Title: Nuclear Astrophysics: Unknown Knowns

Speakers: Isaac Legred

Series: Strong Gravity

Date: October 06, 2022 - 1:00 PM

URL: https://pirsa.org/22100093

Abstract: The advent of precise measurements of neutron star properties has led to an explosion in "nuclear astrophysics": studying the properties of high-density matter using astrophysical phenomena. Remarkably, constraints provided by nuclear theory and experiment and high-energy astrophysical observations are now competitive (and often complementary) in constraining the equation of state (EoS) of matter at supernuclear densities. On the astrophysical side, data have provided a clearer picture how these constraints are affected by the choice of modeling the EoS. Specifically, the nuclear EoS in astrophysical analyses is usually modeled phenomenologically, and often using ad hoc assumptions. I will discuss why these ad hoc assumptions will likely cause problems, considering the deluge of coming neutron-star measurements, by comparing these approaches to a data-driven, "nonparametric", model.

Zoom link: https://pitp.zoom.us/j/94435348102?pwd=OHF2MkNMWStNTlhmdkRQaElNL1M1Zz09

Nuclear Astrophysics: Unknown Knowns

Isaac Legred (Caltech)

PI Strong Gravity Seminar

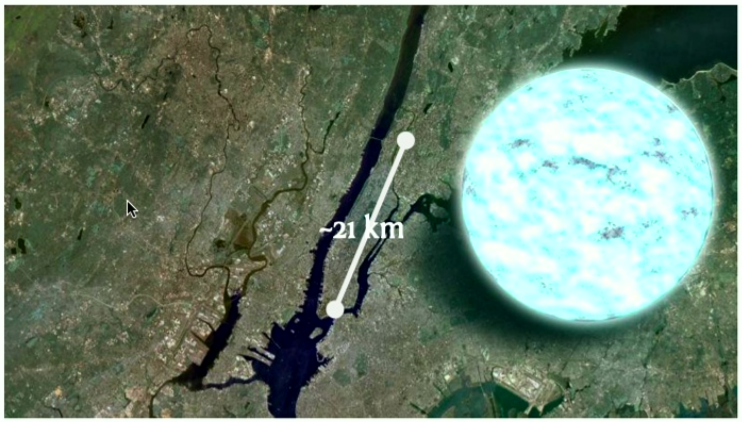
October 5, 2022

Work with: Katerina Chatziioannou,
Reed Essick, and Philippe Landry

Caltech

10.1103/PhysRevD.105.043016
https://arxiv.org/abs/2201.06791

Why Study Neutron Stars?



Source

GR matters when GM/Rc^2 is not small

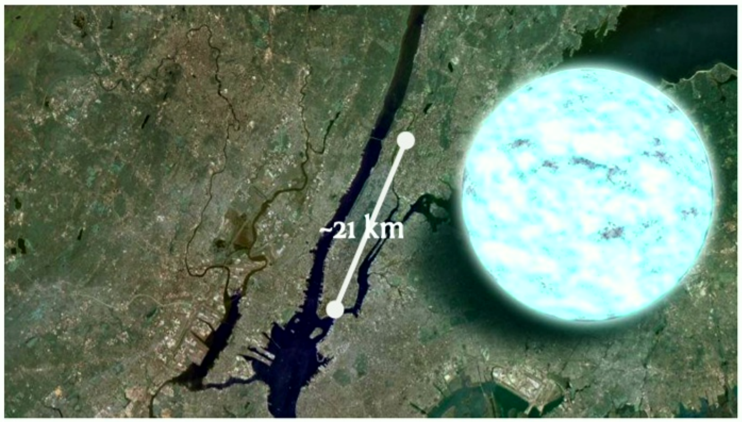
NS: $GM/Rc^2 \sim 1/3$

Behavior of nuclear matter is uncertain when $n/n_{\rm nuc}$ is not small*

NS: $n_{\rm max}/n_{\rm nuc}$ $4-7$?

Neutron Stars give us laboratories to Study nuclear physics along with general relativity

Why Study Neutron Stars?



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(1) Better understand current theories of physics

(2) New physics (beyond SM, beyond GR)

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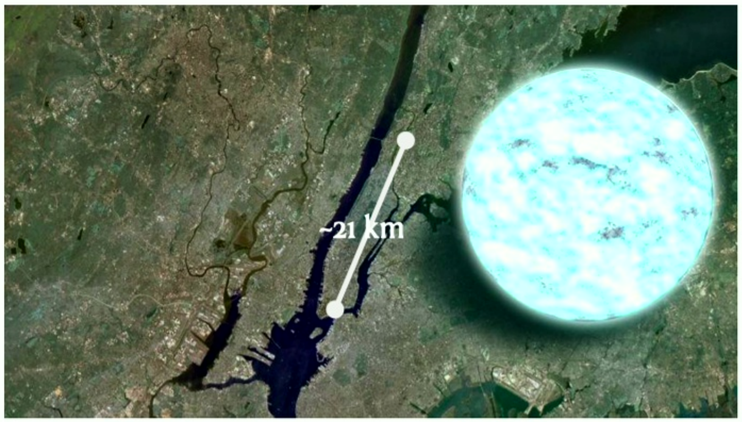

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Unknown Unknowns

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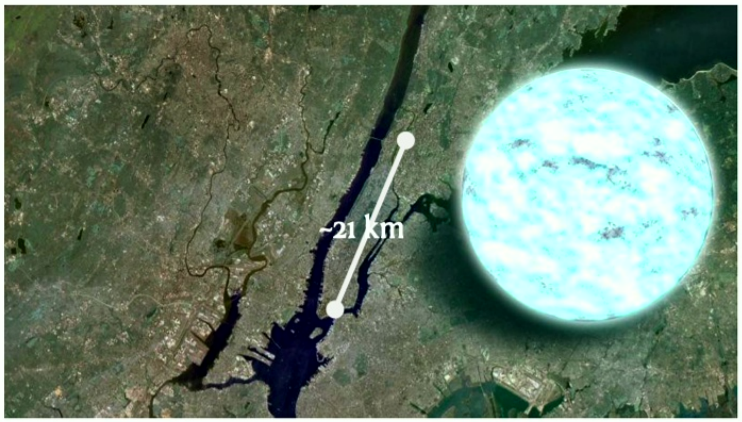

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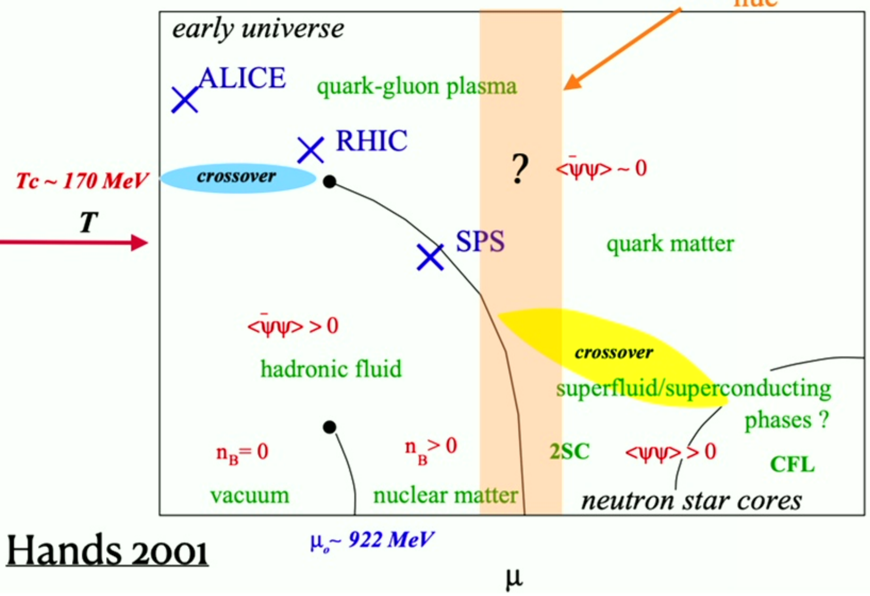


Hands 2001

Inferring the EoS — Motivation

"Microphysics <=> Macrophysics"

EoS

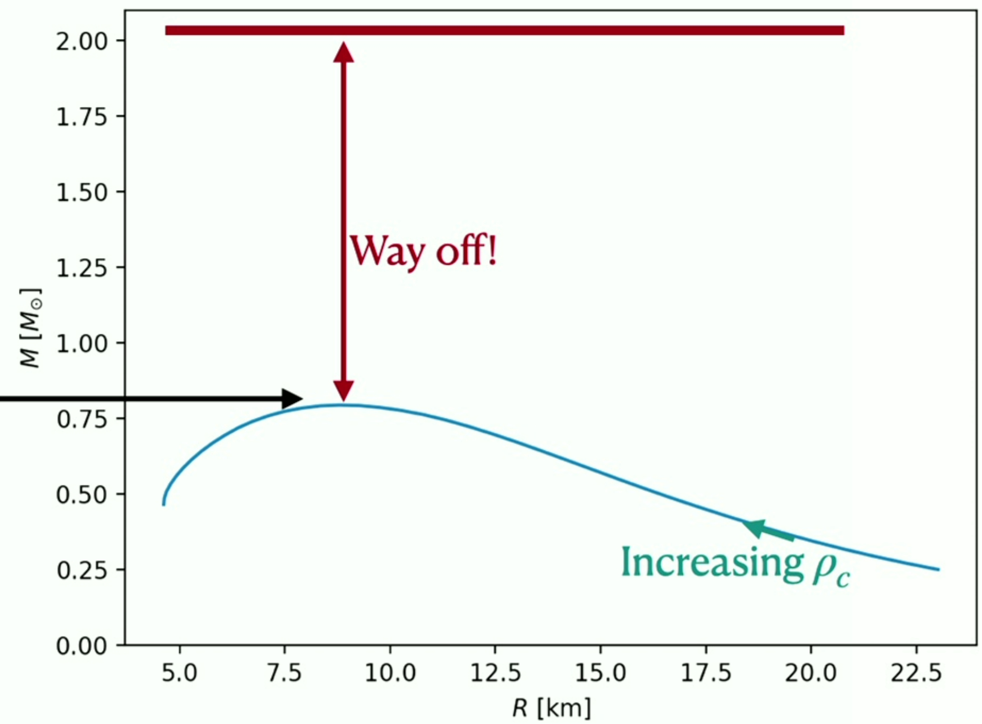
Infer | Compute

NS Properties

Most massive NS w/ this EoS

5/3 Polytrope "degenerate neutrons (Non-Rel)"

Most massive observed pulsar (Fonseca 2021)



Inferring the EoS — In practice

- Want to establish a probability distribution on candidate equations of state given observed astrophysical data

Equation of state candidate

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto \mathcal{L}(d_1, d_2, \ldots \,|\, \varepsilon_i) \times \pi(\varepsilon_i)$$

Prior

Astrophysical data

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Astrophysical data

Prior

Phenomenological

Parametrize a *functional form* (e.g. Spectral, Piecewise-polytrope)

Nonparametric methods, e.g. Gaussian process (GP)

Tabulated models from nuclear theory

Parametric: Models

Put the prior on parameters!

Spectral (Lindblom 2010)

Parametrize the adiabatic index

$$p(\rho) = \rho^{\Gamma(x)} \quad \Gamma(x) = \sum_i \gamma_i x^i; \quad x = \log(p/p_0)$$

Piecewise-polytrope (Read 2008)

A polytrope with multiple segments

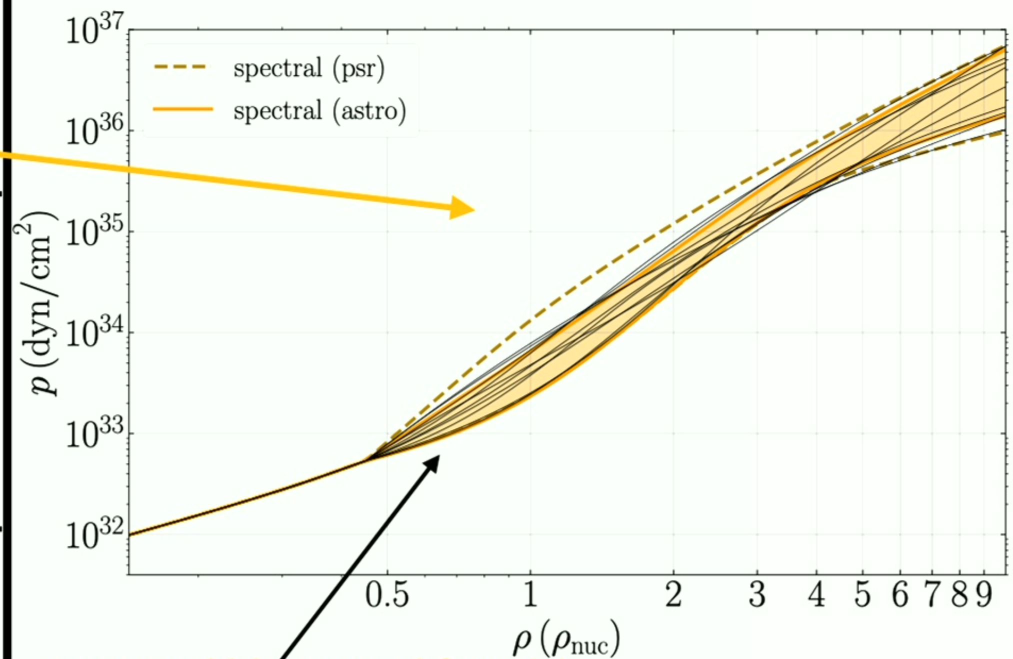
$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1} : \rho < \rho_1 \\ K_2 \rho^{\Gamma_2} : \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3} : \rho_2 < \rho \end{cases}$$

Direct **speed-of-sound** (Greif 2018)

A bump in the speed of sound before asymptotic behavior

$$\frac{c_s^2(z)}{c^2} = a_1 e^{-\frac{1}{2}(z-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + e^{-a_5(z-a_4)}}$$

E.g. for the Spectral Parametrization



90% credible interval for $p(\rho)$

psr -> just heavy pulsar mass measurements (like prior)

astro -> Heaviest pulsar, 2 NICER x-ray, 2 GWs

Parametric: Models

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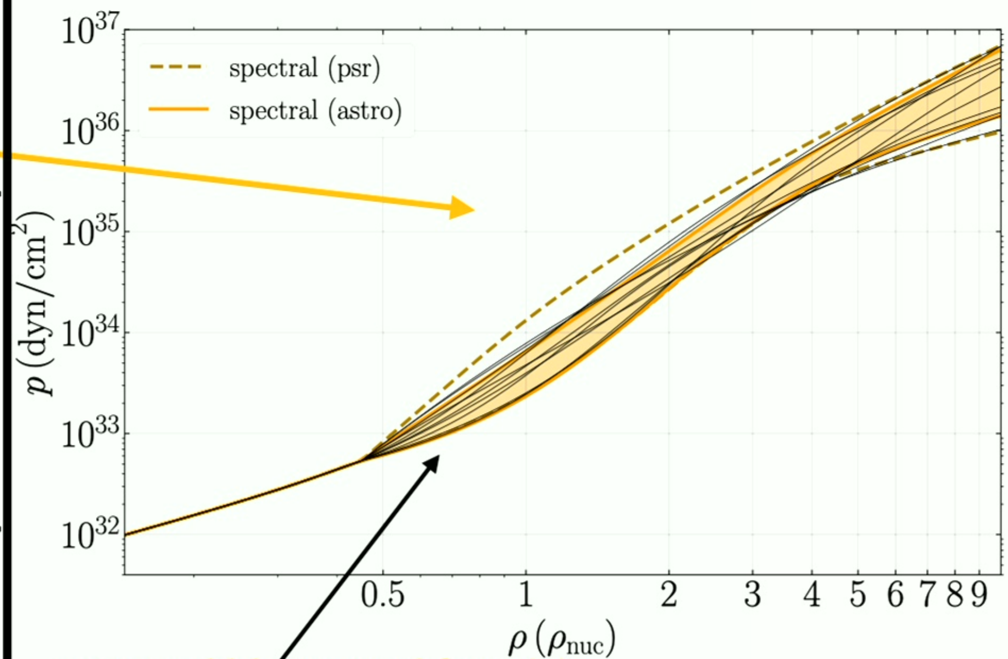
"ad hoc"

E.g. for the Spectral Parametrization



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Nonparametric: Gaussian Process

Put the prior on correlations…

Gaussian Process Regression (Landry and Essick 2018)

Tabulate a draw $\phi(p_i) = \ln(1/c_s^2(p_i) - 1)$ @ Pressures p_i from a multivariate Gaussian

Parameters for the covariance kernel are chosen to Control "shape" of EoS distribution

Model-Agnostic Prior (broadest range of models)

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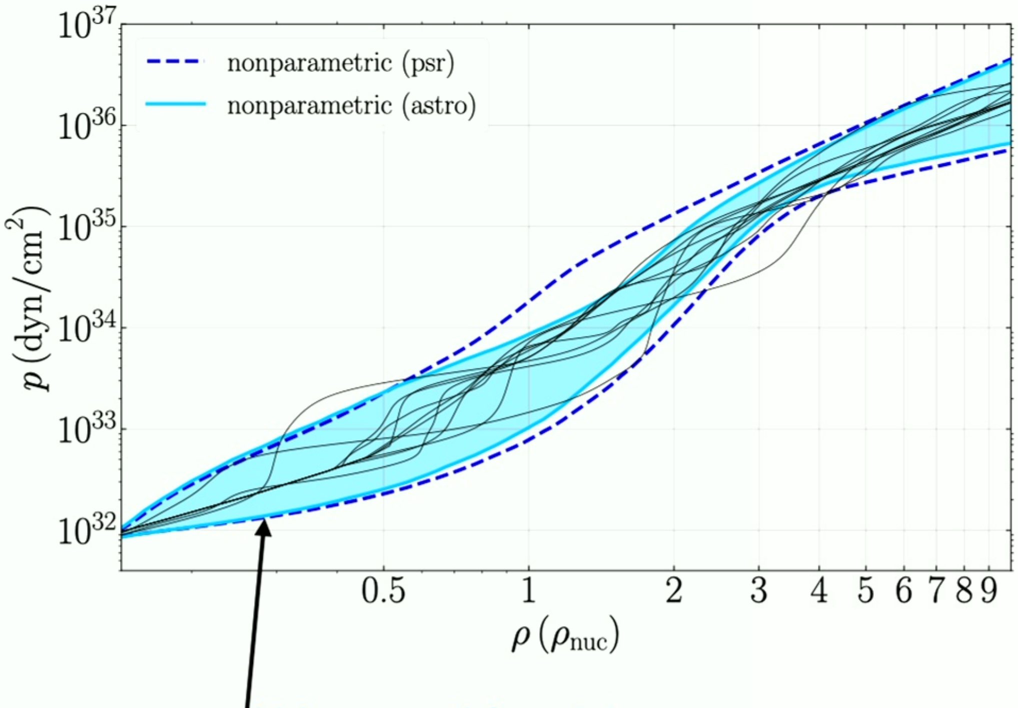

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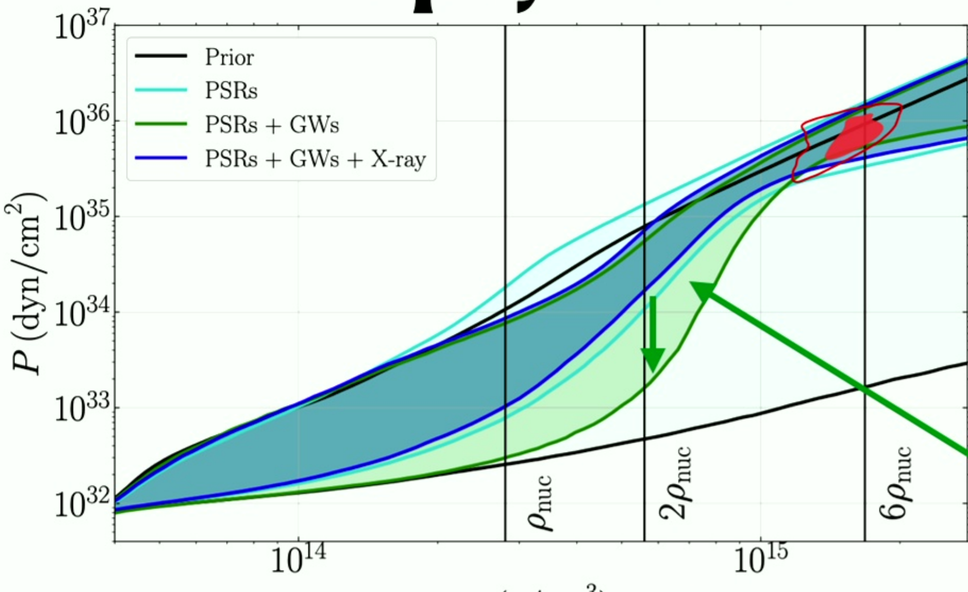
Astrophysical Data and Density Scales



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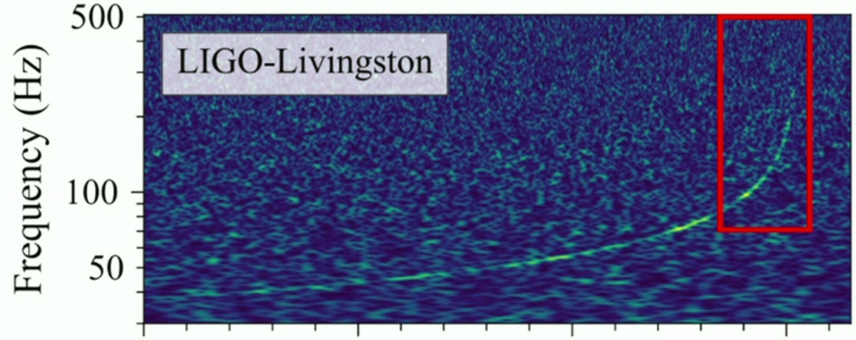
Use tidal effects on **gravitational waves** to measure the stiffness of the EoS at low densities

e.g.

Abbott+ (LVC) 2017



What goes into the likelihood?

$$P(\varepsilon_i | d) = P(\varepsilon | d_1, d_2, \ldots) \propto \boxed{\mathscr{L}(d_1, d_2, \ldots | \varepsilon_i)} \times P(\varepsilon_i)$$

Astrophysical Data and Density Scales

What goes into the likelihood?

What about measuring the NS radius?

Neutron Star Interior Composition Explorer (NICER)
(Currently operating on ISS!)

Measured Mass and radius of two pulsars

J0030 (2019)

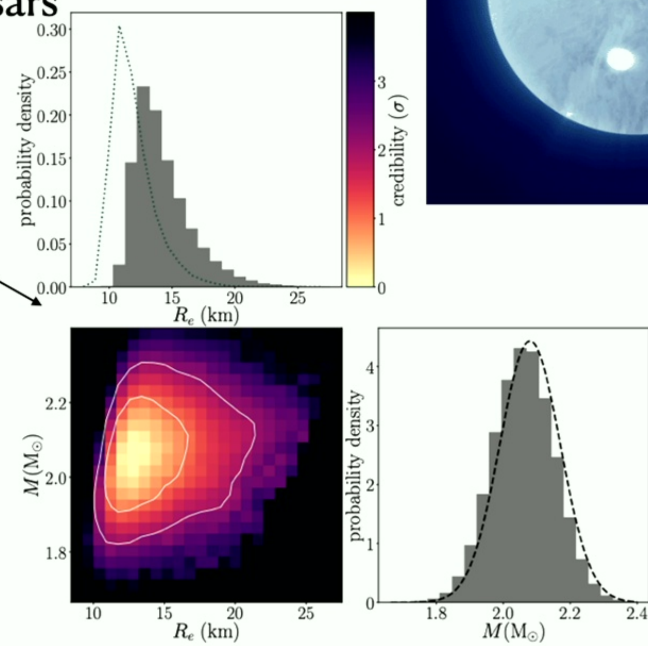
Miller et al.

Raaijmakers et al.

J0740 (2021)

Miller et al.

Riley et al.



Use **gravitational lensing** of x-rays to infer the compactness of the star
Also used XMM-Newton to calibrate pulse rate

Astrophysical Data and Density Scales

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IL+ 2021

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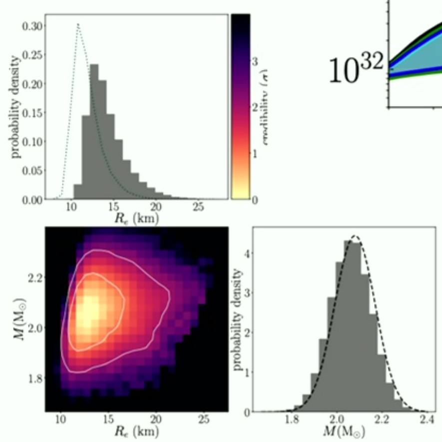

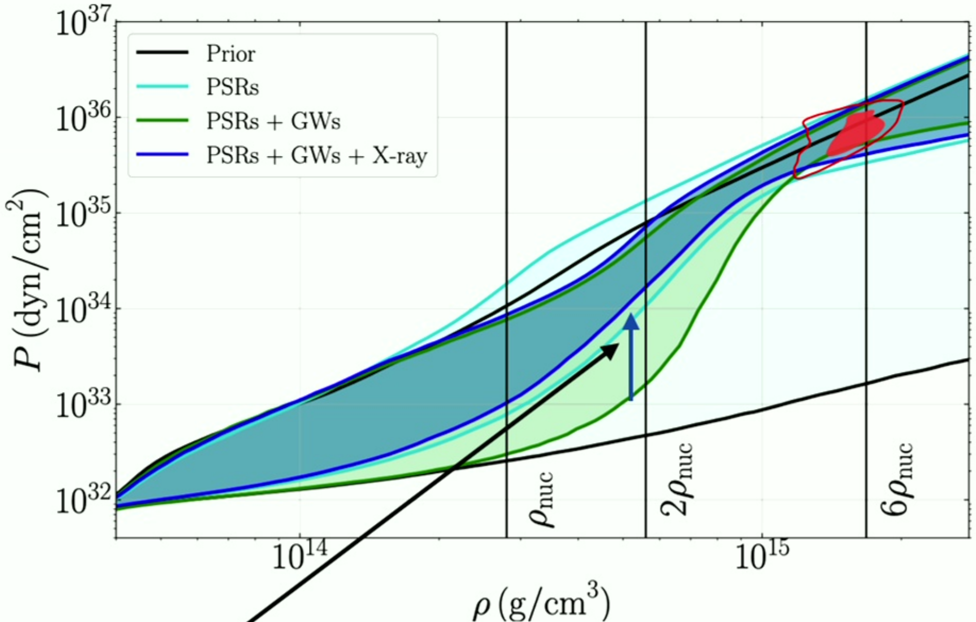

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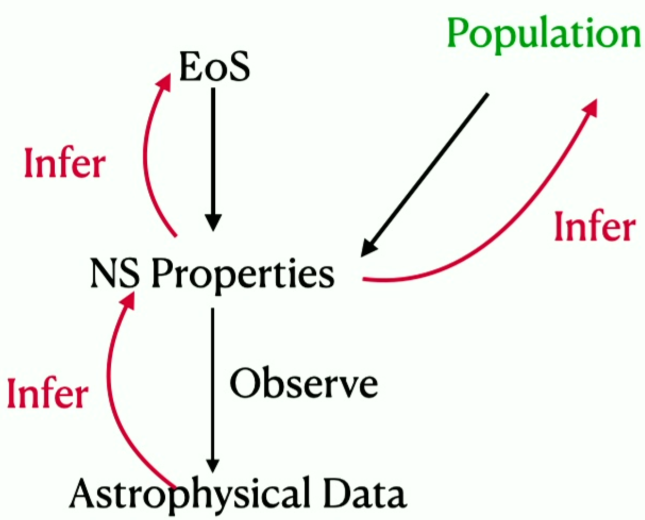
Astrophysical Data (Brief Aside)

Ideally :



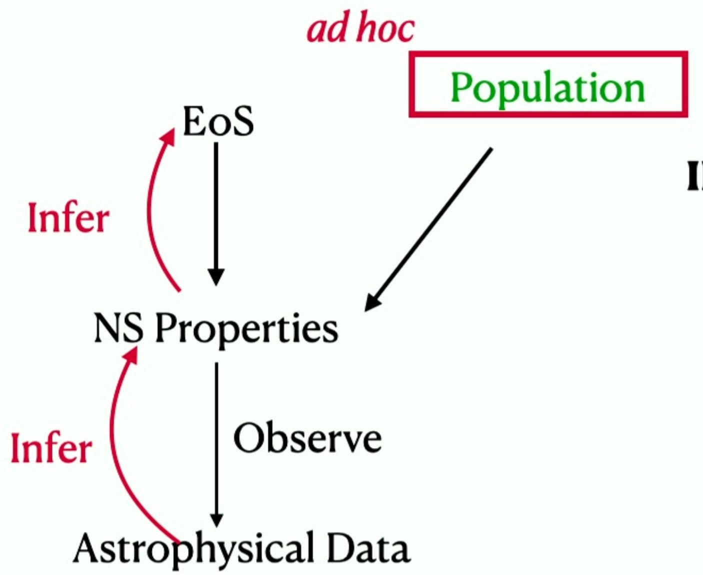
Lots of "details" / Room for improvement

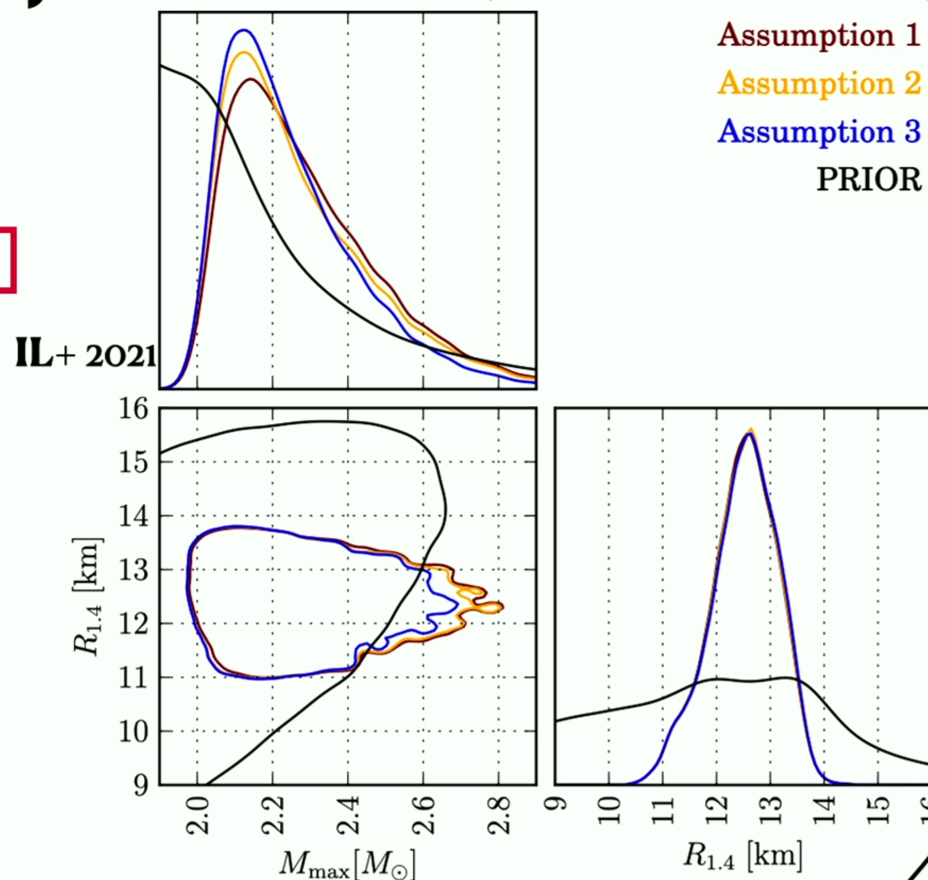
(1) Improvements in detectors/characterization
(2) Interpreting data (GW waveforms, x-ray pulse profiles)
(3) Poorly characterized population of NSs

$$\mathscr{L}(d_1 \mid \varepsilon_i) = \sum_{\text{NS sources}} \mathscr{L}(d_1 \mid \text{NS source}) \times P(\text{NS Source} \mid \varepsilon_i, \text{Population Model})$$

Astrophysical Data (Brief Aside)

In practice:





Assumption 1 BHs are small NSs

Assumption 2 Astrophysically lmtd.

Assumption 3 TOV mass lmtd.

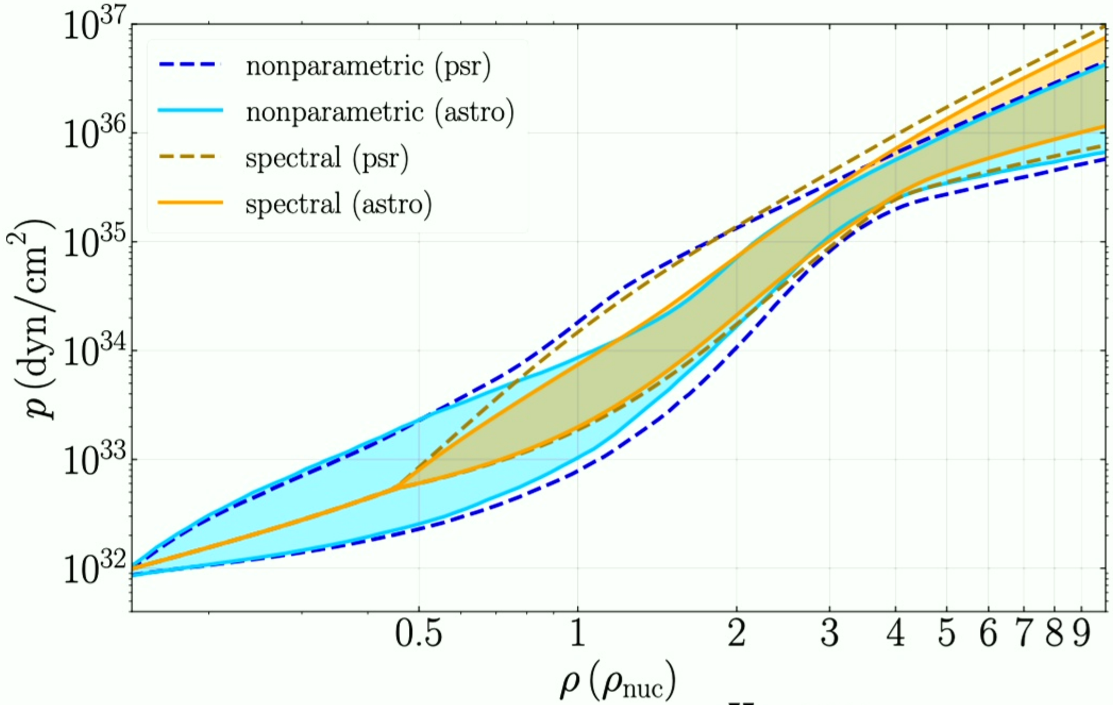
PRIOR

$$\mathcal{L}(d_1 \mid \varepsilon_i) = \sum_{\text{NS sources}} \mathcal{L}(d_1 \mid \text{NS source}) \times P(\text{NS Source} \mid \varepsilon_i, \text{Population Model})$$

Different Priors, Different Results

Posterior is combination of prior and likelihood

We're varying just the EoS prior and examining posteriors



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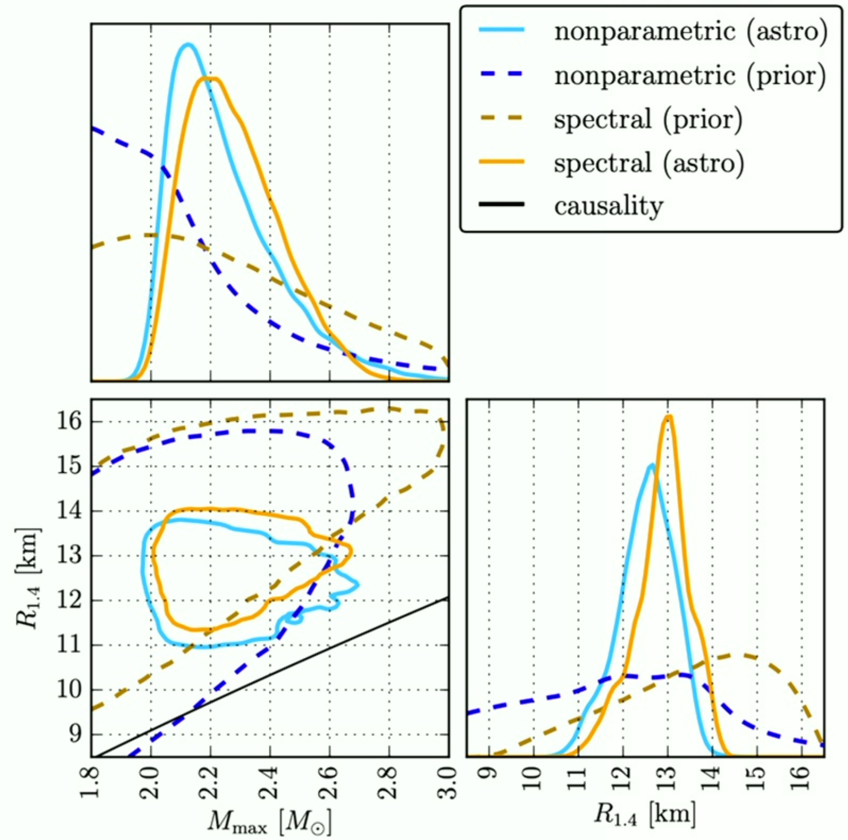
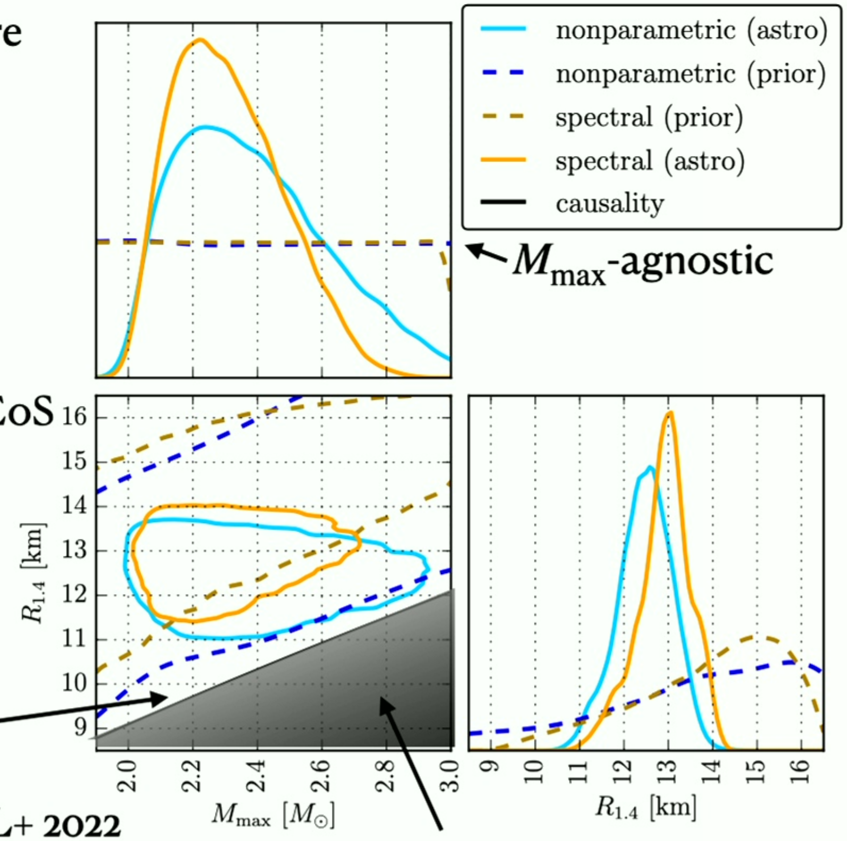


Different Priors, Different Results

TOV maximum mass and radius of a 1.4 solar mass NS are "correlated" among equation of state candidates due to causality

This rules out certain configurations in "$M_{\max} - R_{1.4}$" space

The boundary is "fuzzy" — depends on the low density EoS



$M_{\max}$-agnostic

Parametrized by Stitching density to $c_s^2 = c^2$

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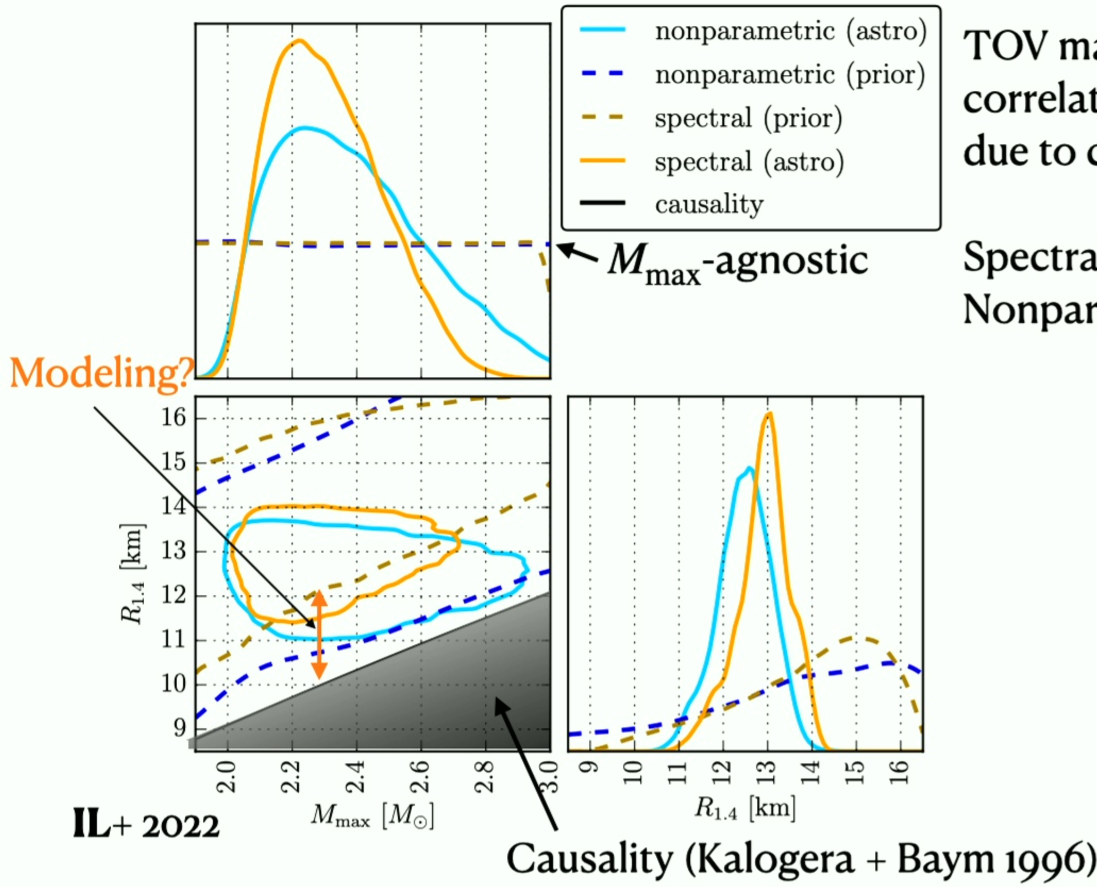
Causality (Kalogera + Baym 1996)

Different Priors, Different Results

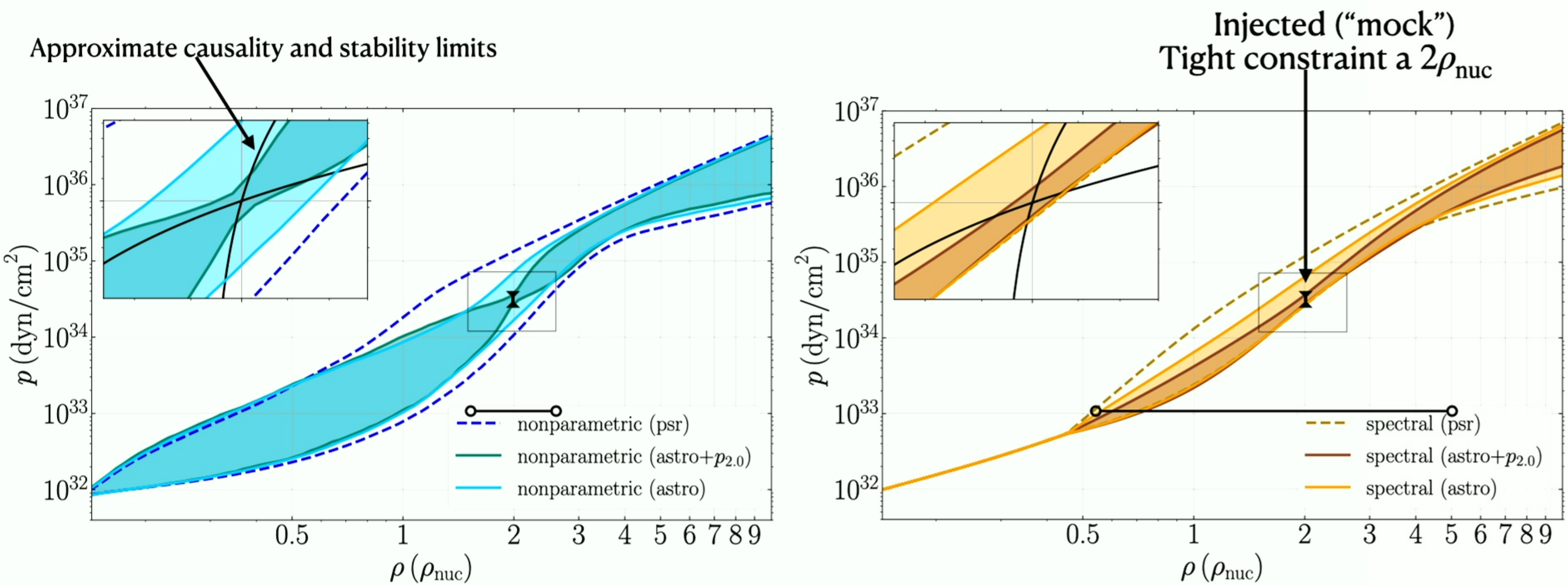


TOV maximum mass and radius of a 1.4 solar mass NS are correlated among equation of state candidates due to causality

Spectral model sees a "tighter correlation" than the Nonparametric model — not likely due to causality!

Correlations

Correlations between astro observables <=> Correlations between density scales



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Implicit Correlations

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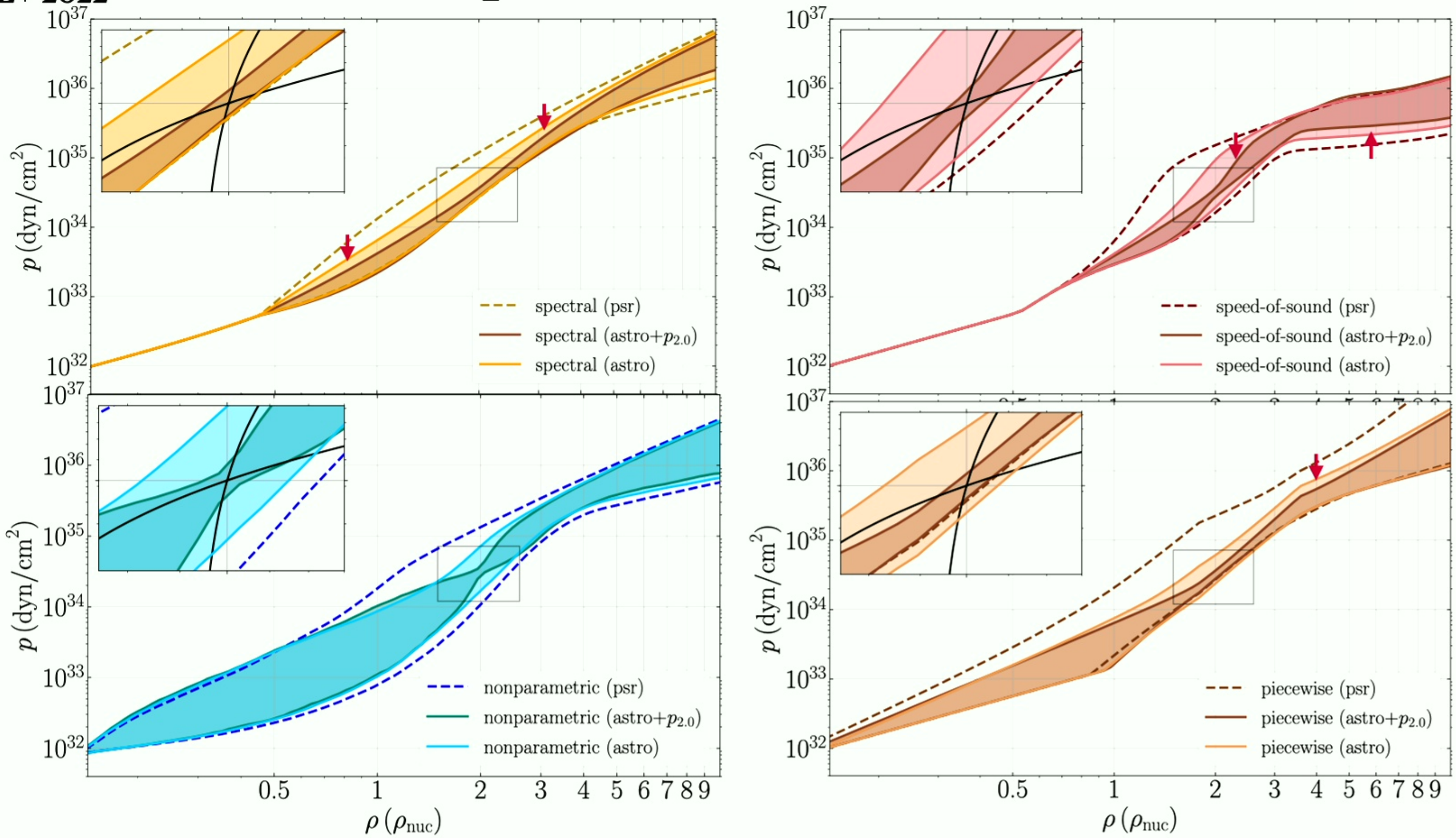

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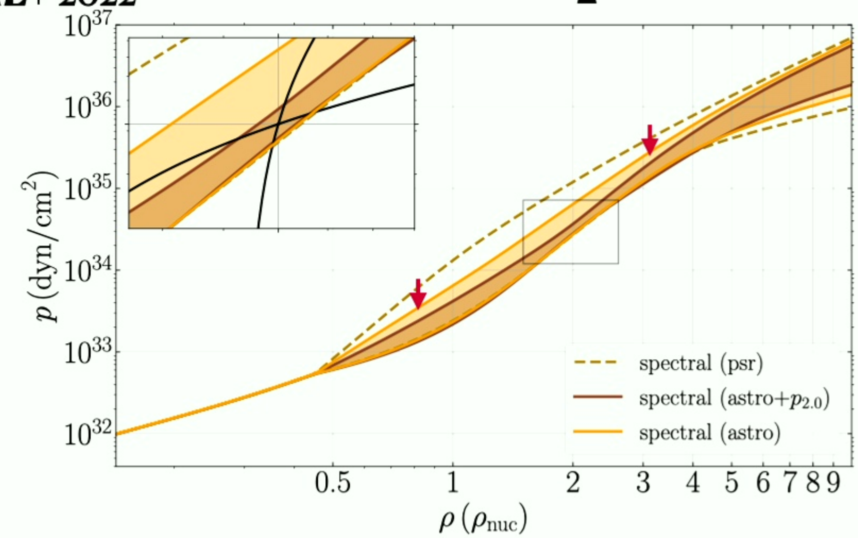
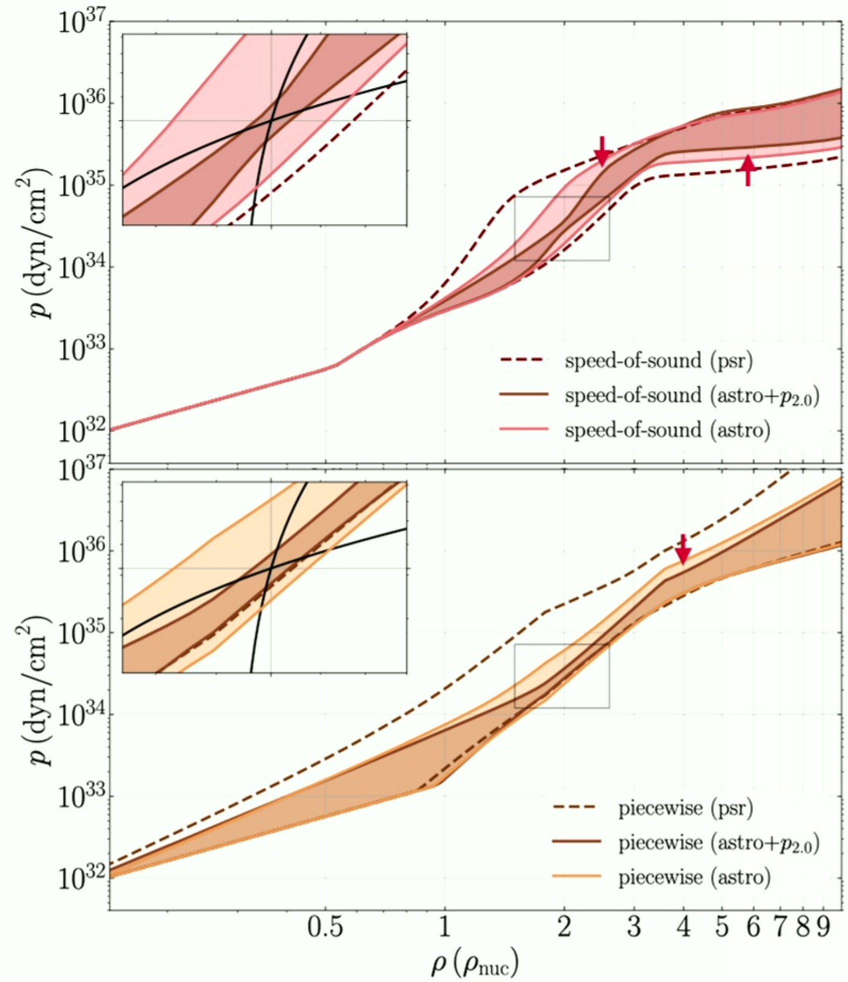


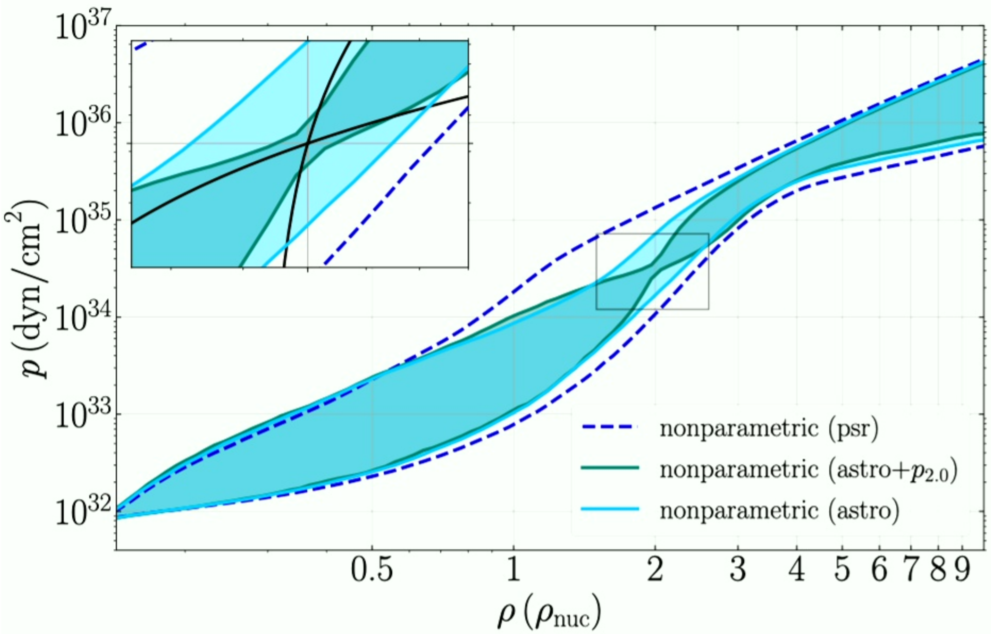


Correlations become more obvious in posterior

Only via model comparison is it obvious they are
Due to the prior

Implicit Correlations

IL+ 2022



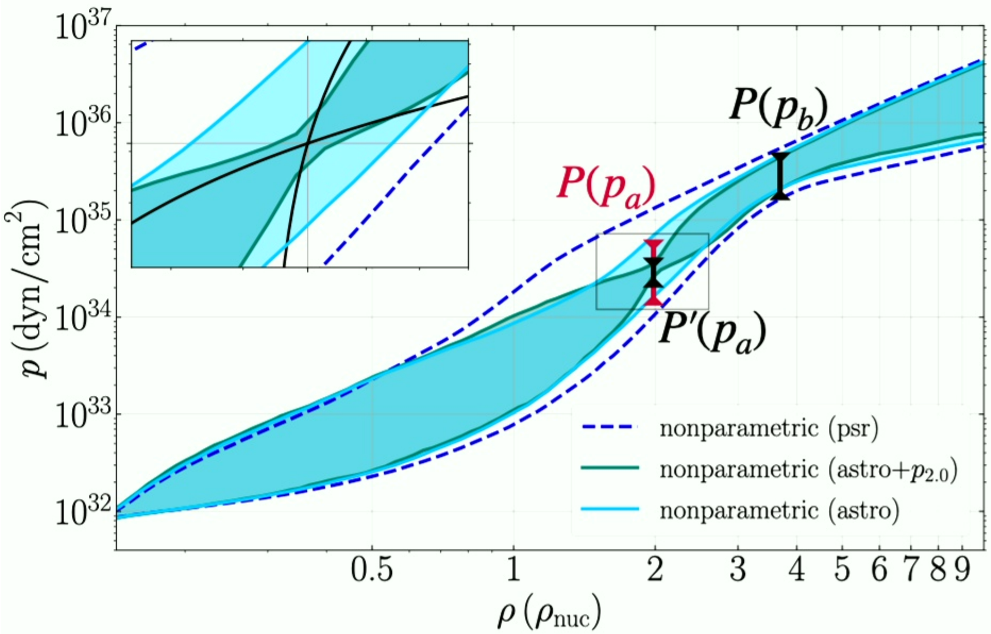
Quantifying correlations — Mutual Information

How much information is gained about other density Scales by knowing the EoS at some fixed density

$$I(p_a, p_b) \equiv \int dp_a dp_b P(p_a, p_b) \ln \left(\frac{P(p_a, p_b)}{P(p_a)P(p_b)} \right)$$

Implicit Correlations

IL+ 2022



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Also a K-L divergence!

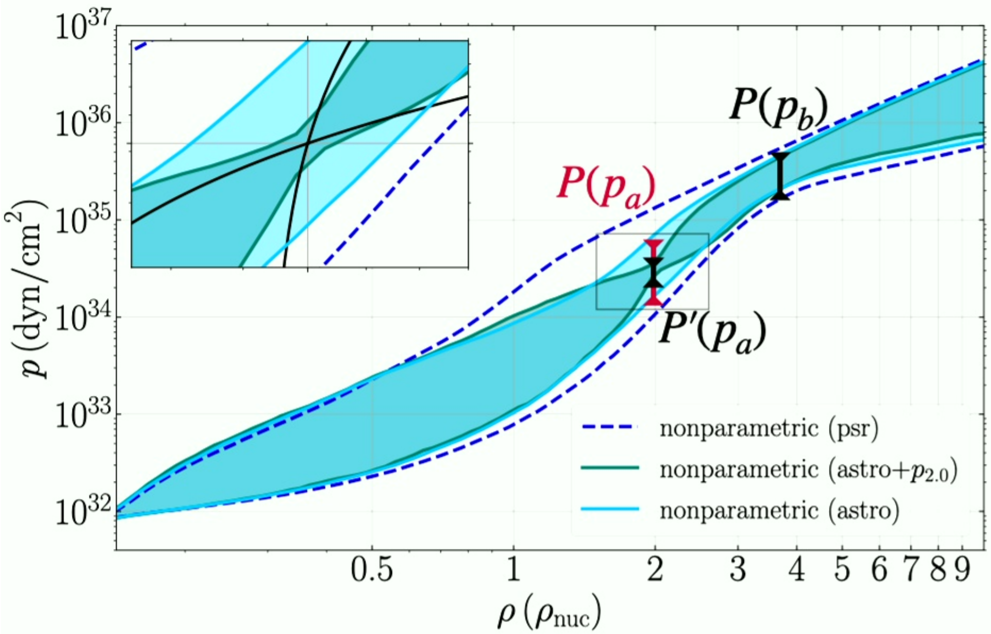
$$I(p_a, p_b) = \int dp_a P(p_a) \int dp_b P(p_b | p_a) \ln \left(\frac{P(p_b | p_a)}{P(p_b)} \right)$$

Difference in knowledge about p_b after learning p_a

Changing this ($P(p_a)$ → $P'(p_a)$) same as adding a tight Pressure "mock-measurement"

Implicit Correlations

IL+ 2022



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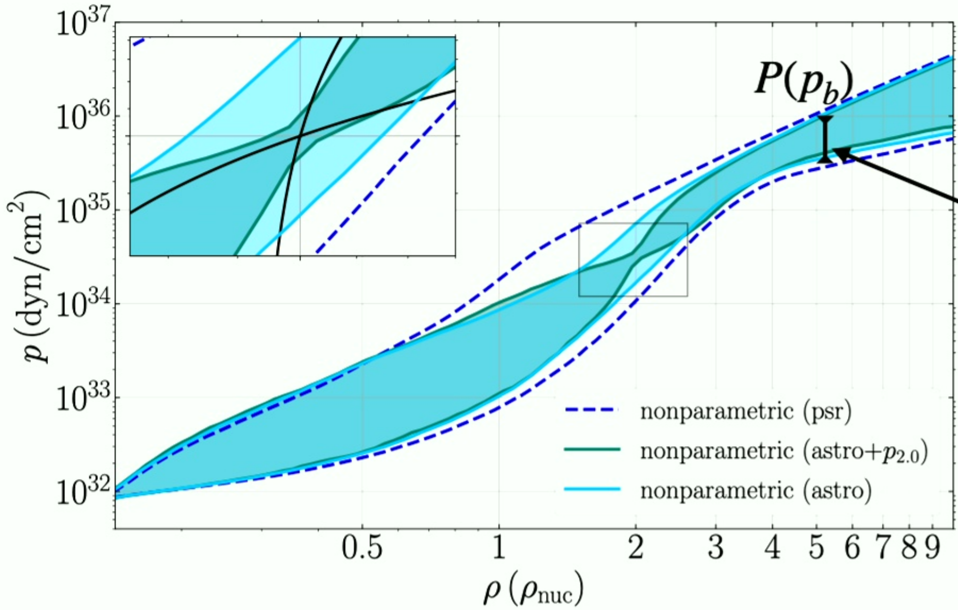


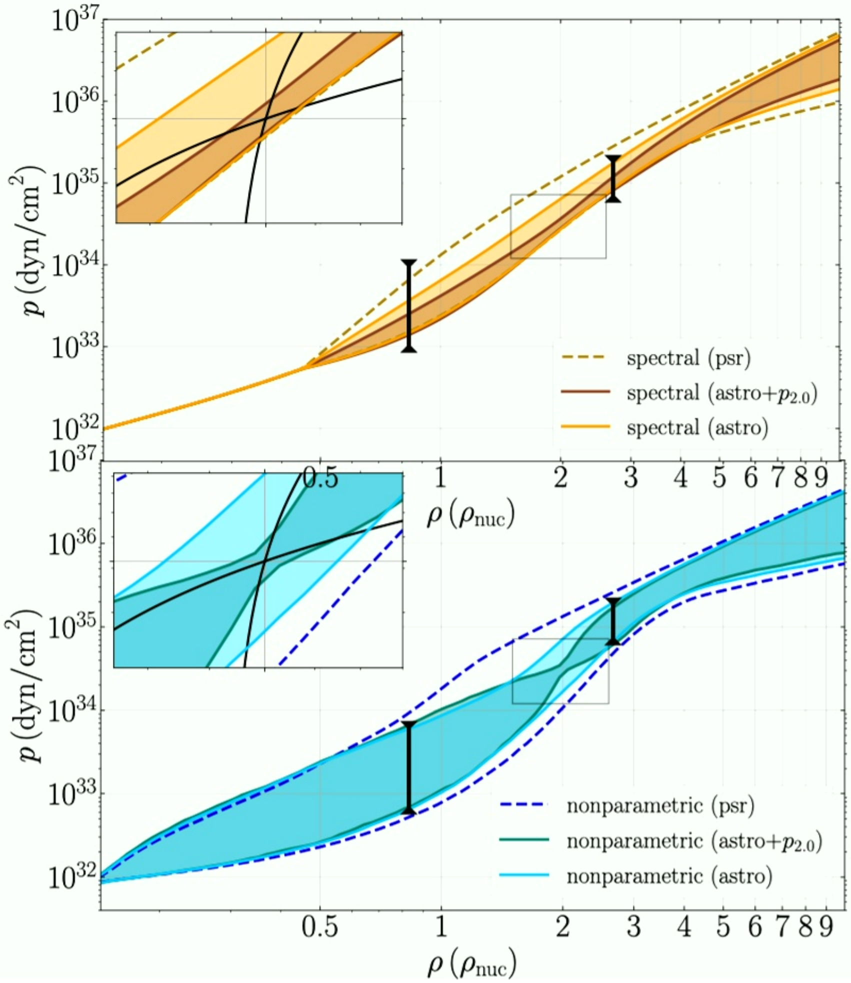
Caveats!

Scales with overall uncertainty of marginal distributions

Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

Implicit Correlations

IL+ 2022



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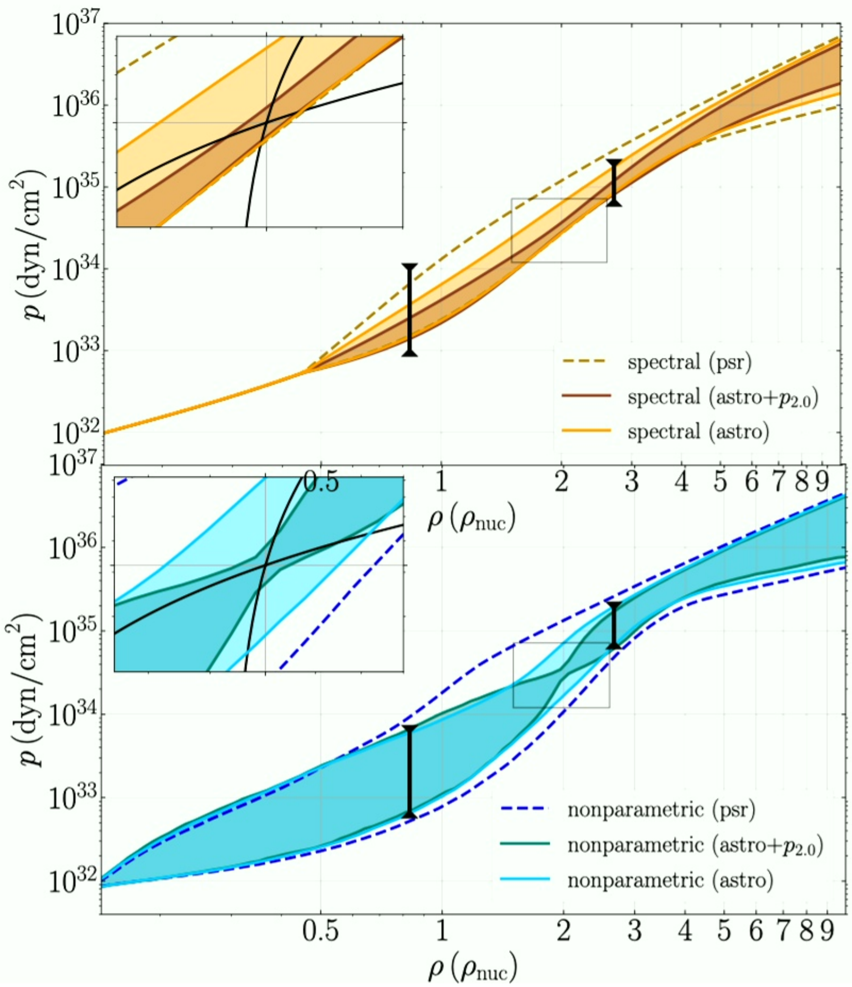
Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

$$I\left(\ln(p_{1.0}), \ln(p_{1.5}), \ln(p_{2.0}), \ln(p_{3.0}), \ln(p_{4.0})\right)$$

	PSR	Astro	Astro+$p_{2.0}$
Nonparametric	3.7	3.1	2.9
Spectral	6.6	5.5	4.7
Polytrope	5.7	4.6	3.8
Speed of sound	5.0	4.7	4.3

Implicit Correlations

IL+ 2022



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Simulated Astrophysical Data

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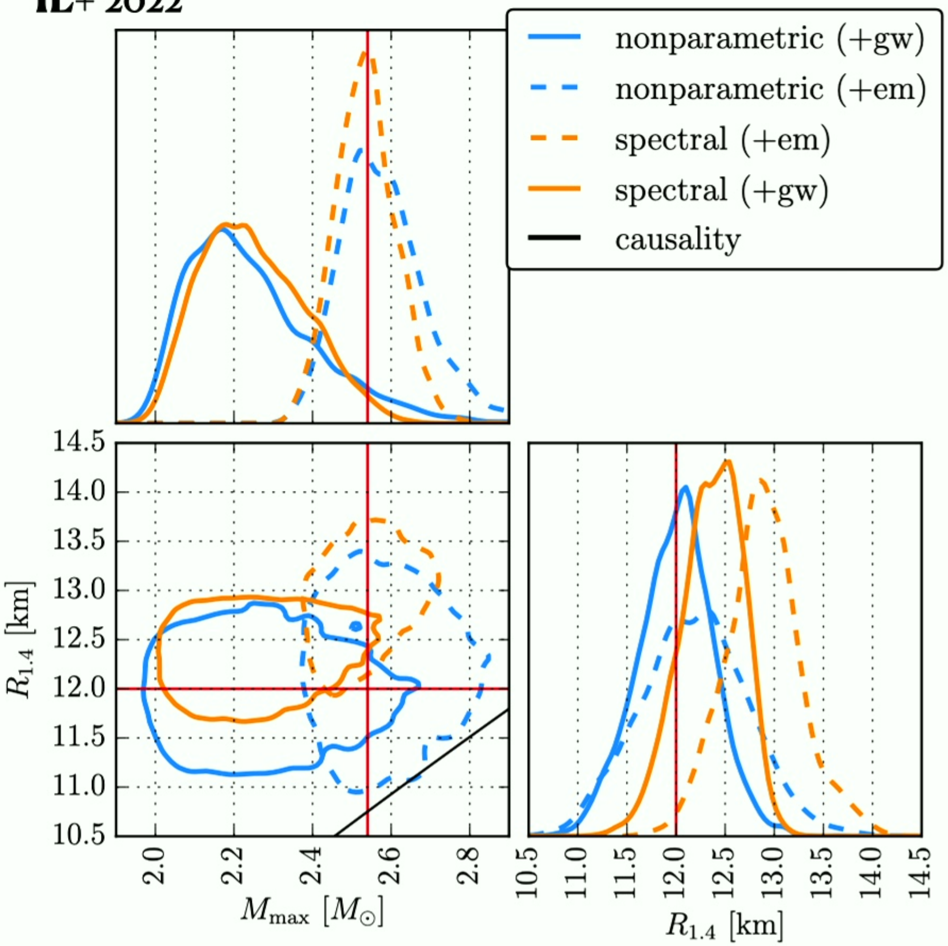


We inject gravitational-wave (gw) and x-ray-radio (em) observations on top of existing constraints

We intentionally choose an EoS that we expect the Spectral model to fail to recover

Gives a sense of tension that may arise from combining constraints using models with unphysical correlations

Modified parametric priors

Why not just modify the parametric models to get more flexibility?

Models are either

(1) fine-tuned => extending them without breaking is difficult (spectral + speed of sound)
(2) Need overhaul-type improvements (piecewise-polytrope + speed of sound)

This is already being done!
Steiner+ 2016 -> More flexible piecewise-polytrope models
Foucart+ 2019 -> Spectral model with easier to interpret parameters

But... Extensions are nontrivial =>
Best to understand limitations of each model while using it

Correlations ≠ Bad!

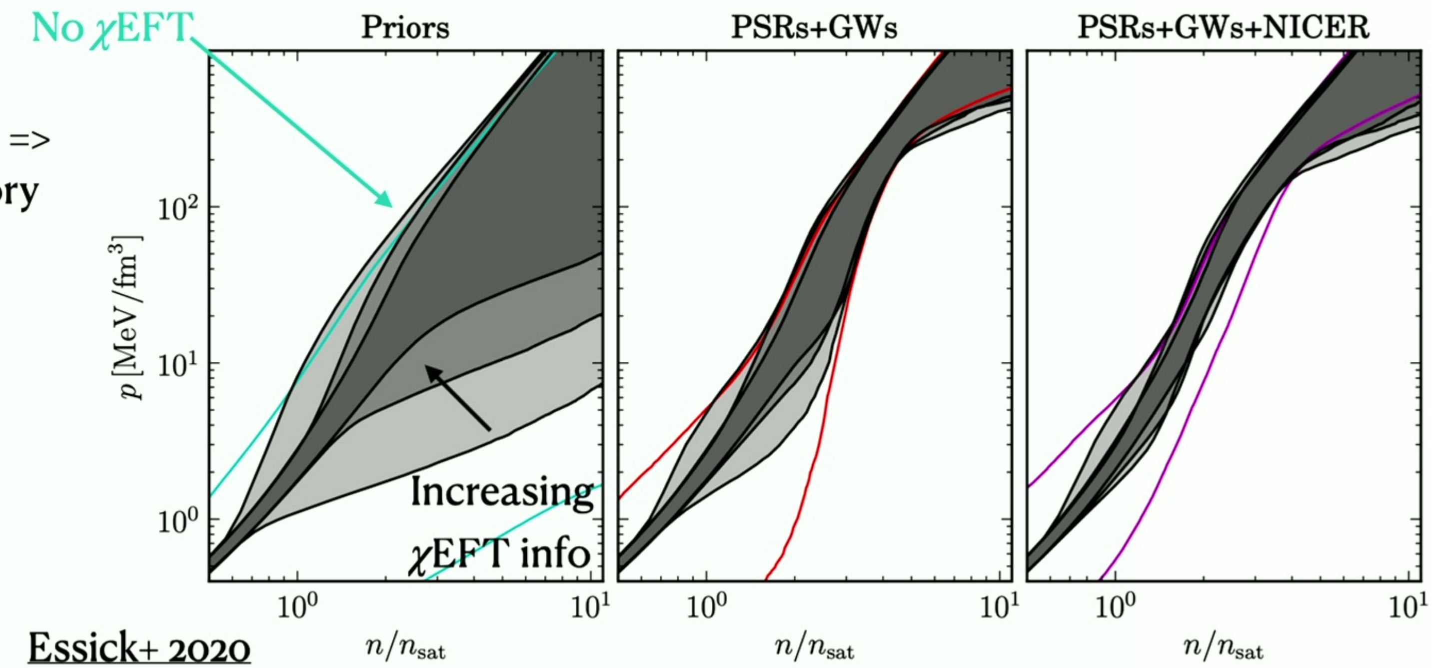
Physical theories have correlations between quantities "F=ma"

Correlated

Goal is to give flexibility in the choice of correlations

Flexibility of Gaussian process =>
Can condition on nuclear theory

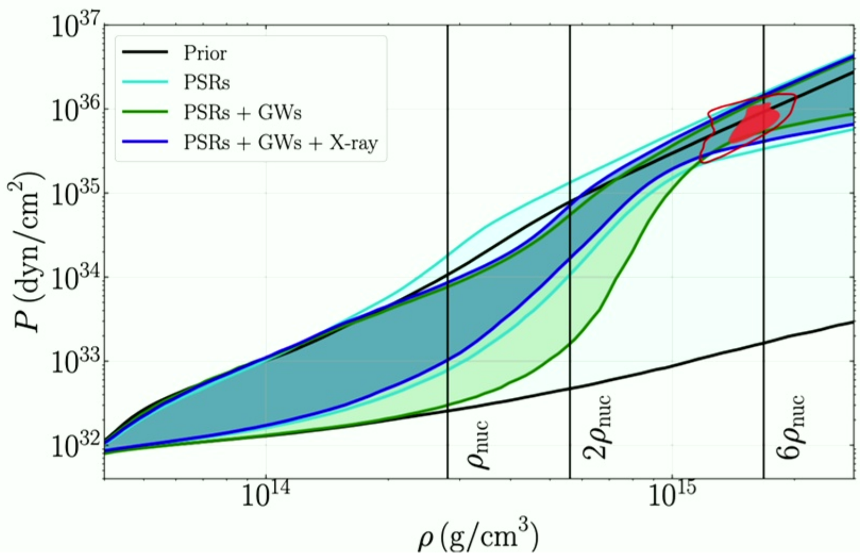
Goal is to *infer* the correlations



Essick+ 2020

Big Caveats

- *a priori* correlations hamper inference => actually can help in Hydrodynamics
- $p(\rho)$ is more "predictable"
- What if the true EoS is not smooth though (e.g. phase transition)?
- Uncertainty in EoS => questions such as "what is the best model for the EoS?" are domain specific



"[The Equation of State model] should be made as simple as possible, but not simpler". —A.E. (maybe)

Conclusions

- Phenomenological models of the nuclear equation of state can build in (often hidden) correlations due to the functional form of the EoS
- Nonparametric models (such as the Gaussian Process model), can provide more flexibility in inference of the EoS
- Care should be taken to guarantee models fit applications, and to understand the limitations of these models