Title: Quantum Theory - Lecture 221004

Speakers:

Collection: Quantum Theory (2022-2023)

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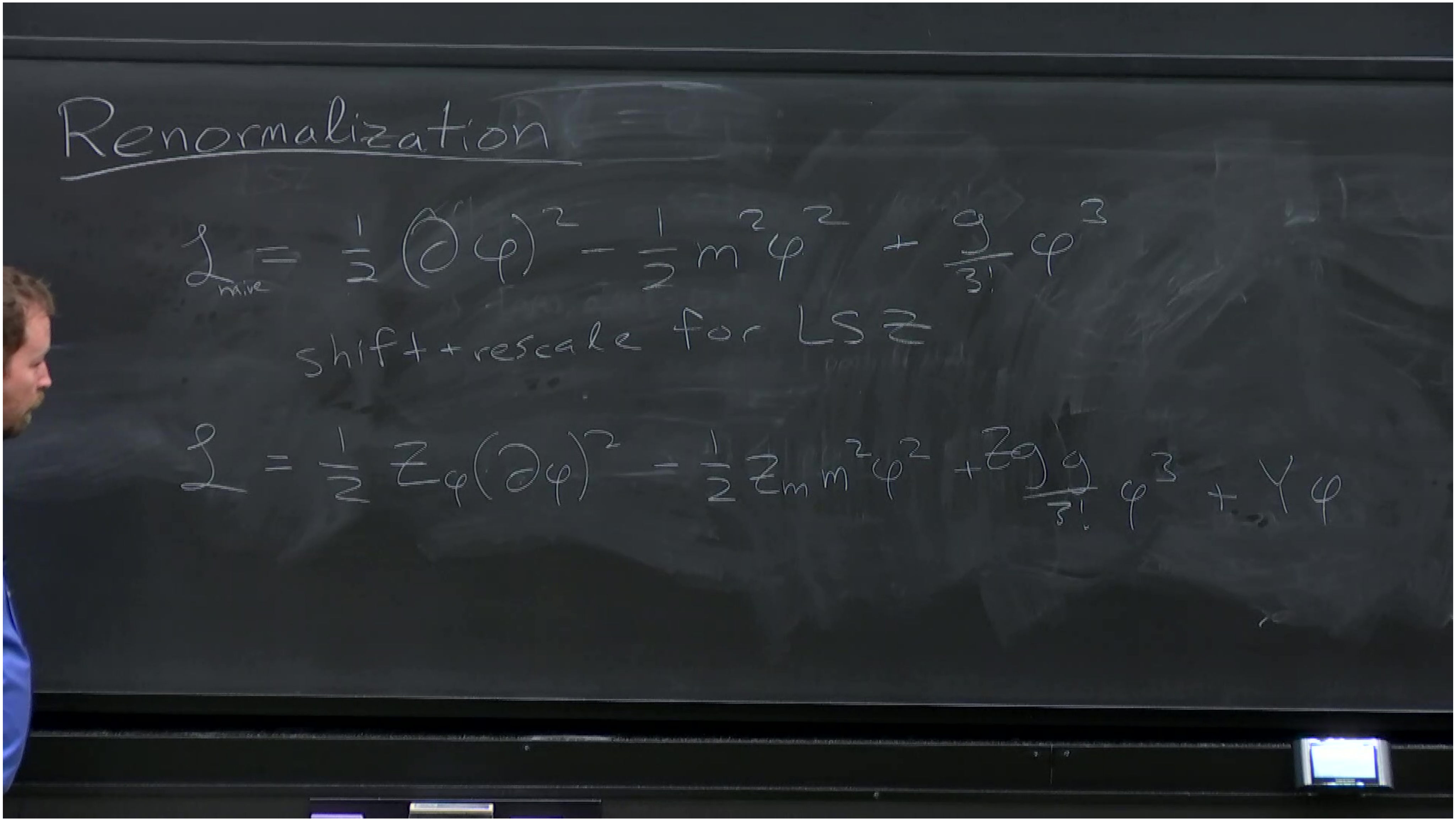
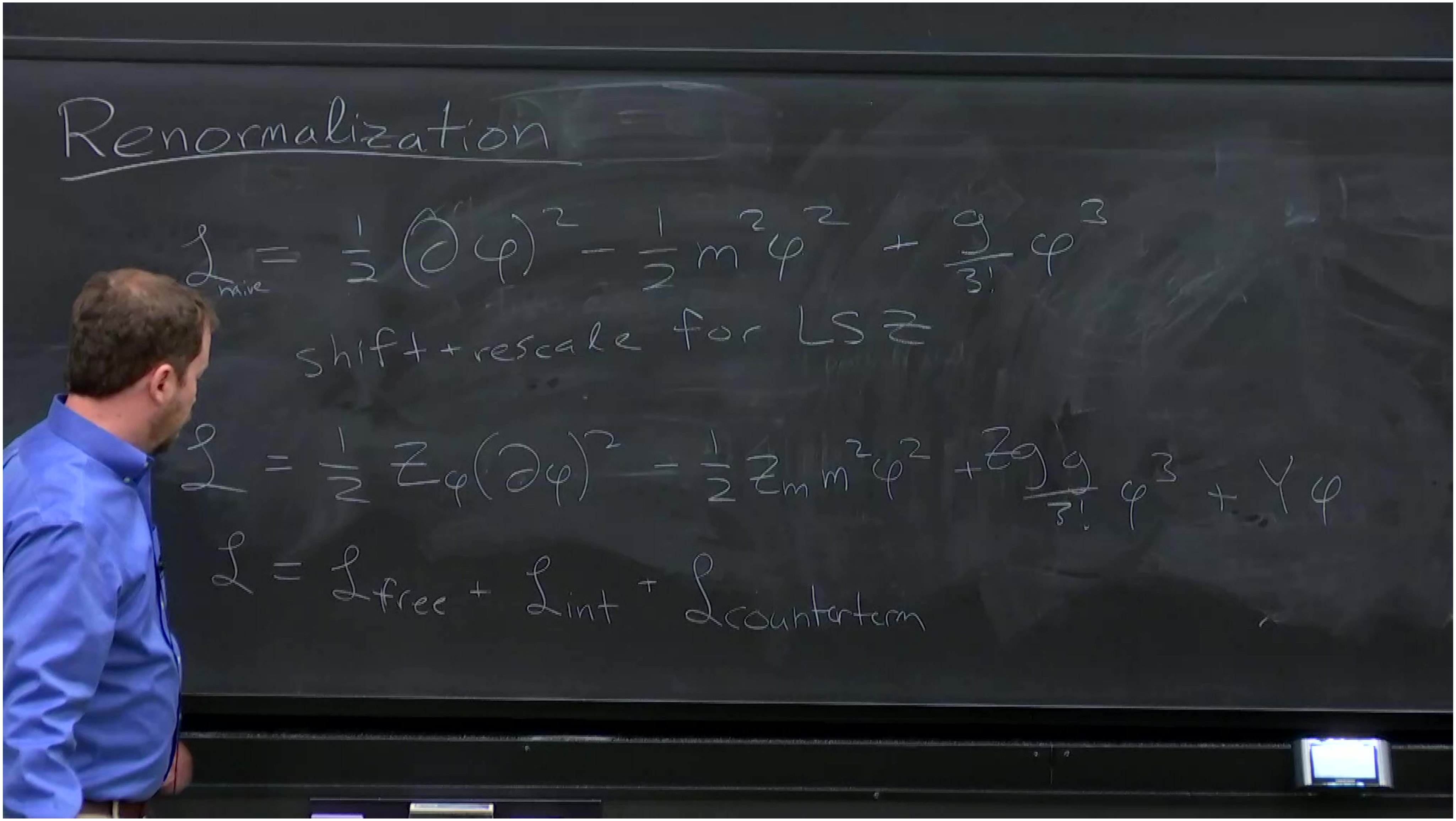
URL: https://pirsa.org/22100099


Renormalization
$\mathcal{L}_{\text{naive}} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{g}{3!}\varphi^3$
shift + rescale for LSZ
$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 + \frac{Z_g g}{3!}\varphi^3 + Y\varphi$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$\mathcal{L} = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}Z_m m^2\varphi^2 + Z_g\frac{g}{3!}\varphi^3 + Y\varphi$
$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{counterterm}}$

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\mathcal{L}_{int} = Z_g \frac{g}{3!}\varphi^3$$

$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$$

use LSZ assumptions

+ new assumptions about experiments to determine Z_i, Y, m, g

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

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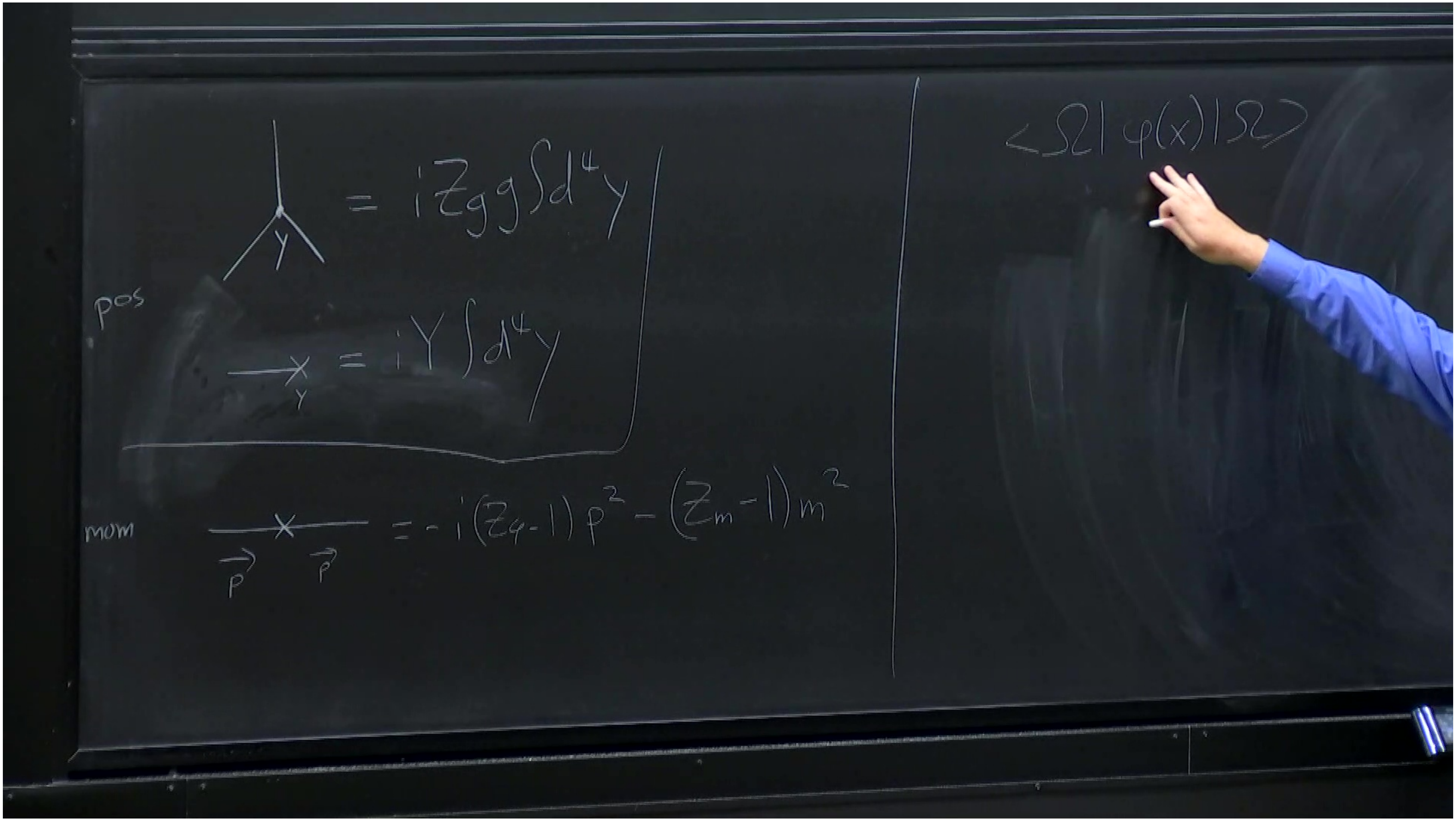
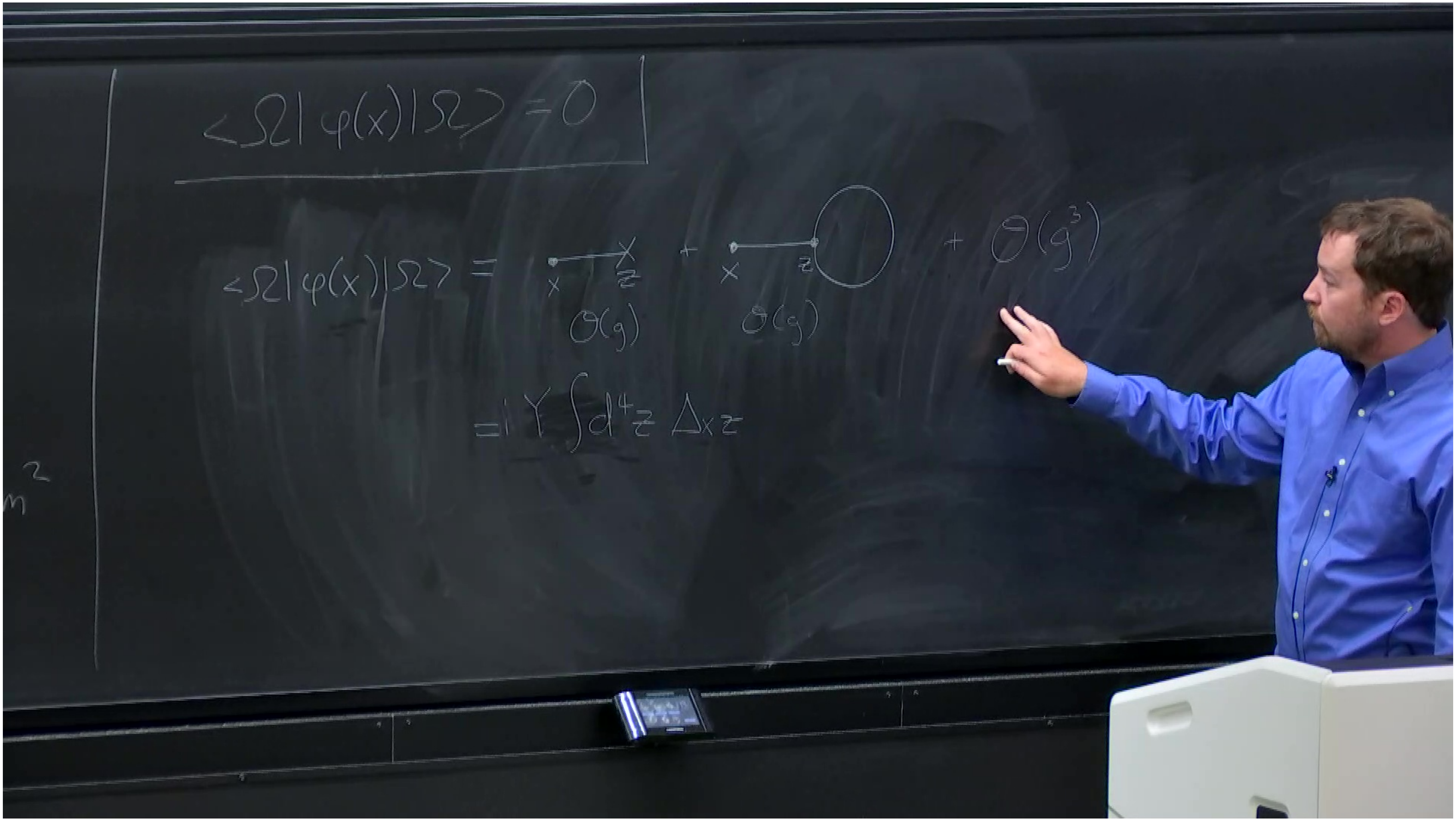
use LSZ assumptions

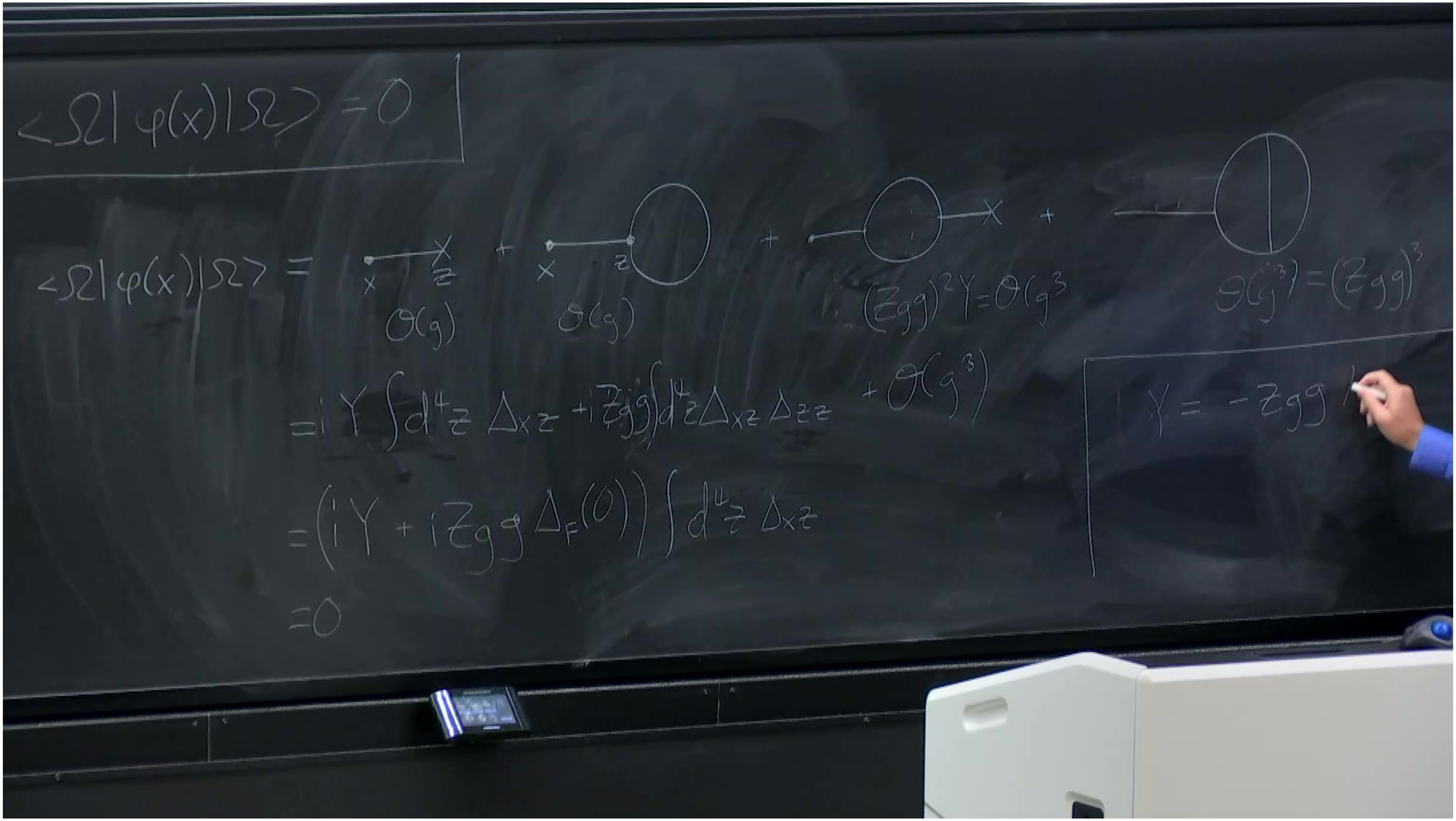
+ new assumptions about experiments to determine Z_i, Y, m, g

expect if $g \to 0$ $\to$ KG

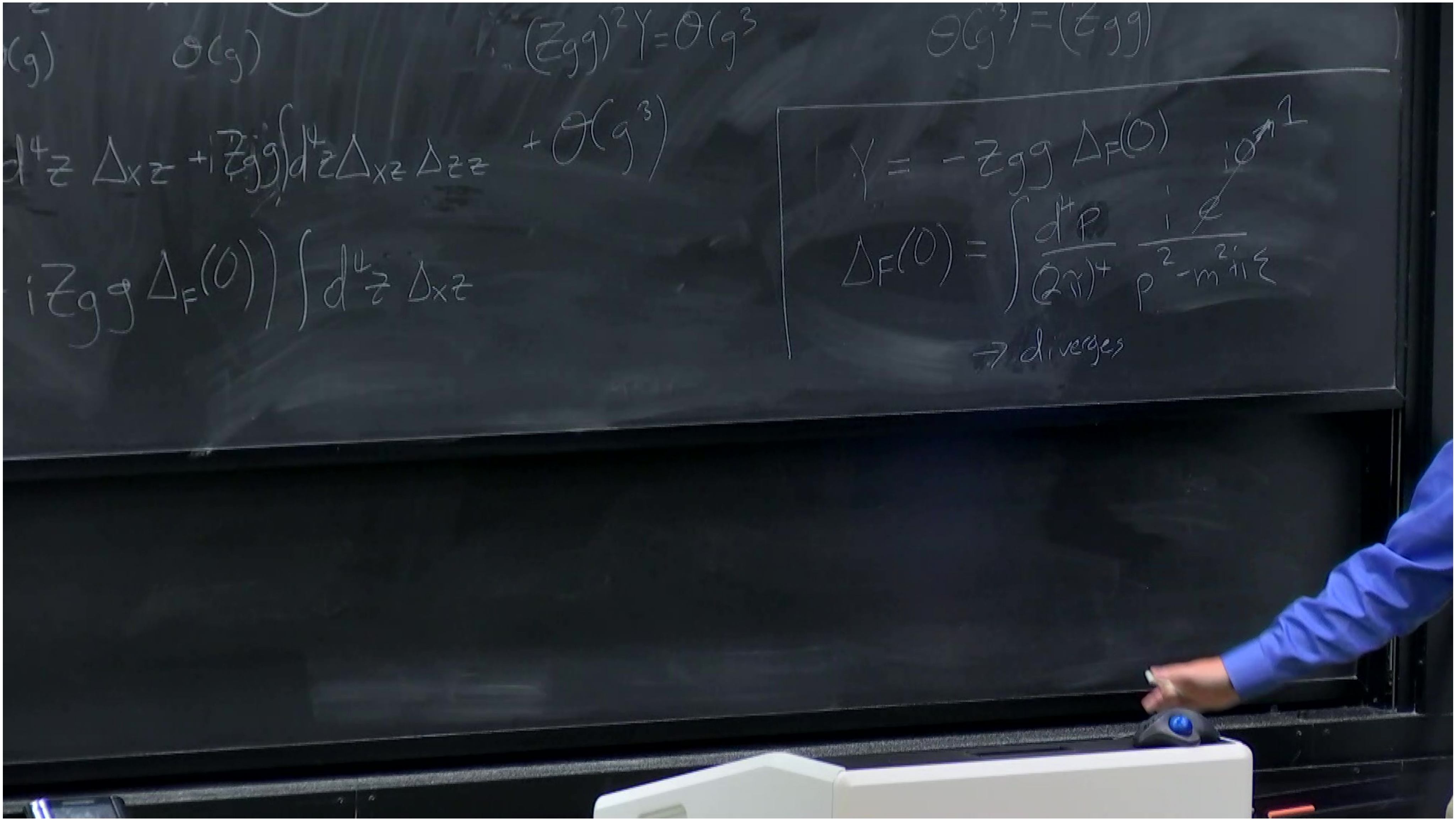
$$\begin{cases} Y = 0 + \mathcal{O}(g_p) \\ Z_i = 1 + \cancel{\mathcal{O}(g)}^{\text{for } \varphi^3} + \mathcal{O}(g_p^2) \end{cases}$$


$= iZ_g g \int d^4y$
pos
$= iY \int d^4y$
mom
$= -i(Z_\varphi - 1)p^2 - (Z_m - 1)m^2$
$\langle \Omega | \varphi(x) | \Omega \rangle$


<Ω|φ(x)|Ω> = 0
<Ω|φ(x)|Ω> =
x
z
+
x
z
+ O(g³)
O(g)
O(g)
= iY ∫ d⁴z Δxz
m²



$\langle \Omega | \varphi(x) | \Omega \rangle = 0$
$\langle \Omega | \varphi(x) | \Omega \rangle =$
x z x z
$\mathcal{O}(g)$
$\mathcal{O}(g)$
$(Zgg)^2 Y = \mathcal{O}(g^3)$
$\mathcal{O}(g^3) = (Zgg)^3$
$= iY \int d^4z\, \Delta_{xz} + iZgg \int d^4z\, \Delta_{xz} \Delta_{zz} + \mathcal{O}(g^3)$
$= \left(iY + iZgg\, \Delta_F(0)\right) \int d^4z\, \Delta_{xz}$
$= 0$
$Y = -Zgg$


$O(g)$
$O(g)$
$(Zgg)^2 Y = O(g^3)$
$O(g^3) = (Zgg)$
$d^4z\, \Delta_{xz} + iZgg \int d^4z\, \Delta_{xz} \Delta_{zz} + O(g^3)$
$iZgg\, \Delta_F(0)) \int d^4z\, \Delta_{xz}$
$Y = -Zgg\, \Delta_F(0)$
$\Delta_F(0) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon}$
→ diverges

LSZ: $\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$

$$G_2(x-y) = \langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle$$

$$G_2(p,p') = \int d^4x \int d^4y \; G_2(x-y)\, e^{i(p\cdot x + p'\cdot y)}$$

$$= (2\pi)^4 \delta^4(p+p')\, G_2(p)$$

LSZ: $\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$

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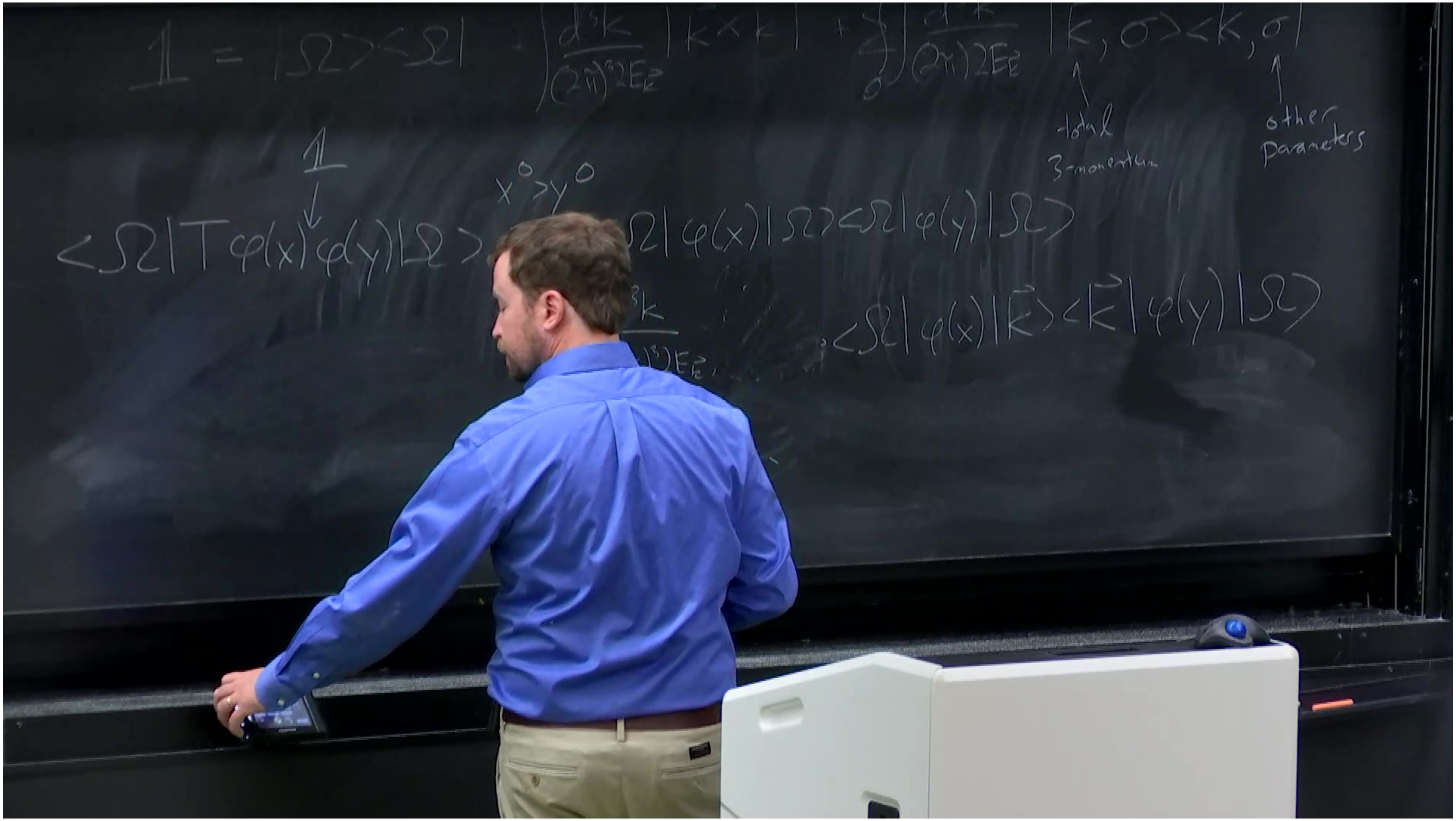
Källen - Lehmann spectral representation

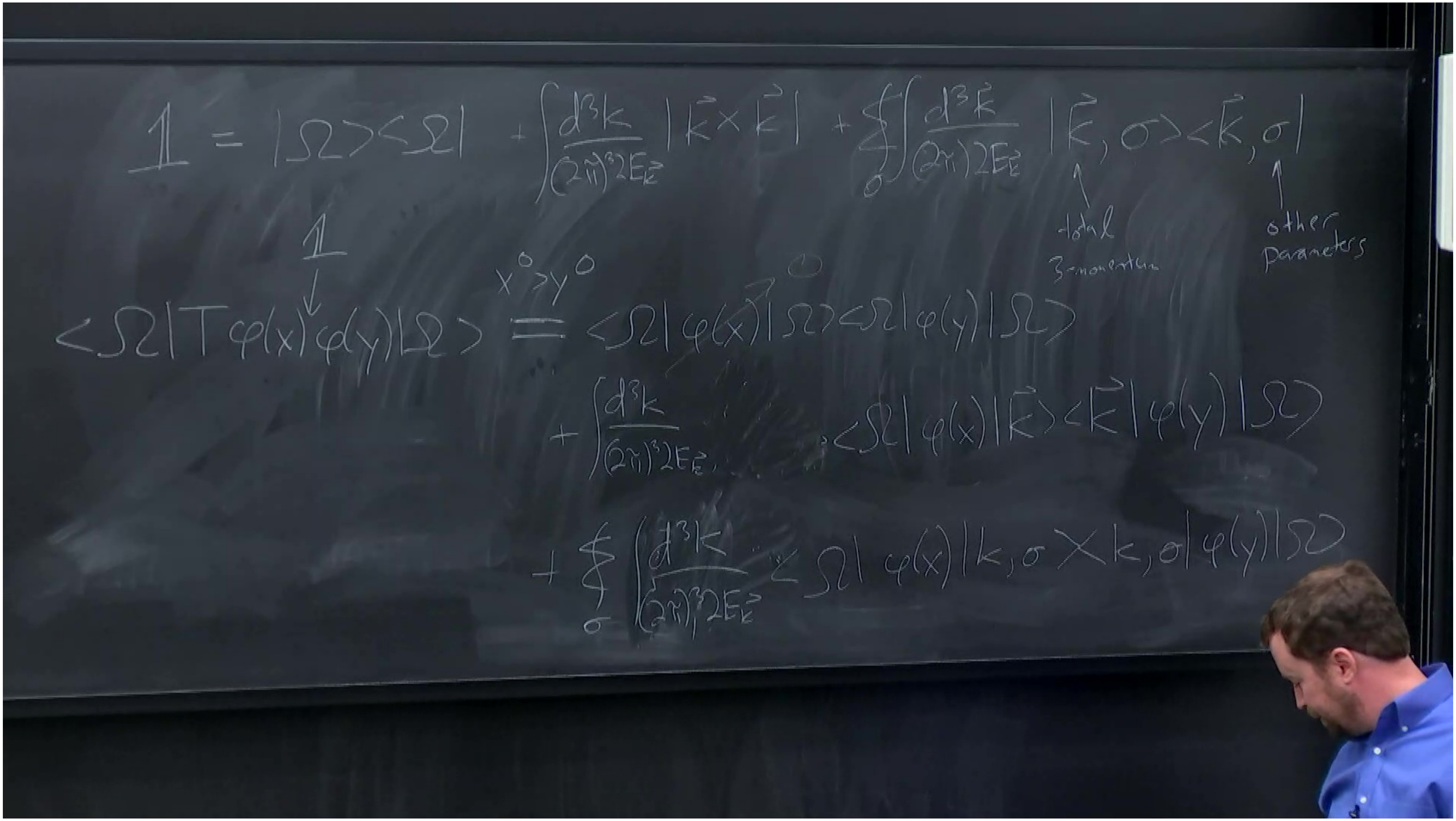
LSZ $\begin{cases} \bullet \ \langle \Omega | \varphi(x) | \Omega \rangle = 0 \\ \bullet \ \langle k | \varphi(x) | \Omega \rangle = e^{ik\cdot x} \end{cases}$

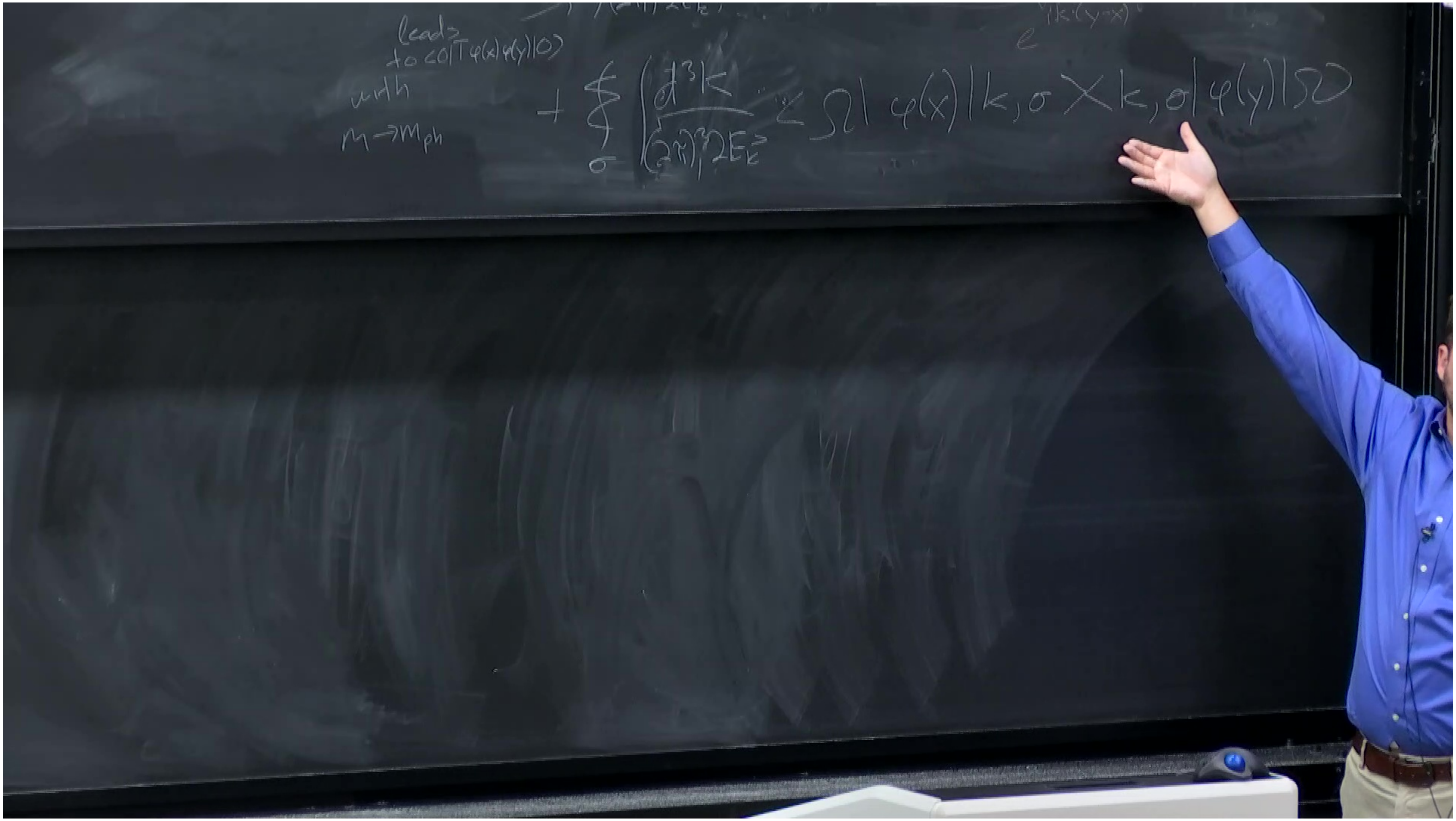
- no bound states

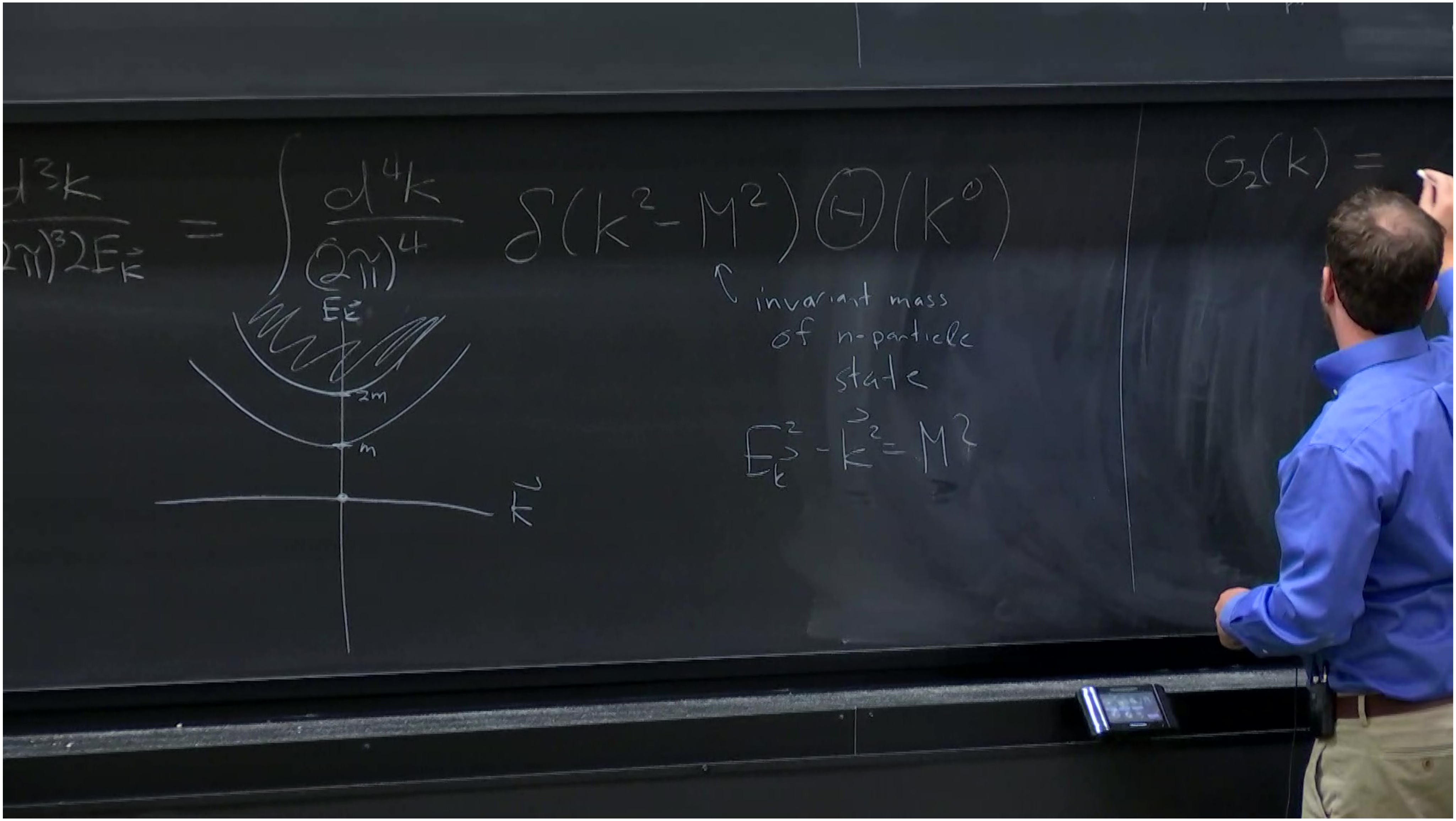
  2-particle states with $E < 2m$

$1 =$

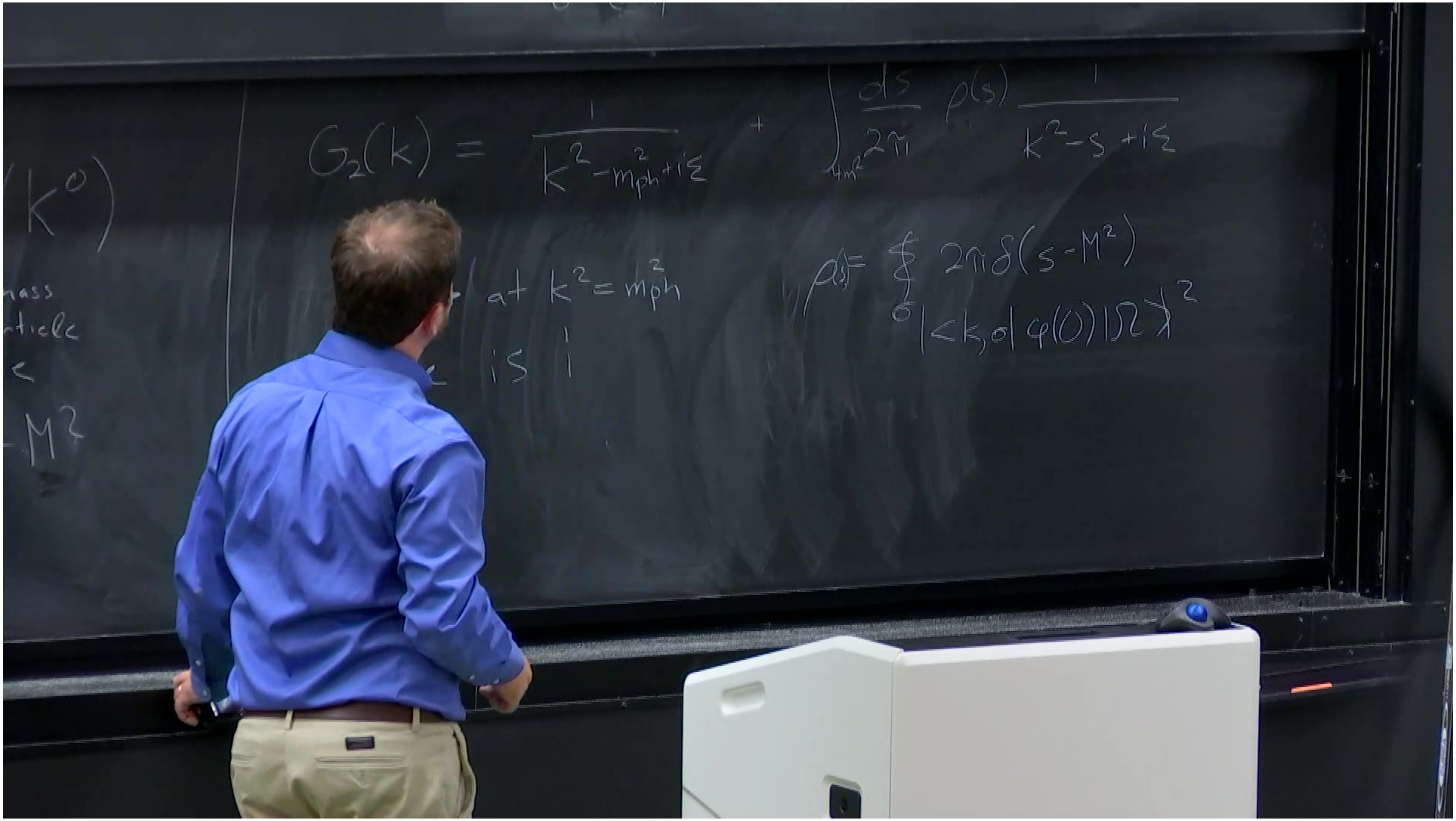

total
3-momentum
other
parameters


$\mathbb{1} = |\Omega\rangle\langle\Omega| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} |\vec{k}\rangle\langle\vec{k}| + \sum_\sigma \int \frac{d^3\vec{k}}{(2\pi)^3 2E_{\vec{k}}} |\vec{k},\sigma\rangle\langle\vec{k},\sigma|$
total 3-momentum
other parameters
$x^0 > y^0$
$\langle\Omega|T\varphi(x)\varphi(y)|\Omega\rangle = \langle\Omega|\varphi(x)|\Omega\rangle\langle\Omega|\varphi(y)|\Omega\rangle$
$+ \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \langle\Omega|\varphi(x)|\vec{k}\rangle\langle\vec{k}|\varphi(y)|\Omega\rangle$
$+ \sum_\sigma \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \langle\Omega|\varphi(x)|k,\sigma\rangle\langle k,\sigma|\varphi(y)|\Omega\rangle$


leads to ⟨0|Tφ(x)φ(y)|0⟩
with
m → m_ph
+ Σ_σ ∫ d³k / (2π)³ 2E_k ⟨Ω| φ(x)|k,σ⟩⟨k,σ| φ(y)|Ω⟩



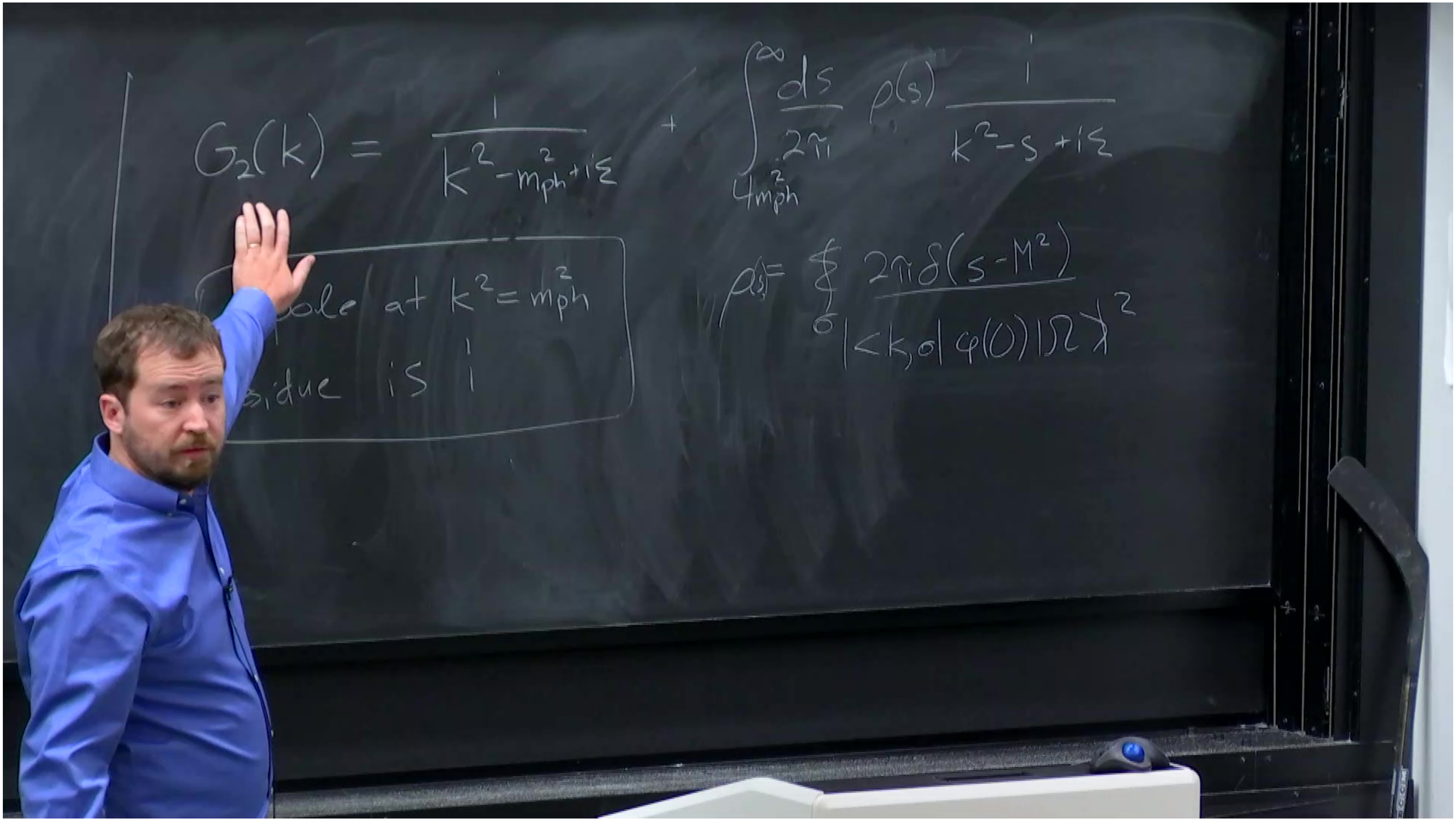
$\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} \delta(k^2 - M^2)\Theta(k^0)$
invariant mass
of n-particle
state
$E_{\vec{k}}^2 - \vec{k}^2 = M^2$
$E_{\vec{k}}$
2m
m
$\vec{k}$
$G_2(k) =$

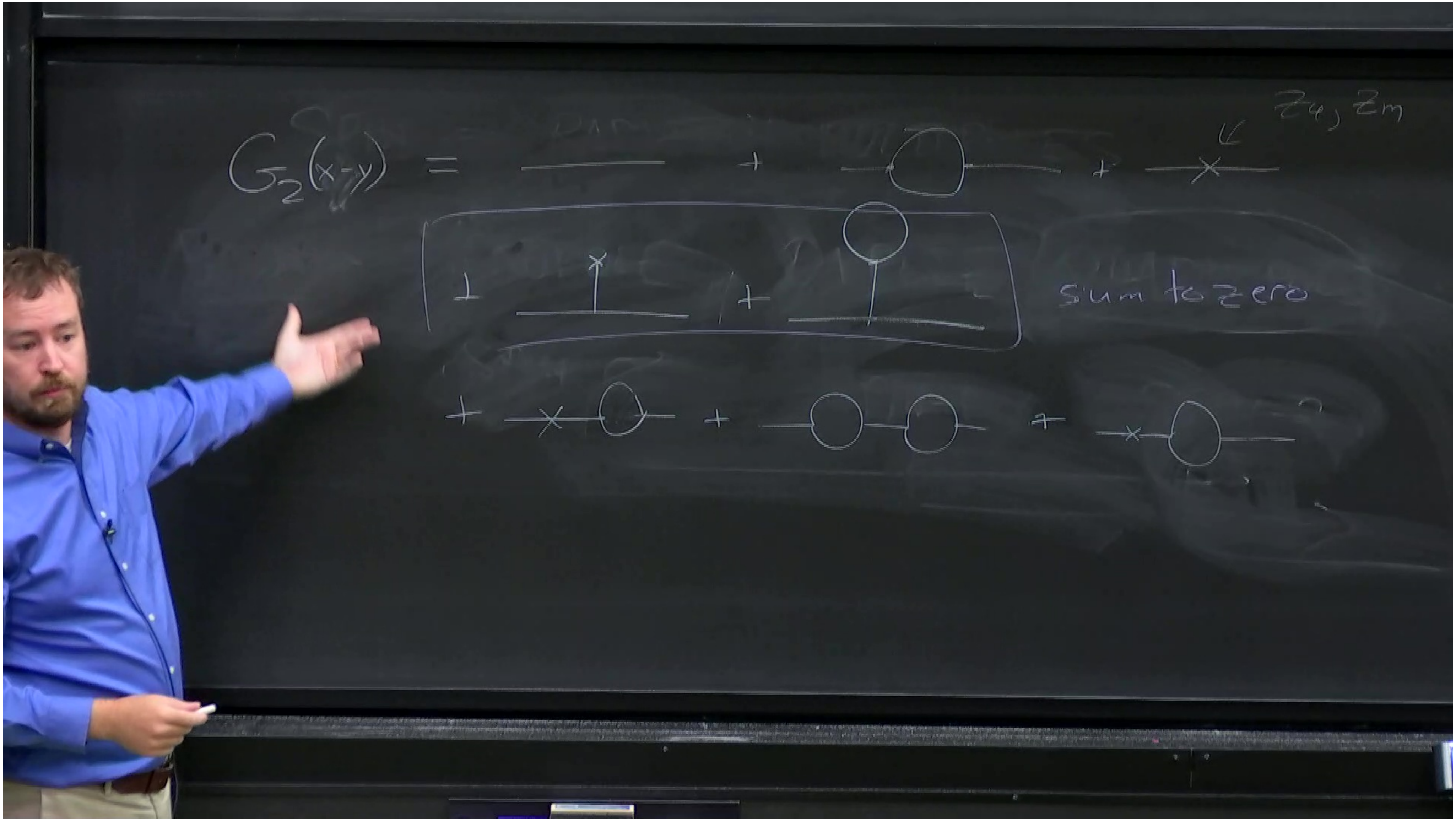

$G_2(k) = \frac{1}{k^2 - m_{ph}^2 + i\varepsilon} + \int_{4m^2} \frac{ds}{2\pi} \rho(s) \frac{1}{k^2 - s + i\varepsilon}$
at $k^2 = m_{ph}^2$
is i
$\rho(s) = \sum_\sigma 2\pi\delta(s - M^2) |\langle k, \sigma | \varphi(0) | \Omega \rangle|^2$

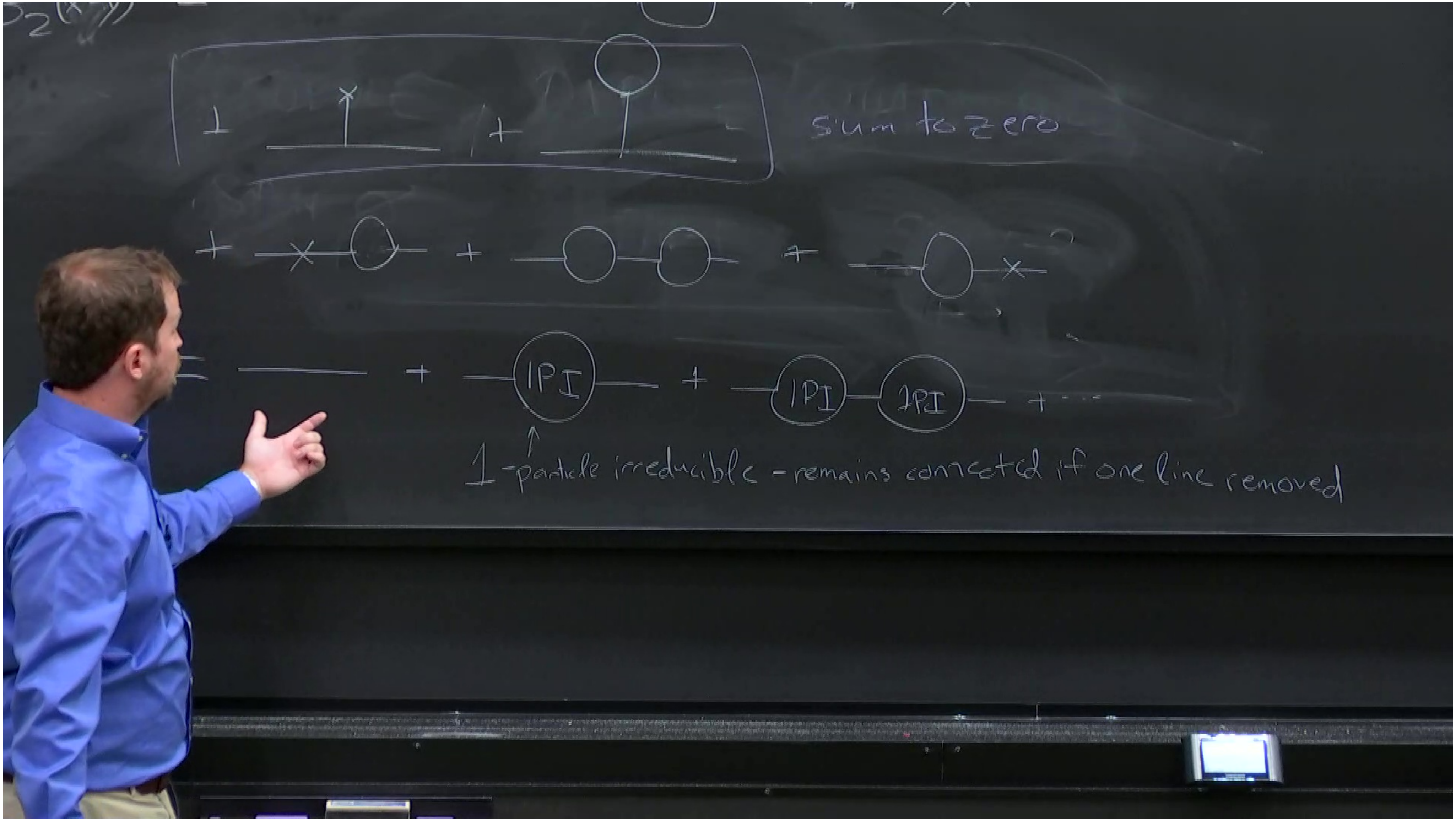
$$\langle 0 | \varphi(x)\varphi(y) | 0 \rangle =$$

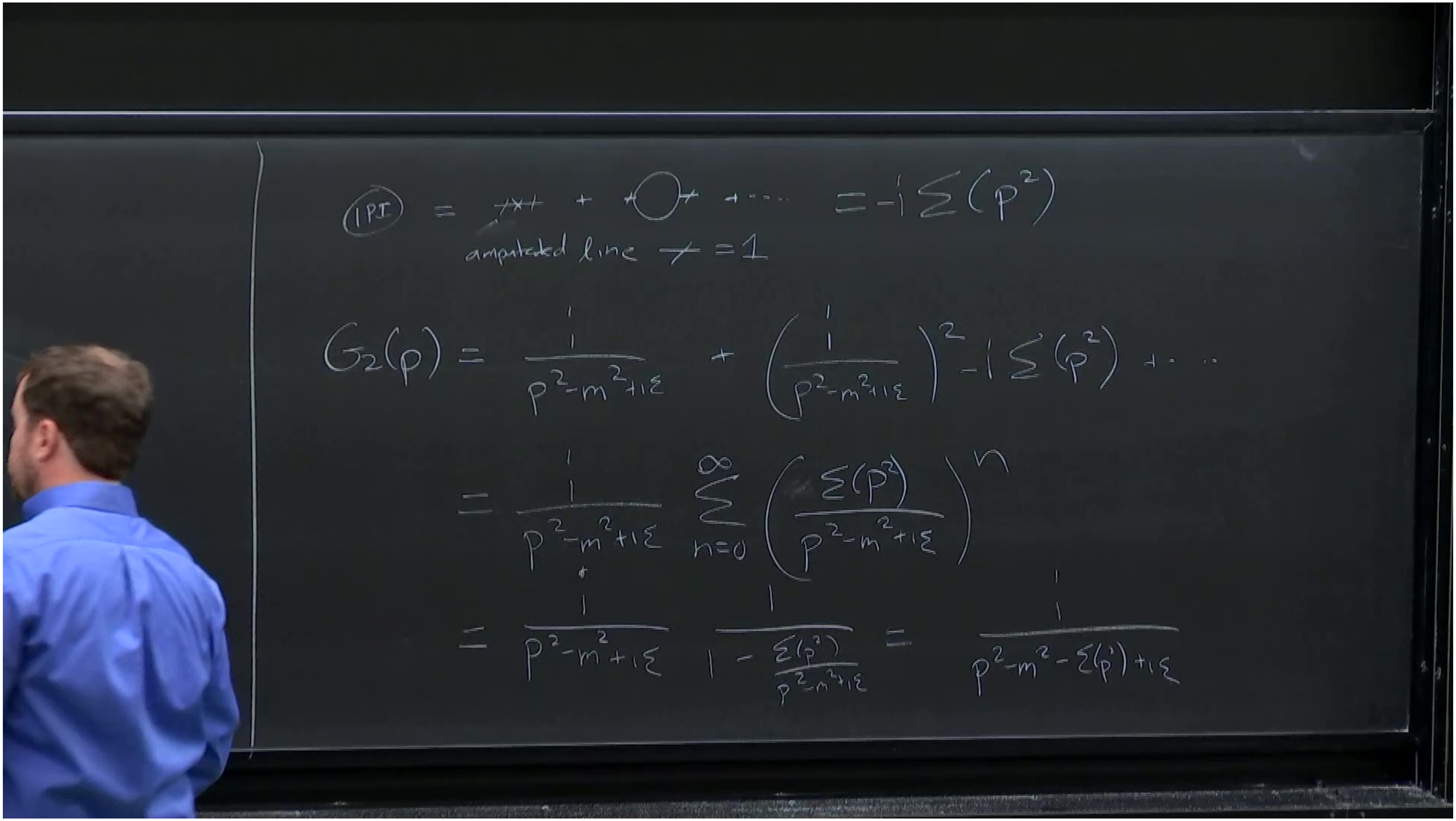
$$\int \frac{d^3k}{(2\pi)^3 2E_k} \int \frac{d^3p}{(2\pi)^3 2E_p} \langle 0 | a_{\vec{k}} e^{-ik\cdot x} a^\dagger_{\vec{p}} e^{+ip\cdot x} | 0 \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} e^{ip\cdot(x-y)}$$


$G_2(k) = \frac{i}{k^2 - m_{ph}^2 + i\varepsilon} + \int_{4m_{ph}^2}^{\infty} \frac{ds}{2\pi}\, \rho(s) \frac{i}{k^2 - s + i\varepsilon}$
pole at $k^2 = m_{ph}^2$
sidue is i
$\rho(s) = \sum_\sigma 2\pi\delta(s - M^2)\, |\langle k,\sigma|\varphi(0)|\Omega\rangle|^2$


G2(x-y) =
Z4, Zm
sum to zero


sum to zero
1PI
1PI
1PI
1-particle irreducible - remains connected if one line removed


1PI = + + ... = $-i\Sigma(p^2)$
amputated line = 1
$G_2(p) = \frac{i}{p^2-m^2+i\varepsilon} + \left(\frac{i}{p^2-m^2+i\varepsilon}\right)^2 (-i\Sigma(p^2)) + \cdots$
$= \frac{i}{p^2-m^2+i\varepsilon} \sum_{n=0}^{\infty} \left(\frac{\Sigma(p^2)}{p^2-m^2+i\varepsilon}\right)^n$
$= \frac{i}{p^2-m^2+i\varepsilon} \frac{1}{1-\frac{\Sigma(p^2)}{p^2-m^2+i\varepsilon}} = \frac{i}{p^2-m^2-\Sigma(p^2)+i\varepsilon}$

1-particle irreducible - remains connected if one lin

$$\frac{i}{p^2 - m^2 - \Sigma(p^2) + i\varepsilon}$$

Assume $m^2 = m_{ph}^2$

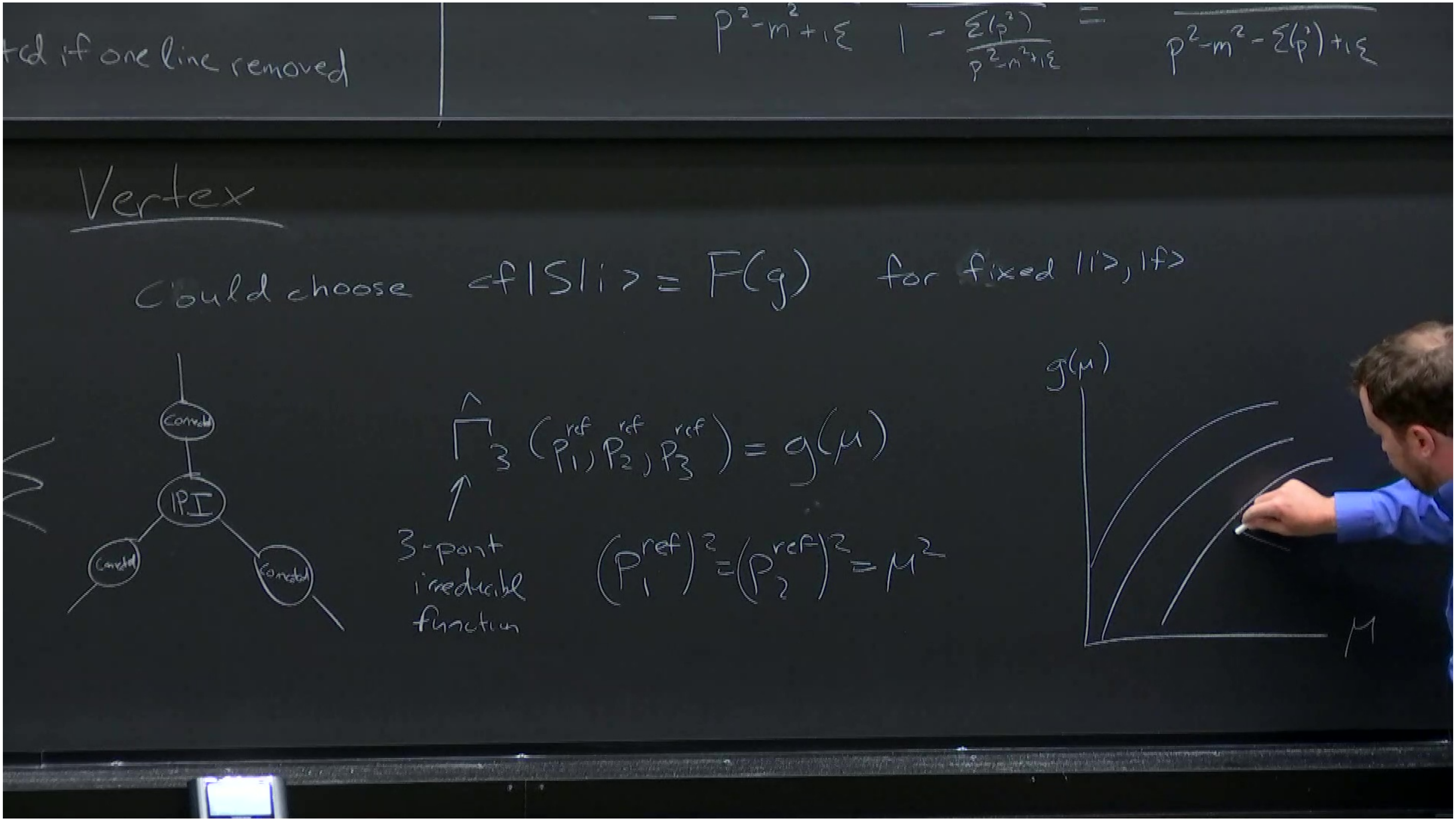
pole at $p^2 = m_{ph}^2$

residue i

$$\Sigma(m_{ph}^2) = 0$$

$$\Sigma'(m_{ph}^2) = 0$$

$$\left.\frac{d\Sigma(p^2)}{dp^2}\right|_{m_{ph}^2} = 0$$

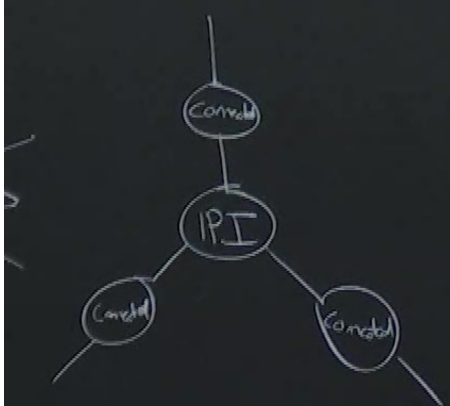

if one line removed
p^2 - m^2 + iε
1 - Σ(p^2) / (p^2 - m^2 + iε)
p^2 - m^2 - Σ(p^2) + iε
Vertex
Could choose <f|S|i> = F(g) for fixed |i>, |f>
Connected
1PI
Γ̂_3 (p_1^ref, p_2^ref, p_3^ref) = g(μ)
3-point irreducible function
(p_1^ref)^2 = (p_2^ref)^2 = μ^2
g(μ)
μ

d if one line removed

$$- \frac{\quad}{p^2-m^2+i\epsilon} \qquad \frac{\quad}{1-\frac{\Sigma(p^2)}{p^2-m^2+i\epsilon}} = \frac{\quad}{p^2-m^2-\Sigma(p^2)+i\epsilon}$$

Vertex

Could choose $\langle f|S|i\rangle = F(g)$ for fixed $|i\rangle, |f\rangle$

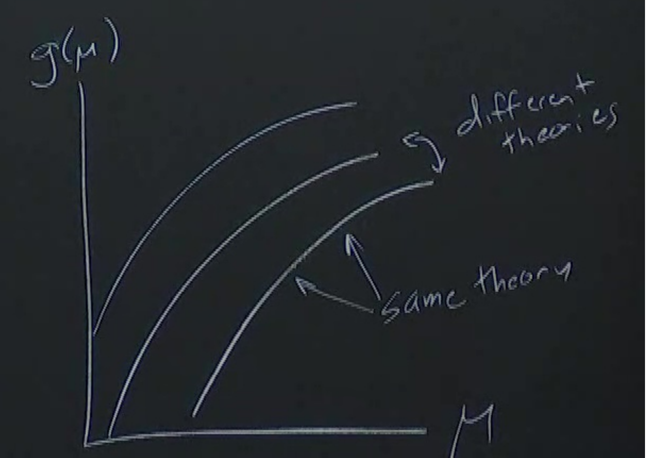


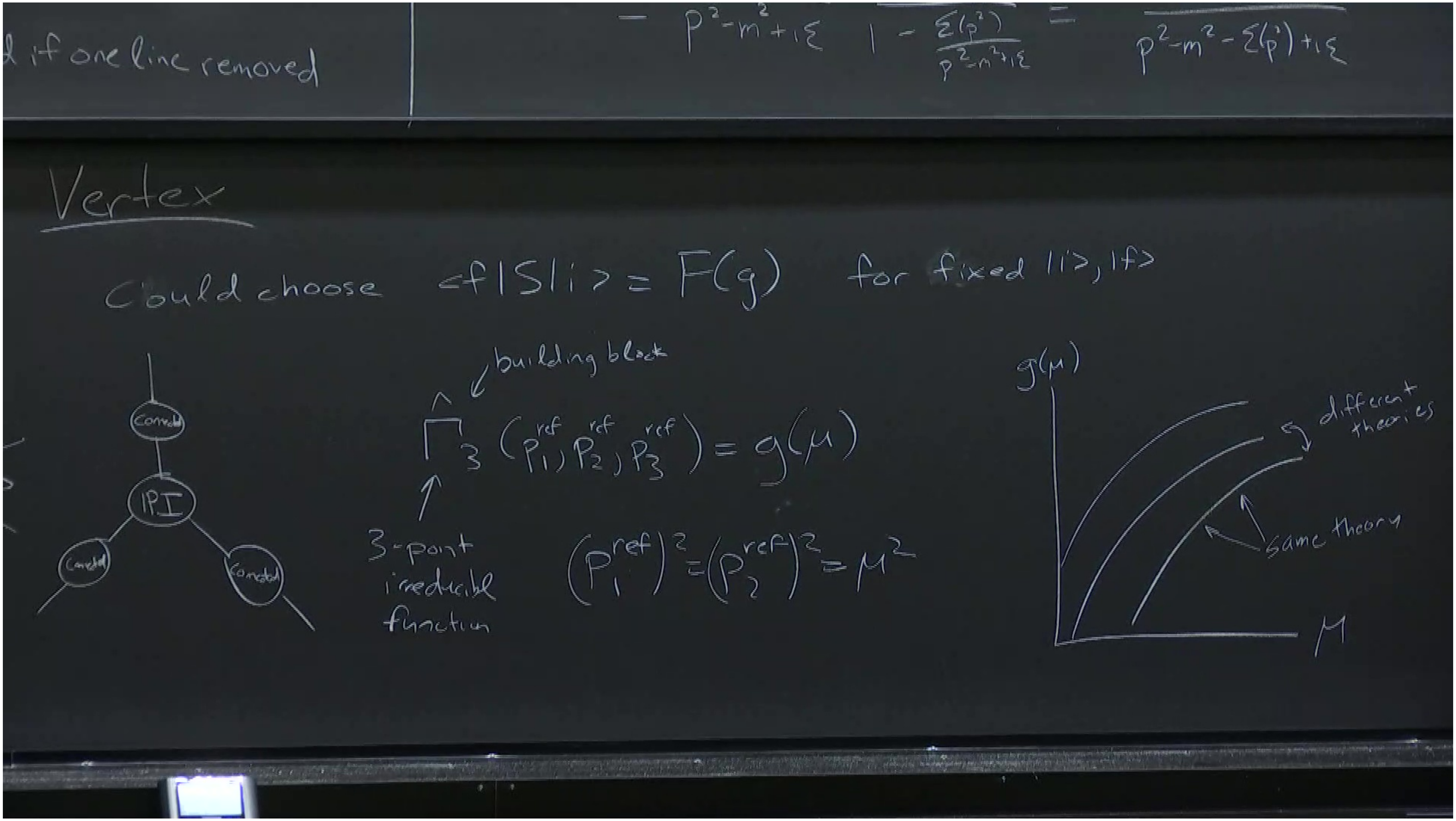
building block

$$\hat{\Gamma}_3(p_1^{ref}, p_2^{ref}, p_3^{ref}) = g(\mu)$$

3-point irreducible function

$$(p_1^{ref})^2 = (p_2^{ref})^2 = \mu^2$$




d if one line removed
$- p^2-m^2+i\epsilon \quad 1 - \frac{\Sigma(p^2)}{p^2-m^2+i\epsilon} = \quad p^2-m^2-\Sigma(p^2)+i\epsilon$
Vertex
Could choose $\langle f|S|i\rangle = F(g)$ for fixed $|i\rangle, |f\rangle$
connected
1PI
building block
$\hat{\Gamma}_3(p_1^{ref}, p_2^{ref}, p_3^{ref}) = g(\mu)$
3-point irreducible function
$(p_1^{ref})^2 = (p_2^{ref})^2 = \mu^2$
$g(\mu)$
different theories
same theory
μ