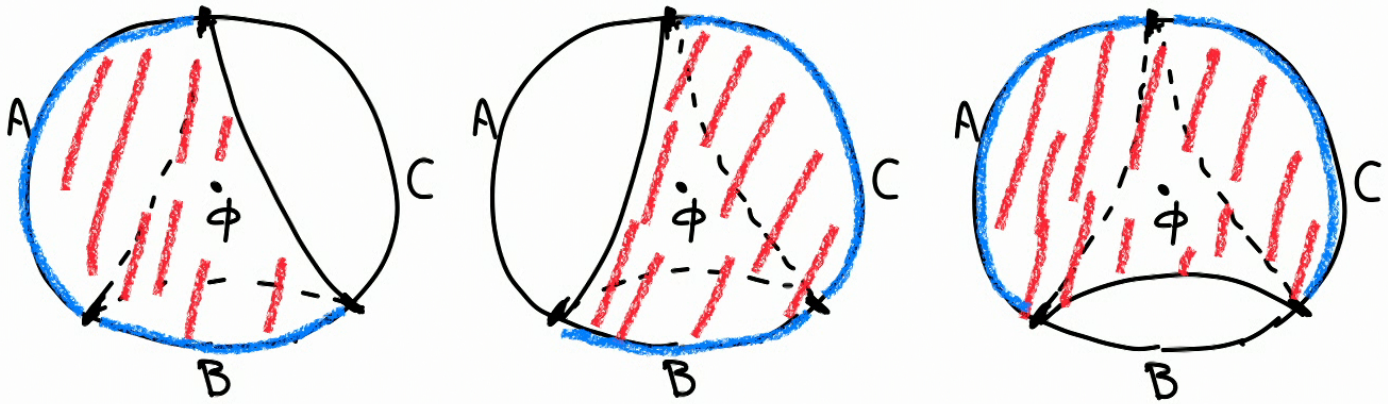
bulk boundary

Holographic Quantum error-correction

$\phi \longrightarrow O_{AB}, O_{BC}, O_{CA}$ (different, but equivalent)

Quantum error-correction:

Losing one subsystem is fine

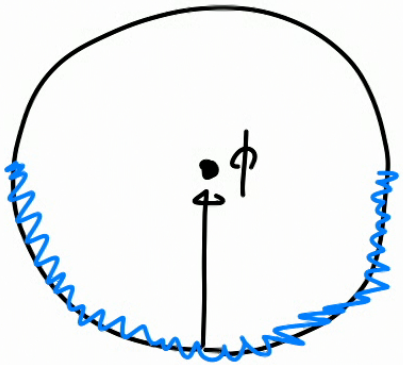


O_{AB} O_{BC} O_{CA}

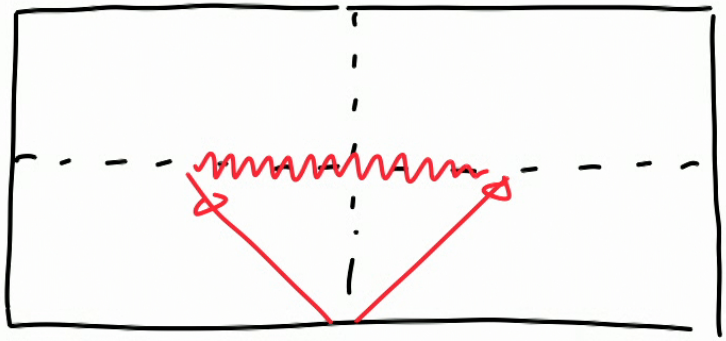
Quantum error-correction and entanglement

Not enough time to delocalize quantum information

→ Entanglement is needed for QEC.

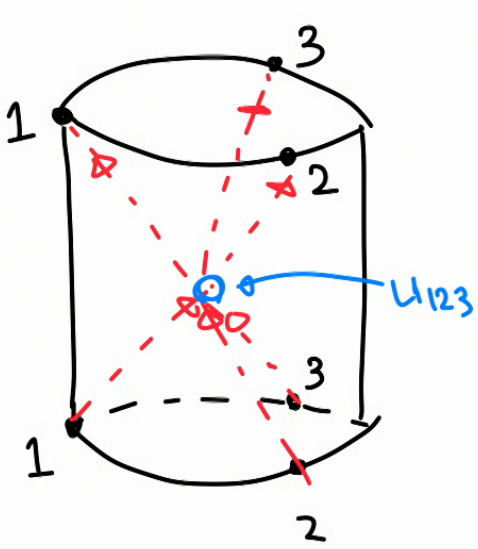


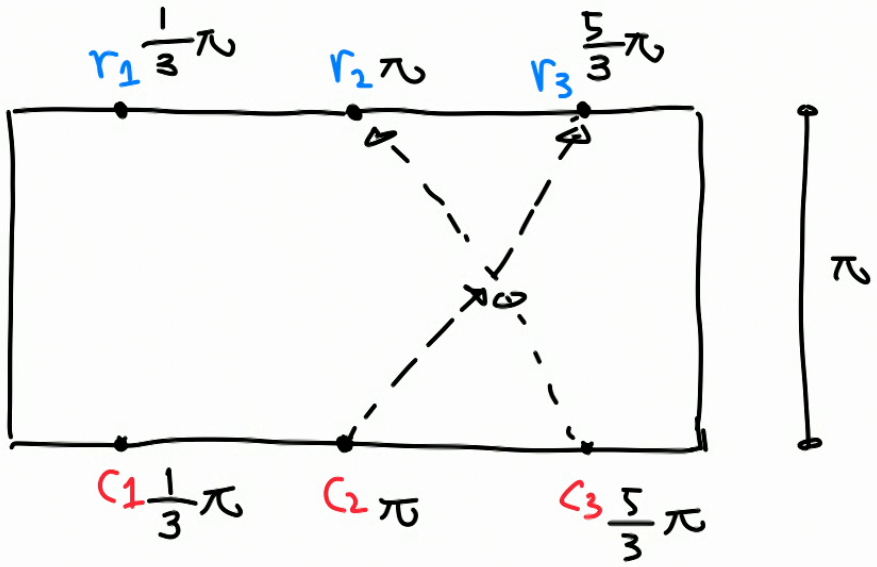
bulk



boundary

3 particle Scattering (or $N \geq 3$)

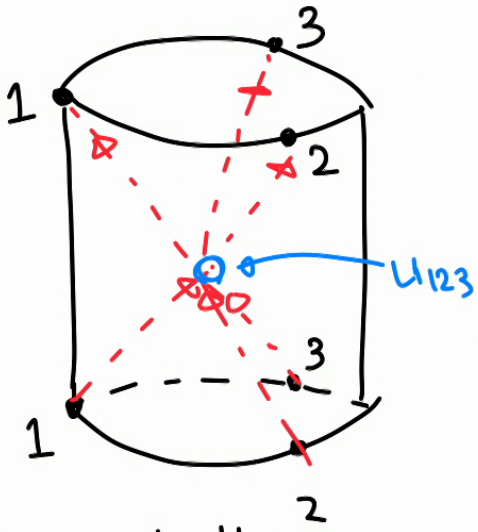


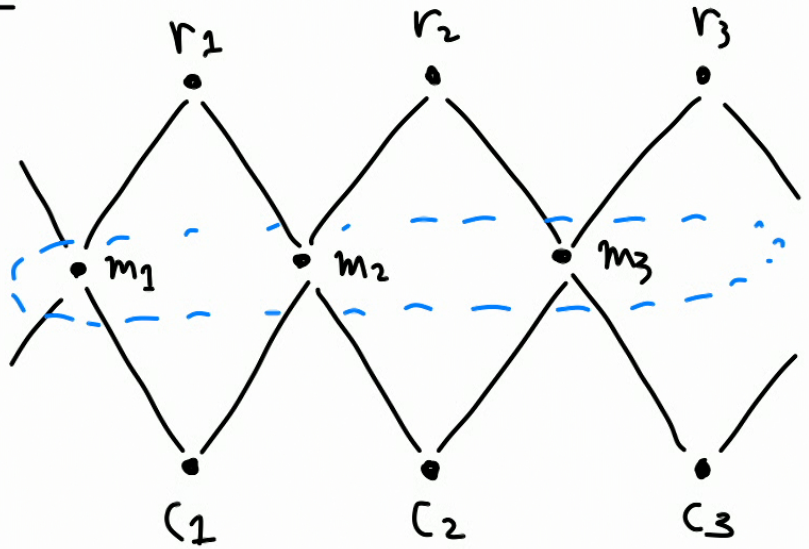


- 2-to-2 Scattering is possible

  3-to-3, not possible ... .

Boundary Causality





bulk

boundary

QECCs ?

Input

C_j

Intermediate

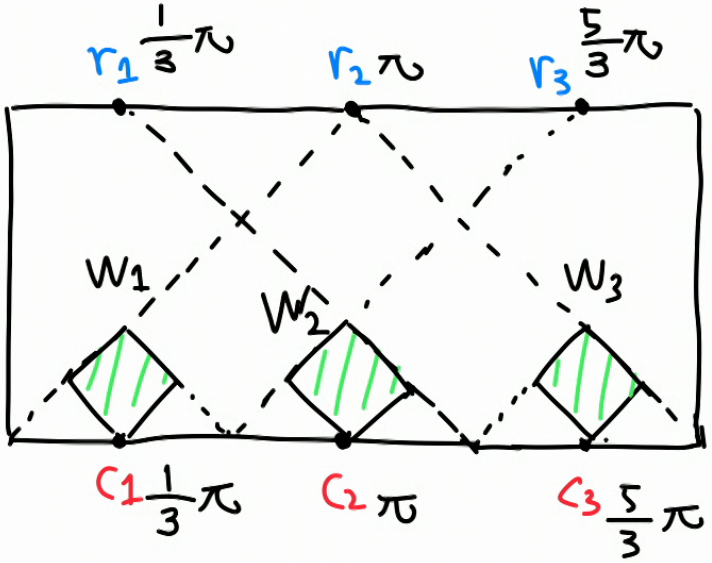
m_j

Output

r_j

Interaction happens here ?

Entanglement for 3 wedges ?



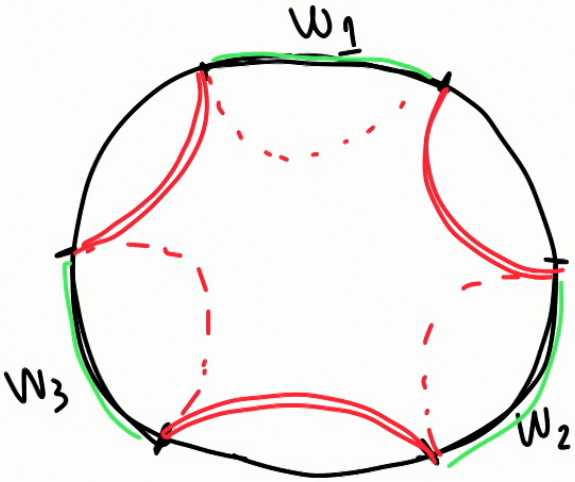
W_j's are entangled

How ??

$$W_2 \equiv J_-(r_1) \cap J_-(r_2) \cap J_-(r_3) \cap J_+(c_2)$$

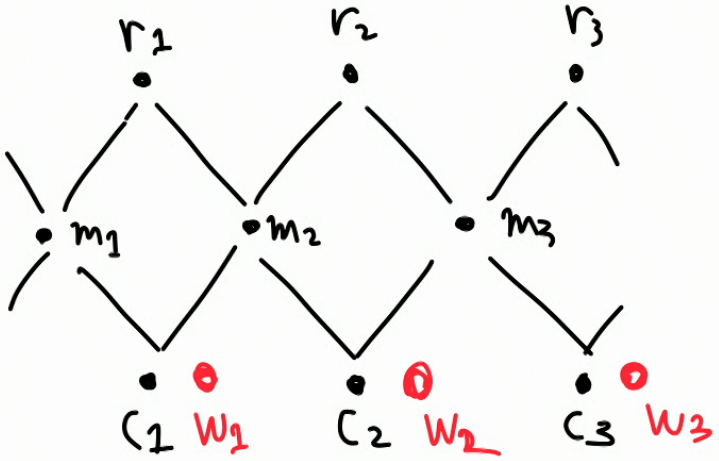
can signal to r_1, r_2, r_3

Connected wedge (Multi-party entanglement)



- $I(W_1, W_2W_3) = O(\frac{1}{G_N})$
- $I(W_1, W_2) \simeq 0$

No EPR pairs !!

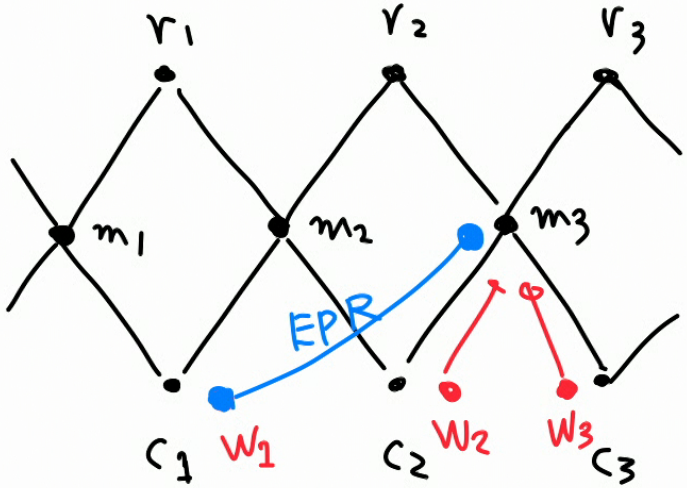


- "Teleportation" does not work ....

Entanglement in spacetime

EPR pair between different time slices

C_1 — m_1, m_2, m_3 encoding is possible!!

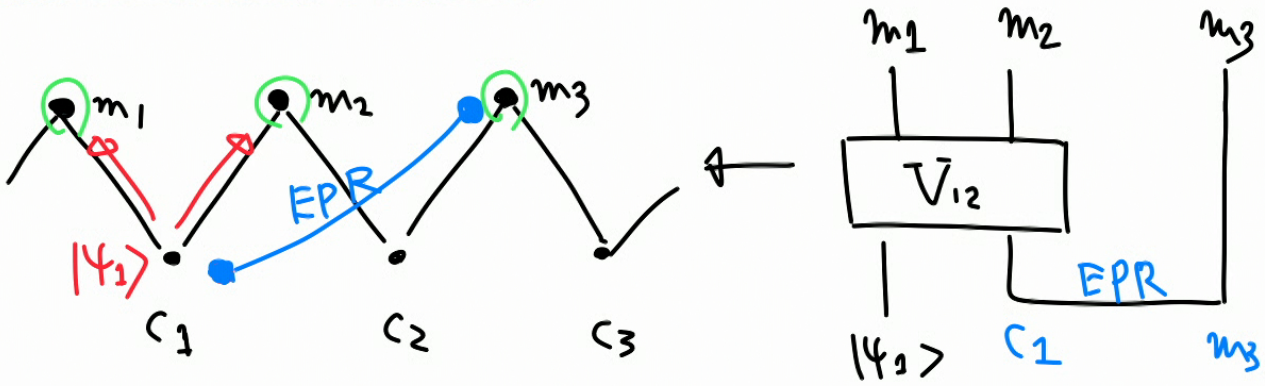


- C_1 and m_3 can share EPR pair

$$I(W_1, W_2 W_3) = O\left(\frac{1}{G_N}\right)$$

Entanglement-assisted QECCs

- Non-local encoding

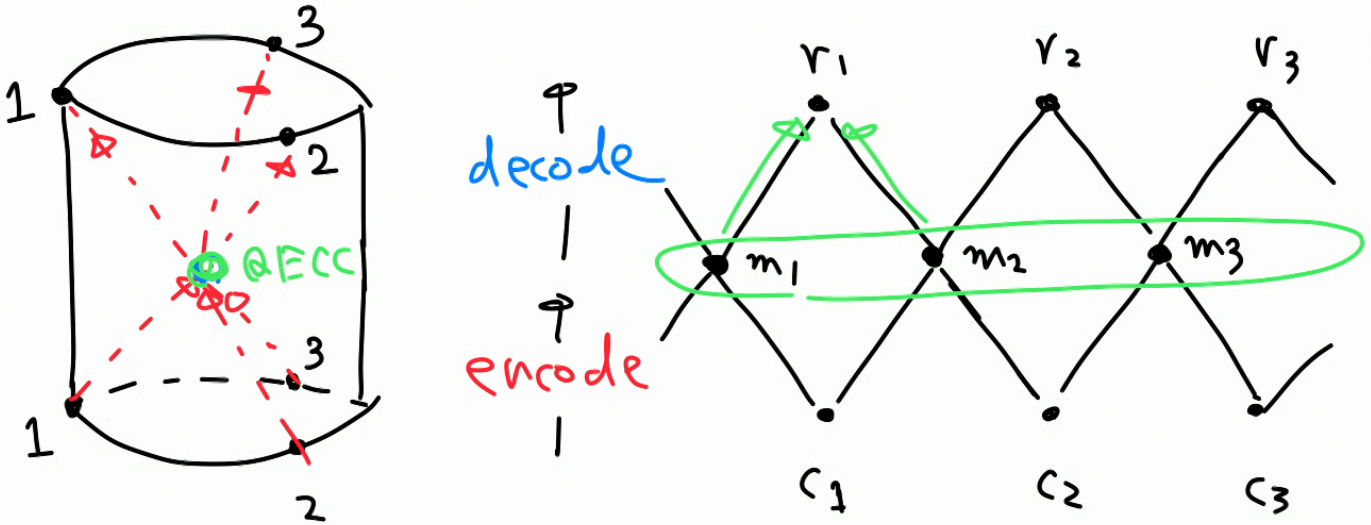


- V_{12} does not need to touch m_3

- Same mechanism as Hayden-Preskill protocol

  (BH – Early radiation entanglement)

Encoding and Decoding



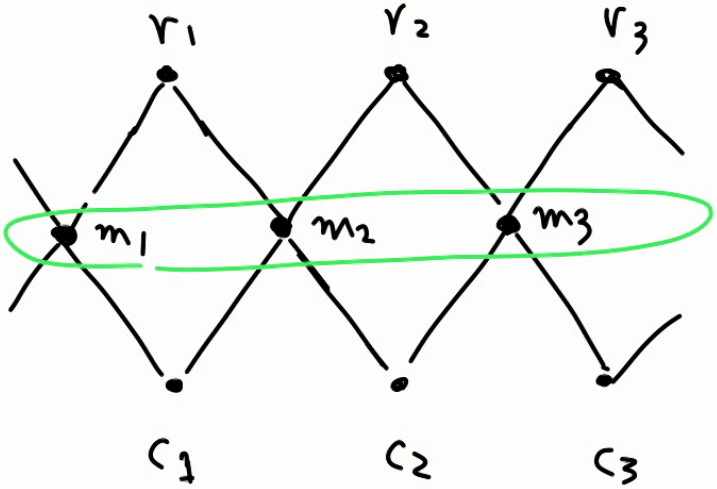
- r_2 has access to m_2 and m_3

  ⟶ $|\psi_1\rangle$ can be reconstructed on r_1, r_2, r_3.

Interaction in

QECCs

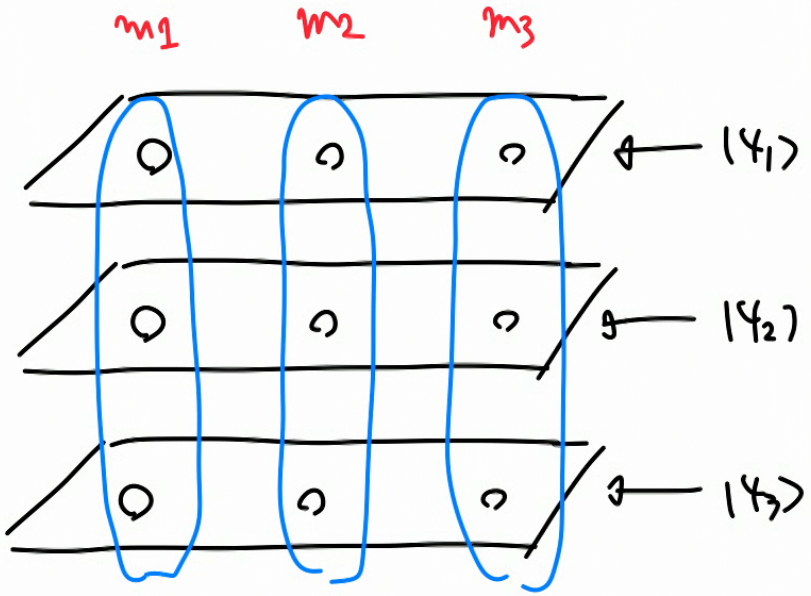
Interaction in QECCs



"Logical operators" act on m_1, m_2, m_3

Transversal logical operators

- Transversal (factorized) logical op only



$V_1 \otimes V_2 \otimes V_3 = V$

$e^{i\theta(x_1x_2x_3 + z_1z_2z_3)}$ is not possible

Implementable Logical gates

- Arbitrary Clifford gate (maps Pauli to Pauli)

  n-qubit $O(n)$ EPR pairs

  Reed-Solomon code (generalization of Qutrit code)

  Random Clifford encoding

- Phase gate

  quadratic

  $$|x_1, x_2, x_3\rangle \longrightarrow e^{i\theta(x_1, x_2, x_3)} |x_1, x_2, x_3\rangle$$

  need $O\left(\log\left(\frac{1}{\theta_{min}}\right)\right)$ EPR pairs

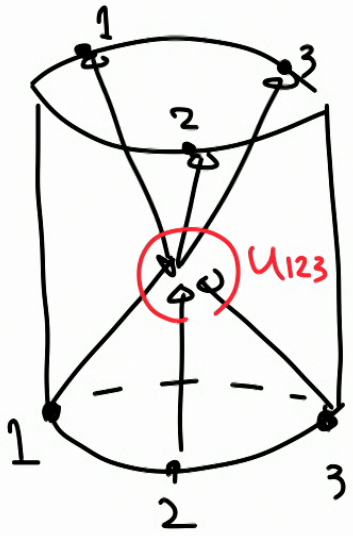
Eastin-Knill theorem

- Transversal gates form discrete groups
  - → Not all U_{123} are transversally implementable
- Approximate transversal gate avoids this no-go theorem
  - → AdS/CFT avoids Eastin-Knill in a clever way?

Summary and Outlook

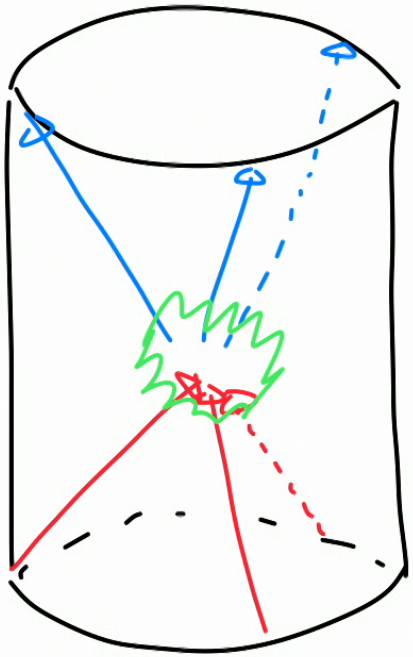
Take home message

- Interaction in bulk AdS emerges from logical op in QECCs.
- QECC encoding is possible due to pre-existing entanglement among wedges W_i's.



- Some U_{123} can be implemented. Arbitrary gate ??

Analogy to fault-tolerant quantum computing



output

decoding from QECC

interaction Transversal logical ops

encoding into QECC

input

Future Problems

- $L = (\nabla\phi)^2 + m_2\phi^2 + m_3\phi^3 + m_4\phi^4 + \cdots$

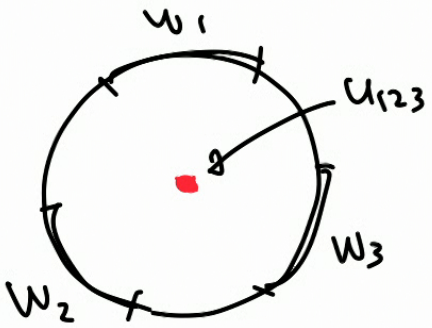
  interaction

  How does this affect $L_{boundary}$?

- Complexity of holographic scattering?

  Restriction on $U_{123\cdots n}$?

- Entanglement wedge reconstruction



Explicit form of bulk op?

- HKLL reconstruction

$$\phi = \int O(x,t)$$

Thank you !!