Title: Local supersymmetry as square roots of supertranslations: A Hamiltonian study

Speakers: Sucheta Majumdar

Series: Quantum Gravity

Date: October 13, 2022 - 2:30 PM

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Abstract: In this talk, I will show that supergravity on asymptotically flat spaces possesses a (nonlinear) asymptotic symmetry algebra, containing an infinite number of fermionic generators. Starting from the Hamiltonian action for supergravity with suitable boundary conditions on the graviton and gravitino fields, I will derive a graded extension of the BMS_4 algebra at spatial infinity, denoted by SBMS_4. These boundary conditions are not only invariant under the SBMS_4 algebra, but lead to a fully consistent canonical description of the supersymmetries, which have well-defined Hamiltonian generators. One finds, in particular, that the graded brackets between the fermionic generators yield BMS supertranslations, of which they provide therefore "square roots". I will comment on some key aspects of extending the asymptotic analysis at spatial infinity to fermions and on the structure of the SBMS_4 algebra in terms of Lorentz representations.

Zoom link: https://pitp.zoom.us/j/95951230095?pwd=eHIwUXB5SUkvd0IvZnVUN3JJMFE1QT09

Local supersymmetry as square roots of supertranslations: a Hamiltonian study

Sucheta Majumdar

ENS de Lyon

Based on ArXiv: 2011.04669 and ArXiv: 2108.07825

Quantum gravity seminar series, Perimeter Institute

October 13, 2022

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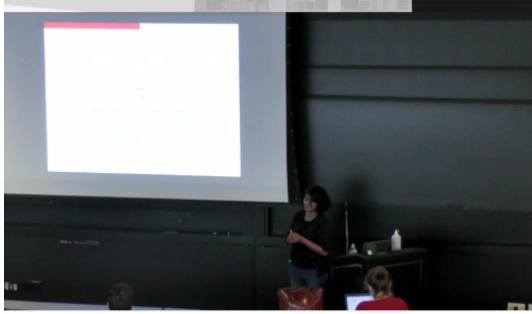

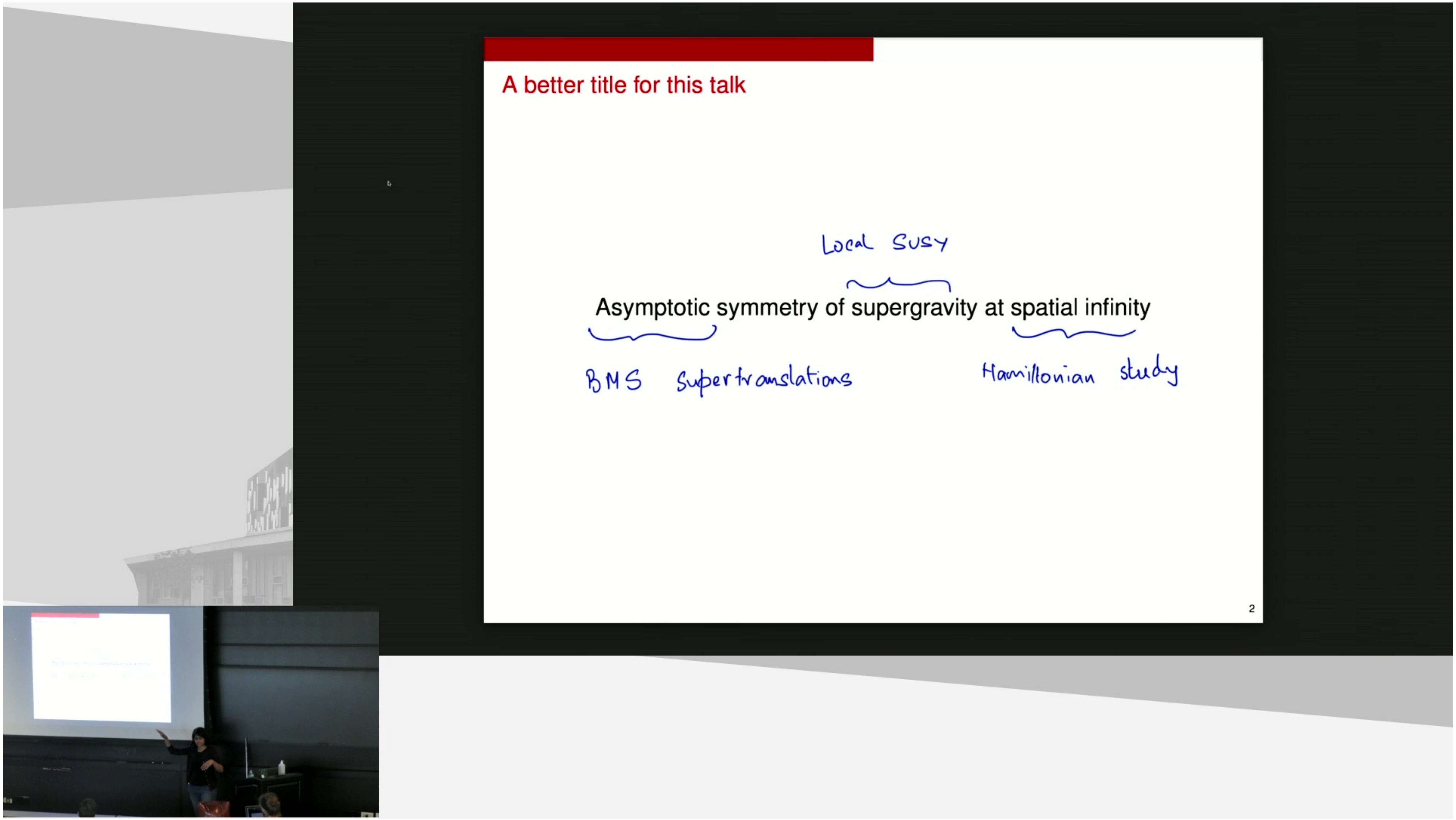
A better title for this talk
Local SUSY
Asymptotic symmetry of supergravity at spatial infinity
BMS Supertranslations
Hamiltonian study
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Motivation

Supersymmetry

- Converts a boson into a fermion and vice-versa

$$Q|B\rangle \sim |F\rangle\,, \quad \overline{Q}|F\rangle \sim |B\rangle$$

Every particle of spin s has a SUSY partner of spin $s \pm \frac{1}{2}$

- Often improves UV behaviour
- Underlying algebra: super-Poincaré $\{P^\mu, J^{\mu\nu}, Q_\alpha, \overline{Q}_\beta\}$

$$\boxed{\{\, Q_\alpha \,,\, \overline{Q}_\beta \,\} \sim \gamma^\mu_{\alpha\beta}\, P_\mu}$$

2 supersymmetries close on spacetime translations

$$a^\mu = i\epsilon_1^T \gamma^\mu \epsilon_2 \quad \rightarrow \quad \text{“Square root of translations”}$$

3

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2 supersymmetries close on spacetime translations

$$a^\mu = i\epsilon_1^T \gamma^\mu \epsilon_2 \quad \rightarrow \quad \text{"Square root of translations"}$$

Supergravity

Theory of gravity (spin 2) with lower spin particles: spin 3/2

- a possible way to reconcile gravity with the other forces
- Better UV behaviour than Einstein's gravity $\rightarrow$ $\mathcal{N} = 8$ theory is finite up to 4 loops

3

Motivation

- Supergravity algebra spanned by $\{P^\mu, J^{\mu\nu}, Q_\alpha, \overline{Q}_\beta\}$

  2 local supersymmetries close on a diffeo:

$$\xi^\mu(x) = i\epsilon_1^T(x)\gamma^\mu\epsilon_2(x)$$

- For asymptotically flat spacetimes, the symmetry group at infinity is BMS

$$\text{BMS}_4 = SO(3,1) \ltimes \text{supertranslations}$$

  Can the fermionic symmetries be enhanced too? "Square root of supertranslations"

- Some super-BMS alegbras with infinite-dimensional fermionic symmetries exist at null infinity [Avery-Schwab 2015, Fotopoulos-Steiberger-Taylor-Zhu 2020, Pano-Pasterski-Puhm 2021]

Focus of this talk

Asymptotic symmetry of supergravity at spatial infinity

1. Graded extensions of the BMS algbera → superalgebras at spatial infinity
2. Infinite-dimensional fermionic symmetries → "square roots" of BMS supertranslations

Based on ArXiv: 2011.04669 and ArXiv: 2108.07825

with Oscar Fuentealba, Marc Henneaux, Javier Matulich and Turmoli Neogi at ULB

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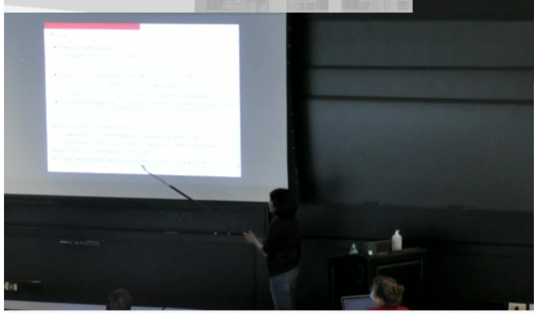

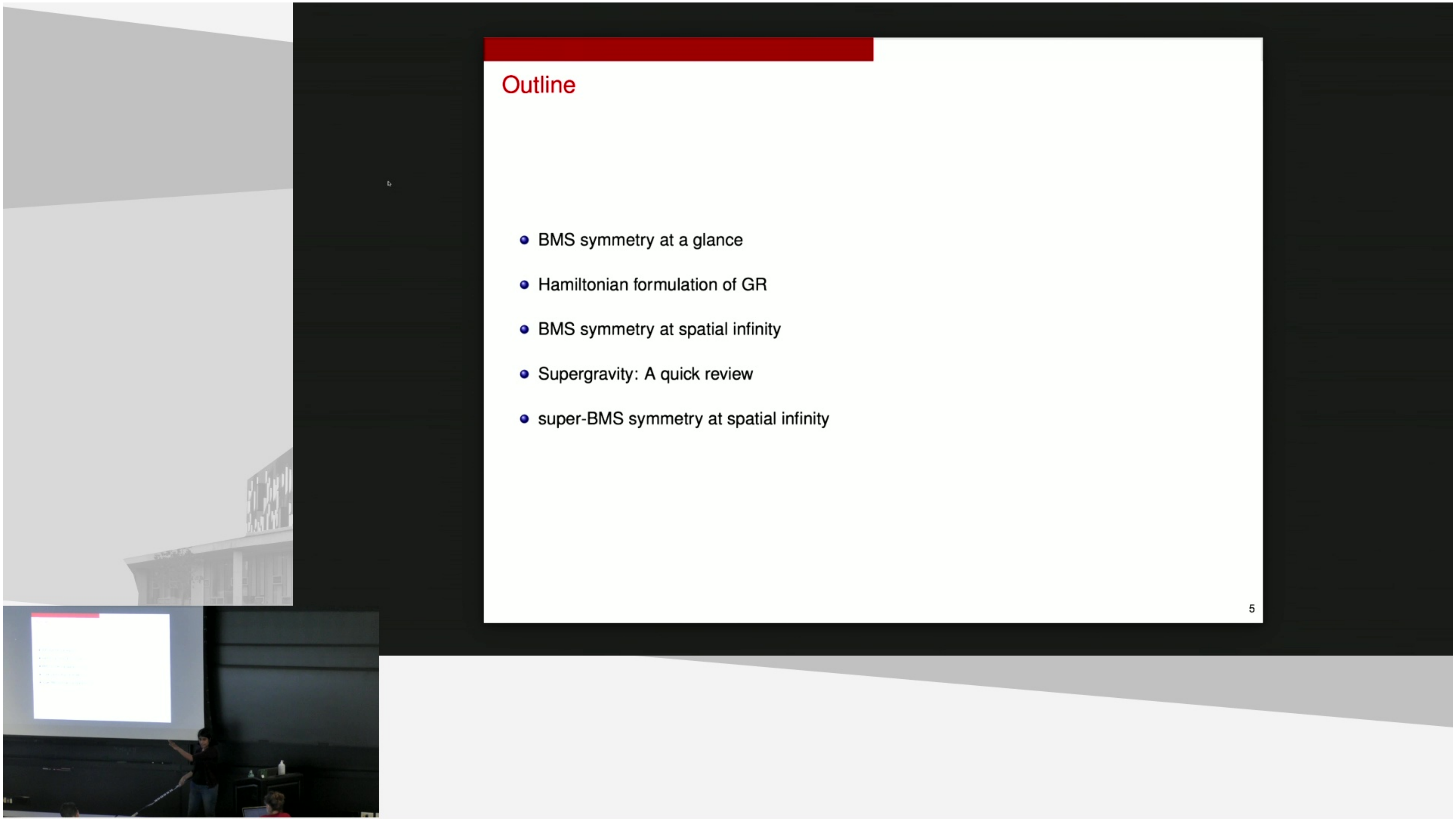
Outline
BMS symmetry at a glance
Hamiltonian formulation of GR
BMS symmetry at spatial infinity
Supergravity: A quick review
super-BMS symmetry at spatial infinity
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What is the BMS symmetry of gravity?

General relativity

$$dS^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\mathcal{S}_{EH} = \int d^4x \sqrt{g}\mathcal{R} \quad \Rightarrow \quad \text{Einstein's equations}$$

Invariance under general coordinate transformations, $x^\mu \to x^\mu + \xi^\mu(x)$

Asymptotic symmetry at null infinity [Bondi-van der Burg-Metzner-Sachs 1962]

All allowed transformations that preserve the form of the metric $g_{\mu\nu}$ at null infinity

- Poincaré in the bulk: $\xi^\mu(x) = \omega^\mu_\nu x^\nu + a^\mu$

  (Lorentz $\omega^\mu_\nu = 3$ rotations $+ 3$ boosts,) + (4 Translations a^μ) : 10 dimensional

$$\downarrow$$

- BMS at null infinity: Lorentz remains the same, a^μ replaced by a function α : ∞-dimensional

  $\to$ Lorentz + "Supertranslations"

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BMS symmetry at a glance

Bondi approach:
BMS as the asymptotic symmetry group at null infinity

- infinite-dimensional enhancement of Poincaré group
- further extended to include superrotations, *Diff*$(\mathbb{S}^2)$, near-horizon symmetries
- connections to soft theorems and on-shell amplitudes, Celestial Holography etc.

[**Bondi-van der Burg-Metzner-Sachs '62**, Barnich-Troessaert, Hawking-Perry-Strominger Compère, Campiglia, Detournay, Donnay, Freidel, Geiller, Grumiller, Laddha, Pasterski, Puhm, Raclariu, Sheikh-Jabbari, Zwikel and many more]

Conformal Carroll approach:

- BMS group as conformal Carroll group
- Further extensions to other Carrollian structures
- Symmetries of null hypersurfaces, Carrollian field theory

[**Duval-Gibbons-Hovarthy '14**, Campoleoni, Ciambelli, Donnay, Fiorucci, Freidel, Flanagan, Heffray, Leigh, Obers, Petropoulos, Ruzziconi and many more]

Hamiltonian approach:
BMS symmetry at spatial infinity

- Based on "$3+1$" Hamiltonian formulation of gravity à la ADM
- Canonical realization of the BMS algebra from an action principle
- a precursor to any quantization methods

[**Henneaux-Troessaert '18**, Fuentealba, Guilini, SM, Matulich, Neogi, Riello, Tanzi, ...]

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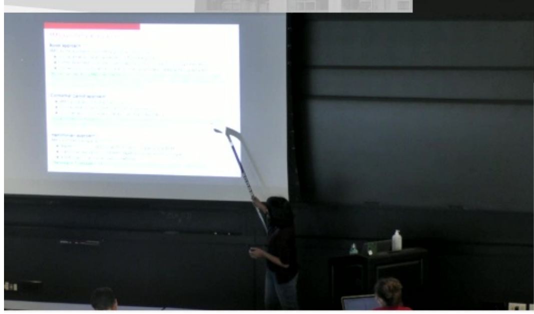
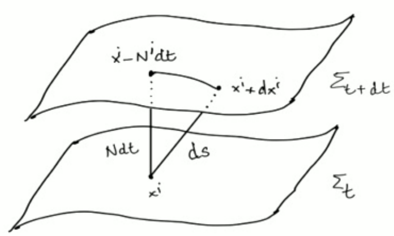
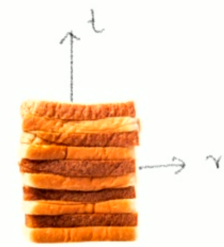


Hamiltonian formulation of GR à la Dirac and ADM

- 3+1 foliation of spacetime by a family of spacelike surfaces Σ_t





- Dynamical variables on Σ_t

  $(g_{ij}, \pi^{ij}) \to$ 3-metric on Σ_t and conjugate momenta

The ADM action for gravity [Dirac '58, Arnowitt-Deser-Misner '62]

$$S_{ADM}[g_{ij}, \pi^{ij}, N, N^i] = \int dt \{ \int d^3x (\pi^{ij}\dot{g}_{ij} - N\mathcal{H} - N^i\mathcal{H}_i) - B_\infty \}$$

Hamiltonian action: $S = \int (p\dot{q} - H)$

Boundary terms B_∞ ensure a good variational principle [Regge-Teitelboim '74]

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Hamiltonian formulation of GR à la Dirac and ADM

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$$\mathcal{S}_{ADM}[g_{ij},\pi^{ij},N,N^i] \;=\; \int dt\{\int d^3x(\pi^{ij}\dot{g}_{ij} - N\mathcal{H} - N^i\mathcal{H}_i) - B_\infty\}$$

Hamiltonian action: $S = \int(p\dot{q} - H)$

- Lagrange multipliers, N and N^i implement the constraints

$$\mathcal{H} \;=\; -\sqrt{g}R + \frac{1}{\sqrt{g}}(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2)\,, \quad \mathcal{H}_i \;=\; -2\,\nabla_j\pi^j_i$$

Constraints generate gauge symmetries

- Symplectic form

$$\Omega \;=\; \int d^3x\; d_V\pi^{ij} \wedge d_V g_{ij}\,, \quad d_V \equiv \text{exterior derivative in field space}$$

Phase space : $\{g_{ij},\pi^{ij}\}$

Poisson bracket: $\{g_{ij}(x),\pi^{kl}(x')\} = \delta^{(k}_{(i}\delta^{l)}_{j)}\delta^{(3)}(x-x')$

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Symmetries of the ADM action

- Diffeomorphisms:

$$
\begin{aligned}
\delta_\xi g_{ij} &= \frac{2\xi}{\sqrt{g}}\left(\pi_{ij} - \frac{1}{2} g_{ij}\pi\right) + \mathcal{L}_\xi g_{ij}\,, \\
\delta_\xi \pi^{ij} &= -\xi\sqrt{g}\left(R^{ij} - \frac{1}{2} g^{ij} R\right) + \frac{1}{2}\xi\sqrt{g}\left(\pi_{mn}\pi^{mn} - \frac{1}{2}\pi^2\right) \\
&\quad -2\xi\sqrt{g}\left(\pi^{im}\pi^{j}_{m} - \frac{1}{2}\pi^{ij}\pi\right) + \sqrt{g}\left(\xi^{|ij} - g^{ij}\xi^{|m}{}_{|m}\right) + \mathcal{L}_\xi \pi^{ij}
\end{aligned}
$$

- Canonical generator for *all* symmetries

$$
G_{\xi,\xi^i} = \int d^3x \left(\xi\mathcal{H} + \xi^i \mathcal{H}_i\right) + Q_{\xi,\xi^i}\,,
$$

a) Gauge symmetry: $Q_{\xi,\xi^i} = 0$

Proper gauge transformations do not affect the physical states

b) True symmetry: $Q_{\xi,\xi^i} \neq 0 \quad \rightarrow \quad$ Noether charges

Improper gauge transformations affect the physical states

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Symmetries of the ADM action

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Improper gauge transformations affect the physical states

E.g. Poincaré symmetry

$$\xi = b_i x^i + a^0, \qquad \xi^i = \omega^i{}_j x^j + a^i,$$

b^i boosts, ω^i_j rotations, a^0 time translation, a^i spatial translations

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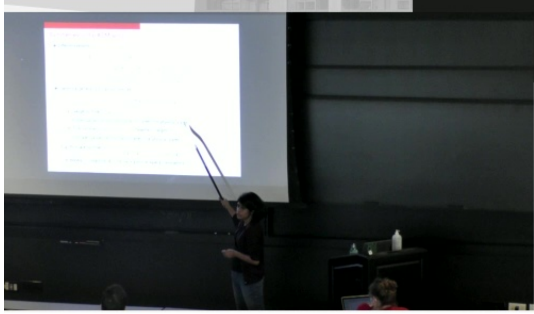

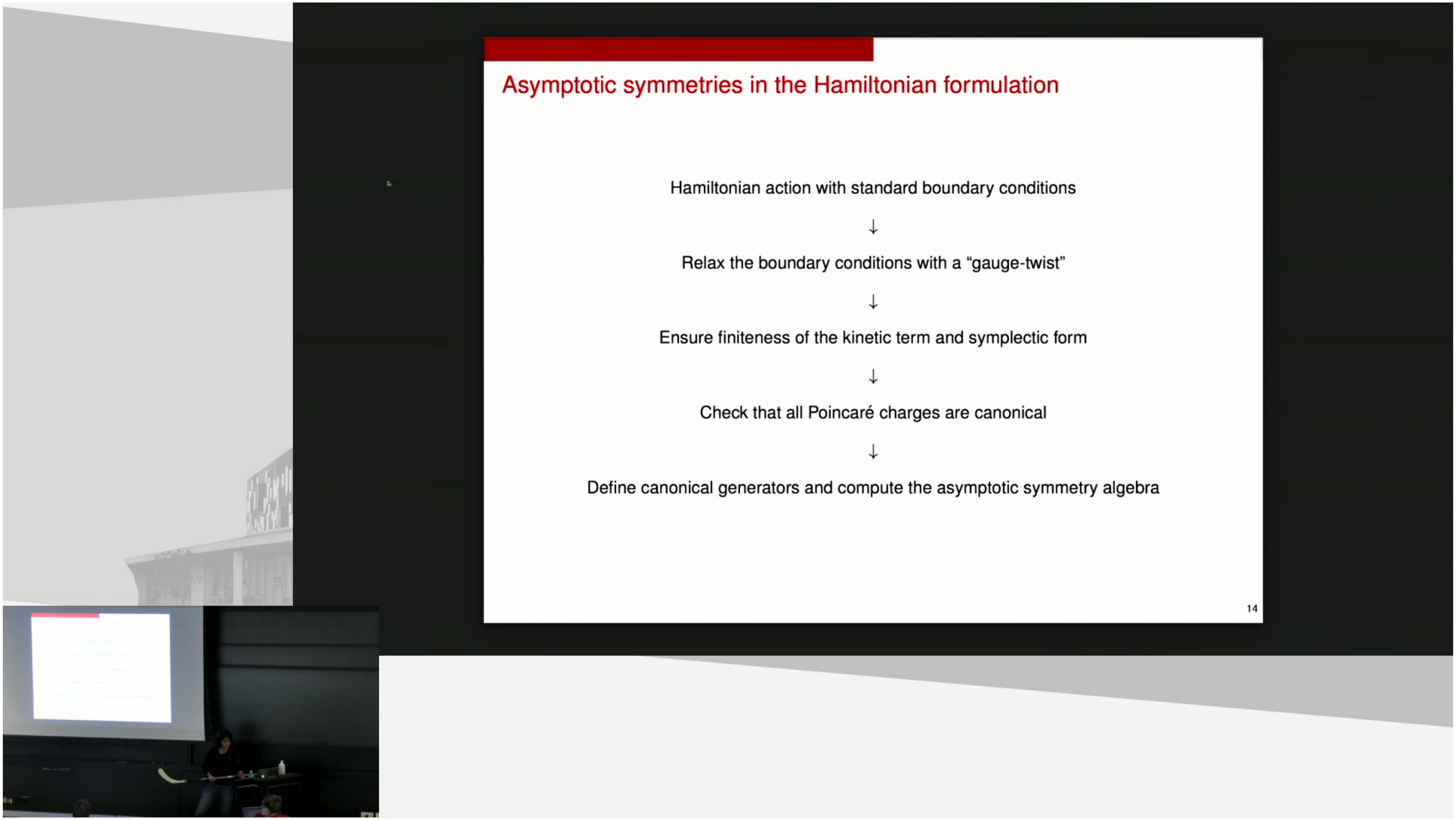
Asymptotic symmetries in the Hamiltonian formulation
Hamiltonian action with standard boundary conditions
↓
Relax the boundary conditions with a “gauge-twist”
↓
Ensure finiteness of the kinetic term and symplectic form
↓
Check that all Poincaré charges are canonical
↓
Define canonical generators and compute the asymptotic symmetry algebra
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Asymptotic conditions I

First ingredient: fall-off conditions

We use spherical coordinates (r, x^A) where x^A are coordinates on the sphere

- Asymptotically flat spacetimes: metric approaches Minkowski as $r \to \infty$

$$
\begin{aligned}
g_{rr} &= 1 + \frac{1}{r}\bar{h}_{rr} + \mathcal{O}(r^{-2}), \\
g_{rA} &= \bar{\lambda}_A + \frac{1}{r}h^{(2)}_{rA} + \mathcal{O}(r^{-2}), \\
g_{AB} &= r^2\bar{g}_{AB} + r\bar{h}_{AB} + h^{(2)}_{AB} + \mathcal{O}(r^{-1}).
\end{aligned}
$$

- Conjugate momenta

$$
\begin{aligned}
\pi^{rr} &= \bar{\pi}^{rr} + \frac{1}{r}\pi^{rr}_{(2)} + \mathcal{O}(r^{-2}), \\
\pi^{rA} &= \frac{1}{r}\bar{\pi}^{rA} + \frac{1}{r^2}\pi^{rA}_{(2)} + \mathcal{O}(r^{-3}), \\
\pi^{AB} &= \frac{1}{r^2}\bar{\pi}^{AB} + \frac{1}{r^3}\pi^{AB}_{(2)} + \mathcal{O}(r^{-4}).
\end{aligned}
$$

Barred quantities, such as $\bar{h}_{ij}$ or the $\bar{\pi}^{ij}$ are functions on the unit sphere

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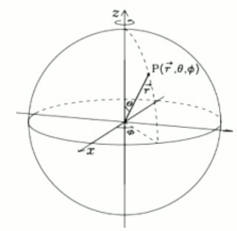
Asymptotic conditions II

Second ingredient: parity conditions on leading terms

"Gauge-twisted" parity conditions: [Henneaux-Troessaert '18]

$$
\begin{aligned}
\bar{h}_{rr} &= \text{even}\,, \\
\bar{\lambda}_A &= (\bar{\lambda}_A)^{\text{odd}} + \overline{D}_A\zeta_r - \bar{\zeta}_A\,, \\
\bar{h}_{AB} &= (\bar{h}_{AB})^{\text{even}} + \overline{D}_A\bar{\zeta}_B + \overline{D}_B\bar{\zeta}_A + 2\bar{g}_{AB}\zeta_r \\
\bar{\pi}^{rr} &= (\bar{\pi}^{rr})^{\text{odd}} - \sqrt{\bar{g}}\overline{\triangle}V\,, \\
\bar{\pi}^{rA} &= (\bar{\pi}^{rA})^{\text{even}} - \sqrt{\bar{g}}\overline{D}^A V\,, \\
\bar{\pi}^{AB} &= (\bar{\pi}^{AB})^{\text{odd}} + \sqrt{\bar{g}}(\overline{D}^A\overline{D}^B V - \bar{g}^{AB}\overline{\triangle}V)\,,
\end{aligned}
$$

Parity: $(r, \theta, \phi) \to (r, \pi - \theta, \phi + 2\pi)$

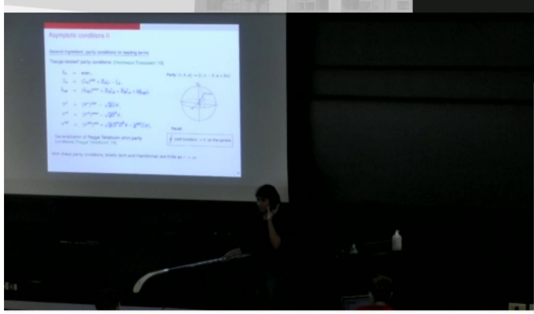


Recall:

$\oint$ (odd function) $= 0$ on the sphere

Generalization of Regge-Teitelboim strict parity conditions [Regge-Teitelboim' 74]

With these parity conditions, kinetic term and Hamiltonian are finite as $r \to \infty$



Canonical realization of Poincare generators

Strict invariance of the symplectic form

$$\Omega \;=\; \int d^3x \; d_V\pi^{ij} \wedge d_V g_{ij} \;,$$

ξ generates a canonical transformation if

$$\boxed{\mathcal{L}_\xi \Omega = \; d_V(\iota_\xi \Omega) = 0 \quad \Rightarrow \quad \iota_\xi \Omega = -d_V G_\xi}$$

G_ξ is the generator associated with this canonical transformation.

- Under Lorentz rotations Y^A

$$d_V(\iota_Y \Omega) = 0 \quad \Rightarrow \quad \iota_Y \Omega = -d_V \mathcal{G}_{Y^A}$$

- Under Lorentz boosts b (in spherical coordinates)

$$d_V\,(\iota_b\Omega) \;=\; -\int d\theta d\varphi \sqrt{\bar{g}}\Big[b\, d_V\bar{h} d_V\left(\bar{h}_{rr} + \bar{D}_A\bar{\lambda}^A\right) - \bar{D}_A b\, d_V\bar{\lambda}^A d_V\bar{h} + b\bar{D}^A d_V\bar{\lambda}^B d_V\bar{h}_{AB}\Big)\Big] \neq 0$$

How to make the symplectic form invariant under boosts?

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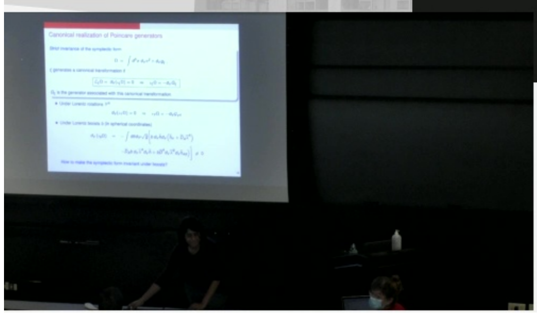

Non-integrability of the boost generators: Resolution

- Perform a gauge transformation

$$\epsilon_{(b)} \equiv bF, \quad \text{F is field-dependent}$$

$$d_V\left(\iota_\xi \Omega\right) + d_V\left(\iota_\epsilon \Omega\right) = -\int d\theta d\varphi \sqrt{\bar{g}} \Big[2b\left(d_V F + \frac{1}{2} d_V \bar{h}\right) d_V\left(\bar{h}_{rr} + \bar{D}_A \bar{\lambda}^A\right) - \bar{D}_A b\, d_V \bar{\lambda}^A d_V \bar{h} + b \bar{D}^A d_V \bar{\lambda}^B d_V \bar{h}_{AB} \Big]$$

$$\boxed{\text{Set } F = -\frac{1}{2}\bar{h}}$$

- Fourth ingredient of asymptotic conditions (Recall: $h_{rA} = \bar{\lambda}_A + \mathcal{O}(r^{-1})$)

$$\int d\theta d\varphi \sqrt{\bar{g}} \left[\bar{D}_A b\, d_V \bar{\lambda}^A d_V \bar{h} - b \bar{D}^A d_V \bar{\lambda}^B d_V \bar{h}_{AB} \right]$$

$$\boxed{\text{Set } \bar{\lambda}_A = 0}$$

$$d_V(\iota_b \Omega) = 0 \quad \Rightarrow \quad \iota_b \Omega = -d_V G_b$$

No need for a surface field in order to define canonical generators: more on this later

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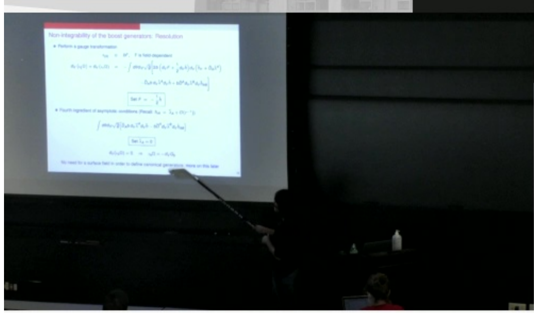

Are there more symmetries?

Yes, symmetries of the form $x^\mu \to x^\mu + \epsilon^\mu(x)$ with parameters

$$\epsilon^0(\theta,\phi) \sim T^{\text{even}}, \quad \epsilon^i(\theta,\phi) \sim \partial_i W^{\text{odd}} \quad \to \quad \text{one single arbitrary function } \boxed{\mathcal{T}(\theta,\phi)}$$

- Time component of gauge parameter

$$\epsilon^0 = \bar{\epsilon}^0 + \mathcal{O}(r^{-1}), \quad \bar{\epsilon}^0 = T_0 + T_2 + T_4 + T_6 + \cdots$$

- Spatial components

$$\epsilon^k = \bar{\epsilon}^k + \mathcal{O}(r^{-1}), \quad \bar{\epsilon}^k = D^k(rW),$$

$$W^r = W, \quad W^A = \frac{1}{r}\bar{D}^A W, \quad W = W_1 + W_3 + W_5 + W_7 + \cdots$$

Where are the spacetime translations? Expand T and W in spherical harmonics

$$\mathcal{T}(\theta,\phi) = \underbrace{T_{0,0}Y_{0,0}}_{\text{time translations } a^0} + \underbrace{\sum_{m=-1}^{1} W_{1,m}Y_{1,m}}_{\text{spatial translations } a^i} + \underbrace{\frac{1}{4}\sum_{m=-2}^{2} T_{2,m}Y_{2,m} + \cdots}_{\text{supertranslations}}$$

[Henneaux-Troessaert 2018; Henneaux-Fuentealba-SM-Matulich-Troessaert 2020]

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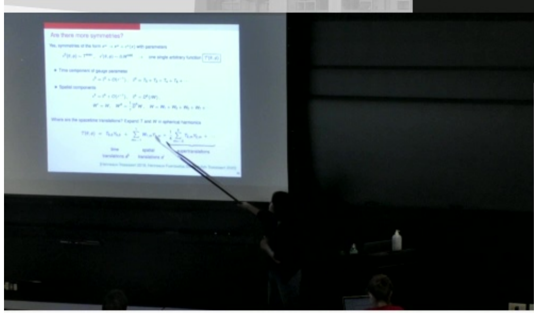

Asymptotic symmetries at spatial infinity

- Canonical generator for BMS

$$G_{\xi,\xi^i} = \int d^3x \left(\xi\mathcal{H} + \xi^i\mathcal{H}_i\right) + Q_{\xi,\xi^i}\,,$$

$$Q_{\xi,\xi^i} = \int d\theta d\varphi \Big\{ b\left[\sqrt{\bar g}\left(-\frac{1}{2}\bar h \bar h_{rr} + \frac{1}{4}\bar h^2 - \frac{3}{4}\bar h_{AB}\bar h^{AB}\right) + \frac{2}{\sqrt{\bar g}}\bar\pi^r_A\bar\pi^{rA}\right] + 2Y_A\bar\pi^{rB}\bar h^A_B$$
$$+2\sqrt{\bar g}T\underbrace{\bar h_{rr}}_{\text{even}} + 2W\underbrace{(\bar\pi^{rr} - \bar\pi^A_A)}_{\text{odd}}\Big\}$$

(T_{odd}, W_{even}) $\rightarrow$ proper gauge transformations

(T_{even}, W_{odd}) $\rightarrow$ improper gauge transformations : Supertranslations

- Poisson bracket algebra [Henneaux-Troessaert '18]

$$\left\{G_{\xi_1,\xi_1^i}, G_{\xi_2,\xi_2^i}\right\} = \hat G_{\hat\xi,\hat\xi^i}\,,$$

Asymptotic symmetry algebra of gravity at spatial infinity

$$BMS_4 = SO(3,1) \ltimes \text{supertranslations}$$

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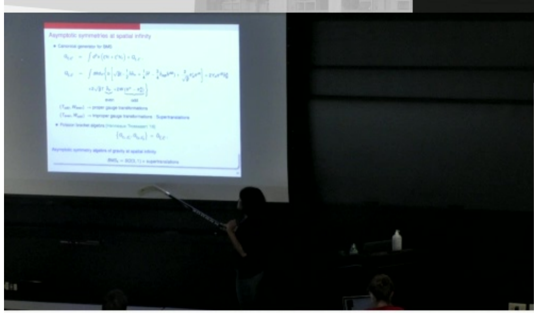

Asymptotic symmetry algebra

- Poisson bracket algebra [Henneaux-Troessaert '18]

$$\left\{ G_{\xi_1,\xi_1^i}, G_{\xi_2,\xi_2^i} \right\} = \hat{G}_{\hat{\xi},\hat{\xi}^i} \,,$$

with the parameters

$$\begin{aligned}
\hat{Y}^A &= Y_1^B \partial_B Y_2^A + \overline{\gamma}^{AB} b_1 \partial_B b_2 - (1 \leftrightarrow 2)\,, \\
\hat{b} &= Y_1^B \partial_B b_2 - (1 \leftrightarrow 2)\,, \\
\hat{T} &= Y_1^A \partial_A T_2 - 3 b_1 W_2 - \partial_A b_1 \overline{D}^A - 2W - b_1 \overline{D}_A \overline{D}^A W_2 - (1 \leftrightarrow 2)\,, \\
\hat{W} &= Y_1^A \partial_A W_2 - b_1 T_2 - (1 \leftrightarrow 2)
\end{aligned}$$

- BMS as the infinite-dimensional enhancement of Poincaré, $G_{\xi,\xi^i} = G_{Lorentz} + G_{T,W}$

$$\left\{ G_{Lorentz}, G_{Lorentz} \right\} = G_{Lorentz} \qquad \left\{ G_{Lorentz}, G_{Lorentz} \right\} = G_{Lorentz}$$

$$\left\{ G_{Lorentz}, G_{a,a^i} \right\} = \hat{G}_{(\hat{a},\hat{a}^i)} \qquad \boxed{G_{a,a^i} \to G_{T,W}} \qquad \left\{ G_{Lorentz}, G_{T,W} \right\} = \hat{G}_{\hat{T},\hat{W}}$$

$$\left\{ G_{a,a^i}, G_{a,a^i} \right\} = 0 \qquad \left\{ G_{T,W}, G_{T,W} \right\} = 0$$

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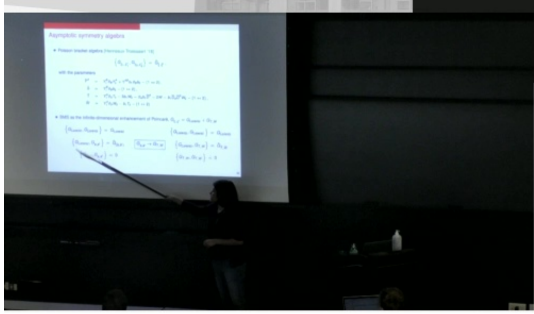
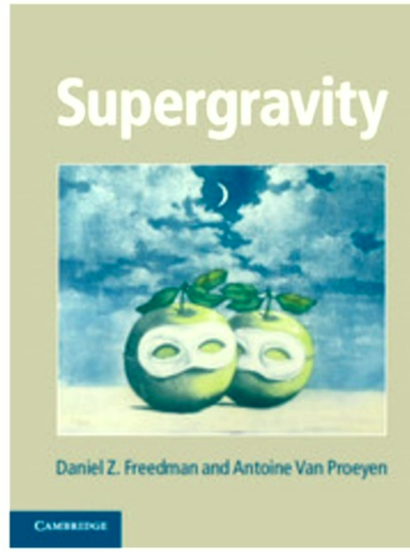


Part II


Supergravity
Daniel Z. Freedman and Antoine Van Proeyen
CAMBRIDGE

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$\mathcal{N} = 1$ supergravity: Hamiltonian formulation

- $\mathcal{N} = 1$ supergravity: Field content $(2, 3/2)$

  We use the vierbein (tetrad) formalism to couple spinors to spin-2

$$g_{ij} = e_i^a e_{aj} \qquad \pi^{ij} = \frac{1}{2} e^{a(i} \pi_a^{j)}\,, \qquad a, b, \cdots = \text{local Lorentz indices}$$

Hamiltonian action of supergravity [Deser-Kay-Stelle '77, Pilati '77]

$$S = \int dt \left[\int d^3x \left(\pi_a^i \dot{e}_i^a + \frac{i}{2} \sqrt{g} \psi_k^T \gamma^{km} \dot{\psi}_m \right) - H \right]$$

Hamiltonian

$$H = \int d^3x \left(N\mathcal{H} + N^i \mathcal{H}_i + i\psi_0^T \mathcal{S} + \frac{1}{2} \lambda_{ab} \mathcal{J}^{ab} \right) + B_\infty^H\,.$$

Fermionic constraint $\mathcal{S}$ generate fermionic gauge symmetry (SUSY)

- Phase space variables: (e_i^a, π_a^i, ψ_i)

  Symplectic form

$$\Omega = \int d^3x \left(d_V \pi_a^i d_V e_i^a + \frac{i}{2} \sqrt{g} d_V \psi_k^T \gamma^{km} d_V \psi_m + d_V \pi_\chi d_V \chi \right)$$

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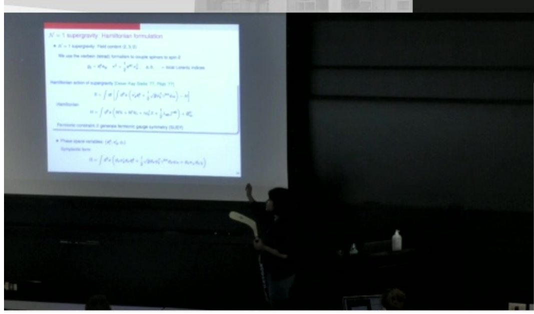

$\mathcal{N} = 1$ supergravity: Attempt I

Boundary conditions I

- Spin-2 fields

$$e^a_i = \delta^a_i + \frac{1}{2}\delta^{aj}h_{ij} + \mathcal{O}\left(\frac{1}{r^2}\right), \qquad h_{ij} = \frac{\overline{h}_{ij}(\theta,\phi)}{r} + \mathcal{O}(r^{-2}),$$

$$\pi^i_a = 2\delta_{aj}\pi^{ij} + \mathcal{O}\left(\frac{1}{r^2}\right), \qquad \pi^{ij} = \frac{\overline{\pi}^{ij}(\theta,\phi)}{r^2} + \mathcal{O}(r^{-3}),$$

"gauge-twisted" parity conditions:

$$\overline{h}_{ij} = (\overline{h}_{ij})^{\text{even}} + (\nabla_i U_j + \nabla_j U_i), \qquad \overline{\pi}^{ij} = (\overline{\pi}^{ij})^{\text{odd}} + (\nabla^i\nabla^j V - \overline{\delta}^{ij}\triangle V),$$

- Spin-3/2 field

$$\psi_k = \frac{\mu_k(\theta,\phi)}{r^2} + \mathcal{O}\left(r^{-3}\right), \quad \text{no parity condition}$$

- Supersymmetry transformations

$$\delta_\epsilon g_{ij} = i\epsilon\gamma_{(i}\psi_{j)} + \cdots, \quad \delta_\epsilon\psi_i = -\nabla_i\epsilon + \cdots$$

$$\epsilon = \epsilon_0 + \mathcal{O}(\frac{1}{r})$$

$\rightarrow$ Fermionic gauge symmetry of the gravitino

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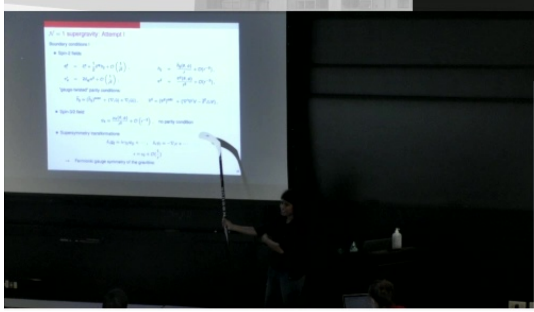

$\mathcal{N} = 1$ supergravity: Attempt I

Canonical generator

$$G_{\xi,\xi^i,\epsilon} = \int d^3x \epsilon^T(\xi\mathcal{H} + \xi^i\mathcal{H}_i + i\mathcal{S}) + Q_{\xi,\xi^i,\epsilon}$$

$$Q_\epsilon = -i\epsilon_0^T \oint d\theta d\phi \sqrt{\bar{g}}\gamma_r\gamma^A\bar{\mu}_A$$

Q_{ξ,ξ^i} BMS charges: same as in Part I

super-BMS_4 algebra: [Gibbons-Awadha-Shaw, 1986]

$$\left\{G_{\xi_1,\xi_1^i}, G_{\xi_2,\xi_2^i}\right\}_{PB} = \hat{G}_{\hat{\xi},\hat{\xi}^i}, \qquad \text{BMS}$$

$$\left\{G_{\epsilon_1}, G_{\xi_2,\xi_2^i}\right\}_{PB} = \hat{G}_{\hat{\epsilon}} \qquad \text{Spinors under Lorentz}$$

$$\{G_{\epsilon_1}, G_{\epsilon_2}\}_{PB} = \hat{G}_{a^0,a^i} \qquad \text{Square root of 4 translations}$$

No square roots of supertranslations with these boundary conditions [Henneaux-Matulich-Neogi, 2020]

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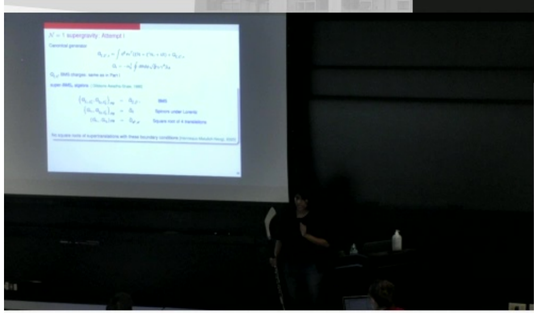

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Meanwhile at null infinity:

Some super-BMS alegbras with infinite-dimensional fermionic symmetries [Avery-Schwab 2015, Fotopoulos-Steiberger-Taylor-Zhu 2020, Pano-Pasterski-Puhm 2021]

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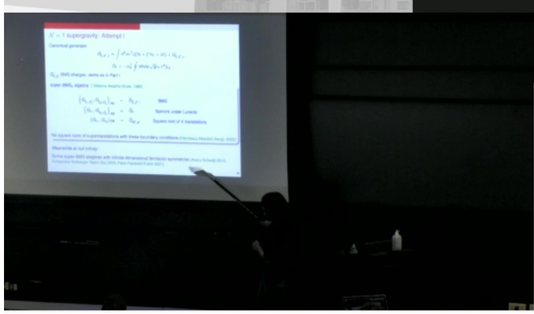

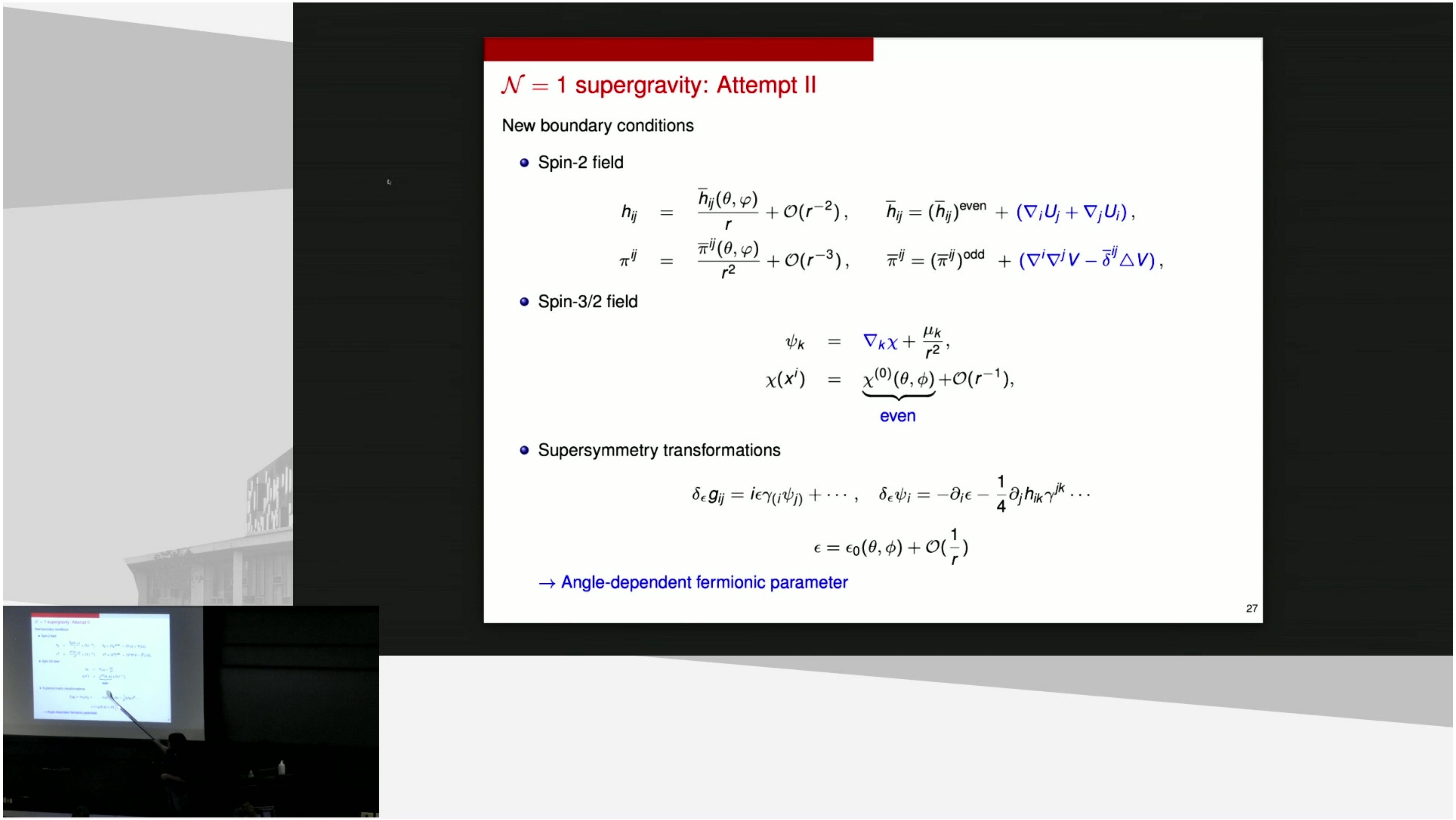
$\mathcal{N} = 1$ supergravity: Attempt II
New boundary conditions
Spin-2 field
$h_{ij} = \frac{\bar{h}_{ij}(\theta,\varphi)}{r} + \mathcal{O}(r^{-2})\,, \quad \bar{h}_{ij} = (\bar{h}_{ij})^{\text{even}} + (\nabla_i U_j + \nabla_j U_i)\,,$
$\pi^{ij} = \frac{\bar{\pi}^{ij}(\theta,\varphi)}{r^2} + \mathcal{O}(r^{-3})\,, \quad \bar{\pi}^{ij} = (\bar{\pi}^{ij})^{\text{odd}} + (\nabla^i\nabla^j V - \bar{\delta}^{ij}\triangle V)\,,$
Spin-3/2 field
$\psi_k = \nabla_k\chi + \frac{\mu_k}{r^2}\,,$
$\chi(x^i) = \underbrace{\chi^{(0)}(\theta,\phi)}_{\text{even}} + \mathcal{O}(r^{-1})\,,$
Supersymmetry transformations
$\delta_\epsilon g_{ij} = i\epsilon\gamma_{(i}\psi_{j)} + \cdots\,, \quad \delta_\epsilon\psi_i = -\partial_i\epsilon - \frac{1}{4}\partial_j h_{ik}\gamma^{jk}\cdots$
$\epsilon = \epsilon_0(\theta,\phi) + \mathcal{O}(\frac{1}{r})$
→ Angle-dependent fermionic parameter
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Asymptotic symmetries in the Hamiltonian formulation

Hamiltonian action with standard boundary conditions

↓

Relax the boundary conditions with a “gauge-twist”

↓

Ensure finiteness of the kinetic term and symplectic form

↓

Check that all Poincaré charges are canonical

↓

Define canonical generators and compute the asymptotic symmetry algebra

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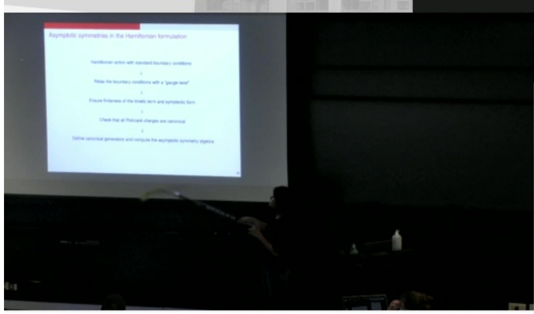

Canonical realization of asymptotic symmetries

- Spacetime translations and Lorentz rotations are canonical ✓
- Under Lorentz boosts b (in spherical coordinates)

$$\delta_b \Omega \;=\; d_V\,(\iota_b \Omega) \;=\; \oint (\chi^{(0)} \text{ terms}) \;\neq\; 0$$

How to make the symplectic form invariant under boosts?

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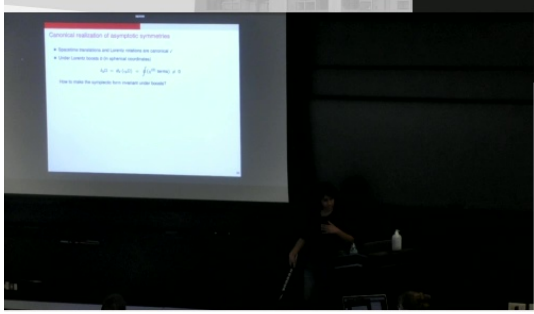

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$$\delta_b \Omega \; = \; d_V\left(\iota_b \Omega\right) \; = \; \oint (\chi^{(0)} \text{ terms}) \; \neq \; 0$$

How to make the symplectic form invariant under boosts?

Resolution: Introduce a boundary field ω

Modifies the action (and symplectic form) by a surface term

$$S' = S + \frac{i}{2} \oint d\theta d\varphi \omega^T \dot{\chi}^{(0)}.$$

Boost transformations of ω cancels the $\chi^{(0)}$ terms in $d_V\left(\iota_b \Omega\right)$

- Extended phase space: $(e^a_i, \pi^i_a, \chi, \mu_i, \omega)$
- The new field comes with its own gauge symmetry,

$$\omega \to \omega + \sigma$$

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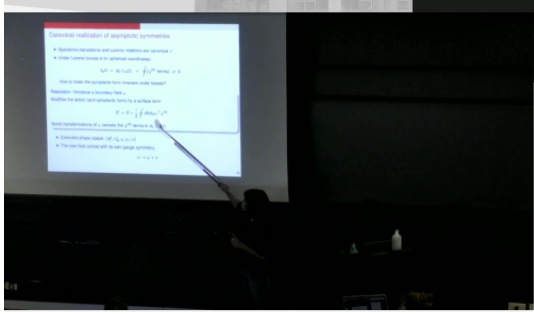

Canonical realization of asymptotic symmetries

Canonical generator for gauge symmetries

$$G_{\xi,\xi^i,\epsilon} = \int d^3x(\xi\mathcal{H} + \xi^i\mathcal{H}_i + i\epsilon^T\mathcal{S}) + Q_{\xi,\xi^i} + Q_\epsilon \,,$$

- Q_{ξ,ξ^i} BMS charges: Same as before
- Q_ϵ Supersymmetry charge

$$\begin{aligned} Q_\epsilon \quad &= -i\oint d^2x\sqrt{\overline{g}}\epsilon_0^T\gamma_r\overline{\gamma}^A\overline{\mu}_A + \tfrac{i}{4}\oint d^2x\sqrt{\overline{g}}\,(\epsilon-\epsilon_0)^T\,\overbrace{\chi^{(0)}\overline{h}_{rr}} - \tfrac{i}{2}\oint d^2x\sqrt{\overline{g}}\,(\epsilon-\epsilon_0)^T\,\omega \\ &- \tfrac{i}{8}\oint d^2x\sqrt{\overline{g}}\,(\epsilon-\epsilon_0)^T\,\gamma_r\overline{\gamma}^A\,\underbrace{\chi^{(0)}\partial_A\overline{h}_{rr}} - \tfrac{i}{2}\oint d^2x\,(\epsilon-\epsilon_0)^T\,\gamma_0\gamma_r\chi^{(0)}\left(\overline{\pi}^{rr} - \overline{\pi}^A_A\right) \end{aligned}$$

Non-linear terms in the supersymmetry charge

- “Extra” gauge symmetry

$$G_\sigma = \frac{i}{2}\oint d^2x\sqrt{\overline{g}}\sigma^T\chi^{(0)}$$

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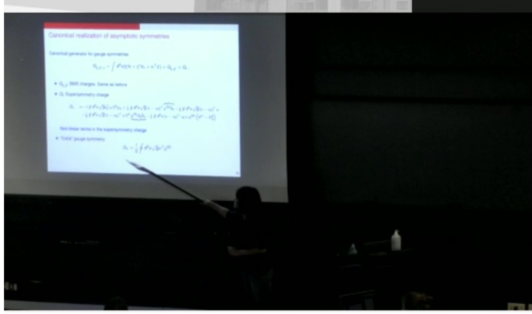

$SBMS_4$ algebra at spatial infinity

- $SBMS_4$ algebra for $\mathcal{N} = 1$ supergravity

$$
\begin{aligned}
\left\{G_{\xi_1,\xi_1^i}, G_{\xi_2,\xi_2^i}\right\} &= \hat{G}_{\hat{\xi},\hat{\xi}^i}\,, && \text{BMS} \\
\{G_\epsilon, G_{\epsilon'}\} &= G_{\hat{T},\hat{W}} && \text{supersymmetry} \\
\{G_\epsilon, Q_\sigma\} &= -\frac{i}{2}\oint d^2x\sqrt{g}\,(\epsilon-\epsilon_0)^T\,\sigma && \text{central charge} \\
\{Q_\sigma, Q_{\sigma'}\} &= 0
\end{aligned}
$$

- Supertranslation parameters $(\hat{T}, \hat{W})$

$$
\begin{aligned}
\hat{T} &= -\frac{i}{4}\epsilon_0^T\epsilon_0' - \frac{i}{4}(\epsilon-\epsilon_0)^T(\epsilon'-\epsilon_0')\,, \\
\hat{W} &= \frac{i}{4}\epsilon^T\gamma_0\gamma_r\epsilon' - \frac{i}{4}\left(\epsilon^T\gamma_0\gamma_r\epsilon_0' - \epsilon'^T\gamma_0\gamma_r\epsilon_0\right)
\end{aligned}
$$

"Local supersymmetry as square root of supertranslations"

- Key features of the algebra

a) Extra infinite-dimensional fermionic symmetry generated by Q_σ

b) Non-linear terms in the generators $\rightarrow$ Jacobi identity with boosts non-trivial

[Henneaux-Fuentealba-SM-Matulich-Neogi, 2021]

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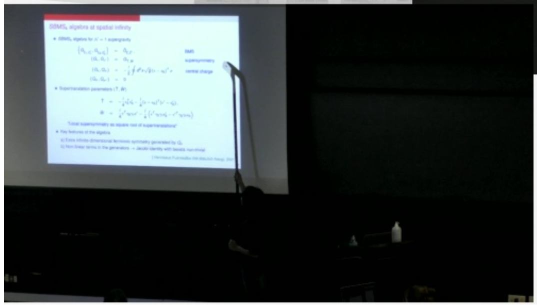

When we need a boundary d.o.f.?

- Spin-1

  → Electromagnetism: Poincaré $\oplus$ angle-dependent $U(1)$
  → Einstein-Maxwell theory : $BMS_4 \oplus U(1)$
  → Electromagnetism in higher dimensions
  → Electromagnetism in the duality-invariant formulation

  Boundary d.o.f. **always** required

- Spin-2

  → Einstein's theory in four dimensions: BMS_4
  → Pauli-Fierz: BMS_4 but proper vs. improper gauge transformations more transparent
  → Einstein's theory in five dimensions: BMS_5

  Boundary d.o.f. **not** required

- Supergravity

  → free spin-$\frac{3}{2}$: Poincaré $\oplus$ angle-dependent fermionic gauge symmetry
  → $\mathcal{N} = 1$ Supergravity: Super-BMS_4

  Boundary d.o.f. **always** required

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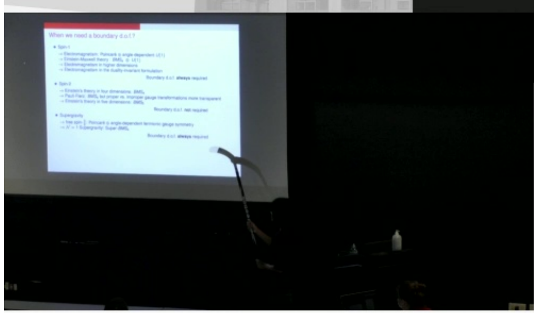

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  Boundary d.o.f. **not** required

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  → $\mathcal{N} = 1$ Supergravity: Super-BMS_4

  Boundary d.o.f. **always** required

A unifying framework to explain this?

→ maybe BV-BFV formalism? [c.f. Kasia Rejzner's talk in "Quantum gravity around the corner"]
→ Connections with covariant phase space, corner modes at null infinity, ...

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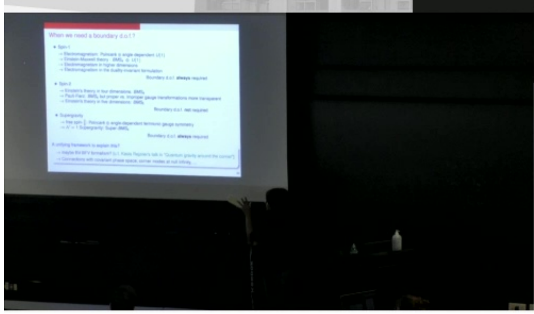

Asymptotic symmetries at spatial infinity

A lot remains to be understood!

- Connections to null infinity results, celestial holography, scattering amplitudes, ...

  Parity conditions $\longleftrightarrow$ Anti-podal matching conditions

  Superrotations, $\mathit{Diff}(\mathbb{S}^2)$ and other extensions at spatial infinity

- Group-theoretic interpretation of the different asymptotic algebras

  a) Non-linear structure of the super-BMS algebra

  See also BMS_5 at spatial infinity [ArXiv:2111.09664]

  b) Infinite-dimensional IRREPs of Lorentz group

  [Harish-Chandra 1947, Naimark 1962, Gel'fand-Minlos-Shapiro 1963]

- Extended supergravity theories, duality-invariant formulation of GR, ...

  super-BMS algebras with duality or exceptional symmetries (e.g. E_7 in $\mathcal{N} = 8$ supergravity)

  $\rightarrow$ Infrared triangle for supergravity?

  $\rightarrow$ Double soft limit of scalars [Arkani-Hamed, Cachazo, Kaplan 2008]

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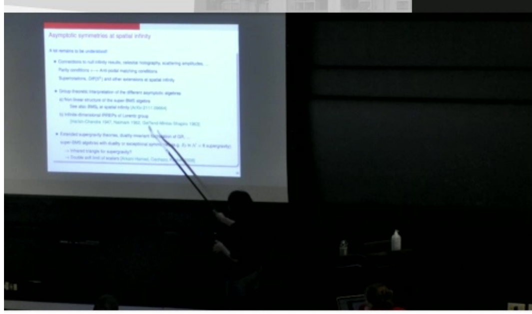


Thank you for your attention!
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