Title: Entropy-Area Law from Interior Semi-classical Degrees of Freedom

Speakers: Yuki Yokokura

Series: Cosmology & Gravitation

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Abstract:

Can degrees of freedom in the interior of black holes be responsible for the entropy-area law? If yes, what spacetime appears? In this talk, I answer these questions at the semi-classical level. Specifically, a black hole is considered as a bound state consisting of many semi-classical degrees of freedom which exist uniformly inside and have maximum gravity. The distribution of their information determines the interior metric through the semi-classical Einstein equation. Then, the interior is a continuous stacking of AdS_2 times S^2 without horizon or singularity and behaves like a local thermal state. Evaluating the entropy density from thermodynamic relations and integrating it over the interior volume, the area law is obtained with the factor 1/4 for any interior degrees of freedom. Here, the dynamics of gravity plays an essential role in changing the entropy from the volume law to the area law. This should help us clarify the holographic property of black-hole entropy. [arXiv: 2207.14274]

Zoom link: https://pitp.zoom.us/j/99386433635?pwd=VzlLV2U4T1ZOYmRVbG9YVlFIemVVZz09

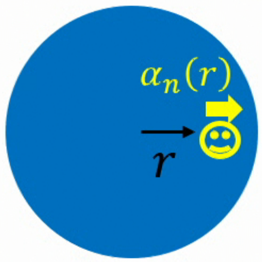
Plan: Self-consistent discussion

- 0. Setup: Assumptions (i) (ii)
- 1. Interior metric $g_{\mu\nu}$
- 2. Local thermal behavior of the d.o.f.
- 3. Derivation the area law
- 4. Self-consistency to $G_{\mu\nu} = 8\pi G \langle\psi|T_{\mu\nu}|\psi\rangle$
- 5. Conclusion and discussions

Self-consistent

Setup: BH as a semi-classical bound state（3/3）

(ii) The acceleration required to stay at r is semi-classically maximum



(i) uniformness

$$\alpha_n(r) \doteq const = O\left(\frac{1}{C\,l_p}\right)$$

$C = O(1) \gg 1$
$= O(a^0)$

⇒Why? Motivation?

(1) BH = maximum gravity,

(cf: $\alpha_n(r)|_{Schwarzschild} = \frac{\frac{a}{r^2}}{2\sqrt{1-\frac{a}{r}}} \to_{r\to a} \infty$ if a horizon exists at $r = a$)

(2) Minimum resolution of spacetime = $l_p \equiv \sqrt{\hbar G}$,

[Parentani and Potting, Rovelli and Vidotto, ...]

while time scale at r = $\frac{1}{\alpha_n(r)}$.

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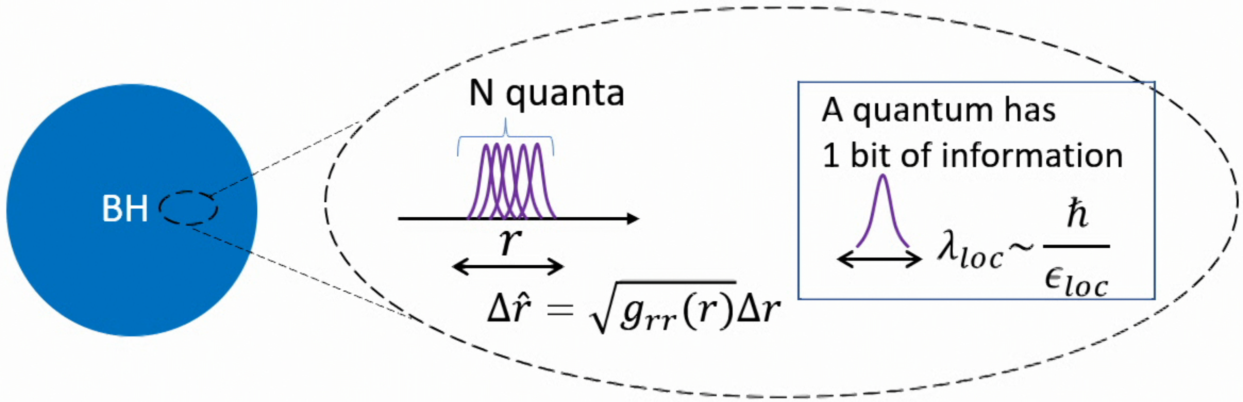
Self-consistent

Interior metric from Interior information (1/3)



- Suppose N quanta with 1-bit of information and local energy ϵ_{loc} around r inside. Then, from a general formula, we can show

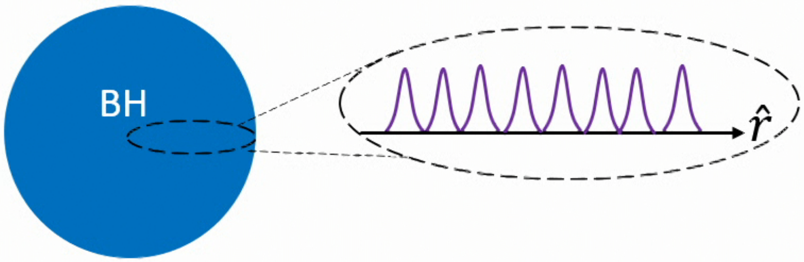
$$\Delta M_{1bit} = \frac{\epsilon_{loc}}{\sqrt{g_{rr}(r)}}$$

Contribution to ADM energy of the part within r in a spherically symmetric system

$$M(r) = 4\pi \int_0^r dr' r'^2 \ \langle -T_t^t(r') \rangle$$

Setup: BH as a semi-classical bound state (2/3)

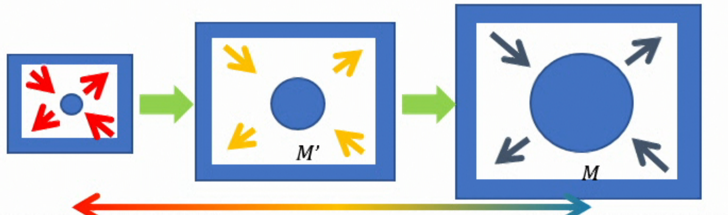
(i)The d.o.f. are distributed inside uniformly in the radial proper length, $d\hat{r} = \sqrt{g_{rr}(r)}dr$.



⇒Why? Motivation?

(1) This is simple!

(2) This should be the most typical configuration.

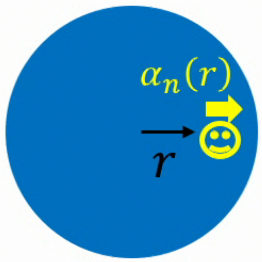


Adiabatic formation of BH
⇒ Uniform interior structure
⇒thermodynamically typical
[Kawai-Yokokura 2015,2021]

Note: The following discussion is independent of the details of the formation process.

Setup: BH as a semi-classical bound state (3/3)

(ii) The acceleration required to stay at r is semi-classically maximum



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$$\alpha_n(r) = const = O\left(\frac{1}{C\,l_p}\right)$$

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(2) Minimum resolution of spacetime = $l_p \equiv \sqrt{\hbar G}$,

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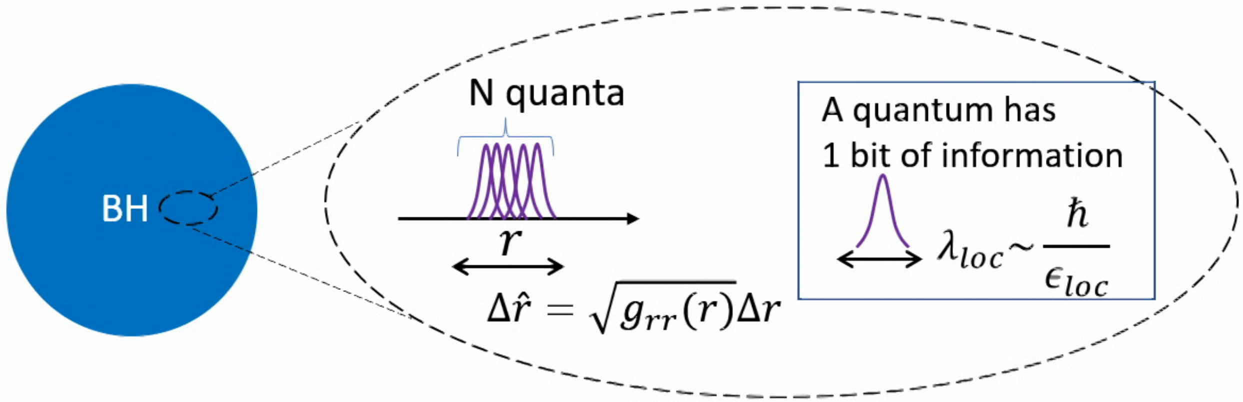
[Parentani and Potting, Rovelli and Vidotto, ...]

Interior metric from Interior information (2/3)



- We can estimate the entropy per unit proper length as

$$s(r) \sim \frac{N}{\Delta\hat{r}} \sim \frac{N\sqrt{g_{rr}(r)}}{r} = const.$$

(i) uniform cond.

$$\Delta\hat{r} \sim \lambda_{loc} \sim \frac{r}{\sqrt{g_{rr}(r)}}$$

- This requires us to set

$$g_{rr}(r) = \frac{r^2}{2\sigma}, \qquad \sigma: const.$$

- Then, the total entropy is given by

$$S = \int_0^R dr \sqrt{g_{rr}(r)} s(r) \sim \int_0^a dr \frac{N g_{rr}(r)}{r} \sim \frac{Na^2}{\sigma} \sim \frac{a^2}{l_p^2}$$

demand

Interior metric from Interior information (3/3)

- Next, we use the condition (ii):

$$\alpha_n(r) \equiv \frac{\partial_r \log\sqrt{-g_{tt}(r)}}{\sqrt{g_{rr}(r)}} \overset{\text{(ii)}}{=} O\left(\frac{1}{Cl_p}\right) \equiv \frac{1}{\sqrt{2\sigma\eta^2}}$$

$$\alpha_n \equiv |g_{\mu\nu}\alpha_n^\mu \alpha_n^\nu|^{\frac{1}{2}},\quad \alpha_n^\mu \equiv n^\nu \nabla_\nu n^\mu,\quad n^\mu\partial_\mu = \left(-g_{tt}(r)\right)^{-\frac{1}{2}}\partial_t$$

$$\begin{cases} C = O(1) \gg 1 \\ \sigma \sim N l_p^2 \end{cases} \Rightarrow N\eta^2 = O(1) \gg 1$$

Integrate

$$g_{rr}(r) = \frac{r^2}{2\sigma}$$

- This leads to

$$g_{tt}(r) = -e^{\frac{r^2}{2\eta\sigma}+A_0}.$$

- (σ, η) will be determined by $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$ later.

Interior metric (1/2)

- We have obtained

$$ds_{in}^2 = -e^{\frac{r^2}{2\sigma\eta}+A_0}dt^2 + \frac{r^2}{2\sigma}dr^2 + r^2 d\Omega^2$$

$\sigma \sim N l_p^2$
$\Rightarrow N\eta^2 = O(1) \gg 1$

→

$\sigma \sim N l_p^2$ (with $N = O(1) \gg 1$)
$0 < \eta < 2$

- **Not fluid**: Using Einstein eq

$$-\langle T^t_t \rangle = \frac{1}{8\pi G r^2}, \qquad \langle T^r_r \rangle = \frac{2-\eta}{\eta}(-\langle T^t_t \rangle) \ll \langle T^\theta_\theta \rangle = \frac{1}{16\pi G \sigma \eta^2}$$

require $\rightarrow 0$

⇒The interior is not fluid.
This supports the d.o.g.

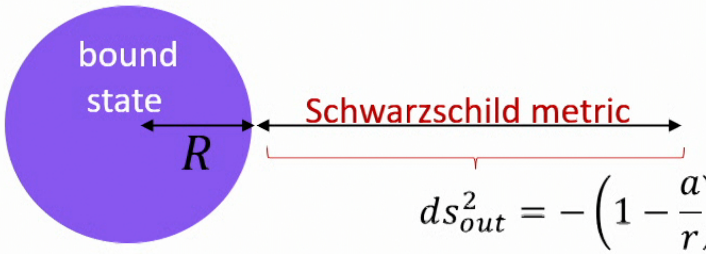
- **No singularity**:

$$R, \sqrt{R_{\mu\nu}R^{\mu\nu}}, \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} = O\left(\frac{1}{\sigma\eta^2}\right) = O\left(\frac{1}{N l_p^2}\right) \ll O\left(\frac{1}{l_p^2}\right) \text{ for } N \gg 1$$

for $r \gg l_p$ semi-classically maximum

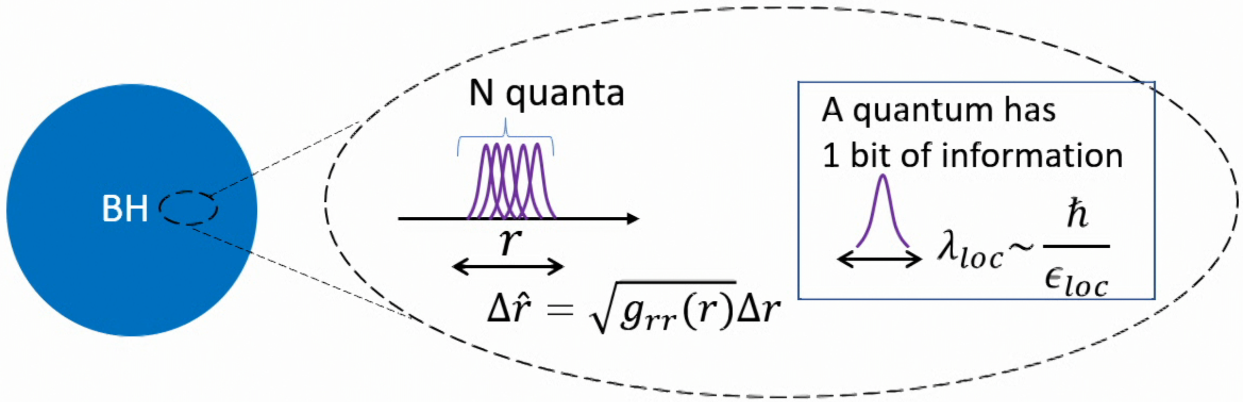
Interior metric (2/2)

- By connecting this to the Schwarzschild metric, we obtain



$$ds_{out}^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \left(1 - \frac{a}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Interior metric from Interior information (2/3)



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demand

Note: At this stage, (N, σ) may depend on a.

- Thus, we reach

$$g_{rr}(r) = \frac{r^2}{2\sigma}, \qquad \sigma \sim N l_p^2$$

Interior metric（1/2）

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$$ds_{in}^2 = -e^{\frac{r^2}{2\sigma\eta}+A_0}dt^2 + \frac{r^2}{2\sigma}dr^2 + r^2 d\Omega^2$$

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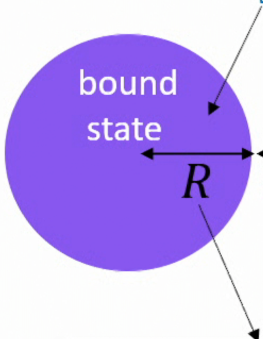
Interior metric

$$ds_{in}^2 = -\frac{2\sigma}{R^2} e^{-\frac{R^2 - r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2$$

[Kawai-Yokokura 2015]

exponentially large redshift
⇒Time is frozen inside!

$\sigma \sim N l_p^2$ (with $N = O(1) \gg 1$)
$0 < \eta < 2,$



Schwarzschild metric

$$ds_{out}^2 = -\left(1 - \frac{a}{r}\right) dt^2 + \left(1 - \frac{a}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Size:

$$R = a + \frac{2\sigma}{a} > a$$

proper distance

$$\Delta\hat{r} = \sqrt{g_{rr}(R)} \frac{2\sigma}{a} \approx \sqrt{\frac{a^2}{2\sigma}} \frac{2\sigma}{a} = \sqrt{2\sigma} = O(\sqrt{N} l_p) \gg l_p$$

⇒physically meaningful
⇒The bound state has no horizon
but looks like a classical BH from the outside.
⇒This dense object is the BH!

[Kawai-Matsuo-Yokokura 2013]

2. Local thermal behavior of the d.o.f.

Interior metric from Interior information (3/3)

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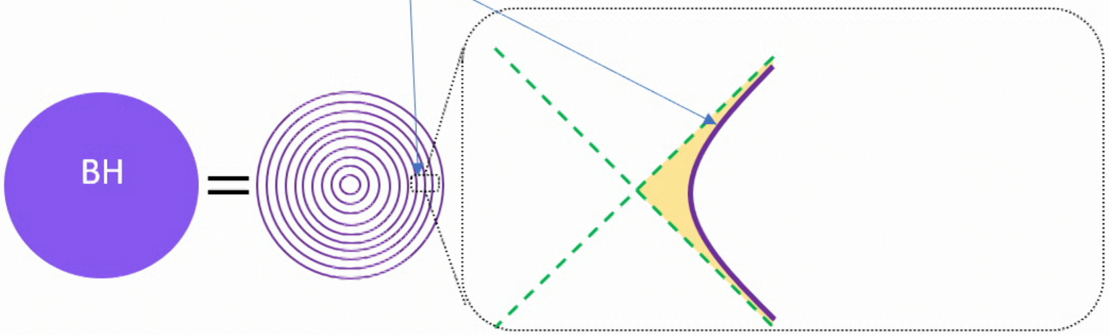
Interior structure

spherical uniform interior = continuous concentric stacking of spherical excitations (like "shells").



- Each one accelerates due to the self-gravity at

$$\alpha_u = \frac{\eta}{2\sqrt{\eta - 1}}\frac{1}{L}.$$

$1 < \eta < 2$

- The metric is locally AdS_2 (of L) $\times$ S^2(of r).

$$R = -\frac{2}{L^2} + O(r^{-2})$$

$$L \equiv \sqrt{2\sigma\eta^2} \sim \sqrt{N} l_p$$

$$ds_{in}^2 = -\frac{2\sigma}{R^2} e^{-\frac{R^2 - r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2$$

$$= \frac{L^2}{z^2}(-dt^2 + dz^2) + r(z)^2 d\Omega^2$$

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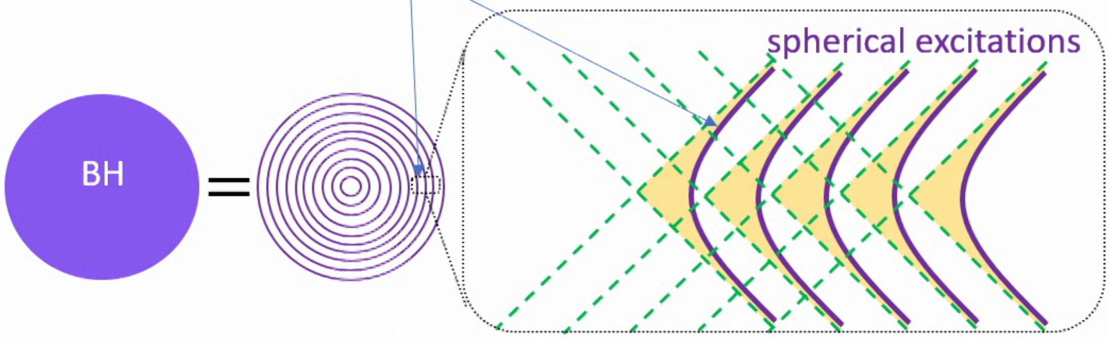

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- The metric is locally AdS_2 (of L) $\times S^2$(of r).

$\Rightarrow$ The interior is a continuous stacking of AdS_2 (of L) $\times S^2$(of r).

Local thermal behavior

- The accelerating shell feels the Unruh temperature:

$$T_U = \frac{\hbar}{2\pi}\sqrt{-\frac{1}{L^2}+\alpha^2} \longrightarrow \frac{2-\eta}{2\sqrt{\eta-1}}\frac{\hbar}{2\pi L}$$

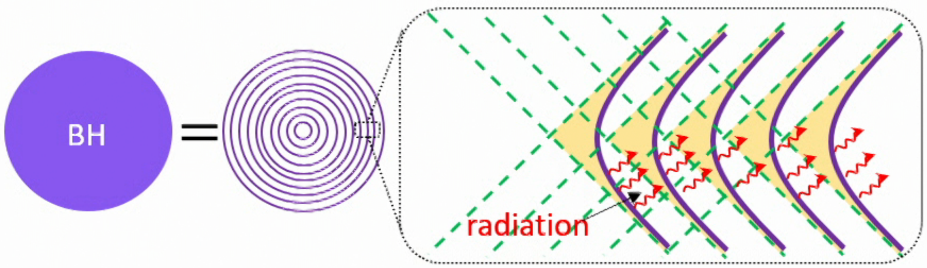
[Deser-Levin, Jacobson]

$$\alpha_u = \frac{\eta}{2\sqrt{\eta-1}}\frac{1}{L}$$
$$L \equiv \sqrt{2\sigma\eta^2} \sim \sqrt{N} l_p$$

- In fact, particles are created around the shells due to the self-gravity as

$$J_{loc} \equiv 4\pi r^2 j^\mu m_\mu = \frac{1}{4G}\frac{2-\eta}{\eta-1} \sim \frac{N}{\hbar} T_U^2,$$ 1D thermal radiation

(similar to the particle creation by a moving mirror)



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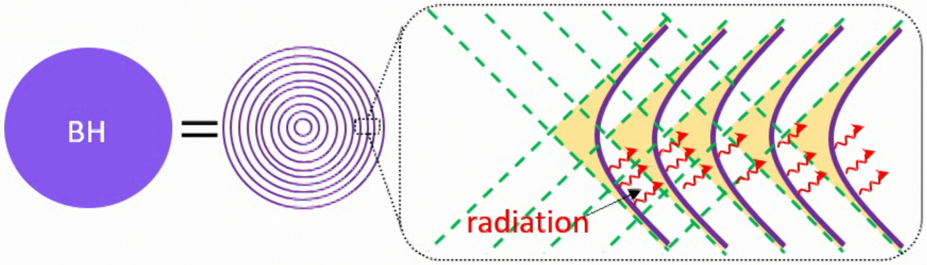

⇒ *Each small region behaves like a 1D subsystem in local equilibrium at*

$$T_{loc} = \frac{\hbar}{2\pi L}.$$ *for any d.o.f.*

Derivation of the area law

- In the local equilibrium, 1D Gibbs relation

$$T_{loc}s = \rho_{1d} + p_{1d}$$

holds and

$$p_{1d} = \frac{2-\eta}{\eta}\rho_{1d},$$

plays a role of the equation of states.

$$\rho_{1d} = 4\pi r^2(-\langle T_t^t\rangle), \quad p_{1d} = 4\pi r^2\langle T_r^r\rangle$$

$$-\langle T_t^t\rangle = \frac{1}{8\pi G r^2}, \quad \langle T_r^r\rangle = \frac{2-\eta}{\eta}(-\langle T_t^t\rangle)$$

from $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$

- Then, we can evaluate the entropy density:

$$s(r) = \frac{\rho_{1d} + p_{1d}}{T_{loc}} = \frac{1}{T_{loc}}\frac{2}{\eta}\rho_{1d} = \frac{2\pi L}{\hbar}\frac{2}{\eta}\frac{1}{2G} = \frac{2\pi\sqrt{2\sigma}}{l_p^2}$$

$$T_{loc} = \frac{\hbar}{2\pi L}, \quad L \equiv \sqrt{2\sigma\eta^2}$$

Here, $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$ plays the essential role.

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Here, $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$ plays the essential role.

⇒Integrating it over the volume reproduces the area law:

$$R = a + \frac{2\sigma}{a}$$

$$S = \int_0^R dr\sqrt{g_{rr}}\,s(r) = \int_0^R dr\sqrt{\frac{r^2}{2\sigma}}\frac{2\pi\sqrt{2\sigma}}{l_p^2} = \frac{\pi R^2}{l_p^2} = \frac{\pi a^2}{l_p^2} + O(1)$$

This holds for any d.o.f.!

Universality of the area law

- Let us review what we have done so far.

Sec1. Demand $S \propto \frac{a^2}{l_p^2}$ for interior d.o.f. $\Rightarrow g_{rr} \propto \frac{r^2}{\sigma}$.

Sec2. QFT on the b.g. $g_{\mu\nu}$ ⇒ Local thermality at $T_{loc} = \frac{\hbar}{2\pi L}$ for any d.o.f..

Sec3. Exact derivation based on thermodynamics at T_{loc}

$\Rightarrow s = \frac{2\pi\sqrt{2\sigma}}{l_p^2}$.

$\Rightarrow \sigma$ cancels out in $S = \int_0^R dr\sqrt{g_{rr}}s(r) = \frac{A}{4l_p^2}$ universally!

Note:

-The above discussion holds thanks to $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$.

Self-consistent solution (1/2)

- Q: Does the metric satisfy $G_{\mu\nu} = 8\pi G \langle\psi|T_{\mu\nu}|\psi\rangle$?

⇒Yes. Indeed, (σ, η) exist satisfying it.

- For simplicity, let's consider conformal matter fields and focus on

$$G^{\mu}_{\mu} = 8\pi G \langle\psi|T^{\mu}_{\mu}|\psi\rangle \doteq 8\pi G\hbar(c_W\mathcal{F} - a_W\mathcal{G} + b_W\Box R)$$

4D Weyl anomaly

$$\mathcal{F} \equiv C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}, \qquad \mathcal{G} \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$c_W, a_W, b_W \sim$ d.o.f. of matter fields.

⇒This eq hold for any state $|\psi\rangle$.

Note:

- Dynamics of the quantum fields are very different in 4D and 2D.
- The self-consistency holds for non-conformal cases. [Kawai-Yokokura 2020]

Self-consistent solution (2/2)

- For the metric, we have

$$G^\mu_\mu = 8\pi G\langle\psi|T^\mu_\mu|\psi\rangle = 8\pi G\hbar\left(c_W C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - a_W \mathcal{G} + b_W \Box R\right).$$

$$\Rightarrow \frac{1}{\sigma\eta^2} = 8\pi l_p^2 c_W \frac{1}{\sigma^2\eta^4}$$

$$\Rightarrow \sigma = \frac{8\pi l_p^2 c_W}{3\eta^2} \text{ with } c_W \sim N \gg 1.$$

[Kawai-Yokokura 2015]

- Furthermore, we can evaluate directly $\langle\psi|T_{\mu\nu}|\psi\rangle$ in the $g_{\mu\nu}$ and an excited state $|\psi\rangle$ at T_{loc} by dimensional regularization and a perturbative calculation to determine η.

[Kawai-Yokokura 2020]

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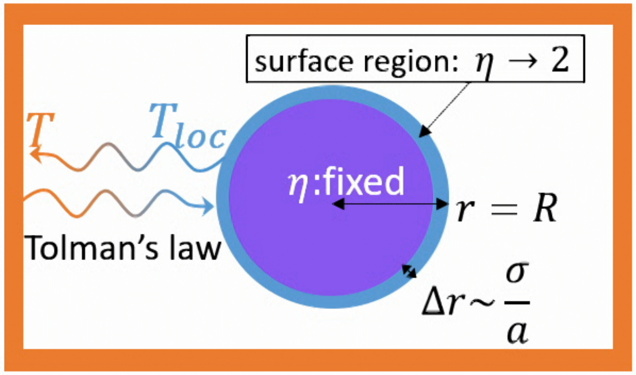
- For the metric, we have
$$G^{\mu}_{\mu} = 8\pi G\langle\psi|T^{\mu}_{\mu}|\psi\rangle = 8\pi G\hbar\left(c_W C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} - a_W \mathcal{G} + b_W \Box R\right).$$
$$\Rightarrow \frac{1}{\sigma\eta^2} = 8\pi l_p^2 c_W \frac{1}{\sigma^2\eta^4}$$
$$\Rightarrow \sigma = \frac{8\pi l_p^2 c_W}{3\eta^2} \text{ with } c_W \sim N \gg 1.$$
[Kawai-Yokokura 2015]

- Furthermore, we can evaluate directly $\langle\psi|T_{\mu\nu}|\psi\rangle$ in the $g_{\mu\nu}$ and an excited state $|\psi\rangle$ at T_{loc} by dimensional regularization and a perturbative calculation to determine η. [Kawai-Yokokura 2020]

- Thus, the full 4D dynamics of the d.o.f induces the large acceleration, pressure, and curvatures self-consistently.
$$\alpha \sim \frac{1}{L^2}, \quad \langle T^{\theta}_{\theta}\rangle \sim \frac{1}{GL^2}, \quad \mathcal{R} \sim \frac{1}{L^2}, \quad L = \sqrt{2\sigma\eta^2} \sim \sqrt{N} l_p$$

Derivation of Hawking temperature

- Suppose that the bound state is in equilibrium in a heat bath.



$-g_{tt} = 1 - \frac{a}{r}$

$R = a + \frac{2\sigma}{a}$

$T_{loc} = \frac{\hbar}{2\pi L}$

$L \equiv \sqrt{2\sigma\eta^2}$

- Then, Tolman's law holds:

$$T\sqrt{-g_{tt}(r \gg a)} = T_{loc}\sqrt{-g_{tt}(r = R)}\Big|_{\eta\to 2}$$

$$\Rightarrow T = \sqrt{1 - \frac{a}{R}}\, T_{loc}\Big|_{\eta\to 2} \approx \sqrt{\frac{2\sigma}{a^2}}\frac{\hbar}{2\pi\sqrt{2\sigma\eta^2}}\Big|_{\eta\to 2} = \frac{\hbar}{4\pi a}$$

Hawking Temperature!

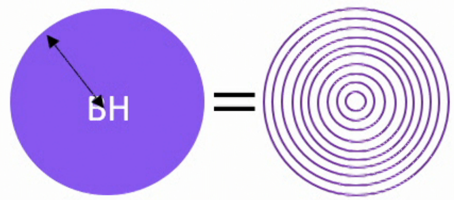
5. Conclusion and discussions

Conclusion

- BH = bound state of many interior semi-classical d.o.f..
- This can be realized as the self-consistent solution of $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$:

$$ds^2 = -\frac{2\sigma}{R^2}e^{-\frac{R^2-r^2}{2\sigma\eta}}dt^2 + \frac{r^2}{2\sigma}dr^2 + r^2 d\Omega^2$$

$$r = R = \mathrm{a} + \frac{2\sigma}{a}$$



No horizon
or singularity
for N>>1

- The area law holds universally for any interior d.o.f.:

$$S = \int_0^R dr\sqrt{g_{rr}(r)}s(r) = \frac{A}{4l_p^2},$$

where gravity changes the entropy from the volume law to the area law.

⇒*The information itself is stored in the "bulk" d.o.f. although its amount is expressed as the "boundary" area.*

⇒Reconsider the meaning of holography!

Discussion

- How plausible is the configuration?

⇒Consider it from a view of the entropy bound. [Bekenstein, Bousso,…]

- Time evolution of the formation [Kawai-Matsuo-Yokokura 2013, Kawai-Yokokura 2020]
- Direct derivation for conformal matters [Kawai-Yokokura 2021]
- Semi-classical stability [Work in progress, Ho-Kawai-Liao-Yokokura]

- A method of constructing a metric $g_{\mu\nu}(x)$ from S and $s(r)$.

1) Modify the interior metric by corrections to the area law.

2) Find interior metrics for other configurations.

- Direct microstate counting? [Kawai-Yokokura 2020]

- Information recovery from the interior structure? [Kawai-Yokokura 2016]

Thank you!

Entropy-Area Law from Interior Semi-classical Degrees of Freedom

RIKEN iTHEMS

Yuki Yokokura

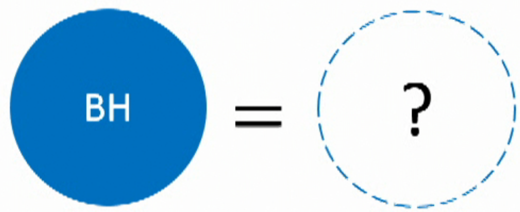
[arXiv:2207.14274]

2022 Oct 11 @ Perimeter Institute

iTHEMS
Interdisciplinary Theoretical and Mathematical Sciences Program

Black hole entropy

- In quantum theory, we don't know yet what a black hole is.



- The notion of spacetime geometry should emerge only under a certain limit.

⇒ Horizon may be an approximated property of black holes.

- The notion of information is quantum mechanical.

⇒A black hole should be characterized more properly by

entropy-area law $S = \log \Omega = \frac{A}{4l_p^2} \cdot \left(= \frac{4\pi M^2}{m_p^2} \right)$

$$A \equiv 4\pi a^2$$
$$a \equiv 2GM$$
$$l_p \equiv \sqrt{\hbar G}, m_p \equiv \sqrt{\hbar / G}$$

BH=bound state of many d.o.f.

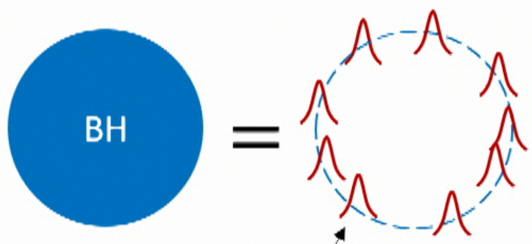
- What is the origin of $S = \log \Omega = \frac{A}{4 l_p^2}$?

⇒Black hole = gravitational bound state of some d.o.f.

responsible for $S = \frac{A}{4 l_p^2}$

Strings/fuzz ball?
[Strominger-Vafa, Mathur...]

brick wall/semi-classical dynamical modes?
[tHooft, Barvinski-Frolov-Zelnikov,...]



discrete spacetime?
[Ashtekar-Baez-Corichi-Krasnov,...]

graviton condensation?
[Dvali-Gomez,...]

+ more approaches...

⇒***Where do the d.o.f. live?***

(i) Around the surface?

BH=bound state of many d.o.f.

- What is the origin of $S = \log\Omega = \frac{A}{4l_p^2}$?

⇒Black hole = gravitational bound state of some d.o.f.

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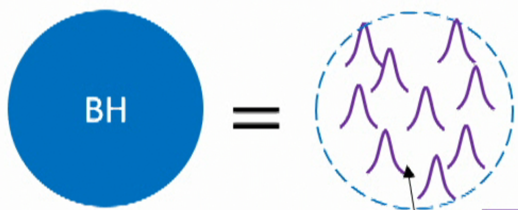


discrete spacetime?
[Ashtekar-Baez-Corichi-Krasnov,...]

graviton condensation?
[Dvali-Gomez,...]

+ more approaches...

Note: The self-gravity of interior d.o.f can change the volume law of the entropy.
ex: spherical thermal radiation with radius R
$S{\sim}R^3 \rightarrow R^{3/2}$
[Sorkin-Wald-Zhang, Oppenheim,...]

⇒***Where do the d.o.f. live?***

(ii) Inside somewhere?

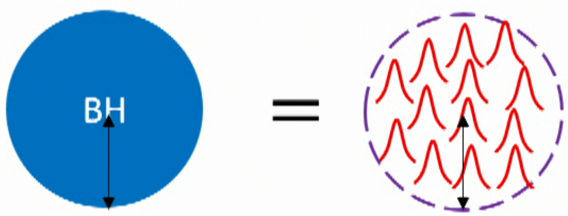
⇒We try to consider case (ii) today.

Setup: BH as a semi-classical bound state (1/3)

- Consider a spherical static BH as a bound state of many interior semi-classical d.o.f. satisfying

semi-classical Einstein eq $$G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle.$$ for any d.o.f.

(gravity = classical metric $g_{\mu\nu}$, matter = quantum fields $\hat{\phi}$)



size: $R \approx a \equiv 2GM$ ←Determined later

mass: M ($\gg m_p$)

- As a simple trial, we focus on a configuration s.t.

(i) Uniform distribution in r-direction

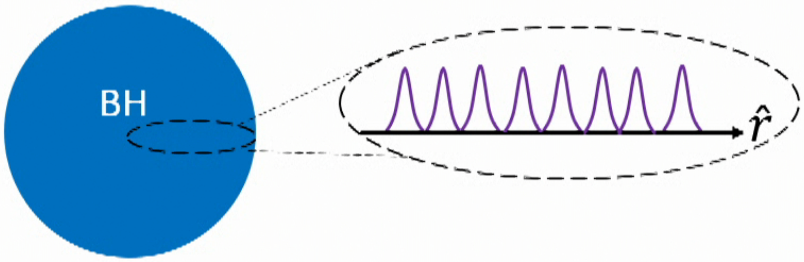
(ii) Semi-classically maximum acceleration

⇒ We will construct the interior metric

$$ds_{in}^2 = g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\Omega^2$$

Setup: BH as a semi-classical bound state（2/3）

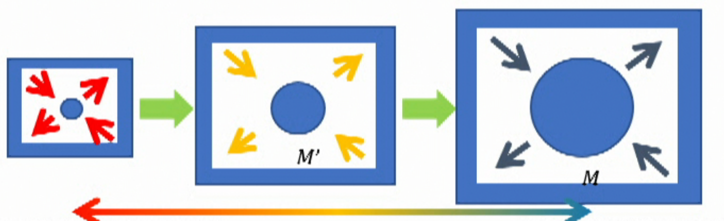
(i)The d.o.f. are distributed inside uniformly in the radial proper length, $d\hat{r}=\sqrt{g_{rr}(r)}dr$.



⇒Why? Motivation?

(1) This is simple!

(2) This should be the most typical configuration.

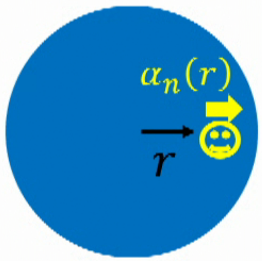


Adiabatic formation of BH
⇒ Uniform interior structure
⇒thermodynamically typical
[Kawai-Yokokura 2015,2021]

Note: The following discussion is independent of the details of the formation process.

Setup: BH as a semi-classical bound state (3/3)

(ii) The acceleration required to stay at r is semi-classically maximum



(i) uniformness

$$\alpha_n(r) = const = O\left(\frac{1}{C\,l_p}\right)$$

$$C = O(1) \gg 1 = O(a^0)$$

⇒Why? Motivation?

(1) BH = maximum gravity,

(cf: $\alpha_n(r)|_{Schwarzschild} = \frac{\frac{a}{r^2}}{2\sqrt{1-\frac{a}{r}}} \rightarrow_{r\rightarrow a} \infty$ if a horizon exists at $r = a$)

(2) Minimum resolution of spacetime = $l_p \equiv \sqrt{\hbar G}$, [Parentani and Potting, Rovelli and Vidotto, ...]

while time scale at $r = \frac{1}{\alpha_n(r)}$.