Title: Quantum Theory - Lecture 221003

Speakers:

Collection: Quantum Theory (2022-2023)

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Feynman diagrams + renormalization

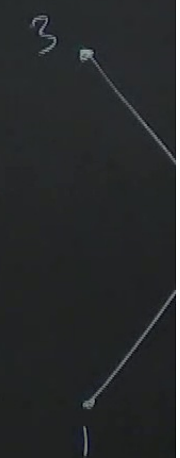
$$\langle f|S|i\rangle \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftrightarrow{\text{Dyson}} \langle 0|T\varphi_1\cdots\varphi_n|0\rangle \xleftrightarrow{\text{Wick}} \Delta_F$$

FD FD

FD

No interference between completely connected + disconnected diagrams

$\mathcal{L}_{int}$



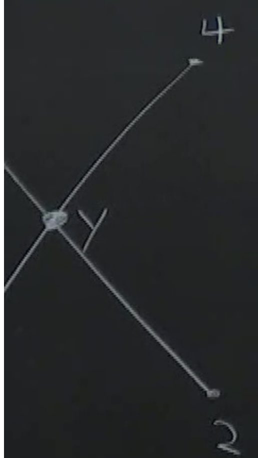
$$= -\frac{\lambda}{4!}\varphi^4$$

$$\Delta_{xy} = \Delta_F(x-y)$$

$$\times_y = -i\lambda\int d^4y$$

$$\frac{1}{S} = \frac{\text{number of Wick contractions}}{(\text{factorials from vertices})(\text{factorials from exp})}$$

$$\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle = \frac{\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4y\,\mathcal{L}]|0\rangle}{\langle 0|T\exp[i\int d^4y\,\mathcal{L}]|0\rangle}$$



$$= -i\lambda\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{3y}\Delta_{4y}$$

$$\langle f|S|i\rangle = i^4\int\left(\prod_i d^4x_i\, e^{-i\lambda_i x_i\cdot p_j}\left(\partial_j^2 + m_{ph}^2\right)\right)\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle$$

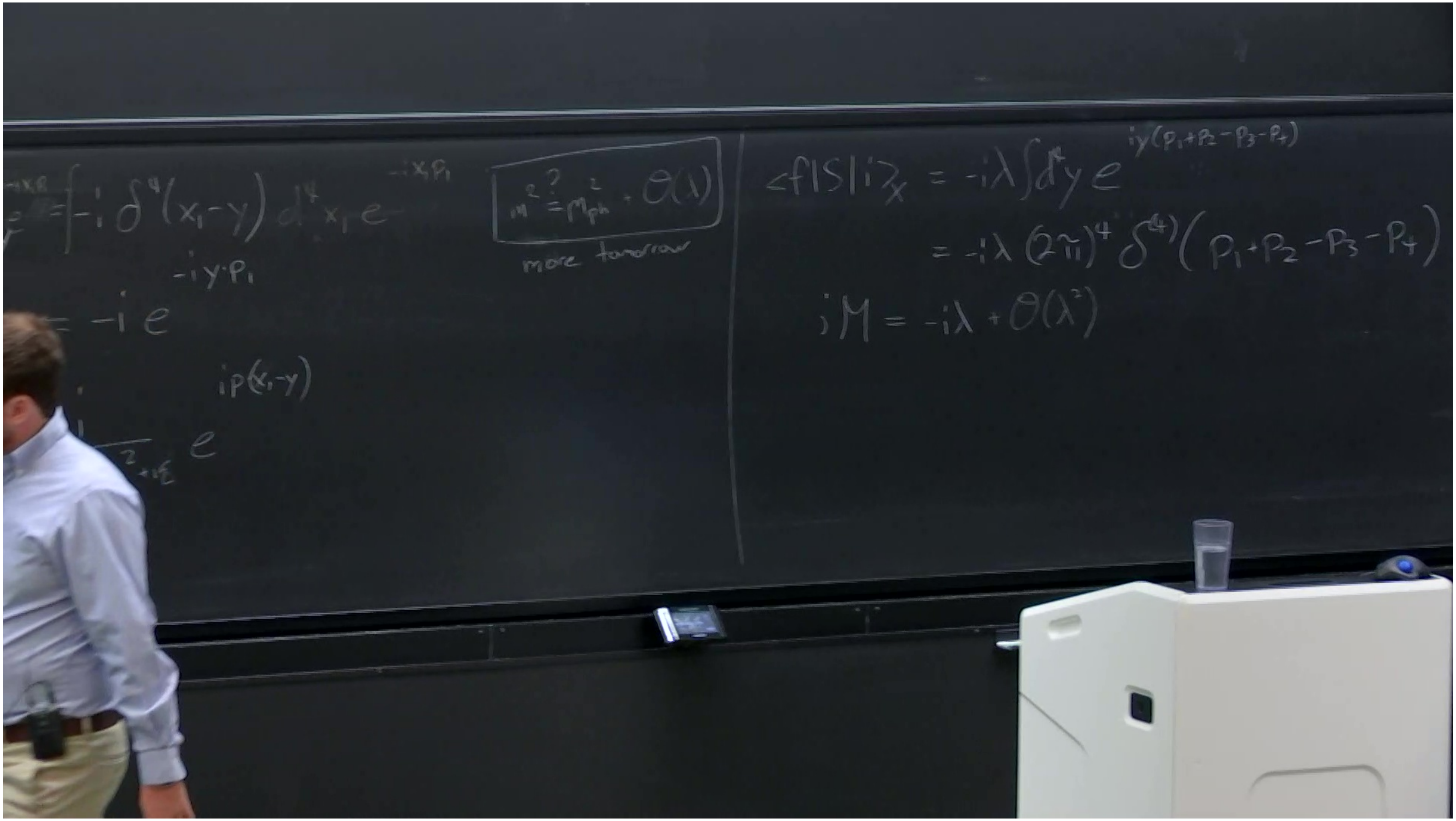
$$\int d^4x_1 \left(\partial_1^2 + m^2\right)\Delta_{1y}\, e^{-ix_1 p_1} = \int -i\,\delta^4(x_1-y)\, d^4x_1\, e^{-ix_1 p_1}$$

$$= -i\, e^{-iy\cdot p_1}$$

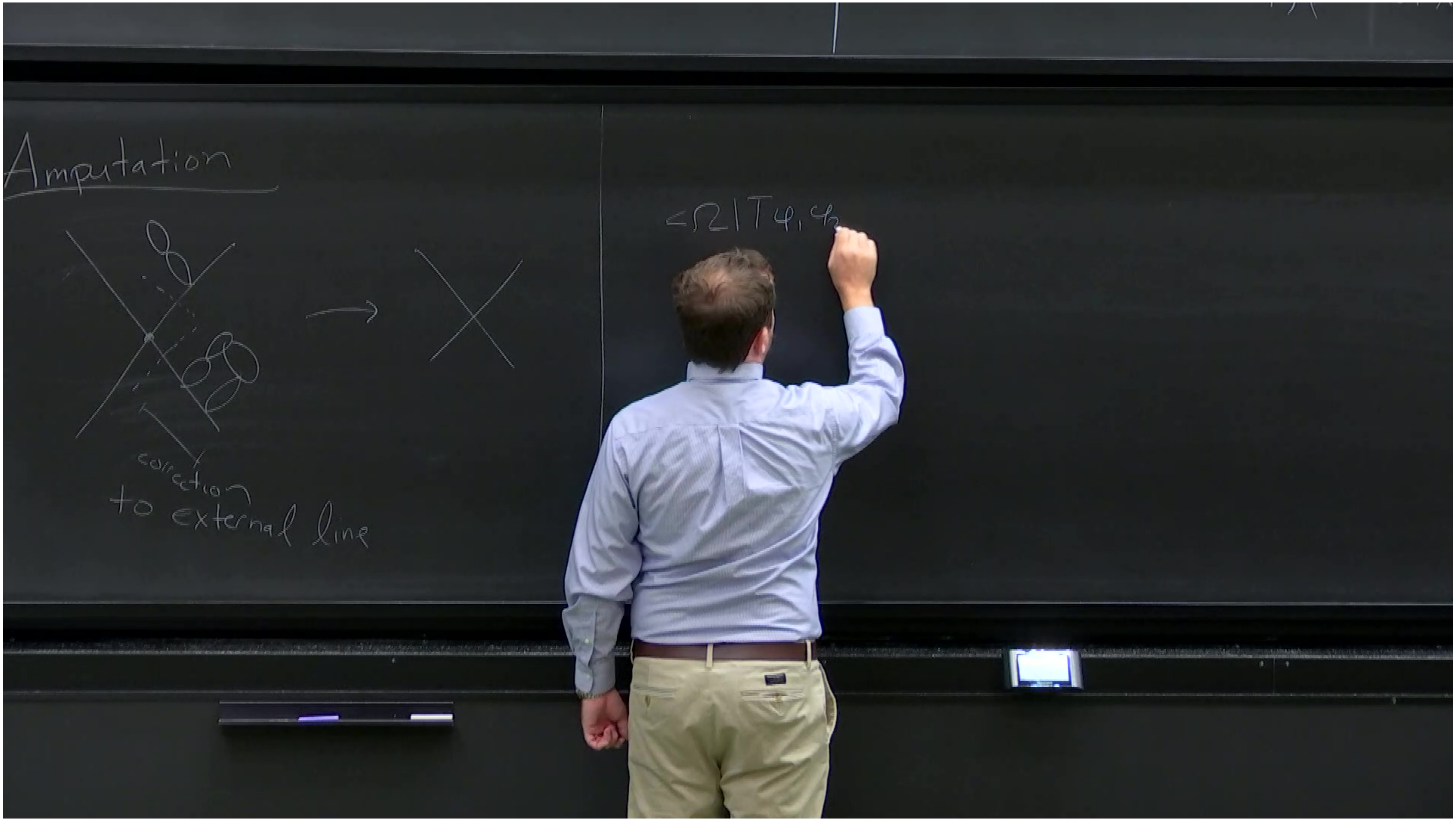
$$\boxed{m^2 \overset{?}{=} m_{ph}^2 + O(}$$

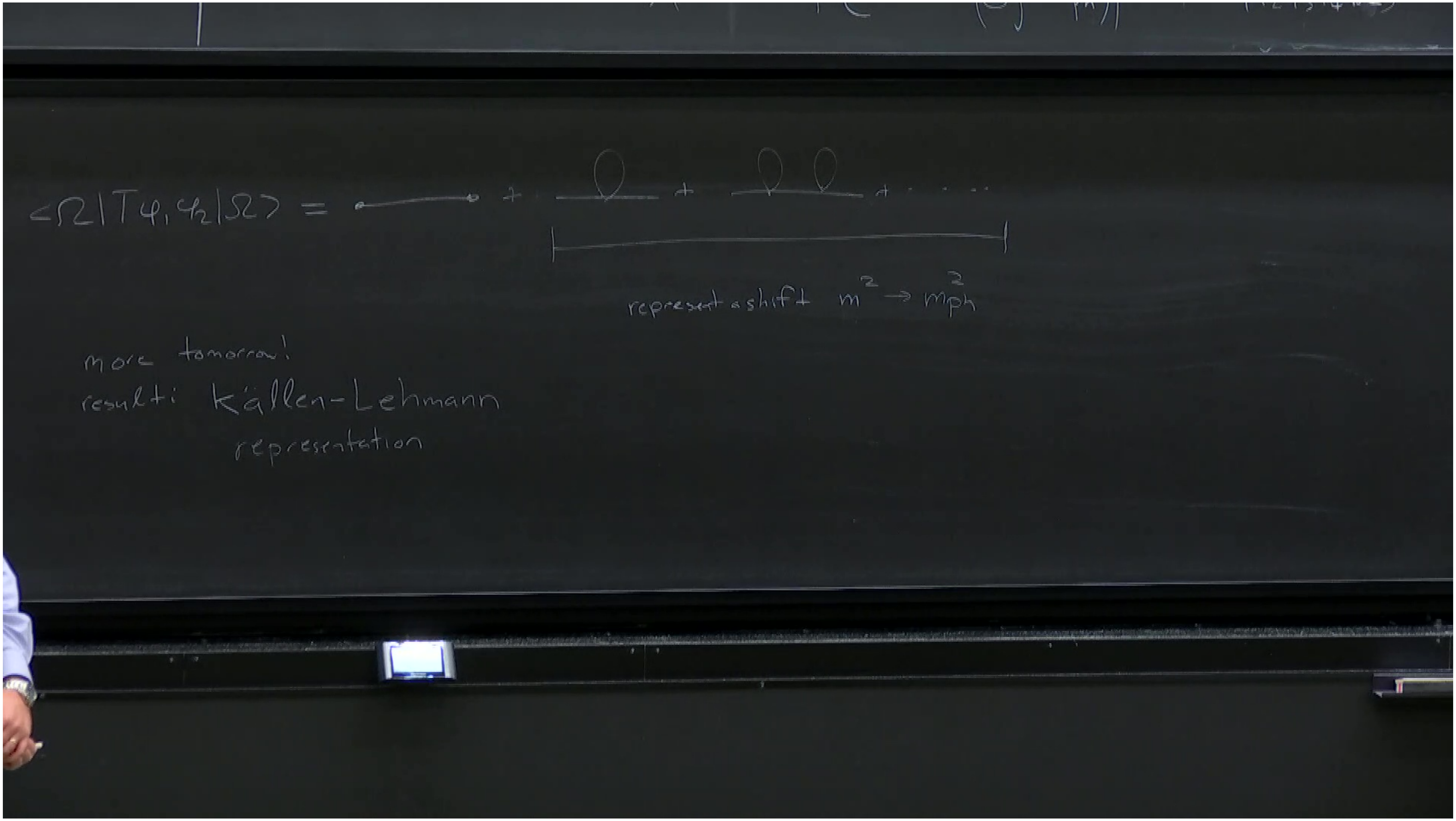
$$\Delta_{1y} = \int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2-m^2+i\epsilon}\, e^{ip(x_1-y)}$$

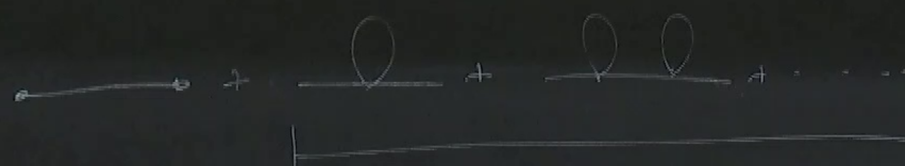

$-ix_1p_1$
$\boxed{m^2 \overset{?}{=} m_{ph}^2 + \mathcal{O}(\lambda)}$
more tomorrow
$\langle f|S|i\rangle_X = -i\lambda\int d^4y\, e^{iy(p_1+p_2-p_3-p_4)}$



m² ?= m_ph² + O(λ)
more tomorrow
⟨f|S|i⟩_X = −iλ ∫ d⁴y e^{iy(p₁+p₂−p₃−p₄)}
= −iλ (2π)⁴ δ⁽⁴⁾(p₁+p₂−p₃−p₄)
iM = −iλ + O(λ²)


Amputation
correction
to external line
<Ω|Tφ₁φ₂

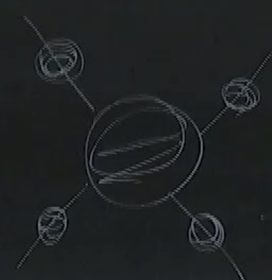

$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle =$ + + + ...
represent a shift $m^2 \to m_{ph}^2$
more tomorrow!
result: Källen-Lehmann
representation

$\langle \Omega | T \varphi_1 \varphi_2 | \Omega \rangle =$ 

represent a shift $m^2 \to m_{ph}^2$

more tomorrow!

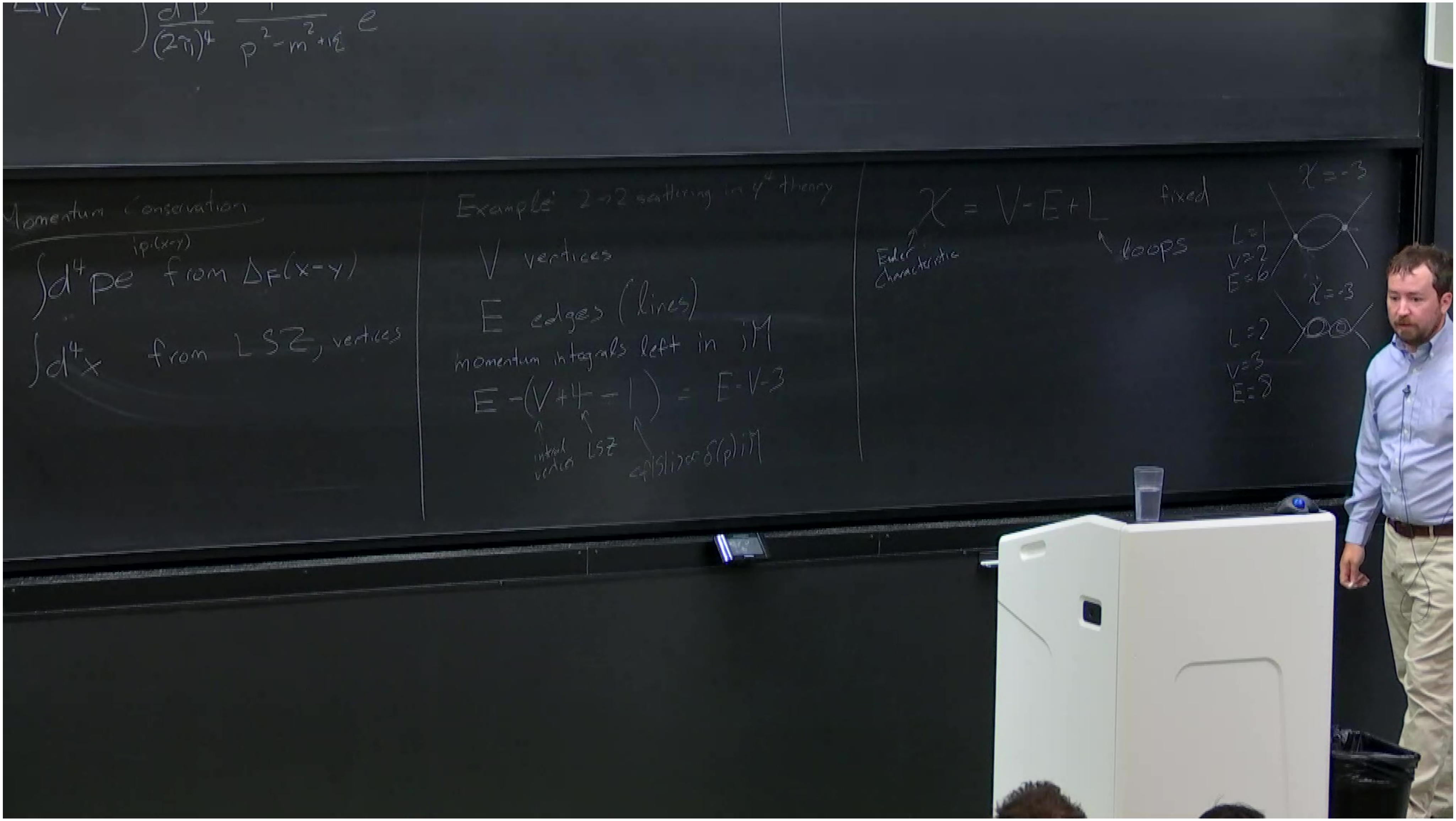
result: Källen-Lehmann representation

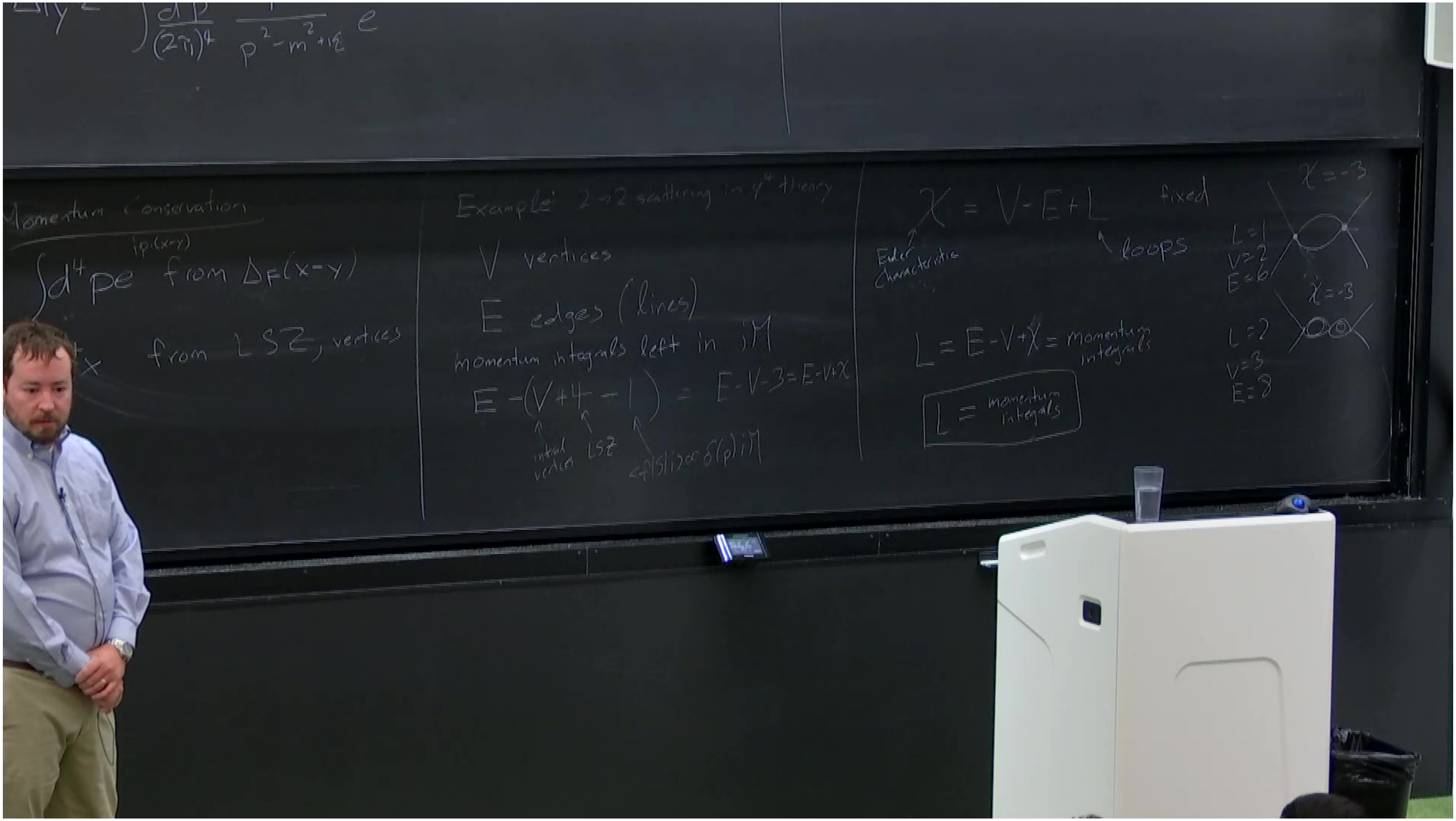


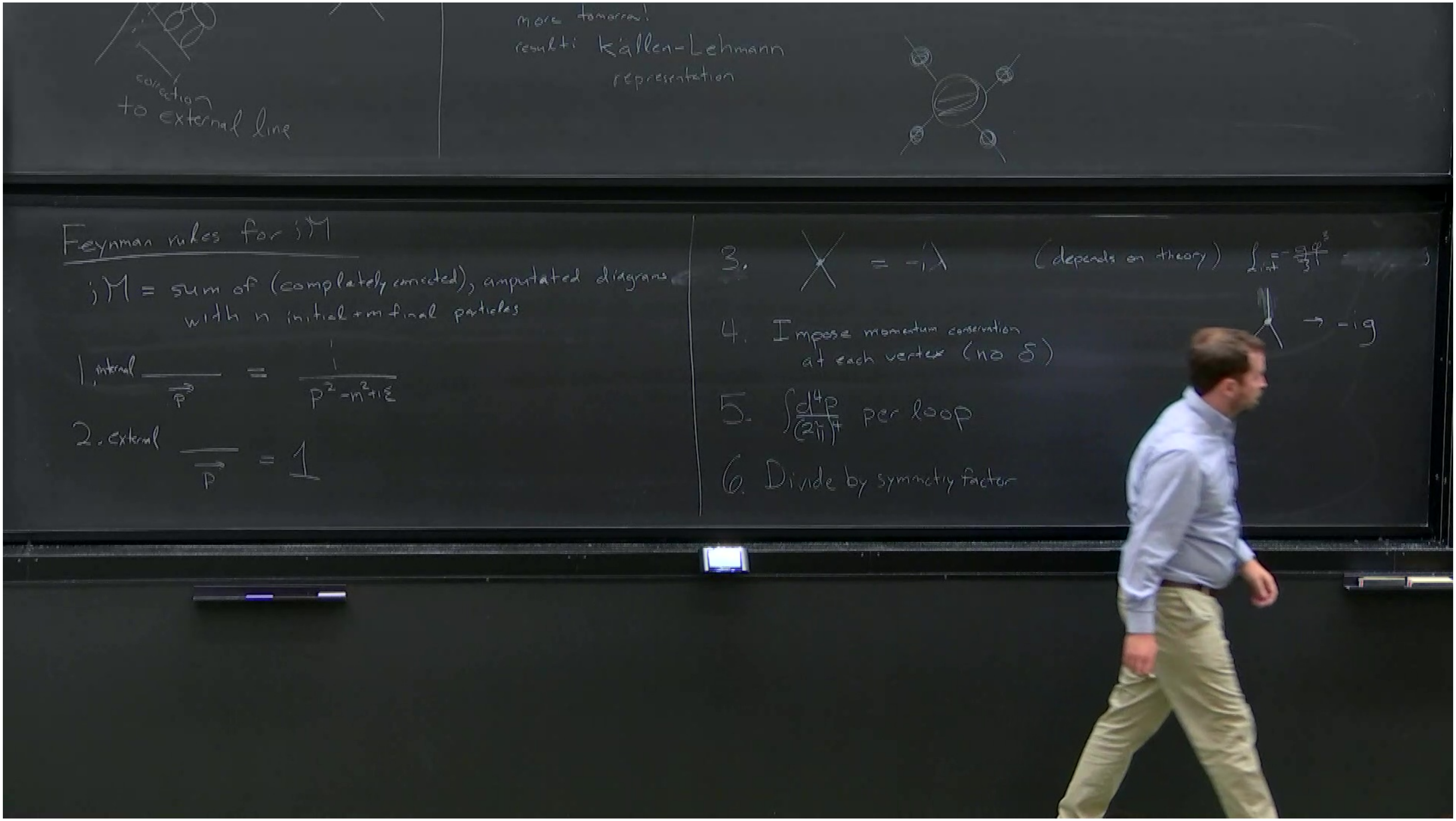
Momentum Conservation

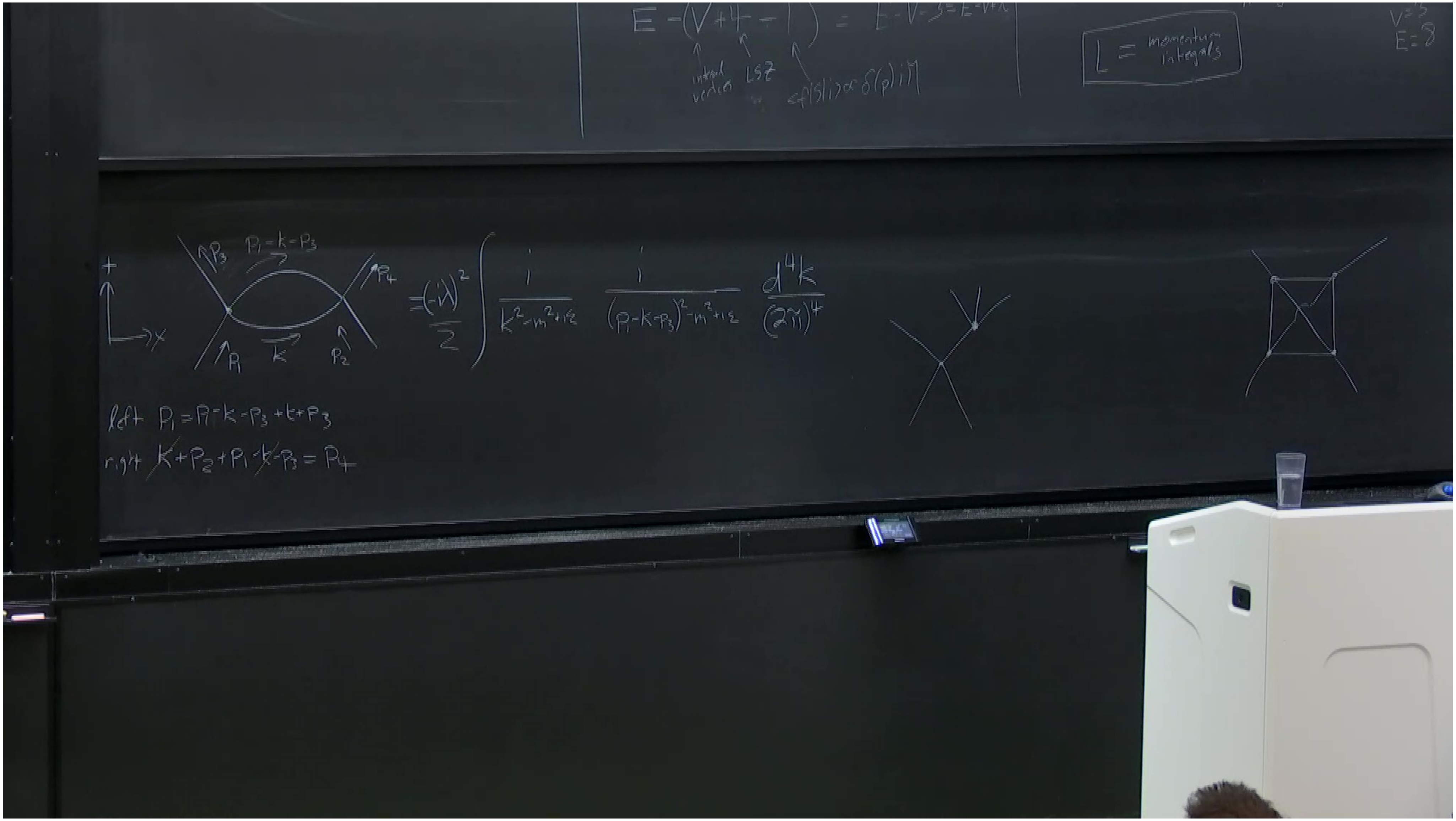
$\int d^4p\, e^{ip\cdot(x-y)}$ from $\Delta_F(x-y)$

$\int d^4x$ from LSZ, vertices

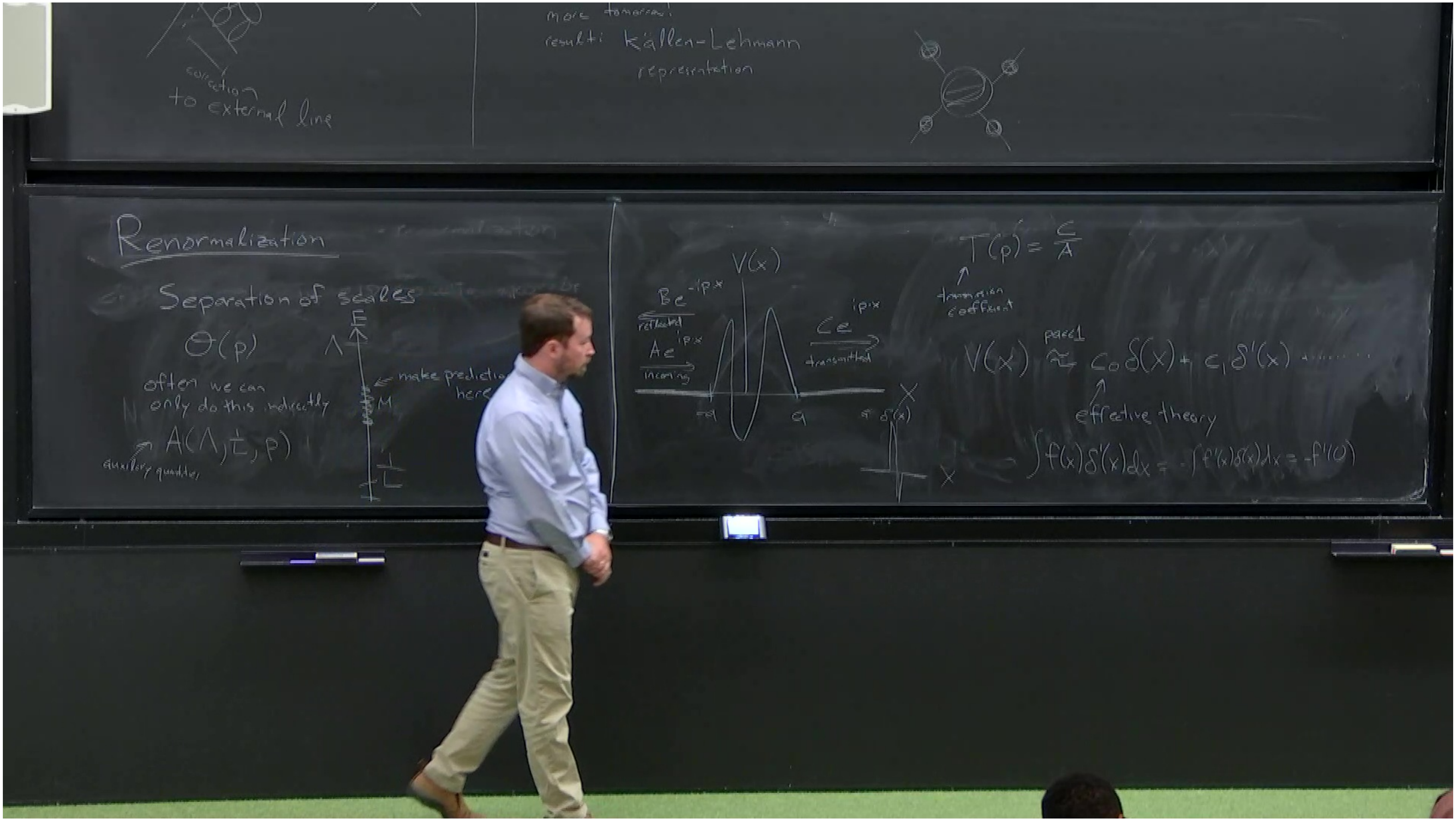

Momentum Conservation
∫d⁴p e^{ip·(x-y)} from Δ_F(x-y)
∫d⁴x from LSZ, vertices
Example: 2→2 scattering in φ⁴ theory
V vertices
E edges (lines)
momentum integrals left in iM
E − (V + 4 − 1) = E − V − 3
LSZ
χ = V − E + L
Euler characteristic
loops
fixed
χ = −3
L = 1
V = 2
E = 6
χ = −3
L = 2
V = 3
E = 8

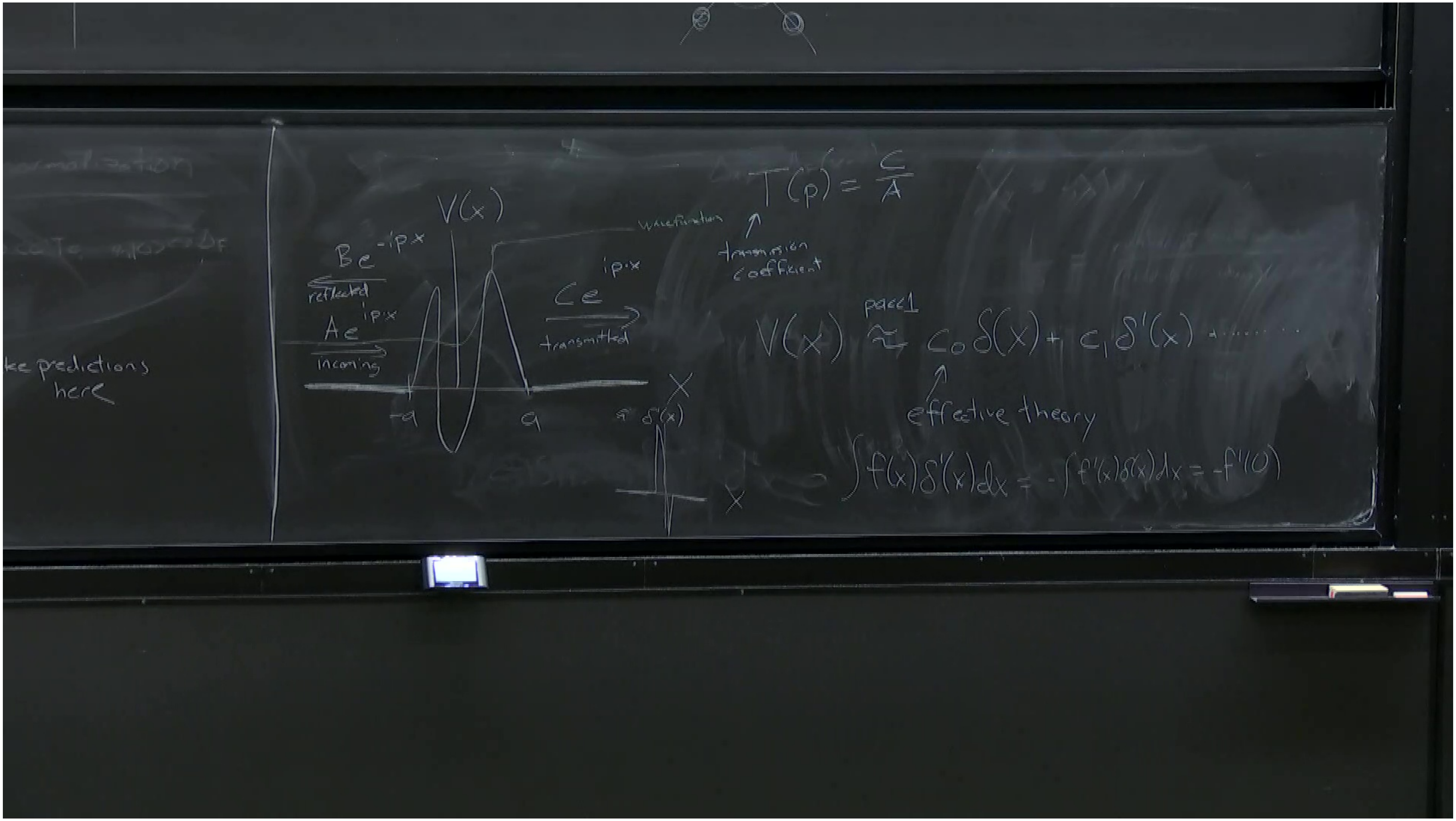

Momentum Conservation
from LSZ, vertices
Example: 2→2 scattering in φ⁴ theory
V vertices
E edges (lines)
momentum integrals left in iM
E-V-3=E-V+χ
χ = V-E+L
fixed
Euler characteristic
loops
L = E-V+χ = momentum integrals
L = momentum integrals
χ=-3
L=1
V=2
E=6
L=2
V=3
E=8

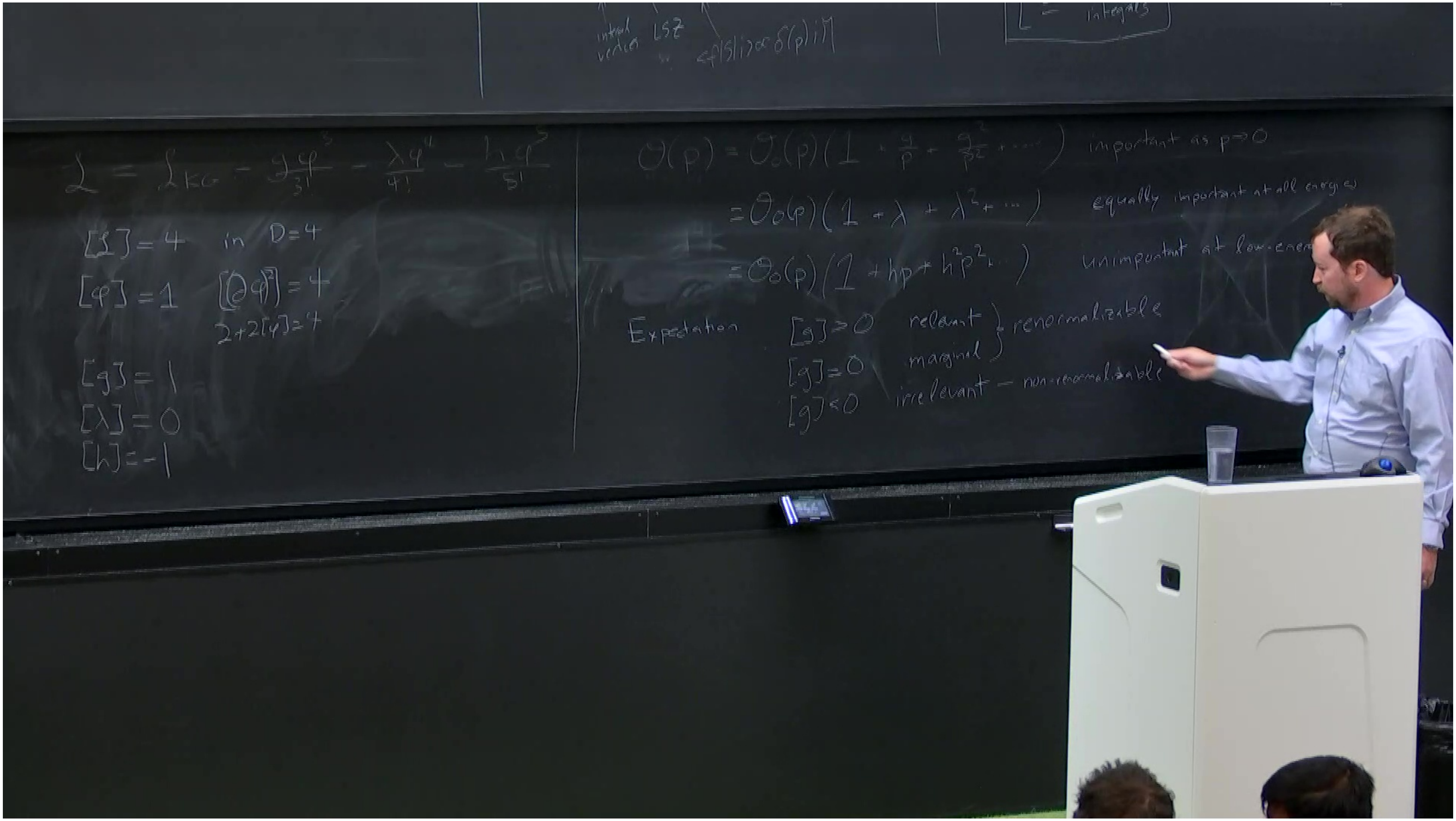

more tomorrow!
result: Källen-Lehmann
representation
correction
to external line
Feynman rules for iM
iM = sum of (completely connected), amputated diagrams
with n initial + m final particles
1. internal
p
= 1/(p^2 - m^2 + iε)
2. external
p
= 1
3.
= -iλ
(depends on theory)
→ -ig
4. Impose momentum conservation
at each vertex (no δ)
5. ∫d^4p/(2π)^4 per loop
6. Divide by symmetry factor



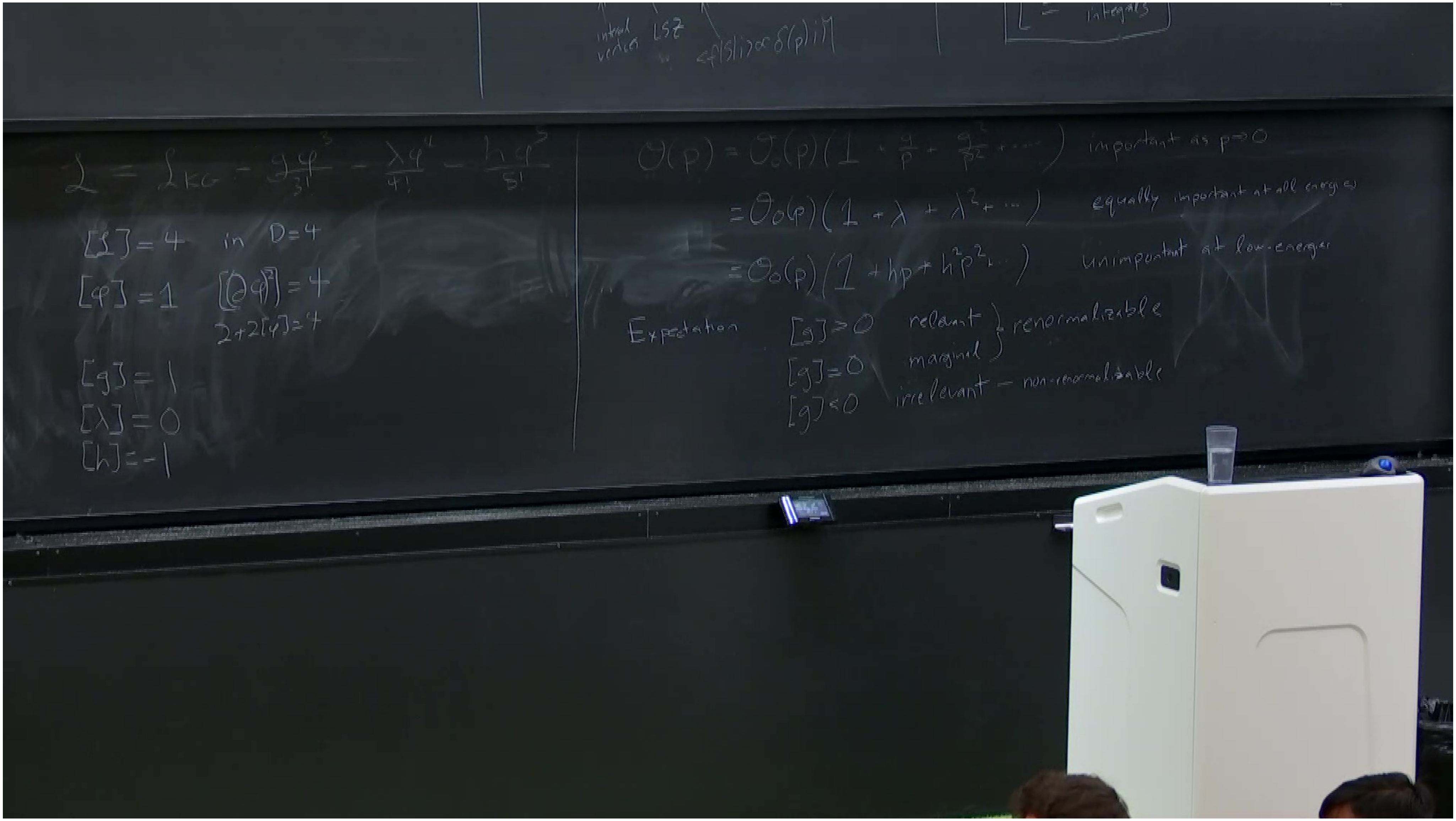
$E - (V + 4 - 1) = E - V - 3$
internal vertices
LSZ
$\langle f|S|i\rangle \propto \delta(p)\, i\mathcal{M}$
$L =$ momentum integrals
p_3
$p_1 - k - p_3$
p_4
p_1
k
p_2
t
x
$= \frac{(-i\lambda)^2}{2} \int \frac{i}{k^2 - m^2 + i\varepsilon} \frac{i}{(p_1 - k - p_3)^2 - m^2 + i\varepsilon} \frac{d^4k}{(2\pi)^4}$
left $p_1 = p_1 - k - p_3 + k + p_3$
right $\cancel{k} + p_2 + p_1 - \cancel{k} - p_3 = p_4$


more tomorrow!
result: Källen-Lehmann
representation
correction
to external line
Renormalization
Separation of scales
E
Θ(p)
Λ
often we can
only do this indirectly
A(Λ, 1/L, p)
auxiliary quantities
make prediction
here
M
1/L
V(x)
Be^{-ip·x}
reflected
Ae^{ip·x}
incoming
Ce^{ip·x}
transmitted
-a
a
δ(x)
T(p) = C/A
transmission
coefficient
part1
V(x) ≈ c₀δ(x) + c₁δ'(x) + ...
effective theory
∫f(x)δ'(x)dx = -∫f'(x)δ(x)dx = -f'(0)


T(p) = C/A
transmission coefficient
V(x)
Be^{-ip·x}
reflected
Ae^{ip·x}
incoming
Ce^{ip·x}
transmitted
-a
a
δ'(x)
V(x) ≈ c₀δ(x) + c₁δ'(x) + ...
effective theory
∫f(x)δ'(x)dx = -∫f'(x)δ(x)dx = -f'(0)
predictions
here



L = L_KG - gφ^3/3! - λφ^4/4! - hφ^5/5!
[L] = 4 in D=4
[φ] = 1 [(∂φ)^2] = 4
2+2[φ]=4
[g] = 1
[λ] = 0
[h] = -1
O(p) = O_0(p)(1 + g/p + g^2/p^2 + ...) important as p→0
= O_0(p)(1 + λ + λ^2 + ...) equally important at all energies
= O_0(p)(1 + hp + h^2p^2 ...) unimportant at low-ener
Expectation
[g]>0 relevant
[g]=0 marginal
[g]<0 irrelevant
renormalizable
non-renormalizable
integrals



intval vertex LSZ
$\langle f|S|i\rangle \propto \delta(p)\, i\mathcal{M}$
integrals
$\mathcal{L} = \mathcal{L}_{KG} - \frac{g\varphi^3}{3!} - \frac{\lambda\varphi^4}{4!} - \frac{h\varphi^5}{5!}$
$[\mathcal{L}] = 4$ in $D=4$
$[\varphi] = 1$ $[(\partial\varphi)^2] = 4$
$2+2[\varphi]=4$
$[g] = 1$
$[\lambda] = 0$
$[h] = -1$
$\mathcal{O}(p) = \mathcal{O}_0(p)\left(1 + \frac{g}{p} + \frac{g^2}{p^2} + \cdots\right)$ important as $p\to 0$
$= \mathcal{O}_0(p)\left(1 + \lambda + \lambda^2 + \cdots\right)$ equally important at all energies
$= \mathcal{O}_0(p)\left(1 + hp + h^2p^2 \cdots\right)$ unimportant at low-energies
Expectation $[g]>0$ relevant } renormalizable
$[g]=0$ marginal
$[g]<0$ irrelevant — non-renormalizable