Title: Dualizability in higher Morita categories

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Series: Mathematical Physics

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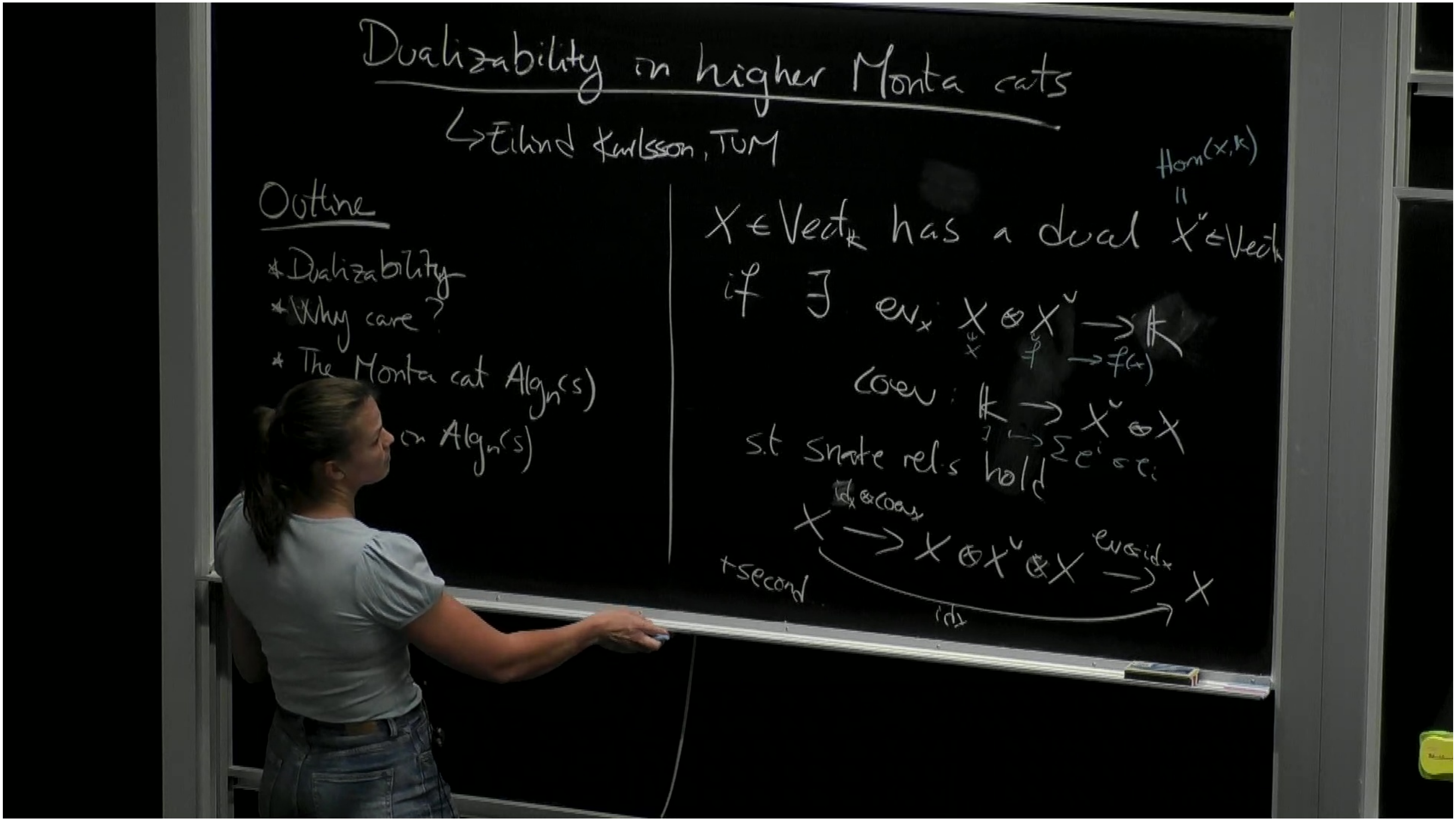
Abstract: The Morita 2-category has as objects associative algebras, 1-morphisms are bimodules and 2-morphisms are given by bimodule homomorphisms. Equivalent objects in this category are exactly Morita equivalent algebras. A vast generalisation of this as a higher category is the so-called higher Morita category, denoted Alg_n. It has two constructions, one due to Haugseng, and one due to Scheimbauer which uses (constructible) factorization algebras. In the latter, Gwilliam-Scheimbauer has proven that every object of Alg_n is n-dualizable. Hence, by the Cobordism Hypothesis, every object gives rise to an n-dimensional (fully extended framed) topological field theory. A natural question to ask is "Which objects of Alg_n are also (n+1)-dualizable?". This talk is on work in progress (for n=2) to prove a conjecture due to Lurie answering this question.

Zoom link: https://pitp.zoom.us/j/98595913913?pwd=Vlo1aWtXVlZBVjYxNFlTM2VZY2s3Zz09


Dualizability in higher Morita cats
↳ Eilind Karlsson, TUM
Outline
* Dualizability
* Why care?
* The Morita cat Alg_n(S)
in Alg_n(S)
X ∈ Vect_k has a dual X^∨ ∈ Vect_k
Hom(X,k)
if ∃ ev_X : X ⊗ X^∨ → k
x ⊗ f ↦ f(x)
coev : k → X^∨ ⊗ X
1 ↦ Σ e^i ⊗ e_i
s.t. snake rel.s hold
X → X ⊗ X^∨ ⊗ X → X
id_X ⊗ coev_X
ev_X ⊗ id_X
id_X
+ second

X 1-dualizable

$\Updownarrow$

X fin. dim.

$\mathcal{C}, \mathcal{D} \in$ ~~Cat~~ $\mathcal{B}$ bicat

1-morp

$F : \mathcal{C} \to \mathcal{D}$ has a left adj $F^L : \mathcal{D} \to \mathcal{C}$ if

$\exists$ nat. ~~transf~~ 2-morp

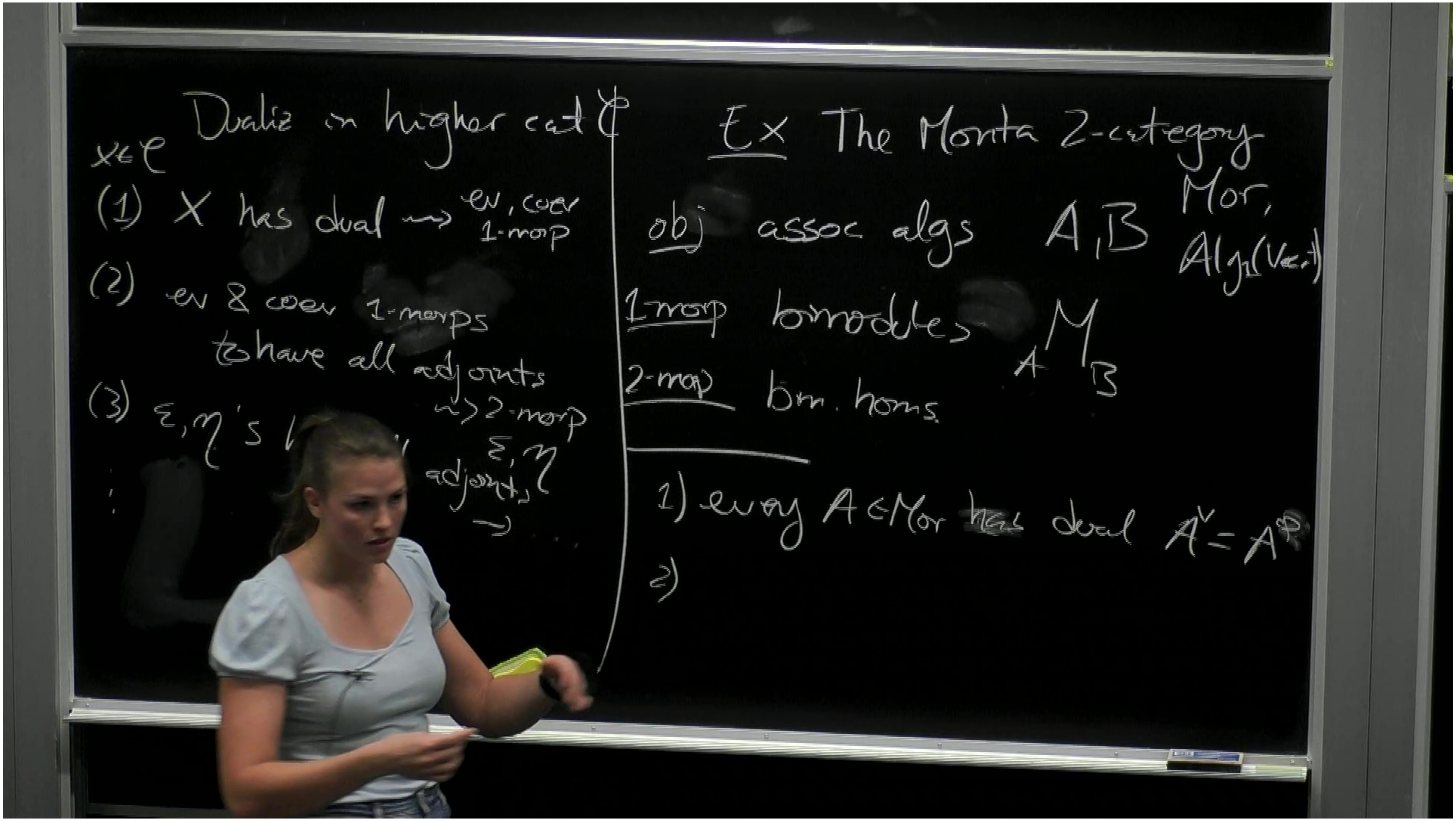
$\varepsilon : F^L \circ F \to id_{\mathcal{C}}$ count

$\eta : id_{\mathcal{D}} \to F \circ F^L$ unit

s.t.

$$\begin{tikzcd} F \arrow[r, "\eta F"] \arrow[rr, bend right, "id_F"'] & F \circ F^L \circ F \arrow[r, "F\varepsilon"] & F \end{tikzcd}$$

+ second..


Dualiz in higher cat $\mathcal{C}$
$x \in \mathcal{C}$
(1) X has dual ⇝ ev, coev 1-morp
(2) ev & coev 1-morps to have all adjoints ⇝ 2-morp
(3) ε, η's
ε, η adjoints ⇝
Ex The Morita 2-category
Mor, $\mathrm{Alg}_1(\mathrm{Vect})$
obj assoc algs A, B
1-morp bimodules ${}_A M_B$
2-morp bim. homs
1) every $A \in$ Mor has dual $A^\vee = A^{op}$
2)

coev 1-morps
have all adjoints
$\rightsquigarrow$ 2-morp
have all adjoints ε, η
$\rightarrow$

<u>1-morp</u> bimodules ${}_A M_B$

<u>2-morp</u> bim. homs

1) every $A \in \mathrm{Mor}$ has dual $A^\vee = A^{op}$

${}_A A_A \rightsquigarrow ev = {}_{A \otimes A^{op}} A_{k}$, $coev = {}_{k} A_{A^{op} \otimes A}$

2) A 2-duali $\Leftrightarrow$ A is seperable, fin. gen & proj over k

Why care?

(1) picks out interesting structures

(2) Cobord hyp [Lur, Schommer-Pries, Baez-Dolan, AF, ...]

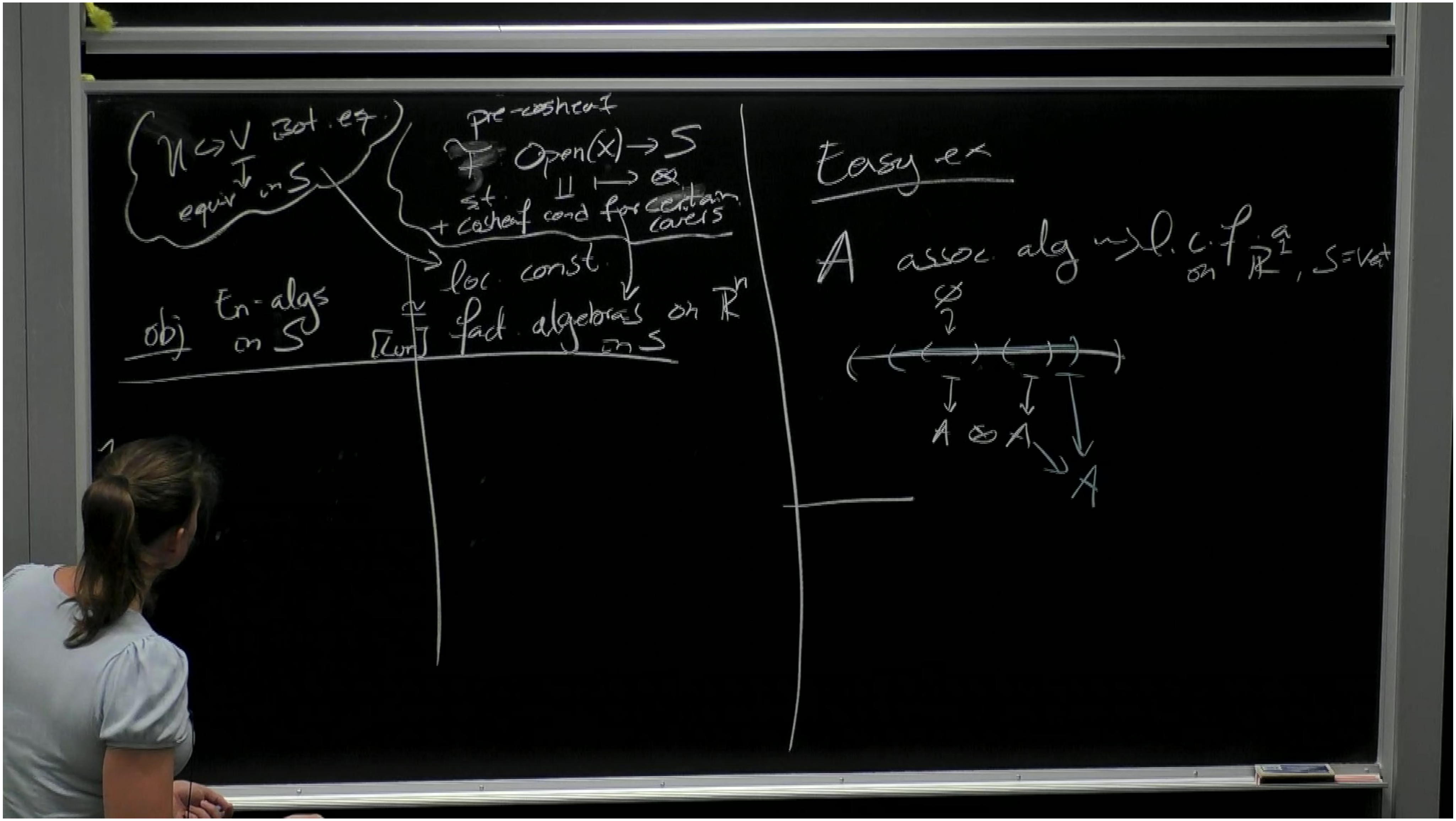
$$\underbrace{dTFT}_{\text{fully ext framed } d\text{-dim TFTs}} = \mathrm{Fun}^{\otimes}(\mathrm{Bord}_d^{fr}, \mathcal{C}) \simeq \mathcal{C}^{\boxed{d\text{-dualiz.}}}$$

$\Rightarrow$ d-dualiz obj gives rise to d-dim TFT.

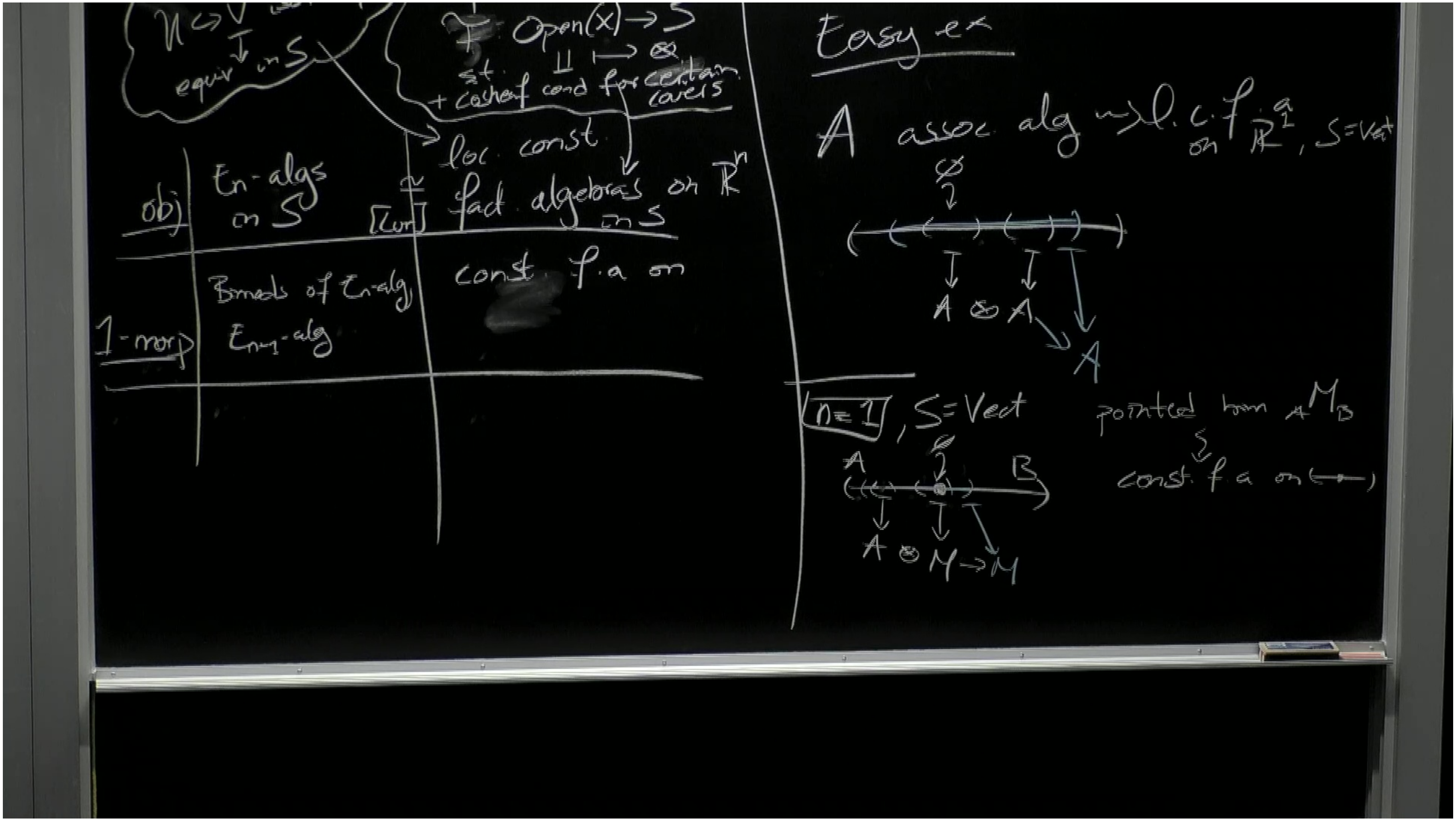
"Good target" = The higher Morita cat $\mathrm{Alg}_n(S)$

Scheimbauer
↳ geometric
↳ pointed

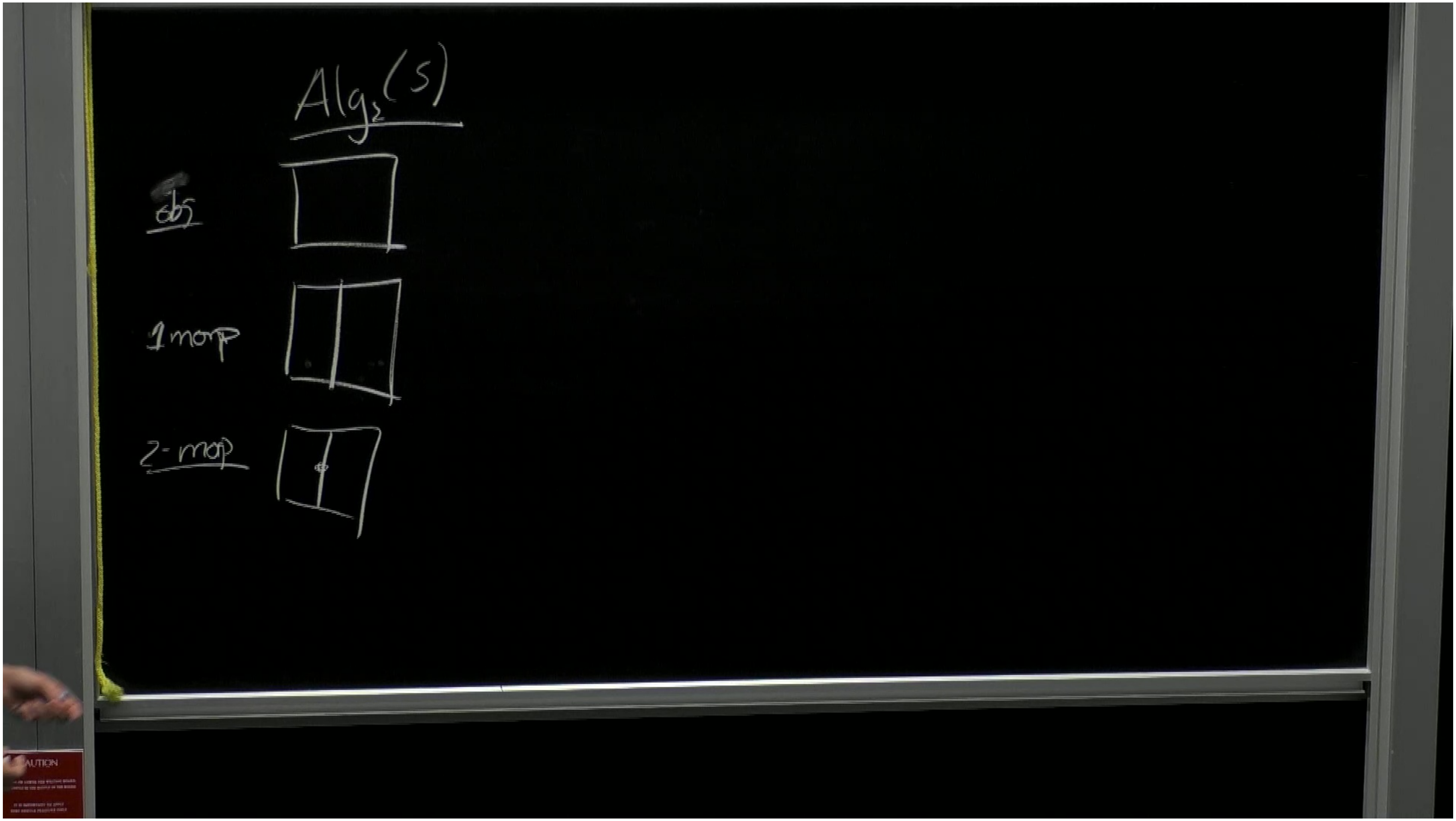
Haugseng
↳ non-symm ∞-operads
↳ unpointed



pre-cosheaf
F: Open(X) → S
st. ⊔ ⟼ ⊗
+ cosheaf cond for certain covers
loc. const
obj
En-algs on S
≃
[Lurie]
fact. algebras on R^n on S
Easy ex
A assoc. alg ⇝ l.c.f. on R^1, S=Vect
∅
A ⊗ A
A



equiv in S
Open(X) → S
st.
+ cosheaf cond for certain covers
loc. const
obj
En-algs on S
[Lur]
fact. algebras on R^n in S
Bimods of En-alg
const. f.a on
1-morp
En-1-alg
Easy ex
A assoc alg
S=Vect
A ⊗ A
A
n=1, S=Vect
pointed bim
const. f.a on
A
B
A ⊗ M → M

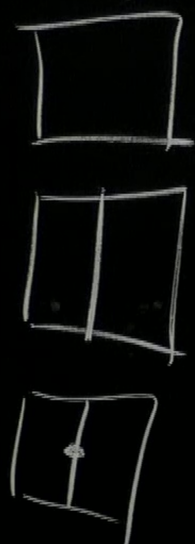

$Alg_2(S)$
obs
1 morp
2-morp

$Alg_2(S)$

obs

1 morp

2-morp

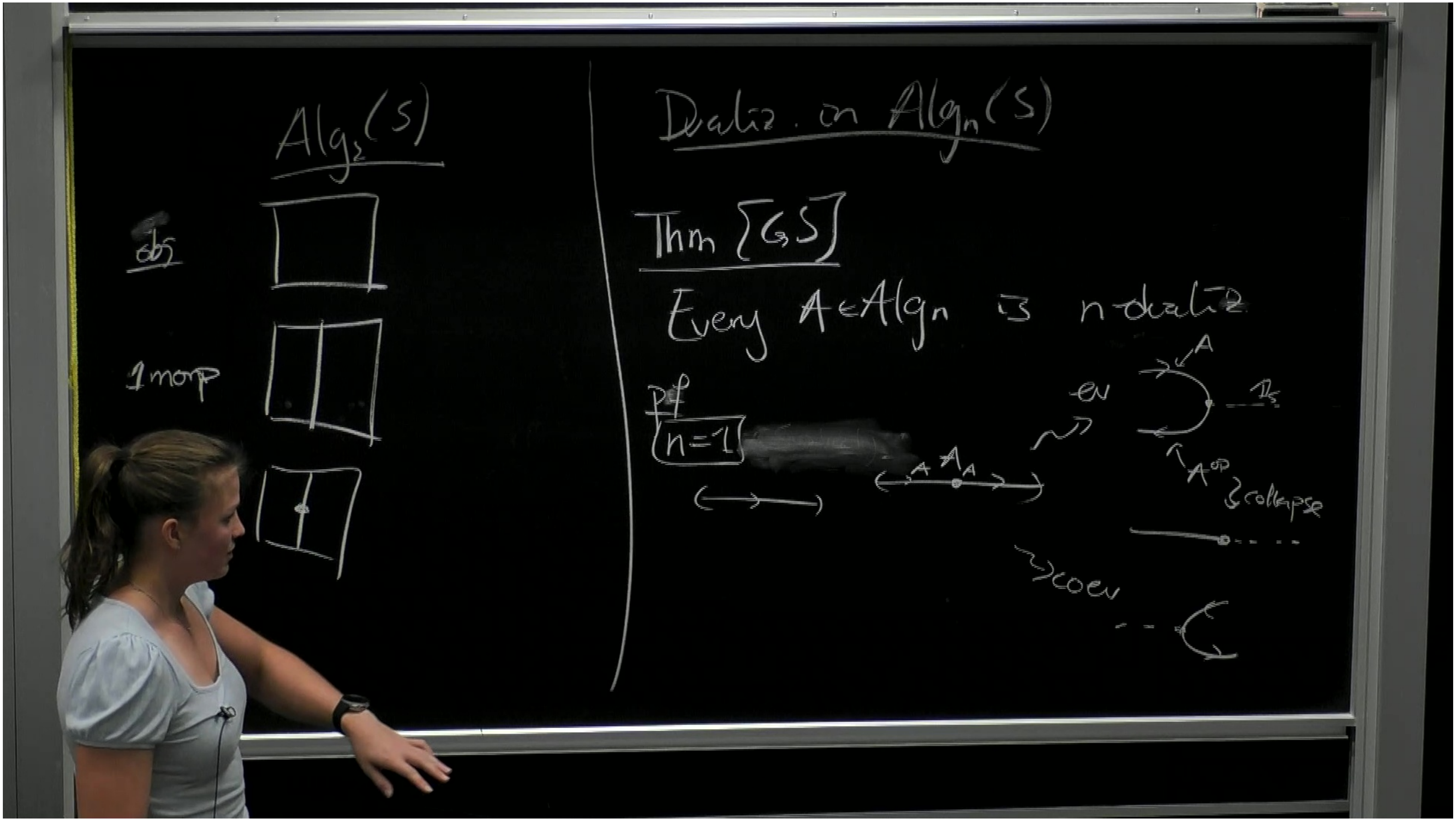


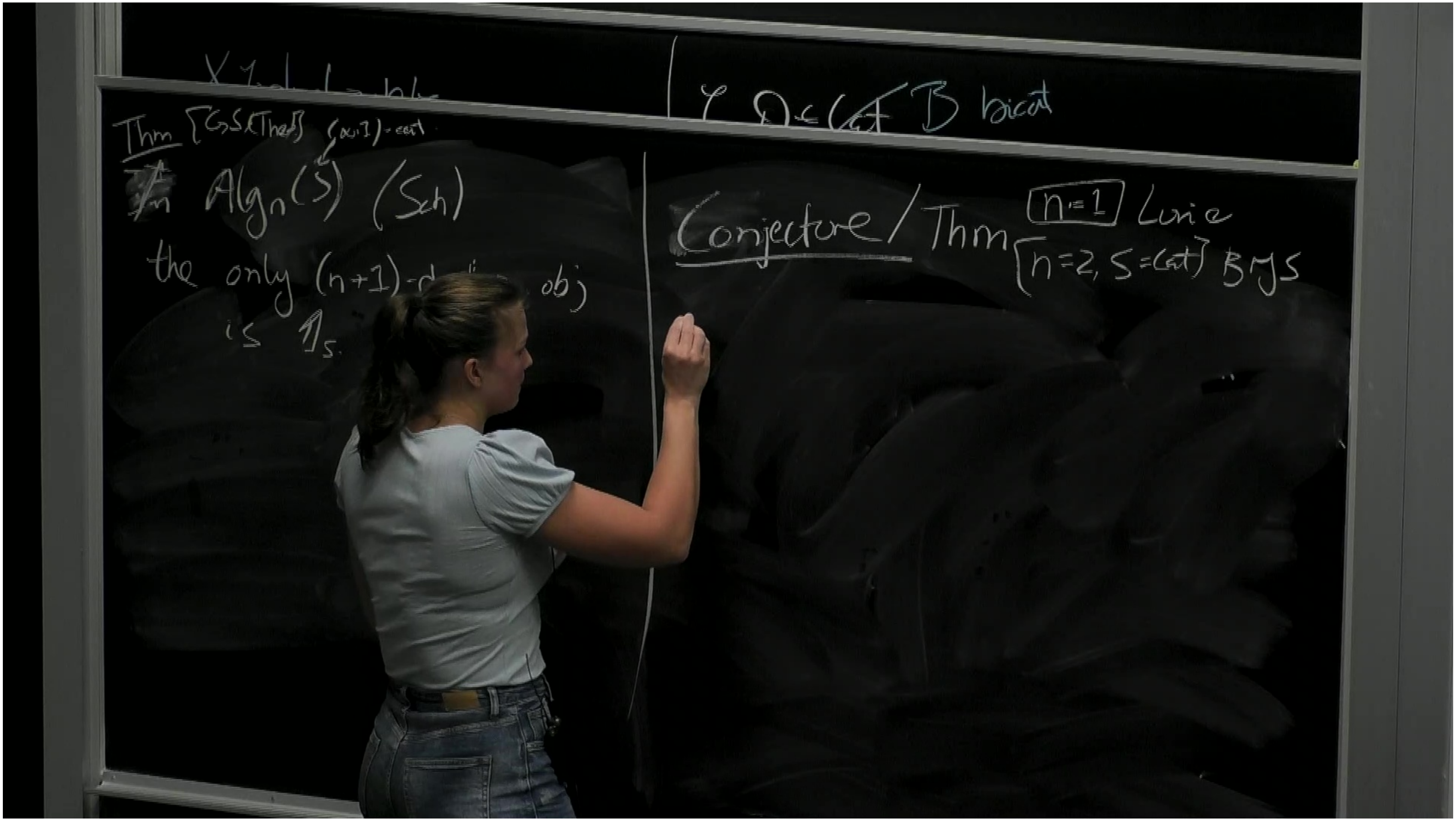
Dualiz. on $Alg_n(S)$

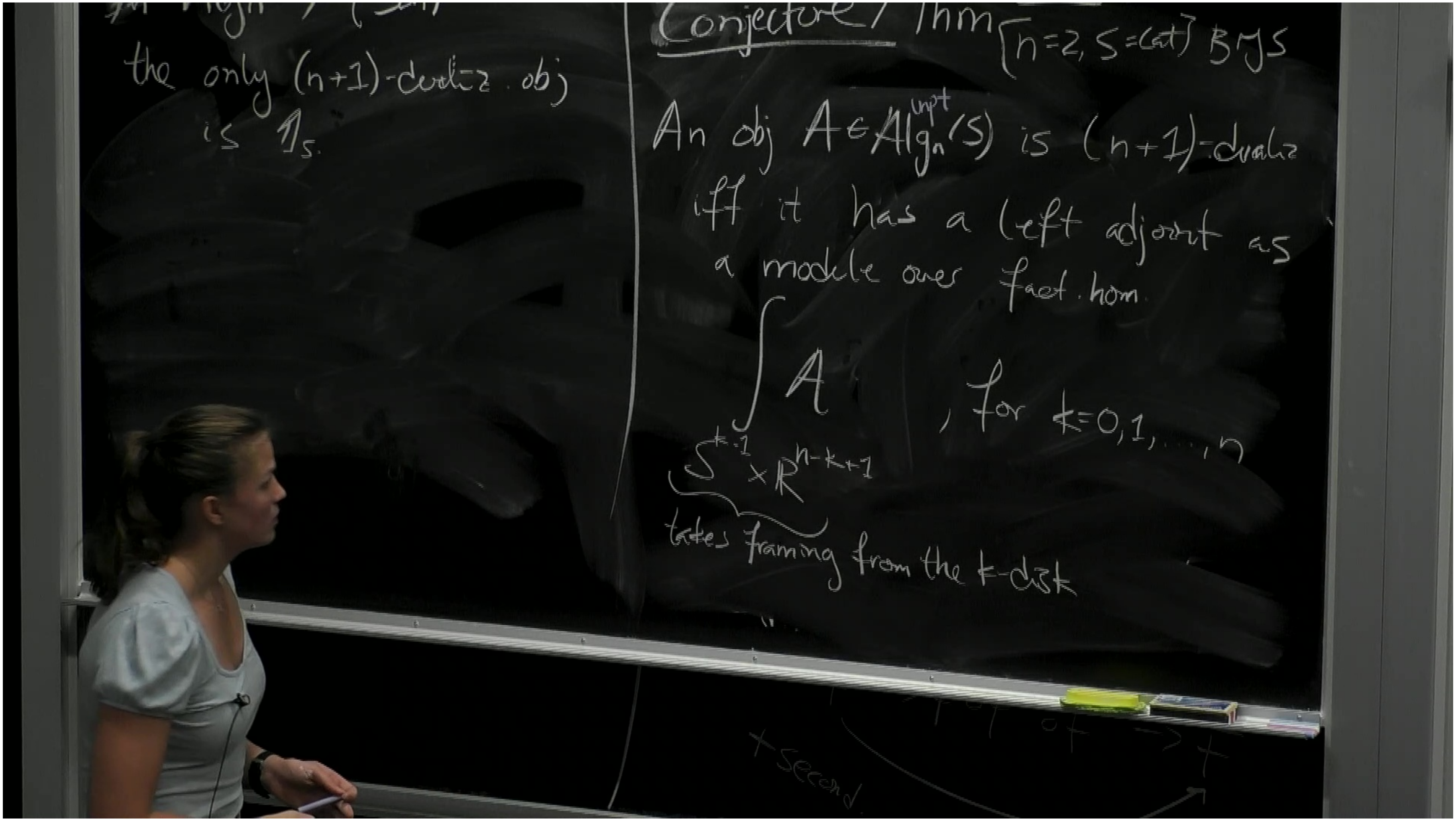
Thm [GS]

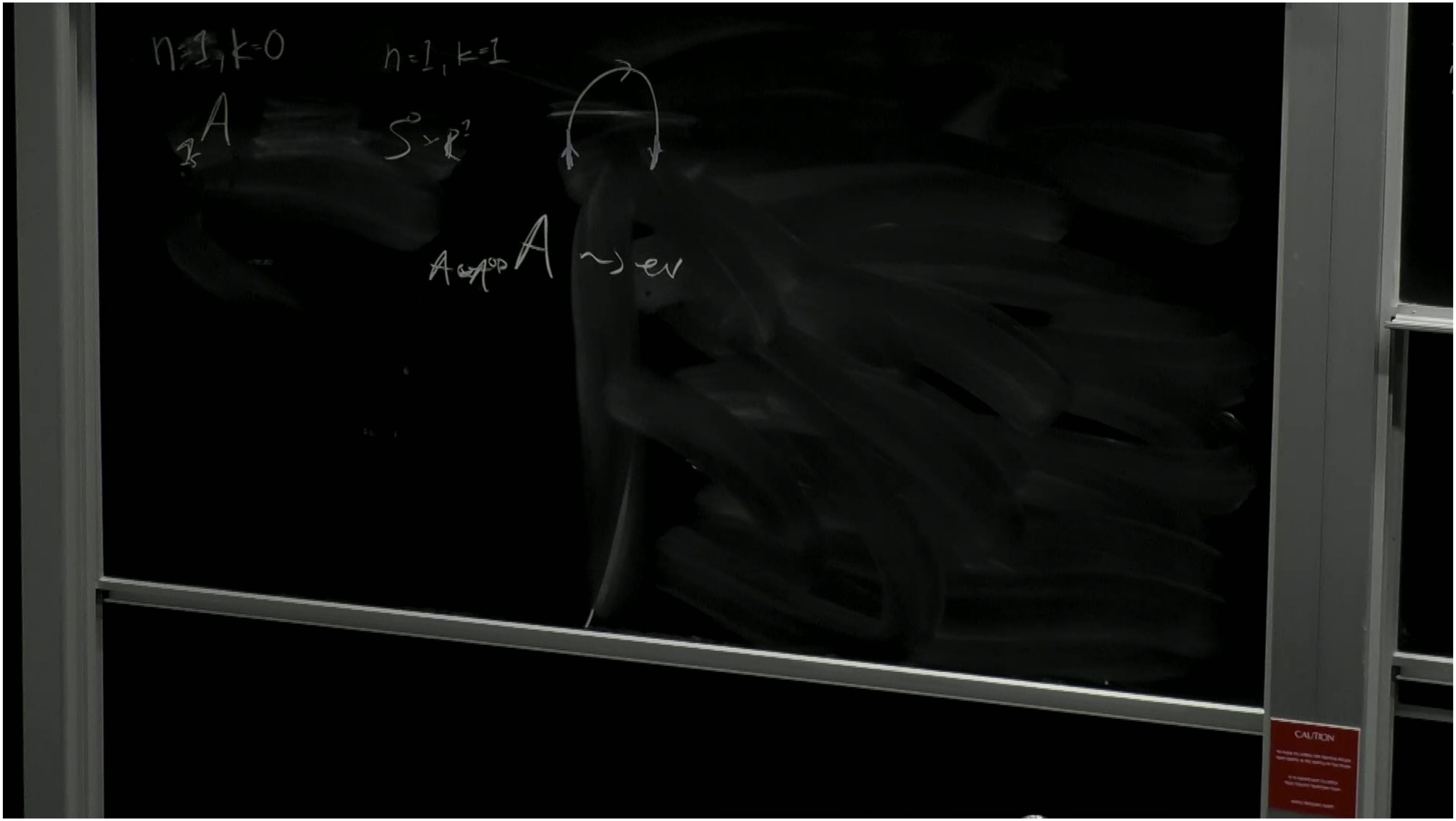
Every $A \in Alg_n$ is n-dualiz.

pf


Alg2(S)
obs
1 morp
Dualiz. on Algn(S)
Thm [GS]
Every A∈Algn is n-dualiz
pf
n=1
ev
A
Aop
collapse
coev


bicat
Thm
the only
obj
Conjecture / Thm
n=1 Lurie


the only $(n+1)$-dualiz. obj
is $\mathbb{1}_S$.
Conjecture / Thm [$n=2$, $S=$ Cat] BJS
An obj $A \in \mathrm{Alg}_n^{unpt}(S)$ is $(n+1)$-dualiz
iff it has a left adjoint as
a module over fact. hom.
$\int_{S^{k-1} \times \mathbb{R}^{n-k+1}} A$, for $k=0,1,\dots,n$
takes framing from the k-disk


n=1, k=0
A
n=1, k=1
ev

$\boxed{n=1}$

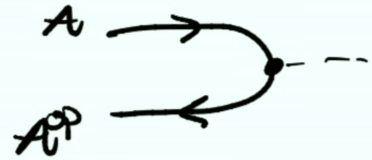
1-dualizability data

[GS]

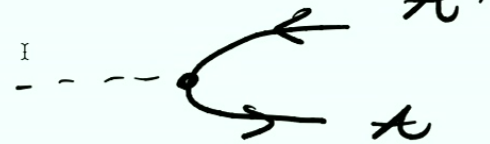
obj $\quad A$

dual $\quad A^{op}$

ev $\quad \cancel{A\boxtimes A^{op}}\ A\ 1_S$



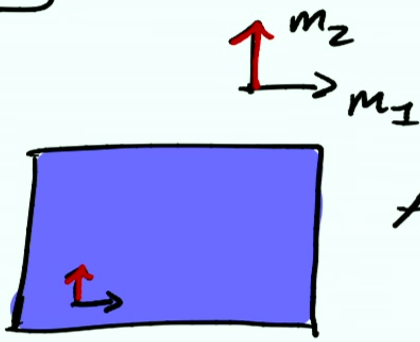
coev



2-dualizability data

$\boxed{n=2}$

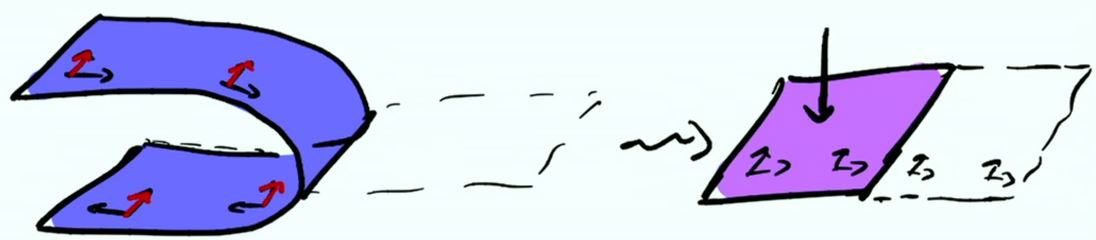
obj



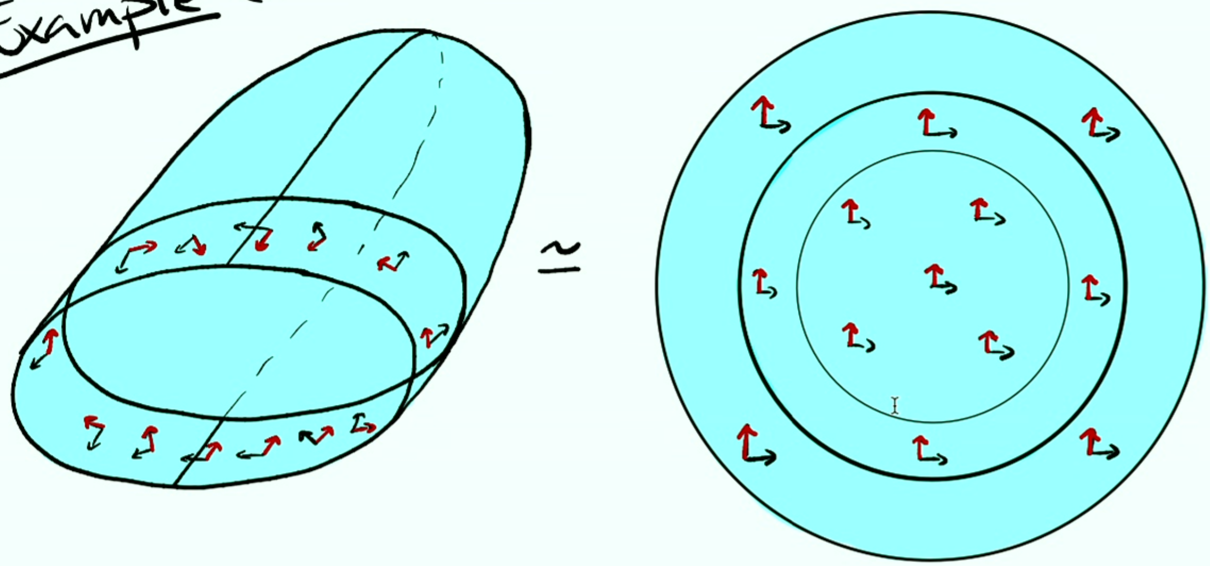
A

evaluation

$A\otimes A^{1op}$

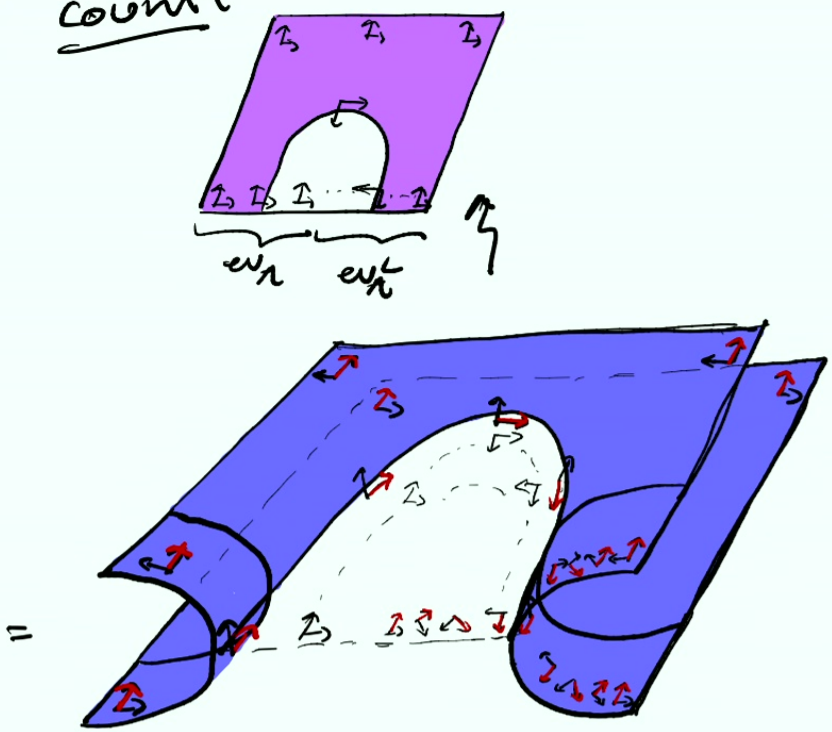


Example $(n=2, k=2)$



2-morphisms for ev_A^L

counit



unit

$\eta_{ev_A^L} =$