Title: On the fate of the LR instability

Speakers: Carlos Herdeiro

Series: Strong Gravity

Date: October 18, 2022 - 1:00 PM

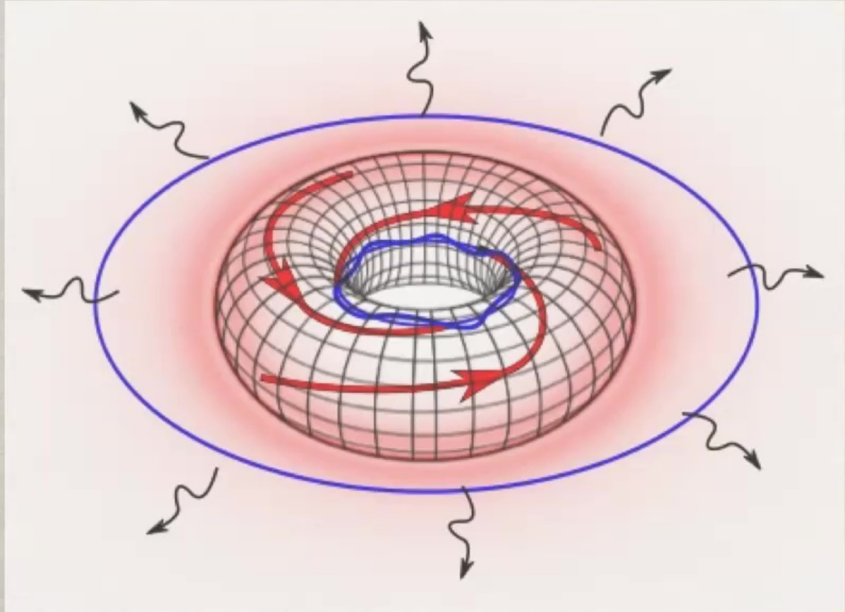
URL: https://pirsa.org/22100139

Abstract: Abstract and

Zoom Link: https://pitp.zoom.us/j/91762985902?pwd=djhwYVdsQ25GVVBRVTlwSkQvaDJ4Zz09

On the fate of the Light Ring instability

C. Herdeiro
Departamento de Matemática e CIDMA, Universidade de Aveiro, Portugal
Perimeter Institute (online), Oct 18th 2022



Based on
2207.13713 with P. Cunha, E. Radu, N. Sanchis-Gual

FCT Fundação para a Ciência e a Tecnologia
The Gravity group @ Aveiro University, Portugal
CIDMA
MSCA RISE

The black hole hypothesis

Evidence 1: Radio galaxies and Active Galactic Nuclei (AGNs)

On the fate of the Light Ring instability

Plan:

1) The "*black hole (BH) hypothesis*": BHs and light rings (LRs)

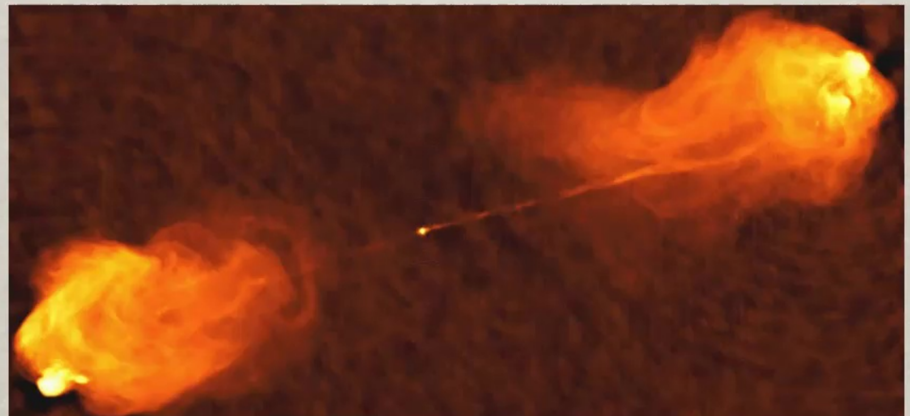
2) The "*exotic compact object (ECO) hypothesis*": ECOs and LRs

3) The LR instability and an explicit test of its fate

4) Discussion and final thoughts

The black hole hypothesis

Evidence 1: Radio galaxies and Active Galactic Nuclei (AGNs)



Cygnus A

Narayan and McClintock, 1312.6698

(Figure from C. Carilli and R. Perley, NRAO)

1040 NATURE March 16, 1963 Vol. 197

3C 273: A STAR-LIKE OBJECT WITH LARGE RED-SHIFT

By Dr. M. SCHMIDT

Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology, Pasadena

THE only objects seen on a 200-in. plate near the positions of the components of the radio source 3C 273 reported by Hazard, Mackey and Shimmins in the preceding article are a star of about thirteenth magnitude and a faint wisp or jet. The jet has a width

Table 1. Wave-lengths and Identifications

λ	$\lambda/1{\cdot}158$	λ_0	
3239	2797	2798	Mg II
4595	3968	3970	Hϵ
4753	4104	4102	Hδ
5032	4345	4340	Hγ

The black hole hypothesis

Evidence 2: X-ray binaries

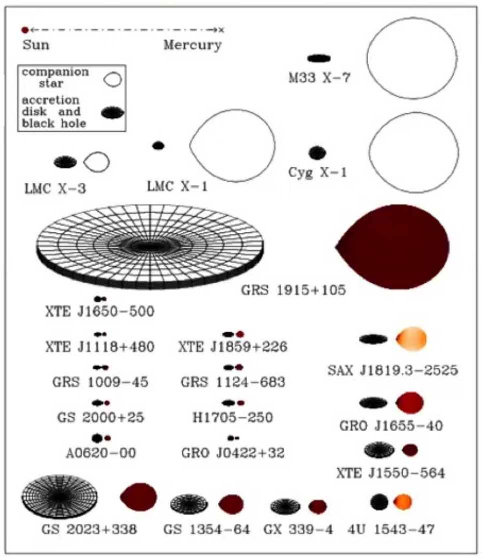
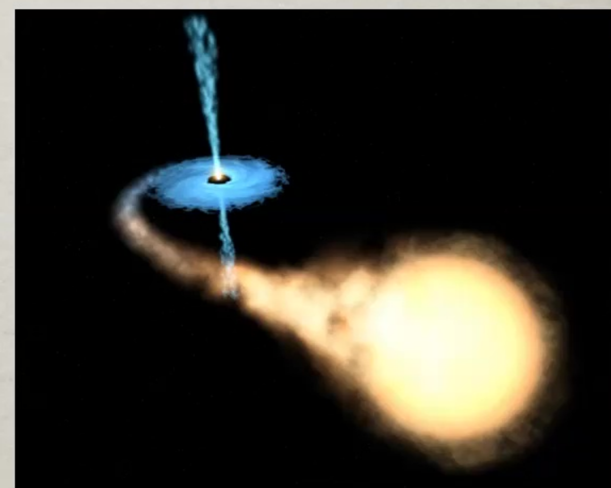


Figure 1.4 Sketches of 21 black hole binaries (see scale and legend in the upper-left corner). The tidally-distorted shapes of the companion stars are accurately rendered in Roche geometry. The black holes are located at the centers of the disks. A disk's tilt indicates the inclination angle i of the binary, where $i = 0$ corresponds to a system that is viewed face-on; e.g., $i = 21°$ for 4U 1543–47 (bottom right) and $i = 75°$ for M33 X–7 (top right). The size of a system is largely set by the orbital period, which ranges from 33.9 days for the giant system GRS 1915+105 to 0.2 days for tiny XTE J1118+480. Three systems hosting persistent X-ray sources — M33 X–7, LMC X–1 and Cyg X–1 — are located at the top. The other 18 systems are transient sources. (Figure courtesy of J. Orosz.)



Artistic impression
of an accretion disk
around a stellar mass BH-star binary

Narayan and McClintock
ArXiv:1312.6698

The black hole hypothesis

Evidence 3: Orbits around Sgr A*

Ghez, A. M. et al., (2008) Astrophysical Journal 689 (2): 1044–1062
Ghez, A. M. et al., (2005) The Astrophysical Journal 620 (2): 744–757

A&A 615, L15 (2018)
https://doi.org/10.1051/0004-6361/201833718

Astronomy & Astrophysics

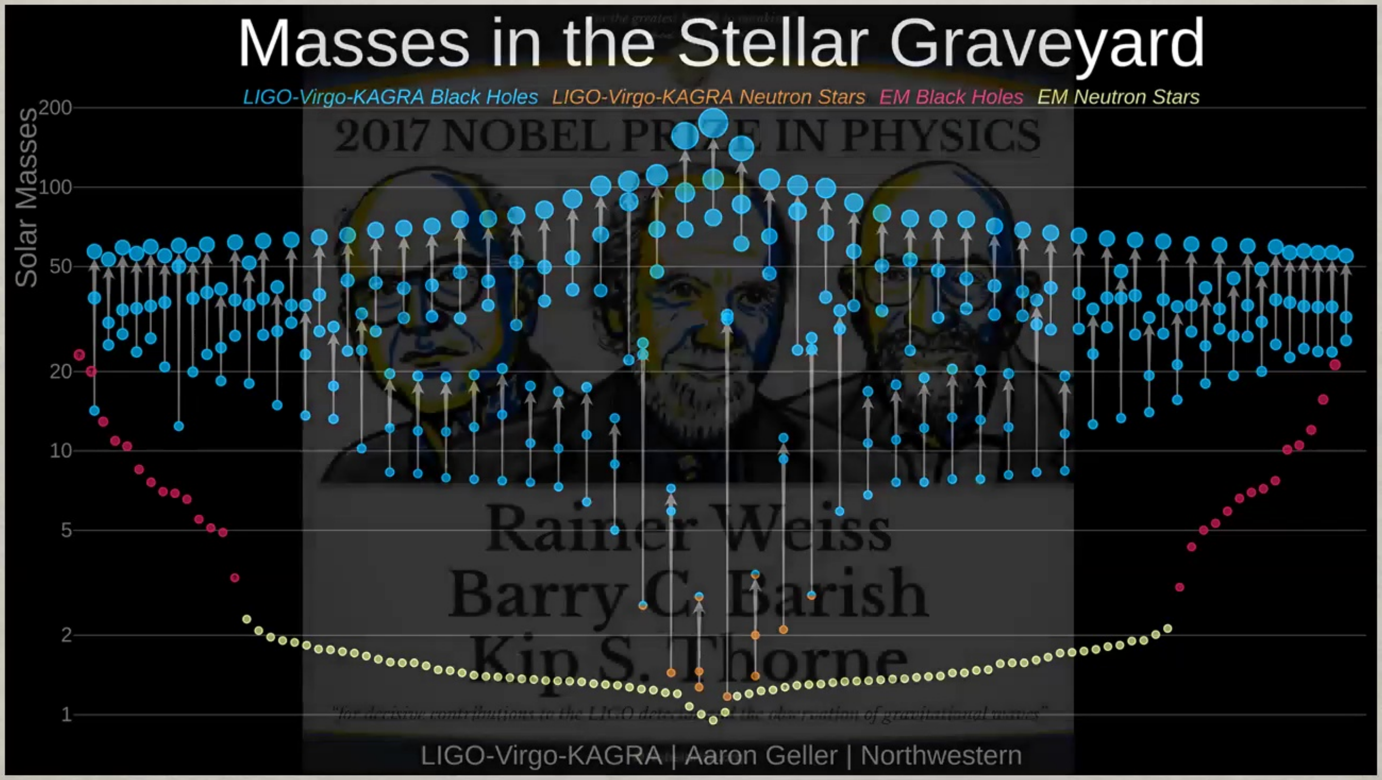
LETTER TO THE EDITOR

Detection of the gravitational redshift in the orbit of the star S2 near the Galactic centre massive black hole*

GRAVITY Collaboration**: R. Abuter[8], A. Amorim[6,14], N. Anugu[7], M. Bauböck[1], M. Benisty[5], J. P. Berger[5,8],

The black hole hypothesis

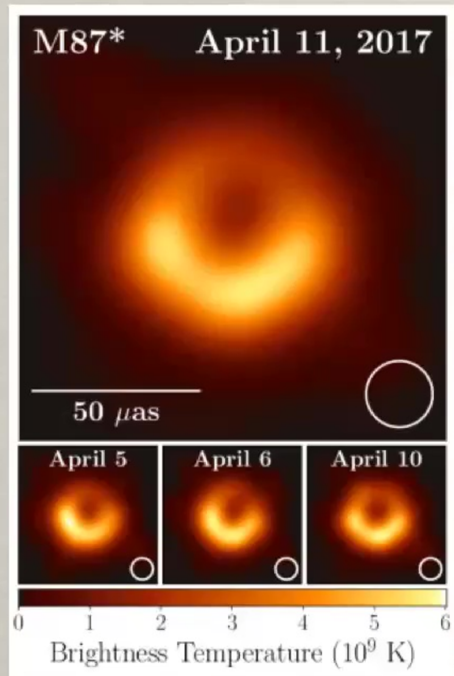
Evidence 4: Gravitational Wave observations



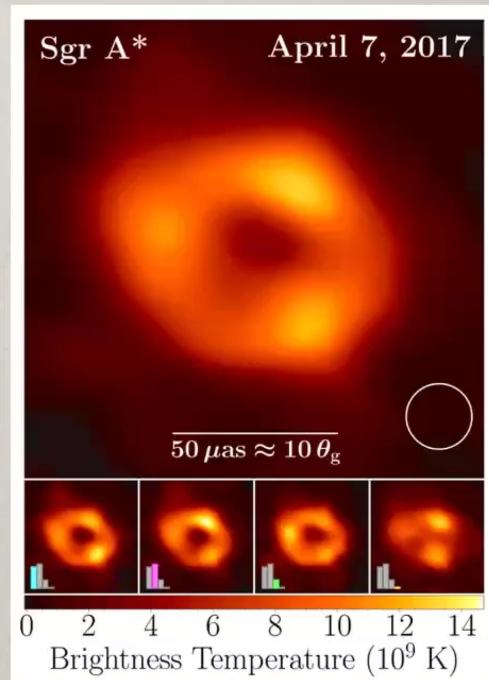
https://www.ligo.caltech.edu/MIT/image/ligo20211107a

The black hole hypothesis

Evidence 5: Black Hole imaging



ApJ Lett. 875 (2019) L1



ApJ Lett. 930 (2022) L12

The (Kerr) black hole hypothesis

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's field equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the Universe."

S. Chandrasekhar, in *Truth and Beauty* (1987)

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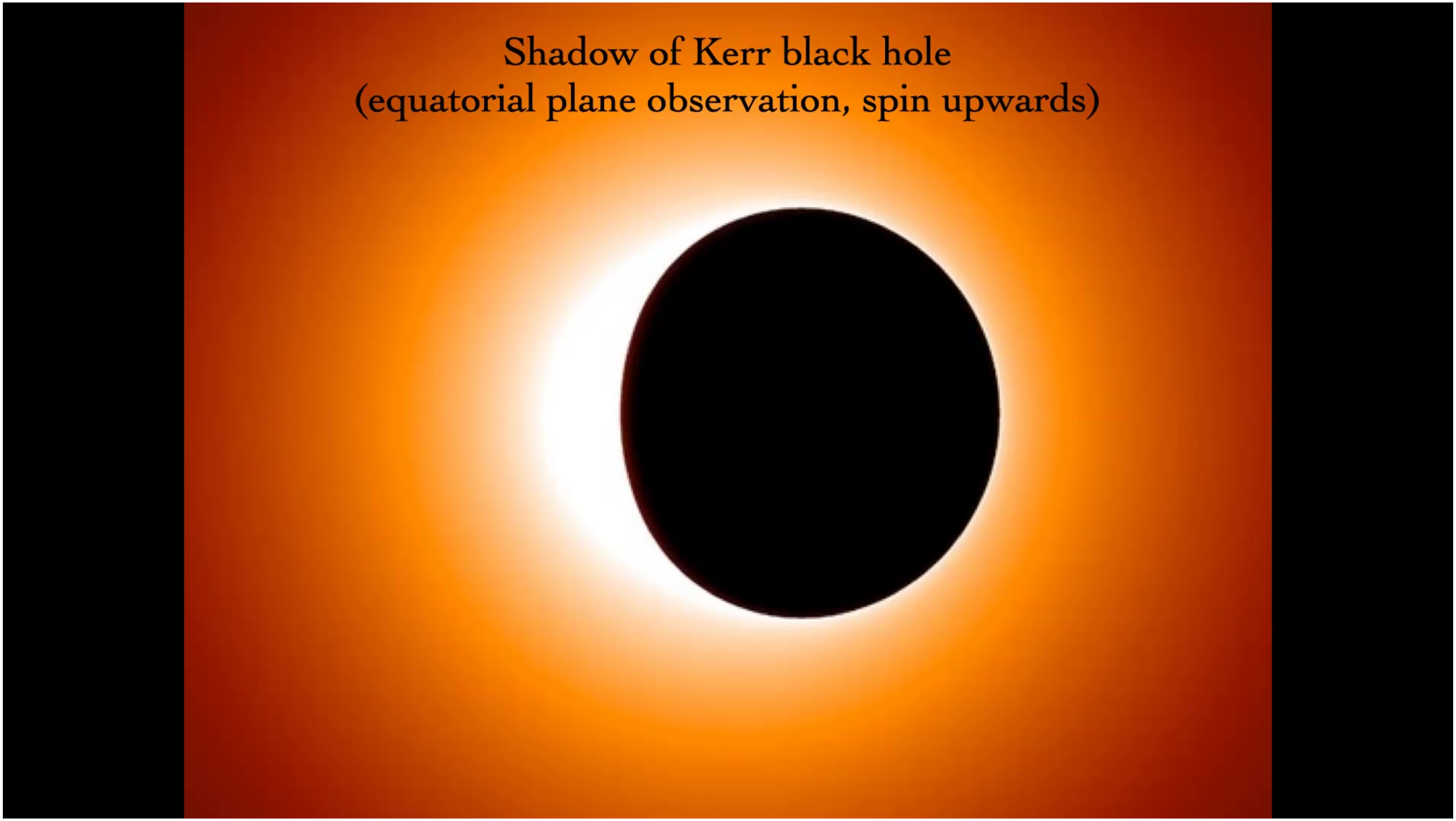
Some critical skepticism:

1) are these untold numbers of massive black holes exactly represented by the Kerr metric ?

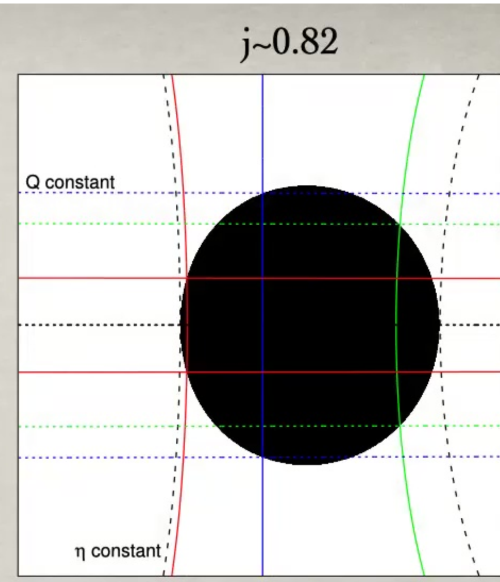
2) are these black holes all of the same type ?

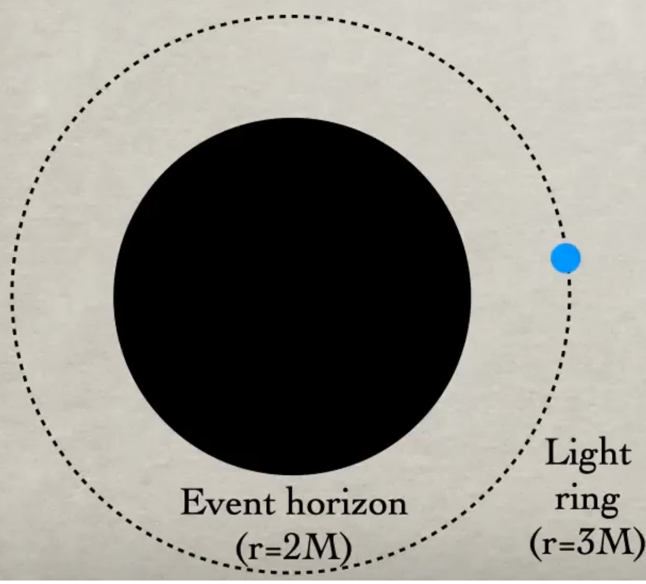
3) are these objects really black holes ?


Shadow of Kerr black hole
(equatorial plane observation, spin upwards)

The edge of the shadow is determined by the
Fundamental Photon Orbits
Cunha, C.H., Radu, PRD 96 (2017) 024039

For Schwarzschild these are all planar light rings

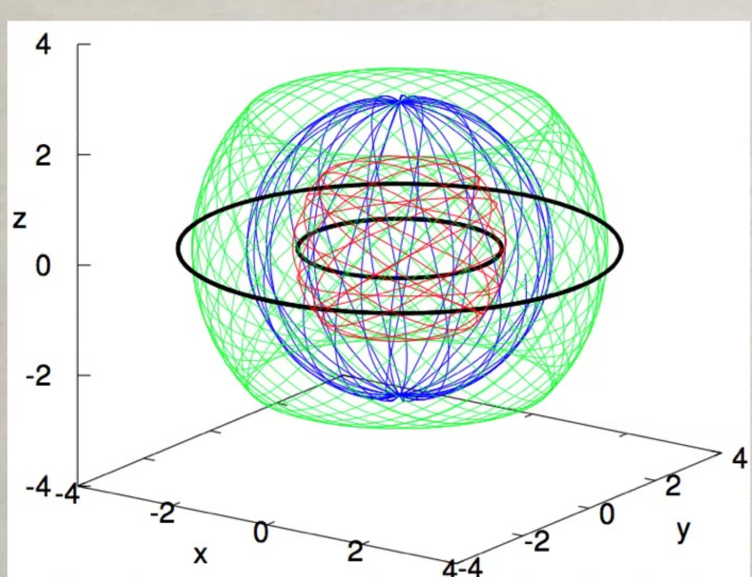




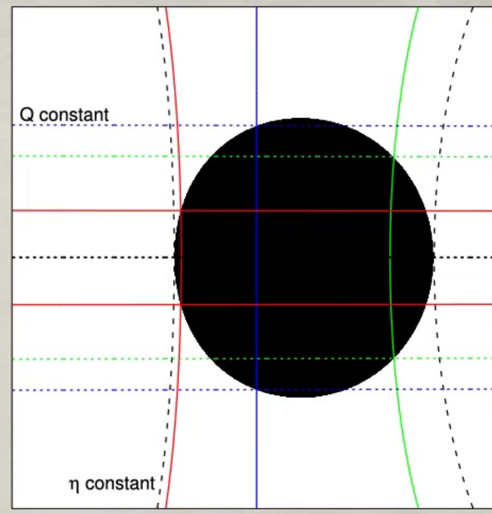
The edge of the shadow is determined by the **Fundamental Photon Orbits**
Cunha, C.H., Radu, PRD 96 (2017) 024039

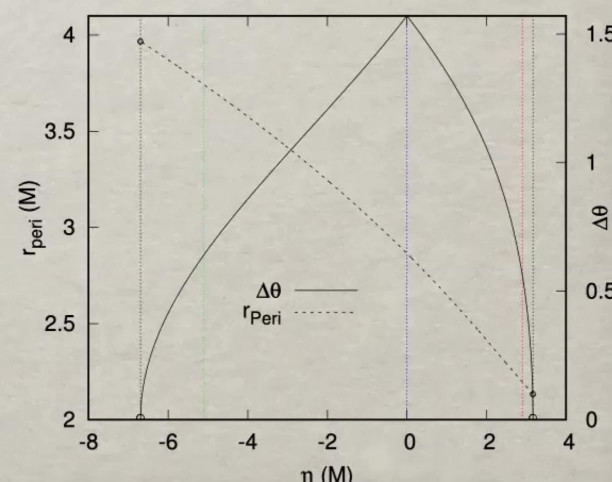
Called spherical orbits in Kerr case
Teo, GRG 35 (2003) 1909

Relate to quasi-normal ringing
Goebel, Astrophys. J. 172 (1972) L95
Cardoso, Franzin and Pani, PRL 116 (2016) 171101



j~0.82





Equilibrium black holes, in general have Light Rings (LRs)

PHYSICAL REVIEW LETTERS **124,** 181101 (2020)

Editors' Suggestion

Stationary Black Holes and Light Rings

Pedro V. P. Cunha[1] and Carlos A. R. Herdeiro[2]
[1]*Max Planck Institute for Gravitational Physics—Albert Einstein Institute, Am Mühlenberg 1, Potsdam 14476, Germany*
[2]*Departamento de Matemática da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*

(Received 19 March 2020; accepted 15 April 2020; published 8 May 2020)

The ringdown and shadow of the astrophysically significant Kerr black hole (BH) are both intimately connected to a special set of bound null orbits known as light rings (LRs). Does it hold that a *generic* [redacted] In this Letter we prove the following theorem. A stationary, axisymmetric, asymptotically flat black hole spacetime in $1+3$ dimensions, with a nonextremal, topologically spherical, Killing horizon admits, at least, one standard LR outside the horizon for each rotation sense. The proof relies on a topological argument and assumes C^2 smoothness and circularity, but makes no use of the field equations. [redacted] establishing that a horizonless ultracompact object must admit an even number of nondegenerate LRs, one of which is stable.

DOI: 10.1103/PhysRevLett.124.181101

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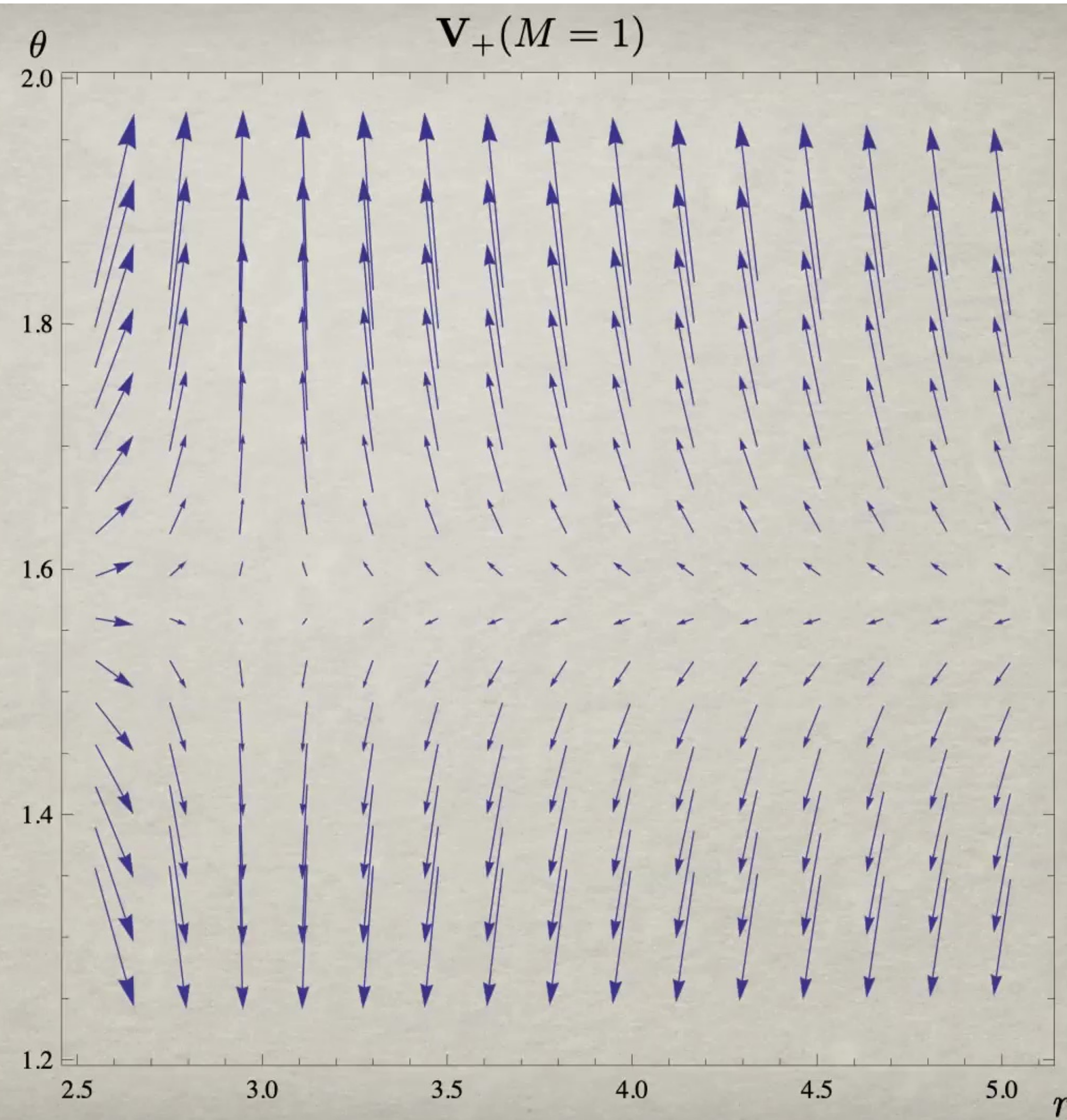
Central idea:
LRs are *critical points* of a potential

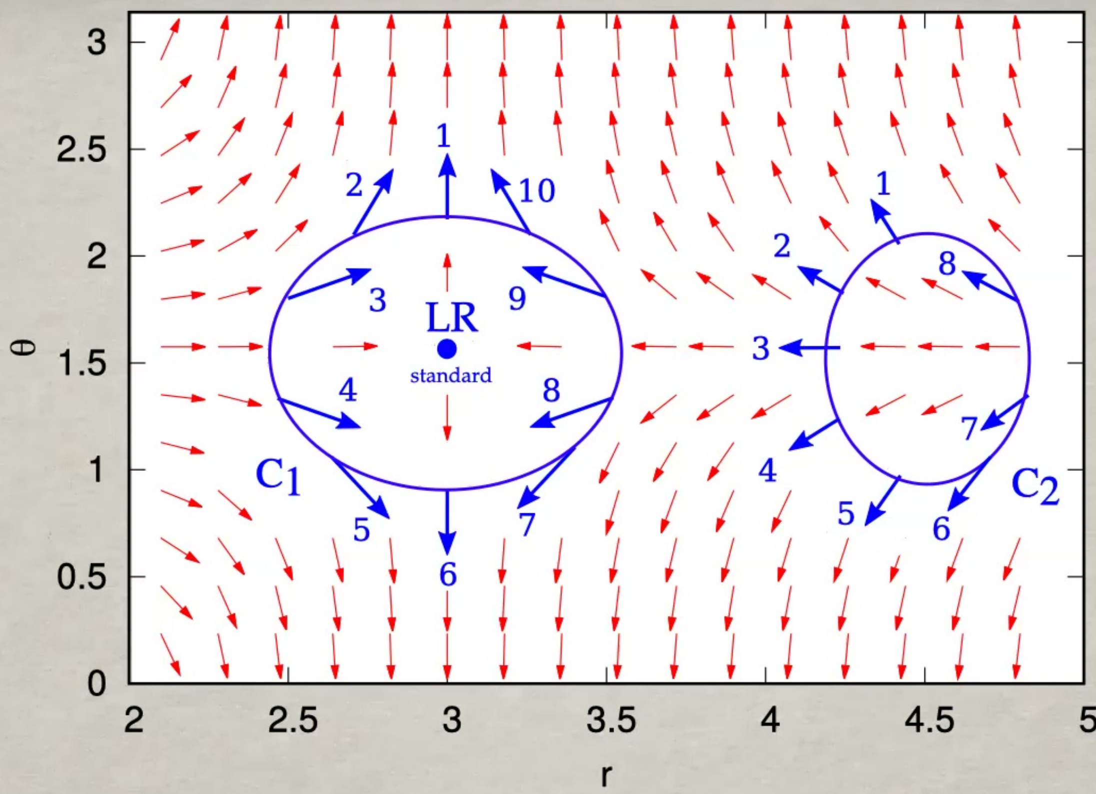
- determined by the Hamiltonian $\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = 0$.
- $$2\mathcal{H} = (g^{ij} p_i p_j) + (g^{ab} p_a p_b), \qquad i \in \{r,\theta\}, \ a \in \{t,\varphi\}.$$
  $$= \quad K \quad + \quad U(r,\theta).$$

For Schwarzschild:

$$H_\pm = \pm\frac{\sqrt{1-\frac{2M}{r}}}{r\sin\theta}$$

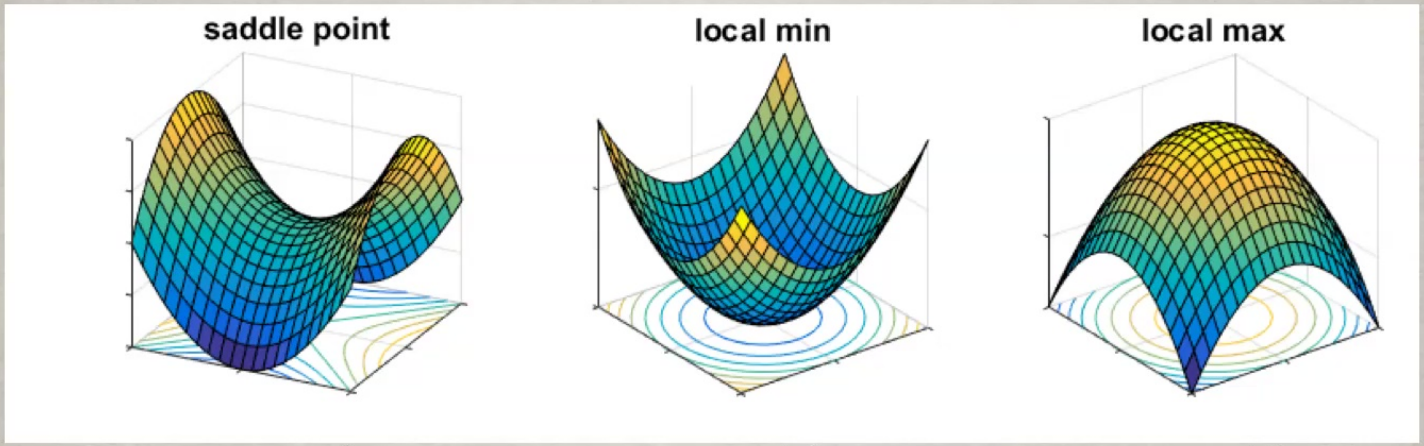
$$\mathbf{V}_\pm = \nabla H_\pm$$





$$\oint_C d\Omega = 2\pi w \ , \qquad w \in \mathbb{Z} \ .$$

Light ring types

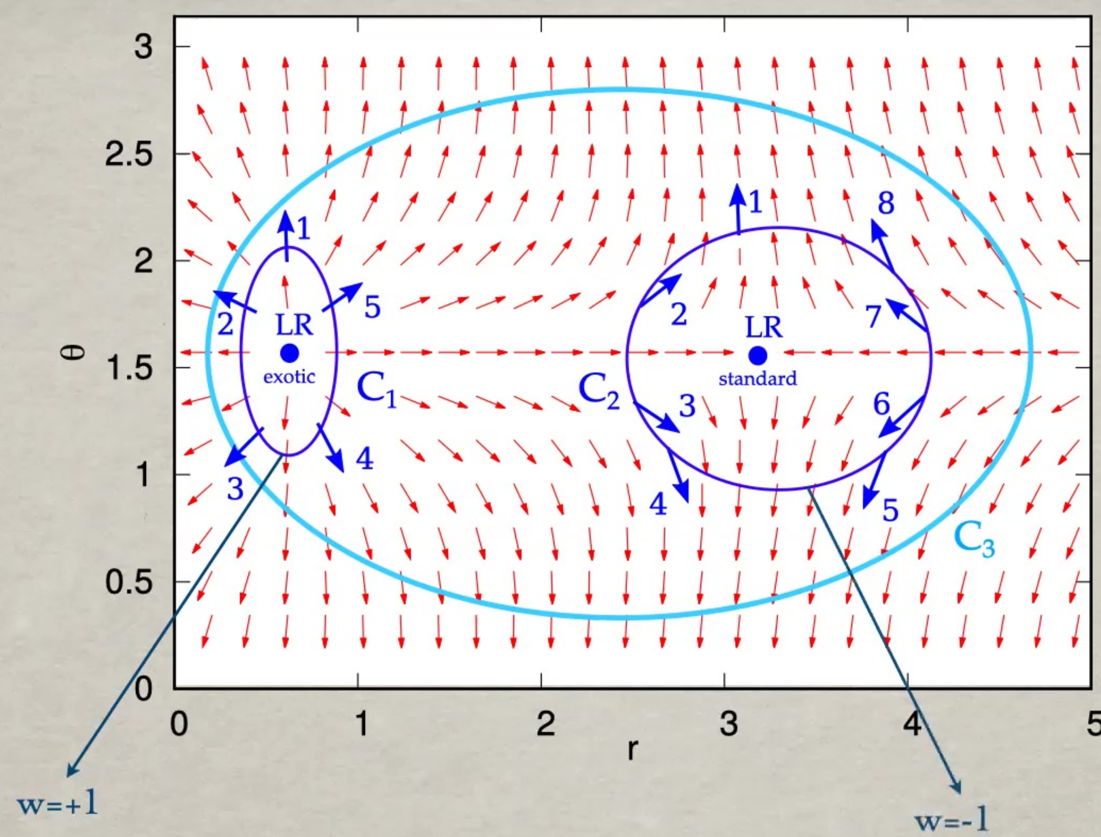


$w = -1$ $w = +1$ $w = +1$

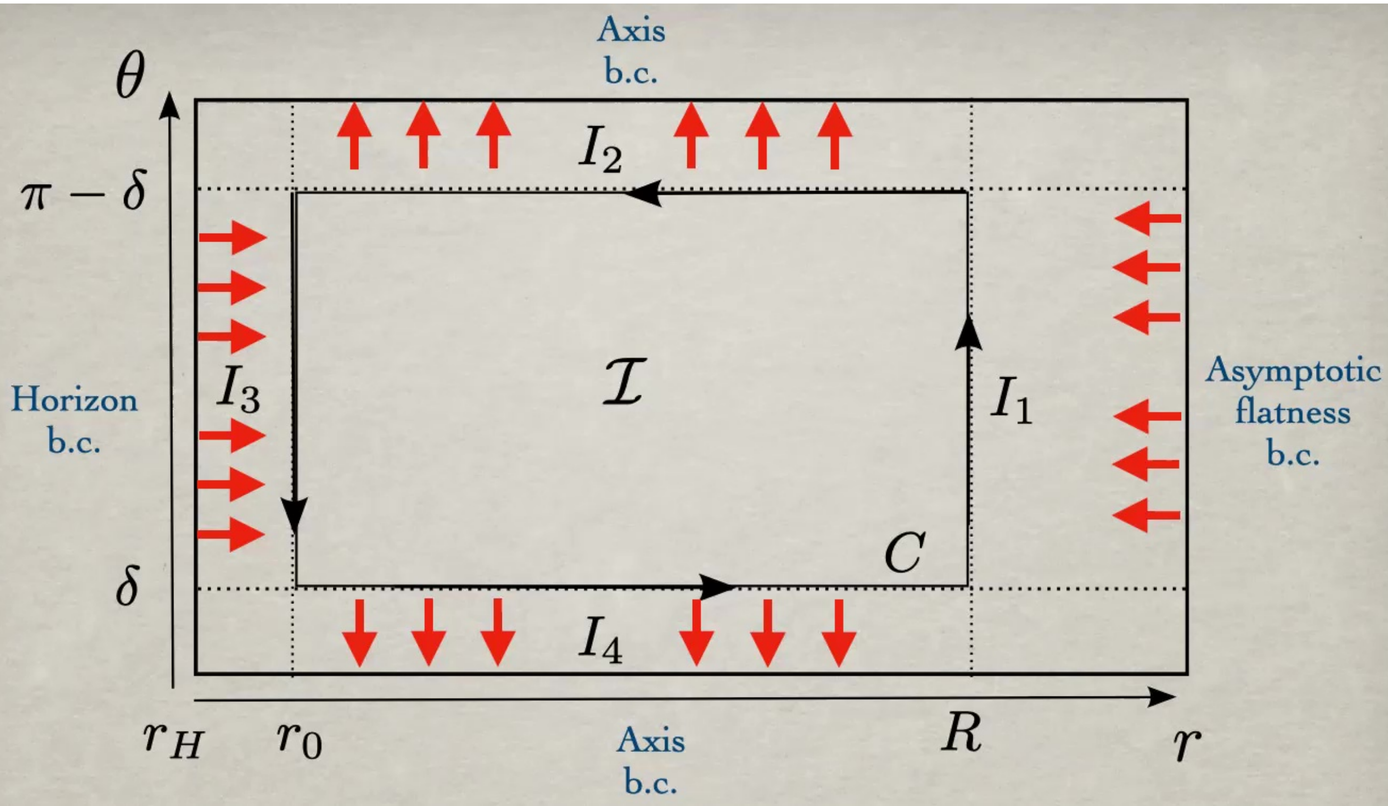
Standard LRs
(Schwarzschild/Kerr like)

Exotic LRs
(Schwarzschild/Kerr unlike)

$$\oint_C d\Omega = 2\pi \sum_i w_i \ , \qquad w_i = -1, 1 \ .$$

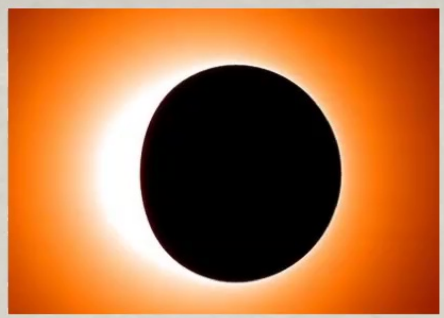


3
2.5
2
1.5
1
0.5
0
θ
0
1
2
3
4
5
r
LR
exotic
LR
standard
C1
C2
C3
1
2
3
4
5
6
7
8
w=+1
w=-1



$$w = \lim_{R \to +\infty} \lim_{r_0 \to r_H} \left(\lim_{\delta \to 0} \oint_C d\Omega \right) = -1$$

The synthetic blurred image is similar do real data, consistent with a Kerr black hole



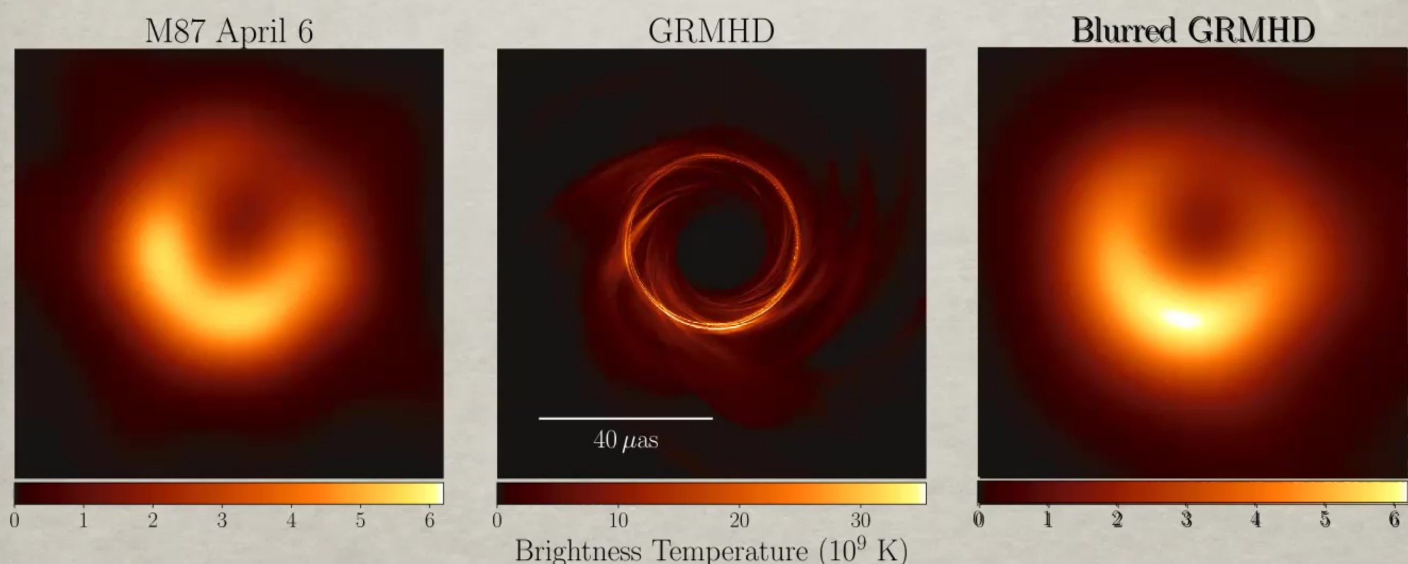


Figure 1. Left panel: an EHT2017 image of M87 from Paper IV of this series (see their Figure 15). Middle panel: a simulated image based on a GRMHD model. Right panel: the model image convolved with a 20 μas FWHM Gaussian beam. Although the most evident features of the model and data are similar, fine features in the model are not resolved by EHT.

ApJ Lett. 875 (2019) L5

The ringdown is associated to the light ring:

THE ASTROPHYSICAL JOURNAL, **172**:L95–L96, 1972 March 15

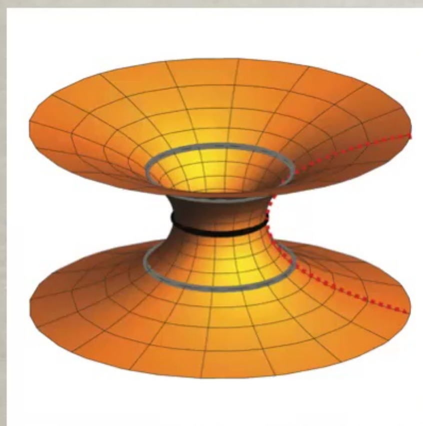
COMMENTS ON THE "VIBRATIONS" OF A BLACK HOLE

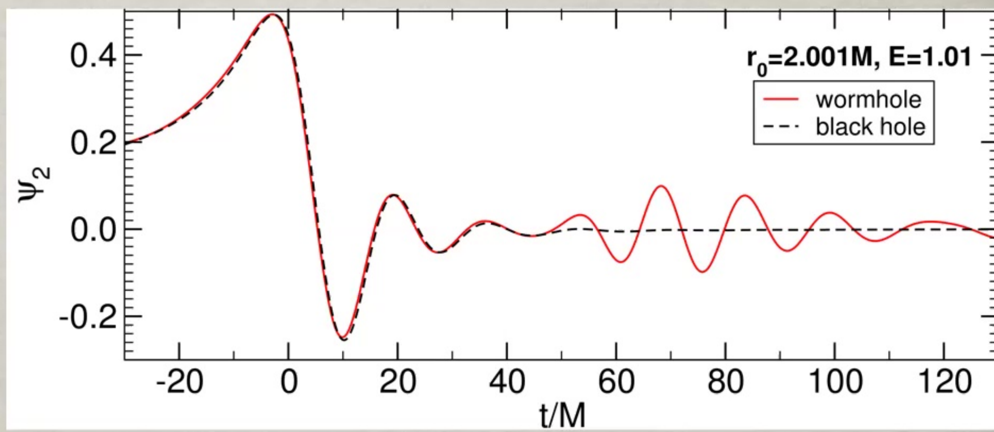
C. J. GOEBEL
University of Wisconsin, Physics Department, Madison
Received 1972 January 4; revised 1972 January 27

ABSTRACT

It is shown that the "vibrations of a black hole" of Press are gravitational waves in spiral orbits close to the well-known unstable circular orbit at $r = 3M$. The corresponding "vibrations" of a spinning black hole are discussed. It is emphasized that these "vibrations" provide, not a source, but only a temporary storage, of high-frequency gravitational radiation.

So, a hypothetical horizonless ultracompact ECO (i.e. with a similar light ring) could vibrate similarly, initially...





Cardoso, Franzin, Pani, PRL 117 (2016) 089902

Are there black hole mimickers with similar light rings
to black holes but no "event horizon"
they could mimic the black hole phenomenology.

Can there be such speculative ultra-compact ECOs?

A theorem for ultracompact ECOs that form from incomplete gravitational collapse

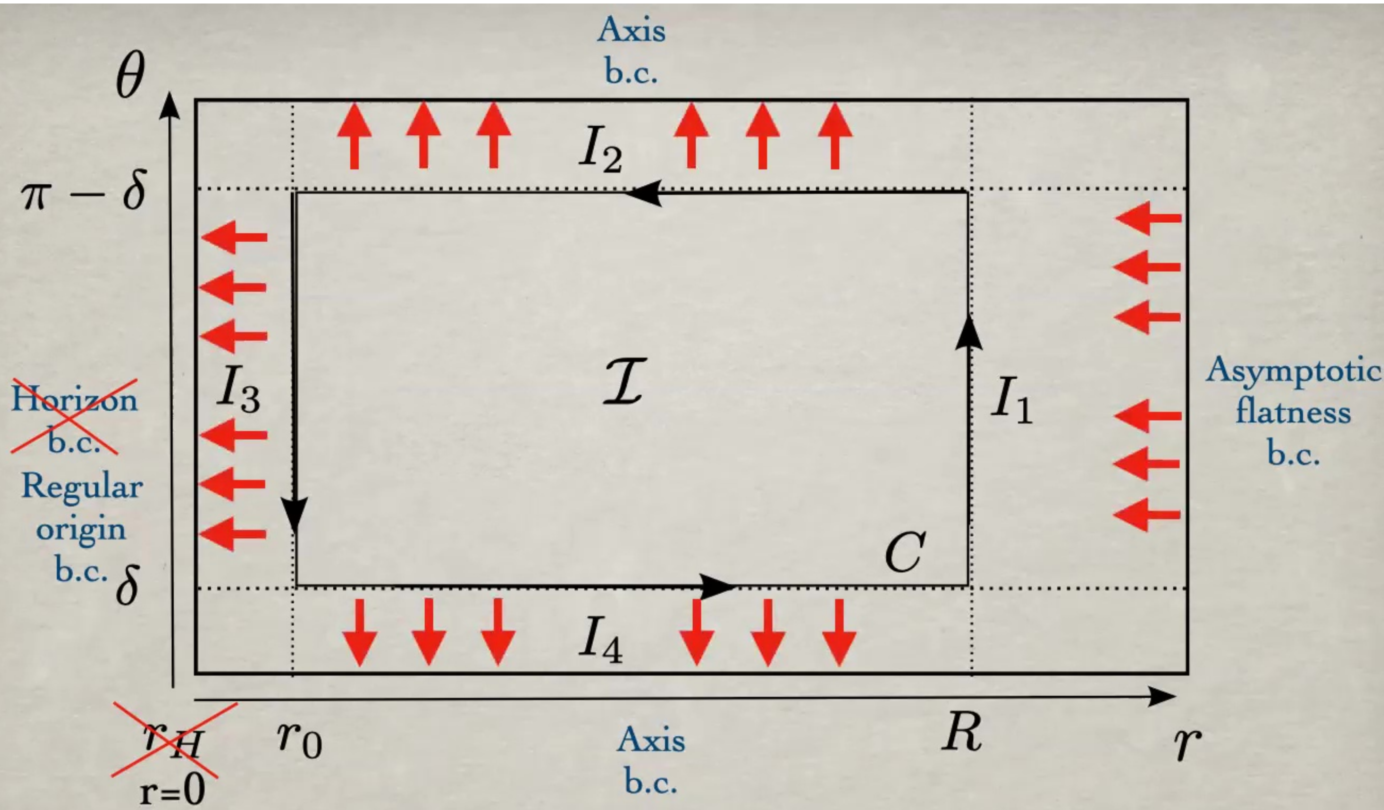
PRL **119,** 251102 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Light-Ring Stability for Ultracompact Objects

Pedro V. P. Cunha,[1,2] Emanuele Berti,[3,2] and Carlos A. R. Herdeiro[1]
[1]*Departamento de Física da Universidade de Aveiro and CIDMA, Campus de Santiago, 3810-183 Aveiro, Portugal*
[2]*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal*
[3]*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*
(Received 3 August 2017; revised manuscript received 18 October 2017; published 18 December 2017)

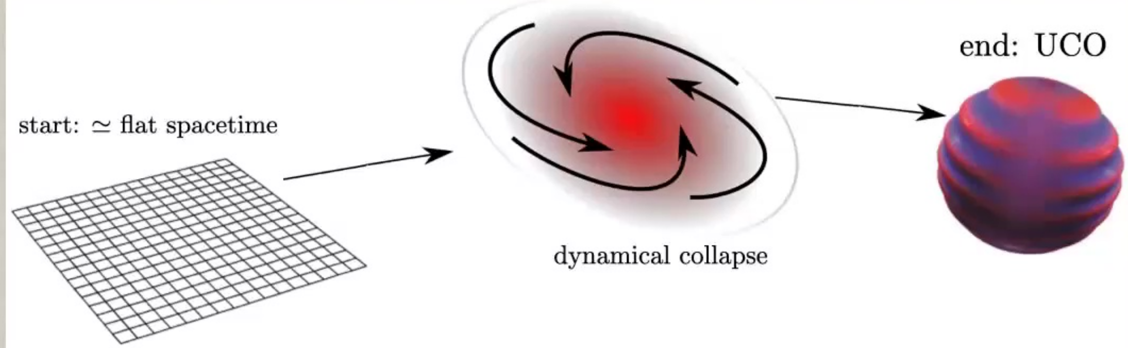
We prove the following theorem: axisymmetric, stationary solutions of the Einstein field equations formed from classical gravitational collapse of matter obeying the null energy condition, that are everywhere smooth and ultracompact (i.e., they have a light ring) must have at least *two* light rings, and one of them is *stable*. It has been argued that stable light rings generally lead to nonlinear spacetime instabilities. Our result implies that smooth, physically and dynamically reasonable ultracompact objects are not viable as observational alternatives to black holes whenever these instabilities occur on astrophysically short time scales. The proof of the theorem has two parts: (i) We show that light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential. This result follows from a topological argument based on the Brouwer degree of a continuous map, with no assumptions on the spacetime dynamics, and, hence, it is applicable to any metric gravity theory where photons follow null geodesics. (ii) Assuming Einstein's equations, we show that the extremum is a local minimum of the potential (i.e., a stable light ring) if the energy-momentum tensor satisfies the null energy condition.

DOI: 10.1103/PhysRevLett.119.251102



$$w = \lim_{R \to +\infty} \lim_{r_0 \to 0} \left(\lim_{\delta \to 0} \oint_C d\Omega \right) = 0$$

A generic dynamical picture



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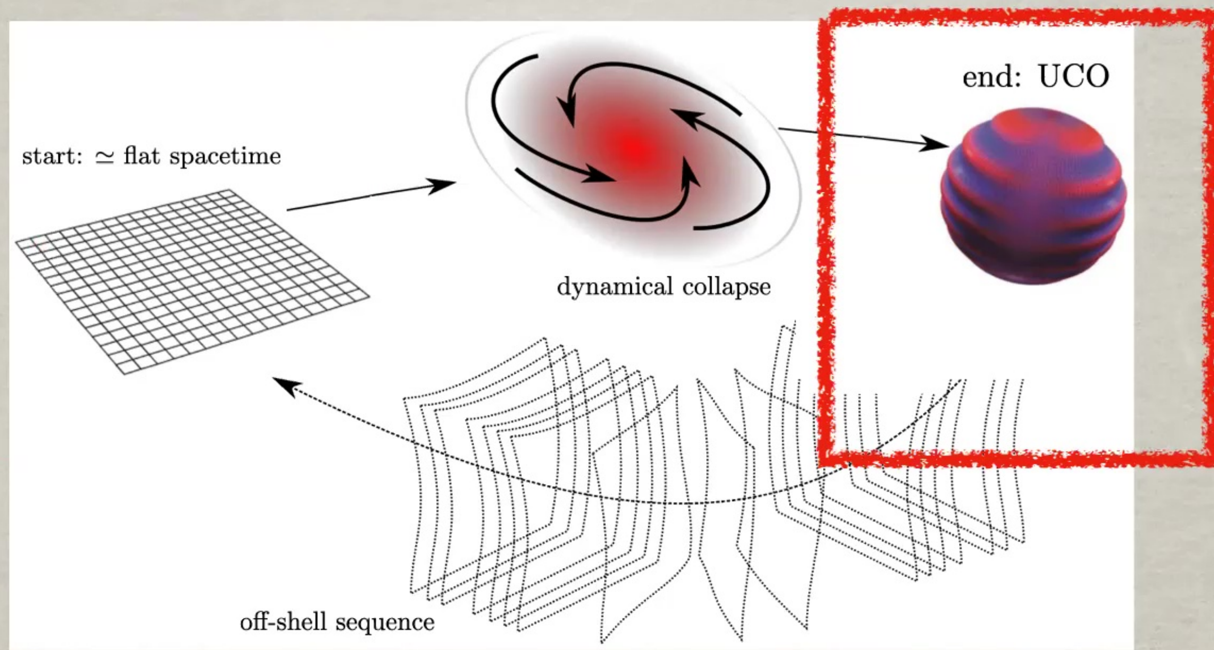
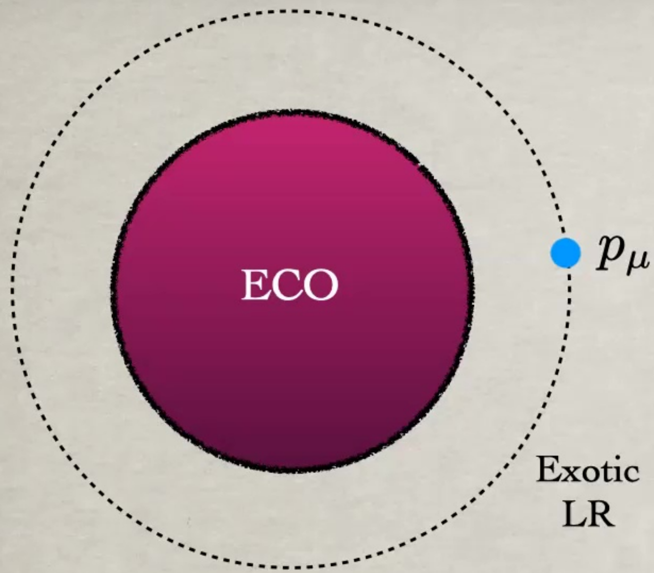




At the LR:

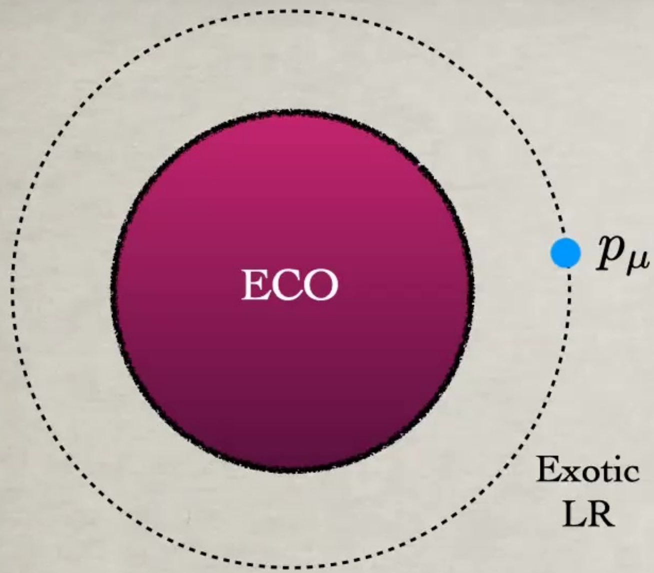
$$G^{\mu\nu} p_\mu p_\nu = \frac{1}{2} \partial_i \partial^i U$$

Assuming GR:

$$T^{\mu\nu} p_\mu p_\nu = \frac{1}{16\pi} \partial_i \partial^i U$$

If the LR is a local maximum of U, then the NEC is violated:

$$\partial_i \partial^i U < 0 \implies T^{\mu\nu} p_\mu p_\nu < 0$$



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If the LR is a local maximum of U, then the NEC is violated:

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If one imposes the null energy condition, the exotic one must be fully stable.

In modified gravity one may proceed similarly in terms of an effective energy-momentum tensor.

"There is a crack in everything, that is how the light gets in"

L. Cohen

It has been suggested: J. Keir, Class.Quant.Grav. 33 (2016) no.13, 135009; Benomio, arXiv:1809.07795

- Treating scalar linear waves as a model for nonlinear perturbations.
- Considering spherically symmetric spacetimes exhibiting stable Light Rings.
- Showing that linear waves cannot (uniformly) decay faster than logarithmically.
- Such slow decay is highly suggestive of a *nonlinear instability*.

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- Considering spherically symmetric spacetimes exhibiting stable Light Rings.
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- Such slow decay is highly suggestive of a *nonlinear instability.*

The existence of a stable light ring is a (potentially) generic obstruction for any ultracompact ECO that can form from classical GR dynamics.

Bosonic stars

Check list:

1) Appear in a well motivated and consistent physical model;

Boson stars: General Relativity minimally coupled to massive bosonic fields

2) Have a dynamical formation mechanism;

Gravitational cooling

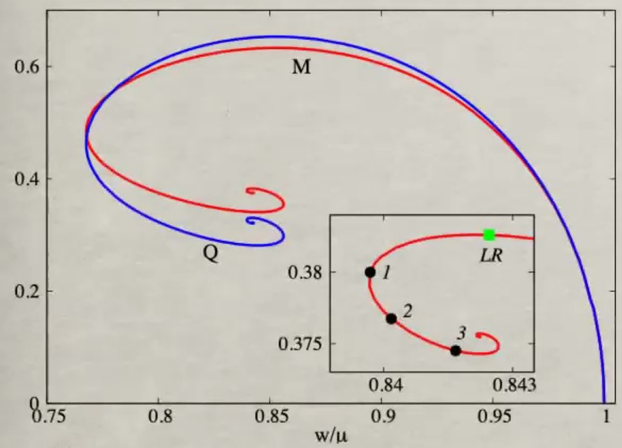
3) Be (sufficiently) stable.

Some solutions are stable

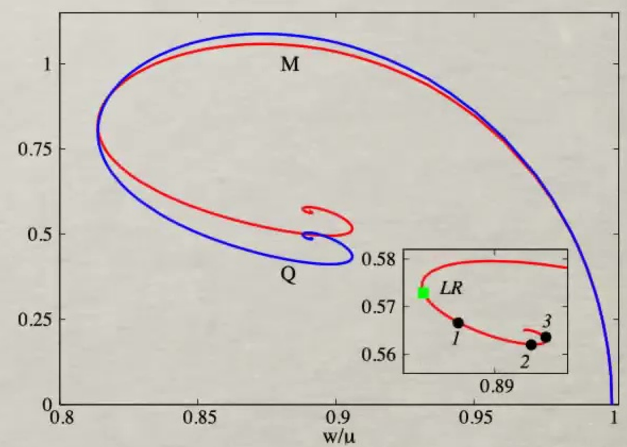
Spherical bosonic stars (no self-interactions) and light rings

Cunha, Font, CH, Radu, Sanchis-Gual, Zilhão, Phys. Rev. D 96 (2017) 104040

scalar

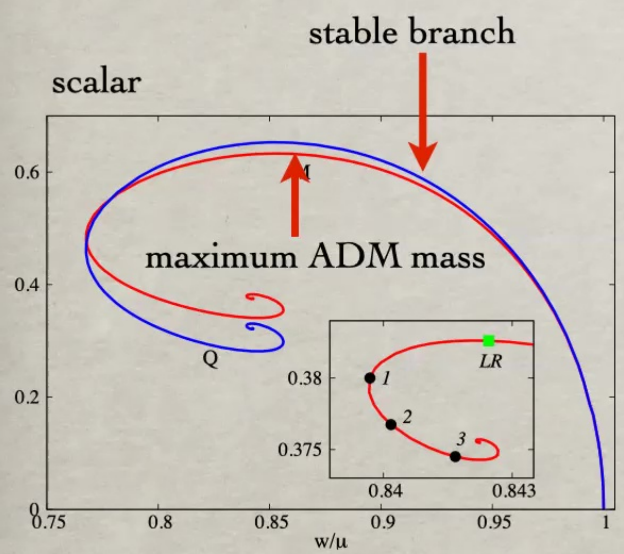


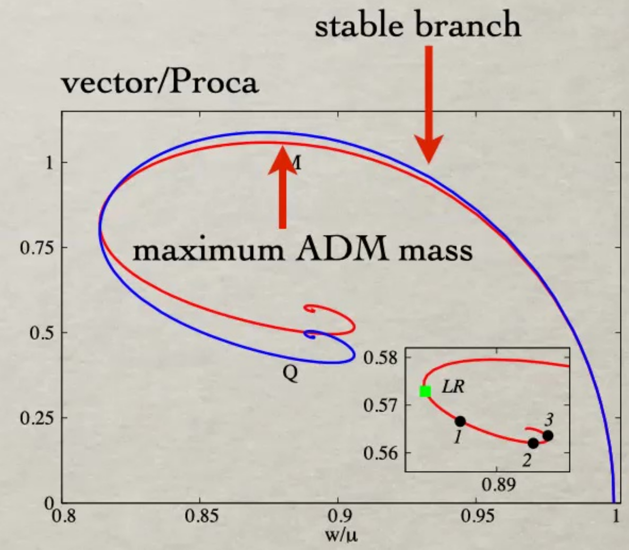
vector/Proca



Spherical bosonic stars (no self-interactions) and light rings

Cunha, Font, CH, Radu, Sanchis-Gual, Zilhão, Phys. Rev. D 96 (2017) 104040

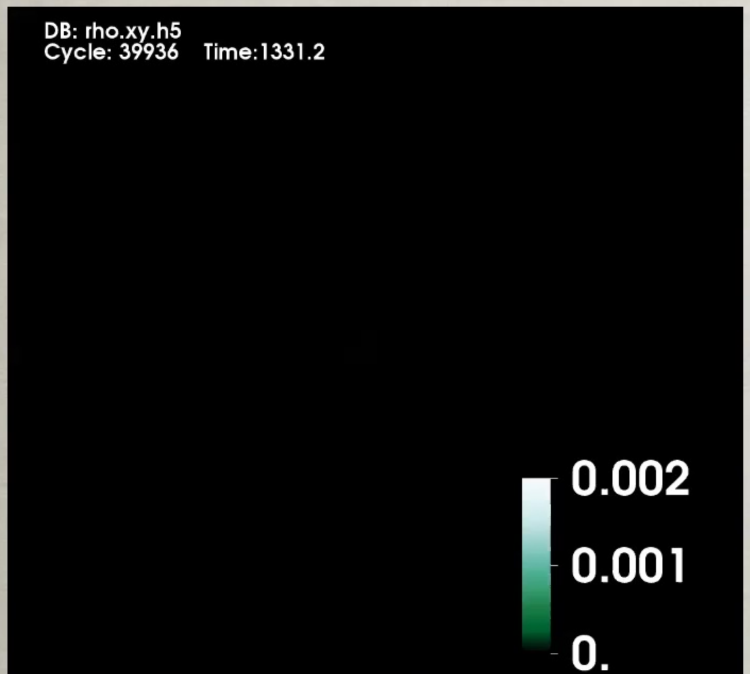


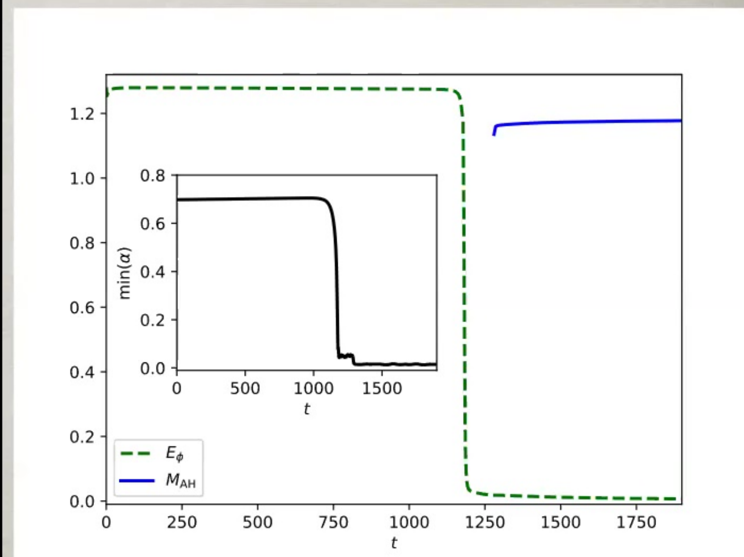


Spinning scalar boson stars have a non-axisymmetric instability

Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

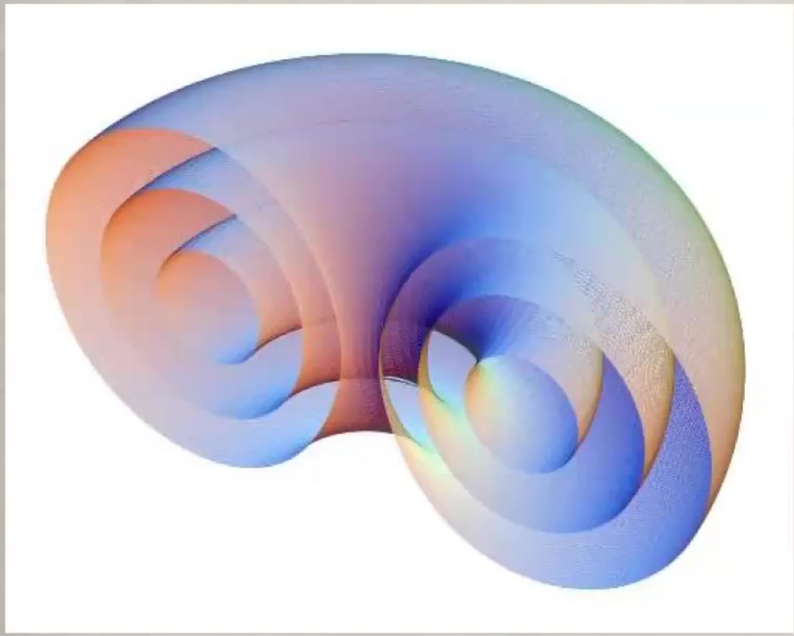
http://gravitation.web.ua.pt/node/1740





Instability may be associated to toroidal structure and is absent in cousin Proca model

Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101
http://gravitation.web.ua.pt/node/1740



Rotating boson stars

Rotating Proca stars

Brito, Cardoso, CH and Radu, PLB 752 (2016) 291
CH, Radu and Rúnarsson, CQG 33 (2016) 154001
CH, Perapechka, Radu and Shnir, PLB 797 (2019) 134845

Our testing ground:

Siemonson and East, Phys. Rev. D 103 (2021) 044022

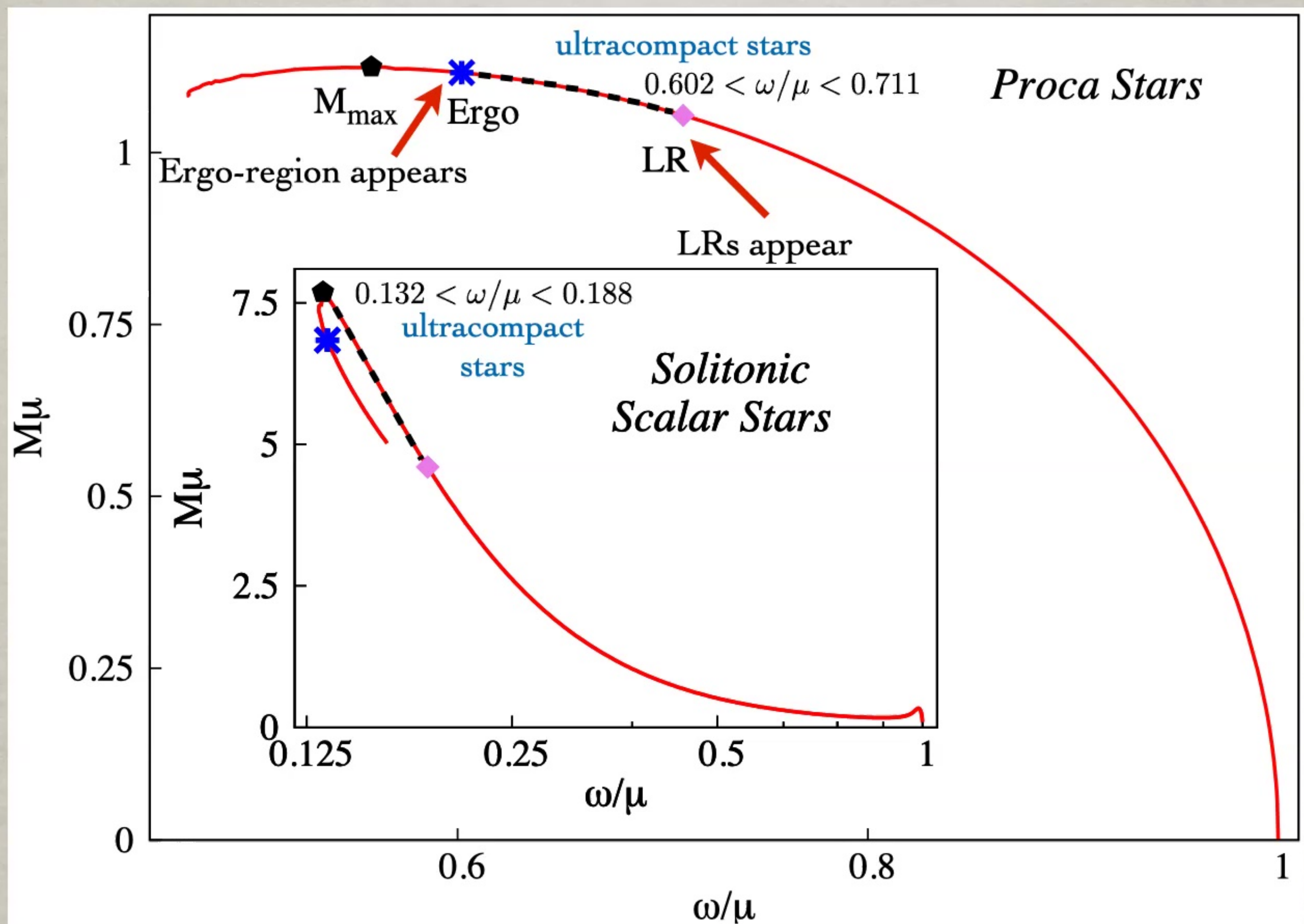
$$\mathcal{L} = \frac{R}{16\pi G} + \mathcal{L}_m$$

$$\mathcal{L}_m = -\partial_\alpha \Phi \partial^\alpha \bar{\Phi} - \mu^2 |\Phi|^2 \left[1 - \frac{2|\Phi|^2}{\sigma_0^2}\right]^2$$

Scalar model has a relativistic (potentially) stable branch

$$\mathcal{L}_m = -\frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2 \mathcal{A}_\alpha \bar{\mathcal{A}}^\alpha$$

Vector model

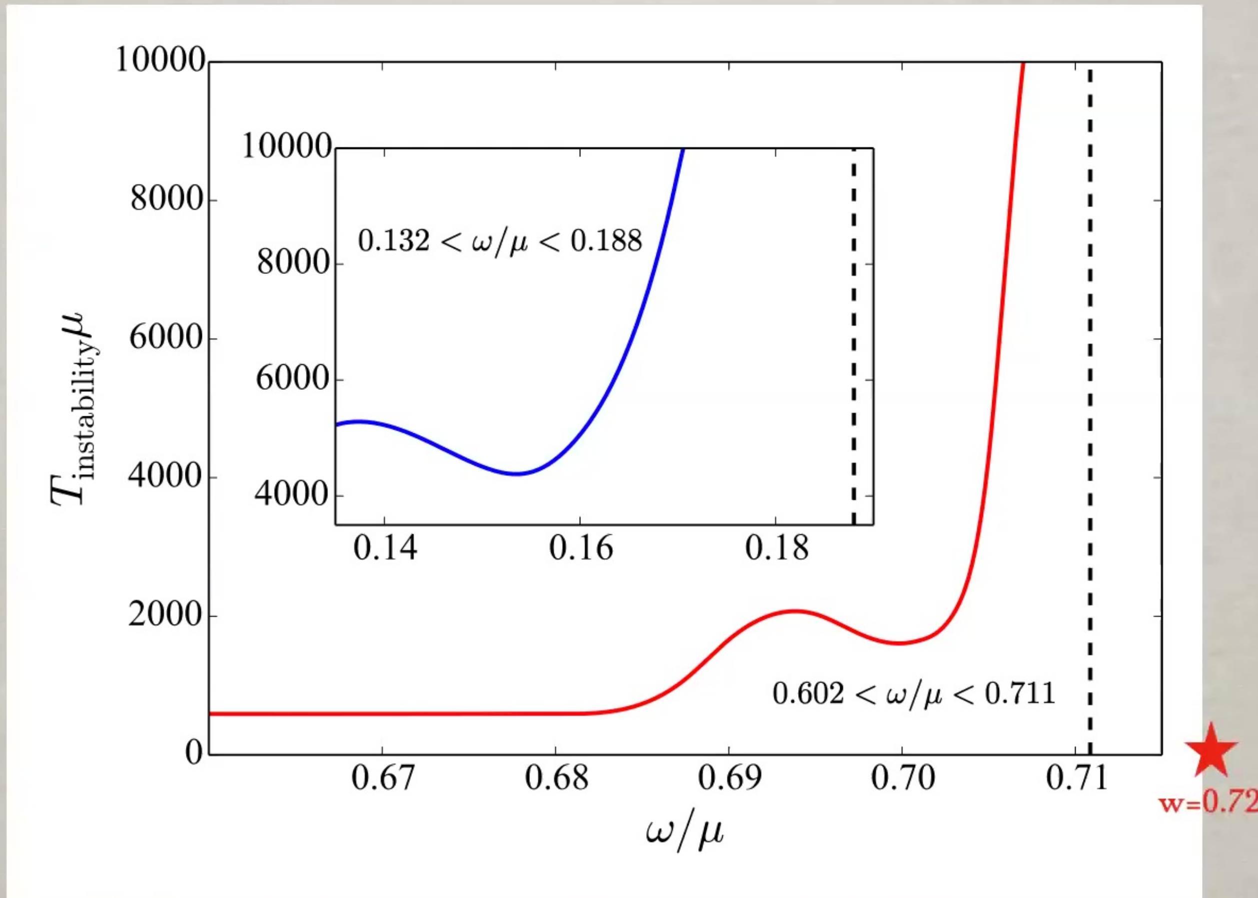


Cunha, CH, Radu and Sanchis-Gual, 2207.13713

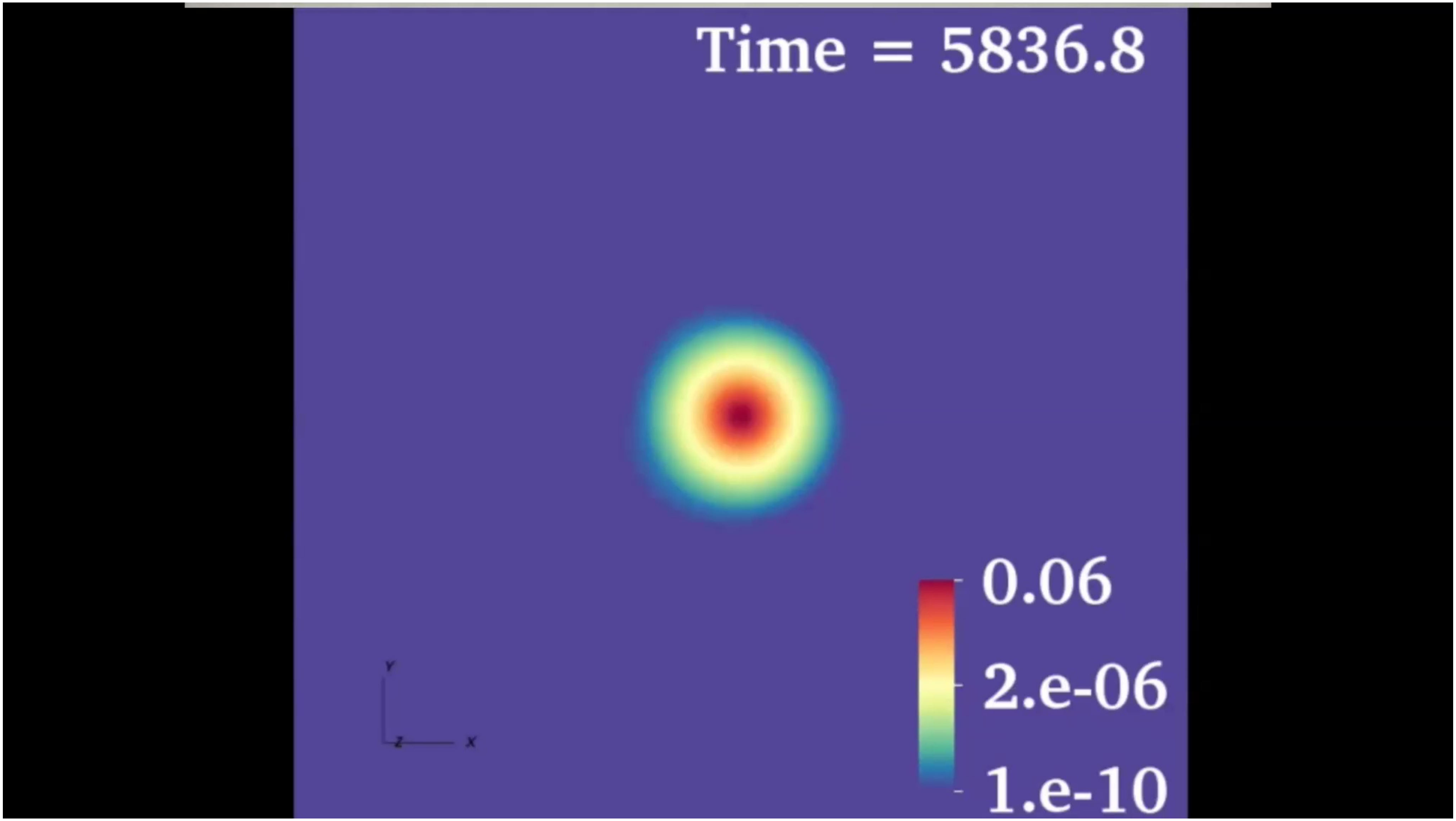
Now we perform fully non-linear numerical relativity evolutions

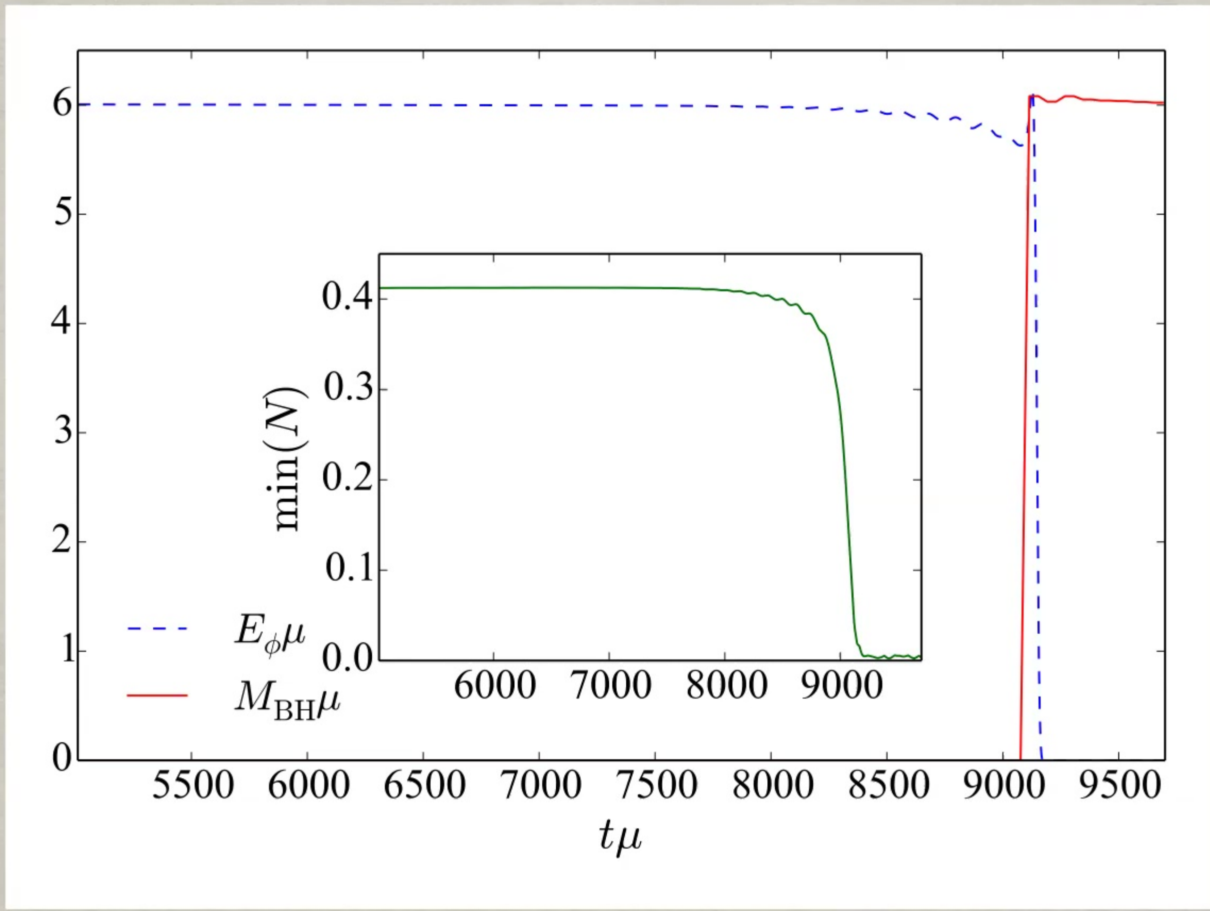
Sanchis-Gual, Di Giovanni, Zilhão, CH, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740



Cunha, CH, Radu and Sanchis-Gual, 2207.13713


Time = 5836.8
0.06
2.e-06
1.e-10
Y
Z
X



6
5
4
3
2
1
0
5500
6000
6500
7000
7500
8000
8500
9000
9500
$t\mu$
$E_\phi \mu$
$M_{\rm BH} \mu$
0.4
0.3
0.2
0.1
0.0
min(N)
6000
7000
8000
9000

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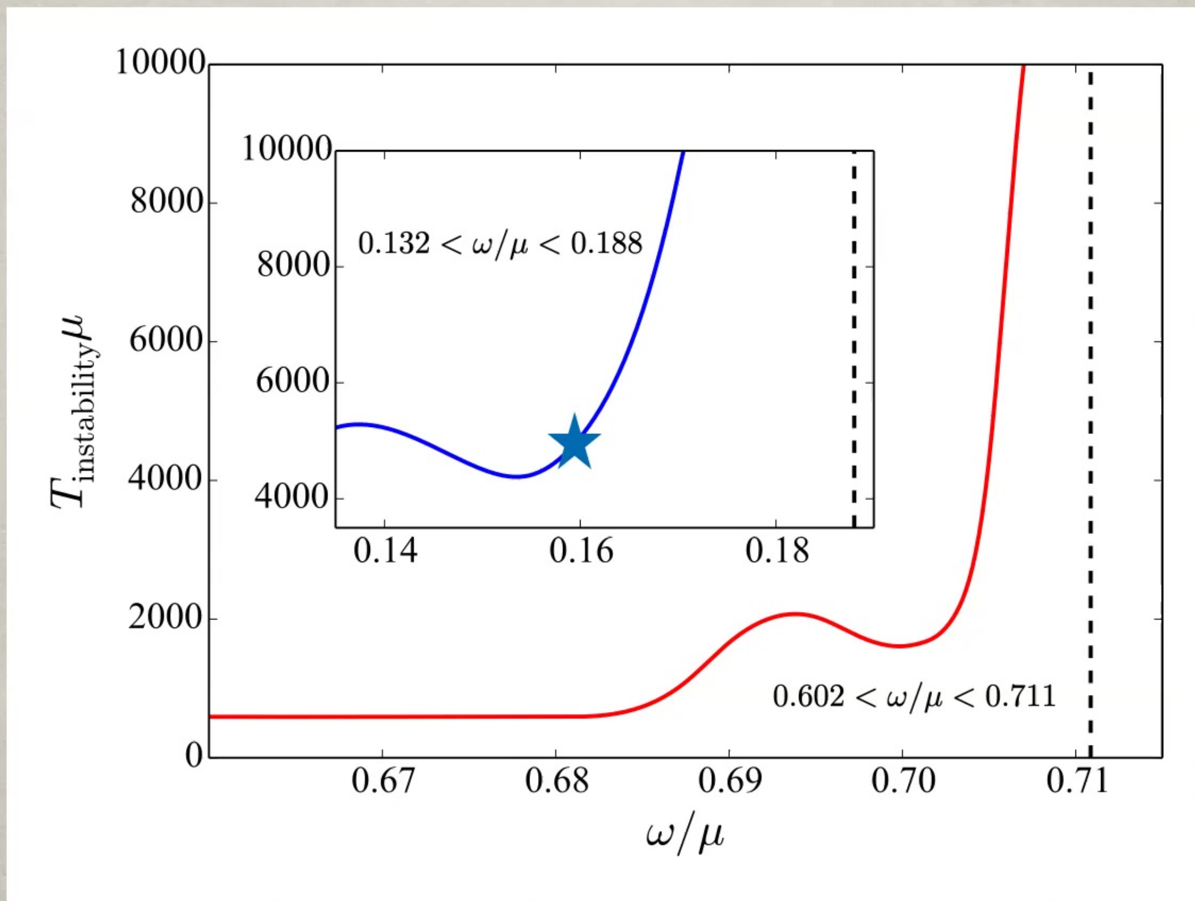
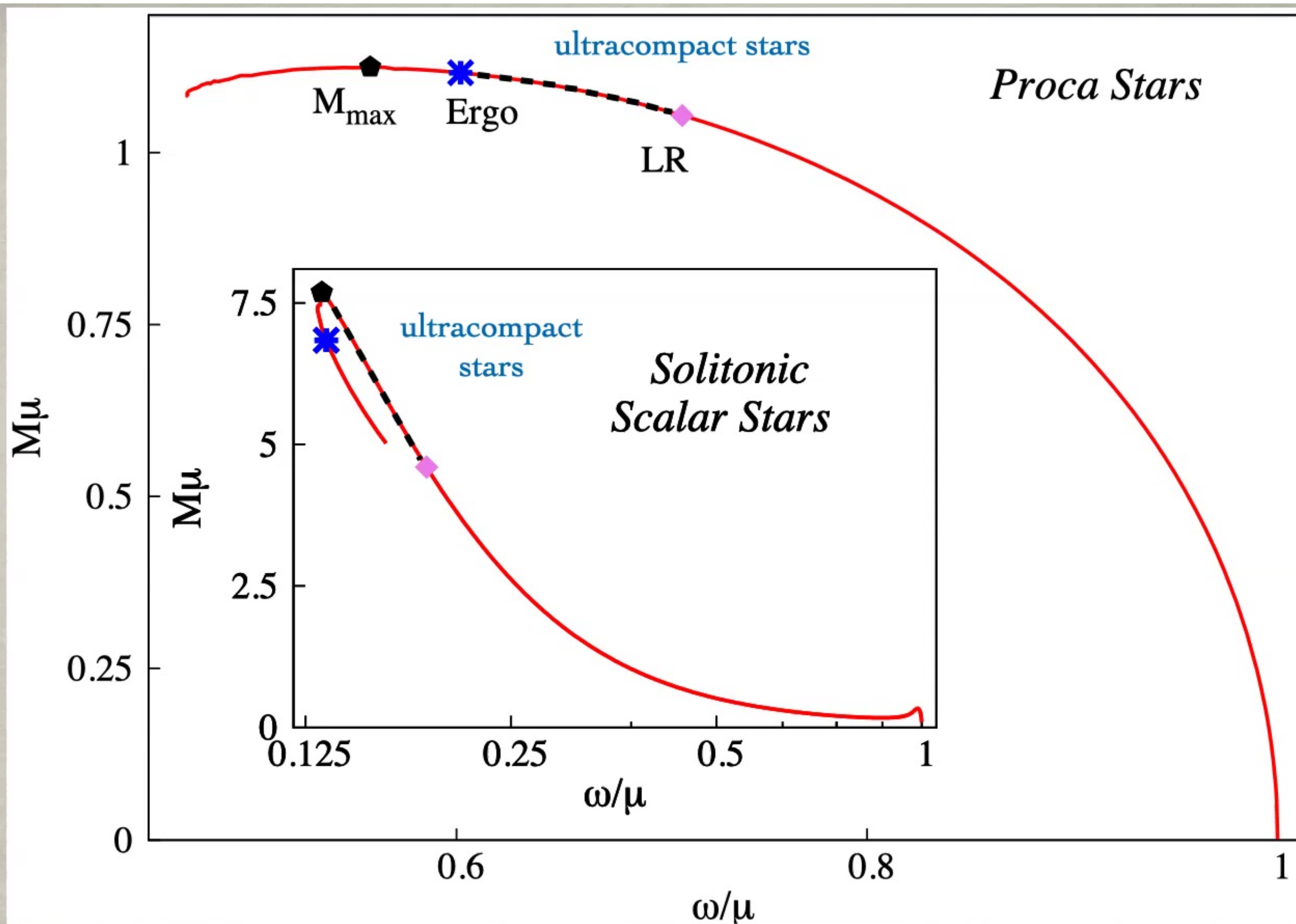
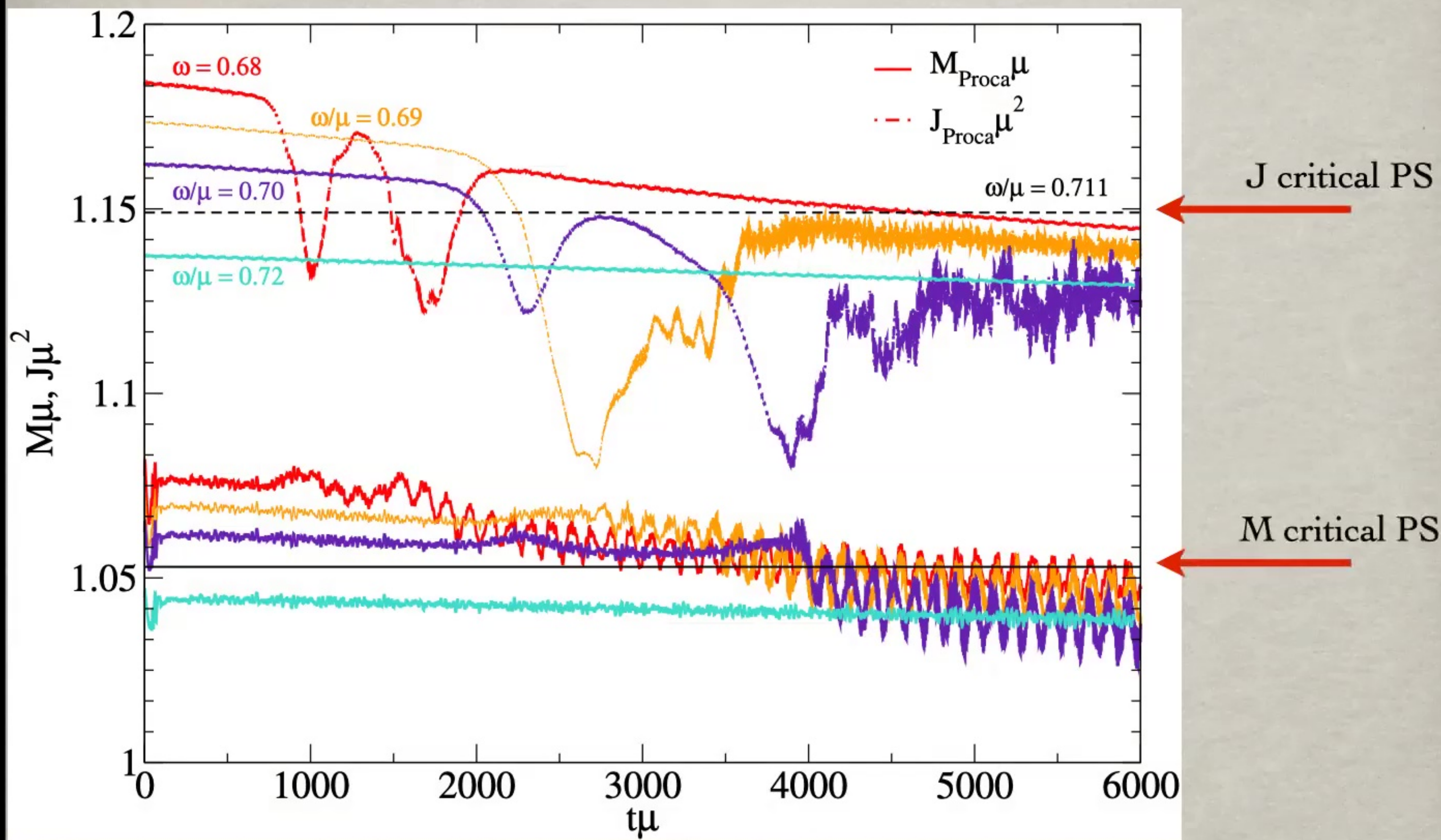
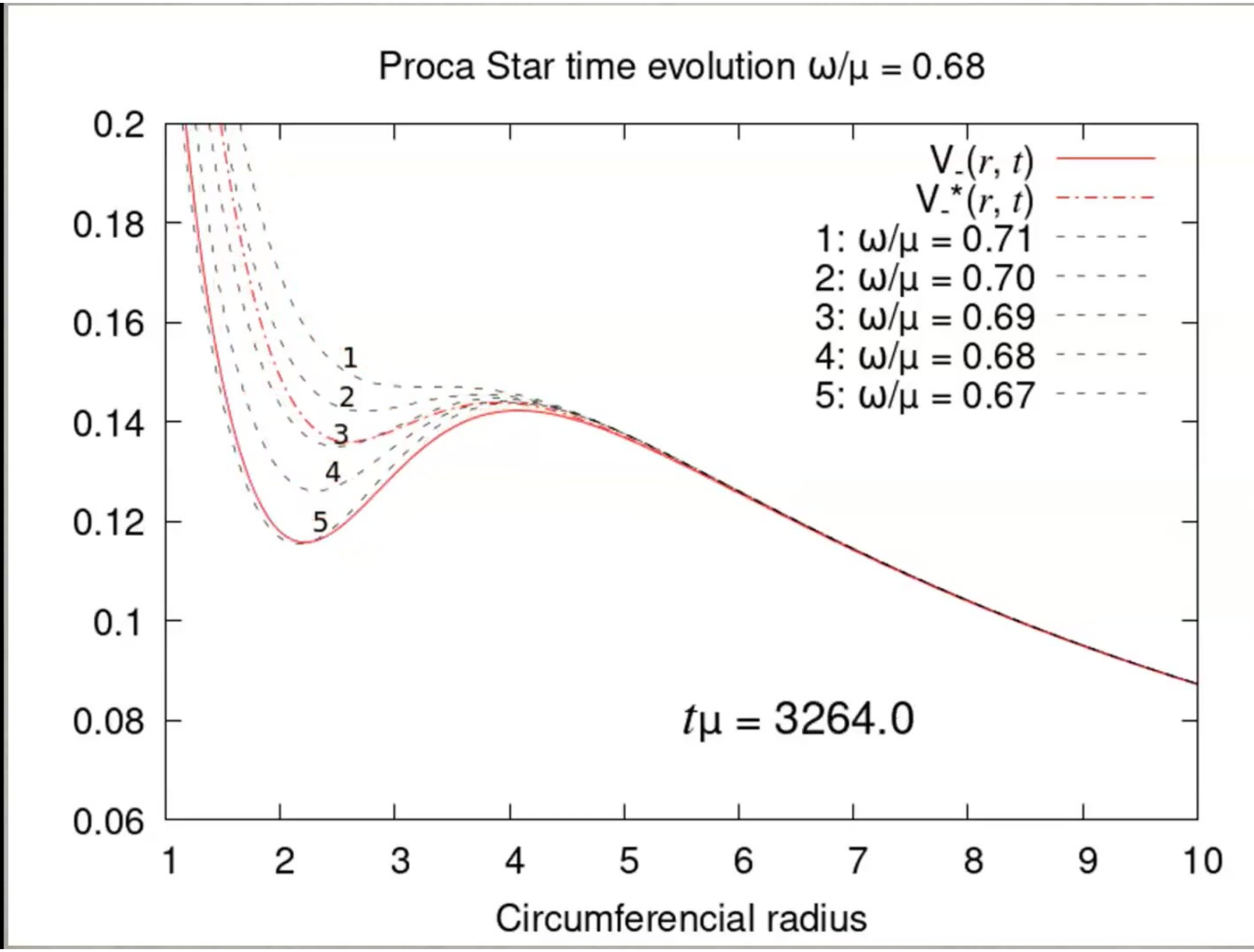


Cunha, CH, Radu and Sanchis-Gual, 2207.13713

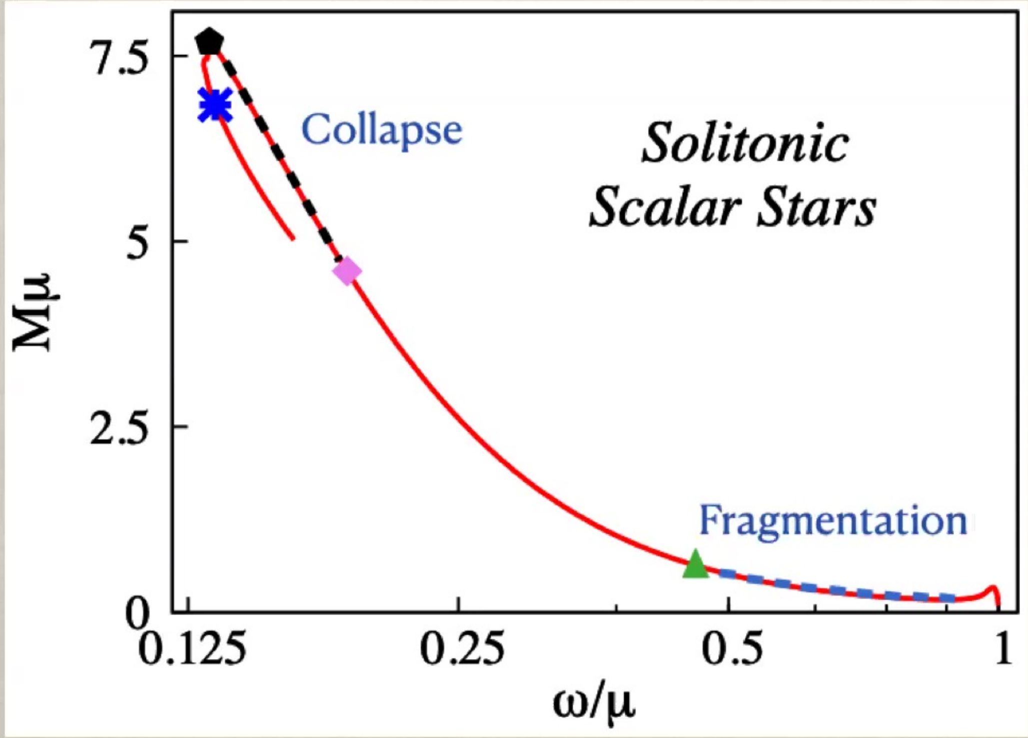


ultracompact stars
Proca Stars
M_{max}
Ergo
LR
1
0.75
0.5
0.25
0
Mμ
0.6
0.8
1
ω/μ
7.5
5
2.5
0
Mμ
ultracompact
stars
Solitonic
Scalar Stars
0.125
0.25
0.5
1
ω/μ


ω = 0.68
ω/μ = 0.69
ω/μ = 0.70
ω/μ = 0.72
ω/μ = 0.711
$M_{Proca}\mu$
$J_{Proca}\mu^2$
Mμ, Jμ²
tμ
J critical PS
M critical PS


Proca Star time evolution ω/μ = 0.68
V-(r, t)
V-*(r, t)
1: ω/μ = 0.71
2: ω/μ = 0.70
3: ω/μ = 0.69
4: ω/μ = 0.68
5: ω/μ = 0.67
tμ = 3264.0
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
1
2
3
4
5
6
7
8
9
10
Circumferencial radius

Are the two NAIs in the (scalar) relativistic branch different? [energetics]



Can there be ultracompact ECOs?

It seems hard:

- The LR instability is real and it needs not be too long lived, except near the critical solution, leading (at least) to collapse or migration.
- This questions the plausibility of ultracompact ECOs, that have a plausible formation mechanism within GR.

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But:

- These are just a couple of families of examples; are we missing some generality?
- There details in the analysis for which a deeper understanding would be necessary (non-monotonic instability time scale, loss of axi-symmetry, non-linear character of the instability, spatial correlation with stable LR,...).
- Are there reasonable ultracompact ECOs without a stable LR? (Drop the assumptions: circularity, axi-symmetry; topological triviality)
- Can ECOs mimic BHs even if they are not ultracompact?

On the fate of the Light Ring instability

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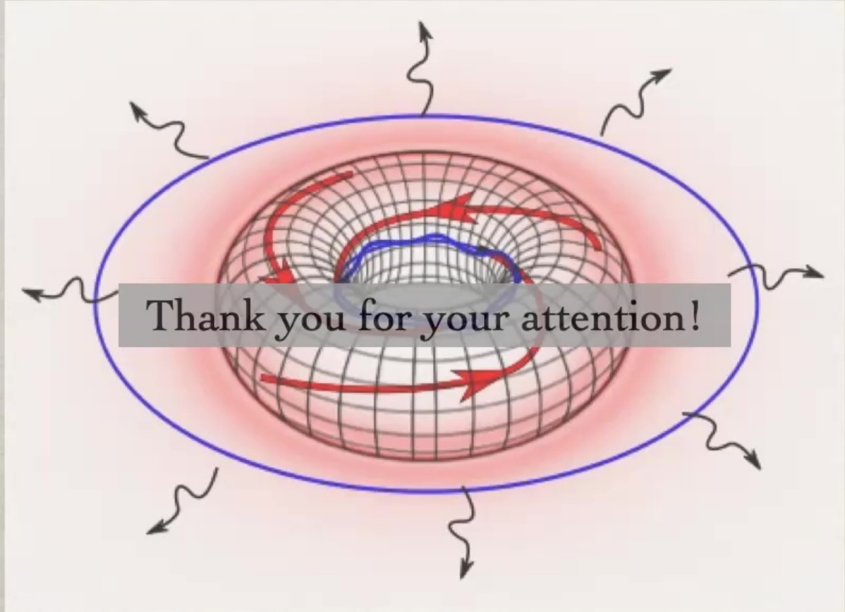
Perimeter Institute (online), Oct 20th 2022



Based on
2207.13713 with P. Cunha, E. Radu, N. Sanchis-Gual

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Gr v
The Gravity group @ Aveiro University, Portugal

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