Title: An Effective Field Theory for Large Oscillons

Speakers: Vasily Maslov

Series: Particle Physics

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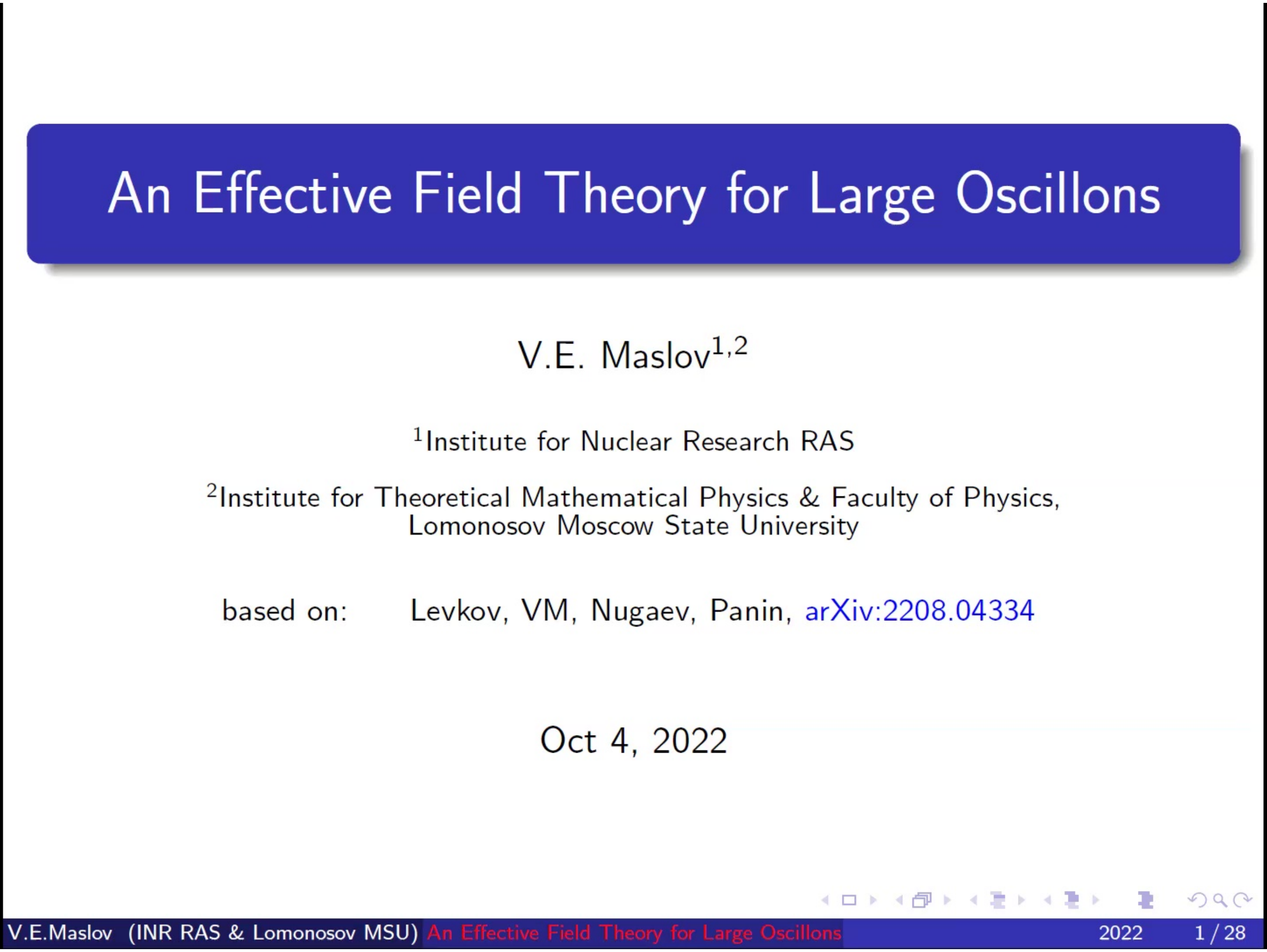
Abstract: Based on arXiv:2208.04334. We consider oscillons - localized, quasiperiodic, and extremely long-living classical solutions in models with real scalar fields. We develop their effective description in the limit of large size at finite field strength. Namely, we note that nonlinear long-range field configurations can be described by an effective complex field ?(t, x) which is related to the original fields by a canonical transformation. The action for ? has the form of a systematic gradient expansion. At every order of the expansion, such an effective theory has a global U(1) symmetry and hence a family of stationary nontopological solitons - oscillons. The decay of the latter objects is a nonperturbative process from the viewpoint of the effective theory. Our approach gives an intuitive understanding of oscillons in full nonlinearity and explains their longevity. Importantly, it also provides reliable selection criteria for models with long-lived oscillons. This technique is more precise in the nonrelativistic limit, in the notable cases of nonlinear, extremely long-lived, and large objects, and also in lower spatial dimensions. We test the effective theory by performing explicit numerical simulations of a (d+1)-dimensional scalar field with a plateau potential.

Zoom link: https://pitp.zoom.us/j/98801138609?pwd=VUJsZm41bnpBQzFoUEFwcUV6SG5Xdz09



An Effective Field Theory for Large Oscillons

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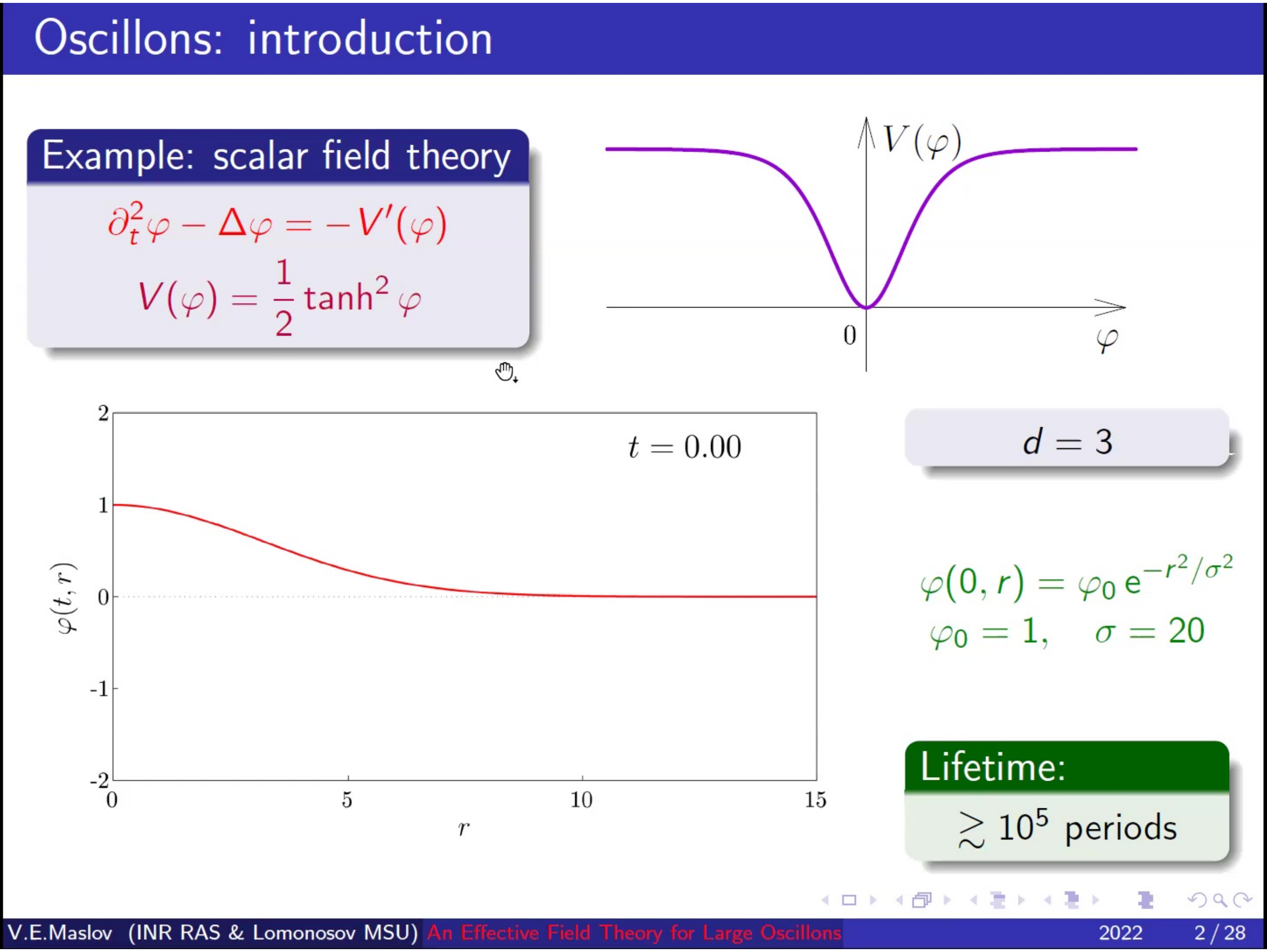
based on: Levkov, VM, Nugaev, Panin, arXiv:2208.04334

Oct 4, 2022



Oscillons: introduction

Example: scalar field theory

$$\partial_t^2 \varphi - \Delta\varphi = -V'(\varphi)$$

$$V(\varphi) = \frac{1}{2}\tanh^2\varphi$$

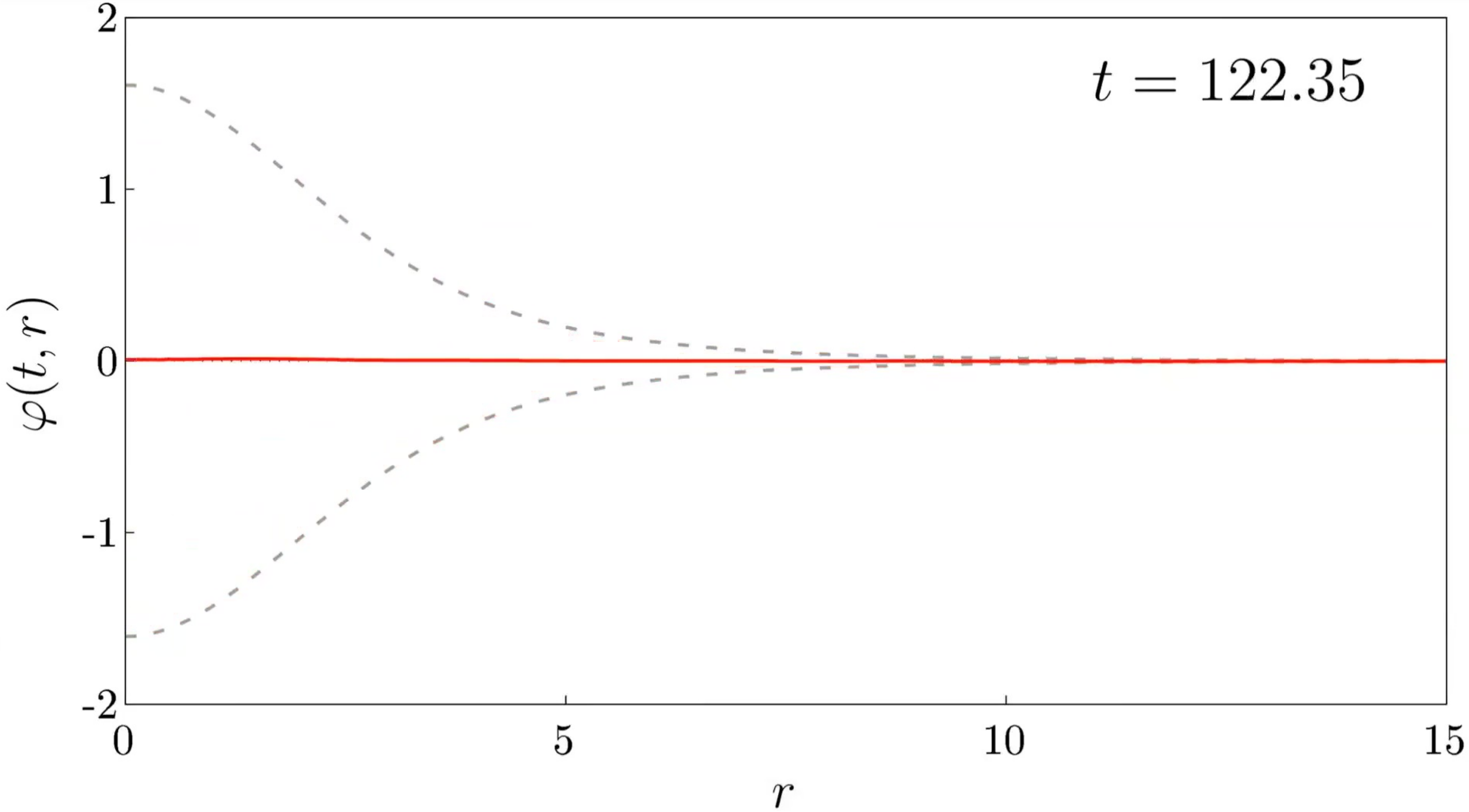
$d = 3$

$$\varphi(0, r) = \varphi_0\, e^{-r^2/\sigma^2}$$

$$\varphi_0 = 1, \quad \sigma = 20$$

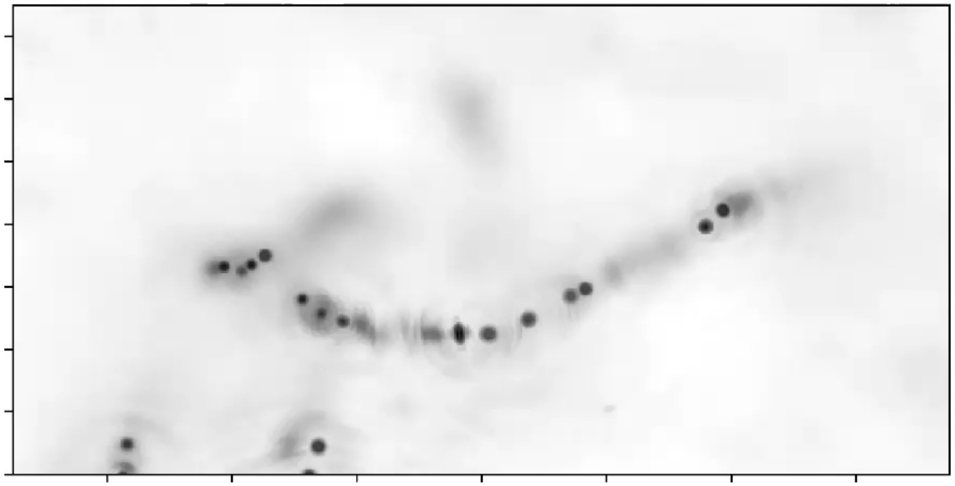
Lifetime:

$\gtrsim 10^5$ periods


$t = 122.35$
2
1
0
-1
-2
$\varphi(t,r)$
0
5
10
15
r

Introduction: oscillons in cosmology

- nucleate during generation of axion or ultra-light DM



Kolb, Tkachev '94

Vaquero, Redondo, Stadler '19

Buschmann, Foster, Safdi '20

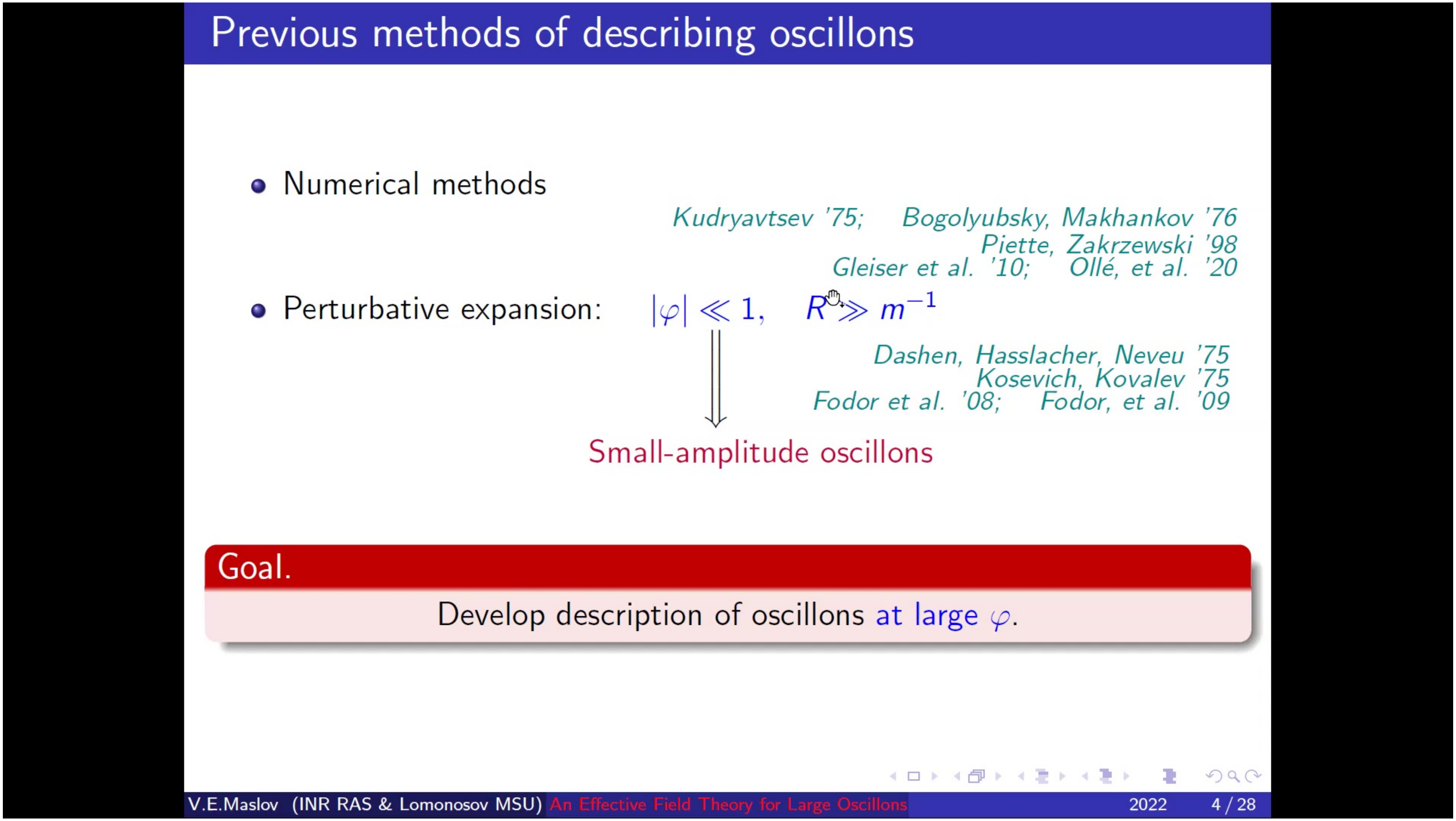
- accompany cosmological phase transitions

Dymnikova, Kozel, Khlopov, Rubin '00
Gleiser, Graham, Stamatopoulos '10

- formed by inflaton field during preheating

Amin, Easther, Finkel, '10
Hong, Kawasaki, Yamazaki '18

Why are oscillons so long-lived? How to describe them?



Previous methods of describing oscillons

- Numerical methods

Kudryavtsev '75; Bogolyubsky, Makhankov '76
Piette, Zakrzewski '98
Gleiser et al. '10; Ollé, et al. '20

- Perturbative expansion: $|\varphi| \ll 1, \quad R \gg m^{-1}$

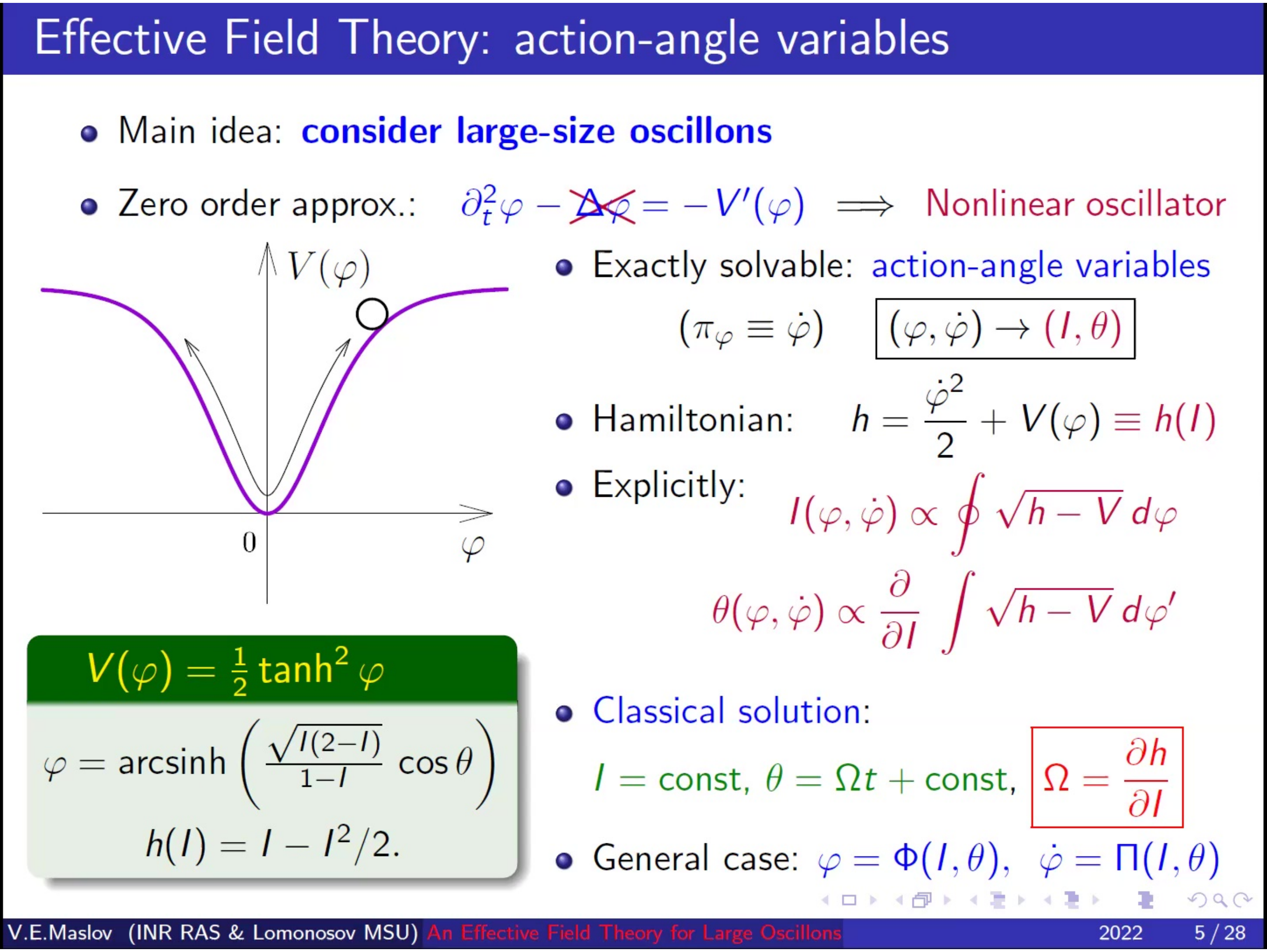
$$\Downarrow$$

Dashen, Hasslacher, Neveu '75
Kosevich, Kovalev '75
Fodor et al. '08; Fodor, et al. '09

Small-amplitude oscillons

Goal.

Develop description of oscillons at large φ.



Effective Field Theory: action-angle variables

- Main idea: **consider large-size oscillons**
- Zero order approx.: $\partial_t^2\varphi - \cancel{\Delta\varphi} = -V'(\varphi) \implies$ Nonlinear oscillator

$$V(\varphi) = \tfrac{1}{2}\tanh^2\varphi$$

$$\varphi = \operatorname{arcsinh}\left(\frac{\sqrt{I(2-I)}}{1-I}\cos\theta\right)$$

$$h(I) = I - I^2/2.$$

- Exactly solvable: action-angle variables

$$(\pi_\varphi \equiv \dot\varphi) \qquad \boxed{(\varphi, \dot\varphi) \to (I, \theta)}$$

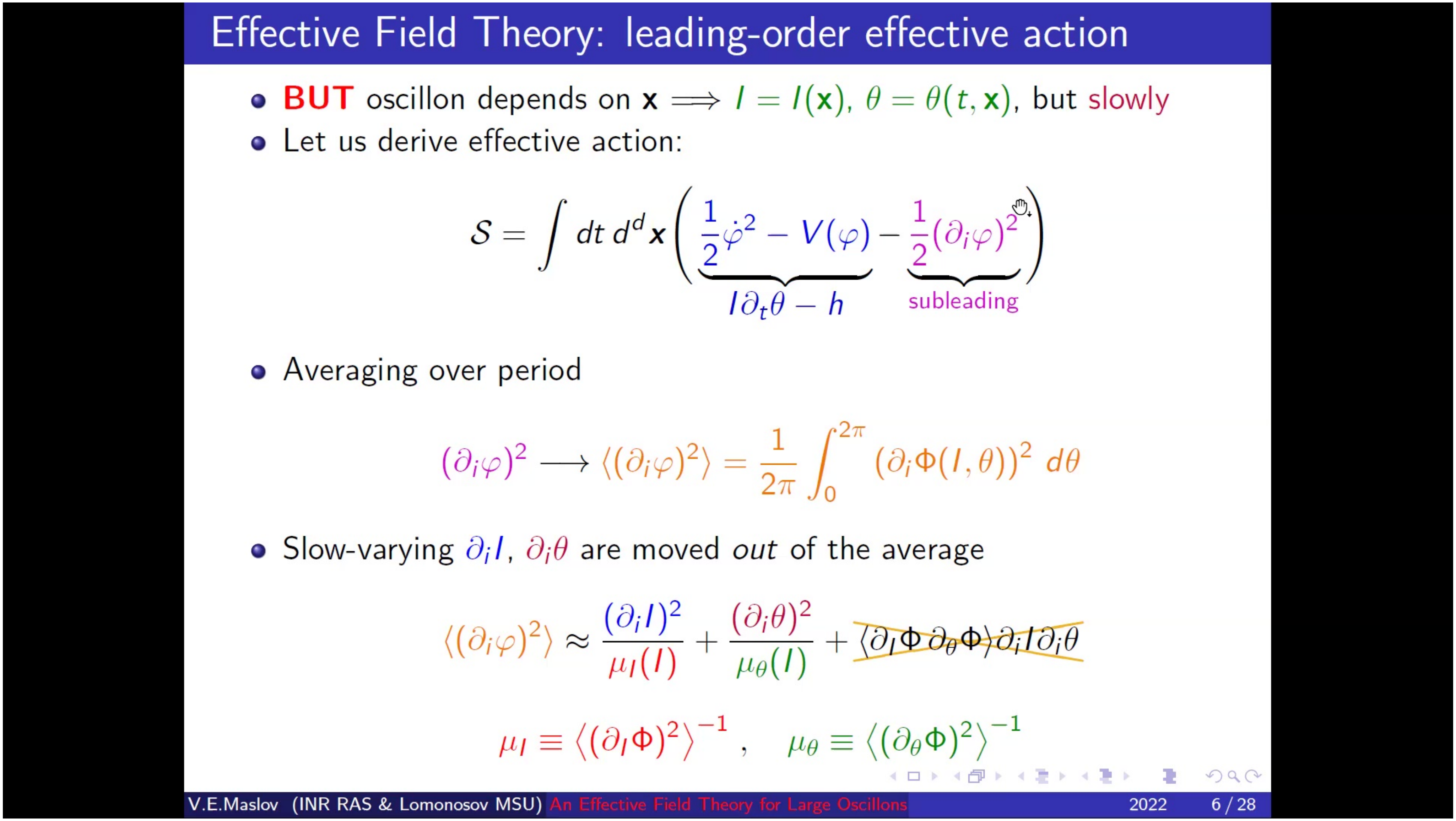
- Hamiltonian: $h = \dfrac{\dot\varphi^2}{2} + V(\varphi) \equiv h(I)$
- Explicitly: $I(\varphi, \dot\varphi) \propto \oint \sqrt{h - V}\, d\varphi$

$$\theta(\varphi, \dot\varphi) \propto \frac{\partial}{\partial I} \int \sqrt{h - V}\, d\varphi'$$

- Classical solution:

$$I = \text{const}, \ \theta = \Omega t + \text{const}, \ \boxed{\Omega = \frac{\partial h}{\partial I}}$$

- General case: $\varphi = \Phi(I, \theta), \ \ \dot\varphi = \Pi(I, \theta)$



Effective Field Theory: leading-order effective action

- **BUT** oscillon depends on $\mathbf{x} \Longrightarrow I = I(\mathbf{x})$, $\theta = \theta(t, \mathbf{x})$, but slowly
- Let us derive effective action:

$$\mathcal{S} = \int dt\, d^d\mathbf{x} \left(\underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \right)$$

- Averaging over period

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} (\partial_i\Phi(I,\theta))^2\, d\theta$$

- Slow-varying $\partial_i I$, $\partial_i\theta$ are moved *out* of the average

$$\langle(\partial_i\varphi)^2\rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i\theta)^2}{\mu_\theta(I)} + \cancel{\langle\partial_I\Phi\,\partial_\theta\Phi\rangle\partial_i I\,\partial_i\theta}$$

$$\mu_I \equiv \left\langle(\partial_I\Phi)^2\right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle(\partial_\theta\Phi)^2\right\rangle^{-1}$$

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$$\mu_I \equiv \left\langle(\partial_I\Phi)^2\right\rangle^{-1}, \quad \mu_\theta \equiv \left\langle(\partial_\theta\Phi)^2\right\rangle^{-1}$$

Effective Field Theory: properties

Effective action in the leading order

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\mathbf{x} \left(I\partial_t\theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i\theta)^2}{2\mu_\theta(I)} \right)$$

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$\mu_I = I(2-I)^2(1-I)^2, \quad \mu_\theta = I^{-1} - 1$

- Introducing complex field $\psi(t, \mathbf{x}) = \sqrt{I}\cdot e^{-i\theta}$ gives

$$\mathcal{S}_{\text{eff}} = \int dt\, d^d\mathbf{x} \left(i\psi^*\partial_t\psi - h - \frac{|\partial_i\psi|^2}{2\mu_1} - \frac{1}{2\mu_2}\left[\psi^{*2}(\partial_i\psi)^2 + \text{h.c.}\right] \right)$$

nonlinear Schrödinger model

Form factors: $\mu_i = \mu_i(|\psi|^2), \quad h = h(|\psi|^2)$

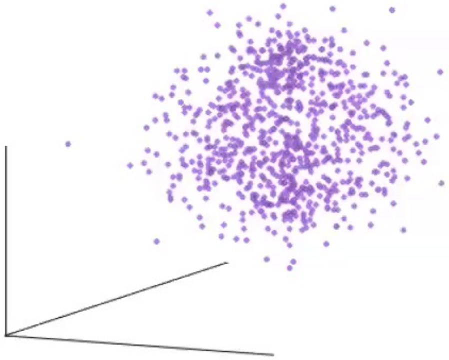
- Global symmetry: $\theta \to \theta + \alpha \quad \Longleftrightarrow \quad \psi \to \mathrm{e}^{-i\alpha}\psi$

$$\Downarrow$$

- Conserved charge: $N = \int d^d\mathbf{x}\, I = \int d^d\mathbf{x}\, |\psi|^2$

Oscillons — solitons in EFT

Attraction + charge conservation = solitons!



- Stationary ansatz:

$$\psi = \psi(r)\,\mathrm{e}^{-i\omega t}, \quad \text{or} \quad \theta = \omega t$$

- Alternatively:

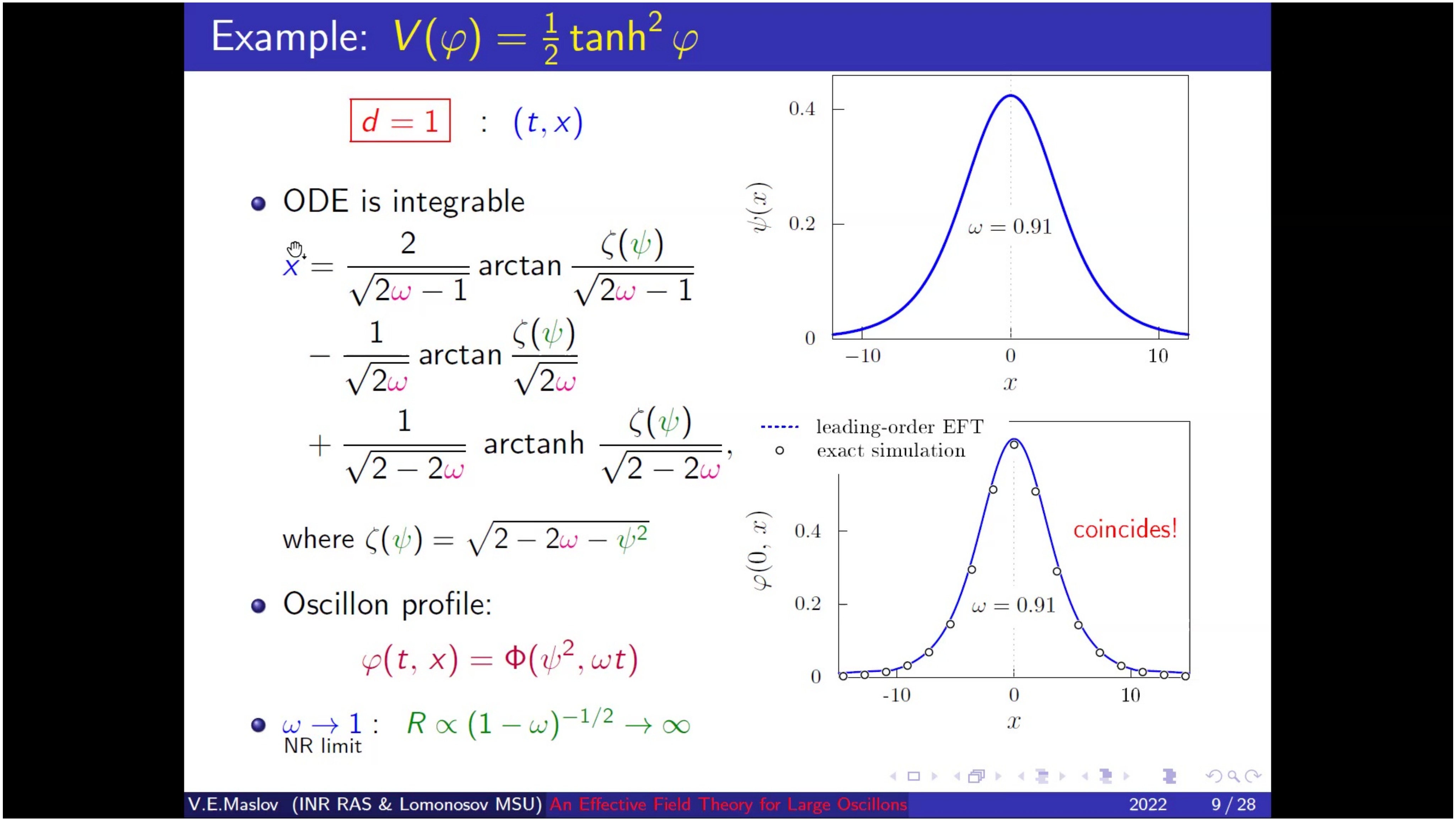
minimize the energy E at fixed charge N.

- Oscillon profile equation

$$\boxed{\Omega = \partial h/\partial I}$$

$$-\frac{2\psi^2}{\mu_I}\Delta\psi - (\partial_i\psi)^2\frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

- At the oscillon: $\delta E = \omega\delta N$ for all variations $\Longrightarrow$ $\omega = dE/dN$.
- Physical interpretation: N — ,,number of particles"
  ω — ,,energy of a particle"



Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

$\boxed{d = 1}$: (t, x)

- ODE is integrable

$$x = \frac{2}{\sqrt{2\omega - 1}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega - 1}} - \frac{1}{\sqrt{2\omega}} \arctan \frac{\zeta(\psi)}{\sqrt{2\omega}} + \frac{1}{\sqrt{2 - 2\omega}} \operatorname{arctanh} \frac{\zeta(\psi)}{\sqrt{2 - 2\omega}},$$

where $\zeta(\psi) = \sqrt{2 - 2\omega - \psi^2}$

- Oscillon profile:

$$\varphi(t, x) = \Phi(\psi^2, \omega t)$$

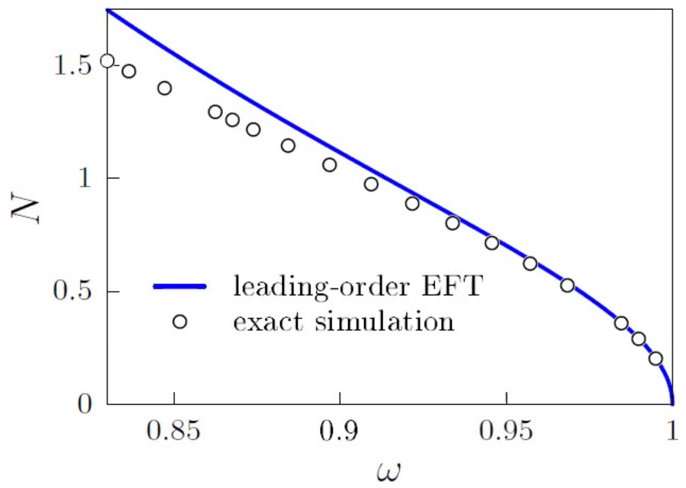
- $\underset{\text{NR limit}}{\omega \to 1}$: $R \propto (1 - \omega)^{-1/2} \to \infty$

Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

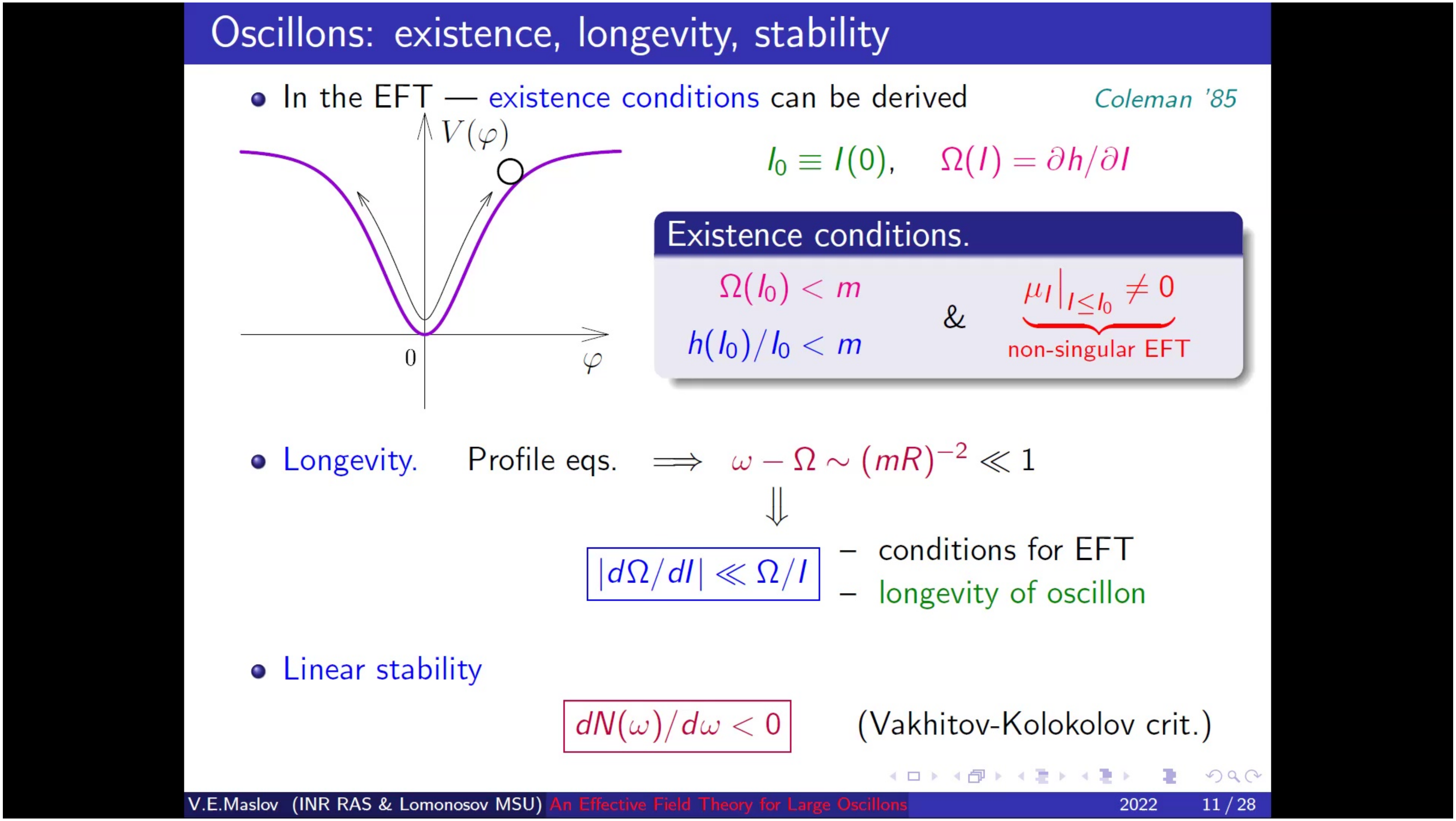
- N and E — also explicitly computed:

$$N = \frac{4}{\sqrt{2\omega - 1}} \arctan\frac{\sqrt{1-\omega}}{\sqrt{\omega - 1/2}} - \frac{4}{\sqrt{2\omega}} \arctan\sqrt{1/\omega - 1}\,,$$

$$E = \frac{4(1-\omega)}{\sqrt{2\omega - 1}} \arctan\frac{\sqrt{1-\omega}}{\sqrt{\omega - 1/2}} + 2\sqrt{2\omega} \arctan\sqrt{1/\omega - 1}\,,$$



- For $d \neq 1$ — profile $\psi(r)$, N and E are found numerically.



Oscillons: existence, longevity, stability

- In the EFT — existence conditions can be derived *Coleman '85*

$$I_0 \equiv I(0), \quad \Omega(I) = \partial h / \partial I$$

Existence conditions.

$$\Omega(I_0) < m \qquad \& \qquad \underbrace{\mu_I \Big|_{I \leq I_0} \neq 0}_{\text{non-singular EFT}}$$

$$h(I_0)/I_0 < m$$

- Longevity. Profile eqs. $\Longrightarrow \omega - \Omega \sim (mR)^{-2} \ll 1$

$$\Downarrow$$

$$\boxed{|d\Omega/dI| \ll \Omega/I}$$

– conditions for EFT
– longevity of oscillon

- Linear stability

$$\boxed{dN(\omega)/d\omega < 0}$$

(Vakhitov-Kolokolov crit.)

Oscillons: restoring $V(\varphi)$

$$\boxed{\varphi(V) = \int_0^V \frac{dh}{\Omega(h)\sqrt{2V-2h}}}$$

2 ways of achieving $|d\Omega/dI| \ll \Omega/I$:

- Small-amplitude approximation:

$$V = \frac{1}{2}m^2\varphi^2 + \frac{g_3}{4}\varphi^4 + \frac{g_5}{6}\varphi^6 + \dots$$

- Special cases of flat $\Omega(I)$, e.g.

$$\Omega = \Omega_0 + \delta\Omega(I), \quad \delta\Omega(I) \ll \Omega_0.$$

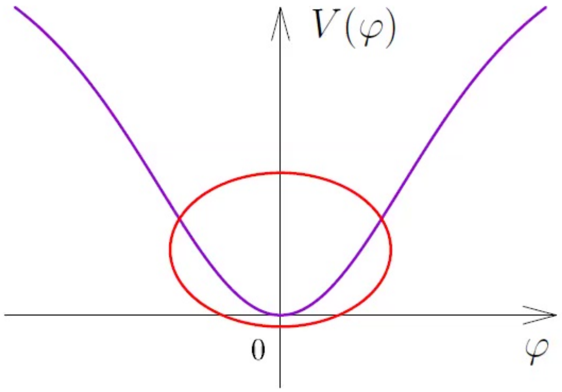
$$\Downarrow$$

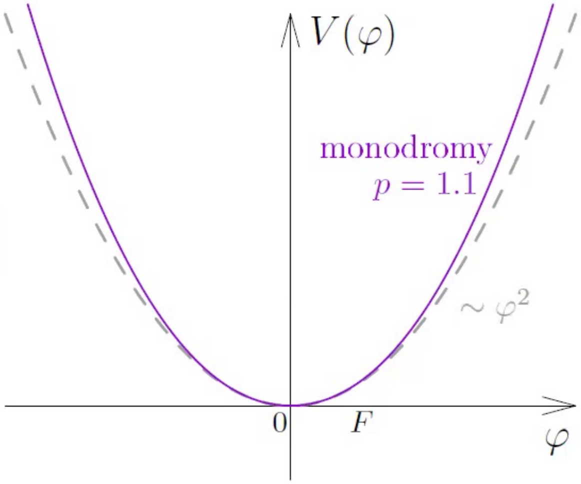
$$V = \frac{1}{2}\tilde{\Omega}^2(\varphi)\,\varphi^2, \quad \tilde{\Omega}^2 \approx \Omega_0^2 + \delta\tilde{\Omega}(\varphi)$$

- E.g. monodromy potential

$$V_p(\varphi) = \tfrac{1}{2p}m^2F^2\left[(1+\frac{\varphi^2}{F^2})^p - 1\right],$$

$p \to 1$ *Ollé et al. '20*





Corrections

- Goal: Develop asymptotic asymptotic expansion in R^{-2}:

$$\mathcal{S}_{\text{eff}} = \underbrace{\mathcal{S}^{(1)}_{\text{eff}}}_{R^0+R^{-2}} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\text{eff}}}_{R^{-4}} + \underbrace{\mathcal{S}^{(3)}_{\text{eff}}}_{R^{-6}} + \ldots}^{\text{corrections}}$$

- Field corrections:

$$I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\textit{fast}}\,, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\textit{fast}}$$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Solve eqs. for δI, $\delta\theta$

$$\delta I \approx \frac{\mathcal{I}[\partial_{\bar{\theta}}\Phi\Delta\Phi]}{\partial_t\bar{\theta}}\,, \quad \delta\theta \approx \frac{1}{\partial_t\bar{\theta}}\left\{\partial_{\bar{I}}\Omega \cdot \mathcal{I}[\delta I] - \mathcal{I}\left[\partial_{\bar{I}}\Phi\Delta\Phi - \langle\partial_{\bar{I}}\Phi\Delta\Phi\rangle\right]\right\}$$

$$\mathcal{I}[f] = \int^{\theta} f(\bar{\theta}')\, d\bar{\theta}' - A \quad \text{— primitive,} \qquad \Phi = \Phi(\bar{I}, \bar{\theta}), \quad \Omega = \Omega(\bar{I})$$

Second-order EFT: oscillons

- Plug δI, $\delta\theta$ into action $\Longrightarrow \mathcal{S}_{\text{eff}} = \mathcal{S}^{(1)}_{\text{eff}} + \mathcal{S}^{(2)}_{\text{eff}}$,

$$\boxed{\bar\theta = \omega t}$$

$$\mathcal{S}^{(2)}_{\text{eff}} = \int dt\, d^d\mathbf{x}\, \left[d_1\,(\partial_i\psi)^4 + d_2\,\psi\Delta\psi(\partial_i\psi)^2 + d_3\,(\Delta\psi)^2\right]$$

Note. Four spatial derivatives

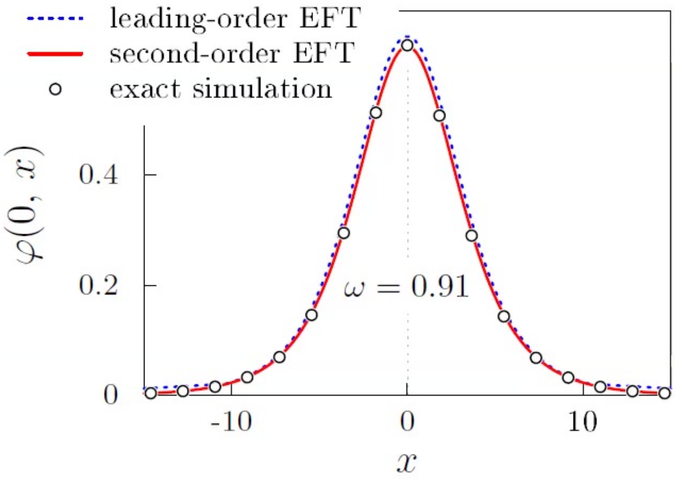
$d_i(\psi^2)$ — form factors

- Oscillon profile $\bar I = \psi^2(\mathbf{x})$ can be found perturbatively:

$$\bar I = \bar I^{(1)} + \bar I^{(2)}, \quad S^{(1)}_{\text{eff}}[\bar I^{(1)}] - \text{min}$$

$$\Downarrow$$

$$\frac{\delta^2 S^{(1)}_{\text{eff}}}{\delta \bar I^2} \cdot \bar I^{(2)} = -\frac{\delta S^{(2)}_{\text{eff}}}{\delta \bar I}$$

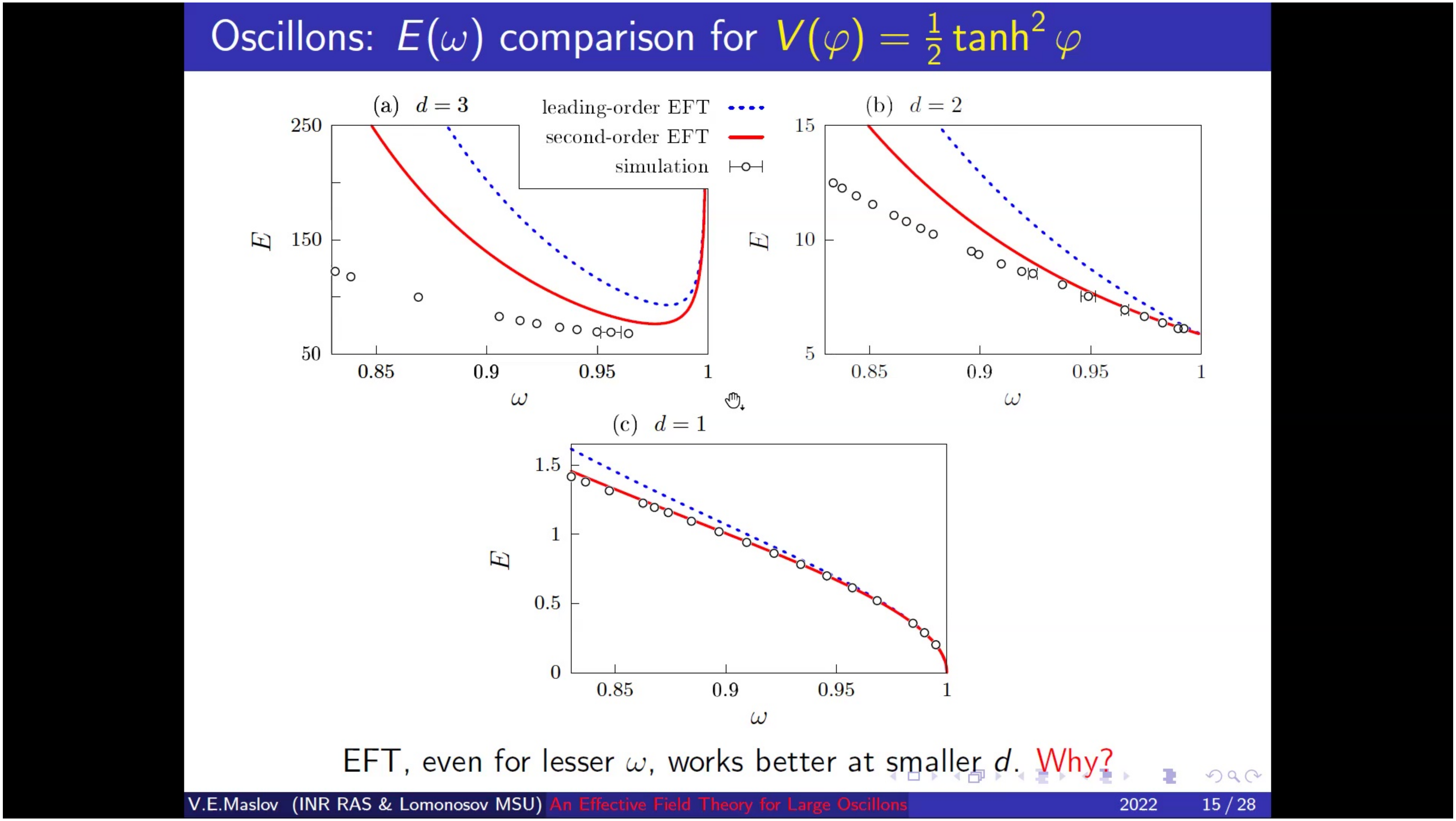


$$V(\varphi) = \frac{1}{2}\tanh^2\varphi, \quad d = 1$$

- Oscillon field:

$$\varphi(t,\mathbf{x}) = \Phi\left(\bar I + \delta I, \omega t + \delta\theta\right),$$

$$\delta I = \delta I(\bar I, \omega t), \quad \delta\theta = \delta\theta(\bar I, \omega t)$$



Oscillons: $E(\omega)$ comparison for $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

EFT, even for lesser ω, works better at smaller d. Why?

Limit $d \to 0$

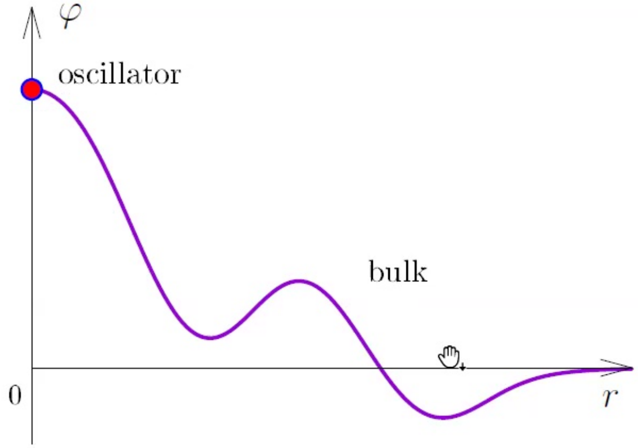
- Analytically continue field equation to $d < 1$:

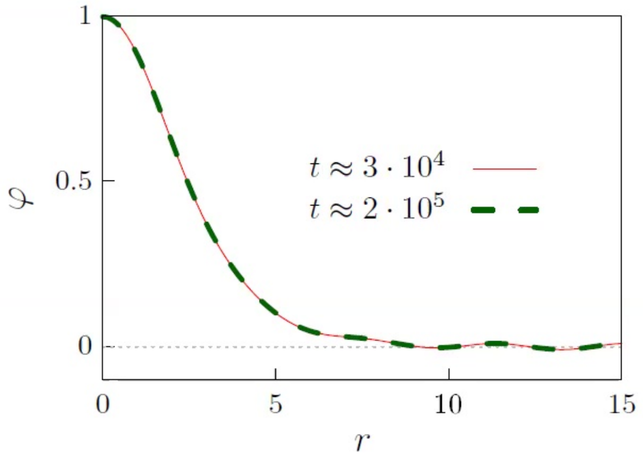
$$\partial_t^2 \varphi - \partial_r^2 \varphi - \frac{d-1}{r}\,\partial_r \varphi = -V'(\varphi)$$

- Substitute $r = 0$: $$\partial_t^2 \varphi(0) \;-\; \underbrace{d\,\partial_r^2 \varphi(0)}_{\text{vanishes at } d=0} \;=\; -V'(\varphi(0))$$

$$\Downarrow$$

Oscillator at $d = 0,\ r = 0$



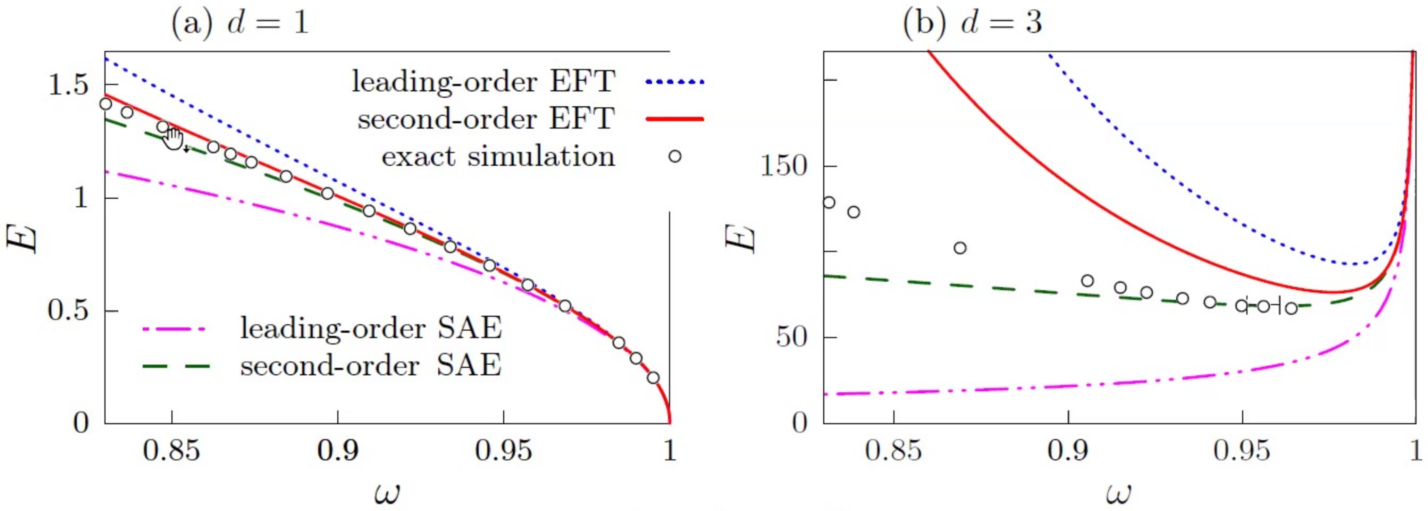


- There are exactly periodic solutions at $d = 0$.

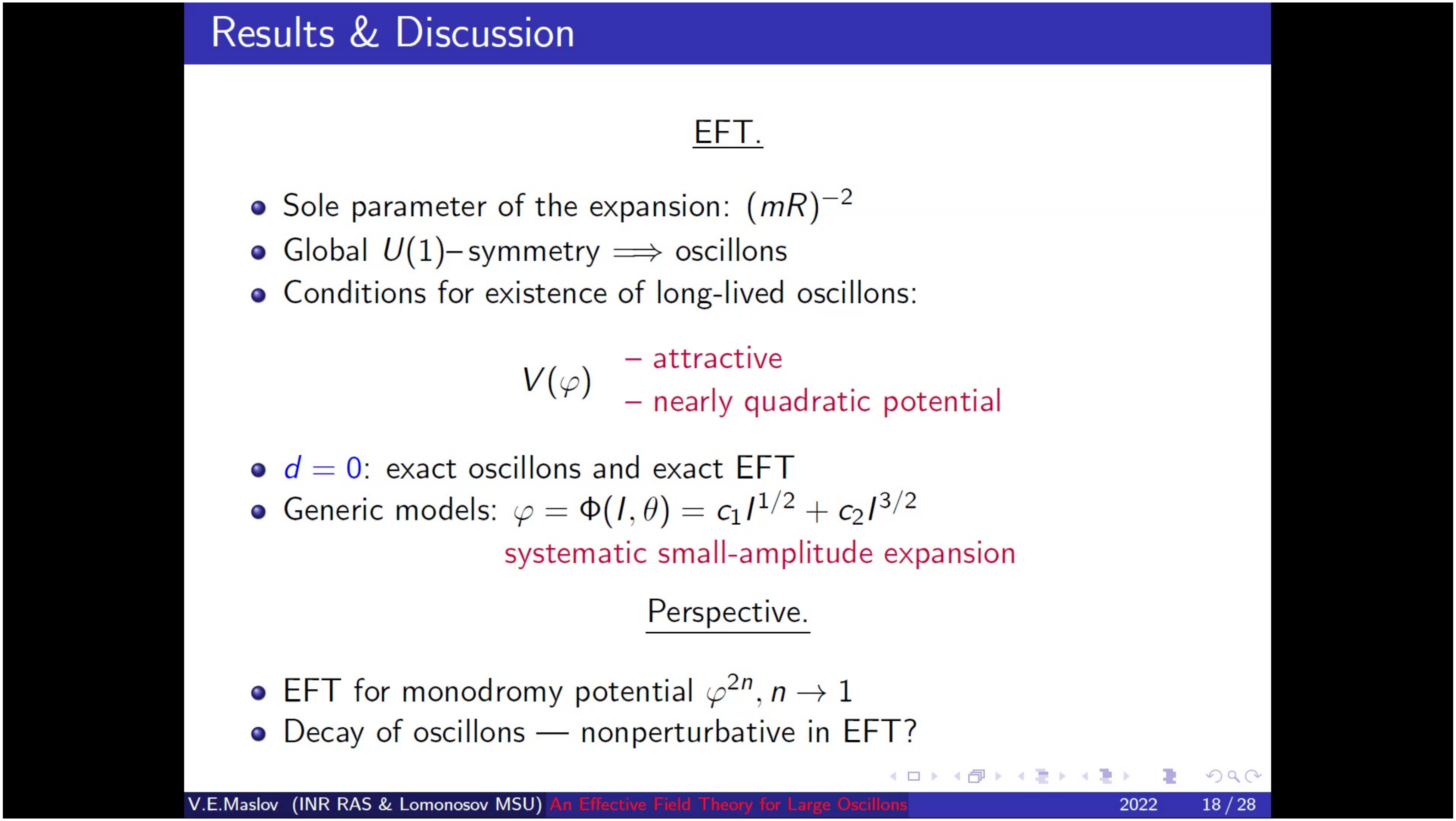
Small-amplitude expansion vs. EFT

Reminder. different expansion parameters:

$R^{-1} \to 0$ at finite φ (EFT), $R^{-1} \sim \varphi \to 0$ (small-amplitude)



$$V(\varphi) = \frac{1}{2}\tanh^2\varphi$$



Results & Discussion

EFT.

- Sole parameter of the expansion: $(mR)^{-2}$
- Global $U(1)$–symmetry $\Longrightarrow$ oscillons
- Conditions for existence of long-lived oscillons:

$$V(\varphi) \quad \begin{array}{l} \text{– attractive} \\ \text{– nearly quadratic potential} \end{array}$$

- $d = 0$: exact oscillons and exact EFT
- Generic models: $\varphi = \Phi(I, \theta) = c_1 I^{1/2} + c_2 I^{3/2}$

systematic small-amplitude expansion

Perspective.

- EFT for monodromy potential φ^{2n}, $n \to 1$
- Decay of oscillons — nonperturbative in EFT?

Thank you for your attention!