Title: Quantum Gravity Demystified

Speakers: Renate Loll

Series: Quantum Gravity

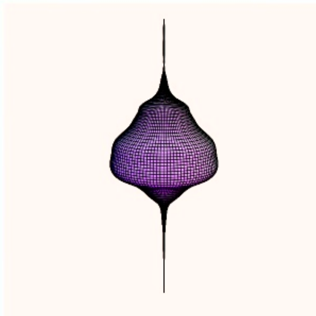
Date: October 27, 2022 - 2:30 PM

URL: https://pirsa.org/22100143

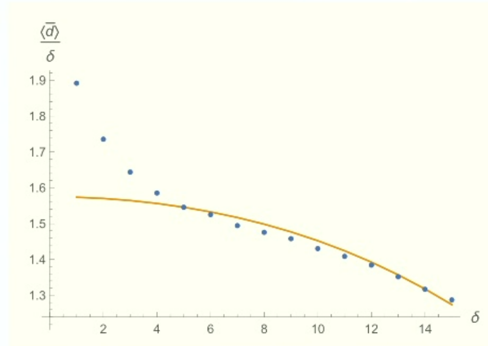
Abstract:

One fruitful strategy of tackling quantum gravity is to adapt quantum field theory to the situation where spacetime geometry is dynamical, and to implement diffeomorphism symmetry in a way that is compatible with regularization and renormalization. It has taken a while to address the underlying technical and conceptual challenges and to chart a quantum field-theoretic path toward a theory of quantum gravity that is unitary, essentially unique and can produce "numbers" beyond perturbation theory. In this context, the formulation of Causal Dynamical Triangulations (CDT) is a quantum-gravitational analogue of what lattice QCD is to nonabelian gauge theory. Its nonperturbative toolbox builds on the mathematical principles of "random geometry" and allows us to shift emphasis from formal considerations to extracting quantitative results on the spectra of invariant quantum observables at or near the Planck scale. A breakthrough result of CDT quantum gravity in four dimensions is the emergence, from first principles, of a nonperturbative vacuum state with properties of a de Sitter universe. I will summarize these findings, highlight the nonlocal character of observables in quantum gravity and describe the interesting physics questions that are being tackled using the new notion of quantum Ricci curvature.

Zoom Link: https://pitp.zoom.us/j/92791576774?pwd=VEg3MEdKOWsxOEhXOHVIQUhPcUt0UT09



J. Ambjørn, J. Jurkiewicz, R.L., PRL 93 (2004) 131301




N. Klitgaard, R.L., EPJ C 80 (2020) 990

# Quantum Gravity Demystified

**Renate Loll**
Radboud University (NL) & Perimeter Institute

**PI Seminar** **27 Oct 2022**

# My perspective on quantum gravity

**Aim**: construct a fundamental theory of quantum gravity as a non-perturbative, diffeomorphism-invariant quantum field theory of dynamical geometry and study its properties in a Planckian regime.

This presents major technical, physical and conceptual challenges: dealing with QFT infinities and the absence of a fixed background spacetime, devising appropriate numerical and renormalization methods, (re-)deriving the classical limit and phenomenology.

***This is possible.*** Major advances towards this goal have been made in the research program of ***Causal Dynamical Triangulations (CDT).*** It sets a concrete frame of reference - beyond "formal" matters - for what we may reasonably expect to be able to achieve in our quest to relate Planckian quantum gravity to "real", observable physics.

# Why should you care?

***quantum gravity:*** nontrivial, unexpected results despite non-exotic ingredients; functioning computational framework (*= our "lab"*) to evaluate quantum observables beyond perturbation theory;
*"CDT is to gravity what lattice QCD is to nonabelian gauge theory"*

***symmetry:*** diffeomorphism symmetry is very different from local gauge symmetry; we finally understand how to implement it consistently in a nonperturbative quantum theory of gravity

***"demystification":*** quantum (field) theory and general relativity are perfectly compatible; CDT provides a bottom-up realization of QG: causal structure is essential, unitarity is realized

***cosmology:*** the most likely phenomenological predictions will involve early-universe quantum physics, but derived from the full theory *without* an a priori symmetry reduction (unlike quantum cosmology)

# What's the problem with quantum gravity?

- General Relativity = theory ***of*** spacetime, not ***on*** (a fixed) spacetime
- quantum theory based on perturbative split $g_{\mu\nu}(x) = \eta_{\mu\nu}^{Mink} + h_{\mu\nu}(x)$ on a fixed Minkowskian background is nonrenormalizable

**M. Goroff, A. Sagnotti, NPB 266 (1986) 709**

- standard relativistic quantum field theory (QFT) not applicable, no blueprint beyond perturbation theory (except nonperturbative lattice QCD, ***but*** this has a fixed background, different gauge symmetry)
- no experiments or observations to guide theory-building
- (nonperturbative) QG ≲ 2000: arguing about the "best approach", however, no-one knows *which observables* to compute, and *how*
- QG ≳ 2000 (post extended-objects era): renaissance of "good old QFT"/the path integral, *we have learned how and what to compute*

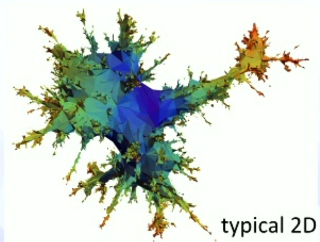
**R.L. et al.: "Quantum Gravity in 30 Questions", arXiv: 2206.06762**

# (Causal) Dynamical Triangulations: the basics

- superposition principle: path integral over metrics on a manifold *M*, a nonperturbative, Lorentzian "sum over (spacetime) histories"

$$Z = \int_{g \in \frac{Lor(M)}{Diff(M)}} \mathcal{D}g \; \mathrm{e}^{i \underbrace{S_{\rm grav}}_{\text{gravitational action}} [g]}$$

- CDT builds on Euclidean (=Riemannian) "**D**ynamical **T**riangulations": a weighted sum over all spherical gluings of *N* equilateral triangles, a piecewise flat implementation of the formal 2D path integral $\int Dg\, e^{-S[g]}$

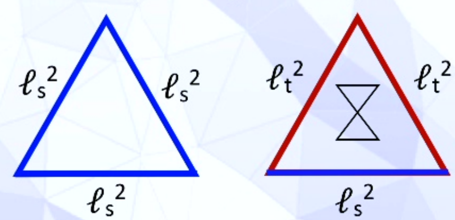


typical 2D random surface (© T. Budd)

- 2D random geometry is a hot topic in maths! a suitable continuum limit $N \rightarrow \infty$ gives rise to the *Brownian sphere* S. Sheffield, arXiv:2203.02470

- there is an *inequivalent*, exactly soluble Lorentzian version, using triangulations with well defined causal structure, J. Ambjørn, R.L., NPB 536 (1998) 407, which also gives interesting results in *D>2*!

elementary 2D triangular building blocks




Euclidean/ Riemannian (DT) | Minkowskian/ Lorentzian (CDT)

# Putting quantum gravity on a lattice, correctly

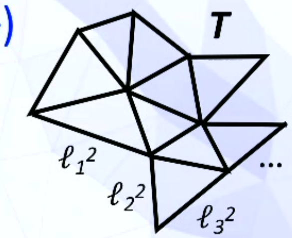
**General strategy:** lattice acts as a <u>regulator</u>, with UV cutoff $a$; search for a continuum limit by approaching a second-order phase transition in the limit $a \to 0$ while renormalizing bare couplings appropriately; attain *universality* (independence of regularization); this is **not** "discrete QG"

• "reaches where other methods don't", subject to numerical limitations; if it exists, continuum theory is essentially *unique*

• "naïve" lattice QG (≥ 1979): put various first-order formulations of GR (tetrad $e_\mu{}^A$ + spin connection $\omega_\mu{}^{AB}$) on a fixed hypercubic lattice; problem: diffeomorphism symmetry badly broken; no interesting results

• "not-so-naïve" lattice QG (≥ 1981): based on "GR without coordinates"
$(M, g_{\mu\nu}(x)) \to (T, \{\ell_i{}^2, i=1,\ldots,n\})$, $S_{grav}[g_{\mu\nu}] \to S^{Regge}(T, \{\ell_i{}^2\})$

T. Regge, Nuovo Cim. A19 (1961) 558




• diffeo-invariance manifest, work directly on $\mathcal{G}(M)$;
CDT ($\ell^2 = \pm a^2$) implementation is labelling-invariant

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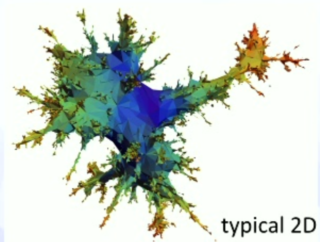

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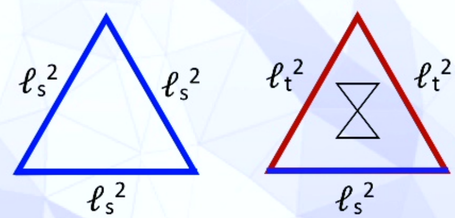


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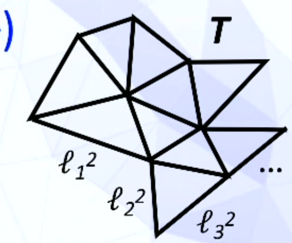


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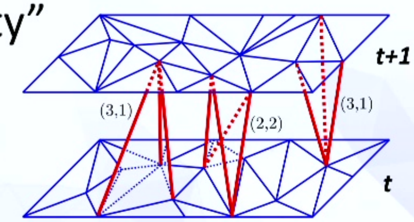
# The path integral according to CDT

simplicial version of action

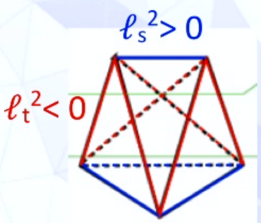
$$Z=\int_{\mathcal{G}(M)}\mathcal{D}g\,\mathrm{e}^{iS[g]},\ S=\frac{1}{16\pi G_N}\int_M d^4x\,\sqrt{|g|}(R[g]-2\Lambda)\xrightarrow{\text{CDT}} Z=\lim_{N\to\infty}\sum_{\substack{causal\\ triang.\,T}}\frac{1}{C(T)}\,\mathrm{e}^{iS^R[T]}$$

geometries = *Lor(M)/Diff(M)* cosmological constant # symmetries of T

- we are not interested in *finite* triangulations, but a regularization-independent continuum limit $N \to \infty$, for finite physical volume $V=N\,a^4$
- CDT spacetimes obey discrete "global hyperbolicity"
- evaluating $Z$ in 4D requires numerical methods, which require an analytic continuation to real $Z$ (*Wick rotation*): in $\ell_t^2=-\alpha\,\ell_s^2$, $\alpha>0$, continue $\alpha\to-\alpha$

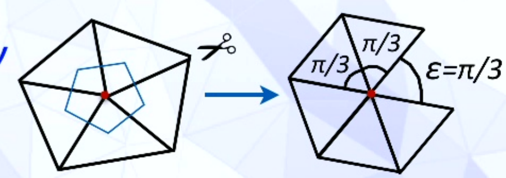


"time layer" of a CDT configuration in 3D



building block of 4D CDT

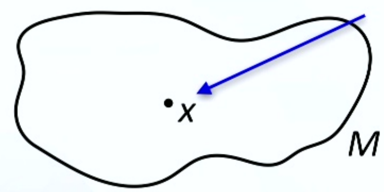
- no coordinate redundancy: *edge lengths + gluing data = geometry*
- curvature captured by deficit angle $\varepsilon$ (but sum diverges in cont. limit)



2D triangulation with Gaussian curvature ε

# CDT quantum gravity: results

- we have a computational framework — what can we do with it?
- physics of ***quantum spacetime*** is captured by expectation values
  $\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z} \int \mathcal{D}g \, \mathcal{O}[g] \, \mathrm{e}^{-S[g]}$ of ***quantum observables*** $\hat{\mathcal{O}}$
- diffeomorphism-invariant observables in pure gravity are *nonlocal* integrals of scalars like $\int_M d^4x \sqrt{g}\, R(x)$

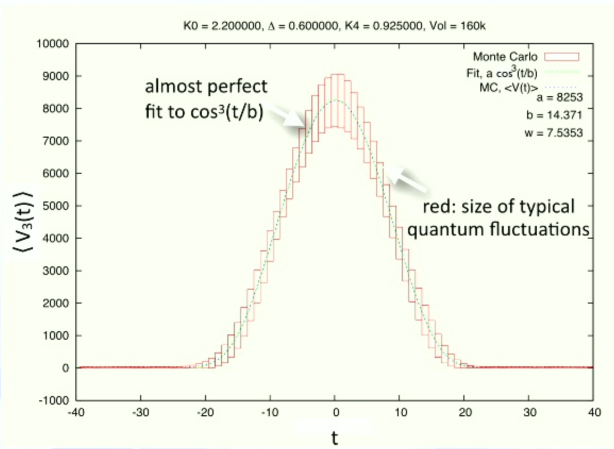



- "expectation management": your favourite (semi-)classical question may not have a Planckian implementation (this is a feature)
- true "quantum signature": CDT predicts a reduction 4 → 2 of the (average) ***spectral dimension*** of spacetime@$\ell_{Pl}$ , **J. Ambjørn, J. Jurkiewicz, R.L., PRL 95 (2005) 171301** — universal in quantum gravity? **S. Carlip, CQG 34 (2017) 193001**
- to understand 4D quantum geometry, must go beyond "dimensions"

# Key result: emergence of classicality from CDT

On sufficiently large scales, the average shape $\langle V_3(t) \rangle$ (spatial volume as a function of proper time) of the quantum spacetime obtained dynamically in CDT matches that of a classical ***de Sitter space***.

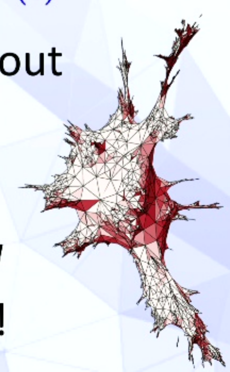
**J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., PRL 100 (2008) 091304, PRD 78 (2008) 063544**




Since global shape is just one mode of the metric, we cannot conclude that this quantum universe *is* a (Euclidean) de Sitter space $S^4$, with line element

$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2_{(3)}, \;\; c = const.$$

Can we say anything about its ***local geometry***?

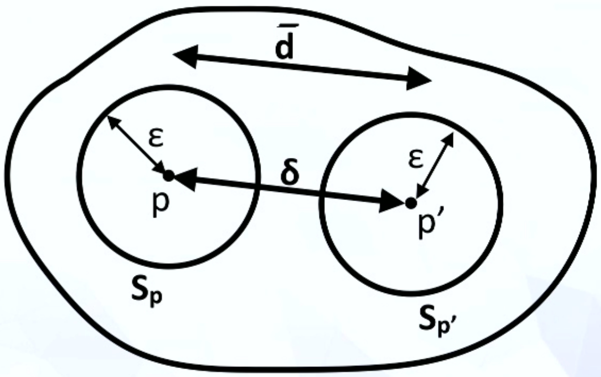


Can we attribute ***local curvature*** to a non-smooth metric space? $R^{\kappa}{}_{\lambda\mu\nu}[g,\partial g,\partial^2 g,x) = ?$ —*Yes,* there is a ***renormalized*** notion of Ricci curvature applicable in a Planckian regime!

# Introducing quantum Ricci curvature

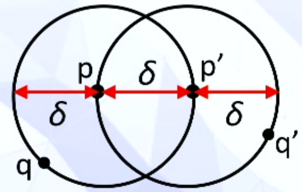
In $D$ dimensions, the key idea is to compare the distance $\bar{d}$ between two *(D-1)*-spheres with the distance $\delta$ between their centres.

The sphere-distance criterion:
***"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres $S_p$ and $S_{p'}$ is smaller (bigger) than the distance $\delta$ of their centres."***




cf. Y. Ollivier, J. Funct. Anal. 256 (2009) 810

Our variant uses the ***average sphere distance*** of two spheres of radius $\delta$ whose centres are a distance $\delta$ apart,

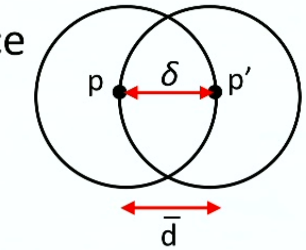



$$\bar{d}(S_p^\delta, S_{p'}^\delta) := \frac{1}{vol(S_p^\delta)} \frac{1}{vol(S_{p'}^\delta)} \int_{S_p^\delta} d^{D-1}q \sqrt{h} \int_{S_{p'}^\delta} d^{D-1}q' \sqrt{h'}\; d_g(q,q'),$$

N. Klitgaard, R.L., PRD 97 (2018) 0460008, N. Klitgaard, R.L., PRD 97 (2018) 106017

# Defining the quantum Ricci curvature (QRC)

From the quotient of sphere distance and centre distance we define the **"quantum Ricci curvature $K_q$ at scale $\delta$"**,




$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad 0 < c_q < 3,$$

where $c_q$ is a non-universal constant depending on the type and the dimension $D$ of the space. For Riemannian manifolds and $\delta \ll 1$:

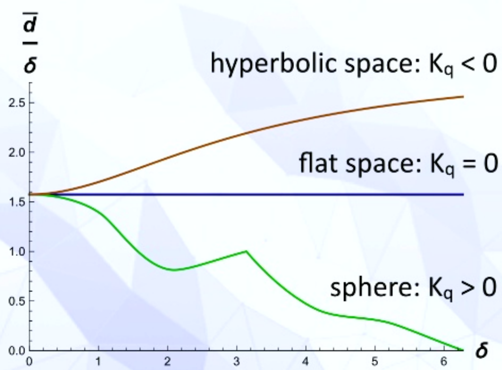
$$\frac{\bar{d}}{\delta} = \begin{cases} 1.5746 + \delta^2 \left(-0.1440\, Ric(v,v) + \mathcal{O}(\delta)\right), & D = 2, \\ 1.6250 + \delta^2 \left(-0.0612\, Ric(v,v) - 0.0122\, R + \mathcal{O}(\delta)\right), & D = 3, \\ 1.6524 + \delta^2 \left(-0.0469\, Ric(v,v) - 0.0067\, R + \mathcal{O}(\delta)\right), & D = 4, \end{cases}$$

- N.B.: this involves only ***distance and volume measurements***
- directional character is captured by the oriented "pair of spheres"; to extract scalar curvature only, set $p=p'$ (coinciding "double sphere")
- the QRC provides a highly nontrivial notion of ***coarse-grained curvature at scale $\delta$***, for non-infinitesimal distance scales $\delta$

# Quantum Ricci curvature for quantum gravity

Our work on QRC on piecewise flat spaces shows it is ***computable***, ***scalable*** (depends on some physical scale), ***renormalizable*** (stays finite in a continuum limit) and ***robust*** (w.r.t. anomalous scaling), and on “nice” classical spaces reproduces standard results.

<u>simplest ***QRC observable***</u>: summing over all pairs *(p′,p)* with $\delta = d(p,p')$ yields a $\delta$-dependent, nonlocal ***curvature profile*** $\bar{d}_{\rm av}/\delta = c_{\rm av}(1 - K_{\rm av}(\delta))$




$\bar{d}/\delta \equiv \bar{d}_{av}/\delta$ for constantly curved spaces in 2D (curvature radius 1)

- new type of observable, characteristic “fingerprint” of an entire universe
- classical constant-curvature spaces have typical deviations from constancy
- can detect anisotropy (ellipsoid vs $S^2$)

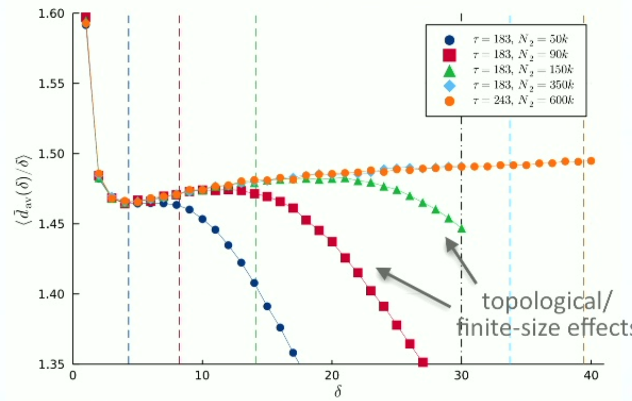
  **G. Clemente, N. Klitgaard, R.L., w.i.p.**
- influence of conical singularities

  **J. Brunekreef, R.L., PRD 103 (2021) 026019**

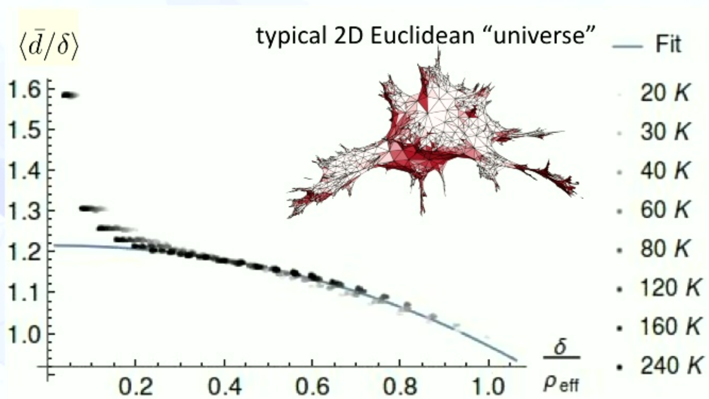
# Finally, measuring *quantum* Ricci curvature!

typical 2D Lorentzian "universe"

The expectation value $\langle \bar{d}_{\rm av}(\delta)/\delta \rangle$ of the curvature profile of 2D Lorentzian quantum gravity on a torus is not flat, but "quantum-flat". **J. Brunekreef, R.L., PRD 104 (2021) 126024**




$<\bar{d}_{av}/\delta>$ in 2D Lorentzian QG on $T^2$, N ∈ [50k,600k]



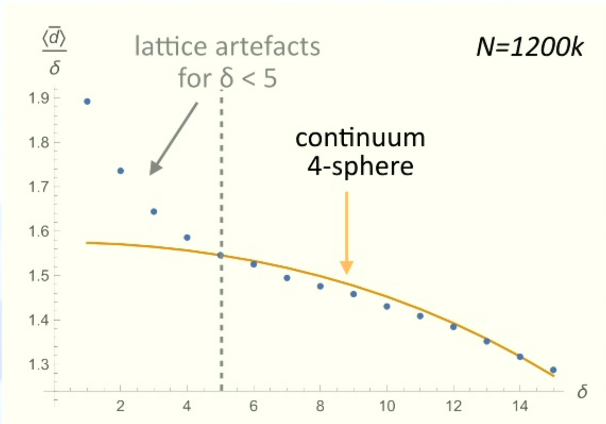

$<\bar{d}_{av}/\delta>$ in 2D Euclidean QG on $S^2$, N ∈ [20k,240k]

The expectation value $\langle \bar{d}_{\rm av}(\delta)/\delta \rangle$ of the curvature profile of 2D Euclidean quantum gravity on a sphere is best matched by a 5D(!) continuum sphere.

**N. Klitgaard, R.L., PRD 97 (2018) 106017**

# Quantum curvature of the de Sitter universe?

Measuring the curvature profile of the quantum universe in 4D quantum gravity from CDT on $M = S^3 \times S^1$ at volumes $N \leq 1.2 \times 10^6$ show that the quantum Ricci scalar is positive, $\langle K_q \rangle > 0$, with a good fit to a Euclidean de Sitter universe $S^4$!




$<\bar{d}_{av}/\delta>$ of the dynamically generated dS universe

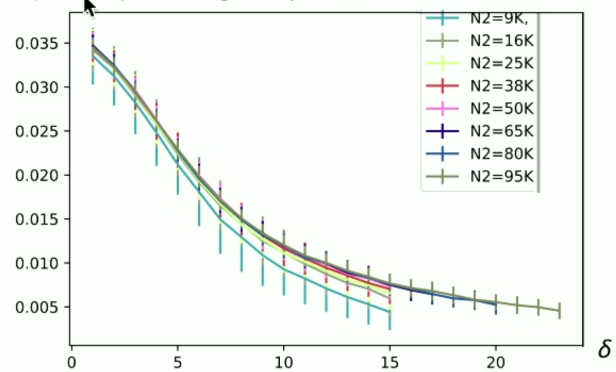
In addition, the quantum *Ricci* curvatures in time- and spacelike directions appear to be the same.

N. Klitgaard, R.L., Eur. Phys. J. C80 (2020) 990

This provides additional support to the interpretation of the emergent quantum universe in terms of a ***de Sitter space***, in the sense of expectation values.

# Opening doors to the early quantum universe

(relative) inhomogeneity for $\mathcal{O}$ = Hausdorff dimension




$\langle \mathcal{H}_{\mathcal{O}}(\delta)/\bar{\mathcal{O}}(\delta) \rangle$ in 2D Lorentzian QG on $T^2$, N ∈ [9k,95k]

***measures of inhomogeneity and anisotropy*** for quantum spacetime, using observables $\mathcal{O}_B$ coarse-grained over geodesic balls $B(x,\delta)$: e.g.

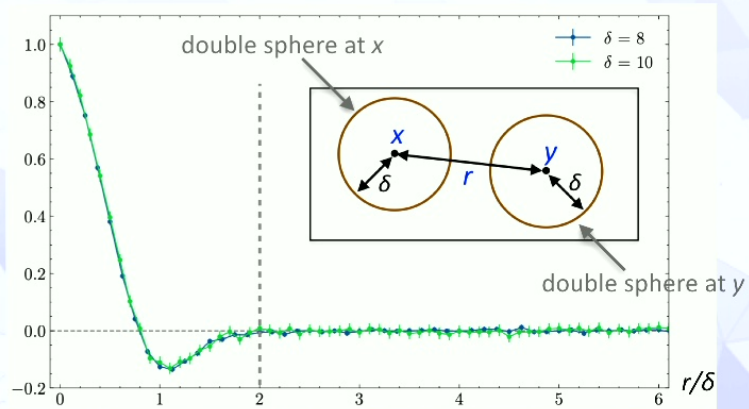
$$\mathcal{H}_{\mathcal{O}}(\delta) = \sqrt{\frac{1}{N_0}\sum_{x\in T}(\mathcal{O}_B(x,\delta) - \bar{\mathcal{O}}_B(\delta))^2}$$

(absolute) inhomogeneity at scale $\delta$

**A. Silva, R.L., w.i.p.**

***2-point curvature correlator*** of the coarse-grained quantum Ricci scalar in 2D Lorentzian quantum gravity: no correlations for $r > 2\delta$

**J. van der Duin, R.L., w.i.p.**




$\langle (K_q(x)-\bar{K}_q)(K_q(y)-\bar{K}_q) \rangle$ in 2D Lorentzian QG on $T^2$, N=100k

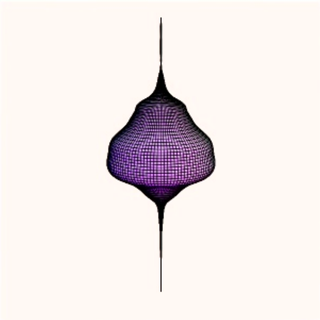
# Summary and outlook

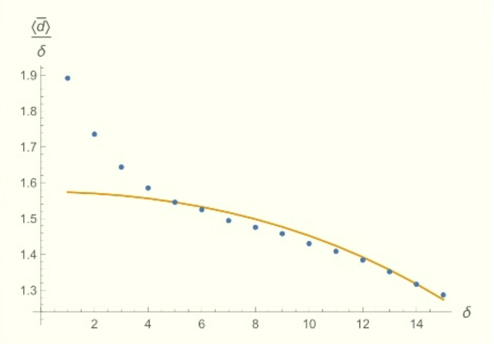
- ***genuine progress*** in applying nonperturbative methods to full 4D quantum gravity: we can compute observables and compare them, e.g. with results obtained by functional RG methods

  A. Bonanno, F. Saueressig et al., Front. in Phys. 8 (2020) 269

- the art is to ***identify (more) observables*** that can be measured in the available scale range and related to macroscopic physics
- CDT provides a rare example of ***spacetime emergence***; the new ***quantum Ricci curvature*** allows us to investigate many interesting properties of the emergent de Sitter universe across scales
- many other ongoing projects in CDT: extended RG flow analysis, roles of matter coupling and global topology, CDT-inspired quantum cosmology J. Ambjørn, Y. Watabiki, early-universe structure formation, ...

CDT reviews: J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., Phys. Rep. 519 (2012) 127, arXiv: 1203.3591; R.L., Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669






# Thank you!

PI Seminar

27 Oct 2022