

Title: Phenomenological thermodynamics with multiple quantities of interest

Speakers: Lidia del Rio

Series: Quantum Foundations

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Abstract: Joint work (in progress) with Ladina Hausmann, Nuriya Nurgalieva and Renato Renner

We can classify contemporary approaches to thermodynamics in roughly four camps:

(1) Top-down microscopic approaches. These are for example resource-theoretical approaches to quantum thermodynamics: they have a microscopic model of states and systems, and which microscopic restrictions implement macroscopic properties. For instance, in the resource theory of quantum thermodynamics, states are represented by density operators, thermal states in particular have a specific micro-canonical form, and constraints like energy preservation are enforced by forcing quantum transformations to commute with a global Hamiltonian. These approaches succeed at deriving thermodynamic laws in general settings that satisfy the microscopic model (like non-relativistic quantum systems).

(2) Bottom-up microscopic approaches. These also start from a microscopic model, but rather than looking for universal restrictions, they search for explicit thermodynamics protocols: this is the case of recent proposals for quantum work extraction or nano quantum heat engines.

(3) Top-down phenomenological approaches. These try to derive thermodynamic laws from first principles independently of a microscopic model. In principle the results derived in this framework can be applied to a wider variety of explicit systems, and the challenge is then to find the right implementations. The first derivations of thermodynamics were naturally phenomenological, and some modern information-inspired derivations follow this approach.

(4) Bottom-up phenomenological approaches. These approaches try to find explicit thermodynamic protocols independently of the microscopic model, based only on operational properties of the systems at hand. It was the case for Carnot's original engines and more recently for some approaches to deriving black hole thermodynamics, or thermodynamics of new materials; some experimental results also fit in this camp.

In this work we generalize top-down phenomenological approaches to the case of multiple conserved quantities. Note that multiple conserved quantities have been studied in top-down and bottom-up microscopic approaches to quantum thermodynamics. We argue that our framework is more general, in that it can be applied to systems for which we don't have an explicit microscopic model; in particular we will apply the results of this framework to black hole thermodynamics. Moreover, having a phenomenological axiomatic approach to thermodynamics allows us to identify which properties are specific to a microscopic model like quantum physics, and which hold in any physical theory: our results can be applied to study the thermodynamics of generalized process theories, and other generalizations and foils of quantum mechanics. This generalization makes us reconsider the second law of thermodynamics, adapting for an exchange of different conserved quantities, for example, energy and angular

momentum, or energy and spin. Our guiding principle here is to use information as a universal token of exchange to convert between different quantities via Landauer's principle.

Zoom Link: <https://pitp.zoom.us/j/96001094153?pwd=YTAhTGpPdEJlNFZMc0FqV1dIRTVyZz09>

# Phenomenological Thermodynamics

Ladina Haussmann, Yuriya Nurgalieva

LdR, Renato Renner

Information + Energy  
+ Angular mom.  
+ Particle number  
+ ....

✓ Energy  
✓ Information



WORK



$$\bar{E} = mgh$$



$$\langle E \rangle = T_f(H\rho)$$

$$\frac{\Delta E}{\langle E \rangle} \ll 1$$

✓ Energy  
 ✗ Information

} Heat



T

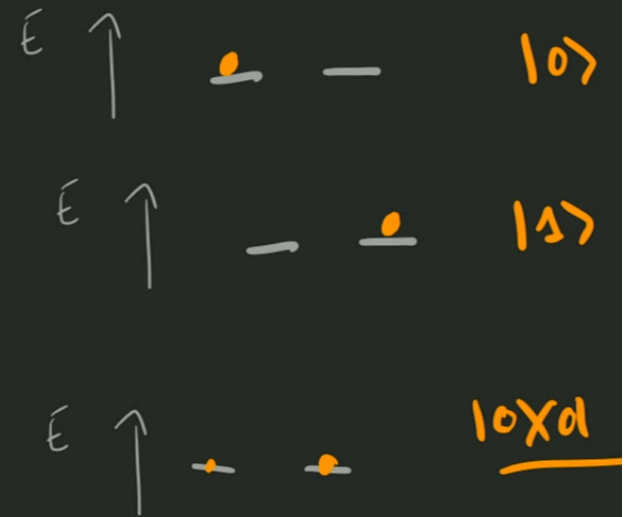


$$\frac{e^{-\frac{1}{k_B} H}}{Z}$$

$$=: \rho_{\text{thermal}}(T, H)$$

X Energy  
✓ Information

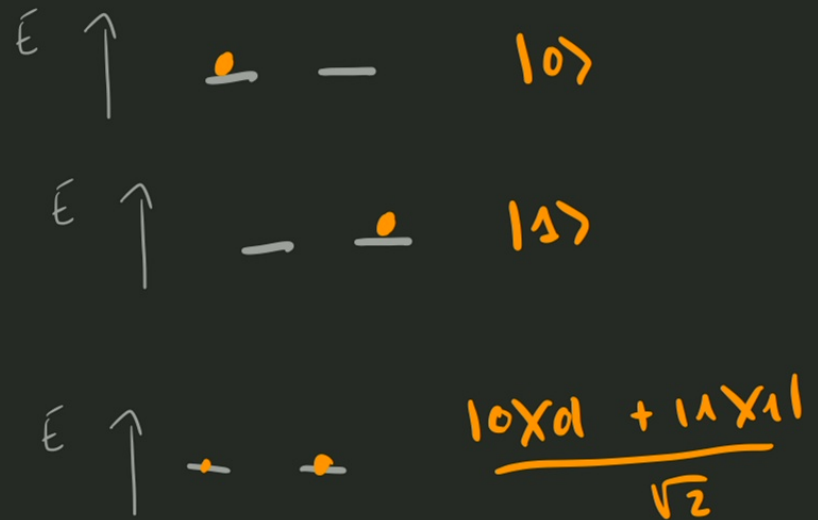
} Landauer  
bitbox



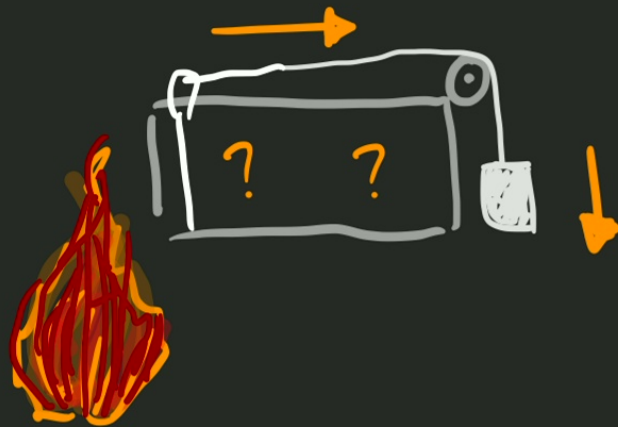
X Energy

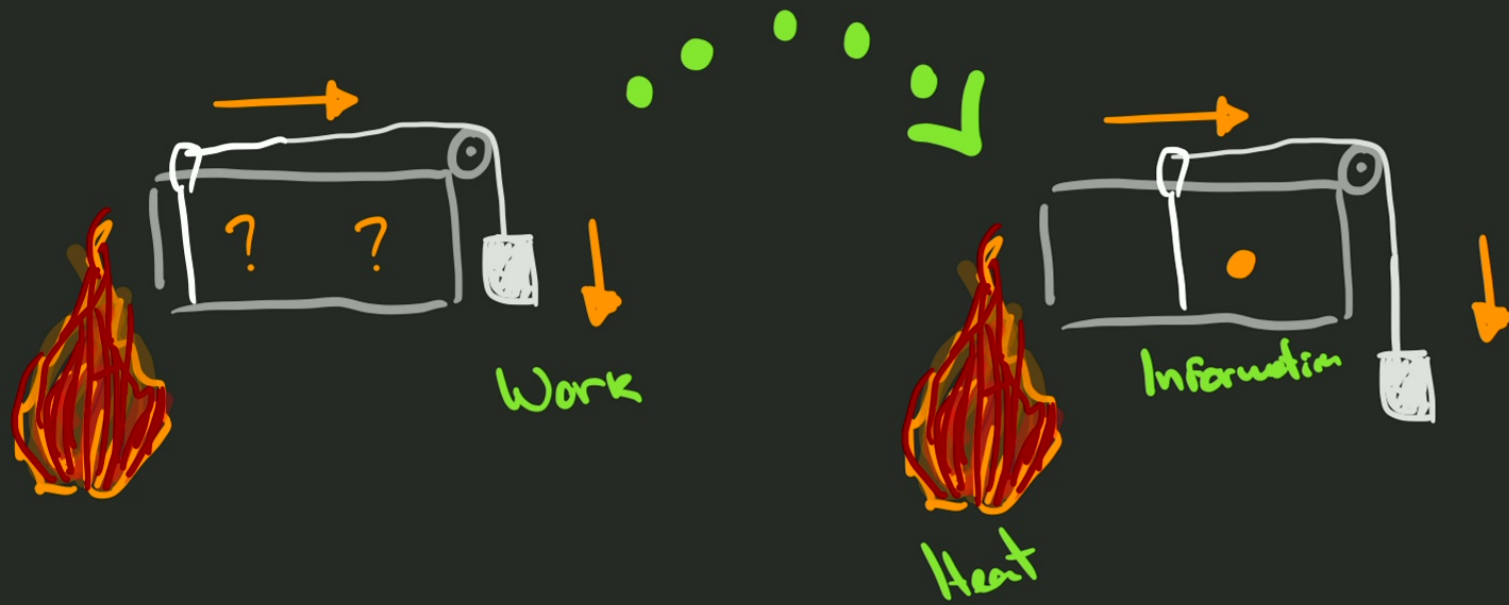
✓ Information

} Landauer  
bitbox

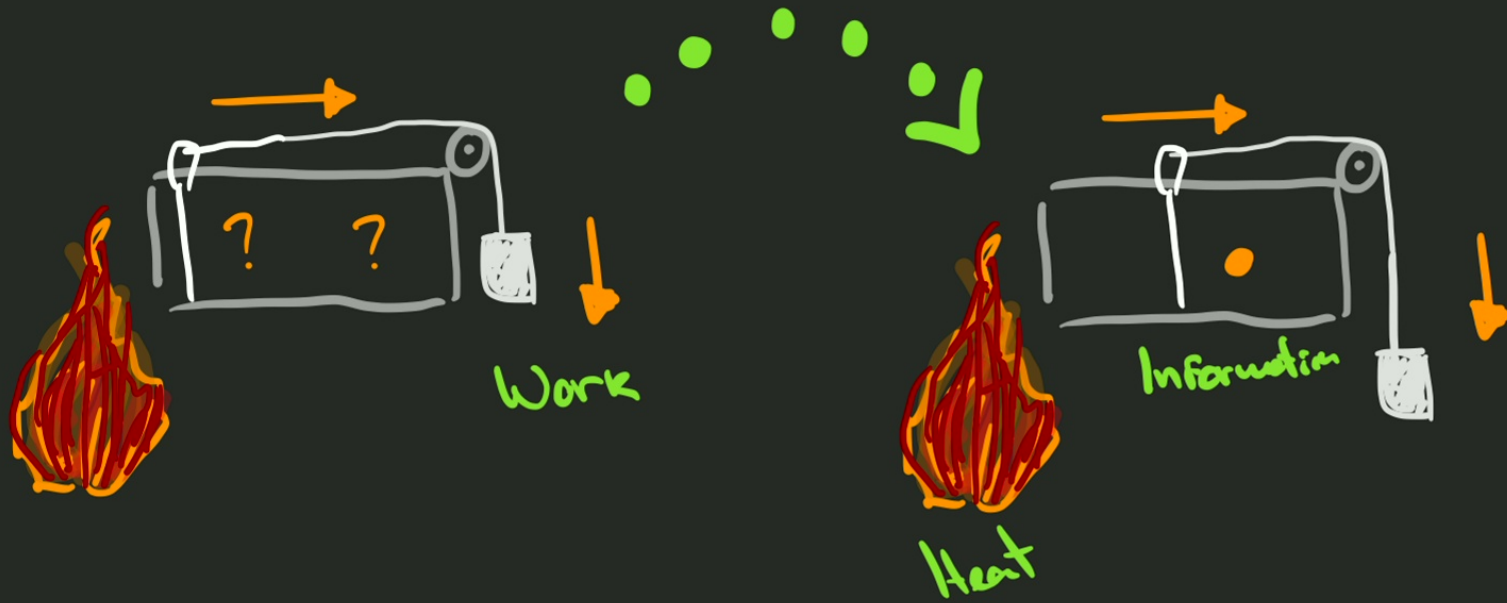








$$W = k_B T \ln 2 \Delta H$$



$$\rho_{th}(T)$$

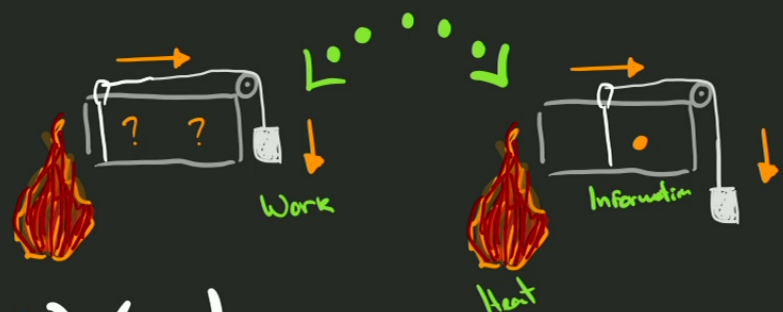
⊗

$$\frac{1}{2}$$

⊗

$$|E \times E|$$

Work



$$\tilde{\rho}_{th}(T_c)$$

Heat

⊗

$$|0 \times 0|$$

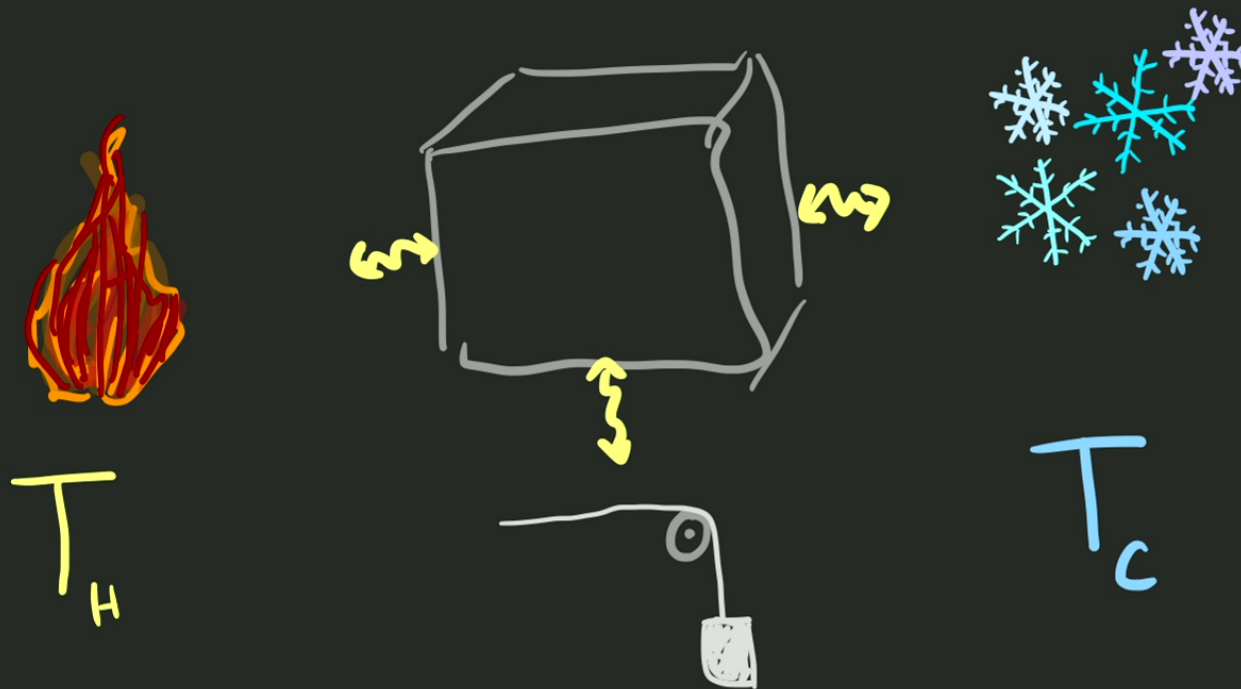
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$$|E - W \times E - W|$$

Information

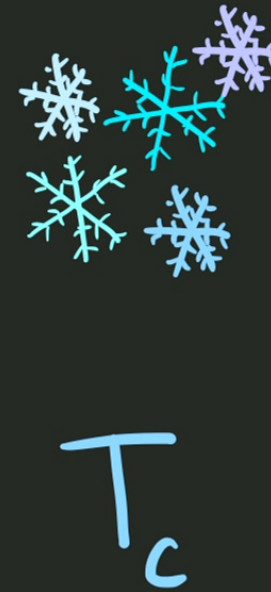
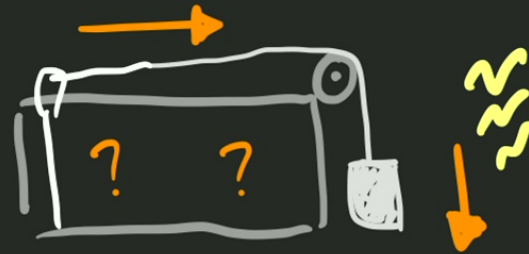
$$\langle E \rangle_{\tilde{p}} - \langle E \rangle_p \approx W$$

# Building a Carnot Engine....



Step 2: Erase BB  $\rightarrow$  spend  $W_2 = -k T_c \ln 2 H$

$$= Q_c$$



$$W = (T_H - T_C) k_B \ln 2 \Delta H$$

$$= (T_H - T_C) \frac{Q_H}{T_H}$$

↳ Carnot's efficiency

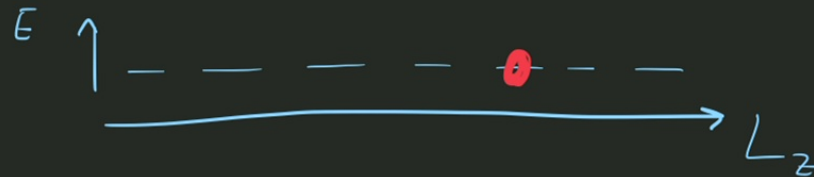
- ✗ Energy
- ✓ Angular momentum
- ✓ Information

"Turn table"

(angular mom. work)



deg. spin lattice



$$\langle E \rangle = 0$$

$$\langle L_z \rangle = \text{Tr} (L_z \rho)$$

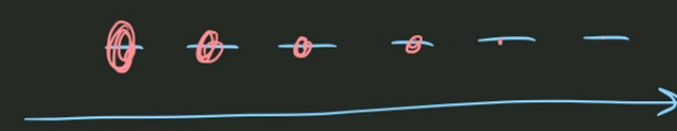
$$\frac{\Delta L_z}{\langle L_z \rangle} \ll 1$$



X Energy  
 ✓ Angular momentum  
 X Information

Angular momentum bath

↑↑↑↑↓↑↑↓↑↑  
 ?  
 $T_L$

$\epsilon \uparrow$   
  

$$\frac{e^{-\frac{1}{k_B} L_z}}{Z} =: \rho_{\text{thermal}}(T_L)$$

$$\rho_{th}(T_L)$$



$$\rho_{th}(T_c)$$

Heat

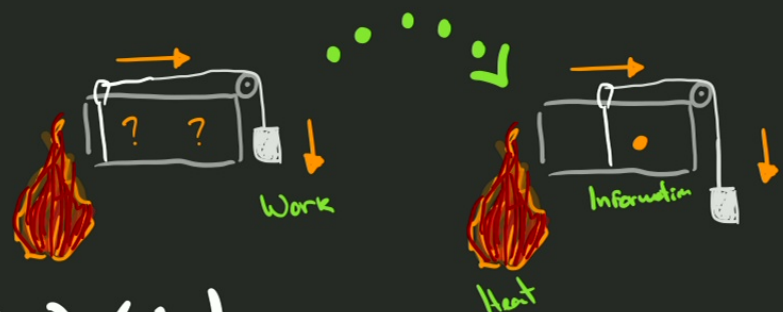
⊗

$$\frac{1}{2}$$

⊗

$$|L \times L|$$

Work



$$\otimes |0 \times 0|$$

⊗

$$|L - W_c \times L - W_c|$$

Information

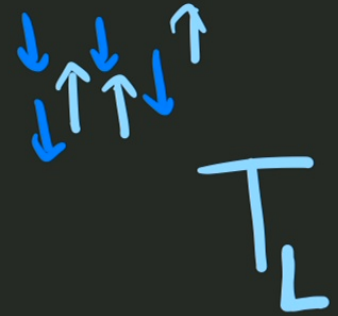
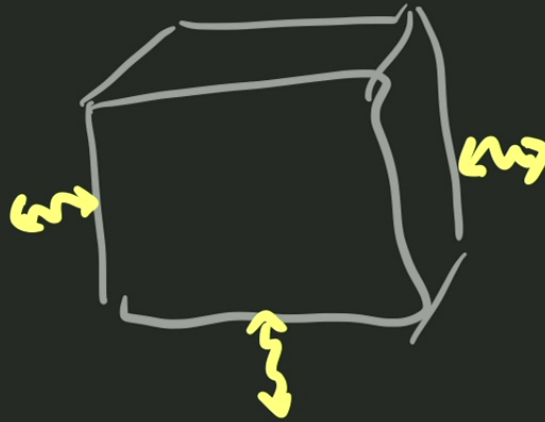
$$W_L = k_L T_L \ln 2 \Delta H$$

" Boltzmann  
ct. "

Temperature

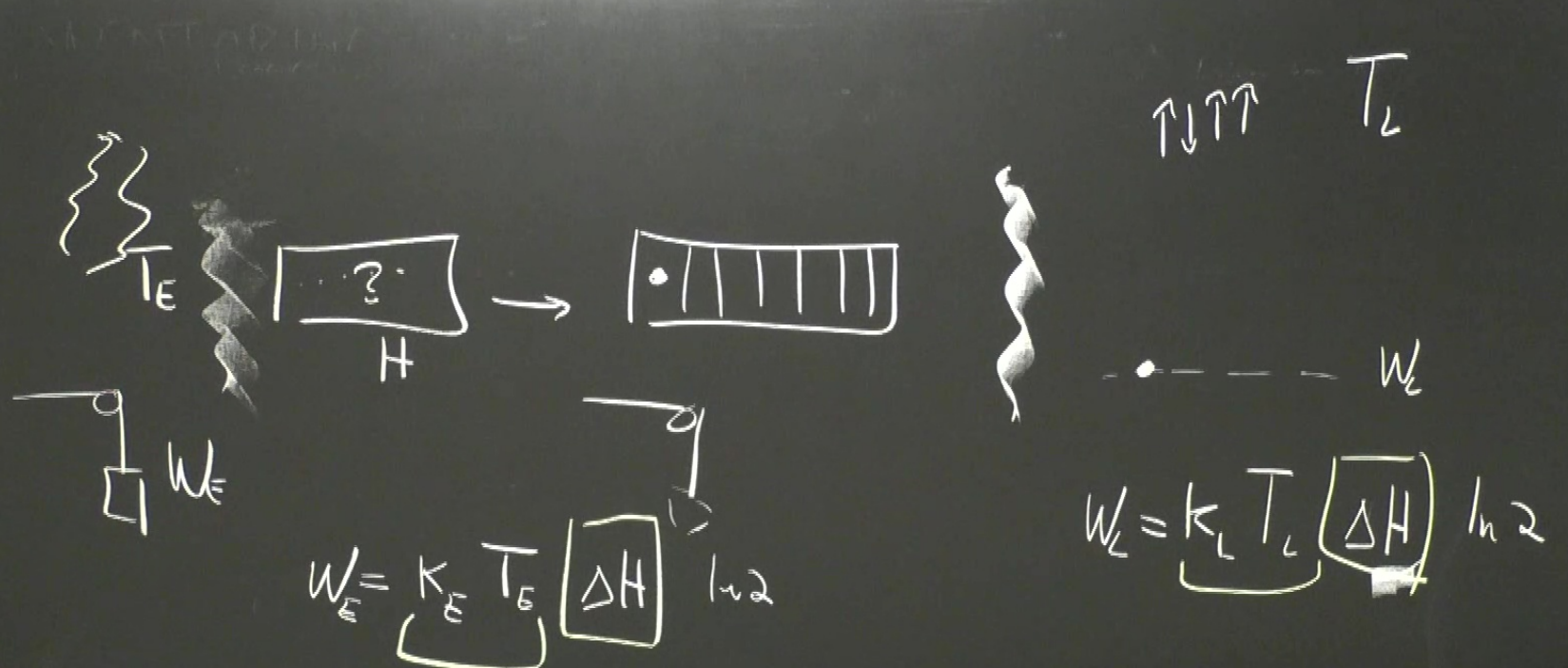
# Trading:

$T_E$



$W_E$



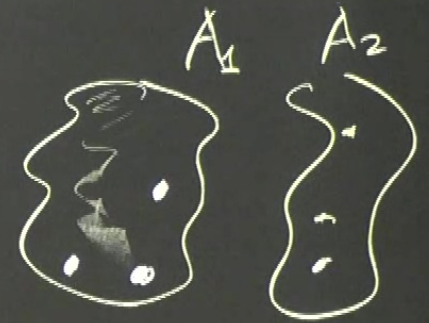




Systems  $S = P(A_1, A_2, A_3, \dots)$

States  $A_i \rightarrow \Sigma_A$

$$S = (A_1, A_2, \dots, A_N) \rightarrow \Sigma_S = \Sigma_{A_1} \times \Sigma_{A_2} \times \Sigma_{A_3} \times \dots$$



$\uparrow \uparrow \uparrow$   $T_L$

# Processes:

$P \Rightarrow \mathcal{P}_S$  processes that act only on  $S$

$$P: \sigma_{\text{initial}} \xrightarrow{P} \sigma_{\text{final}}$$

$$\forall s, \forall \sigma_s \in \Sigma_s, \exists i \in \mathcal{P}_s: L(i) = \uparrow i = \sigma_s$$

Global view of local processes frames

$$\forall S, \forall R: S \cap R = \emptyset$$

$$\forall p \in \mathcal{P}_S, \forall \sigma_R \in \Sigma_R,$$

$$p \times i \in \sigma_R \iff \uparrow p \uparrow_S = \uparrow p \times i$$

$$\uparrow p \uparrow_{SR} = \uparrow p \times i \uparrow_R = \sigma_R$$

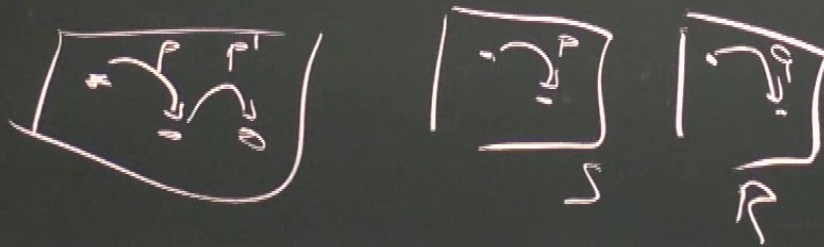


Global view of local processes frames

$\forall S, \forall R: S \cap R = \emptyset$

$\forall p \in \mathcal{P}_S, \forall \sigma_R \in \Sigma_R,$

$$p \times i_{\sigma_R} = \left[ \begin{array}{c} p \\ \sigma_R \end{array} \right]_S = \left[ \begin{array}{c} p \times i \\ \sigma_R \end{array} \right]$$



Conserved quantities  $\mathcal{B} \ni \mathcal{B}$

$\forall S \quad \forall B, p \in P \rightarrow W_S^B(p) \rightarrow$  additive:  $W_S^B(p) = \sum_i W_{A_i}^B(p)$

$S = \sum X A_i$

1st Law

$\forall S,$



$$\forall S \quad \forall B, p \in P \rightarrow W_S^B(p) \rightarrow \text{additive: } W_S^B(p) = \sum_i W_{A_i}(p)$$

### First Law

$$\forall S, \forall p, \sigma \in \Sigma_S, \exists p \in P_S : p \xrightarrow{p} \sigma$$

$$\text{or } \exists p' \in P_S : \sigma \xrightarrow{p'} p$$



$$\sigma_R = (\sigma_1, \sigma_2)$$

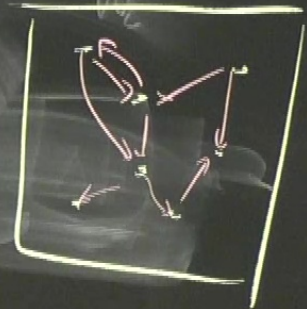
$$\forall S \quad \forall B, p \in P \rightarrow W_S^B(p) \rightarrow \text{additive: } W_S^B(p) = \sum_i W_{A_i}(p)$$

## First Law

$$\forall S, \forall p, \sigma \in \Sigma_S, \exists p \in P_S: p \xrightarrow{p} \sigma$$

$$\text{or } \exists p' \in P_S: \sigma \xrightarrow{p'} p$$

$$\forall p \in P_S, W_S^B(p) = \text{Function}(P_S^T, LP)_S$$



$$\sigma_R = (\sigma_1, \sigma_2)$$



$$\forall S \quad \forall B, p \in P \rightarrow W_S^B(p) \rightarrow \text{additive: } W_S^B(p) = \sum_i W_{A_i}(p)$$

## First Law

$$\forall S, \forall p, \sigma \in \Sigma_S, \exists p \in P_S: p \xrightarrow{p} \sigma$$

$$\text{or } \exists p' \in P_S: \sigma \xrightarrow{p'} p$$

$$\forall p \in P_S, W_S^B(p) = \text{Function}(P_S^T, LP)_S$$



$$\sigma_R = (\sigma_1, \sigma_2)$$

Conserved quantities  $\mathcal{B} \ni B$

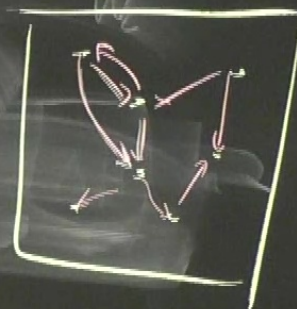
$$\forall S \quad \forall B, p \in \mathcal{P} \rightarrow W_S^B(p) \rightarrow \text{additive: } W_S^B(p) = \sum_i W_{A_i}^B(p)$$

First Law

$$\forall S, \forall p, \sigma \in \Sigma_S, \exists p \in \mathcal{P}_S : p \xrightarrow{P} \sigma$$

$$\text{or } \exists p' \in \mathcal{P}_S : \sigma \xrightarrow{P'} p'$$

$$\forall p \in \mathcal{P}_S, W_S^B(p) = \text{Function}(\{P\}_S, \{L\}_S)$$





Ideal heat bath  $\mathcal{B}' \subseteq \mathcal{B}$

atomic system  $R$

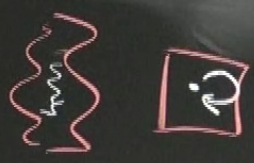
Example.  $U_R^{\mathcal{B}}$  Fully degenerate for  $\mathcal{B} \in \mathcal{B}'$

$(U_R^{\mathcal{B}})_{\mathcal{B} \in \mathcal{B}'}$  injective on set of states

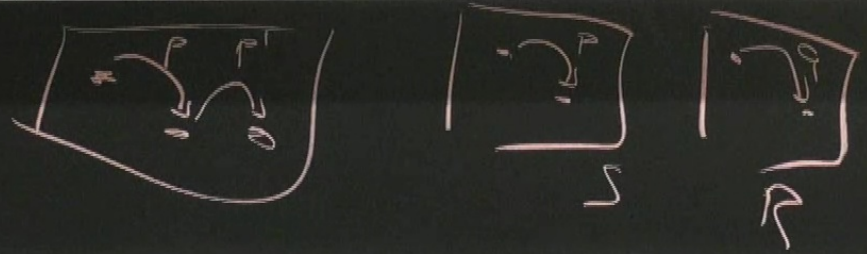
Simple  $\circ U_R^B$  Fully degenerate for  $B \in \mathcal{B}'$

$(U_R^B)_{B \in \mathcal{B}'}$  injective on set of states, each  $U_R^B = \mathbb{R}$

Passive:



$\forall B, \forall S, \forall P \in \mathcal{P}_{RS}$ , if  $[P]_S = [P]_{S'} \Rightarrow W_{RS}^B(P) \geq 0$





both  $\mathcal{B}' \subseteq \mathcal{B}$

system  $R$  that act only on  $S$

is not degenerate for  $\mathcal{B} \in \mathcal{B}'$

injective on set of states, each  $U_R^S = \mathbb{R}$

$\forall \mathcal{B}, \forall S, \forall P \in \mathcal{P}_{RS}$ , if  $\lceil P \rceil_S = \lfloor P \rfloor_S$   
 $\Rightarrow W_{RS}^S(P) \geq 0$

Invariant:

$$\forall P \in \mathcal{P}_{SR}$$

$$\forall \sigma_R \in \Sigma_R$$

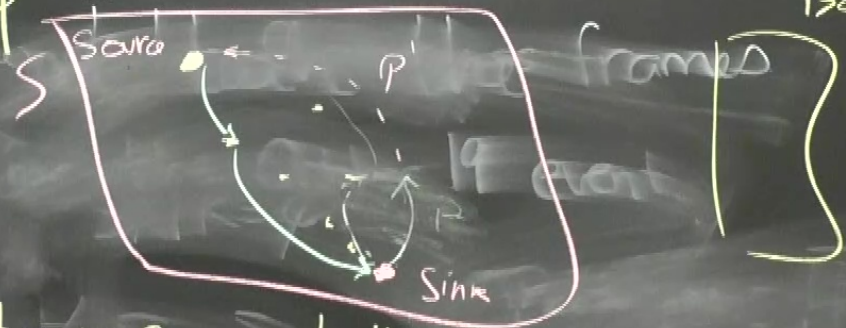
$$\exists P' \lceil P' \rceil_S = \lfloor P' \rfloor_S$$

$$\lceil P' \rceil_R = \sigma_R$$



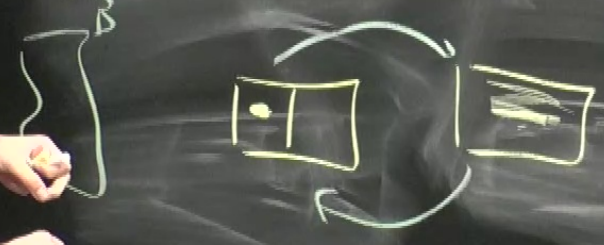
$D, V, S, \& P, \dots, Q_c(P) = 0$  W

Regenerate of Ordered



• You can't extract work from a bath & a sink

$$W = kT \ln 2 \quad A(\text{box})$$



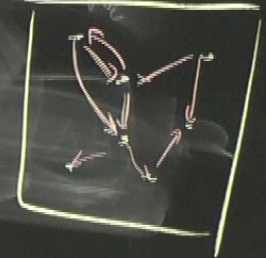


## First Law

$$\forall s, \forall p, \sigma \in \Sigma_s, \exists p' \in P_s : p \xrightarrow{p'} \sigma$$

$$\forall p \in P_s, \exists p' \in P_s : \sigma \xrightarrow{p'} p$$

$$\forall p \in P_s, W_s^B(p) = \text{Function}(P_s, L P_s)$$



$$\Sigma = \Sigma_{A_1} \times \Sigma_{A_2} \times \Sigma_{A_3} \times \dots$$

$$\sigma = (\sigma_1, \sigma_2, \dots)$$

$$\sigma_R = (\sigma_1, \sigma_2)$$

CAUTION

TO AVOID THE RISK OF PERSONAL INJURY, PLEASE DO NOT TOUCH THE BOARD WHEN IT IS BEING USED BY OTHERS.